DOCUMENT RESUME

ED 081 643 SE 016 814

| | |
|---|---|
| AUTHOR | Springer, George |
| TITLE | Report of the Conference on the K-12 Mathematics Curriculum, Snowmass, Colorado, June 21-June 24, 1973. |
| INSTITUTION | Indiana Univ., Bloomington. Mathematics Education Development Center. |
| SPONS AGENCY | National Science Foundation, Washington, D.C. |
| PUB DATE | Jun 73 |
| NOTE | 68p. |
| EDRS PRICE | MF-$0.65 HC-$3.29 |
| DESCRIPTORS | Computers; *Conference Reports; Cooperative Education; *Curriculum; Educational Planning; Evaluation Methods; Mathematical Applications; *Mathematics Education; Problem Solving; Statistics; *Teacher Education |
| IDENTIFIERS | National Science Foundation |

ABSTRACT

Fifty-five participants, representing a wide variety of professional backgrounds and views, met to generate ideas for means of improving the teaching of mathematics in the schools. Topics upon which reports were given include: (1) Goals for Mathematics Curriculums; (2) Teacher Education; (3) Problem Solving; (4) Applications; (5) Statistics; (6) Computers; and (7) Evaluation and Testing. Given under these topics are detailed suggestions for implementing proposals encompassing improved cooperation between the mathematics education community in the university and that in the schools, the examination of societal needs and the delineation of the goals of mathematics education to provide a basis for curriculum development, support for promising innovative pre- and in-service teacher training, implementation of basic research findings into the curricula for teacher education and for school students, preparation of topics with significant application of mathematics suitable for K-12 for use by teachers and authors, instruction in statistics at all grade levels, computer literacy as one of the objectives of mathematics education, and new directions and techniques to assess programs and student performance. (JP)

FILMED FROM BEST AVAILABLE COPY





REPORT OF THE

CONFERENCE ON THE K-12 MATHEMATICS CURRICULUM

SNOWMASS, COLORADO

June 21 - June 24, 1973

Supported by the

National Science Foundation

Washington, D.C.

Under the Direction of the

Mathematics Education Development Center

329 S. Highland

Indiana University

Bloomington, Indiana 47401

Dr. George Springer, Conference Director

REPORT OF THE

CONFERENCE ON THE K-12 MATHEMATICS CURRICULUM

SNOWMASS, COLORADO

June 21 - June 24, 1973

Supported by the

National Science Foundation

Washington, D.C.

Under the Direction of the

Mathematics Education Development Center

329 S. Highland

Indiana University

Bloomington, Indiana 47401

Dr. George Springer, Conference Director

ABSTRACT

The Conference on the K-12 Mathematics Curriculum was held at Snowmass, Colorado, from June 21 to June 24, 1973. Fifty-five participants, representing a wide variety of professional backgrounds and views, met to generate ideas for means of improving the teaching of mathematics in the schools. In spite of the diversity of views of the participants, there were some central themes that emerged from the discussions. Some of these are listed below--the detailed suggestions for implementing each proposal are given in the chapter indicated in parentheses.

1. There is a substantial lack of trust and communication between the mathematics education community in the universities and that in the schools. Efforts need to be instigated to re-establish cooperation. (Chapter I)

2. The public (including a great number of teachers of mathematics) continue to think of mathematics as a set of arithmetic operations (skills). While there skills are important, they constitute only a part of mathematical instruction. Efforts to correct this view must be made before any significant changes in curriculum can be effected. At the present time, there seems to be no clear consensus with regard to the mathematics which should be taught in K-12 and there is an urgent need for a program which will examine societal needs and delineate the goals of mathematics education with sufficient authority to provide a broadly acceptable base for curriculum development. (Chapter II)

3. Efforts toward developing innovative pre- and in-service professional programs for teachers which promise improvements are of utmost importance and should be supported. (Chapter III)

4. Basic research into the nature and meaning of problem solving and mathematical thinking is urgently needed. The findings of such research should coincidentally find their way into the curricula for teacher education and for school students. (Chapter IV)

5. A source of significant applications of mathematics suitable for grades K-12 should be prepared for use by teachers and authors. Such applications, including some open-ended, interdisciplinary situations with quantitative components, should become an important aspect of the curriculum. (Chapter V)

6. Instruction in statistics should be included in all grade levels. (Chapter VI)

7. The computer will make significant changes in teaching and learning. The role of computers in mathematics education (and education in general) must continue to be studied and developed. Computer literacy should be part of every student's education. (Chapter VII)

8. The current practices in evaluation of programs and of the mathematical learning by students are insufficient. New directions and techniques must be generated to assess programs and student performances. (Chapter VIII)

CHAPTER I

# INTRODUCTION

## 1. Pressures for Change

The period following World War II brought with it a wide recognition of the power of mathematics in our new technological age and a realization that our schools were not producing students adequately prepared to meet the needs of science and technology. The curriculum projects of the fifties and sixties concentrated on the mathematical content which would give the high school graduate a deeper understanding of the mathematics he would need in pursuing a professional career. The shifting priorities of the early seventies brought with them a deemphasis on science and technology, a new focus on the problems of society and the environment, and severe limitations on the financial resources available to education.

After two decades of change to what has been called the "New Math", and with changing societal needs, it is only natural to take a critical look at what is going on in the schools to identify shortcomings and to build on strengths of the various mathematics programs. Some discontent has been voiced that a proper balance has still not been achieved in the mathematics curriculum between abstract concepts and arithmetic skills, between mathematical content and applications, and between the needs of the college-bound students and the general student population.

In recent years, there have been interesting developments in learning theory and experimental projects in the teaching of mathematics and science which point toward new directions for the mathematics curriculum; for example, use of computers, use of minipulative materials, development of math labs, use of real-world projects, and the development of approaches more appropriate to the child. On the other hand, there are developments taking place which are widely viewed with alarm by the teaching establishment; for example, the misuse of behavioral objectives, accountability, and the limitation of the curriculum to computational skill at the expense of meaningful understanding through drill approaches, however disguised.

2. The Snowmass Conference

In order to assess the present needs of the country in the teaching of mathematics at the pre-college levels, the National Science Foundation sponsored a Conference on the K-12 Mathematics Curriculum at the Crestwood Lodge in Snowmass, Colorado from June 21 to June 24, 1973. Arrangements for the Conference were made through the Mathematics Education Development Center at Indiana University, Bloomington, Indiana. Participants were selected to include representation not only from mathematicians, mathematics educators, and school personnel, but also from the fields of computer science, statistics, and physical, biological, and social sciences who had experience in curriculum work. Their mission was to identify the current problems in school mathematics, to look for their causes, and to propose actions that can alleviate them.

3. Proposed Topics

In preparation for the Conference, the participants received a list of general topics which could form a basis for discussion at the Conference. These included:

(1) The population whose needs a new curriculum should be designed to meet,

(2) The need for teaching the process of problem solving,

(3) The need for meaningful applications of mathematics in the curriculum,

(4) The emerging roles of computers and calculators,

(5) The scope, organization and sequence of the mathematics curriculum,

(6) The implications of learning theories for curriculum efforts,

(7) The important problem of teacher education.

Participants added the following topics to the original list:

(8) The question of evaluation and testing,

(9) Communications and documentation,

(10) The needs of society and the general goals for the curriculum.

Many of the participants responded in writing to some of these topics and excerpts from their answers are incorporated in this report of the Conference.

4. Operating Procedure

To function more effectively in considering the topics formulated above, the participants were divided into groups of eleven members. The following three questions were suggested to form the basis of their early discussions:

A. What are the problems in elementary and secondary education today? What are the symptoms that suggest a need for change?

B. What are the causes underlying these problems in the schools? Do the following contribute to the trouble: poor content, content inappropriately sequenced, lack of understanding of how children learn, inappropriate teaching strategies, inability of the teachers, wrong philosophical basis for the curriculum? What others can be identified?

C. What should be the goals of school mathematics? (Why are we teaching mathematics in the schools?) How can mathematics be taught to meet societal needs (Practical, cultural, and transferable)? What are the different societal groups with respect to mathematics instruction and what measures must be undertaken to meet their needs?

The groups were to gradually focus their discussions on a number of measures which they thought would improve the teaching of mathematics in the schools. After sufficient discussion of each of their focal topics, each group split into writing subgroups to record their ideas. The final plenary session of the Conference was devoted to a discussion of the reports of the five groups.

5. Group Reports

Although formal votes were not taken at the final plenary session, many of the reports received favorable reactions from a majority of the participants while others only represented the opinions of the specific working groups or perhaps only a small portion of a group (even as small as one person, in cases when these opinions were carefully thought out and formulated over a period of time). Thus the specific ideas and recommendations which are presented in this report are not to be viewed as the opinions of the Snowmass Conference as a whole but as a source of ideas generated at the Conference.

