This volume contains a series of teacher-developed units to supplement the textbook in a high school geometry course. Each unit contains a statement of objectives, content discussion, worksheets, and activity suggestions. Major topics include logic, proofs, ratio and proportion, similarity, and trigonometry. Practical applications are given in each unit where possible. Related volumes in the series are SE 016 through SE 016 617. (LS)
GEOMETRY

MADEHEIM
SAHUARITA HIGH SCHOOL
CAREER
CURRICULUM
PROJECT

COURSE TITLE: GEOMETRY
FIRST QUARTER
BY
JAMES MADEHEIM
Table of Contents
(1st quarter)

Unit 310 Reasoning

Sub-unit 312 Logical thinking
Sub-unit 314 Venn Diagrams
Sub-unit 316 Basic Definitions in Geometry
Sub-unit 318 Induction

Unit 320 Probability

Unit 330 Proofs involving angles

Sub-unit 331 (See Sub-unit 370.1)
Sub-unit 332 Perpendicular lines
Sub-unit 336 Parallel Lines
Introduction to the First Quarter.

The first quarter of geometry will deal with logic procedures, basic geometry, probability, and the beginning of two column proofs.

The logic part will be used to demonstrate that if you have a definite method for solving a problem, instead of haphazardly guessing, the problem is more easily solved.

The unit on probability is an introduction into the terminology of probability and involves examining the outcomes of tossing coins and rolling dice.

The rest of the quarter is spent on developing the ideas necessary for two-column proofs.
Objectives.

During this quarter's work, you will increase your knowledge of careers in transportation and of some of the basic requirements necessary for every job.

You will increase your awareness of skills necessary for methods of logically analysing thoughts and for working out certain types of navigation problems.

Given a logic problem that can be solved systematically, you will be able to set up some kind of procedure to aid your finding the correct solution, and then find the correct solution for at least half the problem.

Given a group of statements that can be represented by Venn diagrams, you will be able to reach a valid conclusion relating to the given statements. Given a test covering the above mentioned skills, you should get 60% of the test questions correct.

Given many groups of objects, you should be able to examine the particular objects of a group and then make a general statement about the entire group. The test for the above skills will be a lab test. You will take the test in groups of three or four students and then the teacher will determine your proficiency.
Given the necessary data, you should be able to find the probability of an event happening, as demonstrated by correctly answering questions asked of you during an oral exam.

Given an angle to measure with a protractor, you will measure the angle within one degree.

Given the measure of two angles of a triangle, you will find the measure of the third angle within one degree.

Given two parallel lines cut by a transversal, you will name eight pairs of equal angles.

Given the height of a pseudo-lighthouse, a theodolite to measure angles with, and a table of angles of depression, you will measure distances from the pseudo-lighthouse to various pseudo-boats to the nearest whole number of feet.

Given a statement to prove - within the framework of unit 332 - you will be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- substitution
- angle addition
- addition property of equality
- angle bisection
- supplementary angles
- complementary angles
- vertical angles
- all right angles are equal
- perpendicular lines form right angles

Given a test covering the above mentioned materials, you will be able to answer 69% of the questions correctly.
Given a statement to prove - within the framework of sub-units 332 and 336 - you will be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- corresponding angles
- alternate interior angles
- the sum of the angles in a triangle is 180°

Given a test covering this material, you will demonstrate your proficiency by getting at least 60% of the problems correct.
S. I. C. S. PAK

STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE

SERIES

EXPLORATION

NUMBER

312

JUSTER

All

AREA

THINKING

TITLE

"Oh, no!!" "I quit. I refuse!!" "It's impossible."
Rationale:

Intuition is fine, sometimes. Guess work is quick, except when you guess wrong. A systematic approach to solving a problem is usually the best way to be sure your problem gets solved correctly.

This unit develops several "systematic approaches" for solving those familiar logic problems that always seem impossible at first glance.

Behavioral Objectives:

Given a logic problem that can be solved systematically, you will be able to set up some kind of procedure to aid your finding the correct solution, and then find the correct solution for at least half the problem.
Pre-evaluation:

Nine men - Brown, White, Adams, Miller, Green, Hunter, Knight, Jones, and Smith - play the several positions on a baseball team. (The battery consists of the pitcher and the catcher; the infield consists of the first, second, and third basemen and the shortstop; the outfield consists of the right, left, and center fielders). Determine from the following data, the position played by each man.

a. Smith and Brown each won $10 playing poker with the pitcher.

b. Hunter was taller than Knight and shorter than White, but each of these weighed more than the first baseman.

c. The third baseman lives across the corridor from Jones in the same apartment house.

d. Miller and the outfielders play bridge in their spare time.

e. White, Miller, and Brown, the right fielder, and the center fielder were bachelors; the rest were married.

f. Of Adams and Knight, one played outfielder position.

g. The right fielder was shorter than the center fielder.

h. The third baseman was brother to the pitcher's wife.

i. Green was taller than the infielders and the battery, except for Jones, Smith, and Adams.

j. The third baseman, the shortstop, and Hunter made $150 each speculating in U.S. Steel.

k. The second baseman was engaged to Miller's wife.
1. The second baseman beat Jones, Brown, Hunter and the catcher at cards.

m. Adams lives in the same house as his own sister but dislikes the catcher.

n. Adams, Brown, and the shortstop lost $200 each speculating in copper.

o. The catcher had three daughters, the third baseman had two sons, but Green was being sued for divorce.

Three men - A, B and C - are aware that all three of them are "perfect logicians" who can instantly deduce all the consequences of a given set of premises. There are four red and four green stamps available. The men are blindfolded and two stamps are pasted on each man's forehead. The blindfolds are removed.

A, B and C are asked in turn: "Do you know the color of your stamps?" Each says: "No." The question is then asked of A once more. He again says: "Np." B is now asked the question, and replies: "Yes." What are the colors of B's stamps?
Of the three prisoners in a certain jail, one had normal vision, the second had only one eye, and the third was totally blind. All were of at least average intelligence. The jailer told the prisoners that from three white hats and two red hats he would select three and put them on the prisoner's head. Each was prevented from seeing what color hat was placed on his own head. They were brought together, and the jailer offered freedom to the prisoner with normal vision if he could tell what color hat was on his head. The prisoner confessed that he couldn't tell. Next the jailer offered freedom to the prisoner with only one eye if he could tell what color hat was on his head. The second prisoner confessed that he couldn't tell.

The jailer did not bother making the offer to the blind prisoner but agreed to extend the same terms to him when he made the request. The blind prisoner then smiled broadly and said:

"I do not need to have my sight; From what my friends with eyes have said I clearly see my hat is..."
Below is a list of all possible ways a hat can be chosen.

<table>
<thead>
<tr>
<th>2 eyes</th>
<th>1 eye</th>
<th>blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 w</td>
<td>w</td>
<td>w</td>
</tr>
<tr>
<td>2 w</td>
<td>r</td>
<td>r</td>
</tr>
<tr>
<td>3 r</td>
<td>w</td>
<td>r</td>
</tr>
<tr>
<td>4 r</td>
<td>r</td>
<td>w</td>
</tr>
<tr>
<td>5 w</td>
<td>w</td>
<td>r</td>
</tr>
<tr>
<td>6 w</td>
<td>r</td>
<td>w</td>
</tr>
<tr>
<td>7 r</td>
<td>w</td>
<td>w</td>
</tr>
</tbody>
</table>

Since only two red hats were used at the most, if two eyes saw two red hats he would know that was white and would have said so. But he didn't know. Therefore # 2 above is not possible.

By the same reasoning 1 eye couldn't tell he was white so he must not have seen two red hats. So # 3 is out.

Also, 1 eye knows he and blind don't both have red hats, because two eye didn't say anything.

Therefore if 1 eye saw a red hat on blind, he would know that he himself must be white, because they both weren't red. But he didn't know this since he didn't speak. Therefore 1 eye couldn't have seen a red hat on the blind man.

Therefore the blind man, who reasoned this all out, must be white.
Benno Torelli, genial host at Hamtramck's most exclusive nightclub, was shot and killed by a racketeer gang because he fell behind in his protection payments. After considerable effort on the part of the police, five men were brought before the District Attorney, who asked them what they had to say for themselves. Each of the men made three, two true and one false. Their statements were:

Lefty: 1 I did not kill Torelli. 2 I never owned a revolver. 3 Spike did it.

Red: 1 I did not kill Torelli. 2 I never owned a revolver. 3 The other guys are passing the buck.

Spike: 1 I am innocent. 2 Butch is the guilty man. 3 Lefty lied when he said I did it.

Dopey: 1 I am innocent. 2 I never saw Butch before. 3 Spike is guilty.

Butch: 1 I did not kill Torelli. 2 Red is the guilty man. 3 Dopey and I are old pals.

WHODUNNIT?

Start this way. If Butch #1 is false, then Butch #2 is true. But this would say that both Butch and Red did the killing, which can't be. Therefore your original assumption is false, and it must be true that Butch did not kill Torelli. Use the same type of reasoning on Dopey #1 and #3. Use it also on Lefty #1 and #3.

Who Dunnit?
The sentences in the problem below are taken out of their paragraph form to make it easier to demonstrate how to solve this problem.

Five men who were buddies in the last war are having a reunion. They are White, Brown, Peters, Harper, and Nash, who by occupation are printer, writer, barber, neurologist, and heating-contractor, though not necessarily in that order. By coincidence they live in the cities of White plains, Brownsville, Petersburg, Harper's Ferry, and Nashville, though not necessarily in that order. But,

1. no man lives in the city having a similar name to his,
2. nor does the name of his occupation have the same initial as his name
3. or the name of the city in which he lives.
4. The barber doesn't live in Petersburg,
5. and Brown is neither heating-contractor nor printer --
6. nor does he live in Petersburg or Harper's Ferry.
7. Mr. Harper lives in Nashville and is neither barber or writer.
8. White is not a resident of Brownsville,
9. nor is Nash, who is not a barber, nor a heating-contractor.

What city does Nash live in. (See the next page for this problem written without numbered sentences.)
Five men who were buddies in the late war are having a reunion. They are White, Brown, Peters, Harper, and Nash, who by occupation are printer, writer, barber, neurologist, and heating-contractor. By coincidence, they live in the cities of White Plains, Brownsville, Petersburg, Harper's Ferry, and Nashville, but no man lives in the city having a name similar to his, nor does the name of his occupation have the same initial as his name or the name of the city in which he lives.

