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AUTHOR	Horowitz, Abraham D.; Rapoport, Amnon
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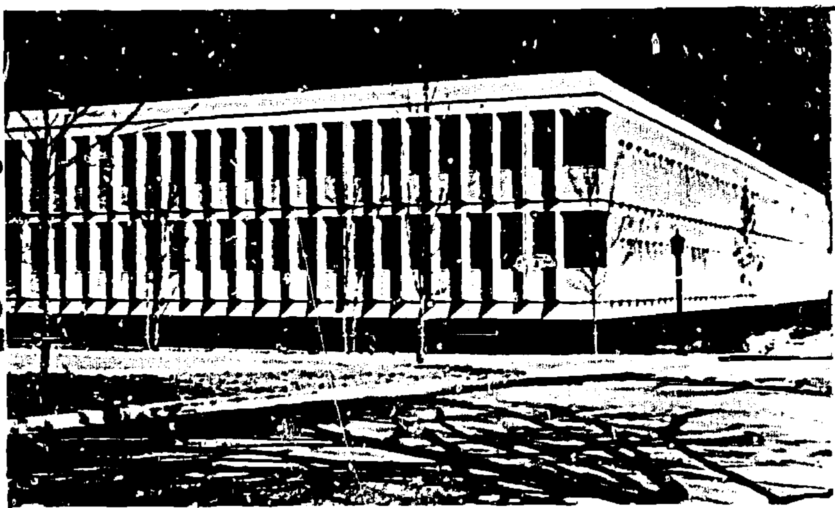
ABSTRACT

Employing a computer-controlled experimental paradigm for studying coalition formation and bargaining, the present study tests three models for n-person games in characteristic function form, namely, the bargaining set and two of its subsets, the competitive bargaining set and the kernel. Twelve groups of subjects participated in several four person and five-person Apex games. The effects of group size, order of communication, learning, and values of the characteristic function were systematically investigated. The final outcomes reject the kernel and support the two bargaining set models; they depend upon group size and order of communication. Models describing the bargaining process, rather than the final outcomes only, are presented, tested, and partially supported. The relationships between the final outcomes of the present study and those of previous studies of Apex games are briefly discussed. (Author)



Test of the Kernel and Two Bargaining Set Models in Four- and Five- Person Games

Abraham D. Horowitz and Amnon Rapoport

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THE L. L. THURSTONE PSYCHOMETRIC LABORATORY
UNIVERSITY OF NORTH CAROLINA

CHAPEL HILL, N. C.
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Test of the Kernel and Two Bargaining Set Models in Four- and Five- Person Games[1]

Employing a computer-controlled experimental paradigm for studying coalition formation and bargaining, the present study tests three models for n-person games in characteristic function form, namely, the bargaining set and two of its subsets, the competitive bargaining set and the kernel.

Twelve groups of subjects participated in several four-person and five-person Apex games. The effects of group size, order of communication, learning, and values of the characteristic function were systematically investigated. The final outcomes reject the kernel and support the two bargaining set models; they depend upon group size and order of communication.

Models describing the bargaining process, rather than the final outcomes only, are presented, tested, and partially supported. The relationships between the final outcomes of the present study and those of previous studies of Apex games are briefly discussed.

An experimental paradigm has been proposed by Kahan and Rapoport (1972) for investigating coalition formation and bargaining processes in small groups. The paradigm is based on considerations of coalition formation through the negotiated division of rewards, or values, available to each coalition that may be formed. Its orientation arises from n-person game theory (see, e.g., Luce and Raiffa, 1957; Rapoport, 1966, 1970; von Neumann and Morgenstern, 1947), in particular from that portion of the theory concerned with formalized models of conflict of interest among n players, which depend only on the respective values of the possible coalitions. Utilization of the paradigm relies heavily upon the development of the digital computer as an instrument of the psychological laboratory for conducting on-line group decision making experiments.

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Whereas Kahan and Rapoport (1972) have studied coalition formation and bargaining processes in the triad, the present study, employing their experimental paradigm, has moved a step further to the case $n \geq 4$. Among the various games that may be investigated, we have focused on a psychologically intriguing game, first introduced by von Neumann and Morgenstern (1947, pp. 473-503) and further explored by Davis and Maschler (1965). The game under consideration, called the Apex game by Horowitz (1971), is a cooperative n-person game, $n \geq 3$, in which the only coalitions assigned positive values are (i) all those coalitions which include a certain player called Apex, and (ii) the coalition formed by the other n-1 players, called Base players.

The Apex game may be cast in terms of the characteristic function of the n-person game, a real-valued set function assigning a real number $v(\underline{X})$ to each nonempty subset $\underline{X}$ of players, where $\underline{X} \subseteq \underline{N}$ and $\underline{N}=\{1,2,\ldots,n\}$. The value $v(\underline{X})$ measures the worth or power which the coalition $\underline{X}$ can achieve when its members act together. For example, consider an Apex game with $n=5$ in which every coalition may win c units, $c>0$. Then assuming that A is the Apex player, and B, C, D, and E denote the four Base players, the characteristic function of this game is

$$v(AB)=v(AC)=v(AD)=v(AE)=v(ABC)=\ldots=v(ADE)=v(ABCD)=\ldots$$
$$=v(ACDE)=v(ABCDE)=v(BCDE)=c,$$

and $v(G)=0$ for any other coalition G.

The Apex's position may be compared to that of a monopolist, with the only limitation that he must find at least one ally. Only the coalition of all other players against him may defeat him (von Neumann and Morgenstern, 1947). The Base's position poses an intriguing dilemma: he must either cooperate with all other Base players, regardless of their number, or he must join the Apex and possibly some other Base players. If the first course of action is chosen, the Base risks being frozen out of a winning coalition, if one or more Base players yield to the temptation of extra gain by forming a coalition with the Apex. On the other hand, if he chooses to negotiate with the Apex, the Base must consider the highly competitive environment produced by the Apex's multitude of choices in stating his demand for his share.

The central issue of the Apex game, as well as any other n-person cooperative game in characteristic function form, has been stated succinctly by Anatol Rapoport: "Given a particular coalition structure, how will the payoffs accruing to each coalition be apportioned among its members [1971, p. 194]?" Answers to this question may be derived from some of the models proposed for n-person cooperative games in characteristic function form, namely, von Neumann and Morgenstern's solution (1947), Shapley's value (1953), Aumann and Maschler's bargaining set (1964), the kernel of Davis and Maschler (1965), and Horowitz's competitive bargaining set (1972) especially developed for Apex games. In attempting to test these models, we have discarded von Neumann and Morgenstern's solution because of the infinite number of imputations it contains, and a special case of it, the main simple solution, which lacks any apparent psychological justification. Shapley's value has been discarded because it is limited to the coalition of all n players, the grand coalition. We have been left, then, with three models to test, the bargaining set, the kernel, and the competitive bargaining set.

The Models

The basic concepts of the bargaining set and kernel models have been presented and discussed by Aumann and Maschler (1964), Davis and Maschler (1965), Horowitz (1972), Kahan and Rapoport (1972), Rapoport (1970), and will not be repeated here. Since familiarity with the competitive bargaining set model, presented by Horowitz in 1972 cannot be assumed, we present its principal ideas below.

Consider a cooperative n-person game in characteristic function form, which consists of a set $\underline{N}=\{1,2,\ldots,n\}$ of n players along with a characteristic function v, assigning the real number $v(\underline{X})$ to each nonempty subset $\underline{X}$ of players, called the coalition $\underline{X}$. $v(\underline{X})$ is assumed to satisfy

(i) $v(\underline{X})\geq 0$ for each coalition $\underline{X}$,

(ii) $v(\{i\})=0$ for each one-person coalition.

Let $\tilde{X}$ be an m-partition of $\underline{N}$ satisfying

$\underline{X}_j \cap \underline{X}_k = \phi$, if $j \neq k$, and $\bigcup_{j=1}^{m} \underline{X}_j = \underline{N}$.

An outcome of the game is represented by a payoff configuration (p.c.)

$$(\vec{x};\underset{\sim}{X}) = (x_1, x_2, \ldots, x_n; \underline{X}_1, \underline{X}_2, \ldots, \underline{X}_m),$$

where $\vec{x} = (x_1, x_2, \ldots, x_n)$ is an n-dimensional real vector, called the payoff vector, representing the realizable distributions of wealth among the n players, x_i is the amount received by player i in the distribution $\vec{x}$, and $\underset{\sim}{X} = \{\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_m\}$ represents the coalition structure which was actually formed.

A p.c. is assumed to satisfy individual rationality, i.e.,

$x_i \geq 0$ for all $i \in N$.

It is further assumed that

$$\sum_{i \in \underline{X}_j} x_i = v(\underline{X}_j), \text{ for } j = 1, 2, \ldots, m.$$

A third, key assumption is that every pair of players who are members of the same coalition ought to be in equilibrium. The concept of equilibrium is crucial to the competitive bargaining set and is defined in terms of the notions of multi-threat and counter-multi-threat.