The topics on which the five groups reported fall into the seven categories which are listed below by Chapter Number in this report:

II. Goals for the Mathematics Curriculum.
III. Teacher Education.
IV. Problem Solving
V. Applications.
VI. Statistics.
VII. Computers.
VIII. Evaluation and Testing.

Since several groups may have reported on the same topic, each of the seven sections contains a synthesis of the various opinions expressed by the different groups, indicating points of disagreement where they exist.

6. Limitations

It was clear to the participants at the Snowmass Conference that they could not settle any fundamental questions about mathematics education in a four-day meeting. For some of these questions, a smaller working group would have to be assembled for a longer period of time. This did not preclude some discussion of a number of these basic questions by the working groups, although no specific results are recorded. One such topic is the preparation of a comprehensive list of goals for school mathematics which meet the needs of society; another is the preparation of a list of the specific mathematical topics and the related pedagogy in the curriculum. Appropriate references are made in the reports to the need for deeper study into certain specific questions.

The Conference mainly focused on the middle 60 percent of the children in school. This does not imply that the future professional user of mathematics is to be ignored, for without continuing effort for improving the upper end, the nation's research and development effort will suffer. Our most important export, many have claimed, is our technological achievements, and technology will play an even greater role in the future. It is now proper to focus on the middle 60 percent while continuing improvement efforts for the upper 20 percent.

There was a concern expressed that schools are no longer looking toward mathematicans and mathematics educators at universities for assistance in their curriculum problems. It was generally felt that efforts must be made to re-establish communication and cooperation between the university community and the school community, and regain the trust that formerly existed.

7. <u>Presentations</u>

Several of the participants made brief presentations to the whole Conference on the opening day. George Springer, the Conference director, welcomed the participants and Lawrence O. Binder set forth the aims of the Conference. John F. LeBlanc presented a brief survey of past curriculum efforts. Morris Kline spoke on the principles of desirable curriculum reform, George Immerzeel on directions for continuing progress in curriculum development, Gail Young on problems of curriculum development in the seventies, and Tom Dwyer and Sylvia Charp on computers in the classroom.

The following individuals offered to make presentations to the Conference participants:

Earle Lomon: Learning through investigation and action on real-world problems.

Ruth Hoffman: Mathematics Laboratories, a slide-tape presentation.

Jay Anderson: A film on Piaget-type interviews of two children.

Seymour Pappert: Directions for curriculum reform and a film on children using computers.

Tom Dwyer: Slides and films on computers in mathematics laboratories and a multimedia presentation of children using computers.

Vivian Howard: Film on children learning mathematics.

Jean Danvers: Video tape of children learning with computers

Further information about these presentations can be obtained directly from the persons who made the presentations.

The summaries of the reports of the working groups now follow in Chapters II to VIII.

# CHAPTER II

## GOALS AND OBJECTIVES FOR THE MATHEMATICS CURRICULUM

The broad and specific goals of instruction in a discipline for given populations of students must be re-examined periodically if a curriculum is to continue to meet the needs of those it serves and remain dynamic and functional. Although conference participants did not attempt to delineate the role of mathematics in the curriculum nor the goals of mathematics education for all students, such a delineation was recognized as a most significant task requiring careful consideration at the present time. Throughout the Conference, discussions continually focused on the goals of instruction for general education, as compared to goals for those who will use mathematics in post secondary studies. There was a clear consensus that future curriculum efforts must be directed primarily toward the wide population of students who may not use mathematics professionally.

Two documents calling for action in this area were prepared at the conference. The reports have a common thrust, yet contain some differences in emphasis and particulars. Both reports are reproduced so that the reader may follow the carefully reasoned arguments on which each is based.

Report A:

1. Introduction

Successful technological development and change requires two essential ingredients: (1) an adequate conceptual base upon which development may build; and, (2) the acceptance and support of society. Thus we recommend that mathematics education in the 1970's have two major concerns: (1) over the short term, activities directed toward felt needs by schools which have or could easily gain broad based societal support and (2) over the long term, more attention to determining current and long term needs and priorities and to establishing new conceptual foundations upon which future developments can be built.

2. A Proposal for the Short Term

Many current suggestions for key ideas in the K-12 mathematics curriculum involve concepts with which teachers have had little or no previous experience. When teachers teach these ideas, often they think that the idea is not mathematics (e.g., using a calculator to do arithmetic) or they feel guilty about taking time to teach some mathematics (e.g., geometry in elementary school) when there is other mathematics upon which their students will be tested. Some ideas which fall into one or both of these categories are:

---problem-identification and problem-solving strategies
--real applications and modeling
--work in a laboratory or with manipulatives

--history of mathematics

--symbolization of non-numerical ideas

--topological concepts

--probability concepts

--stochastic processes and other statistical concepts

--computer ideas and usage

This list is not meant to be exhaustive. Rather it is meant to convey the point that the teaching of many of the new ideas proposed for the curriculum is considered by many teachers and students to be illegal (in the sense of being against the rules or not appropriate for class time). The narrowness of some present curricula is exemplified by most lists of behavioral objectives and standardized tests.

On the other hand, many are questioning the value of topics and emphases now in the curriculum. Some of these have been in the curriculum for decades; others were introduced in the 60's. Among these are:

--rigor in the elementary school

--use of contrived word problems in algebra

--heavy emphasis on proof in geometry

--drill and memorization

--groups and other structures

--mathematical games

--emphasis on sets and properties

Of course, there is no <u>absolute</u> basis upon which one can judge whether an idea is suitable for the curriculum or not. But at present there is not a nationally recognized set of opinions by which one can judge a proposal for curriculum modification. The last widely publicized set of desirable changes was given in 1959 by the Commission of Mathematics of CEEB. It is now not even clear whether the proposals in that report are reasonable for the average student and given the social events and school changes in the past decade, one would not want to base curricular decisions upon that report.

Thus, in light of the established utility of mathematics and mathematical thinking in the physical sciences and engineering, the growing mathematization of the social and life sciences, and the increasing complexity of our technological society, in light of the present malaise and floundering in the curriculum, and in the light of the length of time since national guidance has been given, it is vital to reconsider the priorities for selection of the student's mathematical experiences in grades K-12.

PROPOSAL 1: We propose the establishment of a committee of the highest quality to consider the question: What mathematics should be taught to students in grades K-12?

TASKS OF THIS COMMITTEE: The first task of the committee would be the consideration of the questions, "What is mathematics?" and "What constitutes mathematical thinking?" This is necessary to delineate the boundaries of what the committee believes to be its charge. Also, it is clear that many of the ideas being proposed for the curriculum will require teachers at all levels to expand their notions regarding the teaching activities and content that are to be thought of as reasonable, important, and viable mathematics. One avenue of attack would be to identify those things that are within the realm of mathematics, mathematical thinking, and applications.

Second, the committee should consider the question: "what are the desired outcomes and experiences for the average student of school mathematics?" This would require consideration and identification of priorities.

Third, the committee should turn attention to the interpretation of its recommendations concerning desired outcomes and experiences, considering their suitability for students of different interests, abilities, and social needs. The committee might want to place objectives by grade level or levels.

Fourth, the committee should consider those recommendations which involve new ideas and give suggestions for teacher-education and implementations, including specification of necessary materials and equipment.

BREADTH OF SELECTION OF AND INPUT INTO THE COMMITTEE: The committee must be selected to maximize expertise in its deliverations and the impact of its suggestions. Input from all the groups like those listed below should be considered in the deliberations;

NCTM (for classroom teachers and teacher educators)

ASA (for statisticians)

MAA, AMS (for mathematicians)

ACM, AEDS, AFIPS (for computer associations)

AERA Special Interest Group on Mathematics Education

ASCD (For general curriculum people)

Business and industry

Scientists

social scientists

school administrators

publishers and manufacturers of educational products

test developers

the public

REPORT OF THE COMMITTEE: The report of the committee should be disseminated in two forms: A complete report of the rationale and sequence of the proposed curriculum. A summary report of the specific findings and recommendations for curriculum revision. These reports should be distributed to professional and lay groups, to the media and to the general public in a manner which insures maximal effectiveness of the committee's work. Education of the general public should be an important consideration.

3. A Proposal for the Long Term

In 1958, the Committee on the Undergraduate Program in Mathematics was formed to monitor curriculum development at the college level. During the next decade, operating through a variety of panels, CUPM initiated and supervised extensive revision of the undergraduate course programs in the colleges. It seems to us that now is the appropriate time to establish a National Advisory Board which would function at grades K-12 as CUPM did at the undergraduate level in college.

Since the 1950's a wide variety of topics and approaches has been tried in the mathematics curriculum and there exists a large collection of data and experience. But no qualified body has attempted to synthesize or analyze these developments and data. The National Advisory Board would serve this purpose.

PROPOSAL 2. We propose the establishment of a National Advisory Board in Mathematics Education, consisting of individuals with broad perspective in regard to mathematics, mathematics education, behavioral and education science, and public need.