The barber doesn't live in Petersburg, and Brown is neither heating-contractor nor printer—nor does he live in Petersburg or Harper's Ferry. Mr. Harper lives in Nashville and is neither barber nor writer. White is not a resident of Brownsville, nor is Nash, who is not a barber, nor a heating-contractor.

If you have only the information given above, can you determine the name of the city in which Nash resides?
Due to sentence #1, the following boxes are crossed out.
Due to sentence # 7, the following additional boxes are crossed out.
<table>
<thead>
<tr>
<th></th>
<th>White Plains</th>
<th>Brownsville</th>
<th>Petersburg</th>
<th>Harpers Ferry</th>
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<tr>
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<tr>
<td>Brown</td>
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<tr>
<td>Harper</td>
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<tr>
<td>Mash</td>
<td></td>
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Due to #8 the following box is crossed out.
### BUDDIES

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<th>PEABODY</th>
<th>NASHVILLE</th>
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</thead>
<tbody>
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<td>B</td>
<td>W</td>
<td>B</td>
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<td>P</td>
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<tr>
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<td>C</td>
<td>W</td>
<td>C</td>
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<tr>
<td>P</td>
<td>N</td>
<td>P</td>
<td>N</td>
</tr>
</tbody>
</table>

The following portions are crossed out due to #9.
Due to sentence # 2, the following items are crossed out.
Due to #3 the following items are crossed out.

Because of this last information, you can see that Harper is a printer, and therefore no one else can be a printer. See if you can finish it from here.
Data Brief # 4

The mixture of problems in activity # 4 can be solved using many different approaches. Some of these approaches have already been discussed. In every case possible, set up a table to aid your finding the solution.
Activity # 1

Three men go by turn into a dark closet where hang five hats, three red and two blue. Out they come, each man forbidden to look at his own hat, but permitted to look at the hats of the others in an effort to tell the color of his own. A glances at B and C, and says, "I don't know what color hat I have one." B, who is equally intelligent, looks around and says, "Nor do I know what color hat I have on." What color hat was C wearing and hat was C wearing and how did he figure it out?

Three men are blindfolded and told that either a red or green hat will be placed on each of their heads. After this is done, the blindfolds are removed; the men are asked to raise a hand if they see a red hat, and to leave the room as soon as they are sure of the color of their own hat. All three hats happen to be red, so all three men raise a hand. Several minutes go by until one of them who is more astute than the others, leaves the room. How did he deduce the color of his hat?
Daniel Kilraine was killed on a lonely road, two miles from Pontiac at 3:30 a.m., March 17, 1952. Otto, Curly, Slim, Mickey, and the Kid were arrested a week later in Detroit and questioned. Each of the five made four statements, three of which were true and one of which was false. One of these men killed Kilraine. Whodunnit? Their statements were:

Otto: I was in Chicago when Kilraine was murdered. I never killed anyone. The Kid is the guilty man. Mickey and I are pals.

Curly: I did not kill Kilraine. I never owned a revolver in my life. The Kid knows me. I was in Detroit the night of March 17.

Slim: Curly lied when he said he never owned a revolver. The murder was committed on St. Patrick's Day. Otto was in Chicago at this time. One of us is guilty.

Mickey: I did not kill Kilraine. The Kid never has been in Pontiac. I never saw Otto before. Curly was in Detroit with me on the night of March 17.

The Kid: I did not kill Kilraine. I have never been in Pontiac. I never saw Curly before. Otto lied when he said I am guilty.
Activity # 3

BRAIN TEASER

HERE IS AN INTERESTING ONE WHICH WILL TEST YOUR LOGICAL ABILITY. ALL THE FACTS NEEDED TO ANSWER THE QUESTIONS POSED ARE CONTAINED IN STATEMENTS 1-15 BELOW:

1. There are five houses, each of a different color and inhabited by men of different nationalities, with different pets, drinks and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drink in the green house.
5. The Ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drink in the middle house.
10. The Norwegian lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The lucky strike smoker drinks orange juice.
15. The Norwegian lives next to the blue house.

How, who drinks water? Who owns the zebra?

SEE THE NEXT PAGE FOR AN AID TO THIS PROBLEM
<table>
<thead>
<tr>
<th>Color</th>
<th>Drink</th>
<th>Pet</th>
<th>Cig</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>T</td>
<td>H</td>
<td>C N</td>
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<tr>
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<td>P</td>
<td>U</td>
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<tr>
<td>G</td>
<td>0</td>
<td>O</td>
<td>D S</td>
<td>O L</td>
</tr>
</tbody>
</table>
Smith, Jones and Robinson are the engineer, brakeman and fireman on a train, but not necessarily in that order. Riding the train are three passengers with the same three surnames, to be identified in this problem by a "Mr." before their names.

Mr. Robinson lives in Los Angeles
The brakeman lives in Omaha
Mr. Jones long ago forgot all the algebra he learned in high school.

The passenger whose name is the same as the brakeman's lives in Chicago.

The brakeman and one of the passengers, a distinguished mathematical physicist, attend the same church.

Smith beat the fireman at billiards.

Who is the engineer?
<table>
<thead>
<tr>
<th>Engineer</th>
<th>Brakeman</th>
<th>Fireman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jones</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robinson</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Los Angeles</th>
<th>Omaha</th>
<th>Chicago</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. Smith</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mr. Jones</td>
<td></td>
<td></td>
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<tr>
<td>Mr. Robinson</td>
<td></td>
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</tr>
</tbody>
</table>
The members of a small loan company are Mr. Black, Mr. White, Mrs. Coffee, Miss Ambrose, Mr. Kelly, and Miss Earnshaw. The positions they occupy are manager, assistant manager, cashier, stenographer, teller, and clerk, though not necessarily in that order. The assistant manager is the manager's grandson; the cashier is the stenographer's son-in-law; Mr. Black is a bachelor. Mr. White is twenty-two years old; Miss Ambrose is the tellers step-sister; and Mr. Kelly is the managers neighbor.

Who holds each position?

<table>
<thead>
<tr>
<th>POSITION</th>
<th>ASST. MANAGER</th>
<th>MANAGER</th>
<th>CASHIER</th>
<th>STENO</th>
<th>TELLER</th>
<th>CLERK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. White</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Mr. Black</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mrs. Coffee</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Miss Ambrose</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mr. Kelly</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miss Earnshaw</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

The grid above indicates the positions filled by the members of the loan company.
Mrs. Adams, Mrs. Baker, Mrs. Catt, Mrs. Dodge, Mrs. Ennis and that dowdy Mrs. Fisk all went shopping one morning at the Emporium. Each woman went directly to the floor carrying the article which she wanted to buy, and each woman bought only one article. They bought a book, a dress, a handbag, a necktie, a hat, and a lamp.

All the women except Mrs. Adams entered the elevator on the main floor. Two men also entered the elevator. Two women, Mrs. Catt and the one who bought the necktie, got off at the second floor. Dresses were sold on the third floor. The two men got off at the fourth floor. The woman who bought the lamp got off at the fifth floor, leaving that dowdy Mrs. Fisk all alone to get off at the sixth floor.

The next day Mrs. Baker, who received the handbag as a surprise gift from one of the women who got off at the second floor, met her husband returning the necktie which one of the other women had given him. If books are sold on the main floor, and Mrs. Ennis was the sixth person to get out of the elevator, what did each of those women buy?
## Shopping

<table>
<thead>
<tr>
<th></th>
<th>Book</th>
<th>Dress</th>
<th>Handbag</th>
<th>Nectile</th>
<th>Hat</th>
<th>Lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adams</strong></td>
<td>3 M 6</td>
<td>5 M 6</td>
<td>5 M 6</td>
<td>5 M 6</td>
<td>5 M 6</td>
<td>5 M 6</td>
</tr>
<tr>
<td><strong>Baker</strong></td>
<td>2 M 3</td>
<td>3 M 4</td>
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<td>3 M 4</td>
<td>3 M 4</td>
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<tr>
<td><strong>Catt</strong></td>
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<td>3 M 4</td>
<td>3 M 4</td>
<td>3 M 4</td>
<td>3 M 4</td>
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<td><strong>Fisk</strong></td>
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<td>3 M 4</td>
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</tr>
</tbody>
</table>
A woman recently gave a tea party to which she invited five guests. The names of the six women who sat down at the circular table were Mrs. Abrams, Mrs. Banjo, Mrs. Clive, Mrs. Dumont, Mrs. Ekwall, and Mrs. Fish. One of them was deaf, one was very talkative, one was terribly fat, one simply hated Mrs. Dumont, one had a vitamin deficiency, and one was the hostess.

The woman who hated Mrs. Dumont sat directly opposite Mrs. Banjo. The deaf woman sat opposite Mrs. Clive, who sat between the woman who had vitamin deficiency and the woman who hated Mrs. Dumont. The fat woman sat opposite Mrs. Abrams, next to the deaf woman and to the left of the woman who hated Mrs. Dumont. The woman who had a vitamin deficiency sat between Mrs. Clive and the woman who sat opposite the woman who hated Mrs. Dumont. Mrs. Fish, who was a good friend of everyone, sat next to the fat woman and opposite the hostess.

Can you identify each of these lovely women?
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEAF</td>
<td>TALKATIVE</td>
<td>FAT</td>
<td>HATED</td>
<td>VITAMIN</td>
<td>DEFICIENCY</td>
</tr>
<tr>
<td>ABRAHS</td>
<td>BANJO</td>
<td>CLIVE</td>
<td>ERIWALL</td>
<td>FISH</td>
<td></td>
</tr>
</tbody>
</table>

Diagram of a table layout with six places labeled B, *, 4.
Activity # 4

Three neighbors, Mr. Carpenter, Mr. Mason, and Mr. Painter, have different occupations. By a strange coincidence, their names are the same as their trades, but not necessarily respectively.

Of the following statements, only one is true.

- Mr. Carpenter is not a painter.
- Mr. Mason is not a carpenter.
- Mr. Carpenter is a carpenter.
- Mr. Mason is not a painter.

Who has what occupation?

Acres, Hull, De Maria, and Scott are sports-car drivers whose cars are Mercedes, Austin-Healey, Porsche, and Maserati--though not necessarily respectively. The following statements are true:

- a. Both Hull and the driver of the Maserati have spent week ends with the driver of the Austin Healey.
- b. Acres and De Maria were at the track the day the Porsche driver ran over a Siamese cat.
- c. The Maserati driver who has helped Scott to get membership in the Gear-Box Club is planning to do the same for Acres.
- d. Acres had not yet met De Maria.