Following the notation of Davis and Maschler (1967), let $(\vec{x};\underset{\sim}{X})$ be a p.c. and k and ℓ be two members of some coalition $\underline{X}_j$, $\underline{X}_j \in \underset{\sim}{X}$. Let $\underline{Y}_1, \underline{Y}_2, \ldots, \underline{Y}_t$ be t distinct coalitions, $t \geq 1$, and let $\vec{y}^{(1)}, \vec{y}^{(2)}, \ldots, \vec{y}^{(t)}$ be the associated payoff vectors. The set

$$\left\{(\vec{y}^{(1)};\underline{Y}_1), (\vec{y}^{(2)};\underline{Y}_2), \ldots, (\vec{y}^{(t)};\underline{Y}_t)\right\}$$

is called a multi-threat of player k against ℓ with respect to the p.c. $(\vec{x};\underset{\sim}{X})$ if

(i) $y_i^{(g_1)} = y_i^{(g_2)}$ for all $i \in (\underline{Y}_{g_1} \cap \underline{Y}_{g_2})$, $i \neq k$,

(ii) $k \in \underline{Y}_g$, $\ell \notin \underline{Y}_g$, $k, \ell \in \underline{X}_j$, $g = 1, 2, \ldots, t$,

(iii) $\sum_{i \in \underline{Y}_g} y_i^{(g)} = v(\underline{Y}_g)$, $g = 1, 2, \ldots, t$,

(iv) $y_k^{(g)} > x_k, y_i^{(g)} \geq x_i$ for all $i \in \underline{Y}_g$, $g = 1, 2, \ldots, t$.

In his multi-threat, player k claims that he can gain more in any new coalition $\underline{Y}_g$ that he may form without the consent of player ℓ, and that the new coalition is reasonable because the partners of k in $\underline{Y}_g$ gain at least what they gained in $(\vec{x};\underset{\sim}{X})$.

When t=1, player k may threaten ℓ through one coalition only. In terms of the terminology of the bargaining set model, k is said to have an objection against ℓ. Thus, when t=1, the competitive bargaining set notion of multi-threat reduces to an ordinary objection.

For a coalition $\underline{Z}$ and a payoff vector $\vec{z}$ to its members, the pair $(\vec{z};\underline{Z})$ is called a counter-multi-threat to k's multi-threat against ℓ, if

(i) $\ell \in \underline{Z}$, $k \notin \underline{Z}$,

(ii) $\sum_{i \in \underline{Z}} z_i = v(\underline{Z})$,

(iii) $z_i \geq x_i$ for all $i \in \underline{Z}$,

(iv) $z_i \geq y_i^{(g)}$ for all $i \in (\underline{Z} \cap \underline{Y}_g)$ and g=1,2,...,t.

In lodging his counter-multi-threat, player ℓ claims that he can protect his share in $(\vec{x};\underset{\sim}{X})$ by giving his partners in $\underline{Z}$ at least what they had before in $(\vec{x};\underset{\sim}{X})$, without k's consent. Moreover, if any of ℓ's partners in $\underline{Z}$ is included in the multi-threat of k against ℓ, he would gain at least what he had gained before.

If a counter-multi-threat intersects no more than one of the coalitions $\underline{Y}_1, \underline{Y}_2, \ldots, \underline{Y}_t$, it reduces to the notion of counter-objection of the bargaining set model.

A multi-threat to a p.c. is justified if no counter-multi-threat to it exists; otherwise it is unjustified. An individually rational p.c. is said to be $\underset{\sim}{H}_1^{(i)}$-stable if for each multi-threat of k against ℓ in $(\vec{x};\underset{\sim}{X})$ there exists a counter-multi-threat of ℓ against k. The set of all $\underset{\sim}{H}_1^{(i)}$-stable p.c.'s, which may be empty, is called the competitive bargaining set and denoted by $\underset{\sim}{H}_1^{(i)}$. The superscript of $\underset{\sim}{H}$ indicates individual rationality and the subscript denotes that objections may be made only against one player at a time. Both constraints may be replaced, resulting in more severe requirements of stability.

Thus, one may replace individual by coalitional rationality. Formally, a p.c. is said to satisfy coalitional rationality if for any coalition $\underline{W}$, $\underline{W} \subset \underline{X}_j$, j=1,2,...,m,

$$\sum_{i \in \underline{W}} x_i \geq v(\underline{W}).$$

The set of all coalitionally rational p.c.'s in which no player has a justified multi-threat against any other member of the same coalition is called the competitive bargaining set $\underset{\sim}{H}_1$. It can be shown that $\underset{\sim}{H}_1 \subseteq \underset{\sim}{H}_1^{(i)}$. For the Apex games considered in the present paper $\underset{\sim}{H}_1 = \underset{\sim}{H}_1^{(i)}$.

The bargaining set may be seen as a special case of the competitive bargaining set when t=1. In particular, the bargaining set $\underset{\sim}{M}_1^{(i)}$ is defined to be the set of all individually rational p.c.'s in which no player has a justified objection against any other member of the same coalition. Replacing individual by coalitional rationality yields the bargaining set $\underset{\sim}{M}_1$. For the Apex games considered in the present study $\underset{\sim}{M}_1 = \underset{\sim}{M}_1^{(i)}$.

Horowitz proved that $\underset{\sim}{H}_1^{(i)} \subseteq \underset{\sim}{M}_1^{(i)}$. The proof that the kernel of the n-person game in characteristic function form, denoted by $\underset{\sim}{K}$, is contained in $\underset{\sim}{M}_1^{(i)}$ is given in Davis and Maschler (1965).

It is worth mentioning that the concepts of objection and counter-objection as well as the concepts of multi-threat and counter-multi-threat involve only ordinal preferences of individual players. Interpersonal comparison of utilities is not assumed, thus considerably enhancing the attraction of the two bargaining set models to social scientists. This is not the case with the kernel model.

Predictions derived from the three models may best be demonstrated by an example. Consider a five-person Apex game with the characteristic function

$$v(AB)=v(AC)=v(AD)=v(AE)=v(BCDE)=100,$$

v(G)=0 for any other coalition of G. This particular Apex game, similar to the games investigated in the present study, is not a full Apex game, as any coalition with an Apex player and two or more Base players is assigned the value 0. This, however, does not affect the predictions of the models concerning the division of the payoff for a two-player coalition.

Since the four two-person coalitions which are assigned 100 points each are symmetric, it is sufficient to consider coalition AB. For this coalition there exists a unique $\underset{\sim}{H}_1^{(i)}$-stable p.c., namely,

$$(\vec{x};\underset{\sim}{X})=(75,25,0,0,0;AB,C,D,E),$$

as can easily be shown. A multi-threat from A against B with respect to the p.c. $(\vec{x};\underset{\sim}{X})$ is

$$\left\{(y_A^{(1)},y_C^{(1)};AC),(y_A^{(2)},y_D^{(2)};AD),y_A^{(3)},y_E^{(3)};AE)\right\}$$

where $y_A^{(g)}>75$, $g=1,2,3$, and therefore $y_C^{(1)},y_D^{(2)},y_E^{(3)}<25$, since $v(AC)=v(AD)=v(AE)=100$. Player B can respond to A's multi-threat by a counter-multi-threat

$$(25,25,25,25;BCDE).$$

Note also that any single threat (objection) by B against A with respect to the p.c. $(\vec{x};\underset{\sim}{X})$ can be countered by the latter player. Also, it is seen that for any $\delta>0$ the p.c.

$$(75-\delta,\ 25+\delta,\ 0,0,0;AB,C,D,E)$$

is not $\underset{\sim}{H}_1^{(i)}$-stable, since a multi-threat from A against B in which $y_C^{(1)},y_D^{(2)},y_E^{(3)}>25$ is justified, i.e., it cannot be met by a counter-multi-threat of B.

It can be shown that the only p.c., given the coalition AB, which is contained in the kernel of the Apex game, is

$$(50,50,0,0,0;AB,C,D,E).$$

And finally, the only p.c.'s with the coalition structure (AB,C,D,E) for which no player has an $\underset{\sim}{M}_1^{(i)}$-justified objection against any other consist of the continuum

$$(50\leq x_A\leq 75,\ 25\leq x_B\leq 50,\ 0,\ 0,\ 0;\ AB,\ C,\ D,\ E),$$

where $x_A+x_B=v(AB)=100$.

For the coalition of the four Base players, all three models predict a unique p.c., namely,

$$(0,25,25,25,25;A,BCDE),$$

as can be easily verified.