Tasks of the Advisory Board. The general charge to this board is to examine, integrate, and report on the progress, support, and acceptance of key aspects of research and development in mathematics education, to stimulate and recommend to appropriate organizations and funding agencies the establishment of special panels to work on specific problems in mathematics education, and to strengthen communication with the public concerning the purposes and needs of mathematics education.

The advisory group should be large enough to provide for broad representation. Specific organizations and sources to be considered, among others, in selecting members are listed under Proposal 1.

Broad dissemination of reports and recommendations of the Advisory Group should be given in order that comment, criticism, and debate can occur.

Report B:

The objectives of the secondary school mathematics programs that were developed during the previous decade were focused primarily on the college bound student who would use mathematics in his post secondary studies. Now we in mathematics education need to consider carefully the objectives of secondary school mathematics for general education, for the preparation of knowledgeable voters, and for successful participation in our society. To do so, we need a detailed statement of these objectives. There are a number of crucial issues now facing us which could be attacked more easily if we had such a statement of objectives phrased more broadly than narrowly stated behavioral objectives and yet more specifically than general nostrums such as: preparation for daily living.

Thus, there is today much concern over the use of detailed behavioral objectives at the secondary school level, and over the nature and use of current tests. These questions cannot easily be studied without a clear understanding of what the mathematics program is trying to do. Debates over the content of a curriculum for general education in mathematics would be more sharply focused if such a statement were available. Justifying such a curriculum, and explaining to the public the role and importance of mathematics in our society, would both be facilitated by such a statement.

The possibility of greatly increased use of CAI and other technological aids makes it imperative that we be able to separate what is to be taught from how it is to be taught. There is also the important question of how to describe what is to be taught. Listing topics, for example, has important limitations insofar as specifying what one wants the student to learn.

For these reasons, <u>we recommend support of an effort to prepare a statement of the objectives for general education of mathematics in the secondary school</u>. We emphasize that we are not concerned here with the more formal and technical courses elected by only a proper subset of the students.

We wish to be quite explicit about certain aspects of this proposed statement of objectives.

A. The list of objectives should be comprehensive. It should include all objectives which seem important, more than any school might wish to adopt. It should not appear to be a prescriptive list.

B. Each objective should be stated as clearly and explicitly as possible. Illustrative test items and other instruments for testing competence in the objective should be provided for further clarification.

C. For each objective the cognitive levels of skills, understandings, and applications, as appropriate, should be stated.

D. Affective objectives should not be neglected. We may not know how to teach good attitudes, but we need to avoid developing bad ones.

E. A distinction should be made between those objectives which are intrinsically important and those which we adopt just because we believe they may be prerequisite to others. Research may eventually allow us to discard some of the latter.

F. The list should not be thought of as definitive. Plans should be made to review it from time to time.

G. Compilation of the list need not start from scratch. In addition to the objectives enbodied in the SMSG "Second Round", there are many lists prepared by state and local educational systems that would be well worth reviewing. A few new topics which might be considered are:

1) computer literacy
2) probability and statistics
3) basic ideas, in finite difference form, of differential and integral calculus
4) estimation and approximation
5) linear algebra

We suggest that the preparation of this list of objectives be carried out by a small task force of about a dozen individuals, including mathematics educators, mathematicians, and mathematically literate individuals from other fields. Since the list of objectives is intended to be comprehensive rather than prescriptive, and since it will be used only for purposes of discussion and debate, it does not seem necessary that each segment of the mathematical community be physically represented on the task force.

We suggest that the task force be allowed to meet several times, calling in consultants when necessary, that an initial draft of its report be reviewed by a wide variety of individuals interested in mathematics education, and that time be provided for a revision, based on feedback from the review.

# Chapter III

## TEACHER EDUCATION

The classroom teacher does, and will in the forseeable future, play a significant role in the learning experiences of children. Appropriate teacher education programs in mathematics, both pre-service and in-service, is essential in order to properly affect the future changes that will take place in school mathematics. On the whole current teacher training practices seem not to be producing teachers who can effectively deal with contemporary issues and developments in mathematics education. Thus there is a dire need to develop more satisfactory ways of preparing teachers and efforts to find effective, innovative approaches should be supported.

The discussion on teacher education at the Conference was far reaching, touched every working group and generated a number of ideas. The following discussion emphasizes some general areas of agreement. Where specifics are provided, they are intended as illustrations only. The discussion begins with four brief statements which are followed by amplification and explication.

1. Funding agencies should encourage investigation and development of innovative approaches to better prepare teachers to teach current mathematics curricula.

2. In reforming teacher education, it is necessary to keep the focus on children and how they learn.

3. There should be long-range support of basic research into educating teachers to "think mathematically."

4. There should be on-going support of change mechanisms for mathematics teaching in the schools.

1. Pre-Service Education

Three aspects of teacher education in mathematics can be identified: content, methods, and experience with students. There was a strong feeling that considerably more emphasis should be given to early (pre-student teaching) experience with students which focuses on the student and his or her learning patterns and problems. In particular in the elementary school there was felt to be a need to focus on the child, his or her wants and needs and the implications of child development theory for classroom practices. There was also a feeling for the need to combine and coordinate the content, methods and school experiences so as to take maximum advantage of the inter-relationships, motivation, relevance and focus that each can provide for the others. Two specific proposals to effect this combination were given, one at the elementary level and one at the secondary level.

At the elementary level one can combine the content and methods courses that are normally taken by a prospective elementary school teacher. These can be presented in a laboratory format which emphasizes hands-on, small-group work in an effort to model appropriately a variety of useful teaching strategies. These labs can be coordinated with public school experience where the focus is on working with small groups of children to gain experience with and sensitivity to their learning and behavior patterns. In other words, we do to students in their mathematics courses what they are expected to do to their students. That is, if they have experiences in mathematics themselves, they will teach their students in the same way even though it is not the same level of content.

A proposal that was made for the pre-service training of secondary teachers was for "Didactical Shodow" courses. These ourses are to be taken simultaneously with certain regular mathematics courses and can focus on some of the following: content in the mathematics course that is particularly relevant to the secondary schools, methods of teaching that content, significant applications of that content, thought processes involved in learning that content, historical context for that content, recreation and enrichment, and small-group experiences with high school students. In the group discussion of this "shadowing" concept it became clear that shadow courses could also be created for other areas such as applications and research. One of the intentions of the "shadow" courses is to provide for the specific needs of secondary school teachers without proliferating mathematics courses or diminishing the interaction between mathematics students with other orientations.

In the process of increasing the exposure of the pre-service teacher to children it is important to involve the in-service teacher in pre-service teacher training. This is part of a general need to increase the experienced teacher's role as a professional educator with some responsibility for decision making, long-range planning, apprenticeship and certification. To encourage this professionalism teachers should be given more responsibility for their own learning and they should be encouraged to engage in independent investigation.

There follow several related specific recommendations or comments which are gleaned from various working group reports.

---". . . thought should be given to pre-service curricula which stress that the student's mathematical education will be incomplete upon gradation and which focus on the discipline and technique of self-study."

---". . . the development of courses in colleges that stress open-ended inquiry, heuristic techniques of problem formulation and problem solving should receive encouragement and support."

---"We believe it is important that professional and scholarly organizations such as the MAA, NCTM, and NCATE continue to maintain continuous pressure for higher standards of training . . ." .

2. Focus on Children

There has been much discussion about reforming teacher training and the recognition of the need to keep the focus on children and how they learn. Here we have tried to fuse these objectives because we believe that reform in teacher training must be guided by what we know about children's skills, developmental patterns, and cognitive bents, and especially about how those things differ among children. Our deliberations have led us to make a number of recommendations from this frame of reference; these only typify a more thorough set of the same kind.

Recommendation 1: Teachers should be sensitive to children's having different 1) cognitive styles, 2) modes of conceptualizing and 3) schemes of conceptual development.

Rationale: In the extreme, this means that a teacher should be able to diagnose dyslexia and other pathologies--but our chief intent concerns the normal.

The question of what is "normal" is very complex. The complexity of this issue is brought to the fore by considering theoretical approaches to the problem of cognitive development. To simplify matters we select two such approaches: the S-R or Behaviorist and the Structural-Stage views of development. In many ways the Behaviorist's view of the child is the child as a miniature adult. By exposing the child to adult ideas and experiences, we build his knowledge. Exposure to a given stimulus takes effect through the mechanisms of association. The more associations, the greater the adult approximation and reinforcement. Positive reinforcement stamps in the desired associations and negative reinforcement stamps out or redirects the nondesired associations. In contrast the stage theorist models the child in a different way. The child is seen as moving through qualitatively different stages. Each stage is characterized in terms of an internal structure which serves in part to define the interpretation of a stimulus and therefore the child's response. Development from stage to stage proceeds as the child confronts the environment and tries to understand it. Thus, for example, researchers in psycholinguistics report that preschoolers fuse the meaning of more and less. Accordingly, the young child who is asked to compare pairs of arrays of elements and indicate which has more and less, can be expected to give random responses--at least from the adult point of view. From the child's point of view, he is always correct as long as there is a difference in the number of elements in the represented arrays. As he begins to see the need to make further use of numbers, he begins to work out the ambiguity of terms like more and less. In many ways, these two positions have different implications for teaching. We focus on the concept of definition of an "error" and the notion of motivation.

