The recent marked increase in voter-taxpayer rejection of school budget and school bond issues at polls across the United States -- a phenomenon popularly called the "taxpayers' revolt" -- has given rise to the widespread claim that public school finance is in a state of crisis. This paper develops a simplified model of a political marketplace, linking together individual demand for a public good and the vote in fiscal elections to set the supply and the tax-cost of such a good, where both the system of elections and the system of taxation are taken as givens. Data from school budget elections in Oregon are applied to the model and the demand for education under varying tax-cost conditions and overtime in Oregon is estimated. The resultant analysis leads to a markedly different interpretation of the increase in rejection of school finance proposals than that of a "taxpayers' revolt." Moreover, the analysis traces directly the linkage between demand for education and the tax-cost of education and indirectly the linkage between demand for education and the quality or valuation of education. Such findings have major implications in assessing the present state of public school finance and in formulating public school finance policy. (Author)
THE SUPPLY AND COST OF EDUCATION AND THE VOTE

A Political-Economic Theory of School Finance Elections

Michael Boss
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INTRODUCTION

The recent marked increase in voter-taxpayer rejection of school budget and school bond issues at polls across the United States—a phenomenon popularly called the "taxpayers' revolt"—has given rise to the widespread claim that public school finance is in a state of crisis. According to the popular view, the increase in election failures indicates that present means of school finance, primarily the local property tax, are no longer adequate to satisfy the "demand" for quality education. Consequently, it is argued, alternative or supplementary means of school finance are required immediately. There is no quarrel here over the noxiousness of the property tax—or for that matter any tax. Further, the argument that the voter-taxpayer, as a "rational actor," prefers to pay less in taxes if possible is well taken. The objection to the prevailing view is that there is little explicit or consistent link between individual demand for education, the vote in fiscal elections, the supply of education, and the tax-cost of such education. Most significantly, any consideration that what the voter-taxpayer may be willing to vote for at the polls depends at least in part upon what he prefers and gets for his tax dollar is usually omitted.

This paper takes the view that the vote in fiscal elections is primarily a means for adjusting or limiting the supply of a public good so as to satisfy, if only imperfectly, individual demand for that public good.
in a democratically governed system of public finance. The vote as a means of protesting taxes is not precluded but considered only secondary. This paper develops a simplified model of a political marketplace, linking together individual demand for a public good and the vote in fiscal elections to set the supply and tax-cost of such a good and where both the system of elections and system of taxation are taken as givens. Data from school budget elections in Oregon are applied to the model and the demand for education under varying tax-cost conditions and over time in Oregon is estimated. The resultant analysis leads to a markedly different interpretation of the increase in rejection of school finance proposals than that of a "taxpayers' revolt." Moreover, the analysis traces directly the linkage between demand for education and the tax-cost of education and indirectly the linkage between demand for education and the quality or valuation of education. Such findings have major implications in assessing the present state of public school finance and in formulating public school finance policy.
A Model of Supply and Demand in the Political Marketplace

*Ordinary goods* are goods or services available on an individual basis, the supply of which can be regulated by individual demand for and consumption of such goods in the economic marketplace. *Public goods* are goods or services provided through collective organization and available to an entire collectivity. Public goods are supplied equally and indivisibly to all individuals in a collectivity regardless of individual demand for such goods. The costs of providing public goods are imposed, although seldom equally, on all individuals in a collectivity regardless of individual consumption of such goods. Consequently, the supply of public goods, unlike the supply of ordinary goods, can only be regulated by some collective expression of individual demand in the political marketplace.

