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ABSTRACT

Intended for teachers of the mentally gifted in grades 1 through 3, the guide distinguishes between the verbally gifted and the mathematically gifted and discusses subject matter content, development of intellectual skills and creativity, and gives teaching suggestions. Discussed are a different emphasis for the mathematically talented, the opportunities of unstructured programs, and the need for sequence and continuity. Also considered for determination of subject content are suggestions for the verbally gifted and broad applications of mathematics. Stress is put on the development of understanding, generalizations, and basic principles. Recommended for the improvement of mathematical skills are quantitative questions, open-ended problems, and individualized programs. The discovery method of teaching is encouraged for development of higher intellectual skills such as analysis-evaluation and synthesis-evaluation. A rich mathematical environment and a teacher who enjoys mathematics is suggested to develop creativity in mathematics. Mathematics instruction is seen to encourage the full development of the gifted child's human potential. Teaching suggestions include ways to use the number line, primitive number systems, nonmetric geometry with geoboards, and problem solving. (DB)

Teaching Gifted Children Mathematics in Grades One Through Three

Prepared for the

DIVISION OF SPECIAL EDUCATION
California State Department of Education

By VIRGINIA WALKER

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FOREWORD

You who teach gifted children have awesome, yet exciting, responsibilities that lead you along uncharted paths, that defy traditional approaches to teaching, that take you and those you lead down Robert Frost's "other road." And you who help the gifted find avenues of learning represent the key to successful educational programs for the intellectual leaders of yet another era.

The author of this publication says that the teacher is the "key to whether the program (for gifted children) encourages creativity or supresses individual initiative." She also says that the success of programs in mathematics for the intellectually gifted is dependent on three crucial factors: (1) the teacher's knowledge and understanding of mathematics; (2) the teacher's understanding of the full range of mathematical performance; and (3) the teacher's attitude and understanding of the discovery approach. I would add a fourth factor, which I am certain the author would accept: the teacher's love for children.

The *Mathematics Framework for California Public Schools* emphasizes the importance of the teacher as a "guide who conducts his pupils into regions uncharted to them." The framework also calls for the establishment of a climate that is "pupil-oriented, self-directed, and nonauthoritarian." And the authors of the framework say that the teacher is one who "frames questions, plans work that excites curiosity, and encourages pupils to exploit what they know and intuitively feel about the situation at hand."

I agree with the authors of the framework and with the author of this publication: the teacher is the key to our developing meaningful, successful educational programs for all children regardless of their abilities or handicaps.

We who are school administrators must help establish climates of open communication among teachers and children and parents. We must encourage our teachers to create a spirit of discovery for children, to accept parents as partners in the education of youth, to be leaders unafraid of roads not yet taken, to be open to innovation without closing the door to custom.

The intellectually gifted have been specially endowed with talents that need--and deserve--nurturing by well-trained, sensitive teachers. Let us do our best to give each child that teacher. I pledge my energies to the task. I seek your help and cooperation.

Wilson Riles

Superintendent of Public Instruction

PREFACE

This publication is one of the products of an education project authorized and funded under provisions of the Elementary and Secondary Education Act, Title V. It is intended for use by the teachers of pupils whose mental ability is such that they are classified as mentally gifted. It is also recommended for use by administrators, consultants, and other professional personnel involved in helping gifted children.

Teaching Gifted Children Mathematics in Grades One Through Three is one of a group of curriculum materials designed for use by teachers of the mentally gifted in grades one through three, four through six, seven through nine, and ten through twelve. These materials were prepared under the direction of Mary N. Meeker, Associate Professor of Education, and James Magary, Associate Professor of Educational Psychology, both of the University of Southern California.

Also developed as part of the education project is a series of curriculum guides for use in the teaching of mentally gifted minors in elementary and secondary schools. The guides, which contain practical suggestions that teachers can use to advantage in particular subject areas, were prepared under the direction of John C. Gowan, Professor of Education, and Joyce Sonntag, Assistant Professor of Education, both of California State University, Northridge.

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Introduction

Those who enjoy mathematics commonly hold that young children start out liking mathematics and enjoy it until someone infects them with his own dislike for this abstract subject. In fact, far too many children, and particularly the gifted, come to regard mathematics as a dull, hated subject in school. Today, educators hold that this attitude should not prevail for a number of reasons. The child who enjoys mathematics while he learns acquires knowledge and a capacity for viewing the modern world logically that should serve him well. If properly instructed, the child should gain a respect for accuracy and logic. He should acquire skills in problem solving and with these skills acquire a feeling of confidence that does not exist in those who have learned to avoid mathematics.

The creative teacher of mathematics can help gifted pupils realize their full potentialities. If taught with care and imagination, the gifted child who is not talented in quantitative thinking can develop a high degree of skill in computation, problem solving, and logical thinking and can learn to appreciate the cultural aspects of mathematics. The mathematically talented child can, in addition, be stimulated to solve problems in a variety of ways and to explore the world of numbers and space.

Gifted children vary significantly in their interest in, and talent for, mathematics. Therefore, a basic theme running through the various sections of this publication is an emphasis on the need for a variety of approaches to meet the interests and skills of each child. Too many times the program in primary mathematics for gifted pupils has taken a course of either "more of the same," which is deadly, or acceleration for all gifted pupils regardless of their interests and abilities. The end product of both courses is large numbers of children who dislike mathematics.

In each section of this document, along with a discussion of the philosophical basis for the pertinent aspects of the mathematics program, a number of teaching suggestions are made. Since teachers, as well as pupils, differ in their talents and interests, some of the more imaginative and unstructured teaching suggestions should appeal to mathematically talented teachers, while other suggestions should interest teachers willing to try materials and ideas that are not too adventurous and that can be structured to fit into other parts of the program.

In summary, the premises on which the statements of goals, content, teaching philosophy, and techniques are based in this publication are the following:

- All gifted children can learn to enjoy mathematics, even though their abilities, interests, and levels of attainment differ.
- Gifted children differ in their abilities to think quantitatively and to perceive spatial relations, so the mathematics program should be designed to meet their differing needs.
- Teachers also differ in their abilities and interests. Therefore, a variety of approaches must be suggested.

Chapter 1

Subject Matter Content

Facts, skills, concepts, principles, and generalizations selected for the content of a mathematics program for gifted pupils in the primary grades are, of necessity, influenced by the following considerations:

- For the gifted child who is creative and highly motivated in mathematics, a different emphasis in mathematical content is required than for the gifted child who learns his mathematics lessons well when properly guided but who is not particularly interested in quantitative ideas.
- The child in the primary grades is a very young child, full of wonder and curiosity, but his need for the tangible, the concrete, and the real must be considered when selecting mathematical content for his program even if he is gifted.
- The structure of mathematics requires greater accommodation to the demands of sequence and continuity than do the other subjects.
- The need for a well-rounded, balanced program that allows the gifted pupil to explore all areas of the curriculum limits the mathematical content that can be introduced to children in the primary grades.