For the Behaviorist, the child who (in a given context) gives a response that differs from the one expected by the adult has made an error. And the child who gives the expected response is correct. Reinforcements are doled out appropriately. For the stage theorist, the unexpected is taken as symptomatic of a different way of thinking about the problem. Furthermore, the child determines if it is an error and what he will do about it.

The current thinking of developmental psychologists reflects a recognition of the fact that some kind of complex stage theory of learning is necessary. When it comes to teacher training then, it is essential to make the teacher familiar with the different stages a child can pass through and the implications of these stages vis-a-vis the child's motivation, and what might be "normal" for children of different ages. A teacher needs to be able to use several strategies when presenting a portion of content or when solving a mathematical problem. He should also be able to recognize and accommodate different strategies children use in trying to solve the same problem.

Recommendation 2: Each teacher should be provided with the necessary information and skills to allow him to have available and be comfortable with multiple strategies.

Recommendation 3: Mathematics teachers should have resource persons available to them.

There are several reasons for resource specialists to be available to teachers; these reasons should sufficiently delineate his duties and skills:

1. A resource specialist should be a channel to more central resources, such as to make Recommendation 2 feasible in a progressing and developing educational environment.

2. We cannot make every elementary school mathematics teacher into a full-fledged mathematician. The teacher will therefore find it easier to encourage a child to explore far outside the rest of the class if there is readily available the resource to accommodate that child.

3. The teacher should be able to identify his own styles, modes of presentation, and so on, so that he can resolve whatever difficulties may be consequent in a particular teaching environment.

4. Some children may represent particular problems of analysis or even diagnosis: unusual skills and styles may be harder to recognize than distinct cognitive gaps, for example.

Particular technological developments should be provided with responsive resources available to the teacher. Examples are computers and computation, the new calculators, as well as TV.

<u>Recommendation 4:</u> The "modules" or "atoms" of content should be matched with what is known about childrens cognitive reasoning and representational (e.g. linguistic) developments.

It is not only the case that children pass through different stages of cognitive development. Individual children and adults are recognized to use different modes of representation. Terms such as <u>symbolic</u>, <u>spatial</u>, <u>visual</u>, etc. are used by psychologists to reflect this. With respect to mathematics, the challenge is to match the teaching style and content to the child's mode and style of representation. The child who thinks visually may have great difficulty with a lesson in algebra--not because of a lack of mathematical capacity but because he is forced to deal in a mode of representation that is not readily available to him.

This raises a research problem--i.e., the extent to which different subjects in mathematics can be readily presented in different modes. For example, there is the question of whether each mathematics topic which is generally based on spatial assumptions can be presented in other modes and, if not, how it can be accommodated by a sequence of other modes.

We see at least two ways in which the above recommendations need to be implemented. One is in the preservice training experiences of future elementary teachers. The other is in the form of information and training of the larger group of current teachers. For this latter group the availability of consortia of teaching centers provides a natural vehicle for the implementation of these and some of the other recommendations of this report.

Recommendation 5: Teachers should be encouraged to let children in a class act as teachers themselves for that class and/or a portion of it.

Most of us recognize that we often truly understand a concept of point when we have to teach it. But more than that, children are obviously sometimes a good deal more sensitive to the difficulties and sticking points of other children than someone who not only has solved the problem years before, but thinks much faster along different and more advanced lines.

We do not mean to advocate mere role playing (for which there may be other arguments). Teaching by a ch[illegible] to a child is in fact likely to be more useful to the teaching child than the taught child. Learning from peers rather than from an authority figure extends participation and involvement for both partners.

We realize that dispersal of informative or instructive flow can be thought of as disruptive in some classroom situations; and that the tactic must be used sensitively enough to avoid categorizing and ranking the children in ways offensive to them.

Observation: Prob[illegible]y the most potent forces for positive change in mathematics education will arise outside the traditional educational infra-structure--i.g., mass media (TV), computer technology.

School districts are emphasizing cost effectiveness (or even just cost) more than excellence and innovation. As population growth slows, or becomes negative, the teaching profession will emphasize maintaining power, improving status, etc., as primary goals instead of derivative ones.

3. Mathematical Thinking

Considerable attention was given to the topic of mathematical thinking. Many feel that there is a special kind of thinking that mathematicians engage in, that this kind of thinking is definable, generalizable, transferable, and desirable and that this kind of thi king can and should be taught, developed and encouraged in the general public. At the same time, it is also strongly felt by some that there is no special quality of being "mathematically minded," despite common opinion to the contrary. Those at the Conference who addressed themselves to this matter felt that it is a matter of overriding importance, and yet a matter to which insufficient concerted attention has been given. It was recommended that long-range, basic research be undertaken to delimit the concept of mathematical thinking and to devise and validate techniques for teaching it. It is hoped that this will become part of the education of teachers and of children in the future. The problem of current teachers, current children and current school practices is discussed in Chapter IV.

4. Teacher Centers

In addition to the need for innovation in current pre-service teacher education practices and for research in mathematical thinking there is a need for a long-range mechanism for support of improvement of teaching and of innovation in the classroom. One proposal for meeting this need was for the support and encouragement of teacher centers. Teacher centers were proposed, either directly or indirectly, by several groups and there was considerable variance as to specifics. There follow some of the thoughts shared by several participants.

The center should be permanent and should engage in helping teachers solve their teaching problems; introduction of innovation through workshops, follow up, and local popular support; provision of information and material from national and international sources; provision of released time and other support for teacher-generated innovation; consultation with school personnel concerning selection, implementation and evaluation of mathematics programs.

The center should at least coordinate pre-service school experiences. There was also mention of the center having responsibility for all teacher training and for that matter, research on learning and graduate intern experiences. There was some disagreement on the extent to which the center should be university-based or school-based and, therefore, on how much of the university's normal function should go on there.

In terms of staffing, the center should have direction from individuals with the perspective and competence of experienced mathematics educators; and should be continuously staffed by persons with sufficient experience and sensitivity to support teachers in their daily functions. The center should be open from 8:00 a.m. to 8:00 p.m. when it can be used by teachers.

Safeguards should be present to maintain the focus of the center. There was concern that such centers might go the way of many bureaucracies and become self-serving and resistant to the very changes that they were designed to facilitate. There was hope that they would "possess professional-level independence of action..." rather than be the "...agents of certain administrations..." Some safeguard could be provided by an independent, broadly-based board of directors of the center. Membership on the board should have short term appointments and could include people from among mathematicians, mathematics educators, school teachers, school administrators, politicians, and parents.

# Chapter IV
## PROBLEM SOLVING

Throughout the Conference, the topic of problem solving was a continual and pervasive subject of discussion. There was a general consensus that problem solving experiences of a variety of types should be a more central part of the mathematics curriculum. There was general agreement that there is a need to develop problem solving experiences that can be used successfully and gain acceptance in a curriculum designed for broadly based use.

It was also evident that there was little consensus on the meaning and nature of problem solving. For some, the words "Problem solving" and "applications" were close enough to be used interchangeably; for others, the terms "problem solving" and "mathematical thinking" were synonymous. For many, problem solving had several meanings ranging from the simple arithmetical examples usually found in elementary texts to the more formal mathematical problem sets associated with abstract mathematics. Several other sections of this report, notably those on Applications and on Computers, deal directly with some aspects of problem solving.

Within the discussion of problem solving, it became clear that the process of solving problems of any description has not been given due attention in curriculum materials. The reason for this seemed to be either that curriculum developers were unaware of the techniques and procedures that develop the process, or that such techniques and procedures are not available. There have been efforts to make explicit many of the basic principles which are frequently useful in attacking problems of a mathematical nature. The books of Polya set forth in great detail some of these principles, particularly those which are appropriate at the secondary school level. There have also been articles describing some principles which are useful for problem solving at the elementary level. In addition to articles on problem solving which have appeared in journals on mathematics education, there is the work of the testing agencies in this direction, for the development of effective test items is intimately related to an understanding of the problem solving process. Some of their results seem to indicate that beyond the principles which are already known, the problem solving process becomes an intensely individual thing. A student was successful in a particular problem solving activity because he was able to mobilize his own previous experience and insights to establish connections which were important in solving the problem. Thus caution must be exercised in undertaking new projects to be sure that they will lead to the kind of understanding of the problem solving process which will be of use in mathematics education.

Several instances of successful and innovative experiments in particular directions related to problem solving were reported on at the Conference. Some of these made imaginative use of the computer. Some have developed problems and are observing children as they solve these problems. Other individuals have recently been developing problems for young children to solve and have tried to develop generalizeable and teachable techniques for solving these problems. These two approaches suggest the need for support in two directions related to problem solving.