A basic model of the political marketplace in which decisions to provide public goods are made directly by the individuals in a collectivity is considered here. The requirement of the model is that any proposal to establish or to increase the supply of a public good be approved by a simple majority of the individual voter-taxpayers voting in the collectivity. If a simple majority of the voter-taxpayers vote to accept a proposal, the proposal is effected for the entire collective. If less than a simple majority of the voter-taxpayers vote to accept the proposal, the proposal is altered and resubmitted for approval. The model is common to many budget, bond, serial levy and other fiscal elections in the United States.
Individual demand. A "rational" voter-taxpayer who is able to and does make some personal valuation of a public good and some estimate of the personal tax-cost incurred for a public good is taken as a fundamental assumption here. Given this, individual demand curves for public goods analogous to demand curves for ordinary goods that indicate the quantity of a public good that would be optimally desired by the individual voter-taxpayer at each of a series of tax-prices per unit can be conceptualized. At the time of an election to establish or to increase the supply of a public good, then, the voter-taxpayer first makes some estimate of his individual tax liability for that public good (based upon existing and proposed tax provisions), the resultant tax-price per unit for that public good at the proposed level of supply, and the quantity of that public good that is optimally desired at the given tax-price per unit. If the quantity of the public good that is optimally desired by the voter-taxpayer is equal to or greater than the proposed supply of that good, the voter-taxpayer is faced with a potential condition of undersupply and will vote to accept or to effect the proposed supply of the public good. If the quantity of the public good that is optimally desired by the voter-taxpayer is less than the proposed supply of that good, the voter-taxpayer is faced with a potential condition of oversupply and will vote to reject the proposed supply of the public good. Given the votes of individual voter-taxpayers on a sufficient number of proposals with varying supply and attendant tax-cost provisions, individual demand curves for a public good might be mapped.
Collective demand. All of the voter-taxpayers in a collectivity can be arrayed conceptually according to the quantity of a public good that each optimally prefers to yield a collective preference distribution as illustrated in Figure 1 (see page 6). The proposed supply of the public good and the median preference (quantity preferred by the median voter-taxpayer in the collectivity) for the public good can be represented by $Q_s$ and $Q_m$, respectively, along the collective preference distribution. All of those voter-taxpayers lying between 0 and along the collective preference distribution are in a potential condition of oversupply and are expected to vote to reject the proposed supply of the public good. All of those voter-taxpayers lying between $Q_s$ and 00 along the collective preference distribution are in a potential condition of undersupply and are expected to vote to accept or to effect the proposed supply of the public good. Under conditions where the proposed supply of the public good is less than or equal to the median preference for that good ($Q_s \leq Q_m$), a majority of voter-taxpayers will vote to accept and thus effect the proposed supply of that good. In the fiscal model under consideration here, where simple majority vote is the decision rule, the median preference sets the maximum limit of supply of a public good for a collectivity, or as a corollary, the quantity of a public good that the collectivity is willing to purchase. Further, given prior constitutional agreement to simple majority vote as the decision rule, the median preference also expresses the quantity of a public good optimally preferred by the collectivity. The median preference for a public good thus represents the collective demand for a public good.
oversupply undersupply

QUANTITY OPTIMALLY PREFERRED

Figure 1: Collective Preference Distribution
If the proposed supply of a public good is exactly equal to the median preference for that good ($Q_s = Q_m$), one-half of the voter-taxpayers will vote to accept the proposed supply and one-half of the voter-taxpayers will vote to reject the proposed supply of the good. As the proposed supply of the public good decreases (or increases), the proportion of the voter-taxpayers voting to accept the proposed supply will increase (or decrease) accordingly, the exact rate of increase determined by the shape of the collective preference distribution. Given some estimate of the shape of the collective preference distribution, then, the median preference for a public good for a collectivity or the collective demand for the public good can be determined from the proposed supply of that good and election results.

The median tax-cost of a proposed supply of a public good (tax-cost incurred by the median voter-taxpayer in a collectivity as a taxpayer and regardless of the quantity of the public good optimally preferred) can usually be directly estimated (i.e. tax rate multiplied by median housing value or median income). If tastes are assumed independent of tax-costs incurred, the estimated median tax-cost per unit of a public good (the quantity of the public good supplied divided by the median tax-cost of that proposed supply) then corresponds to the quantity of the public good optimally preferred by the collectivity. Given a set of estimates of the quantity of public good optimally preferred by a collectivity derived from election results and the corresponding median tax-prices per unit for the goods, the collective demand curve or function for that public good can be mapped.
The Data

To construct empirically based collective demand curves for education, data were collected for each school district for each year during the period from 1963 through 1970 for a random sample (without replacement) of forty-nine out of a total of 138 unified school districts in the state of Oregon. The data include:

1. Quantity of Education Supplied \( (Q_{i,j}) \) -- Actual annual dollar expenditures per student for operational costs of education, excluding capital outlays and adjusted for inflation, in district \( i \) in year \( j \).

2. Tax-Price per Unit of Education \( (Y_{i,j}) \) -- Estimated cost to the owner of the median valued home in dollars in order to supply one unit (dollar) of education, based upon property tax rates for educational purposes and adjusted for inflation, in district \( i \) in year \( j \).

3. Proportion of Voter-Taxpayers Rejecting Proposed Supply of Education \( (P_{i,j}) \) -- Proportion of voters casting a vote to reject the proposed budget of the total voters voting in the first annual school budget election, regardless of whether that budget is accepted or rejected, in district \( i \) in year \( j \). Eighteen observations were excluded due to missing data. Reported analyses are based upon a total of 374 observations in the 49 school districts.

Collective preference distributions for the schools districts are not known. Collective preference distributions for education were assumed to approximate a normal distribution.