The competitive bargaining set model was developed as a result of what Horowitz (1972) considered to be a shortcoming of the bargaining set model, namely, that the latter model implies noncompetitive bargaining behavior by those members of the counter-objection, other than ℓ, who are not members of the objection. It is assumed by the bargaining set model that player i, $i \neq \ell, k$, is willing to join a counter-objection for at least his previous gain x_i, regardless of his relative power as defined by the characteristic function of the game. But, why should player i help ℓ to form a coalition $\underset{\sim}{L}$ for a return z_i, $z_i \geq x_i$, ignoring other potential coalitions in which he may obtain a higher payoff?

It seems that the noncompetitive bargaining behavior implied by the bargaining set model results from its assumption that only a single objection may be expressed at any one time. Underlying the competitive bargaining set model is the assumption that since threats are often tacit (see, e.g., Schelling, 1960), their number should not be restricted to one. Rather, threats are assumed to be perceived and considered simultaneously even though their simulteneous implementation is impossible. The resulting outcome, for the particular example considered above and the Apex games considered below, is that the competitive bargaining set theory yields a unique solution located at one extreme of the continuum of solutions prescribed by the bargaining set model. The other extreme point is the kernel. For the Apex games considered in the present study, $\underset{\sim}{M}_1$ reflects various degrees of competitive bargaining behavior, where $\underset{\sim}{K}$ represents a minimal amount of competition, and at the other end $\underset{\sim}{H}_1$ represents a maximal amount. For a more detailed comparison of the three models see Horowitz (1972).

Employing the experimental paradigm of Kahan and Rapoport (1972), one of the purposes of the present experiment is to test the three models in Apex games. An additional, equally important, purpose is to develop and test models accounting not only for the final outcomes but also for the bargaining process. Additionally, the present experiment looked at the effects of group size, order of communication among the n players, practice, and ratio of the value of the Apex coalition to the Base coalition on the final outcomes of the game and the bargaining process.

Method

Subjects

Sixty undergraduate male students at the University of North Carolina participated in the experiment. They were recruited by an advertisement in the student newspaper which promised financial reward. The subjects were divided into 12 groups of five subjects, each group participating in two three-hour sessions.

Design

A 2x2x2x2 factorial design was employed, with repeated measures on two of the four factors. One factor, $\underline{O}$, was the order of communication, in which the Apex player either communicated before all the Base players ($\underline{O}_1$) or after them ($\underline{O}_2$). A second factor, $\underline{V}$, concerned the value of a coalition between the Apex player and a single Base player, hereafter called the Apex coalition. This value was either 72 ($\underline{V}_1$) or 108 ($\underline{V}_2$). The value of the coalition of all n-1 Base players, hereafter called the Base coalition, was always 72 points. The third factor, $\underline{N}$, was the size of the group, either a quartet (condition $\underline{N}_1$, an Apex plus three Bsse players) or a quintet (condition $\underline{N}_2$, an Apex plus four Base players). Each of the eight games defined by the Cartesian product of the three factors $\underline{N}$, $\underline{V}$, and $\underline{O}$, was played twice to yield a fourth factor of runs, $\underline{R}$, with first ($\underline{R}_1$) and second ($\underline{R}_2$) plays as levels.

Three groups of five subjects each were assigned to each of the four $\underline{O}$x$\underline{V}$ combinations. Repeated measures on factors $\underline{N}$ and $\underline{R}$ were employed. Each group participated in two three-hour sessions, which typically took place within a single week. The first was a practice session. In the second session each group played two four- and two five-person Apex games, with a single passive player in the former case, who could observe the bargaining but neither send nor receive any messages. Players were labelled A, B, C, D, and E, and were required to send typed messages in alphabetical order. The Apex was therefore player A in condition $\underline{O}_1$ and player E in condition O_2. The design of the experiment as well as the characteristic functions of the game are presented in Table 1.

Procedure

The first session started by having the subjects of each group read a set of instructions. Since the instructions are given in Horowitz (1971) and are also summarized in Kahan and Rapoport (1972), they will not be presented here. Essentially, they present the bargaining game as a three-stage process, consisting of an offer stage, in which the potentials of various coalitions may be explored, an acceptance stage, in which a particular p.c. is seriously considered, and a ratification stage, in which the agreement on a division of value becomes binding. Communication takes place through the use of six keywords, OFFER, ACCEPT, REJECT, RATIFY, PASS, and SOLO, allowing players to propose various p.c.'s, accept, reject or ratify them, make no communication, or withdraw from the game.

Table 1

Research Design

Order	Value	Groups	Size	Characteristic Function
$\underline{O}_1$	$\underline{V}_1$	1,2,3	$\underline{N}_1$	v(AB)=v(AC)=v(AD)=v(BCD)=72
			$\underline{N}_2$	v(AB)=v(AC)=v(AD)=v(AE)=v(BCDE)=72
	$\underline{V}_2$	4,5,6	$\underline{N}_1$	v(AB)=v(AC)=v(AD)=108, v(BCD)=72
			$\underline{N}_2$	v(AB)=v(AC)=v(AD)=v(AE)=108, v(BCDE)=72
$\underline{O}_2$	$\underline{V}_1$	7,8,9	$\underline{N}_1$	v(EB)=v(EC)=v(ED)=v(BCD)=72
			$\underline{N}_2$	b(EA)=v(EB)=v(EC)=v(ED)=v(ABCD)=72
	$\underline{V}_2$	10,11,12	$\underline{N}_1$	v(EB)=v(EC)=v(ED)=108, v(BCD)=72
			$\underline{N}_2$	v(EA)=v(EB)=v(EC)=v(ED)=108, v(ABCD)=72

Note.- The characteristic functions are the same in the two levels of factor $\underline{R}$.

The written instructions were followed by a verbal explanation, in which the experimenter reiterated the main rules of the bargaining game. Then each subject entered a separate cubicle containing a teletypewriter connected to a PDP-8 computer to play two example games. While playing the two games under the experimenter's supervision, the subjects were encouraged to ask questions about the rules of the game and the operation of the teletypewriter and to employ all the options provided by the computer program. At the end of the first session they were told that the number of games to be played in the second session was fixed. These instructions were provided to discourage the subjects from fast bargaining in order to increase the number of games played and, consequently, the amount of money earned.

Subjects returned a few days later for the experimental session, entered their respective cubicles without communicating with one another, and started immediately to play. The order of the games was randomized for each group, and the Apex role was not assigned more than once to a given subject. Roles were reassigned for each game to prevent sequential effects between games and to assure that experimental effects would be attributed to role and not to bargaining strategies of individual players. An interrogation of the subjects following the second session revealed that subjects did not form any hypotheses about roles assigned to the players in successive games, nor could they successfully guess the identity of the other players in a particular game.

At the end of the second session, each subject was paid $4.50 for participation in the first three-hour session, plus 5c per point earned in the second session, plus a fixed sum of 75¢ per hour in the second session.

The experiment was administered with a set of PDP-8, on-line, computer programs called Coalitions. A non-technical brief description of the main program is provided by Kahan and Rapoport (1972). For a complete, technical description see Kahan and Helwig (1971).

Results

The basic data consist of the typed messages sent during the experimental session, starting with the first OFFER or PASS and ending with the last RATIFY. Conceptually, it is convenient to analyze these data in terms of (i) the final outcomes, (ii) the initial phase of negotiations, and (iii) the bargaining process. To simplify the ensuing presentation of the results, a system of terminology and notation is first presented. Let $G_{g,r,n}$ denote a game, where g, g=1,2,...,12, is the group number, r, r=1,2, is the run number, and n, n=4,5, is the group size. For example, $G_{7,2,4}$ denotes the game played by group 7 in quartet form (condition $\underline{N}_1$) on the second run (condition $\underline{R}_2$). An offer is a p.c.. When accepted by all its members it is called a _tentative coalition_. The final tentative coalition in a game is called the _ratified coalition_. A tentative coalition is _dissolved_ when one of its members rejects it explicitly, or, equivalently, enters another tentative coalition. A Base player included in a ratified coalition is a _Base winner_, otherwise a _Base loser_.

The main dependent variable is the number of points, x_β, allocated to a Base player (or to all the Base players in the case of a Base coalition) in an offer, tentative coalition, or ratified coalition. Since the value of the Apex coalition, as given by the characteristic function, is known (either 72 or 108), the Apex's share, x_α, may be obtained by subtraction.

Final Outcomes

The final outcomes of the 24 quintet and 24 quartet games are presented in Tables 2 and 3, respectively. Table 2 shows that in 23 of 24 quintet games an Apex coalition was formed, yielding the Base winner a mean of 20.1 points. The range of the Base winner's payoffs was from 18 to 25. The payoffs x_β predicted by the bargaining set model fall in the closed interval $18 \le x_\beta \le 36$ for both conditions $\underline{V}_1$ and $\underline{V}_2$. The competitive bargaining set lies at the one extreme of 18, and the kernel at the other, 36. All the final outcomes fall in the bargaining set $\underset{\sim}{M}_1$. In particular, they provide strong support for the competitive bargaining set compared to the kernel.