1. There is a need for a center or centers for basic research in how children (students) solve problems. In the spirit of the Geneva Institute, children's thinking should be observed in the hopes both that the nature of the growth of the problem solving process can be discovered and that an appropriate taxonomical order for problems to develop this process can be determined.

2. There is a need for this same center to develop curriculum materials for both school students as well as teachers. The center should use what is known (very little, it appears) in the development of these materials. At the same time, these developmental activities should be evaluated and disseminated. Eventually the materials and techniques should be incorporated into the general curriculum.

Many curriculum efforts now claim a problem solving thrust. Yet the nature of this process and the techniques for developing this process are not well publicized. The importance of this process suggests that a massive effort be supported to define and develop problem solving for use in the teaching of mathematics.

# Chapter V

## APPLICATIONS

1. Introduction. Efforts in mathematics curriculum reform in the 1960's were primarily directed toward increasing the role of logic and structure in school mathematics. The present day curriculum, with its emphasis on the structure of mathematics and genuine understanding of that structure on the part of students, while representing a large step forward over mere rote learning, exhibits a lack of concern for the important role of applications. In particular it has been critized on the following grounds:

a) There is a dearth of applications that students feel are important to them now or even will be important to them in the future.

b) In most texts now available, the problems tend to be of one type only: drill exercises on the immediately preceding the material. Rarely are the problems of the sort that are encountered outside of a textbook, and even more rarely are they truly open-ended.

c) The present curriculum does not sufficiently motivate students to study mathematics. The interplay between mathematics and other school subjects is frequently not apparent.

d) The present curriculum does not take account of the impact of new devices--calculators, mini-computers, computers, etc.--nor does it utilize their potentialities.

The phrase "Applications of Mathematics" as used here refers to much more than the mere translation of words into symbols as required by the typical word problems of school mathematics. It is even more than mathematical problem solving. It includes the identification and formulation of specific problems in unstructured situations, the solution of a problem posed in mathematical form, relevant computations, comparison of the results with observations, and reformulation as needed, and the drawing of appropriate conclusions. Thus, the mere presentation of problems originating outside of mathematics which involve only the solution of a mathematical problem is not sufficient to illustrate the application of mathematics.

Of course, the process of applied mathematics as described here is intimately related to the general process of problem solving. Thus, the comments and recommendations of the section of problem solving are also of interest with regard to applications.

Finally it is recognized that there are important interrelations between various fields within mathematics. These connections may arise in purely mathematical discussions or they may arise in the mathematical arguments used to study a real problem. They constitute applications of mathematics in that they involve the use of the mathematics created in one context but used in another, and they should not be neglected. However, they are not the sort of applications of primary interest here.

Specific recommendations for suggested action on the difficulties cited above follow. The first is concerned with the creation of source material for teachers and authors. The second proposes some modifications of the current curriculum which would lead to instruction giving more appropriate weight to applications. The third suggests more fundamental reform and associated research.

2. Source Materials. The recommendations proposed in this section depend for their implementation on the existence and dissemination of an ample supply of source materials for the use of teachers and authors.

Recommendation 1. There be constituted a task force made up of mathematicians, mathematics educators, representatives from industry, teachers, and students to develop and disseminate a comprehensive collection of examples of applications suitable for grades K-12.

Discussion. It is suggested that these applications be organized in terms of the usual strands in the curriculum, i.e., number, geometry, operations on rational numbers, solving algebraic equations, etc., and that they be organized according to grade level and type of application. The styles of presentation of the examples should vary from simple structured problems to open-ended unstructured situations that may require an inter-disciplinary approach, i.e., the student may need to use information from fields other than mathematics.

A number of such applications are already available. For instance, certain of the SMSG materials (second version of the Junior High School Program, some publications in the Supplemental and Enrichment Series, and some volumes in the Studies in Mathematics Series), materials produced by the Madison Project, and some NCTM publications contain appropriate examples. However, these materials are incomplete, e.g., insufficiently diverse, and not adequately organized for easy classroom use. Also many teachers are simply unaware of their existence.

This collection of examples would not only directly help the teacher in the classroom, but would have the indirect benefit of providing incentive and direction for authors of school textbooks. In the long run it is anticipated that applications of the form discussed here will be incorporated in standard textbooks.

An important function of the task force will be to see that the materials developed are classroom tested and if necessary modified on the basis of the results of this testing. It is intended that the materials produced for teachers include comments and teaching hints derived from classroom experiences.

3. Suggestions for the Modification of the Current Curriculum

It is possible to meet some of the criticisms cited in the introduction to this section within the framework of the current mathematics curriculum. Some suggested actions are provided here.

Recommendation 2. Current mathematics courses be restructured to include significant applications of mathematics.

Discussion. At the present time the typical mathematics course does not contain applications in the sense described in the introduction. At most it includes only that aspect of applications involving the solution of mathematical problems obtained by direct translation from words to symbols. Part of the reason for the neglect of the other aspects of the process of applied mathematics is that, for the most part, the exercises in textbooks are given by fiat--no identification and formulation stages are involved. Exercises are frequently artificial, completely formulated, routine, limited situation problems given in a context that are 'made to come out'. Knowing that a particular problem comes from a specific chapter of a given text constitutes at least 90% of the solution. In actual situations things are usually not so simple. The situation itself may be ill defined and specific problems may be difficult to identify. Also, once specific problems are identified finding the right tools to solve them can involve more work than actually obtaining the solution. Actual situations usually contain irrelevant and redundant information and may lack certain necessary information. Occasionally, one even gets misinformation.

Since the identification and formulation stages in the analysis of an incompletely defined situation are common to both pure and applied mathematics (as well as other fields), it would be useful to students if mathematics were taught in a manner emphasizing all aspects of the analysis of situations. These aspects are useful organizing concepts for mathematics instruction at all levels.

Some of the applications should involve situations in which the direction of attack is fairly clear. Students gain confidence by studying such situations and the instruction can be easily managed. However, it is important that some situations be considered in which the approach is not clear and direct, i.e., which are open-ended. Also, situations should be considered in which the open-endedness results from an ambiguity as to what constitutes a solution to the problem.

It will not suffice to interject just a few applications or open-ended problems in the course of an academic year. The real concern is with the attitude pervading the teaching of mathematics. The objective is to bring a greater vitality to mathematics by involving the students more actively on a continuing basis, and by constantly reinforcing the relations between mathematics and the rest of both the "real" and the "academic" worlds.

Recommendation 3. The curriculum should be modified so as to include the study of open-ended interdisciplinary situations which have quantitative components.

Discussion. Some of the concerns now being expressed, though perhaps focused on mathematics, are not unique to mathematics education. They are representative of society's reaction to education as a whole. It is quite clear that society in general feels that schools should respond to student needs and interests, and that they have operated and are operating in partial isolation from both of these. In addition students need to know and feel that what they are doing in school is of value to them. Without this feeling little permanence of learning will be present.

Efforts are being made to meet this need. A battery of real problems has been (and is being) developed, each intended to lead to recommendations, based on a careful analysis, which may in fact produce changes in the students' environment or new procedures associated with the students' lives. These problems have scientific and mathematical components, but also involve social or political elements. A number of examples are given in the Report of the Estes Park Conference on Learning through Investigation and Action on Real Problems in Secondary Schools (Education Development Center, 55 Chapel Street, Newton, Massachusetts, 02160).

There are certain aspects of this approach which the intending user should keep in mind. A reasonably complete analysis of a real problem is likely to be a lengthy project, for which time must be provided. Not all such projects will have enough mathematical content or diversity to satisfy subject-matter criteria. It would be difficult to creat such problems on an ad hoc basis, and use would probably have to be made by the teacher of prepared materials. Real and significant problems accessible to elementary pupils are limited in number, and a given problem may lose its significance after having been used a few times in a given school. Genuine cooperation is required between teachers of several disciplines.

While these difficulties must be mentioned, we do not intend to imply that they are insuperable, and indeed a number of schools across the country are now using such materials.

Any materials that are developed must be thoroughly tested in school situations. It is vitally important, in these materials particularly, that teachers should carefully observe what happens when children deal with larger problems than are present in most current materials. Teacher's guides to the use of the materials should include the results of their observations of what the children are doing. Decisions must be made on what resources other children will need to do similar problems better and this information must be disseminated. Research should be done on how teachers could handle the range of applications and problem solving. The professional education of teachers should involve studying curricula which emphasize applications and problem solving. It is particularily important that teachers deal with open-ended questions themselves, gaining experience with children in internship and student teaching situations. A very helpful procedure would be to explore the use of aides and paraprofessionals in this role, in providing nondirective positive feedback short of giving answers. Such aides could include professional and non-professional industrial arts people.

All these considerations clearly involve all subject areas and mathematics equally, and consequently much effort must be taken with educators in the other disciplines. A good starting point might be to correlate these efforts with some of the consortium plans generated at the Estes Park Conference.

Recommendation 4. Applications should be presented in a manner which recognizes the important role of calculators and computers in school mathematics instruction.