Different Emphasis for Mathematically Talented

Certain difficulties are inherent in selecting mathematical content for the gifted program at the primary level. The young child who is gifted in mathematics is fascinated by the world of numbers. He is constantly considering how many, how big, how long, and how small. He revels in quantitative and spatial ideas as other children at this age revel in words. However, he is often deprived of opportunities to explore in this area – seldom does the program at this level capitalize upon this great interest by motivating the child's efforts in various subjects. On the other hand, many gifted pupils have little interest or particular ability in quantitative thinking, but when these children show evidence of giftedness in the language arts area, they are often judged, mistakenly, to be gifted in mathematics also. As a result, they are given less careful sequential instruction than they require, and they frequently retreat from mathematics because they become afraid of it.

Unstructured Program Opportunities

Because of his immaturity, the child in the primary grades needs mathematical content that does not draw him into too rigorous or too structured a program at an age when he must be allowed the opportunity to explore, to build, and to reflect. The young child, even the gifted, needs careful instruction in the basic program, but he also needs enough time for individual exploration and guided study. A balance must be maintained, dependent upon the child's degree of giftedness.

Need for Sequence and Continuity

The third consideration for selecting appropriate mathematical content is that demands are different in the subject area of mathematics because of its sequential and cumulative nature. In no other subject are the concepts and skills so interrelated with previously learned concepts and skills. In mathematics the pupil cannot venture very far unless he has grasped prerequisite concepts and skills. The child who is highly gifted in quantitative thinking can bridge most gaps himself, but the less able child cannot, even when he has been identified as gifted.

It is important that the very young child be encouraged to explore all aspects of the world about him. His skills and aptitudes are not yet well defined. He is seldom formally identified as gifted at this age because so little is known about him. For these reasons it is unwise to overemphasize any one subject area. However, in the case of mathematics, such an overemphasis seldom occurs because of the language arts orientation of most teachers in the primary grades.

Facts and Concepts

Gifted children in the primary grades should be exposed to and instructed according to that part of the second Strands Report which is applicable to pupils at the primary level. The goals are stated in general terms for each category of the report. Those goals applying to pupils in the primary grades are the following:

1. *Numbers and Operations*

   To use effectively the fundamental operations of arithmetic . . . to understand and utilize the properties of the operations (commutative property and so forth) and the properties of order and absolute value . . .

To read and understand mathematical sentences involving operations . . . and to formulate and use such sentences in the analysis of mathematical problems

2. *Geometry*

To recognize and use common geometric concepts and configurations . . .

3. *Measurement*

To make measurements, to understand the notion of unit of measurement, and to use and interpret various units; to understand the degree of accuracy of an approximate measurement; to estimate measurements and the results of simple calculations involving measurements . . .

4. *Applications*

To analyze concrete problems by using an appropriate mathematical model; to employ . . . sentences, formulas, computations, and reasoning in studying the mathematics of such a model; to interpret mathematical consequences in concrete terms; and to examine the concrete results of such an analysis in terms of reasonableness and accuracy

5. *Statistics and Probability*

To construct and read ordinary graphs
To collect and organize data by means of graphs and tables . . .

6. *Sets*

To understand and use routinely the basic set concepts . . .

7. *Functions and Graphs*

To use the coordinate plane to display relations and to organize data . . .

8. *Logical Thinking*

To understand, to appreciate, and to use precise statements . . .

9. *Problem Solving*

To devise and apply strategies for analysis and solution of problems, and to use estimation and approximation to verify the reasonableness of the outcome.[1]

In general the program for gifted pupils should have greater depth, a different emphasis, and a higher content level than that which has been set forth for all children in the Strands Report. However, it is

[1]*Mathematics Framework for California Public Schools, Kindergarten Through Grade Eight: The Second Strands Report.* Prepared by the Statewide Mathematics Advisory ...ce. Sacramento: California State Department of Education, 1972, pp. 109-10.

not possible to say how deep, which emphasis, or what content the program should have unless the individual pupil is considered. The Strands Report is a frame of reference for a rich, creative mathematics program for all children. In addition to such a program, guidance to higher levels of understanding should be provided for the gifted pupil. If he is highly talented and motivated in mathematics, his program requires academic acceleration.

Chapter 2

Content of Mathematics Gifted Program

The content of the gifted program might differ from the program for most children, as shown in the following outline:

Group A – All children

1. Content of the state program based upon the second Strands Report
2. Quantitative aspects of science and social science

Group B – Gifted children not particularly talented in mathematics

1. Same as item 1 for Group A, with emphasis upon structure, logic, geometry, and problem solving
2. Same as item 2 for Group A
3. Cultural aspects of mathematics
   a. Primitive number systems
   b. Development of the abacus
   c. Geometry in the world about us
   d. Music and mathematics
   e. Development of measurement systems

Group C – Children highly gifted in quantitative thinking

1. Same as item 1 for Group A, with emphasis upon structure, logic, geometry, problem solving, graphing, patterns, and relationships
2. Same as item 2 for Group A, in greater depth
3. Same as item 3 for Group B, with less historical emphasis
4. Signed numbers
5. Acceleration of basic content when appropriate
6. Recreational mathematics (puzzles, brainteasers)
7. Probability

Suggestions for the Verbally Gifted

The verbally gifted pupil frequently enjoys making reports on the development of our number system. Some suggested topics are the following:

Early man's counting system
Egyptian numbers
The Indian time system
Tally systems
Early counting devices
The Japanese Sorobon
The Hindu-Arabic place value system

These topics are not mathematics as such, but their study may help to put mathematics in perspective for the child who needs this orientation. He needs to know why he is learning each phase of mathematics. Some time spent in studying historical topics can help the verbally gifted child get an overall view of mathematics and fit the part he is learning into the broad scope of this subject. Such a perspective can give him confidence as well as sharpen his appreciation for mathematics.

The Broad Application of Mathematics

Geometry is one of the major strands set forth in the Strands Report. Children at many levels of ability like to study geometry in the world about them. They can be guided to observe the profusion of geometric shapes and concepts in nature and in the modern technological world. Teachers should offer for observation freeway patterns, honeycombs, sunflowers, seashells, bridges, space shots, butterfly wings, and many other examples of geometric shapes. Enjoyment in this case can be artistic as well as mathematical. The opportunities for art activities accompanying mathematic programs are unlimited.

Much of the relationship between music and mathematics is above the level of pupils in primary grades, but rhythm patterns are not. In its most elementary phase, a pattern is a simple counting exercise, but many creative activities can be developed from this beginning. Children enjoy plotting the beat with designs. First they establish the count, as in a waltz, with the following pattern:

1 2 3 1 2 3 1 2 3

Then they make designs such as

1 2 3 1 2 3 1 2 3 or

/ / / / / / / / /
1 2 3 1 2 3 1 2 3.

From these they choose their favorite rhythm patterns and develop them through the use of a variety of art media – chalk, paint, cut paper, or clay.