Question: Who drives which car?
Five men are in a poker game: Brown, Perkins, Turner, Jones, and Reilly. Their brands of cigarettes are Luckies, Kools, Old Golds, and Chesterfields, but not necessarily in that order. At the beginning of the game, the number of cigarettes possessed by each of the players was 20, 15, 8, 6, and 7, but not necessarily in that order.

During the game, at a certain time when no one was smoking, the following conditions obtained:

a. Perkins asked for three cards.

b. Reilly had smoked half of his original supply, or one less than Turner smoked.

c. The Chesterfield man originally had as many more, plus half as many more, plus 2 1/2 more cigarettes than he now has.

d. The man who was drawing to an inside straight could taste only the menthol in his fifth cigarette, the last one he smoked.

e. The man who smokes Luckies had smoked at least two more than anyone else, including Perkins.

f. Brown drew as many aces as he originally had cigarettes.

g. No one had smoked all his cigarettes.

h. The Camel man asks Jones to pass Brown's matches.

How many cigarettes did each man have to begin with, and of what brand?
Andy, Bob, Charlie, Don, and Ed started together for their vacation resorts driving a baby Austin, Buick, Cadillac, Ford, and Oldsmobile. Each agreed to send a postcard to the home of all of the drivers each time he crossed a new state line. The clues are:

a. Don went to a different resort this year to avoid playing golf with the Olds driver.

b. When the group arrived at a crowded ferryboat, Andy, though the last one in line, was the only one to get across on that trip. He received five more cards than he mailed.

c. Don, brother of the Buick driver, dropped back from the group because his daughter was holding hands with the banker's son in the Cadillac ahead.

d. The name of Charlie's car has as many letters as the number of cards he sent out, plus half of that many, plus the fraction 1/2. Andy sent out twice as many cards as Charlie.

e. When the Olds driver had half his number of states, he signaled to Bob that Andy was entering his vacation place.

f. Ed wished the Ford driver luck on the remainder of his journey.

g. Bob sent out the same number of postcards as he received but the Ford driver did not do so well on this score.

Questions:

Identify each driver's car.

How many states did each enter?
Three neighbors, Mr. Carpenter, Mr. Mason, and Mr. Painter, have different occupations. By a strange coincidence, their names are the same as their trades, but not necessarily respectively.

Of the following statements, only one is true.

Mr. Carpenter is not a painter.
Mr. Mason is not a carpenter.
Mr. Carpenter is a carpenter.
Mr. Mason is not a painter.

Who has what occupation?
Post-Test:

1. Joe and the third baseman lived in the same building.
2. Bob, Joe and Frank and the catcher were beaten at golf by the second baseman.
3. Ed was a very close friend of the catcher.
4. The center fielder was taller than the right fielder.
5. The shortstop, the third baseman and Frank each liked to go to the races.
6. The pitcher's wife was the third baseman's sister.
7. Bill's sister was enraged to the second baseman.
8. Bob and Harry each won $5 from the pitcher at poker.
9. The catcher and the third baseman each had two children.
10. Jim decided to get a divorce.
11. All of the battery and the infield, except Harry, Joe, and Ed were shorter than Jim.
12. Bill and the outfielders like to play Gin Rummy together whenever they could.
13. Jack was taller than Frank. Tom was shorter than Frank. Each weighed more than the 1st baseman.
14. One of the outfielders was either Tom or Ed.
15. Bill, Bob and Jack, the center fielder and the right fielder were bachelors. The others were married.
16. Bob, Ed and the shortstop were teetotalers.

What is the name for the man at each position?
In a certain mythical community, politicians always lie, and non-politicians always tell the truth. A stranger meets three natives, and asks the first of them if he is a politician. The first native answers the question. The second native then reports that the first native denied being a politician. Then the third native asserts that the first native is really a politician.

How many of these natives are politicians?

On a certain train, the crew consists of three men, the brakeman, the fireman, and the engineer. Their names listed alphabetically are Jones, Robinson, and Smith. On the train are also three passengers with corresponding names, Mr. Jones, Mr. Robinson, and Mr. Smith. The following facts are known:

a. Mr. Robinson lives in Detroit.
b. The brakeman lives halfway between Detroit and Chicago.
c. Mr. Jones earns exactly $20,000 a year.
d. Smith once beat the fireman at billiards.
e. The brakeman's next-door neighbor, one of the three passengers mentioned, earns exactly three times as much as the brakeman.
f. The passenger living in Chicago has the same name as the brakeman.

What was the engineer's name?
SUB-UNIT 314

Given a group of statements that can be represented by Venn diagrams, you will be able to reach a valid conclusion relating to the given statements.

Given a test covering the above mentioned skills, you should get 60% of the test questions correct.

Do W/S 314.2
Do W/S 314.4
Do W/S 314.6
Do W/S 314.7

Take Test 314
IN EACH DRAWING, GIVE A FULL DESCRIPTION OF WHERE THE X'S ARE.

A

B

C

A

B

C

A

B

C
DRAW THREE INTERLOCKING CIRCLES FOR EACH PROBLEM. LABEL THEM A, B, C.

PUT X’S IN

1. \(A \cap B\)
2. \(A \cap C\)
3. \(\overline{B}\)
4. \(\overline{B} \cap \overline{A}\)
5. \(\overline{B} \cup \overline{A}\)
6. \(A \cup \overline{B} \cup C\)
7. \(A \cap B \cup \overline{C}\)
8. \(\overline{A} \cup B\)
9. \(\overline{A} \cup (A \cap B)\)
10. \(A \cup (A \cap \overline{B})\)
11. \(A \cup (A \cap B)\)
12. \(A \cap [(C \cup B) \cup A]\)

LET \(O\) STAND FOR ORANGES AND LET \(C\) STAND FOR CITRUS FRUIT. NOW DRAW TWO CIRCLES THAT STAND FOR \(O, C\), SHOW THIS TO THE TEACHER,!!!

DRAW TWO CIRCLES THAT STAND FOR APPLES AND FOOD SHOW TO THE TEACHER!!!!
Draw three circles that stand for oranges, citrus fruit, and food.

Draw two circles that stand for the phrase "some mushrooms are poisonous".

Draw two circles that stand for "some men are in the army".

Draw two circles that stand for "all the girls that are boys".

Draw three circles that stand for everything that is blue, every car, every Pontiac.
EVERYTHING GREEN
EVERY LEAF
EVERYTHING THAT IS EDIBLE

ALL GIRLS
ALL BOYS
HIGH SCHOOL STUDENTS

Draw a Venn diagram of these statements:

If I have a lemon, then I have a citrus fruit.
Locate the spot within your Venn diagram that represents
the fact that I have a citrus fruit but it isn't a lemon. Mark
it with an X.

If I have a lemon, what can you conclude from your diagram?
If I have no citrus fruit, what can you conclude?
If I have no citrus fruit but do have a lemon, what can
you conclude?
In some of the following problems, there will be times when you can reach no conclusion from the given statements. If this is so, you must state “No conclusion.”

Given: All Edsels are lemons.

Statement: I have an Edsel.
Conclusion?

Statement: I have a lemon.
Conclusion??

Statement: I have a Ford.
Conclusion??

Statement: Joe’s father sells Edsels and he is going to get me a good one.
Conclusion??

Given: All hippies have long hair.

Statement: Joe is a hippie.
Conclusion??

Statement: Bill does not have long hair.
Conclusion??

Statement: I have long hair.
Conclusion??
Given: Anyone who has long hair and a beard is a no good bum
(Show your diagram of this statement to the teacher.)*

Statement: Jack's hair reaches his shoulders, and he hasn't shaved for six months.
Conclusion??

Statement: John is a no good bum.
Conclusion??

Statement: Marie is a no good bum.
Conclusion??

Statement: "Give a physical description of Jesus Christ"
Conclusion??

What are your comments on this last conclusion and on the 'given'
at the top of the page.

Statement: Mike has a beard.
Conclusion??

Statement: Bob has long hair.
Conclusion??

Statement: Susan has long hair.
Conclusion??
Given: An employer states that if you want to work for me you have to dress nicely and you have to be clean shaven.

Statement: Sally's clothes haven't been washed in 6 weeks and she sleeps in them also.
Conclusion??

Statement: Joe is one of the best dressed men in the city. He even trims his beard every day.
Conclusion??

Statement: Billy drives a new Cadillac convertible and is clean shaven
Conclusion??

Statement: Jack wants to work of this man, but he has a beard and never wears shoes or a shirt.
Conclusion??

Statement: Henry is a pretty good dresser, most of the time. He has a rather neat beard and his father is the boss of the man who set the employment rules. Henry applies for the job.
Conclusion??
GIVEN: If the car is in the garage, then the door is closed.

GIVEN: If Jones is at home, then the car is in the garage.

Using the two above statements draw a Venn diagram that illustrates their meaning. (You will have to use three circles). Check your drawing with the teacher before going any further.

According to the information given above:
"If the door is closed Jones is at home" is this statement true or false or you don't know.

"If the door is closed and the car is in the garage, then Jones is at home." Is this statement true, false, or you can't tell?

"Jones is at home" from this statement what can you conclude?

"The car is in the garage" what can you tell from this?
DETERMINE THE VALIDITY OF THE CONCLUSION FOR EACH PROBLEM

1. "If your daughter Ainslie looks like Grace Kelly, she is beautiful. But you say she does not look like Grace Kelly? Alas, then your daughter Ainslie is not beautiful."

2. "The dormouse has not buried the worm, for if he was buried the worm would be dead, and the worm is not dead."

3. "If and only if the priest is wearing red vestments, the Mass is for a martyr. Now, we know that this Mass is for a martyr. Therefore we know that the priest is wearing red vestments."

4. "Of course, this man is married, for if a man is married, he has a wife, and this man has a wife."

5. "That orangutan is not irritated. How do I know? Listen, my friend, if an orangutan is irritated, it growls, and this orangutan is not growling."
1. "Either you are keeping a pterodactyl in the bathtub or you are meeting secretly with a cipher clerk from the embassy. It is impossible that you can be doing both. Now, we have found out that you have been meeting secretly with a cipher clerk from the embassy. It is evident, therefore, that you are not keeping a pterodactyl in the bathtub."

7. "Either she loved me or she was deceiving me. I have confirmed the fact that she loved me. So I know she was not deceiving me."

8. "It can't be that the train has passed. If the train has passed, the green flag is up, and the green flag is not up."