Discussion. The computer should be a vital part of any mathematics curriculum, and it finds some of its most fruitful uses in the teaching of applications. First, the use of computers makes it possible to consider situations having a much greater complexity than would be possible if the associated arithmetical operations were to be carried out by hand. The same is true to a lesser extent of calculators. The computer has the additional capability of doing the logical operations which are important in some problems. Secondly, the use of a computer helps keep the student (and teacher) honest. If he can program his argument, then he can usually determine whether it is valid or not in a very short time. Thirdly, he must really know exactly what he is doing to get correct responses from the computer. Just as many teachers have noted that you never really learn a subject until you teach it, students using a computer also get the benefits of this type of experience. Indeed, when he develops an algorithm and writes a program for his problem, he is essentially teaching the computer to perform the calculations. Additionally, he is free to concentrate on the ideas involved in the calculation and not in the tedious arithmetic which generally is counter-productive. Finally, when something goes wrong in his program, he is well motivated to 'debug' it. "The idea of debugging itself, for example, is a very powerful concept--in contrast to the helplessness promoted by our cultural heritage about gifts, talents and aptitudes. The latter encourages 'I'm not good at this' instead of 'How can I make myself better at it?'" (M. Minsky and S. Papert) Further good reasons are given in W. Standberg, "Computing in the high school--Past, present, and future--and its unreasonable effectiveness in the teaching of mathematics," and W. Koetke , "The Impact of Computing on the Teaching of Mathematics," (Spring Joint Computer Conference Proceedings, 1972, pp. 1051 - 1058, 1043 - 1049).

4. Suggestions for Fundamental Changes in the Current Curriculum

Although there have been significant changes in content and emphasis, the sequence of mathematics courses in the high school curriculum today is roughly the same as thirty years ago: algebra I, plane geometry, algebra II, and some sort of senior mathematics. A typical senior mathematics course taught today might include some coordinate geometry, trigonometry, topics from solid geometry, and to a more limited extent finite mathematics (linear algebra, probability), statistics, and calculus. Major goals have been the development of a modest facility in algebra and geometry and an appreciation of the role of structure and proof in modern mathematical thought. There is some evidence that in the achievement of these goals mathematics instruction has failed to provide the sort of mathematical experiences which have the maximum future utility.

Recommendation 5. The current curriculum be studied with a view toward introducing fundamental changes; specifically, the introduction of new applications oriented courses at the 9th and 10th grade levels.

Discussion. Before proposing significant changes in the basic high school mathematics program, it is essential that it be clearly understood what portions of the current courses provide essential knowledge for future work (in academia or everyday life) and where this knowledge is used. On the basis of what is known now there have been proposals that a radical restructuring is in order. The specific recommendation cited above is made with the object of influencing the maximum number of students as far along as possible in their mathematical education. It envisions replacing the basic courses with large enrollments (general mathematics, algebra I, plane geometry) with new courses which are applications (or applications/computer) oriented. These new courses would consist of applications which are carefully selected to have important and useful mathematical content and whose solution provides insight into significant scientific and human problems.

Such a revision might be viewed as leading to a mathematics curriculum consisting of three levels. Grades K-8 would be devoted to developing most of the essential mathematical content and skills and the associated culture which every citizen must know to function effectively in everyday life. The 9th and 10th grades would be used to study situations in which mathematical approaches are useful. In these courses there would be new mathematics developed as well as applications of the K-8 material. Mathematics courses in the final two years of high school would provide the remaining mathematics necessary for college bound students with interests in the sciences.

As in the earlier recommendations, it will be necessary to prepare materials for student and teacher use. Since in this case the applications will be the vehicle for teaching the mathematics, the writing task is even more complex. With this approach it is likely that the computer will play a significant role, both as a means of facilitating computation and as a device for simulating complex systems. Some computer based materials have been developed (primarily in connection with projects in other disciplines) and offer a starting point for materials preparation for the proposed project.

# Chapter VI

## STATISTICS

The processing of numerical information is an essential activity in everyday life as well as in many academic fields outside mathematics. Numerical data and statistics are encountered in newspapers and magazines, on television, in experimental work in the social and life sciences, in business, and in medicine. However, few people feel comfortable with such information, much less know how to interpret it properly and use it effectively. Therefore, it is proposed that at all grade levels there be instructional units in statistics, and that these units be tied into the study of mathematics, of the biological, physical, and social sciences, and even into the study of the humanities where appropriate.

In elementary school these units would be primarily concerned with descriptive statistics: the collection and organization of numerical data, the construction, reading and interpretation of tables, charts and graphs, and statistical measures such as the median and range. Later, with the acquisition of elementary concepts of probability, the study of statist cs can be extended to cover inference.

From the beginning students should be involved with their own data. Imaginative materials relating to student interests in everyday problems as well as other school subjects need to be developed. An excellent beginning has been made with the four volumes Statistics By Example (Addison-Wesley, 1973) prepared and edited by the Joint Committee on the Curriculum in Statistics and Probability of the ASA and NCTM, Frederick Mosteller, Chairman. This material progresses from the simplest descriptive aspects of statistics (requiring only arithmetic, rates, percentages) to the formulation of rather sophisticated models to describe real phenomena. In order to consider significant examples it is necessary to use calculators or computers to carry out the arithmetical operations. In addition, the use of computers permits the simulation of complex situations which may be difficult or impossible to observe in the real world. Thus statistics provides another means of illustrating and exploiting the role of computing in mathematics and science.

The above proposal continues the spirit of previous recommendations on instruction in statistics. In the 1940's and before, statistics instruction took place almost exclusively in graduate schools. With the postwar trend toward the increased use of numerical data and quantative methods, it was recognized by faculty and students that this was much too late to begin the study of statistics and appropriate courses began to appear in the college curriculum. In the 1950's and 60's at the suggestion of the Commission on Mathematics of the CEEB, some high schools introduced a 12th grade course in probability and statistics. These beginnings had two shortcomings: 1) the courses were reserved for the mathematical elite among high school students; 2) the emphasis in the courses tended to be on probability theory. Statistics, if treated at all, was viewed as an application of probability. This proposal recommends that all students receive instruction in basic statistical concepts and techniques, and that the statistics be firmly related to significant problems.

## Chapter VII

## THE ROLE OF THE COMPUTER IN THE MATHEMATICS CURRICULUM

It is obvious that the computer has had a great impact on our society. This impact has brought about both quantitative and qualitative changes in the lives of everyone and has prompted much concern about the influence the computer will have on future societal developments. These facts suggest that the computer should play a vital role in any proposed mathematics curriculum and should be an integral part of it. Indeed, the computer (and the variety of calculators currently on the market) should be used as a tool not only for the learning of mathematics but also for other school subjects and in later life. Participants in the conference cited several reasons for this point of view. Recent efforts to use the computer in mathematics instruction, both in a student-controlled instructional mode and a computer-controlled instructional mode,* have been encouraging. The computer appears to be a strong motivational device for many students, including those who have been identified as educationally disadvantaged and it broadens the scope of the mathematical content that can be included in the curriculum. Also, it adds a new dimension to mathematical problem solving by allowing the student to focus on the essential mathematics of a problem without getting

---

*student-controlled vs. computer-controlled instructional modes deserve clarification. In a student-controlled mode the student actually programs the computer to perform certain operations. A computer-controlled mode involves the presentation of instructional materials by the computer to the student. A simulation mode might possibly be thought of as a combination of these two modes. However, in the strictest sense, simulated activities are computer-controlled.

bogged down in computational drudgery. Finally, it is clear that the computer will continue to be an overwhelming influence in the rapid change and increase in knowledge in our society. For these reasons a serious effort must be made to explore the potential uses of the computer in mathematics instruction and identify feasible means of implementing these uses in the schools.

## Potential Uses of the Computer

Several areas in which the computer has a potential for making a direct, positive impact were discussed in detail by various conference participants.

1. The computer can be used to simulate a variety of complex problem solving activities which involve mathematics.

2. The computer can be used as a student record-keeping instrument, thereby enabling the teacher to have more accurate and up-to-date information on the progress of his students.

3. Although past efforts have not achieved the degree of success desired, computer-assisted-instruction (i.e., computer-controlled instruction) is still viewed as a potentially powerful force in mathematics curriculum revision and a source of more effective individualization of instruction.

4. The computer may enable the teacher to devote more attention to modeling problems and problem solving activities by removing the constraints (e.g. computational tedium) inherent to a consideration of key mathematical topics (e.g., use of recursive processes, modeling processes, and contruction of algorithms).

5. As a problem solving tool, the computer can help organize the student's thinking. The belief is that the student must really know exactly what he is doing in order to get correct responses from the computer. The student who develops an algorithm and writes a program for a problem is essentially teaching the computer to perform the calculations thereby freeing him to concentrate on the ideas involved and not on the tedious arithmetic which generally is counter-productive.