Although the mathematically talented child is interested in the historical and cultural aspects of mathematics, he is impatient to get to the numbers themselves. He is happiest when given tasks that require investigation and the collection of data. Such a child enjoys the study of number series, prime numbers, figurate numbers, modular arithmetic, and signed numbers. He likes to consider very large numbers and infinity. Materials that encourage the discovery of number relationships and patterns are particularly suited to this pupil, who revels in them. A case can be made for using such materials for an informal determination of the mathematical talent of a given pupil. Wirtz[1] has suggested materials, for instance, that are greeted with enthusiasm by young mathematicians.

On the other hand, gifted children who lack talent in quantitative thinking usually shun such specialized materials. With encouragement those children not gifted in mathematics can acquire an interest and some skills, but they will actually learn to hate mathematics if thrown into a competitive situation with their mathematically talented counterparts in this type of study. They feel threatened by the swiftness with which talented pupils grasp patterns. A differentiated program is crucial to the development of each child's pleasure and confidence in mathematics.

If an individual pupil is found to be highly talented in mathematics, means should be found to allow him to work at a higher level. This action should be taken with caution and not as an automatic reaction to his giftedness, nor should it replace a rich, exciting program at his age level with narrow acceleration in the basic skills and concepts. On the other hand, the highly talented cannot "go anywhere" in their discoveries without appropriate skills, and they are frequently frustrated for lack of these skills at their own grade level. The special needs of those pupils who are highly talented in mathematics might best be served by (1) advanced placement for mathematics only; (2) guided study with access to advanced textbooks; or (3) contact on a regular basis with a person who loves mathematics (an older student, another teacher, or a volunteer from the community or the local college).

The modern program for all children should emphasize the relationships between mathematics and the sciences, including the social sciences. The teacher can set up many open-ended situations in

[1] Robert W. Wirtz and Others, *Games and Enrichment Activities: Math Workshop.*): Encyclopaedia Britannica Inc., 1964.

which this relationship is expressed and in which the class begins an investigation and the mathematically talented student takes it much further.

Importance of Scientific Orientation

A task of the scientist is to observe, to record his observations, and to try to set up a mathematical model that facilitates prediction about future events. Very young children can be led to observe carefully. They can collect quantitative data to make their observations more precise, and they can record these observations in systematic ways. They usually cannot develop a mathematical model, but they can make and check informal predictions based upon recorded data.

The American Association for the Advancement of Science has developed a program that goes far in this direction. The program is known as "Science–A Process Approach." Children in this program are guided through eight basic processes: (1) classifying; (2) observing; (3) using numbers; (4) using space/time relationships; (5) communicating; (6) measuring; (7) inferring; and (8) predicting.[2]

If children are encouraged early in both the science and social science programs to observe carefully, to respect fact, to record data, and to report accurately, then a major step will have been taken in upgrading the mathematical content of these subject areas. This is an aspect of early education that has long been neglected.

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[2]John R. Mayor, "Science and Mathematics in the Elementary School," *The Arithmetic Teacher,* XIV (December, 1967), 629-35.

Chapter 3

Understanding, Generalizations, and Principles

The development in pupils of the ability to generalize and to state principles is an integral part of modern mathematics instruction for all children. For gifted pupils the degree to which this guidance should be different must vary with each child.

As pupils work independently or with the teacher using some of the modern materials, they are constantly exploring, discovering, generalizing, and stating principles. The gifted child learns these processes easily. If the gifted child is mathematically talented, he readily grasps concepts and finds subtleties and abstractions not seen by children who are mathematically less able.

The degree to which generalizations and principles are verbalized in teaching young children has met with differing opinions among pioneers in modern mathematics education. In the earlier stages of the "new math" revolution, precise vocabulary and much verbalization were in favor, even for very young children. More recently, some educators, including Dienes,[1] Wirtz,[2] and others have urged a much less verbal approach. They argue that much verbalization with precise mathematical vocabulary limits the young child's development. Such verbalization places too much restriction on his thinking and encourages him to lean too heavily on the "rule" once it has been stated. These educators go so far as to urge a completely nonverbal approach for part of the instruction.

It would appear that a balance must be maintained. The "nonverbal" advocates emphasize creativity and enjoyment of mathematics. They remember that the child is young and that mathematics can be an exciting adventure. However, not all children are "freewheeling," and even when they are gifted, they want to know the rules and to talk about them. A balanced program offers much opportunity to explore nonverbally, but it also leads pupils to statements of basic mathematical principles.

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[1] Zoltan P. Dienes, *Building Up Mathematics*. London: Hutchinson Educational, Ltd., 1960, pp. 7-124.

[2] Robert W. Wirtz and Others, *Games and Enrichment Activities: Math Workshop*. Chicago: Encyclopaedia Britannica Inc., 1964.

Chapter 4

Mathematics Skills

What are the mathematical skills we wish to develop in young gifted children? They are skills that mathematically talented children appear to possess inherently. Weaver and Brawley have listed ten traits that are typical of the mathematically gifted child. This list can be used by the teacher to identify such a child and can also be used as a roster of skills that all children can be encouraged to develop to varying degrees. The ten traits include the following:

1. Sensitivity to, awareness of, and curiosity regarding quantity and the quantitative aspects of things within the environment
2. Quickness in perceiving, comprehending, understanding, and dealing effectively with quantity and the quantitative aspects of things within the environment
3. Ability to think and work abstractly and symbolically when dealing with quantity and quantitative ideas
4. Ability to communicate quantitative ideas effectively to others, both orally and in writing, and to receive and assimilate quantitative ideas in the same ways
5. Ability to perceive mathematical patterns, structures, relationships, and interrelationships
6. Ability to think and perform in quantitative situations with creativity, originality, self-direction, independence, eagerness, concentration, and persistence
7. Ability to think and reason analytically and deductively, to think and reason inductively, and to generalize
8. Ability to transfer learning to new, unusual, and "untaught" quantitative situations
9. Ability to apply mathematical learning to social situations and to other curriculum areas
10. Ability to remember and retain that which has been learned[1]

Quantitative Questions

If the teacher makes a point of asking a number of quantitative questions during the discussion periods, the skills listed can emerge

[1] J. Fred Weaver and Cleo Fisher Brawley, "Enriching the Elementary School Mathematics Program for More Capable Children," *Journal of Education*, CXVII (October, 1959), 1-40.

quickly. For instance, if some pupils in the first grade are talking about the school, and the teacher asks such questions as "Who is our principal?" "What does he do?" and "Who are our helpers?" she might also ask the class, "How many rooms are there in our school?" "How many helpers does the principal have?" and "How many bathrooms are there in our building?" The verbal child will answer, "Lots!" But there is a child in the first grade who will, a week later, report just how many there are. He will have counted them! He will also have acquainted himself with the school plant and personnel in the process.

Open-Ended Problems

During the mathematics period the alert teacher can discover other abilities children have by leaving problems open to a variety of solutions. It is interesting to observe a typical classroom situation with the teacher at the chalkboard working with the subtraction algorithm.

An example she might use is the following:

$$\begin{array}{r} 25 \\ \underline{-13} \end{array}$$

The class repeats with the teacher: "Five minus three equals two, two minus one equals one. The answer is 12."

One boy whose reputation for divergence has already gained him a reputation will blurt out, "You can do it by adding." The teacher says, "Oh, Jimmy, you know this is subtraction. Now keep your mind on the work at the board!"