Table 2

Final outcomes of quintet games

	$\underline{O}_1$				$\underline{O}_2$			
	g	r	x_α	x_β	g	r	x_α	x_β
$\underline{V}_1$	1	1	54	18	7	1	47	25
		2	54	18		2	54	18
	2	1	53	19	8	1	0	*
		2	52	20		2	47	25
	3	1	50	22	9	1	50	22
		2	53	19		2	54	18
$\underline{V}_2$	4	1	90	18	10	1	86	22
		2	90	18		2	83	25
	5	1	90	18	11	1	83	25
		2	90	18		2	89	19
	6	1	90	18	12	1	89	19
		2	86	20		2	90	18
	Mean			18.8	Mean			21.5
	S.D.			1.3	S.D.			3.1

*Ratified coalition: (9,21,21,21;ABCD)

Table 3

Final outcomes of quartet games

	O_1				O_2			
	g	r	x_α	x_β	g	r	x_α	x_β
V_1	1	1	48	24	7	1	36	36
		2	52	20		2	0	*
	2	1	47	25	8	1	0	*
		2	48	24		2	42	30
	3	1	47	25	9	1	40	32
		2	47	25		2	40	32
V_2	4	1	90	18	10	1	80	28
		2	90	18		2	78	30
	5	1	83	25	11	1	80	28
		2	83	25		2	80	28
	6	1	86	22	12	1	84	24
		2	84	24		2	83	25
	Mean			22.9	Mean			29.3
	S.D.			2.7	S.D.			3.5

*Ratified coalition: (24,24,24;BCD)

An Apex coalition was formed in 22 of the 24 quartet games yielding the Base winner a mean of 25.8 points. The range of the Base winner's payoffs was from 18 to 36. The predicted payoffs in M_1, H_1, and K for the Base winner are $24 \le x_\beta \le 36$, $x_\beta = 24$, and $x_\beta = 36$, respectively. As in condition N_2, the mean final outcome supports the competitive bargaining set relative to the kernel. The bargaining set is supported by 18 of the 22 final Apex coalition outcomes. Both M_1 and H_1 are also supported by the results of the two Base coalitions that were formed in games $G_{7,2,4}$ and $G_{8,1,4}$.

To assess the effects of the four experimental conditions on the final outcomes, a 2x2x2x2 analysis of variance with repeated measures on factors N and R (employing a multivariate approach) was conducted on the Base winner payoffs presented in Tables 2 and 3. The significant group size effect ($F=53.4$, $p<.001$) is predicted by neither the kernel nor the bargaining set model. The competitive bargaining set model, however, predicts a difference of 6 points between quartet and quintet games for the Base winner's payoff; the observed mean difference was 5.7.

The second significant main effect was due to factor O ($F=10.2$, $p<.02$), with the Base player winning significantly more in condition O_2 than in O_1. This effect is inconsistent with all three models, since none of them incorporates any consideration of order of communication. The other two main effects, V and R, did not contribute significantly to the final outcomes.

The only significant interaction was the two-way interaction $O \times N$ ($F=6.5$, $p<.05$), which accounts for the different effects of order of communication in quartet and quintet games. This interaction is related to the theoretical predictions in the following way. While outcomes in the quintet games supported the competitive bargaining set, the outcomes of the quartet games supported it only when the Apex player communicated first. The means of the quartet games were 22.9 and 29.3 in conditions O_1 and O_2, respectively. A .99 confidence interval computed for condition O_1 yielded the range $20.9 \le x_\beta \le 24.9$, excluding most of the continuum of the bargaining set and including H_1. A similar confidence interval in condition $\underline{O}_2$ was $26.4 \leq x_\beta \leq 32.2$, covering about two thirds of $\underset{\sim}{M}_1$ and excluding $\underset{\sim}{H}_1$ as well as $\underset{\sim}{K}$.

The Initial Phase of Negotiations

The initial phase of negotiations is defined here as the first two rounds of communication, i.e., either the first eight or ten messages in conditions $\underline{N}_1$ and $\underline{N}_2$, respectively. It was extensively analyzed mainly for two reasons. First, the initial tentative coalition was formed during the first two rounds of negotiations in 47 of 48 games. Secondly, 25 of the 48 initial tentative coalitions were ratified, indicating that the initial phase of negotiations strongly affected the final outcomes.

The inital phase of negotiations may be further divided into three parts that will be analyzed below: the initial orientation of the Base players, the relation between first offers and final outcomes, and the relations between responses to initial offers and final outcomes.

Base's orientation. On the first round of negotiations a Base player might attempt to either cooperate with the other Base players or form a coalition with the Apex player. A measure of initial orientation of the Base players is provided by the percentage of these players who addressed the Apex with an offer, or accepted an offer he made on the first round of negotiations. These measures, ranging between 0 and 100, were obtained for each group and subjected to a 2x2x2x2 analysis of variance (using again the multivariate approach) with repeated measures on factors $\underline{N}$ and $\underline{R}$. The analysis yielded a significant group size effect ($F=14.3$, $p<.01$). Whereas 82% of the Base players in the quintet games negotiated with the Apex on the first round of negotiations (in which each player could send only a single message), only 58% did so in the quartet games.

Another source of significant variation was the two-way interaction $\underline{N}\times\underline{O}$ ($F=10.5$, $p<.01$). When the Apex was the first player that could send a message (condition $\underline{O}_1$), the percentages of the Base players who negotiated with him were 75 and 77 for conditions $\underline{N}_1$ and $\underline{N}_2$, respectively. When the Apex was the last player to communicate (condition $\underline{O}_2$), the respective values were 41% and 85%.

The significant interaction is the same as the one found in the analysis of final outcomes. Since neither of the other main effects, $\underline{V}$, $\underline{R}$, or $\underline{O}$, nor any of the interactions were significant, the results point again to the size of the group and order of communication as the two critical variables in Apex games.

Initial offers. Tables 4 and 5 present the initial offers made in each game, the responses to them by Base or Apex on the first or second round of negotiations, and the final outcomes for conditions $\underline{N}_2$ and $\underline{N}_1$, respectively. For each group size results are presented separately for conditions $\underline{O}_1$ and $\underline{O}_2$. The initial offers and the final outcomes are stated as before in terms of x_β. The letters W and L indicate that the Base player addressed by the Apex in his first offer was a winner or loser in the game, respectively.

An inspection of the column "Offer by Apex" in both tables for condition $\underline{O}_1$ reveals strong run effect in Apex's initial offers; Apex's offer in the second run was never larger than in the first run. This finding strongly suggests that an Apex player in the second run learned that the Base coalition was unlikely to form, therefore demanding at least what another player in the Apex role had demanded (but not necessarily had accepted) in the first run

Tables 4 and 5 further show that the distribution of the Apex's initial offers to Base was bimodal, with the major mode falling close to H_1, and the minor mode falling close to K. The medians of the Apex's initial offers to Base were within one or two points of H_1, and the means were 4.9 and 2.4 points higher than H_1 for quintets and quartets, respectively. A second consistent trend emerged in condition $\underline{O}_1$ when Apex's initial offer to Base was compared to the final outcome. The winner Base's final outcome was never larger than his share in the Apex's initial offer. Stated differently, the bargaining that ensued in condition $\underline{O}_1$ lowered x_β in the direction of H_1. Moreover, the bargaining also reduced the variability around the mean final outcome.

Tables 4 and 5 show that the standard deviation of Apex's initial offers to Base was larger than that of the final outcomes.