11. "Either it's raining or it's not raining. It's not raining. Therefore, it's raining."
14. (1) If you like De Kooning's pictures, you have admirable taste. (2) If you don't like abstract art, you don't have admirable taste. Conclusion: If you don't like De Kooning's pictures, you don't like abstract art.

15. (1) Either that dugong is sick or it is nervous. (2) If it is not nervous, it will make a good pet for the children. Conclusion: Either it's not nervous or it will make a good pet for the children.

16. (1) Either that Jaguar does not belong to Francoise or she borrowed it from Alain. (2) If she borrowed it from Alain, she will get into trouble. (3) Either she did not borrow it from Alain or she won't get into trouble. Conclusion: That Jaguar belongs to Francoise.

17. (1) If the title is secure and a bank loan is withheld, we cannot go ahead with construction. (2) We are going ahead with construction. (3) The title is secure. Conclusion: The bank loan is not withheld.
10. (I) If you have the Torah, you have wisdom. Conclusion: If you have not wisdom, you have not the Torah.

11. (I) Either the beta particles are not penetrating the metal or the electron counter is off. (2) The beta particles are penetrating the metal. Conclusion: The electron counter is off.

12. (I) Either the man's Harvard graduate or he is not worth knowing. (2) He is worth knowing. Conclusion: He is not a Harvard graduate.

13. (I) If being a martyr implies a saint, then Margaret Clitheroe should be canonized. (2) It is not the case that one can be a martyr and not be a saint. Conclusion: Margaret Clitheroe should be canonized.
18. (1) If Swann loves Odette, he is willing to endure the Verdurins. (2) Either Swann is jealous or he loves Odette. (3) If Swann is willing to endure the Verdurins, he is jealous. (4) Either Swann is not willing to endure the Verdurins or he loves Odette. Conclusion: Swann loves Odette.

19. (1) If your mother comes, I leave. (2) If the children don't keep quiet, I leave. (3) If I leave, your mother does not come. (4) If the children keep quiet, then either I leave or your mother comes. (5) Either the children are not keeping quiet or I'm not leaving. Conclusion: If the children keep quiet, then I'm not leaving.
For each question circle either True, False, or Don't Know.

Given:

1. All Communists are Atheists
2. All Russian Politicians are Communists
3. Jones says that he is an American and believes in no religion

(Draw Venn diagrams for these statements)

1. Jones is a Commie. True, False, Don't Know
2. All atheists are Communists
   True, False, Don't Know
3. Vladimir SlovoVitsky is a Commie
   True, False, Don't Know
4. Vladusky is an atheist, a Russian and is not mixed up in politics. But he must be a communist.
   True, False, Don't Know
5. Vishnesvsky is an atheist and a politician, therefore he is a Commie
   True, False, Don't Know
6. Jones lied. His real name is Kusevinsky and he is a Russian Premier. Therefore I can't tell if he is an atheist or not.
   True, False, Don't Know
GIVEN:

If the robber came in the door, then it is an inside crime.

If it's an inside crime, then the butler did it.

The robber came in thru the window, therefore the butler couldn't have done it.

The butler can prove he was in a restaurant at the time of the crime, but since the front door was open, it must be an inside job.

The door was closed and the robber did not come in that way. Therefore it couldn't be an inside job.

The butler didn't do it. What can you conclude from this statement?
Either it's sunny outside or I'm not going to school. If I don't go to school I'll flunk. It's sunny outside, therefore I won't flunk. True or False

If you are intelligent and beautiful (girl), you will be married before you are 30.

Maria is intelligent and beautiful and 41. What can you conclude?

Alicia is beautiful but dumb. Therefore she is not married. Is this a valid conclusion?

Joan is intelligent and married. Therefore she is beautiful. Is this a valid conclusion?
Draw Venn diagrams to illustrate each set of statements and then write down your conclusion, arrived at from these diagrams. If there is no valid conclusion, state this.

1. All A is B
   All C is A.
   Conclusion?

2. All dogs bite mailman.
   Spot bit the mailman yesterday.
   Conclusion?

3. All girls like to go to the movies.
   Jane is a girl scout.
   Conclusion?

4. All boys like cars.
   Sandy doesn't like cars.
   Conclusion?

5. All dogs wag their tails and bite.
   Fido wags his tail and bites.
   Conclusion?

6. Anyone who protests is a communist.
   Kurtnevsky is a communist.
   Conclusion?

7. John is either a hippie or he is lazy.
   He is a hippie.
   Conclusion?
Sub-unit 316-318

316 This unit is a review of basic definitions and ideas that should be already familiar to the student. It is all lecture and there is no test.

318 Given many groups of objects you should be able to examine the particular objects of a group and then make a general statement about the entire group.

The test for the above skills will be a lab test. You will take the test in groups of three or four students and the teacher will determine your proficiency.

Materials: Geometry textbook

Lecture 316.2
- Do w/s 316.2

Lecture 316.4
- Do assigned problems

Read p. 53-55
- Do oral and written exercises with the teacher p. 55-7

Take test 318 (lab test)
Do the following written exercises.

P. 24-5 # 1-12
P. 29 # 1-12, 25-32
P. 32 # 1, 2, 3, 7
P. 38 # 11-19 ALL
UNIT 320

Given the necessary data, you should be able to find the probability of an event happening as demonstrated by correctly answering questions asked of you during an oral exam.

Materials: Insurance Company Booklet

Read each section and answer all questions. Show each set of answers to the teacher. Do pages 1-11. Do not start the section entitled "More On Sample Spaces" on page 11.

Do v/s 320.4

Take Oral Test
A) Event ($\bar{C}$)

B) Sample Space

C) Probability of your event (theoretical): $\frac{A}{B} \ P(\bar{E})_T$

D) Number of times you attempt your event

E) Number of times your event is successful

F) Probability of your event (actual): $\frac{E}{D} \ P(\bar{E})_A$

Probability of the complement of your event (theoretical)

$(1 - P(\bar{E})_T)$
Sub-unit 332

Given a statement to prove - within the framework of this unit - you must be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- Substitution
- Angle addition
- Addition property of equality
- Angle bisection
- Supplementary angles
- Complementary angles
- Vertical angles
- All right angles are equal
- Perpendicular lines form right angles

Given a test covering the above mentioned materials, you should be able to answer 60% of the questions correctly.

Materials: Geometry textbook

Read p. 123-7
- Do oral ex. with the teacher p. 129 #1, 2, 9, 10
- Do w/s 332.2
- Do w/s 332.4
- Do written ex. p. 129-30 #15-20, 21, 23, 25

Read p. 130-4
- Do oral ex. with the teacher p. 135 1-12
- Do written ex. p. 135-6 #1-16, 23-32

Read p. 137-9
- Do oral ex. with the teacher p. 140
- Do written ex. orally with the teacher p. 141
- Do written ex. p. 142 21-28, 35, 36, 37
- Do w/s 332.8

Take test 332
1. Why does \( \angle 5 + \angle 6 = \angle 7 \)?

2. \( \angle 4 = \angle 2 \) \hspace{1cm} \text{GIVEN}
   \( \angle 6 = \angle 2 \) \hspace{1cm} \text{GIVEN}
   \( \angle 4 = \angle 6 \) \hspace{1cm} ?

3. \( \angle 5 = \angle 2 \) \hspace{1cm} \text{GIVEN}
   \( \angle 5 + \angle 6 = \angle 2 + \angle 6 \) \hspace{1cm} ?
   \( \angle 6 = \angle 1 \) \hspace{1cm} \text{GIVEN}
   \( \angle 5 + \angle 6 = \angle 2 + \angle 1 \) \hspace{1cm} ?
   \( \angle 5 + \angle 6 = \angle 7 \) \hspace{1cm} ?
   \( \angle 2 + \angle 1 = \angle 9 \) \hspace{1cm} ?
   \( \angle 7 = \angle 9 \) \hspace{1cm} ?
If ∠QOX = 115°
∠ROX = 85°
∠SOX = 45°
∠TOX = 25°
Which angle measures
20°
40°
70°
30°
90°
60°

If ∠QOX = A°
∠ROX = B°
∠SOX = C°
∠TOX = D°
Which angle measures (B - C)°
(A - C)°
(B - D)°

If B - C = C - D, what two angles have the same measure?

Name two angles of which ray OQ is a common side.
For each problem on the following pages, you must draw a picture which illustrates the problem, label it, and then write down what is given and what is to be proved.

(Do not do the actual proof)

Example: In a right triangle the longest side is opposite the right angle.

Example: Prove that the sum of the angles in a triangle is 180°.

Example: If two sides of a triangle are equal, the angles opposite those sides are equal.
1. If two angles of a triangle are equal, the sides opposite those angles are equal.

2. If the three angles of a triangle are equal, the three sides of the triangle are equal.

3. If a segment joins the midpoints of two sides of a triangle, its length is one-half the length of the third side.

4. If a point is on the perpendicular bisector of a segment, then the point is equidistant from the end points of the segment.

5. The segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.
6. The bisectors of two adjacent supplementary angles form a right angle.

7. If the four sides of a quadrilateral are equal, any diagonal will bisect the two angles whose vertices it connects.

8. The diagonals of a rhombus are perpendicular.

9. In an isosceles triangle, a line drawn from the vertex to the midpoint of the base, is perpendicular to the base.

10. If the base angle bisectors of a triangle are equal, the triangle is isosceles.
11. The perpendicular bisectors of two chords intersect at the center of the circle.

12. The sum of the squares of the sides of a right triangle equal the square of the hypotenuse.
1. Name the angle which is the union of ray AB and ray AC.

2. If OA and OP are opposite rays, what kind of an angle is \( \angle AOB \)?

3. If ray DX bisects \( \angle CDE \), name two angles which have equal measure.

4. T F The angle addition theorem is the authority for writing \( \angle SOT + \angle ROS = \angle ROT \).

5. T F The measure of \( \angle SOT \) is \( Y \).

6. In the above figure, \( \overline{AB} \parallel \overline{BC} \). Find 1 in degrees if the ratio of \( \angle 1 \) to \( \angle 2 \) is 3:2.

In the proof which follows supply the reasons which have been omitted. Although you may not know the reason for a particular statement, you may fill in later spaces.