Had the teacher's response been, "How, Jimmy?" he would have shown that he understood inverse operations and missing addends intuitively before they were presented to the class. He would have said, "Add up two plus three, that equals five; one plus one equals two."

Need for Individualized Programs

Since the mathematically talented child does possess mathematical skills quite extensively, he needs a somewhat different teaching approach from that used with the highly verbal child who does not exhibit such skills. The mathematically talented child needs to have his abilities recognized early and to be encouraged to use them. This means that the primary teacher must have a flexible, individualized program that allows this pupil time to explore the mathematical world about him, that guides him in these explorations, and that gives him recognition for his endeavors.

The verbal child who is gifted but not mathematically talented, on the other hand, should be protected from early competition with the talented child whose very quickness in the mathematical skills is a threat to him. What he needs is not a "diet of deadly drill and practice" but consistent guidance in the discovery approach and consistent verbalizing of generalizations. This nonmathematically gifted child needs stated rules and principles once he understands the concepts underlying them. He needs the security of much more talk about mathematics, while his talented counterpart is impatient to work with the number ideas and appears to absorb rules and principles somewhat instinctively and nonverbally.

It would be unfair to the verbally gifted child to expect from him the degree of skill or interest in mathematics that the mathematically talented child has. However, the verbally gifted child can be taught thoroughly and imaginatively so that mathematical ideas do not become something to be avoided throughout his life.

It would be equally unfair to impose highly verbalized, repetitive instruction on the mathematically talented child. He, too, can learn to hate the mathematics period, but for a very different reason. Its very slowness and (for him) dullness can leave him frustrated and unhappy.

Chapter 5

The Higher Intellectual Skills

Modern creative instruction in mathematics encourages the use of the higher intellectual skills. The effective teacher leads pupils continuously from the concrete to the abstract, from a low level of understanding to the subtleties and logic of a higher degree of understanding, and from specific learning to generalizations stated with precision. Children who are gifted in quantitative thinking do this easily; others become proficient when given careful instruction.

To help teachers set appropriate objectives so they can guide their pupils effectively, Avital and Shettleworth in *Objectives for Mathematics Learning*[1] have set forth a model based on the *Taxonomy of Educational Objectives*.[2] They outline three levels of thinking and five corresponding categories of mathematical performance, as shown in the following chart:

Thinking process	*Taxonomic level*	*Guilford*
1. Recognition, recall	1. Knowledge	1. Memory
2. Algorithmic thinking, generalization	2. Comprehension	2. Cognition
3. Open search	3. Application	3. Convergent product
	4. Analysis	4. Evaluation and convergent product
	5. Synthesis	5. Evaluation and divergent product

The Knowledge–Memory Level

At the *knowledge* level the pupil is involved in memorization and recall of facts, definitions, and rules. He is expected to recognize a

[1] Shmuel Avital and Sara J. Shettleworth, *Objectives for Mathematics Learning*. Bulletin No. 3. Toronto: Ontario Institute for Studies in Education, 1968, pp. 1-33.

[2] *Taxonomy of Educational Objectives: The Classification of Educational Goals – Handbook I: Cognitive Domain*. Edited by Benjamin S. Bloom. New York: David McKay Co., Inc., 1956.

rule stated in the way it has been presented. At this level the pupil in the primary grades would be asked to do the following:

1. State the basic combinations.
2. Recognize various sets and identify them with appropriate symbols.
3. Write the numerals for given numbers.
4. State the principles of commutativity and associativity.

The inclusion of the fourth activity leads to an interesting problem. Much of modern mathematics instruction at the elementary level is inductive rather than deductive. Children do not learn rules and basic principles until they have been led from many specific examples to state the generalization and from this to state the mathematical law or principle. Therefore, item four in the preceding list is typical of a knowledge-level activity only when this process is complete and the basic laws are part of the pupil's knowledge.

The Comprehension–Cognition Level

At the level of *comprehension*, the process of generalization or simple transfer is used. The child is asked to produce examples that illustrate given definitions or statements and to be able to change from words to symbols and vice versa, provided the items are part of the pupil's knowledge.

The Application–Convergent Product Level

Generalization and transfer underlie *application* as well as comprehension. The difference lies in the amount of novelty in the situation. Problems that require more than one step, problems from real life, and problems from scientific phenomena are usually considered application problems. It is foolish for the teacher to spend an exorbitant amount of time deciding which category of objectives is pertinent. A good rule of thumb is to set up many problem situations in science, mathematics, and social science that require recognition of a basic principle, transfer of the basic principle to mathematical symbols, and selection of the proper algorithm for solution. When this is done, some of these steps will lie in the category of application and some in the category of comprehension.

The Analysis–Evaluation and Convergent Product Level

At the levels of *analysis* and *synthesis*, no straightforward procedure or algorithm provides a complete solution. In these categories the open search level of thinking is called upon.

Nonroutine manipulation of previously learned material and the discovery of relationships are expected.

In analysis the pupil breaks down the problem into its parts to discover the internal relationships between the parts. A child at the primary level might be asked, after being given limited information, to decide the numerical value of some imaginary numerals. He must analyze the relationships among the various symbols (see Table 1). The use of imaginary symbols requires him to employ open search thinking because it thrusts him out of the familiar structure. A creative alternative to this type of problem is to ask the mathematically gifted child to design a similar problem using his own symbols.

Synthesis–Evaluation and Divergent Product Level

Synthesis is a higher level of open search thinking than the application and analysis levels. Synthesis enables the pupil to put given elements together in a new way. The new way will not, in all probability, be new for a specialist in the field, but it will be a genuine invention for the child. It is at this level that the gifted child begins to appreciate the "man-madeness" of mathematics. As he realizes that mathematics is entirely invention, he can appreciate man's greatness in its creation and realize that he, too, can add to its invention.

Development of Higher Level Skills

Higher level mathematical thinking depends on: (1) accumulation of a well-known and practical body of knowledge; (2) ability to draw together seemingly unrelated concepts; and (3) ability to see solutions to a problem or to see the generalization of an idea by means of a new method.

The child must have a body of information and skills upon which to draw when he is attempting higher level thinking. This points up the difficulties a teacher faces when deciding upon a method of instruction in a particular situation. At what point should the teacher encourage work at the open search level? Should it begin after thorough instruction at the lower levels? Should it be encouraged in all cases? This approach would appear to be at variance with the exhortations to "discovery" teaching proffered by most advocates of the "new math."

At this point the primary teacher probably needs help in sorting out what appears to be conflicting advice in order to plot a consistent, thorough, yet exciting mathematical program for all pupils.