The computational method is given in the appendix.
The Collective Demand Function for Education

The estimated collective demand functions for education in Oregon for each year from 1963 through 1970 are plotted for one standard-deviation about the mean tax-price per unit of education in Figure 2 (see page 10). It is evident from the downward slopes of the curves that demand for education as expressed by the voter-taxpayers in the political marketplace, similar to individual demand for ordinary goods expressed by the consumers in the economic marketplace, is influenced by the tax-price per unit of such goods. As the tax-price per unit of education increases, the quantity of education optimally preferred by the collectivity clearly decreases. Given evidence of this function, it becomes an imperative to examine collective demand for education (and presumably other public goods), whether for analytic or policy purposes, with reference to the collective demand function in order to take into account the tax-cost component.

Policy Implications. The evidence of the downward sloping collective demand function for education has direct relevance to school finance policy, particularly so given the current plethora of educational tax reform proposals. Consideration of any specific proposal is beyond the scope here. All of the proposals, however, alter in some way the distribution of "apparent" tax-costs for education by providing alternative or supplementary means of educational finance. Such proposals may therefore also alter the tax-price per unit of education, and given evidence of the collective demand function, the level of optimal (or as
Figure 2: Collective Demand Functions for Education in Oregon by Year
school governors may view it, maximum) supply of education for a collect-
itivity as well. It can be seen, for example, that supplementary educational
support in the form of state or federal aid may not only provide additional
sources of monies for education, but by reducing the apparent tax-price
per unit of education locally, may increase the collective demand for
education. Similarly, tax-relief measures may not only alleviate pur-
ported inequities in local educational tax structures, but by reducing the
apparent tax-price per unit, if such measures are effective at the median,
may increase collective demand for education. It is apparent then,
that because of the collective demand function for education, the effects
of educational tax reform proposals are amplified considerably.

Changing Demand for Education: A Revolution? It is evident from
direct examination of the curves plotted in Figure 2 that the quantities
of education optimally preferred by the voter-taxpayers and as expressed
by the collective demand functions have increased rather continuously
and constantly during each year of the eight year period. The specific
characteristics of the popular view of the taxpayers' revolt are not
entirely clear but certainly the view suggests that some radical decrease
or other abrupt change in voter-taxpayer preferences for education as
indicated by the collective demand functions for education would be
evident. Something quite the opposite appears to be the case. Indeed,
the overall increase in demand for education indicated by the collective
demand functions suggests that if there has been a revolution, it has
been a revolution for more education. The explanation of the increase in
rejection of school finance issues at the polls offered by the taxpayers'
revolt thesis thus appears highly suspect.
Supply, Demand, and the Vote: A Predictive Model

The foregoing conclusion that collective demand for education in Oregon increased continuously during the period from 1963 through 1970, which is in apparent contradiction to the taxpayers' revolt thesis, necessitates an alternative explanation of the marked increase in voter-taxpayer rejections of school finance issues at the polls in evidence during the same period. The initial model of the political marketplace provided a means for estimating collective demand for a public good from the supply of that good and election results. An inversion of the model allows the prediction of election results given the collective demand function, the proposed supply, and the tax-price per unit of a public good. Such a model is presented graphically in Figure 3 (see page 13).

The graphic form of the predictive model consists of a rectangular coordinate system with the tax-price per unit of a public good represented by the abscissa and the proposed supply of that public good represented by the ordinate. The collective demand function for that public good (represented by the heavy curve 50-50 in Figure 3) has been plotted upon the coordinate system. At any given tax-price per unit, exactly 50 percent of the voter-taxpayers are expected to vote to accept (vote yes) a proposed supply of a public good at the level of the collective demand function; less than 50 percent of the voter-taxpayers are expected to vote to accept a proposed supply of a public good greater than the level of the collective demand function; and more than 50 percent of the voter-
Figure 3: Proposed Supply, Tax-Price Per Unit and the Vote
taxpayers are expected to vote to accept a proposed supply of a public good less than the level of the collective demand function. One-half decile deviations from the collective demand function have been plotted upon the coordinate system to indicate the expected or predicted vote at levels of proposed supply greater than or less than the collective demand function. The predicted vote in a fiscal election is then given by that point on the coordinate system indicating the proposed supply and tax-price per unit of the public good for the election. It can be seen in Figure 3, for example, that at the proposed supply and tax-price per unit for a public good indicated by point A the predicted vote in a fiscal election would be about 87 percent yes (to accept the proposed supply) and 13 percent no.