**Given:** \( \overline{AO} \perp \overline{OB} \), \( \overline{CO} \perp \overline{OD} \)  **Prove** \( \angle 1 = \angle 2 \)

**Proof**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \overline{AO} \perp \overline{OB} ); ( \overline{CO} \perp \overline{OD} ).</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle AOB ) and ( \angle COD ) are rt. ( A ).</td>
<td>b.</td>
</tr>
<tr>
<td>c. ( \angle AOB = \angle COD ).</td>
<td>c.</td>
</tr>
<tr>
<td>d. ( \angle 1 + \angle 3 = \angle AOB ); ( \angle 2 + \angle 3 = \angle COD ).</td>
<td>d.</td>
</tr>
<tr>
<td>e. ( \angle 1 + \angle 3 = \angle 2 + \angle 3 ).</td>
<td>e.</td>
</tr>
<tr>
<td>f. ( \angle 1 = \angle 2 ).</td>
<td>f.</td>
</tr>
</tbody>
</table>
12. If \( \angle 1 \) and \( \angle 2 \) are both supplementary to \( \angle 3 \), what relation exists between \( \angle 1 \) and \( \angle 2 \)?

13. How many pairs of vertical angles are formed when two lines intersect?

14. An angle has a measure of \( 2x - 10 \). Express the measurement of its complement.

15. If two angles are both equal and complementary, find the measure of each.

16. In the above figure \( R, S, \) and \( T \) are collinear and \( \angle 1 \) and \( \angle 3 \) are complementary. What kind of angle is \( \angle 2 \)?

17. Given \( A = x^\circ \). By what number of degrees does the supplement of \( \angle A \) exceed the complement of \( \angle A \)?

In Questions 18 and 19 a statement is given accompanied by a figure. If you were attempting to prove the statement, what would be the given and to prove in terms of the figure?

18. Statement: If two sides of a triangle are equal, the angles opposite those sides are equal.

19. Statement: The diagonals of a parallelogram bisect each other.
QUESTIONS 20 THRU 25 REFER TO THE FIGURE BELOW. CLASSIFY EACH STATEMENT AS TRUE OR FALSE.

20. If $\angle 2 > \angle 1$, then $\angle 2 > \angle 3$.  
21. If $\angle 1 = \angle 5$, then $\angle 2 = \angle 6$.  
22. If $\angle 2 > \angle 6$, then $\angle 1 > \angle 5$.  
23. If $\angle 5 = \angle 7$, then $\angle 8$ is a right angle.  
24. If $\angle 4 = \angle 7$, then $\angle 2 = \angle 6$.  
25. If $\angle 6 > \angle 5$, then $\angle 8 > \angle 7$.

In the following proof write the reasons which have been omitted. Although you may not know the reason for a particular step, you may fill in those for later steps.

Given: $\overline{AB}, \overline{CD}, \overline{CE}$ intersecting as shown, $\angle 1 = \angle 2$.

To Prove: $\angle 4 + \angle 3 = 180^\circ$.

PROOF

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle 1 = \angle 2$.</td>
<td>$\theta$ Given.</td>
</tr>
<tr>
<td>$\angle 2$ and $\angle 3$ are supplementary.</td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\angle 2 + \angle 3 = 180^\circ$.</td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\angle 1 + \angle 3 = 180^\circ$.</td>
<td>$\theta$</td>
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<tr>
<td>$\angle 1 = \angle 4$.</td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\angle 4 + \angle 3 = 180^\circ$.</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>
Sub-unit 336

Given a statement to prove - within the framework of this unit and sub-unit 332 - you must be able to form a rigorous proof of this statement using the following phrases in your reasoning:

Corresponding angles
Alternate interior angles
The sum of the angles in a triangle is 180°

Given a test covering this material you must demonstrate your proficiency by getting 60% of the problems correct.

Materials: Geometry textbook

Read p. 153-55
- Do oral ex. with teacher p. 155
- Do written ex. p. 156 # 3, 6-12, 15, 16, 17, 19, 21

Read p. 157-9
- Do oral ex. with the teacher p. 160
- Do written ex. p. 160-3 # 1-11, odd, 27

Read p. 163-5
- Do written ex. p. 166 # 1, 2, 4, 5

Read p. 166-8
- Do oral ex. p. 168-9 with teacher
- Do written ex. p. 169 # 1, 2, 5-19

Read p. 171-2
- Do oral ex. p. 173 # 4, 5
- Do written ex. p. 174-5 # 1-19 odd only

Read p. 176-7
- Do written ex. p. 178 # 1, 2, 3, 4, 7, 9, 11, 13, 15

Take test
TEST 336

USE THE FIGURE BELOW FOR THE FIRST FOUR QUESTIONS

1. **Name four sets of corresponding angles.**
   - ________

2. **Name two sets of alternate interior angles.**
   - ________

3. **Name two sets of alternate exterior angles.**
   - ________

4. **Name two sets of interior angles on the same side of the transversal.**
   - ________

USE THE FIGURE TO THE RIGHT FOR 5 AND 6.

5. \( \angle 1 = (2x + 30)°; \angle 3 = (3x + 20)° \) \( x = ? \)
   - ________

6. \( \angle 2 = (x + 42)°; \angle 3 = (5x - 8)° \) \( x = ? \)
   - ________

COMPLETE THE DEMONSTRATION

**Given:** \( j \parallel k; \angle 1 = \angle 4 \)

**Prove:** \( \angle 2 = \angle 3 \)

**Proof**

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
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<tbody>
<tr>
<td>( j \parallel k. )</td>
<td>Given.</td>
</tr>
<tr>
<td>( \angle 2 = \angle 1. )</td>
<td>7.</td>
</tr>
<tr>
<td>( \angle 1 = \angle 4. )</td>
<td>8.</td>
</tr>
<tr>
<td>( \angle 4 = \angle 3. )</td>
<td>9.</td>
</tr>
<tr>
<td>( \angle 2 = \angle 3. )</td>
<td>10.</td>
</tr>
</tbody>
</table>
Complete the demonstration.

Given: $a \parallel b; c \parallel d$.

To prove: $\angle 13 = \angle 4$.

11. **PROOF**

<table>
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</table>

What lines can you conclude are parallel (if none, write none) in the figure shown, if it is given that

12. $\angle 1 = \angle 5$?  
13. $\angle 2 = \angle 3$?  
14. $\angle 5 = \angle 6$?  
15. $\angle 2 = \angle 7$?  
16. $\angle 1 = \angle 4$  
17. $v \parallel w$ and $\angle 1 = \angle 7$?
18. In $\triangle ABC$ and $\triangle RST$: $\angle A = \angle R$ 
$\angle B = \angle S$. What can you conclude?

19. Can an exterior angle of a right triangle be acute?

20. In $\triangle ABC$: $\angle A = 60^\circ$, $\angle B = 40^\circ$
What is the measure of the exterior angle at $B$?

21. In the above picture, if $\angle 3 = 90^\circ$ and $\angle 2 = 45^\circ$, $\angle 1 = ?$

Complete the demonstration.

Given: $j \parallel n$; $k \parallel n$.

To Prove: $j \parallel k$.

**PROOF**

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
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<tbody>
<tr>
<td>$j \parallel n$; $k \parallel n$.</td>
<td>.33.</td>
</tr>
<tr>
<td>$\angle 1 = \angle 3$.</td>
<td>.33.</td>
</tr>
<tr>
<td>$\angle 2 = \angle 3$.</td>
<td>.24.</td>
</tr>
<tr>
<td>$\angle 1 = \angle 2$.</td>
<td>.25.</td>
</tr>
<tr>
<td>$j \parallel k$.</td>
<td>.26.</td>
</tr>
</tbody>
</table>
Complete the demonstration.
Given: $AB \perp AD; CD \perp AD$.
To Prove: $\angle B = \angle C$. 

? 7.
What are the chances of picking from a deck of cards, the

1. The Ace of Clubs?

2. The Four of Diamonds?

3. The King of Hearts?

4. A Queen

5. A 10

6. An Ace

7. Any Club

8. Any Spade

What are your chances of winning a car that is being rifled off if 4000 tickets are sold and you buy

9. One ticket

10. 2 tickets

11. 5 tickets

12. 25 tickets
SAHUARITA HIGH SCHOOL
CAREER CURRICULUM PROJECT

COURSE TITLE: GEOMETRY
SECOND QUARTER
BY
JAMES MADEHEIM
Table of Contents
(2nd quarter)

Unit 340  Proofs involving Triangles

Unit 350  Ratios and Similarity

Sub-unit 352  Ratio and Proportion
Sub-unit 356  Similar Triangles
Introduction to the second Quarter.

The first part of this quarter covers the standard geometry material dealing with triangles and two column proofs.

The second part deals with ratio and proportion. The student will have a choice in selecting which career oriented problems he wants to do to develop his knowledge of the use or ratios and proportions.
Objectives.

During this quarter's work you will increase your knowledge of careers in transportation and construction.

You will increase your awareness of skills necessary for mechanics, architects, electricians, masons, carpenters, and engineers.

Given a statement to prove - within the framework of Sub-unit 310 and Unit 330 - you will be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- sss
- sas
- hl
- asa
- aas
- ll
- ha
- la
- reflexive property
- corresponding parts of congruent triangles
- base angles of an isosceles triangle are equal

Given a test covering this material, you will be able to answer 60% of the questions accurately.

Given two numbers, you will be able to express their ratio in simplest form.

Given three members of a proportion, you will be able to find the fourth member.
Given a test covering the above mentioned skills, you will be able to answer 60% of the questions accurately.

Given two triangles, you will be able to prove them similar by the angle-angle theorem.

Given the ratio of the sides of two triangles and all the measurements of one of them, you will be able to determine all the measurements of the other triangle.

Given a test covering the above mentioned skills, you should be able to answer 60% of the questions accurately.
SUB-UNIT 340

Given a statement to prove - within the framework of this unit and Unit 339 - you must be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- SSS
- SAS
- HL
- ASA
- AAS
- LL
- HA
- LA
- Reflexive property
- Corresponding parts of congruent triangles
- Base angles of an isosceles triangle are equal

Given a test covering this material, you should be able to answer 60% of the questions accurately.

Materials: Geometry textbook

Read p. 189-91
  Do written ex. # 1, 5, 6

Read p. 193-4
  Do oral ex. with the teacher p. 195
  Do written ex. p. 196-7 # 1, 3, 5, 7, 11, 13, 15

Read p. 198
  Do oral ex. with teacher # 7-16
  Do written ex. p. 200 # 1-15 odd only

Read p. 201
  Do written ex. p. 203 # 1-17 odd only

Read p. 204
  Do written ex. p. 204-6 # 1-17, 21-27 odd only
Read p. 207-9

Do written ex. p. 217-2  #1-7, 11-21 odd only

Do w/s 340.5

Read p. 217-19

Take test
Prove the statements listed below using these definitions.