TABLE 1

Higher Intellectual Skills

Category	Thinking process	Types of classroom activities for gifted pupils: Mathematically talented	Types of classroom activities for gifted pupils: Nonmathematically talented
Knowledge (memory) (Guilford)	Recognition, recall	Write and read numerals for given numbers: $0 \rightarrow$ infinity.	Same. Limit to numbers expected of all pupils.
		State the principles of commutativity and associativity.	Same.
		Recognize and write the symbols for simple fractions.	Recognition only.
		Construct number lines with whole numbers, fractions, and signed numbers.	Construct number lines with whole numbers only.
		Recognize and construct basic geometric shapes, angles, and line segments.	Recognition only, unless interested. (Emphasize informal activities.)
Comprehension (cognition)	Algorithmic thinking, generalization	Translate a word problem into a number sentence.	Same, with less complex problems.
		Use the proper algorithm to add (or subtract) a series of numbers.	Same.
		Give examples of the basic laws, using numbers or other symbols.	Same.
		Demonstrate understanding of base ten numeration system with whole numbers of any size. Use expanded notation or abaci.	Same, with numbers to 10,000.
Application (convergent product)	Open search	Take care of class monies. Keep records and make change (as maturity allows). Measure growth of plants. Be the class weatherman and keep records. Graph the results. Make a device to "teach" place value for classmates.	Pair with a mathematically talented pupil in application activity. (The nonmathematically talented often makes a good partner with the mathematically talented. The former often reads and records better; he enjoys and learns much in the process while his talented partner is testing his own prowess with quantitative challenges.)

Analysis (evaluation and convergent product)	Open search	Analyze two- and three-step word problems to determine what is given and what is asked. Then translate into proper number sentences.	Same.
		If $a = 7$ and $b = 10$, show the value of y and z in: $y + z = a$ $y \times z = b$ $y + y = b$ (By analysis one may determine that y and z are 2 and 5. But the third equation shows that $y = 5$; therefore $z = 2$.)	Not usually interested in this type of activity.
		Design similar problems using any symbols.	Of little interest.
Synthesis (evaluation and divergent product)	Open search	Work with simple equations requiring discovery of negative numbers such as, "Is there a solution to $2 - 3 + \square$?	Develop a tally system for a tribe of Indians with six fingers on each hand.
		Play "What's my rule" games. (See Wirtz and Sawyer.)	Work with a mixed class in playing, "What's my rule." (The teacher should vary the difficulty so that all children can meet success part of the time.)
		Explore other activities related to attribute games and blocks. (See list of suggested materials.	Explore other activities related to attribute games and blocks (insofar as interested).
		Demonstrate how much five (or any number) halves is. Use Cuisennaire rods or ruler.	

Source: Compiled by Virginia Walker.

Discovery Method of Teaching

Glennon has clarified the notion of discovery teaching when he points out that there must first be a balance between *telling* (authoritative identification) at one end of a continuum and *unguided discovery* at the other end. Most of the pupils of the teaching-learning process favor *guided* discovery through which the learner is led to discover cognitive material that is new to him. Glennon further points out that cognitive material is not all one of a kind. He categorizes cognitive material as follows:

1. Arbitrary association ($2 + 3 = 5$; 5 is the numeral for the set: $\left\{ \begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \end{matrix} \right\}$).
2. Concepts (A numeral is a symbol for a number. Division can be viewed as repeated subtractions.)
3. Understanding (Numbers are inventions of man.)[3]

Pure discovery for learning arbitrary associations, used alone as a teaching technique, would be a waste of time. Concepts and understandings, however, fit easily into the discovery method of teaching.

The relationship between guided study and personality traits should be considered. Some children cannot tolerate the anxiety inherent in the discovery approach. Others are too quick in their group and are slowed down when they must wait to discover. These are eager learners who want to know *now* so they can get more accomplished. To deny this experience to such learners is to kill their motivation.

A balance should be maintained between *telling* and *guiding*. At the recognition, recall level, telling is an efficient and usually the most productive method to use. In algorithmic thinking and generalization, a balance between guided discovery and telling would be a better approach. In open search the pupil is thrust into unguided discovery. He is asked to use all of his knowledge to analyze a problem, put together ideas, and then arrive at a solution. A certain beauty can be seen in mathematics at this higher level when the pupil joyously realizes he has reached a solution. The "aha!" experience is manifest, the pieces fit together, and the pupil "knows he knows!"

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[3]Vincent J. Glennon, *Some Perspectives in Education*. Twenty-Seventh Yearbook of the National Council of Teachers of Mathematics, Washington, D.C.: National Council of Teachers of Mathematics, 1963, pp. 17-40.

Inservice Training of Teachers

Another conflict that lies in the development of the higher intellectual skills in a primary mathematics program is the extremely limited nature of the usual primary teacher's mathematical knowledge and insights. The primary teacher is most frequently a verbally oriented teacher who fears and dislikes quantitative ideas and who, as well, is a product of the lock-step-drill approach to mathematics. This type of teacher fails to recognize the mathematically talented child and to nurture him. She fails, also, to bring an appreciation of the world of numbers and facility in number manipulation to the other pupils. It is here that inservice training of teachers, in which a modified discovery approach is used, is essential. With such training, negative attitudes can change to positive ones. The teacher who enjoys mathematics and feels confident teaching the subject can stop following the basic textbook as if it were a bible and can stop giving pages and pages of drill material as punishment, seatwork, or "enrichment" homework for the gifted.

The crucial factors determining success in developing higher intellectual skills in children enrolled in mathematics programs in the primary grades are (1) the primary teacher's knowledge and understanding of mathematics; (2) the primary teacher's understanding of the full range of mathematical performance; and (3) the primary teacher's attitude and understanding of the discovery approach. Any programs that do not take these issues into account are unrealistic in their expectations and can have little effect upon the gifted program.

Chapter 6

Creativity

Mathematics can inspire creativity and encourage individual discovery as can few other subjects. If the young child is allowed to enjoy and manipulate a rich mathematical environment, if he is guided into the abstract world of symbols as an adventure, and if he is encouraged to explore many approaches to a problem, he can learn to find deep satisfaction in trying and testing his own mathematical ideas. The sensitive teacher must pause here and reflect on these "ifs." By using timed drills and tests that add a dimension of anxiety to mathematics, the teacher is likely to block creative approaches and preclude achieving any of these "if" qualifications.

Mathematics is an invention. Primary teachers need to understand this very basic notion. Man invented, and continues to invent, numbers and number ideas so that he can understand the rhythm in his environment, suit his growing needs, and fulfill his soaring imagination. As a child realizes this, he sees mathematics as an abstract but useful game to which he may add some variations if he is clever.

How can mathematics in the primary grades be taught so that its intrinsic creativity is not lost in the process? A number of approaches that might be taken simultaneously are the following:

1. Provide a rich mathematical environment.
2. Select a teacher who enjoys mathematics and young children.
3. Develop an instructional plan that encourages and nurtures numerical exploration.
4. Organize for creativity.
5. Provide regularly scheduled opportunities for interaction of interested mathematically talented pupils of various achievement levels.

A Rich Mathematical Environment

A rich environment can be provided in a classroom, in an instructional materials center, or in a separate mathematics laboratory; in each, a variety of manipulative materials, games, puzzles, filmstrips, tapes, kits, books, and worksheets can be made available. In the classroom or instructional materials center, one part of the room can be set up as a mathematics learning center. The mathematics laboratory differs only in scope and space.