Table 4

Initial Offers, Responses to Initial Offers, and Final Outcomes in Quintet Games

		O_1						O_2		
g	r	Offer by Apex	Base's re-sponse	W or L	Final x_β	g	r	Demand by Base(s)	Apex's response	Final x_β
1	1	22	c.o.l.	L	18	7	1	22,27	c.o.m.25	25
	2	18	ign.	L	18		2	18,26,36	acc. 18	18
2	1	32	acc.	L	19	8	1	19,20,37	acc. 19	*
	2	32	acc.	L	20		2	25,30,32	acc. 25	25
3	1	22	acc.	W	22	9	1	20,20,26,28	acc. 20	22
	2	20	c.o.l.	W	19		2	18,18,20,20	acc. 18	18
4	1	18	c.o.m.	L	18	10	1	18,20,20,22	acc. 22	22
	2	18	acc.	W	18		2	20,21,25	acc. 25	25
5	1	20	acc.	L	18	11	1	25,25,26,30	acc. 25	25
	2	**	-	-	18		2	19,23,24,25	acc. 19	19
6	1	30	acc.	W	18	12	1	19,28,30,54	acc. 19	19
	2	20	acc.	W	20		2	14,17,18,25	acc. 18	18
Mean		22.9			18.8	Mean		24.0		21.5
S.D.		5.6			1.3	S.D.		7.0		3.1

* Base coalition formed

** pass

Table 5

Initial Offers, Responses to Initial Offers, and Final Outcomes in Quartet Games

g	r	Offer by Apex	Base's response	W or L	Final x_β	g	r	Demand by Base(s)	Apex's response	Final x_β
1	1	28	c.o.l.	W	24	7	1	24	acc.	36
	2	20	acc.	W	20		2	24	acc.	*
2	1	36	acc.	L	25	8	1	-	off 25	*
	2	24	acc.	W	24		2	-	off 32	30
3	1	25	acc.	W	25	9	1	-	off 32	32
	2	25	acc.	W	25		2	15,24,30	acc. 15	32
4	1	18	acc.	W	18	10	1	30	off 20	28
	2	18	acc.	W	18		2	30	acc. 30	30
5	1	38	acc.	L	25	11	1	40	c.o.m.28	28
	2	**	-	-	25		2	20,25,25	acc. 20	28
6	1	30	acc.	L	22	12	1	21,24	acc. 24	24
	2	28	c.o.l.	L	24		2	20,25	acc. 25	25
Mean		26.4			22.9	Mean		25.1		29.3
S.D.		6.6			2.7	S.D.		5.8		3.5

* Base coalition formed

** pass

This effect is significant as tested by the ratio of the variance for quintets and quartets ($F=19.6$, $p<.01$, and $F=3.8$, $p<.01$, respectively).

The results of condition $\underline{O}_2$, presented on the right-hand sides of Tables 4 and 5, are less regular and different than those of condition $\underline{O}_1$. A comparison of the initial offers and final outcomes in Table 5 shows that the winner Base obtained on the average 29.3 points, 4.2 points more than his average initial demand. This difference between the two means was significant ($t=2.0$, $p<.05$). Quintet games, however, did not exhibit such a significant trend. This finding may be explained in terms of the $\underline{N}x\underline{O}$ interaction, which was significant in the previous analyses. Perhaps due to the relatively few initial offers made by Base to Apex in quartet games in condition $\underline{O}_2$, the Apex reduced his demand in order to avoid a formation of the Base coalition.

Responses to initial offers. When either a Base or an Apex player was made an offer he had to select exactly one of five possible ways of responding: (i) reject the offer (rej.), (ii) ignore the offer by either passing or, if the player was Base, addressing the Base coalition (ign.), (iii) counter-offer and demand more than initially offered, (c.o.m.), (iv) counter-offer and demand less than initially offered, (c.o.l.), or (v) accept the offer (acc.). An inspection of the column labeled "Base's response" of condition $\underline{O}_1$ in Tables 4 and 5 shows that the Base chose the latter two ways in 20 out of 22 games (16 acceptances and 4 counter-offers for less points). The two single cases of "resistance" to Apex led to the Base's elimination from the ratified coalition (games $G_{1,2,5}$ and $G_{4,1,5}$).

The particular payoff tentatively accepted by the Base is crucial for his chances to be a winner. The column "W or L" in Tables 4 and 5 indicates that a tentative agreement in the higher half of $\underset{\sim}{M}_1$ on the kernel's side led finally to the exclusion of the Base player from the ratified coalition. Note the agreements involving, for a Base player, 32 points in quintet games and 36, 38, and 30 points in quartet games. Recall that $\underset{\sim}{K}=36$ points for both quartets and quintets. On the other hand, Base players who accepted a payoff within four points of H_1 typically ended the game as winners.

It is instructive to describe two exceptions in which the Base players lost though they accepted initial offers located two points higher than H_1. In game $G_{1,1,5}$, the Base player counter-offered for less, 20 instead of 22, and the Apex accepted. Later in the game the Base did not agree to ratify the agreement, demanding 25 points instead of 20. He ended a loser. In game $G_{5,1,5}$, the original agreement which assigned 20 points for the Base player was disrupted by the Apex. However, in the ensuing negotiations the first Base player passed at a crucial moment after a disruption of another Base coalition, and later he reentered the Base coalition possibly set as a trap by another Base player. It seems that in both games the Base players who had been addressed initially by the Apex could have won if they had been loyal to the Apex through all phases of the negotiations.

The Bargaining Process

The two bargaining sets M_1 and H_1 can be represented as sets of solutions of conjunctive-disjunctive systems of linear inequalities involving the final outcomes as unknowns. The predictions derived from these models, which have been tested above, concern only the final outcomes of the negotiations among the n players. The models are mute with respect to the characterization of the bargaining process which leads to ratification of the tentative coalitions. But, clearly, players do not solve conjunctive-disjunctive systems of linear inequalities in order to form coalitions and disburse their values. Rather, a coalition is ratified and its value is disbursed among its members after a lengthy process of negotiations involving offers, counter-offers, acceptances, rejections, and passes, which reflect only in part the threats, counter-threats, promises, bluffs, and other negotiation steps actually considered by the players. From a psychological viewpoint, it is the bargaining process with all its intricacies rather than the final outcomes which is of primary interest. We attend to an analysis of it in the present section.

The bargaining process may be modelled in several different ways. The alternative chosen here has been motivated by the success of the bargaining set $\underset{\sim}{M}_1$ and its extreme point $\underset{\sim}{H}_1$ in accounting for the final outcomes of Apex games. If both models provide an adequate description of final outcomes in Apex games, and we are unwilling at this juncture to prefer one or the other, a model of the bargaining process should converge to either $\underset{\sim}{M}_1$ or $\underset{\sim}{H}_1$ as its final outcome.

Depending whether convergence to either $\underset{\sim}{M}_1$ or $\underset{\sim}{H}_1$ is sought, the two bargaining models described below amount to testable dynamic interpretations of the bargaining set model and the competitive bargaining set model, respectively. Both $\underset{\sim}{M}_1$ and $\underset{\sim}{H}_1$ can be described as p.c.'s in which every objection (appropriately defined for each of the two models) has a counter-objection (also appropriately defined for each case). If, for a given p.c., a player k can sustain a justified objection against player ℓ, a reasonable negotiation move might be for all players included in the objection to accept it. The resulting p.c. may be subjected to further negotiations. If the objection is unjustified, the bargaining continues.

Figure 1 diagrams the proposed structure of the bargaining process. On the first stage of the process a coalition (p.c.) is tentatively formed; it may be either an Apex or a Base coalition. The tentative coalition, denoted by $\underline{X}$ in Fig. 1, may either result from the initial negotiations among the n players or may follow the dissolution of a previous tentative coalition. Players are assumed to search for justified objections against $\underline{X}$. If none exists, the tentative coalition is eventually ratified. If at least one justified objection exists, an objection to the tentative coalition, denoted by $\underline{Y}_s$ in Fig. 1, is expressed through an offer (p.c.).

The objections involve only integral units and are made one at a time, as dictated by the rules of the game. The identity of the player expressing an objection is irrelevant. Thus, for example, if a tentative coalition X=AB is formed, an objection $\underline{Y}_s$=AC by A against B may be made by A through an offer to C, or by C