Parallelogram: A four-sided figure with opposite sides parallel.

Rhombus: A parallelogram with all sides equal.

Prove opposite sides of a parallelogram are equal.

Prove if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

A diagonal of a rhombus bisects two angles of the rhombus.

The diagonals of a rhombus are perpendicular to each other.
1. Suppose that a correspondence is set up between the vertices of \( \triangle ABC \) and \( \triangle JXN \) so that

Point \( A \) corresponds to point \( J \);
Point \( B \) corresponds to point \( X \);
Point \( C \) corresponds to point \( N \).

a. The three pairs of corresponding angles are __________

b. The three pairs of corresponding sides are __________

c. \( \overline{JN} \) corresponds to the side lying opposite __________ in \( \triangle ABC \).

d. \( \angle A \) corresponds to the angle included between sides __________ of \( \triangle JXN \).

2. When one plane figure fits exactly over another plane figure, the figures are said to ________.

3. When two triangles are congruent, how many parts of one triangle are equal to corresponding parts of the other triangle?

Write the abbreviation for the postulate, theorem, or corollary you would use to prove that \( \triangle ABC \cong \triangle RST \).

a. Given: \( \angle A = \angle R \); \( \angle B = \angle S \); \( AB = RS \).

b. Given: \( \angle A = \angle R \); \( \angle B = \angle S \); \( BC = ST \).

c. \( \angle C \) and \( \angle T \) are rt. \( \Delta \); \( AB = RS \); \( \angle A = \angle R \).

d. \( \angle C \) and \( \angle T \) are rt. \( \Delta \); \( AC = RT \); \( BC = ST \).

e. \( \angle C \) and \( \angle T \) are rt. \( \Delta \); \( AB = RS \); \( BC = ST \).

f. \( AB = RS \); \( BC = ST \); \( AC = RT \).
5. Complete the demonstration.

Given: \(\triangle ABC\) and \(\triangle ABD\) are rt. \(A\); \(AC = AD\).

To Prove: \(\triangle ABC \cong \triangle ABD\).

**PROOF**

<table>
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<th>REASON</th>
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6. Complete the demonstration.

Given: \(RT \parallel QS\); \(RX = SX\).

To Prove: \(\triangle RXT \cong \triangle SXQ\).

**PROOF**

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</table>
7. Complete the demonstration.

Given: \( XP = XQ; YP = YQ \).
To Prove: \( \angle P = \angle Q \).

**PROOF**

**STATEMENT**

**REASON**

8. Complete the demonstration.

Given: \( AB \parallel CD; AB = DC \).
To Prove: \( BP = CP \).

**PROOF**

**STATEMENT**

**REASON**

9. Complete the demonstration.

Given: \( XI = ZK; XY = ZW; \angle X = \angle Z \).
To Prove: \( \angle Y = \angle W \).

**PROOF**

**STATEMENT**

**REASON**
10. Quadrilateral $ABCD$, with diagonals $AC$ and $BD$, is shown. Select the most specific name (square, rectangle, rhombus, parallelogram, trapezoid, quadrilateral) that applies to figure $ABCD$ if it is given that:

- a. $AB \parallel CD; AD \parallel BC; \angle A$ is a rt. $\angle$.
- b. $BC \parallel AD; BC = AD$.
- c. $CD \parallel AB; CD \neq AB$.
- d. $AC \perp BC$.
- e. $DC \parallel AB; AD$ is not $\parallel$ to $BC$.
- f. $ABCD$ is a rhombus; $AC = BD$.
- g. $\triangle ADC \cong \triangle CBA$.
- h. $AC = BD; \overline{AC}$ and $\overline{BD}$ bisect each other.
- i. $\overline{AC}$ and $\overline{BD}$ are bisectors of each other.
- j. $ABCD$ is a parallelogram and $\overline{AC}$ bisects $\angle DAB$.

11. Complete the demonstration.

**Given:** Trapezoid $RSTQ$ with bases $RS$ and $TQ$; $RQ = ST = QT$.

**To Prove:** Ray $RT$ bisects $\angle QRS$.

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $RSTQ$ is a trapezoid.</td>
<td>a. Given.</td>
</tr>
<tr>
<td>b. $QT \parallel RS$.</td>
<td>b.</td>
</tr>
<tr>
<td>c. $\angle 1 = \angle 2$.</td>
<td>c.</td>
</tr>
<tr>
<td>d. $RQ = TQ$.</td>
<td>d.</td>
</tr>
<tr>
<td>e. $\angle 1 = \angle 3$.</td>
<td>e.</td>
</tr>
<tr>
<td>f. $\angle 2 = \angle 3$.</td>
<td>f.</td>
</tr>
<tr>
<td>g. Ray $RT$ bisects $\angle QRS$.</td>
<td>g.</td>
</tr>
</tbody>
</table>
SUB-UNIT 352

Given two numbers you should be able to express their ratio in simplest form.

Given three members of a proportion, you should be able to find the fourth member.

Given a test covering the above mentioned skills, you should be able to answer 60% of the questions accurately.

MATERIALS: Geometry Textbook

Read p. 229-31

Do Oral ex. with the teacher
Do written ex. p. 232, #1, 2, 3, 5, 6, 7, 13, 14A

Read p. 234-5

Do oral #4 with teacher
Do written p. 236 # 2, 3, 5, 7, 8, 9, 10, 11
Do W/S 352.2
Do W/S 352.4 or .5 or .6

TAKE TEST 352
1. What is the ratio of 9 inches to 1 foot?
2. What is the ratio of \( 1\frac{2}{3} \) yds. to \( 2\frac{1}{2} \) yds?
3. Given \( A = 30^\circ \), what is the ratio of the complement of \( A \) to the supplement of \( A \)?
4. If \( 5x = 6y \), what is the ratio of \( x \) to \( y \)?
5. Solve for \( x \): \( 3:5 = x:4 \).
6. A regular pentagon and a regular hexagon have equal perimeters. What is the ratio of a side of the pentagon to a side of the hexagon?
7. If \( 3ab = 2xy \), which of the following proportions is correct?
   
   \[(A) \quad \frac{3a}{2y} = \frac{x}{b} \quad \quad (B) \quad \frac{a}{y} = \frac{2x}{3b} \quad \quad (C) \quad \frac{a}{2y} = \frac{x}{3b}\]
8. Find the ratio of \( x \) to \( y \).
9. If \( \frac{x-2y}{x+3y} = \frac{2}{5} \), find the ratio of \( x \) to \( y \).
10. If \( \frac{a}{b} = \frac{c}{d} = \frac{d}{f} = \frac{5}{7} \), find \( \frac{a+c+e}{b+d+f} \).
1. Gunpowder contains two parts of sulphur, three parts of charcoal, and 15 parts of saltpeter. Find the number of pounds of saltpeter required to make 2000 lbs. of gunpowder.

2. Express as a decimal to two places the ratio of the weight of 1 cu. ft. of white pine, which is 25lb, to the weight of 1 cu. ft. of water which is 62.5lb. (Explain why the wood will float).

3. A gasoline engine cylinder has a volume of 75 cu. in. at the bottom of its piston stroke and 10 cu. in at the top of its piston stroke. Find the compression ratio. (Compression ratio equals full volume divided by compressed volume)

   If interested - explain how increasing the bore of the cylinder would effect compression. How about increasing the stroke? How about planing off the head? How about using thin-head gaskets? What does a high-dome piston do?

4. If your really interested in automobiles try this one.

   If your car has a 4.56 rear end ratio, this means that for one revolution of the rear wheels, the driveshaft (or engine) turns 4.56 times. This is done by two gears in the rear end (differentiai). How many teeth are there on each of the gears? (For instance, if one gear had two teeth, the other gear would have to have 9.12 teeth, and how do you make a gear with fractional amounts of teeth?)
WHEN AN ARCHITECT OR A MACHINE DESIGNER MAKES A DRAWING OF A STRUCTURE OR A MACHINE, HE CANNOT MAKE IT TO FULL SIZE. HE MUST SCALE IT DOWN, THAT IS, HE MUST MAKE THE DRAWING SMALLER THAN THE ORIGINAL OBJECT SO THAT IT WILL FIT AN ACCEPTABLE SIZE OF PAPER AND NOT BE UNWIELDY ON THE JOB. TO DO THIS, HE MUST SELECT A RATIO OF DRAWING SIZE TO FULL SIZE.

1. ON A PIECE OF PAPER (THIS SAME SIZE) MAKE A SCALE DRAWING OF A TABLE TOP THAT IS 6.5' x 10.25'. USE A SCALE OF 1" TO 12". (RECTANGULAR TABLE TOP)

2. NOW MAKE A SCALE DRAWING OF A RECTANGULAR TABLE TOP THAT IS 14' x 8'. WHAT SCALE ARE YOU GOING TO USE?

3. MAKE A DRAWING OF A CIRCULAR GARDEN PLOT 20' IN DIAMETER, AND AROUND THE OUTSIDE OF THE OF THIS GARDEN IS A CEMENT WALK 4' WIDE. WHAT SCALE ARE YOU GOING TO USE?

4. MAKE A DRAWING OF A TRIANGULAR CHIP OF SELENIUM THAT IS .1" LONG ON EACH SIDE, AND HAS A HOLE CENTERED IN IT THAT IS .01" IN DIAMETER. WHAT SCALE ARE YOU GOING TO USE?

5. IN MAKING A LAYOUT FOR A CIRCLE WHOSE DIAMETER IS .001", A MACHINE DESIGNER USES A SCALE OF 1" = .001". DRAW THE CIRCLE, WHAT SCALE ARE YOU GOING TO USE?