Whether the center is elaborate or modest, it is essential that ongoing activities be arranged in such a way that pupils can receive "feedback" from their participation. The center should offer many opportunities for random, casual exploration, but it should offer more than this. The teacher should provide guided study as needed.

Many teachers find job cards useful for setting up independent learning activities in the mathematics learning center. The teacher highlights a few materials at a time and makes cards with simple instructions available for pupil selection. The cards are designed to cover a wide range of abilities in order to meet the needs of all children. For feedback, procedures may be established as necessary, such as providing answer forms for self-checking, designating a place to leave worksheets or answers for correction, or having an aide available to check results with the pupils.

A variety of materials can be used to enrich the mathematical environment. Some suggested materials are the following:

Games and puzzles (Most are available at department stores and toy shops.)

Dominoes (double twelves)
Tri-Ominoes (Pressman Company)
Tryptic, Three-Dimensional Tic-Tac-Toe (Invento Products)
Kadoodle (Miller)
Flash cards
Number Lotto
Multiplications (eight crystalline cubes – Museum of Modern Art, New York)
Rondo (Invento Products)
Twixt (3M Company)
Towers of Hanoi (teacher-made)

Measuring devices (Creative Playthings manufactures many of these items for the use of very young children.)

Tape measures
Yardsticks
Rulers
Metric weights
Liquid containers of various sizes
Calipers
Protractors
Child-sized trundle wheel
Pan balance scale
Thermometers
Clock
Stopwatch

Manipulative materials

Dienes Logical Blocks (Creative Playthings)
Cuisenaire rods (Cuisenaire)
Developing Number Experiences, Kit A (Holt, Rinehart & Winston)
Stern kits (Houghton Mifflin Company)
Attribute Games and Problems (McGraw-Hill, Webster Div.)
Fraction blocks, pies, and so forth
Abaci (a variety)
Counters (a variety, including beans, play money, and macaroni)
Cubes, blocks
Hand calculators
Hundred boards
Kaleidoscopes
Flash cards, Number Lotto
"Treasure chest" – an old, decorated box full of discarded clocks, egg beaters (gears), speedometers, watches, and other things to take apart

Geometry

Moby Lynx (Kendry Company)
Rubber Parquetry (Creative Playthings)
Flexagons (Creative Playthings)
Geoboards
Geometric solids
Compasses

Audiovisual aids

Film loops
Filmstrips
Tapes
Pictures

Books

A variety of reference and picture books on historical and cultural aspects of mathematics; and collections of brainteasers and riddles

Science

Pulleys
Levers
Gears
Prisms
Magnets
Magnifying glasses
Microscopes

Construction materials:

Scissors	Felt pens
Paste	Colored pencils
Paper	String
Graph paper	

Equipment

Shelves	Filmstrip viewer
Display space	Film loop projector
Work table(s)	Listening center

How can the teacher encourage creativity in the mathematics center? Mary Meeker's suggestions for "teaching" creativity to the preschool child apply equally to the gifted child in the primary grades. Some of her suggestions are as follows:

> The major difference between a divergent curriculum and a convergent one lies in the experiences afforded the child. In the convergent curriculum, we are telling-teaching and the learning is controlled; in the divergent curriculum, we are allowing intellectual and tactile privileges of exploration within an environment organized to that purpose.[1]

According to Mary Meeker, the environment of a mathematics center should provide an atmosphere conducive to creativity, with some of the following conditions:

1. "Unlimits" are set.
2. Each child is allowed to be alone with the materials regardless of what other children are doing.
3. The child can have an uninterrupted one-to-one relationship with the teacher when needed, with the resulting feeling that what he is doing is important. The desire to want to work is thus established.
4. Many models are set around the room which the child may emulate, or he may take off on his own.
5. The child is allowed to use the materials as he wishes, with the exception of destroying them, since the materials are not his.
6. There is much time provided for each child to be listened to.[2]

A Teacher Who Enjoys Mathematics

The teacher is the real key to whether the program encourages creativity or suppresses individual initiative. Few teachers of primary

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[1] Mary Meeker, "What About Creativity for the Pre-School Child?" *The Gifted Child Quarterly*, IX (Autumn, 1965), 144.

[2] *Ibid.*

grades have sufficient mathematics backgrounds to guide the creative efforts of highly gifted children.

Ideally, the teacher should be organized, flexible, and imaginative and know and enjoy mathematics thoroughly, but also be equally knowledgeable concerning the teaching-learning process with young children. Since these "ideal" qualities are not always to be found in every person in primary education, a realistic approach is to select an excellent primary teacher who enjoys teaching arithmetic and who is willing to participate in inservice training in which "new math" materials and approaches are used. Such training can help in the teacher's efforts to encourage divergent thinking in a child. This teacher must be sufficiently aware of other solutions and other ways of computing in order to refrain from holding back the child who sees a problem differently. Such a teacher would not feel threatened by the adventurous student or by the materials that should be used with him but will be constantly asking, "Is there another way? How did *you* do it?"

An Instructional Plan

An instructional plan goes beyond the daily lesson plan, unit of work, or even projected plans for a school year. It consists of selecting an approach to teaching and then developing ways to use that approach consistently in the presentation of subject matter content.

In mathematics the instructional plan might require the use of the three levels of thinking (recognition, generalization, and open search) described at length in Chapter 5. At the level of open search thinking, the teacher would devise questions and plan activities that encourage analysis and synthesis and that call upon individual originality and inventiveness.

If Guilford's intellectual operations are followed, the teacher would lead the students from memory to cognition to convergent and divergent thinking, and finally to evaluation skills. Gallagher offers a good system of mathematics instruction for teachers who are attempting this strategy for the first time.[3] At the divergent production level, the teacher would be asking such questions as the following:

1. How did *you* solve it?
2. Is there another way?
3. Can you think of a number that does the same thing?
4. If we put another number in that position, what would happen?

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[3] James J. Gallagher, *Teaching the Gifted Child*. Boston: Allyn & Bacon, Inc., 1964, pp. 177-96.

In evaluative thinking students would be guided to check their "other ways" with more traditional methods or by another process. Many of the modern textbooks provide ideas for working at the divergent evaluative levels verbally and nonverbally.

Guided discovery is another plan that leads to creative thinking in mathematics. It is an inductive approach in which students are guided to generalize from a number of specific instances. They are then asked to check their generalizations by applying them to other situations.

Whatever plan is selected, it should be followed consistently, yet leave room for flexibility. The teacher should also stimulate idea fluency and originality. For this the popular "brainstorming" technique is very useful in mathematics. The teacher calls for every possible solution to a problem, writing them all on the chalkboard without comment. Then, teacher and pupils together evaluate the solutions and decide which ones are usable. At other times the teacher may not go this far, but she should consistently invite alternative approaches and listen to them when they are offered. She should keep out of her own mind, and the child's mind, too, the notion that there is one correct approach to a problem.