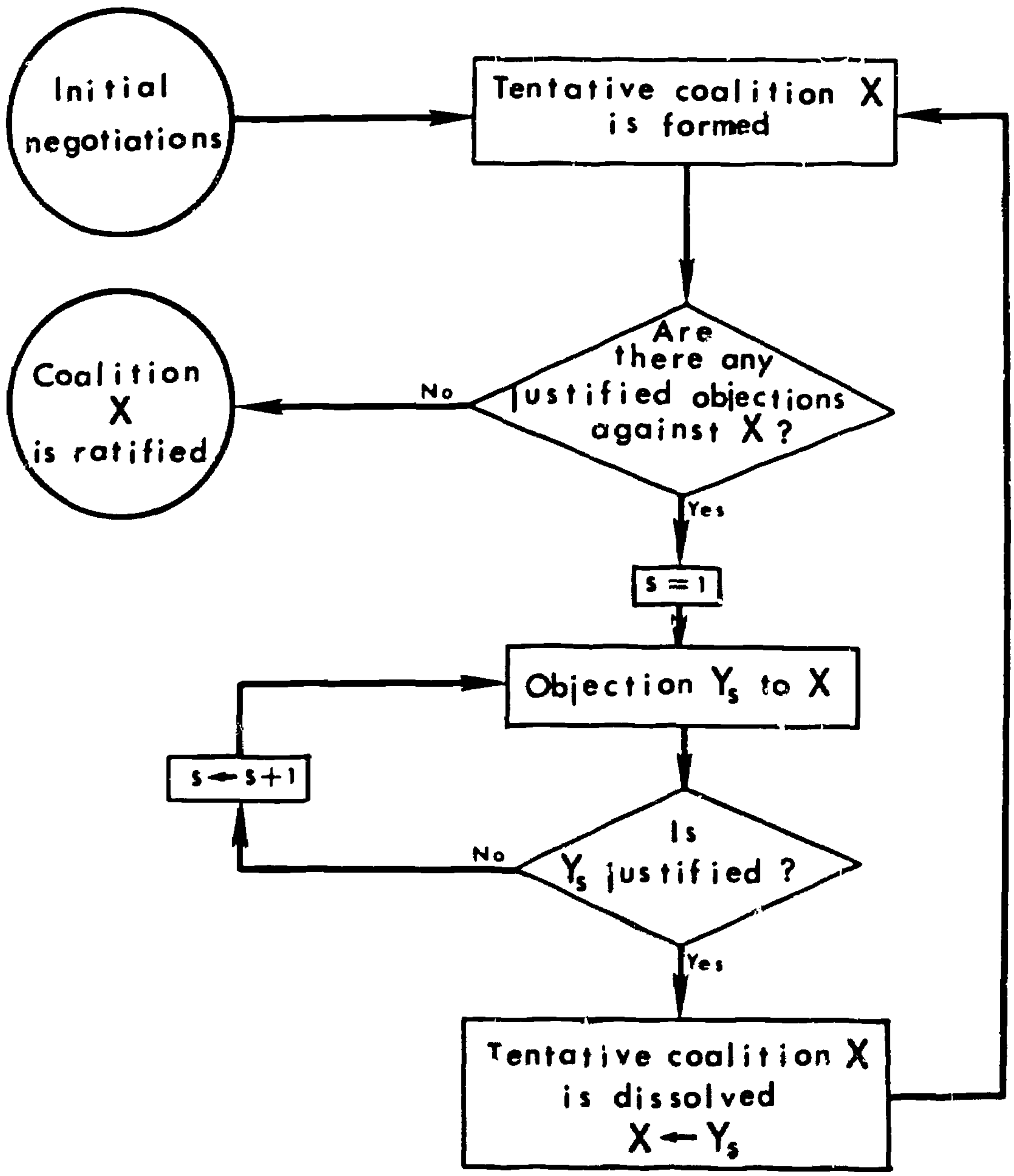

Figure 1. A proposed bargaining process converging to the bargaining sets $\underset{\sim}{M}_1$ or $\underset{\sim}{H}_1$.

through an offer to A. This requirement is compatible with the definition of objection (Aumann and Maschler, 1964).

If the objection $\underline{Y}_s$ is justified, the tentative coalition $\underline{X}$ is dissolved and the objection is accepted, thus resulting in a new tentative coalition ($\underline{X} \leftarrow \underline{Y}_s$ in Fig. 1). If, however, the objection is unjustified, the tentative coalition $\underline{X}$ is retained, and a new objection against $\underline{X}$, ($s \leftarrow s+1$ in Fig. 1) is made. If a player has only unjustified objections when it is his turn to play, his new objection is necessarily unjustified. However, it is assumed that at least one of the players possessing both justified and unjustified objections against $\underline{X}$ will eventually express a justified objection. The latter assumption, though admittedly strong, is required to insure convergence.

The proof for Apex games that the bargaining process described above converges to a p.c. in the appropriate bargaining set is too detailed to be presented here. Essentially, it is based on the idea of dividing all the p.c.'s in terms of x_α into five mutually exclusive and collectively exhaustive classes: (1) $0 \leq x_\alpha < c/(n-1)$, (2) $c/(n-1) \leq x_\alpha < c/2$, (3) $c/2 \leq x_\alpha \leq c(n-2)/(n-1)$, (4) $c(n-2)/(n-1) < x_\alpha \leq c$, (5) $x_\alpha = 0$, where c is the value of the Apex coalition. While the first four classes involve the Apex coalition only, the fifth assumes the formation of the Base coalition. The proof proceeds to show that the bargaining process described in Fig. 1, assuming $\underset{\sim}{M}_1$-justified or $\underset{\sim}{M}_1$-unjustified objections, converges in a finite number of stages to the two classes comprising the bargaining set $\underset{\sim}{M}_1$, that is, either class (3) or class (5) (with an equal split of the value of the Base coalition). The proof for $\underset{\sim}{H}_1$ is quite similar. Stearns (1967) proved convergence of an entirely different transfer scheme to either $\underset{\sim}{M}_1$ or $\underset{\sim}{K}$ for the general n-person game in characteristic function form.

To exemplify the bargaining process leading to M_1, consider a four-person Apex game defined for players A, B, C, D, where $v(AB)=v(AC)=v(AD)=v(BCD)=72$. Suppose that after a tentative Base coalition is formed with a p.c. (0,34,19,19;A,BCD), the Apex player A, offers 21 points to C. Since this offer is an $\underset{\sim}{M}_1$-justified objection by C against B ("Base against Base"), the

Base coalition is dissolved and the p.c. (51,0,21,0;AC,B,D) forms after C accepts A's offer. The second iteration continues, for example, by an offer from D of 62 points to A, which is an objection by A against C (i.e., "Apex against Base"). However, it is not M_1-justified because C can counter-object. Since the p.c. (51,0,21,0;AC,B,D) is not M_1-stable, the game proceeds. For example, it may continue with a "Base against Apex" objection by C against A (0,22,28,22;A,BCD) made by B. Since it is M_1-justified it becomes the new tentative coalition. Finally, for example, an objection by B against C, $(72-x_B, 24 \leq x_B \leq 27, 0, 0; AB, C, D)$ is both M_1-justified and stable, since C cannot counter-object. Any such objection should be ratified.

To describe the bargaining process leading to H_1, the conditions under which an offer is interpreted as a multi-threat should be specified. The bargaining process model assumes that an objection by player k against ℓ may be viewed as a multi-threat when other objections of k against ℓ are implied and not explicitly stated. The multi-threat is, therefore, a tacit threat accompanying the actual objection. Thus, consider again the previous example. Suppose that A and B agree on the p.c. (45,27,0,0;AB,C,D), which is H_1-unstable. Player C may object by offering 46 points to A, i.e., the p.c. (46,0,26,0;AC,B,D). This is an objection of the type "Apex against Base." The multi-threat is assumed to consist of the actual objection plus the implied (tacit) objection (46,0,0,26;AD,B,C). Since the multi-threat is H_1-justified, the bargaining process continues with the dissolution of the Apex coalition, AB, until, finally, an Apex coalition is formed in which A gets 48 points.

Any of the following three events disconfirms the two bargaining process models: (i) an unstable tentative coalition not followed at least once by a justified objection, (ii) a justified objection which is ignored, i.e., does not dissolve the tentative coalition to which it is addressed, and (iii) an unjustified objection which is accepted, resulting in a new tentative coalition. The protocols of the 48 games (which appear in Horowitz, 1971) show that each of these violations occurred. With regard to the first event mentioned above, analysis of the protocols shows that of 25 M_1-unstable tentative coalitions that were formed, 17 were followed at least once by M_1-justified objections but 8 were not. Of 77 H_1-unstable tentative coalitions that were formed, 59 were followed by H_1-justified objections, but 18 were not.

A frequency analysis of objections in Apex games warrants a distinction among three types of objections: Apex against Base, Base against Apex, and Base against Base. Clearly, when an Apex coalition is tentatively formed, Apex against Base or Base against Apex type of objections may be expressed. If a Base coalition is tentatively formed, only a Base against Base type of objection may be stated.

Regardless of the type of objection, and consistent with the assumptions of the bargaining process models leading to M_1 and H_1, two cases were distinguished in the classification of the objections presented in Table 6. The first is when players stated only unjustified objections, indicated by UJ in Table 6. The second case is when at least one of the stated objections was justified, indicated by J. In the former case, the unjustified objections were classified as either accepted or ignored, depending on whether one of them dissolved the tentative coalition or not. The same classification was maintained in the latter case, depending on whether one of the justified objections dissolved the tentative coalition or not.

The two bargaining process models leading to M_1 or to H_1 may be compared to each other only when the stated objection is of the type "Apex against Base." The two models yield the same predictions when a "Base against Apex" or "Base against Base" objection is stated. Table 6 shows that there were no cases where a player stated an M_1-justified "Apex against Base" objection. Of the 69 tentative coalitions for which all objections were M_1-unjustified, in 50 cases the objections were ignored. The results for model H_1 were even more impressive. Of the 27 tentative coalitions, in 25 cases the unjustified objections were ignored. However, of the 42 coalitions against which players expressed M_1-unjustified but H_1-justified objections, only 16 coalitions were

Table 6

Frequencies of tentative coalitions followed by unjustified objections only, or by at least one justified objection

Who against whom	Objection status		Objection accepted	Objection ignored
Apex against Base	M_1	Only UJ	19	50
		At least one J	0	0
	H_1	Only UJ	2	25
		At least one J	16	26
Base against Apex	M_1 & H_1	Only UJ	7	40
		At least one J	6	7
Base against Base	M_1 & H_1	Only UJ	9	2
		At least one J	4	0
Total	M_1	Only UJ	35	92
		At least one J	10	7
	H_1	Only UJ	18	67
		At least one J	26	33

dissolved. These 16 objections will be discussed in more detail below. The order of communication significant effect that was found above is reflected in the finding that 13 of these 16 cases occurred in condition $\underline{O}_1$ and only 3 in condition $\underline{O}_2$. And conversely, of the 26 objections that were ignored, thus discounting the bargaining process model leading to $\underset{\sim}{H}_1$, only 7 occurred in condition $\underline{O}_1$ and 19 in condition $\underline{O}_2$.