6. A central feature of computer programing in "debugging." When a program does not work, the errors must be found and eliminated. The student is forced to analyze the problem very carefully and is usually highly motivated to do so because the work is entirely his responsibility. Thus, the computer can serve as a motivational device to force students to analyze their work carefully.

Unresolved Issues and Problems

Several issues must be given serious attention if the full potential of the computer in mathematics instruction is to be realized. Computer hardware and software must be designed that fit the needs of students and the classroom environment instead of vice-versa. Computers should be designed specifically for instructional purposes and the cost should be brought down to make them generally accessible to schools. Recent developments in the use of time-sharing systems are promising and seem headed toward helping to solve problems of appropriate hardware and software.

A shift of emphasis is needed from using the computer to provide drill and practice in computational skills to applications of the computer's calculational ability in problem solving activities. Although, many good ideas for using the computer in a problem solving mode are proposed, there is still a tendency to view the computer as little more than a glorified calculator.

Perhaps the most pressing issue is that of societal uses and implications of the computer. At the present time the average citizen and teacher have a very hazy picture of the various roles of the computer in our society. A determined effort must be made to improve the computer literacy* of the citizenry.

*The term "computer literacy" is used in the sense described in the report of the Conference Board of the Mathematical Sciences "Recommendations Regarding Computers in High School Education." In the Report "computer literacy" includes the following aspects: some direct experience with computers and knowledge of a simple programing language, knowledge of the uses of computers in society, and knowledge of the effects of computers on society.

Recommendations

In light of the potential advantages of using the computer in the classroom and the unresolved issues and problems, the following recommendations were made:

1. The computer should be an important part of any future mathematics curriculum efforts.

2. Emphasis should be placed on using the computer to involve students in problem solving activities. Computer use for drill and practice on computational skills should receive less attention.

3. Certain readiness concepts about the use of computers should be included in the elementary grades (1 - 6). These should involve the use of calculators and an exposure to algorithmic approaches.

4. In grade seven, students should be taught a programing language which is appropriate for the level of students involved. In this grade students should become familiar with information processing and the computer should be used as an integral part of the mathematics course.

5. The mathematics curriculum in grades 7 - 12 should be studied and revised in order to make optimal use of the computer as a tool* in the mathematics courses.

---

*"Use of the computer as a tool" refers to a variety of considerations. Specifically, reference is being made to simulation of problem solving activities, student programing to solve mathematics problems, and construction of algorithms by the student.

6. A one-semester computer science course should be offered in grade 12 which may be selected as an option.

7. Societal uses and implications of the computer should be studied at some point in the school program, possibly in the 10th or 11th grade. The writing of modules on this subject that can be inserted in a social studies course is encouraged. Another possibility might be the development of a course "Mathematics and the Computer in Society."

8. There should be continued funding of efforts to investigate uses of the computer in a variety of instructional modes until more data are available regarding the value of these modes. Funded projects which explore the potential of different uses of the computer in education are encouraged.

9. If computers are to be systematically employed in the above ways in the schools, then the implications of this for widespread computer-access and teacher education should be effectively faced, spelled-out, and dealt with.

# Chapter VIII
## EVALUATION

The subject of evaluation was considered by several groups and individuals. Two broad areas came under discussion. The first area was the evaluation of students' learning together with the evaluation of current programs in instruction.

Underlying the discussion in this first area was a concern over the present nature and use of evaluation. Strong concerns were expressed over the undue control evaluation exerts on what is taught and learned; the frequent negative effect of evaluation on schooling in general, the emphasis on using evaluation to linearly order students, and the limitation of evaluation to narrow instructional goals. Of particular concern was the current emphasis on training pupils to perform on narrowly-defined or "atomized" pieces of content with the assumption that ability to perform reflects or is more important than comprehension of content underlying performance. It was felt that such approaches overlook broader or higher cognitive level goals of instruction. The problem of measuring new programs with instruments reflecting different objectives was identified. It was noted that major decisions regarding the success of curriculum projects, as well as the progress of individual students are often based on the results of current standardized tests which are considered inappropriate for the most significant goals of newer programs.

Rather than criticizing and then ignoring the current state of affairs, discussants felt that dealing positively with the issue could give better direction to a current educational force and focus attention on a broader set of learning outcomes. Two recommendations emerged from discussions.

Recommendation 1: A project should be funded to develop sample evaluation instruments for measuring students' progress towards selected goals for school mathematics.

Discussion: Just as the curriculum projects of the sixties produced sample mathematics textbooks that stimulated textbook publishers to revise their products, so an evaluation project would produce sample mathematics tests (and other devices) that might stimulate a similar response from the publishers of mathematics achievement tests. An evaluation project would demonstrate to those concerned with the school mathematics program (1) that current tests are inadequate, (2) that a broader array of instruments and techniques is needed to assess many of the most important objectives, and (3) that the development and use of such instruments and techniques is a matter of major concern.

The products of such an evaluation project would include new kinds of tests for measuring what have been termed "the higher cognitive levels" of mathematics achievement: tests to measure such things as the ability to formulate a mathematical problem, the ability to make different kinds of estimations, the ability to visualize three-dimensional figures, or the ability to detect a mathematical pattern. Other products might include more sensitive inventories for measuring attitudes toward various facets of mathematics, scales for measuring a student's self-concept regarding mathematics, interview techniques for assessing the quality of a student's thinking about mathematical concepts and problems, and observation schedules for studying classroom activities in mathematics.

Such an evaluation project would not have to be as large in scale as the above list of products might imply. Its major purpose would be to illustrate what ought to be done either in measuring an individual student's proficiency in mathematics or in judging a curriculum project's success in meeting certain goals. The project would provide samples of what might be done; it would not attempt to cover any one set of goals in a comprehensive way. Such a project could draw upon the experience of mathematicians, mathematics educators, and psychologists who have participated in various mathematics test development projects or who have conducted clinical studies of mathematics teaching and learning. It has been noted that the construction of a test item to measure the higher cognitive levels of mathematics achievement has all of the characteristics of a research project in mathematics. This task should not be left, by default, to people who neither understand the goals for school mathematics nor appreciate their importance.

Recommendation 2: A small group of mathematicians, mathematics educators, teachers, and users of mathematics (natural and social scientists, consumer representatives, etc.) should be formed to take responsibility for improving the testing situation in mathematics. Such a group would make specific proposals for improving testing in mathematics and be responsible for implementing and supervising specific approaches such as those discussed below.

Discussion: Possible directions for more comprehensive evaluation programs are presented as indices of ways of improving the usefulness and validity of tests.

1. Tests should not be limited to pencil and paper situations. In some cases the test should consist of immersing a student in an appropriate situation and observing his behavior. Observation might center on such question as; "Does the student solve real problems when the verbal barrier is not present?" or "Does the student want to talk about mathematics with other students.?"

Clinical interviews can be a most valuable technique for gathering information on an individual's thought processes and levels of comprehension.

2. Large pools of items should be available for tests in each area. Such pools should be available for teachers to use in "teacher-made" tests as well as for state or national tests. The pools should cover every important topic in the area and should be made public. Then a stratified random sample of the items would constitute each test. Such a large pool of items would remove the severe secrecy conditions surrounding tests and allow open consideration of the goals being measured, thus providing for criticism of overly skill-oriented tests.

3. Since time is seldom of the essence in the use of mathematics by ordinary citizens outside of the classroom, time should not be the dominant factor in tests. When time limits must be imposed they should be sufficiently liberal so that the great majority of students finish before the time has elapsed. Further, since real life situations generally allow use of books, other sources of information, and helpful apparatus (such as calculators), use of such aids in test situations should be allowed in general.

4. Tests should include some mandatory items, some in which the student has a choice (for example "Answer any 10 of the following 30 questions"), and a completely permissive section in which the student is allowed to demonstrate his knowledge of anything he chooses.

While it is recognized that any approach to evaluation may be misused, it is felt that the flexibility represented by the above items can promote better and innovative approaches to evaluation.

The second area of discussion relating to evaluation dealt with the role of evaluation in new curriculum developments. One concern dealt with the danger of applying the results of speculation resulting from work in learning theory, directly to the classroom. A second concern was the frequent lack of adequate evaluation plans as an integral part of curriculum projects. In such efforts the initial emphasis is placed on content and methods. Participants often become convinced of the validity of an idea and evaluation then appears redundant. The lack of an adequate evaluation plan can hamper a good curriculum project or fail to detect the weaknesses of a poor project.

<u>Recommendation</u>: Funding agencies should require that a plan for evaluation be submitted as an integral part of any proposal for a curriculum development project in mathematics. The objectives of the project should be specified, and some indication should be given as to how progress toward these objectives is to be evaluated. Some component of the evaluation should be made by people not connected with the development part of the project.

<u>Discussion</u>: Although the participants in the curriculum project may view evaluation as either a trivial or redundant part of their work, the increasing competition for limited resources makes comprehensive plans for evaluation more important than ever before. As guides to curriculum developers, a funding agency might wish to support the development of illustrative evaluation plans. These plans would suggest various instruments and devices that evaluators might want to use, depending on the nature of the project.