6. A RECTANGULAR BUILDING IS 200' x 135' AND IS DRAWN TO SCALE OF \(\frac{1}{200}\) = 1'-0". WHAT IS THE SIZE OF THE RECTANGLE THAT WILL REPRESENT THIS BUILDING ON A BLUEPRINT?
7. MAKE A DRAWING OF THE MACHINE BAR SHOWN BELOW TO A SCALE OF 6" = 1'-0"

8. A LAYOUT IS TO BE MADE TO A SCALE OF 1" = 1'-0". FIND THE DIMENSIONS ON THE DRAWING THAT WILL CORRESPOND TO THE FOLLOWING ACTUAL DIMENSIONS: (A) 5'-0", (B) 4'-0", (C) 3'-9", (D) 2'-3", (E) 2'-6", (F) 18".
Some dimension in construction drawings (for example, distance from base line and elevation of surfaces) are given in engineer's instead of carpenter's measure. Engineer's measure is expressed in feet and decimal parts of a foot or in inches and decimal parts of an inch, such as 100 1/10 feet or 11.14 inches. Carpenter's measure is expressed in yards, feet, inches and even-denominator fractions of an inch, such as 1/16, 1/8, 1/4, inch, inch, and inch. To convert engineer's measure to carpenter's measure, you can use ratio and proportion.

Let us say that you want to convert 102.14 feet to the nearest 1/16 inch. The 102 you do not need to convert, since it is already in feet. What you do first is to find out how many twelfths of a foot (inches) there are in a foot. Set this up as a proportional equation as follows:

\[
\frac{x}{12} = \frac{14}{100}
\]

You want to find \(x\), which stands for how many twelfths (or inches) is equivalent to 14 100's.

Cross multiplying will yield

\[
100x = 12(14)
\]

\[
100x = 168
\]

\[
x = 168/100
\]

\[
x = 1.68 \text{ inches}
\]

This means \( \frac{1.68}{12} = \frac{1.4}{100} \)

Therefore 102.14 feet equals 102' 1.68". Now the .68 has to be converted to 16's because we specified to the nearest 1/16 inch.
We want to know how many 16's equals 68-100's, or

\[ \frac{x}{16} = \frac{68}{100} \]

Cross multiplying yields

\[ 100x = 16(68) \]
\[ 100x = 1088 \]
\[ x = \frac{1088}{100} \]
\[ x = 10.88 \text{ (sixteenths)} \]

We have to know a whole number of sixteenths. 10.88 rounded off to a whole number is 11. Thus \( .6\bar{8} = \frac{11}{16} \).

Therefore 102.14 feet = 102'1\frac{11}{16}''

In the following problems convert engineer's measure to carpenter's measure, (to the nearest 16th of an inch)

105.75'  87.79'  13.7'
348.1'  3.5'  19.26'

Convert to the nearest 8th of an inch

108.23'  19.125'  653.54'

Convert the figures below to engineer's measure to the nearest 100th of an inch. Think !!!!!!!

105'11''  56'1''  85'4''
156'3\frac{5}{8}''  1823'6\frac{1}{2}''  45'1\frac{3}{16}''
There are many other practical applications of ratio and proportion in the construction field. Suppose, for example, that a table tells you that, for the size and type of brick wall you happen to be laying, 12,321 bricks and 195 cubic feet of mortar are required for every 1000 square feet of wall. How many bricks and how much mortar will be needed for 750 square feet of the same wall? You set up the equation as follows:

12,321 is to 1000 as x is to 750 or

\[
\frac{12321}{1000} = \frac{x}{750}
\]

Cross multiply.

\[
1000x = 9,240,750
\]

\[
x = 9,240.75
\]

\[
x = 9,241 \text{ bricks}
\]

How many bricks and how much mortar is needed for walls of the following dimensions?

1200 sq. ft.
69 sq. ft.
800 sq. ft.
846 sq. ft.
1045 sq. ft.
Suppose, for another example, that the ingredient proportions (by volume) for the type of concrete you are making are 1 cubic foot of cement to 1.7 cubic feet of sand to 2.8 cubic feet of coarse aggregate. Suppose you know by reference to a table that these ingredients combined in these amounts will yield 4.07 cubic feet of concrete. How much of each ingredient will be required to make 27 cubic feet (this is one cubic yard. Why?)

Cement

\[
\frac{1 \text{ cu.ft cement}}{4.07 \text{ cu.ft concrete}} = \frac{x \text{ cu.ft cement}}{27 \text{ cu.ft concrete}}
\]

\[
\frac{1}{4.07} = \frac{x}{27} \quad x =
\]

Sand

\[
\frac{1.7}{4.07} = \frac{x}{27} \quad x =
\]

Coarse aggregate

\[
\frac{2.8}{4.07} = \frac{x}{27} \quad x =
\]

Using the information in the above paragraph, how much of each ingredient will be required to make

1 1/2 cu.yd. 35 cu.ft. 3 cu.yd.

18 cu.ft.
You're driving an automobile that can do 100 miles per hour, not just on the speedometer - that can be anything from 75 up - but an honest 100 by a stop watch. You are at the beginning of a five-mile straight stretch of concrete, so you stick your foot into it, hit and hold the 100 mark. Ahead of you there is a parked car, white, low and mean-looking. As you pass, the fellow behind the wheel guns his engine and starts after you. You keep your foot hard down, and well before you've covered a mile, you hear a brutal scream, a roar that sears your eardrums, a whoosh, and the white car has passed you. Another quarter of a mile and he's out of sight.

That, gentlemen, is acceleration. Let us have no nonsense about how fast your car is away from the lights; never mind the time you spun the wheels in second gear on dry concrete. The man in the white car spotted you 100 miles per hour and a running start and almost blasted you off the road when he went by you within a single mile. What manner of automobile can this be? A thirty-five-year-old model, the Type W125 Mercedes-Benz. Cylinders? Eight. Horsepower? Maximum 646. Speed? Something over 200 miles per hour. Was it a freight-car size monster? Not at all. With fuel and water and driver aboard it weighed less than 2500 pounds - half a ton less than the ordinary sedan. It was designed to be raced on ordinary two-lane roads and did so with great success. (It didn't just travel in a straight line for one short burst of speed, like a dragster). Only in the summer of 1951 did the pre-war records of the Mercedes-Benz begin to be cracked.*

* This story is taken from The Kings of the Road by Ken W. Purdy.
\( X = \) rear end ratio
\( R = \) engine speed (rpm)
\( Z = \) wheel speed (rpm)
\( r = \) rolling radius of wheel (ft)  Distance from road to wheel hub center.
\( q = \) rolling radius (inches)
\( A = \) car speed (mph)

\[ X = \frac{R \text{ rpm}}{Z \text{ rpm}} \]

This means that with a rear end ratio of 4.3:1, if your tach says 4300 rpm, then your rear wheels are turning 1000 rpm.

\[ A \text{ mph} = \frac{A(5280) \text{ ft}}{60 \text{ min}} = \frac{88A \text{ ft}}{\text{min}} \]

But \( 2\pi r \text{ ft} = 1 \text{ rev} \) Therefore \( 1 \text{ ft} = \frac{1 \text{ rev}}{2\pi r} \)

\[ Z \text{ rpm} = \frac{88A \text{ ft}}{\text{min}} \cdot \frac{2\pi r \text{ min}}{1 \text{ rev}} = \frac{88A \text{ rev}}{2\pi r} \]

\[ Z \text{ rpm} = \frac{88A \text{ rev}}{2\pi r \text{ min}} = \frac{44A \text{ rpm}}{\pi r} \]

So \( x = \frac{R \text{ rpm}}{44A \text{ rpm}} = \frac{\pi r R}{44A} \quad r = q/12 \)

\[ X = \frac{\pi q R}{12(44)A} = \frac{\pi q R}{528A} = \frac{.006q R}{A} \]
Use of this formula assumes that your transmission is in its top gear and it has a 1:1 ratio. We will now generalize the formula so that it can be used no matter what your transmission gear ratio is.

Let \( q = 12\)", \( R = 3000\)rpm, \( A = 70\) mph, Then \( x = 3.09\)

Suppose you know that your second gear ratio is 2.5:1 and you run this same test in second gear. This 2.5 ratio means your engine is going to work harder (faster) at the same speed, or that your engine will stay at the same speed and you will go slower. In other words, at 3000 rpm you will not be able to go 70 mph while in second gear. You will go 2.5 times as slow or 70/2.5 = 28 mph.

You can put this ratio into the formula by either multiplying it times \( R \) or dividing it into \( A \).

E.G. If \( x = 3.09 \), \( R = 3000 \), \( q = 12 \) and \( T = 2.5 \), find \( A \) using both formulas below.

\[
X = \frac{0.006qRT}{A} \quad \text{OR} \quad X = \frac{0.006qR}{AT}
\]

In either case if \( T = 1 \), you will get the original 70 mph.

But suppose your transmission ratio is given as 4.3 to 1.6. This can also be written as \( 4.3/1.6 \), which converts to \( 2.69:1 \). (how???)

If your transmission ratio is 2:1 and your rear end ratio is 4:1, what is the ratio of revolutions of engine to revolutions of rear wheels?

Suppose \( T = 2.5:1 \) and \( x = 3.06:1 \), what is the overall ratio?

If you travel at 3200 rpm at 64 mph, and your rear end ratio is 3.02, what is the transmission ratio for the gear you are in?
By this time you should have realized that your overall ratio is found by multiplying the transmission ratio times the rear end ratio. If you are in any doubt what-so-ever, see the teacher about this.

Overall ratio = T x X

Don Garlits' rear engined swamp rat - 1R has a 4.10 rear axle ratio, 16.2 rolling radius on his tires and the engine can turn 9500 rpm. If his engined is peaked out as he goes thru the lights and his wheels aren't slipping, how fast can he go in a quarter mile.

What would be the result of changing the rear axle to a 3.91, assuming that the engine would still reach 9500 rpm?

He tries a 4.26 rear axle, and the engine won't go above 9500 rpm. What is the result?

If you have a dragster that has a 4.96 rear axle, 15" rolling radius tires, and is just reaching 91 mph in a quarter mile. What is the engine rpm.

You figure your engine is good for 7000 rpm, and you install a 5.11 rear axle. Now what is your final speed?
If A is the driver and turning at 1200 RPM, find the speed of each of the other wheels.

If C is the driver and turning at 1299 RPM, find the RPM of the other gears.

Do the same if D is turning at 1200 RPM and D is the driver.

In each problem above multiply the RPM times the number of teeth for each particular gear. Do each problem separately. What do you find?
PEAR 1
TEETH × RPM = TEETH × RPM

If Gear 1 has 16 Teeth 1000 RPM
Gear 2 has 32 Teeth ?

Gear 1
17 Teeth
750 RPM

Gear 2
23 Teeth
? RPM

23 Teeth
? RPM

31 Teeth
1900 RPM

4200 RPM
1867 RPM

The gear ratio of gear 1 to gear 2 is 3.42. Gear 1 turns 3420 RPM what does gear 2 turn?

Gear 1 has 24 teeth. How many teeth on gear two?