The model further allows a direct assessment of the effects of changes in proposed supply or tax-price per unit of a public good upon the predicted vote. It can be seen in Figure 3, for example, that if from the level indicated by point A either the tax-price per unit is increased to the level indicated by point B or the proposed supply is increased to the level indicated by point C, the margin in the predicted vote narrows to about 67 percent yes and 33 percent no. If either, or a combination of both the proposed supply and tax-price per unit of a public good, are increased sufficiently to the level indicated by point D, for example, the margin in the predicted vote reverses to about 43 percent yes and 57 percent no. Given a downward sloping collective demand function, increases in the proposed supply or the tax-price per
unit of a public good can thus result in narrower and even reversed margins in the vote without any change whatsoever n the collective demand function.

**The Supply of Education.** Utilization of the predictive model to assess or to explain changes in the outcome of the vote in school finance elections during the period from 1963 through 1970 requires the additional correction or control for changes evident in the collective demand function for education during the same period. The corrected model is presented graphically in Figure 4 (see page 16). A segment of the collective demand function for education based upon all years has been plotted upon the coordinate system. The mean supply of education and one standard deviation from the mean for all district by year, corrected for the difference between demand by year and demand for all years, at the effective tax-price per unit has also been plotted on the coordinate system for each year during the period from 1963 to 1970. 23 The effects of changes in supply and tax-price per unit education indicated by the corrected yearly plots can be interpreted directly from the model.

It is evident from the plots in Figure 4 that supplies of education in relation to the collective demand function for education have generally increased from year to year with mixed change in tax-price per unit. 24 The indication in this is that school budget planners, either by accident or by design, have increased the proposed supply of education offered to the voter-taxpayers during the period so as to more closely approximate the
Figure 4: The Supply of Education, the Tax-Price Per Unit of Education, and the Budget Election Vote.
optimally preferred quantity indicated by the collective demand function for education. The consequence, however, is that the margin in the predicted vote narrows considerably. The centroids of the plots in Figure 4, for example, indicate predicted votes of about 65 percent yes and 35 percent no at the beginning of the period, but indicate predicted votes of only about 51 percent yes and 49 percent no at the end of the period. The penalty is a smaller margin for "error," and given this smaller margin, school budget planners undoubtedly more often offer the voter-taxpayers proposed supplies of education that exceed the collective demand function for education and are therefore rejected by a majority at the polls. During the period covered by the plots in Figure 4, for example, increasing proportions of the plots fall above the collective demand function for education into the area where the predicted vote is less than 50 percent yes. The model fully indicates the noted increase in rejection of school finance issues at the polls. But the model also demonstrates that the source of the rejections, far from some kind of taxpayers' revolt, is a predictable and designed response of a democratic fiscal system to increases in the proposed supply of a public good to levels beyond the optimal preferred by a collectivity.

*Policy Implications.* How close to the collective demand function for education school budget planners should set proposed supplies of education is a normative question. Repeating the earlier argument, however, prior constitutional agreement upon simple majority vote as the decision rule for collective decisions regarding the supply of education
suggests that the optimally desirable condition is one in which one-half of the voter-taxpayers are satisfied by the supply of education or exactly at the level indicated by the collective demand function for education. The indicated increases in supplies of education to nearer the quantities indicated by the collective demand functions for education may thus be desirable even if this also results in increased numbers of elections in which the proposed supply of education is rejected by the voter-taxpayers. It is, of course, little solace to beleaguered school budget planners or voter-taxpayers weary of marching to the polls, but the increase in rejection of school finance issues at the polls may indicate that the present systems of school finance are operating in a very desirable fashion. The present systems of school finance, particularly the tax structure, may well warrant overhaul for other reasons, but not just because school finance issues are being defeated more often at the polls.

**Valuation and Demand for Education**

The analyses thus far have concentrated upon the tax-cost component and largely ignored the equally important valuation component in the examination of individual and collective demand for public goods. Voter-taxpayer valuations of public goods, similar to consumer valuations of ordinary goods, first vary with tastes basic to the individual.\(^{27}\) Taking these tastes as givens, it is the assumption here that voter-taxpayer valuations of public goods, again similar to consumer valuations of ordinary goods, are likely to be higher if these public goods are judged
to be of high or desirable quality. The quantity of a public good optimally preferred at a given tax-price per unit—the outcome of valuation—thus depends upon the combination of tastes and quality that enter into the valuation of the public good. It can be seen then that the voter-taxpayers in one collectivity may optimally prefer more (or less) of a public good than the voter-taxpayers in another collectivity at comparable tax-prices per unit because of differences in either tastes for that good or the judged quality of that good or a combination of the two.