A related discussion focused on the value of an awareness of past attempts to deal with an educational problem by those involved in present innovative approaches to the problem. Many of the plethora of innovation in education throughout history have been heralded as <u>the</u> solution to educational problems but none has successfully achieved that goal. Perhaps many innovations fail even to achieve their natural potential because too much is expected of them and too little caution and judgement has been used in implementing them.

An example of an innovation that appears to hold great promise but could easily fall far short of that promise is the approach to elementary school mathematics sometimes described as "an activity approach" or "mathematics laboratory approach." In such an approach, children actually manipulate objects, play games, or participate in activities from which they are expected to abstract certain mathematical concepts. In so far as such an approach helps the child to understand the mathematics and strengthens the ties between reality and mathematics in the child's mind it can be a most constructive influence. In so far as such approaches have no clear relevance to appropriate mathematical concepts in the mind of the teacher and ultimately in the minds of the children, they may be simply a waste of time and money. Given the unfortunate history of the Progressive Education movement, Froebel's activity centered approach and similar movements that depended upon incidental learning of mathematics, great care should be taken in planning, implementing and evaluating innovations of this sort, so there is some assurance that appropriate mathematics will be learned.

Thus it is recommended that mathematics educators should become familiar with prominent innovations and write short articles emphasizing both the promise of the innovation and the hazards which must be avoided. Such articles should include a discussion of similar innovations that have occured in the past with some indication of the history of these innovations, and should also include lists of places where such innovations are presently being tried with some apparent success. Appropriate professional Journals, such as The Mathematics Teacher and The Arithmetic Teacher should actively solicit such articles.

## List of Participants

Dr. Jay Anderson
Biological Sciences Curriculum Study
P.O. Box 930
Boulder, Colorado 80302

Dr. Richard Balomenos
Department of Mathematics
University of New Hampshire
Durham, New Hampshire 03824

Dr. Edward G. Begle
School of Education

Stanford University
Stanford, California 94305

Dr. Laurwnce O. Binder
National Science Foundation
Division of Pre-College Education in Science
1800 G. Street, N.S.
Washington,D.C. 20550

Dr. Truman Botts
Conference on Board of the Mathematical Sciences
2100 Pennsylvania Ave., N.W., Suite 834
Washington, D.C. 20037

Dr. Peter Braunfeld
Department of Mathematics
University of Illinois
Urbana, Illinois 61801

Dr. Alphonse Buccino
Division of Pre-College Education in Science
National Science Foundation
Washington, D.C. 20550

Dr. Sylvia Charp
Director of Instructional Systems
Board of Education
Philadelphia Public Schools
Fifth and Luzerne Street
Philadelphia, Pennsylvania 19140

Dr. Robert B. Davis
Curriculum Laboratory
University of Illinois
1210 W. Springfield
Urbana, Illinois 61801

Dr. Robert P. Dilworth
Department of Mathematics
California Institute of Technology
Pasadena, California 91109

Dr. Thomas A. Dwyer
Department of Computer Sciences
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

Dr. Rochel Gelman
Psychology Department
University of Pennsylvania
3813-15 Walnut
Philadelphia, Pennsylvania 19104

Dr. James F. Gray
Director of Education
Marianist Provincialate
Society of Mary, Marycliff
Glencoe, Missouri 63038

Dr. Herbert J. Greenberg
Department of Mathematics
University of Denver
Denver, Colorado 80210

Dr. Carole Greenes
School of Education
Boston University
765 Commonwealth Avenue
Boston, Massachusetts 02215

Dr. Ruth Hoffman
Mathematics Laboratory
Department of Mathematics
University of Denver
Denver, Colorado 80210

Dr. Vivian Howard
Department of Mathematics
Virginia State University
Petersburg, Virginia

Dr. George Immerzeel
School of Education
University of Northern Iowa
Cedar Falls, Iowa 50613

Dr. David C. Johnson
Department of Mathematics
330 Peik Hall
University of Minnesota
Minneapolis, Minnesota 55455

Dr. J. Arthur Jones
National Science Foundation
Office of Dubget, Programming and Planning Analysis
1800 G. Street, N.W., Room 428
Washington, D.C. 20550

Dr. Donald R. Kerr, Jr.
Mathematics Education Development Center
329 S. Highland Ave.
Bloomington, Indiana 47401

Dr. Jeremy Kilpatrick
Department of Mathematics Education
Box 18 Teachers College
Columbia University
New York, N.Y. 10027

Dr. Murray Klamkin
Ford Science Laboratory
P.O.Box 2053
Dearborn, Michigan 48121

Dr. Morris Kline
Department of Mathematics
New York University
251 Mercer Street
New York, N.Y. 10012

Dr. Mary M. Kohlerman
Division of Pre-College Education in Science
National Science Foundation
Washington, D.C. 20550

Dr. Donald Kreider
Department of Mathematics
Dartmouth College
Hanover, N.S. 03755

Dr. John LeBlanc
Mathematics Education Development Center
329 S. Highland Ave.
Bloomington, Indiana 47401

Dr. Frank Lester
School of Education
Indiana University
Bloomington, Indiana 47401

Dr. William J. LeVeque
Department of Mathematics
Claremont Graduate School
Claremont, California 91711

Dr. Richard Link
Artronic Information Systems, Inc.
155 E. 44th Street
New York, N.Y. 10017

Dr. Earle L. Lomon
Educational Development Center
55 Chapel Street
Newton, Massachusetts 02160

Dr. Daniel Maki
Department of Mathematics
Indiana University
Bloomington, Indiana 47401

Dr. Andrew R. Molnar
National Science Foundation
Computer Innovation in Education Section
1800 G. Street, N.W.
Washington, D.C. 20550

Dr. John W. Moore
Department of Chemistry
Eastern Michigan University
Ypsilanti, Michigan 48197

Dr. Gottfried Noether
Department of Statistics
University of Connecticut
Storrs, Connecticut 06268

Dr. Seymour Papert
Artificial Intelligence Laboratory
545 Technology Sq.
Cambridge, Massachusetts 02139

Dr. George Pedrick
Department of Mathematics
Hayward State University
Hayward, California 94542

Dr. Billy E. Rhoades
Department of Mathematics
Indiana University
Bloomington, Indiana 47401

Dr. Joseph M. Scandura
Graduate School of Education
University of Pennsylvania
Philadelphia, Pennsylvania 19104

Dr. Claude Schochet
Department of Mathematics
Indiana University
Bloomington, Indiana 47401

Dr. Judah Schwartz
Physics Department
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Dr. Oliver Selfridge
Lincoln Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Dr. George Springer
Department of Mathematics
Indiana University
Bloomington, Indiana 47401

Dr. Marc Swadener
School of Education
University of Colorado
Boulder, Colorado 80302

Dr. Maynard Thompson
Department of Mathematics
Indiana University
Bloomington, Indiana 47401

Dr. Paul Trafton
School of Education
Indiana University
Bloomington, Indiana 47401

Dr. Zalman Usiskin
Graduate School of Education
University of Chicago
5835 S. Kimbark Ave.
Chicago, Illinois 60637

Dr. Irvin E. Vance
New Mexico State University
Department of Mathematics
Las Cruces, New Mexico 88001

Dr. Marian Visich
Polytechnic Institute of Brooklyn
Graduate Center
Rt. 110
Framindale, N.Y. 11735

Dr. David Wells
Supervisor for Mathematics
Oakland County Schools
2100 Pontiac Lake Road
Pontiac, Michigan 48054

Dr. Hassler Whitney
Institute for Advanced Study
Princeton, N.J. 08540

Dr. Stephen Willoughby
Department of Mathematics
Education
New York University
New York, N.Y. 10003

Dr. Lauren Woodby
Department of Mathematics
Wells Hall
Michigan State University
East Lansing, Michigan 48823

Dr. Gail S. Young
Department of Mathematics
University of Rochester
Rochester, N.Y. 14627

Dr. Jerrold R. Zacharias
Physics Department
Massachusetts Institute
Cambridge, Massachusetts 02139

## Working Group Assignments

**Group 1**

Dilworth, Chairman
Greenes
Hoffman
Johnson
Kreider
LeBlanc
Moore, Recorder
Papert
Scandura
Usiskin
Woodby

**Group 2**

Begle
Botts, Recorder
Buccino
Greenberg
Kilpatrick
Schochet
Schwartz
Trafton
Vance
Whitney
Young, Chairman

**Group 3**

Anderson
Braunfeld
Davis
Dwyer
Howard
Kerr
Klamkin, Chairman
Lester
Molnar
Noether
Pedrick, Recorder

**Group 4**

Binder
Charp
Gray, Chairman
Kline
Link
Springer
Swadener, Recorder
Thompson
Visich
Willoughby
Zacharias

**Group 5**

Balomenos, Recorder
Gelman
Immerzeel
Jones
Kohlerman
LeVeque
Lomon
Maki
Rhoades
Selfridge
Wells, Chairman