A frequency analysis of "Base against Apex" objections shows that of the 13 cases in which players expressed at least one justified objection, only six were accepted. Of the 47 tentative coalitions against which all the objections made were unjustified, the objections were ignored in 40 cases. There were seven cases where the Base dissolved the Apex coalition in favor of the Base coalition for an unequal apportionment of its value. In five of these seven cases the Base disruptor ended as a loser.

There were only 15 tentative coalitions followed by "Base against Base" objections, 13 of which were dissolved. Recalling that the Base coalition was ratified in only three of 48 games, the high percentage of accepted objections provides additional evidence to the instability of the Base coalition. The latter was dissolved by almost any objection, whether justified or not. It is worth noting the relationship between the "Base against Base" objections that were accepted and winning or losing the game. Six of the 9 Base players who dissolved the Base coalition through an unjustified objection ended as losers, whereas all 4 players who dissolved it through a justified objection ended as Base winners.

The frequencies of objections that were either accepted or ignored, summed over the three types, are presented in the lower part of Table 6. The frequencies are shown separately for the two bargaining process models. The null hypothesis of no interaction between the two factors of each table was rejected ($\chi^2=5.44$, $p<.02$, for model $\underset{\sim}{M}_1$, and $\chi^2=7.56$, $p<.01$, for model $\underset{\sim}{H}_1$). Inspection of the frequency tables shows that for both models, when a player had only unjustified objections, an objection was about three times more likely to be ignored than accepted. When he had both justified and unjustified objections, a justified objection was stated and accepted in only approximately half of the cases.

As stated above, there were 16 M_1-unjustified but H_1-justified "Apex against Base" objections dissolving the tentative coalitions, nine in condition N_2 and seven in condition N_1. These are portrayed in Fig. 2. The bottom axis in each of the two halves of the figure shows the payoff to the Base player in the coalition that was dissolved. The middle axis shows the Base's payoff in the dissolving objections, and the top axis displays the payoffs to Base in the ratified coalitions. Note that in some cases more dissolutions occurred between the middle and top levels. The general pattern of results displayed in Fig. 2 indicates that Apex's dissolving objections reduced the large range of payoffs to Base in the tentative Apex coalitons (practically the whole continuum of M_1) to a considerably smaller range around H_1. Three of four games in which objections resulted in Base payoffs outside M_1 ended in M_1 as a result of later disruptions.

Discussion

The final outcomes of the Apex games strongly support the bargaining set model; 45 of the 48 final outcomes were included in M_1. As noted earlier, for the characteristic functions presented in Table 1, M_1 comprises an interval of p.c.'s rather than a unique solution. One may hold the view that stronger predictions, constituting subsets of M_1, are not possible, since extra-game-theoretical considerations such as "standards of behavior" in groups of college students, the nature of the communication channels or the "bargaining abilities" of the players determine particular outcomes within M_1. Von Neumann and Morgenstern (1947) presented a similar argument in defending their "solution." But if one is dissatisfied with the multitude of solutions in M_1 wishing to achieve a higher level of predictability, this, presumably, being one of the reasons for developing other solution concepts such as the kernel (Davis and Maschler, 1965), the nucleolus (Schmeidler, 1969), and the competitive bargaining set (Horowitz, 1972), alternative models should be investigated.

Both the competitive bargaining set and the kernel models predict unique payoff vectors for Apex games, the extreme points of M_1. The present experiment was designed to make the range

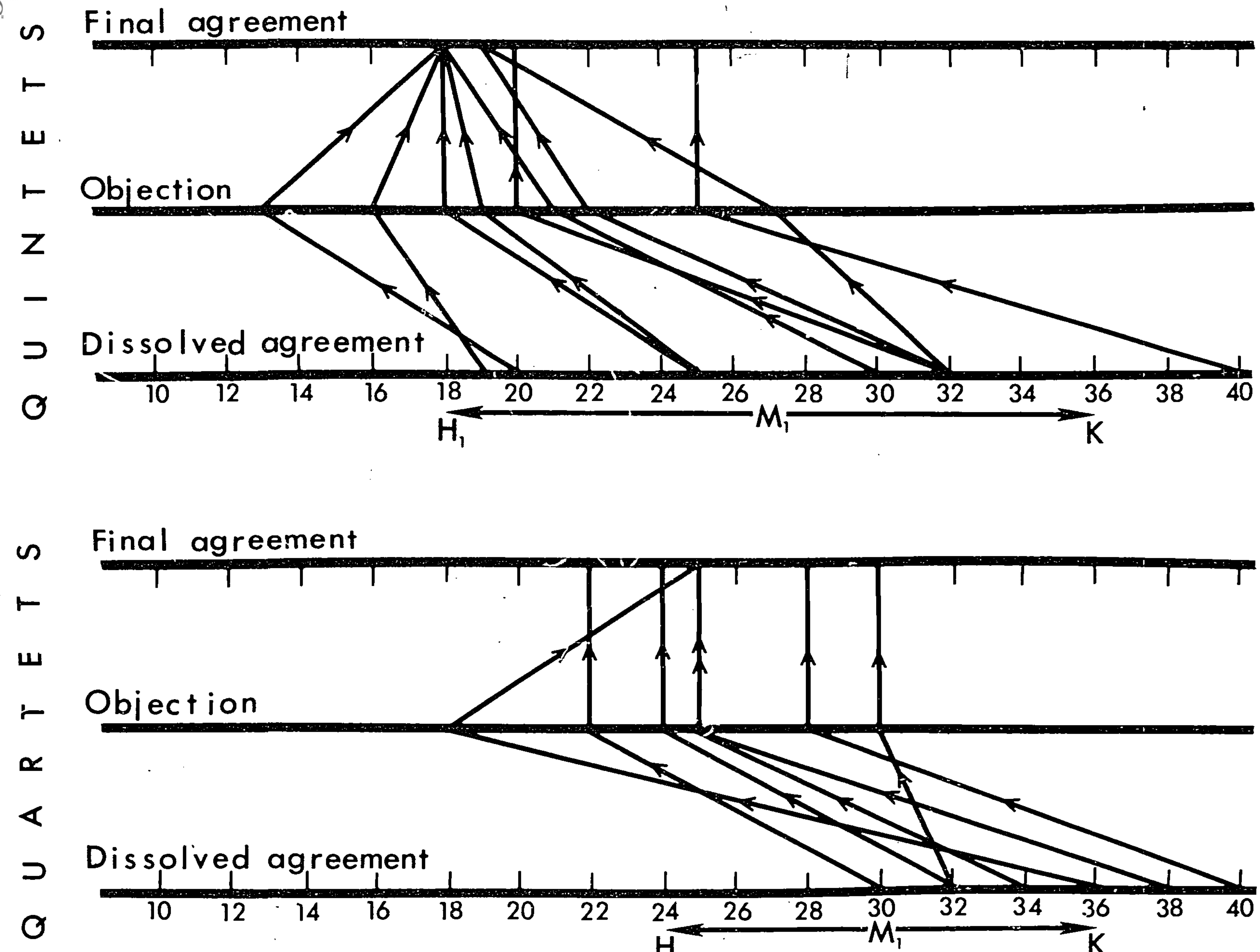

Figure 2. Dissolutions of tentative agreements as a result of $\underset{\sim}{M}_1$-unjustified but $\underset{\sim}{H}_1$-justified "Apex against Base" objections, in terms of Base's payoff.

between the two extreme points sufficiently large to allow powerful statistical tests of the models even when the number of groups is relatively small. The final outcomes, however, were unambiguous, requiring no sophisticated statistical analysis of the data. Fourteen of the 48 final outcomes were included in $\mathbf{H}_1$ in comparison to only a single outcome in $\mathbf{K}$. Moreover, the mean final outcome for Base was within 2.1 points from $\mathbf{H}_1$ for both group size conditions, whereas its distance from $\mathbf{K}$ was approximately five to eight times larger. The analysis of variance results, however, showed that the success of the competitive bargaining set in accounting for final outcomes depended upon the group size and order of communication. Whereas the final outcomes of the quartet games in condition O_1 and those of quintet games for both conditions of order of communication supported $\mathbf{H}_1$ relative to $\mathbf{K}$ and most of $\mathbf{M}_1$, the final outcomes of the quartet games in condition O_2 were approximately uniformly distributed within $\mathbf{M}_1$.