While it is impossible to deal directly with taste and quality in the valuation of education here, indirect analysis does allow some assessment of the valuation component in the collective demand for education. The predicted level of supply of education optimally preferred by a collectivity, \( \hat{Q}_m \), at a given tax-price per unit in a given year can be estimated from the collective demand function for education. The collective demand function is based upon valuations of education by voter-taxpayers in all collectivities in the sample. If valuations of education by the voter-taxpayers in a given collectivity are higher (or less) than in the average collectivity, the measured collective demand for education in that collectivity, \( Q_m \), will be greater (less) than the predicted demand, \( \hat{Q}_m \). Any differences between measured demand and predicted demand, \( Q_m - \hat{Q}_m \), then, can be attributed to valuation as well as to error. If error is assumed to be random, any stability in the differences between measured demand for education and predicted demand for education in a collectivity over time must be due to valuation and such stability thus provides and indirect measure of the valuation of education, \( V_{m_l} \).
Expressing collective demand for education as a function of the components tax-price per unit, valuation, year, and error, the estimated propositions of the variance attributable to each component are given in Table 1 (see page 21).

Policy Implications. It is evident in Table 1 that the valuation of education by the voter-taxpayers in a collectivity appears to be a significant source of variation in collective demand for education, exceeding substantially the tax-cost of education. This evidence of the valuation component suggests that to equalize the tax-cost of education would not result in equal demand, and therefore spending for education under the present political marketplace system. And conversely, equalizing spending for education, would deny the voter-taxpayers the possibility to set spending for education in accordance with their valuations. The significance of the valuation component is perhaps surprising, given that so much attention directed to educational finance has focused upon the tax-cost of education while the valuation of education has been largely ignored. School policy-makers may take tastes for education as (undesirable) given to live with, and most certainly such tastes can only be cultivated over time, if at all. Given the significance of the valuation component indicated here, however, there may be great potential for increasing individual and collective demand for education through improved educational packages that better satisfy the voter-taxpayers. Unfortunately, further examination of the valuation of education is precluded here by limitations in the data. The valuation of education including both the
Table 1  
Sources of Variance in Collective Demand for Education

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>Percentage of Total Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax-price per Unit</td>
<td>21.1</td>
</tr>
<tr>
<td>Valuation</td>
<td>30.8</td>
</tr>
<tr>
<td>Time</td>
<td>23.7</td>
</tr>
<tr>
<td>Error</td>
<td>24.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.1</strong>*</td>
</tr>
</tbody>
</table>

*Total exceeds 100 percent due to rounding.
tastes for and the quality of education is clearly an area that warrants further exploration by both researchers and policy-makers.

Conclusions

Taking a simple democratic fiscal system as a base, a model of a political marketplace was developed. The model demonstrated that individual voter-taxpayer demand for a public good can be aggregated (imperfectly) through the vote to express the collective demand for that good and that such collective demand sets the maximum and the optimal level of supply of the public good for the collectivity. Application of the model to the analysis of school budget elections in Oregon indicated that collective demand for education in Oregon increased continuously, but the supply of education increased even more rapidly during the period from 1963 through 1970. The indication is that as school finance planners set proposed supplies of education nearer the estimated level of the collective demand function for education, instances in which the proposed supply of education exceeded the level of the actual collective demand function for education, thus resulting in budget failure, increased. The latter was interpreted as normal operation of a democratic system of public finance and even desirable. The analysis further indicated that collective demand for education is constrained by the tax-cost of such education and that collective demand for education increases as the tax-price per unit of education decreases. Educational tax reform proposals, in addition to any direct or
intended effects, may thus indirectly affect collective demand for education by altering the apparent tax-price per unit of education locally. The analysis finally indicated that the valuation of education is a major component underlying the collective demand for education. While it was not possible to resolve the valuation of education into taste and quality dimensions, improved educational packages that better satisfy the voter-taxpayer might increase collective demand for education.

The voter-taxpayer certainly opts to pay less for the same amount of education if possible. Moreover, the voter-taxpayer occasionally tires of repeated elections on varying proposed supplies of education. Alternative or supplementary systems of school finance may therefore seem attractive. A finance system giving rise to a political marketplace in which voter-taxpayers collectively control the supply of a public good through the use of the vote, vote reflecting individual valuation and tax-costs incurred for the public good, is now in effect. The voter-taxpayer should be wary of any reforms in the system of public school finance that might impinge upon or eliminate this individual voter-taxpayer control over the supply of education in the political marketplace.
APPENDIX

COMPUTATIONAL METHOD*

The difference between one-half of the total vote and the vote rejecting the proposed supply is the equivalent of the area represented under the collective preference distribution between the level of the proposed supply and the collective demand or

\[ \frac{N}{2} - N_r = \int_{Q_s}^{Q_m} F(x) \, dx, \quad (1) \]

where \( F(x) \) is the collective preference distribution, \( x \) is the quantity of the public good optimally preferred by each individual voter-taxpayer, \( N_r \) is the number of voter-taxpayers voting to reject the proposed supply of the public good, and \( N \) is the total number of voter-taxpayers voting in the collectivity.