Organization for Creativity

To organize for creativity would appear to be a contradiction in terms. It cannot be stated too strongly, however, that good teaching for creativity is the antithesis of the laissez-faire, disorganized situations that sometimes are erroneously regarded as creative learning environments. The classroom or school that is badly organized, that lacks properly working equipment, and in which procedures are poorly developed will not foster creativity. The teacher who is ill prepared in subject matter content and who does not take time to prepare for each day with the children is consistently caught off guard when moments that invite creativity arise.

A case in point is the use of a mathematics center, which is discussed in this chapter. Children in grades one through three must be given help in learning how to use the center effectively. Teachers should introduce a few items at a time; as each set of items is introduced, it becomes the focus of instruction for a given period of time. Then another set of items is introduced. If job cards or worksheets are used, children must know what to do with them, where to put them, and how to find out if their work is correct. Teachers should discuss the proper care of the items. Children enjoy taking care of things if they feel they are valuable.

A plea for organization in the classroom has frequently been misinterpreted as an appeal for a strict, rigid classroom atmosphere.

This can happen when it becomes an end in itself, but the opposite has been the intent of this discussion. Teachers who "set the stage" can be relaxed, accepting, and challenging leaders. This calls for good organization. When the teacher and the plans are well prepared, divergent thinking and creative ideas from gifted pupils will not upset flexible long-range objectives but should enhance them.

Interaction of Mathematically Talented Pupils

The opportunity the highly talented child has for interaction with his mathematical peers needs to be considered. The gifted child needs contact with others who enjoy mathematics. This contact could take the form of a schoolwide mathematics club. It could be a one-to-one contact between a younger pupil and an older one, arranged so that they meet at regularly scheduled intervals. Whatever form it takes, the contact should help to relieve the young, talented child of some of the frustration he meets in his classroom and should encourage him to be creative in mathematics.

In summary, the gifted child can be stimulated to be creative in mathematics if he receives regular instruction in a rich mathematical environment according to a system which encourages divergent thinking and if he has access to people who enjoy mathematics. He is particularly likely to develop his creative abilities if he has a well-organized, hard-working, creative teacher who enjoys mathematics and young children.

Chapter 7

Full Development of Human Potential

The purpose of this publication is to outline ways in which good mathematics instruction can help gifted children become fully functioning human beings. A consistent effort has been made to differentiate the needs of the mathematically talented child from the needs of the gifted child who is not particularly interested or capable in mathematics.

Certainly, there are gifted children who have other kinds of needs. There are, for example, the underachieving, the autistic, the emotionally handicapped, the neurologically handicapped, and the blind. In most cases the comments that apply to verbal, nonmathematical gifted children apply also to the other groups of gifted children. For some children there are also underlying emotional problems that must be dealt with before appreciable learning can take place. For the handicapped many special techniques can be used. These techniques have not been discussed here, but the general content and techniques described for gifted normal children apply equally to gifted handicapped children after their special learning needs have been met.

In Chapter 2 it is suggested that the goals, content, and teaching pedagogy set forth in the second Strands Report can form the basis of an exciting program for gifted children. Indeed, a basic effort in many classrooms can very well be an attempt to establish a program like that in the report. The second Strands Report itself would provide a far better gifted mathematics program than in many cases is provided for gifted children. If a program based on the second Strands Report were established, with particular attention being given to the needs of a variety of gifted children, such a program would help gifted children realize their potential. The importance of the realization of gifted children's potential cannot be emphasized enough. Good mathematics instruction is exciting, creative, and satisfying. A gifted program is not "frosting on a dull cake." It is an elaboration of an essentially interesting program worked out in greater depth and guided to higher levels of abstraction.

If young gifted children are to realize the full development of their potential, then their common immaturity as well as their differences must not be forgotten. Since these children are young and have short attention spans, they have a need for the spiral approach to learning. In concept development it is better to stop when attention lags and to return a few days later with a different approach.

The young child needs to manipulate the world of numbers. This need varies with each gifted child, but even the most talented can profit from opportunities to explore with concrete objects if the materials and activities suit his needs.

For the mathematically talented child, a creative mathematics program opens up to him the world of abstraction, the beauty of mathematical structure, and the fun of mathematical invention. If his social science and science programs include the quantitative aspects of those subjects, he is motivated to broaden his interests and to make his unique contributions to these programs.

A number of these mathematically talented children (certainly not all) do not perform well in the language arts and appear to lack auditory and visual memory for words. The same child (highly developed in memory and a knowledge of symbols) who can remember the first 15 prime numbers in order and can visualize the freeway system of a large city frequently forgets names and places he reads about and has difficulty in language usage unless he is commensurately able in semantics. If the mathematics program is satisfying for him, he may be intrigued into being helped through a mathematics program in which word usage or reading is taught. A child of this type often responds best to a businesslike discussion of his difficulties and is able to make plans to correct them. He needs the teacher's patience and help in this area so that he can realize his potential. He should not be expected to perform as well as his verbally gifted counterpart.

The verbally gifted child will need proficiency in calculus as an adult if he is to become a graduate student in most subjects, including linguistics. If he is highly artistic, his interests and abilities may not be in academic achievements. In that case less proficiency will be needed. For all of these children and their less able peers, however, the world in which they will live will be highly technical. A rich, happy mathematics experience, not overly competitive, can give gifted children an appreciation for the inner workings of that world and more confidence in living in it.

The ideas and suggestions presented here are not intended as a plea for a mathematics program that outshines all other aspects of the ng gifted child's program. It is hoped that his differing needs are studied and met in an imaginative way. He is young and must taste, feel, touch, smell, and think about the world around him. This plea has been to make his early mathematics instruction as challenging and creative as the other subject areas. Then he should be on his way to realizing his full potential.

Chapter 8

Teaching Suggestions

So many outstanding supplementary textbooks and teaching guides have been developed in the past few years that it would be a waste of time for most teachers to develop special written materials of their own. The equivalent time would be better spent learning to make use of the unusual materials listed in Chapter 6.

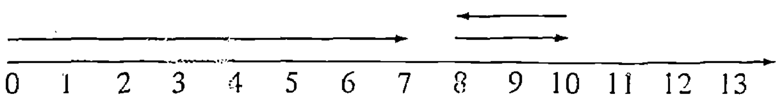
A few teaching suggestions that illustrate content and methods are the following:

Number Line

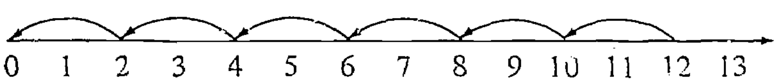
Activities and questions which illustrate the levels of thinking discussed in Chapter 5, Higher Intellectual Skills, are as follows:

1. Recognition, recall (Bloom: knowledge)
   a. Write the counting numbers on a number line, 1 through 25.
   b. Find certain numbers on the number line.
   c. Draw a red circle around each even number on the number line segment, 25 through 35.
   d. Draw a blue circle around each odd number on the same line segment. Are there more odd numbers or more even numbers?

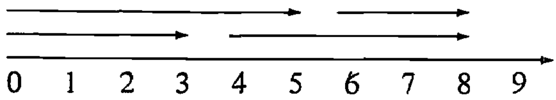
2. Algorithmic thinking, generalization (Bloom: comprehension, application)
   a. With arrows, show that 7 + 3 = □.



   b. On a number line, show how many 2s there are in 12.



c. Show that 3 + 5 = 5 + 3, using the number line.



d. In 1(a), why are there more odds than evens?