From the four factors that were manipulated in the experiment, the group size and order of communication emerged as the most prominent factors. Run effects were only noted when Apex's initial offers were analyzed, whereas the value of the Apex coalition significantly affected behavior in none of the analyses we have conducted. Factors N and O, beside significantly affecting the final outcomes, also affected the Base's bargaining behavior at the outset of the game. Since only two group size conditions were run, the results are presently not generalizable beyond n=5. They suggest, however, that the larger the n the less cohesive are the Base players on the first round of negotiations in Apex games. This hypothesis is testable in other experiments in which n>5. The results also suggest that when n is small, cohesion among the Base players increases if, rather than letting the Apex player attempt to form an Apex coalition, the communication rules present the Base players with the opportunity to briefly negotiate with one another before the Apex's intervention.

The importance of group size and order of communication is supported by two additional statistical tests, unrelated to the final outcomes. Two 2x2x2x2 analyses of variance, employing a multivariate approach as before, were conducted on (i) the total number of messages sent during each game, and (ii) the number of tentative coalitions formed in each game. The only significant effects in both tests were again attributable to $\underline{N}$, $\underline{O}$, or their interaction.

The following picture emerges, then, regarding the bargaining behavior of the players. If the Apex player communicates first when n=4, he controls the game during the first phase of the experiment, making negotiations among the Base players very unlikely. But if the Apex communicates last, after the Base players, the latter are more likely to communicate with one another, thus exerting moderate pressure on the Apex, who, in turn, responds by initiating or accepting a less favorable share to insure the formation of the Apex coalition. Since almost all the first tentative coalitions were formed on the first two rounds of negotiations, and more than half of them were later ratified, presumably because the penalty for dissolving a tentative coalition was high, Apex's mean final outcome did not differ significantly from his mean demand in condition $\underline{O}_1$, but was significantly smaller in condition $\underline{O}_2$. Increasing the number of Base players from three to four almost completely prevented cooperation among the Base players, thus decreasing the likelihood of the formation of the Base coalition. Hence, when n=5, the order of communication did not affect the initial orientation of the players and, consequently, the final outcomes.

With respect to Base's behavior, the following very simple policy seems to enhance his chances to win. When Apex's initial offer to him is high relative to what he could obtain from a symmetric apportionment of the value of the Base coalition, the Base should counter-offer demanding less for himself. Otherwise, he should accept Apex's offer immediately, and remain in coalition with the Apex until ratification. The analysis of the bargaining process showed that most of the Base players who dissolved a tentative coalition with an unjustified objection ended as losers. Hence, Base's best policy is to adhere to the prescriptions of the two bargaining process models, which are the same for his bargaining behavior.

The only data directly reflecting the bargaining process consisted of the messages typed and transmitted by the subjects. Such data provide only occassional glimpses of the bargaining process, partly because the experimental design allowed the players to communicate with only a small set of legal messages, and, more importantly, because the threats, counter-threats, promises, and other, more subtle, negotiation moves that the players might have considered, could not be reflected in the messages they sent. Additional information about the bargaining process may, perhaps, be obtained by requiring the players during the game to state the reasons for their moves and explain in as much detail as possible their thought processes. The talking aloud procedure, which has been proved useful in some problem solving studies, may provide equally fruitful information in bargaining studies.

Notwithstanding the limitations of the analysis, the results supported the two bargaining process models. In particular, the analysis showed that whether a tentative coalition was dissolved by an objection depended on whether the objection was justified. Additionally, this dependence was affected by the type of the objection. Both models were supported when the objector possessed only unjustified objections, unless the objection was of the type "Base against Base." The support given to the bargaining process leading to $\underset{\sim}{H}_1$ is particularly impressive since, it may be recalled, $\underset{\sim}{H}_1$ consists of only a single p.c..

Both models of the bargaining process suffer from several deficiencies. The first weakness concerns the proposed test which, if answered negatively, leads to ratification (see Fig. 1). To perform this test, the players are supposed to search for justified objections against $\underline{X}$. Although the set of objections they are assumed to consider is finite, it may be very large, making an effective search unfeasible. To allow for an effective search, the set of objections considered by the n players should be restricted. A second, more serious weakness, is that the "distance" of $(\vec{x};\underset{\sim}{X})$ from $\underset{\sim}{M}_1$ (or $\underset{\sim}{H}_1$) does not affect its dissolution by a justified objection. A more reasonable model would require the dissolution of the tentative coalition $\underline{X}$ by a justified objection

$\underset{\sim}{Y}$ to be probabilistically determined with the probability of dissolution increasing monotonically as the "distance" of $(\vec{x};\underset{\sim}{X})$ from $\underset{\sim}{M}_1$ (or $\underset{\sim}{H}_1$) increases.

The results of the present experiment may be compared to results obtained in two other experiments that employed Apex games.

Maschler (1965) employed two Apex games among several three- and four-person games in characteristic function form played by Israeli high school students. The first, Game I, was defined by the characteristic function v(AB)=v(AC)=v(AD)=v(BC)=v(BD)=v(CD)=50, v(BCD)=111, and v(G)=0 for any other coalition G. The second, Game II, was like Game I, with the only difference being that v(BCD)=120. Each of the two games was played once by each of five different quartets of players. As in the present study, the Base coalition, BCD, was formed in only one of 10 cases. In eight of the nine cases that the Apex coalition was formed, the final outcome was included in $\underset{\sim}{M}_1$ but never in $\underset{\sim}{K}$ or $\underset{\sim}{H}_1$. The predicted payoffs for Game I in $\underset{\sim}{M}_1$, $\underset{\sim}{K}$, and $\underset{\sim}{H}_1$ for the Base winner are $37 \leq x_\beta \leq 43$, $x_\beta = 43$, and $x_\beta = 37$, respectively. The respective predictions for Game II are $40 \leq x_\beta \leq 47\frac{1}{2}$, $x_\beta = 47\frac{1}{2}$, and $x_\beta = 40$. The final outcomes for the Base winner were 38, 40, 40, 40, and 45 for Game I, and 41, 41, 45, and 45 for Game II.

In an experiment conducted by Selten and Schuster (1968), 12 groups of five subjects each played a five-person game with the characteristic function v(AB)=v(AC)=v(AD)=v(AE)=v(ABC)=v(ABD)= v(ABE)=v(ACD)=v(ACE)=v(ADE)=v(ABCD)=v(ABCE)=v(ABDE)=v(ACDE)= v(ABCDE)=v(BCDE)=DM40, where DM40 equals approximately $10. The Base coalition was formed in two of 12 cases. A two-person Apex coalition was formed in eight of 12 cases, yielding the Base winner the payoffs 12, 15, 15, 15, 15, 15, 20, and 22. The predictions of $\underset{\sim}{M}_1$, $\underset{\sim}{K}$, and $\underset{\sim}{H}_1$ are $10 \leq x_\beta \leq 20$, $x_\beta = 20$, and $x_\beta = 10$, respectively. The median payoff, 15, fell in the middle of $\underset{\sim}{M}_1$. In one game a coalition among the Apex and three Base players was formed, and in another game coalition ABCDE was formed for an equal split of its value.

All three experiments were designed to test the bargaining set and kernel models. The final outcomes clearly support the former model and reject the latter. Another common finding is the rarity of the Base coalition. This finding is of interest, because it seems to refute the widespread conviction, as well as the predictions of some social psychological theories, that the weak Base players are likely to unite against the strong Apex player instead of the other way around.

With regard to a comparison between $\mathbf{M}_1$ and its extreme point $\mathbf{H}_1$, the results of the three studies are less consistent. Whereas most of the final outcomes in the present study were distributed in one half of $\mathbf{M}_1$, with an average very close to $\mathbf{H}_1$, the final outcomes of the quartet games in condition O_2 and those of the experiments by Maschler (1965) and Selten and Schuster (1968) were distributed over the entire set of solutions comprising $\mathbf{M}_1$, with an average very close to its middle. Because of the differences among the studies in the values of the characteristic functions, the experimental designs, and the nationality and the age of the subjects, the discrepancy between the final outcomes may be attributed to a variety of factors. In particular, the discrepancy between the central tendencies of the final outcomes of the three studies in comparison to $\mathbf{M}_1$ may be very likely attributed to the difference among the studies in the form of communication. Whereas Maschler as well as Selten and Schuster allowed free, face to face negotiation, the present experiment limited the communication to a small preselected set of formal messages, without allowing the players to see or hear one another or even know the identity of the other players. Face to face contact allows the communication of intentions, gestures, and emotions, probably enhancing the salience of social norms of equity. It is a reasonable conjecture that allowing direct negotiations in experimental n-person games in characteristic function form will result in a more egalitarian apportionment of the final outcomes.

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