Assuming that the collective preference distribution for education is normal, then

\[ F(x) = \frac{N}{\sqrt{2\pi} \delta} \left[ 1 - \frac{(x - c)^2}{2\delta^2} \right]. \]

Let

\[ z = \frac{x - c}{\delta}, \quad \text{where} \ c = \delta. \]

Then simplifying

\[ F(x) = \frac{N}{\sqrt{2\pi}} \left[ 1 - \frac{z^2}{2} \right]. \]

*Note: Subscripts are omitted for clarity except where there is specific reference to the collected data.
Since
\[ Z = \frac{x - x_0}{c} \]
\[ Q_m = 0 \quad \text{and} \quad Q_s = -cZ. \]

Equation (1) above then takes the form
\[
\int_{-cZ}^{0} \frac{-N}{\sqrt{2 \pi}} e^{-\frac{(z)^2}{2}} \, dZ - \left( \frac{N}{2} - Nr \right) = 0.
\]

Dividing both sides of the equation by \( N \)
\[
\int_{-cZ}^{0} \frac{1}{\sqrt{2 \pi}} e^{-\frac{(z)^2}{2}} \, dZ - \left( \frac{1}{2} - \frac{N_r}{N} \right) = 0.
\]

\( \frac{N_r}{N} \) is the proportion of voters voting to reject the proposed supply of education or \( P_{r_{ij}} \).

Substituting
\[
\int_{-cZ}^{0} \frac{1}{2} e^{-\frac{(z)^2}{2}} \, dZ - \left( \frac{1}{2} - P_r \right) = 0. \quad (2)
\]

While equation (2) is indefinite, numerical solution (by computer) gives an approximation of \( Z_{ij} \) for any \( P_{r_{ij}} \).
Then by definition

\[ Q_m = Q_s + cZ. \]  \hspace{1cm} (3)

Let the tax-price per unit be represented by \( Y \). Then collective demand can be expressed as a function of tax-price per unit

\[ Q_m = G(Y). \]  \hspace{1cm} (4)

Combining (3) and (4) above

\[ Q_m = Q_s + cZ = G(Y). \]

Transposing

\[ Q_s = G(Y) - cZ. \]  \hspace{1cm} (5)

Expanding \( G(Y) \) into the general form of a polynomial, (4) above takes the form

\[ Q_m = G(Y) = a + b_1 Y + b_2 Y^2 + \ldots + b_n Y^n \]  \hspace{1cm} (6)

and (5) above takes the form

\[ Q_s = a + b_1 Y + b_2 Y^2 + \ldots + b_n Y^n - cZ. \]  \hspace{1cm} (7)

\( Q_s \), \( Y \), and \( Z \) in (7) are known. The data were formulated into an equation of form (7) and \( a \), \( b \), and \( c \) estimated using the ordinary least squares method.
Estimates for a, b, and c in (7) using the ordinary least squares method are biased unless \( \Sigma Z_{ij} = 0 \). Solution of (7) using the ordinary least squares method assumes that \( \Sigma O_m_{ij} = \Sigma O_s_{ij} \), \( \Sigma O_m_{ij} = \Sigma O_s_{ij} \) only if \( \Sigma Z_{ij} = 0 \). If \( \Sigma Z_{ij} \neq 0 \), unbiased estimates for a, b, and c are obtained by correcting \( Z_{ij} \) to \( Z_{ij} + \bar{Z} \). Since \( \Sigma Z_{ij} > 0 \), \( O_m_{ij} \) was estimated using the corrected formulation of (3)

\[
Q_m = Q_s + cZ + c\bar{Z} \tag{8}
\]

where \( \bar{Z} = \frac{\sum Z_{ij}}{N} \). The demand functions for education \( G(Y) \) based upon all school districts were then estimated for each year from 1963 through 1970 from (6) using the ordinary least squares method.
NOTES


2. "Demand" for education in the popular view is derived from the "public interest" rather than the "individual interest" and therefore based upon an economic efficiency function. See for example Burkhead, Public School Finance, pp. 10-14. Demand for education for an entire collectivity is therefore not linked to individual demand for education. James M. Buchanan and Marilyn Flowers, "An Analytical Setting for a 'Taxpayers' Revolution,'" Western Economic Review, VII (December 1969), 349-359, are a notable exception in proposing that the taxpayers' revolution reflects both supply and tax-cost considerations of education. Also see an extension of the latter, Raymond Jackson, "A 'Taxpayers' Revolution' and Economic Rationality," Public Choice X (Spring 1971), 93-96.