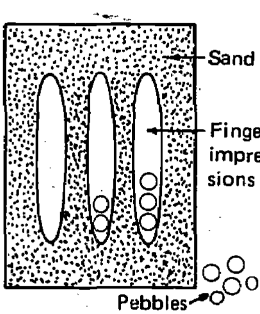
3. Open search (Bloom: analysis, synthesis)

a. On a number line, 0, 1, . . . , 100, circle the 0, skip one; circle the 2, skip two; circle the 5, skip three; circle the 9, skip four. (Write this on the chalkboard without talking.) Ask that the sequence be continued.
b. In 1(a), what could you do to the line so that there would be the same number of odds and evens?
c. Are there numbers on the other side of 0?
What is 2 − 6 = □?

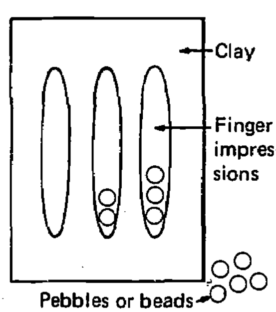
Primitive Number Systems

The teacher should have the younger children make tally systems using knotted ropes, hash marks, or a bag of pebbles. Other children will enjoy collecting samples of tally symbols designating groups or objects.

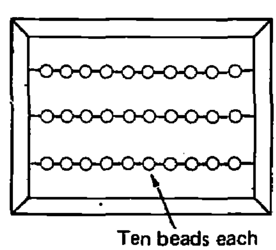
The children may make abaci of sand and pebbles, of clay and pebbles, or of beads strung on wire. Simple bead frames can be made by stringing ten beads – commercial or salt-and-flour variety – each on three wires and fastening them to a wooden frame.



Sand abacus



Clay abacus



Wire-and-bead abacus

Children in the third grade may be introduced to the early Egyptian and Roman numeration systems. An interesting project is ...e a frieze showing early measuring systems. Pupils may also make individual bas-relief clay plaques using various numerals and mathematical symbols for designs.

Nonmetric Geometry with Geoboards

A lesson suggested by Wirtz in *Games and Enrichment Activities: Math Workshop*[1] has been adapted to illustrate the development of a concept using Guilford's intellectual operations (memory, cognition, convergent and divergent thinking, and evaluation) Each pupil is given a geoboard, various colored rubber bands, and string. When distributing these supplies, the teacher may discuss with the children the ways in which they find that string differs from rubber bands.

Memory, Cognition

Children are allowed free play at one or two sessions, using geoboards and one rubber band. They may construct patterns such as the following:

The children can enjoy sharing these activities while the alert teacher begins the process of naming and describing the constructions: point, line, triangle, square, diamond, flat, five-sided, and angle vertex. The teacher may ask questions such as, "Who can show us a point? Can you touch a point with your finger? Who can show us a line? Can you make a line on your geoboard?"

Convergent Thinking

The children may compare with each other the ways in which they made their figures or constructions. The teacher might ask, "How did you make a line? What is a line? What is a triangle? Is this (points to a child's geoboard) a triangle?"

Divergent Thinking

The teacher asks the children to use three rubber bands, each a different color, and allows free play for a while. Then the teacher asks, "How many triangles can you make?" Some children may find many triangles in their patterns; others may find only one or two. The teacher continues, asking, "What would happen if we used string instead of rubber bands? Would string work as well for making

[1]Robert W. Wirtz and Others, *Games and Enrichment Activities: Math Workshop.* Chicago: Encyclopaedia Britannica Inc., 1964.

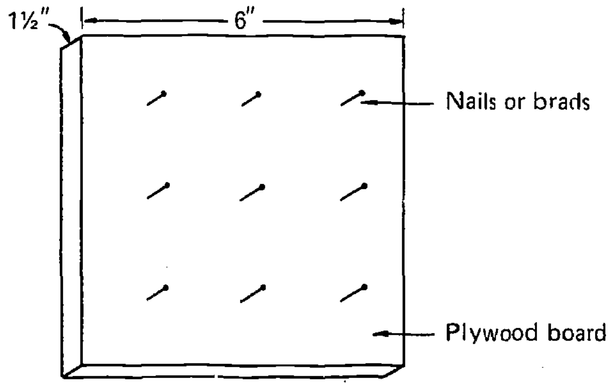
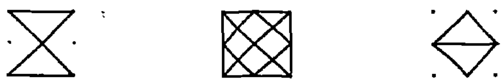


Illustration of a geoboard

triangles as rubber bands? Would the edges of the shapes be straight? Would the vertices be sharp?" Various patterns may evolve from this discussion, such as the following:



Evaluation

In mathematics, values and moral judgments are not part of evaluation, which is rather the level at which ideas developed at the divergent and convergent thinking levels are checked. For example, here the teacher distributes the pieces of string, and the conjectures regarding string are checked by manipulation and discussion.

This lesson could be followed at a later time by another lesson with the string in which open and closed curves are developed.

Other Recommendations

Geometry for Primary Grades by Hawley and Suppes[2] and *Experience with Geometry* by Eicholz and O'Daffer[3] offer follow-up to discovery techniques for nonmetric geometry. Suppes's materials are particularly good for the less mathematically talented child who is happier in a structured situation. The lessons encourage precision,

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[2]Newton Hawley and Patrick Suppes. *Geometry for Primary Grades*. San Francisco: Holden-Day, Inc., 1961.

[3]Robert E. Eicholz and Phares G. O'Daffer. *Experiences with Geor. etry*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1966.

logic, and the opportunity to develop fine motor skills. The Eicholz materials are less formal and more adventurous. They provide many opportunities to construct geometric shapes, both plane and solid.

Problem Solving

Much improvement in problem solving can be achieved if the emphasis is on *translation*. Once children see mathematics as another language and realize that it is possible to translate from English to mathematical symbols and from the symbols to English, they can develop a powerful tool for word problem attack.

To help gifted children gain this insight, the teacher may introduce simple games, such as the following:

1. Ask a child to make up "stories" about a given open sentence, such as $2 + 4 + \square = 10$. Encourage elaboration and embedding of the story because the child will be more able, at a later time, to detect the extraneous material in textbook problems. He might say, "My mother came home from the store with four bags of groceries and gave my younger brother and me a little bag. It had two bubble gums, four Tootsie-rolls, and some jawbreakers. There were ten things. How many jawbreakers did we get?" Let young children dictate their stories to the teacher as she types them on a primary typewriter. Ask each child to read his story to the class. Older children can write their own stories.
2. Ask the children to translate word problems to number sentences. Encourage them to write the symbols in the same order as they are presented in the problems. For example: John had some matchbook cars. His mother gave him three more. Then he had six. How many did he have in the beginning? The child should write: $\square + 3 = 6$, rather than $3 + \square = 6$. The latter is not incorrect, but the logic is clearer in the first example.

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