3. There is no concern here with the traditional questions of public finance. The provision of education by the state is taken as a given. How the tax-costs of education should be distribution is not considered. The efficiency of a political marketplace based upon an election system for determining the supply of a public good for a collectivity and attendant welfare questions are only considered incidentally.
4. The ideal model would require that a proposal to supply zero quantity of a public good be approved by a simple majority of the voter-taxpayers. In practice, proposals to supply zero quantity of a public good are usually effected by default.


6. It could be assumed that the voter-taxpayer who is faced with a potential condition of undersupply will vote to reject the proposed supply, hoping that the new proposed supply would be incremented upward. Under such conditions, only the voter-taxpayer whose optimally preferred quantity of the public good is exactly equal to the proposed supply of the good will then vote to accept the proposed supply of the public good. The likely consequence is an Arrow voting paradox, a majority of voter-taxpayers opposing any proposed supply that might be offered. In practice, however, it is generally the case that budget officials offer the highest reasonable proposed supply of a public good to the voter-taxpayers first and nearly always reduce the proposed supply if it is rejected. The voter-taxpayer who is faced with a potential condition of undersupply of a public good thus will usually vote to accept the proposed supply of the good lest the proposed supply be reduced and the potential state of undersupply increased further. The Arrow voting paradox is thus avoided.

7. The voter-taxpayer who abstains from voting is ignored here. If the effects of the supply and tax-cost of a public good are less than the costs of voting, the voter-taxpayer will presumably abstain. Anthony Downs, *An Economic Theory of Democracy* (New York: Harper and Row, 1957) develops a model that includes abstaining voters. Empirical investigation of a similar model is currently in progress.
8. Except in the unusual condition that all voter-taxpayers optimally prefer the exact same quantity of a public good (and then there is no problem on supply), most voter-taxpayers in a collectivity are likely to be dissatisfied by any particular supply of a public good. If individual dissatisfaction (or cost) is expressed by the absolute difference between the quantity optimally preferred and the quantity supplied, then the total of dissatisfaction for an entire collectivity, $D_s$, is given by

$$D_s = \int F(x) |Q_s - x| \, dx.$$ 

It can be seen that $D_s$ is minimized if $Q_s - x$ is minimized or if $Q_s = x$ at the median of $x$. The estimate of the median preference derived directly from the vote with a simple majority rule thus approximates the (Pareto optimal) condition where total dissatisfaction in a collectivity is minimized. Presumably this consideration enters into the constitutional agreement upon simple majority vote as the decision rule. James M. Buchanan and Gordon Tullock, *The Calculus of Consent* (Ann Arbor: Ann Arbor Paperbacks, The University of Michigan Press, 1962) examine the basis for agreement on a decision rule in detail.

9. Initially, assumptions that tastes are randomly distributed are usually necessary—although control for tastes is presumably possible in advanced analyses. It seems likely that the more wealthy might desire to consume not only more private goods but more public goods as well. Further, tax-costs are generally (but not always) higher among the more wealthy. There is a possibility then that the quantity of a public good optimally preferred and tax-costs for that good are related. The effect is to reduce the apparent constraint of tax-cost upon demand for a public good within a collectivity. This does not affect the analysis here directly since it is cross-sectional.

10. All reported variances are corrected for sampling without replacement.
11. The availability of data dictated the use of actual dollars per student for education rather than proposed dollars per student for education. Comparison, where possible, indicated only minor differences between the two figures.

Dollars per student for education is an incomplete measure of the supply of education, excluding all considerations of quality. Further, dollars per student for education is a measure that confuses the individual purchase with the collective cost. A complete conceptual scheme would include the final package of educational goods.

Unfortunately, an operational measure of the package of education is not available. Quality is treated indirectly in a later section.

12. The median tax liability for education, $T_m$, was computed from tax rates applicable to education including the local extended, IED, and county school fund levies and the estimated median housing valuation from the 1960 and 1970 Census of Housing reports and interim changes in district valuation per student. Renters are ignored. Then the estimated effective tax-price per unit of education, $Y$, is given by

$$Y = \frac{T_m}{O_s}.$$ 

Indirect tax-costs resulting from state and federal support to education are ignored since the voter-taxpayer can expect to incur these costs regardless of the supply of education locally.
13. The first budget election each year was judged most comparable both cross-sectionally and longitudinally, the later budget elections being subject to numerous idiosyncratic factors. Bernard F. Saalfeld, Taxpayers and Voters: The Political Economy of Public Education (Unpublished Ph.D. dissertation, University of Oregon, 1973).

14. The collective preference distributions could be determined by mapping the individual demand functions for education for a sample of voter-taxpayers from each collectivity.

15. The analyses may underestimate the slopes of the collective demand functions for education. First, the analyses assume that valuations of education and the tax-price per unit of education are randomly distributed or independent of each other. The tax-price per unit of education is slightly higher than the average for Oregon in the urban school districts. Further, there is some evidence that tastes for education are greater in urban areas than in nonurban areas. See for example Jerry Minor, Social and Economic Factors in Spending for Public Education (Syracuse: Syracuse University Press, 1963), p. 99. Also see note 9. Second, the analyses assume that the supply of education is set within each collectivity and is not affected by any exogenous factors. There are state statutory minimums for education (i.e., number of days of school per year, number of pupils per teacher, classroom space per pupil) that set the lower limit of supply of education regardless of local valuations of education or the local tax-price per unit of education. The effects of the above may result in the over-estimation of the level of the true collective demand function for education in the lower part of the curve. The result is to flatten the curve and reduce the apparent effect of tax-price per unit upon the demand for education. Control for the above estimation error is possible through the use of an analysis of covariance design but is precluded here by sample size.
16. The most direct application is to make comparisons of differences in collective demand for a public good either cross-sectionally or longitudinally in terms of deviation from the collective demand function for that good.

17. Any attempt to catalogue the educational tax reform proposals presently under consideration would be heroic. The President, several members of Congress, many state governors, members of states legislatures, school officials, and citizens groups have all entered into the educational tax reform gambit.

18. The term "apparent" is emphasized here since ultimately the voter-taxpayers must bear the cost of education in one way or another. The particular method of taxation, i.e., addition altering the distribution of tax-costs, may also affect and even make it impossible for the voter-taxpayer to estimate the tax-cost incurred to provide a public good. Buchanan, Public Finance, is excellent in treatment of this point.

19. Tax-relief measures that are effective only at the extremes appear likely to have little impact upon demand for education when compared with even more modest measures but measures that have greater breadth.

20. The function of the difference in the collective demand functions for education by year approximates a monotonic function.

\[ F \left[ \int_{t_1}^{t} G_t(Y) \, dY \right] = 5.3t + k \]

with .976 proportional reduction in error.

Whether the function represents change depends upon the base from which change is measured (i.e., the base may be fixed, a linear function, a logarithmic function, etc.) The appropriate base here is the actual educational price deflator, an index that is not available. The indicated stability of any change is the important point here. C.W. Harris, ed., Problems in Measuring Change (Madison: University of Wisconsin Press, 1963) provides extended treatments of the problem here.
21. In 1963, proposed school budgets were accepted by the voters in the first school budget election in 88 percent of the districts included in the study sample. By 1970, proposed school budgets were accepted by the voters in the first school budget election in only 56 percent of the districts, a decrease of over 30 percent.

22. The predicted vote is more precisely expressed as a function

$$P_{a_{ij}} = F \left\{ \left[ \frac{G_{ij}(Y) - Q_{si}}{L_{ij}} \right], U, V, W, \ldots \right\}$$

where $U, V, W, \ldots$, are additional variables than may enter the predictive equation.

23. The offset of the centroid of each supply plot from the collective demand function is the equivalent of the difference between $Q_s$ and $Q_m$. The distribution about the centroid is properly an ellipse but is plotted as a line for clarity.

24. The point is incidental here but note that between 1966 and 1967, the year of greatest shift in the vote, an increase in supply and an increase in the tax-price per unit of education is evident. Interestingly, this was the time that state level property tax relief was eliminated in Oregon.

25. The term "error" is used here somewhat facetiously. It appears both school officials and voter-taxpayers tend to view offering a proposal to the voters that is rejected as a blunder of some kind. The view here is that the defeat of proposals at the polls is only a normal expectation in a democratic system of public finance.

26. The question of how close to the collective demand function for education school budget planners do set proposed supplies of education is itself interesting. There is a slight correlation evident between school district size (number of students) and the supply-demand differential ($Q_s - Q_m$) indicating in larger, more bureaucratized school districts that school budget planners set proposed supplies of education nearer to the level of the collective demand function for education.
27. Tastes for public goods, while taken as givens here, can presumably be shaped and cultivated through proper propaganda much as tastes for ordinary goods appear to be.

28. Expressed mathematically,

\[ v_{mi} = \frac{\xi (Q_{mi} - \hat{Q}_{mi})}{j} \]

Note that this represents a conservative estimate of stability.

The function

\[ v_{mij} = F (Q_{mij} - \hat{Q}_{mij}) \]

is an alternative estimate of stability and one that reduces the residual error.