Remember, a gear ratio of 3.79 means 3.79:1.
Gear E = 16 Teeth
Gear C = 48 Teeth
Gear F = 23 Teeth
Gear D = 16 Teeth.

Suppose gear E is the main drive gear in the transmission and is therefore attached to the engine and turns at the speed of the engine. Gears C, F, D are all attached to the main shaft at the rear of the transmission and spin at the same rate as the driveshaft. Gears C, F, and D are all welded together.

If the driveshaft turns once and C and E are meshed, how many times did the engine turn?

What is the gear ratio (1st gear)?

What is gear ratio when C and F are meshed?

If E turns 2809 times, how many times does F turn when E and F are meshed? (2nd gear)

What is the gear ratio for third gear?
In each problem below the rear-axle ratio will be 3.51, the maximum RPM of the engine will be 5000. How fast can you go in each gear. The rolling radius of a tire is 12".

<table>
<thead>
<tr>
<th>Chevvy 3-speed truck</th>
<th>1st gear</th>
<th>2.85</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd gear</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>3rd gear</td>
<td>DIRECT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chevvy 4-speed truck</th>
<th>1st gear</th>
<th>6.55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd gear</td>
<td>3.58</td>
</tr>
<tr>
<td></td>
<td>3rd gear</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>4th gear</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chevvy 5-speed truck</th>
<th>1st gear</th>
<th>6.70</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd gear</td>
<td>4.02</td>
</tr>
<tr>
<td></td>
<td>3rd gear</td>
<td>2.49</td>
</tr>
<tr>
<td></td>
<td>4th gear</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>5th gear</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chevvy 5-speed truck (with overdrive)</th>
<th>1st gear</th>
<th>5.71</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd gear</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>3rd gear</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>4th gear</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>5th gear</td>
<td>0.85</td>
</tr>
</tbody>
</table>
In each problem below the rear-axle ratio will be 3.99.
The maximum rpm will be 5000. How fast can you go in each gear?
The rolling radius of a tire is 12.1".

<table>
<thead>
<tr>
<th></th>
<th>1st Gear</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ford 3-speed</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Gear</td>
<td>2.42</td>
<td></td>
<td>1.61</td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td>1.61</td>
<td>1.00</td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td>DIRECT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1st Gear</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ford 3-speed</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Gear</td>
<td>2.99</td>
<td></td>
<td>1.75</td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td>1.75</td>
<td>1.00</td>
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<tr>
<td>3rd</td>
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<tbody>
<tr>
<td><strong>Maverick</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1st</td>
<td>3.41</td>
<td></td>
<td>1.86</td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td>1.86</td>
<td>1.00</td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td>1.00</td>
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<tr>
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<th>3rd</th>
<th>4th</th>
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</thead>
<tbody>
<tr>
<td><strong>Ford 4-speed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>2.78</td>
<td></td>
<td>1.93</td>
<td>1.36</td>
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<tr>
<td>2nd</td>
<td></td>
<td>1.93</td>
<td>1.36</td>
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<tr>
<td>3rd</td>
<td></td>
<td></td>
<td>1.36</td>
<td>DIRECT</td>
</tr>
<tr>
<td>4th</td>
<td></td>
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<td></td>
<td>DIRECT</td>
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<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
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</thead>
<tbody>
<tr>
<td><strong>Ford 4-speed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>2.32</td>
<td></td>
<td>1.69</td>
<td>1.29</td>
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<tr>
<td>2nd</td>
<td></td>
<td>1.69</td>
<td>1.29</td>
<td>DIRECT</td>
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<td>3rd</td>
<td></td>
<td></td>
<td>1.29</td>
<td>DIRECT</td>
</tr>
<tr>
<td>4th</td>
<td></td>
<td></td>
<td></td>
<td>DIRECT</td>
</tr>
</tbody>
</table>
GEAR E IS ATTACHED DIRECTLY TO THE ENGINE AND SPINS AT 2000 RPM. GEAR S, A AND B ARE FIXED PERMANENTLY TOGETHER, AND ARE ATTACHED DIRECTLY TO THE DRIVESHAFT. SUPPOSE E IS MOVED TO POSITION E'. WHAT SPEED WILL THE DRIVESHAFT TURN AT AND IN WHICH DIRECTION. (THIS MEANS E AND C ARE MESHE D).

WHAT SPEED WILL THE DRIVESHAFT TURN AT WHEN E AND A ARE
A simple transformer consists of two coils of wire, usually one coil wrapped around the other, but electrically insulated from it. Below is a schematic drawing of a transformer.

P means primary and voltage is input here.

S means secondary and is the voltage output.

If there are twice as many windings in the secondary as in the primary then there is twice as many volts in the secondary.

The figure below illustrates this principle.

![Diagram](image)

If a primary winding has 500 turns and 120 volts, how many volts will the secondary have with 2500 turns?

If a primary has 120 volts, 2000 turns, and you want 6 volts in the secondary, how many turns will you need?

A transformer has a turns ratio of 1:12 (pri. to sec.). If there are 250 turns in the primary, how many turns will there be in the secondary?

In a transformer the voltage ratio, primary to secondary, is 20 to 1, and the primary voltage is 50 volts. What is the turns...
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In a transformer the voltage ratio, primary to secondary, is 20 to 1, and the primary voltage is 50v. What is the turns
You never get something for nothing!!! If you put 20 volts into a transformer and get 80 volts out of it you get four times as much voltage as you started with. You have to give up something: it's current.

In Fig 357.62, if there was 1 amp in the primary, there would have to be 1/4 amp in the secondary. As far as power or energy are concerned, you have not gained anything, you have only changed around the quantities of current and voltage.

In every transformer voltage x current in the primary equals voltage x current in the secondary. Voltage x current is power.

In electronics for TV and radio current is usually measured in milliamps (ma). 1ma = .001a.

Secondary voltage =
Secondary current =

A transformer has 120v in the primary and 6v and 3ma in the secondary. What is the primary current?

A transformer has 300 volts in the secondary and 290v and 6ma in the primary. What is the secondary current.

I_p = primary current   E_p = primary voltage (or V_p)
I_s = secondary current   E_s = secondary voltage (or V_s)
\[ E_p \cdot I_p = E_s \cdot I_s \]

\[ \frac{E_p}{E_s} = \frac{I_s}{I_p} \]

1. \( I_p = 24 \) \( E_s = 100 \text{V} \) \( I_s = 14 \) \( E_p = ? \)

2. \( E_p = 200 \text{V} \) \( E_s = 4 \text{V} \) \( I_s = 150 \text{mA} \) \( I_p = ? \)

\[ N_p = \text{Primary Turns} \]
\[ N_s = \text{Secondary Turns} \]

3. \( N_p = 300 \) \( N_s = 150 \)
\( E_p = 120 \text{V} \) \( E_s = ? \)
\( I_p = 3 \text{mA} \) \( I_s = ? \)

4. \( N_p = 1100 \) \( N_s = ? \)
\( E_p = 200 \text{V} \) \( E_s = 20 \)
\( I_p = ? \) \( I_s = 10 \text{mA} \)
Sub-unit 356

Given two triangles, you should be able to prove them similar by the Angle-Angle Theorem.

Given the ratio of the sides of two triangles and all the measurements of one of them, you should be able to determine all the measurements of the other triangle.

Given a test covering the above mentioned skills, you should be able to answer 60% of the questions accurately.

Materials: Geometry textbook

Read p. 238-41
Do oral # 1-5 with teacher.
Do written p. 242 # 1, 2, 3, 8, 10-22

Read p. 244-6
Do oral ex. 5-8 with teacher
Do written ex. p. 247 # 2, 5, 7, 8, 9, 11, 13, 15, 17
21, 27, 32

Read p. 251, thm. 34
Do written p. 255-6 # 1-6, 11, 15, 19

Read p. 260 thm 37
Do written p. 263 # 5, 6, 7, 11, 21

Take Test 356
1. Given $\triangle DEF$ and $\triangle RST$ with $\angle D = \angle S$ and $\angle E = \angle T$. Which one of the following correctly indicates the correspondence by which the triangles are similar?

(A) $\triangle DEF \sim \triangle RST$  
(B) $\triangle DEF \sim \triangle TRS$  
(C) $\triangle DEF \sim \triangle STR$  
(D) $\triangle DEF \sim \triangle STR$

2. In the adjacent figure $\triangle RST \sim \triangle RXY$. Which one of the following extended ratios is correct?

(A) $\frac{RS}{XY} = \frac{ST}{XR} = \frac{TR}{YR}$  
(B) $\frac{RS}{XY} = \frac{ST}{XR} = \frac{TR}{YR}$  
(C) $\frac{RS}{XY} = \frac{ST}{XR} = \frac{TR}{YR}$  
(D) $\frac{RS}{XY} = \frac{ST}{XR} = \frac{TR}{YR}$

Questions 8 through 13 refer to the adjacent figure in which $DE \parallel ST$. In each question find the length of the indicated segment.

8. $RD = 4, DS = 2, RE = 6, ET =$ ________________
9. $RD = 5, RS = 8; RE = 7, ET =$ ________________
10. $RD = 3, RS = 4, RE = 4, RT =$ ________________
11. $RD: DS = 2:1, RT = 9, RE =$ ________________
12. $RD = 4, DS = 2, DE = 6, ST =$ ________________
13. $RD = a, DS = b, RE = c, ET =$ ________________
Questions 14 through 17 refer to the adjacent figure in which $AX \perp BC$ and $CY \perp AB$.
In each question name the triangles you would have to prove similar in order to establish the given equality.

14. \[ \frac{AB}{BC} = \frac{AX}{CY} \]
15. \[ \frac{CT}{AT} = \frac{CX}{AY} \]
16. \[ \frac{CY}{CX} = \frac{CB}{CT} \]
17. \[ (TY)(AB) = (AT)(XB) \]

18. Supply the reasons for each statement in the proof shown.

Given: $TS \perp MN$,

To Prove: $(MS)(MN) = (MT)(MO)$.

PROOF

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $TS \perp MN$; $NO \perp MO$.</td>
<td>a. Given.</td>
</tr>
<tr>
<td>b. $\angle S = 90^\circ$; $\angle O = 90^\circ$.</td>
<td>b.</td>
</tr>
<tr>
<td>c. $\angle S = \angle O$.</td>
<td>c.</td>
</tr>
<tr>
<td>d. $LM = LM$.</td>
<td>d.</td>
</tr>
<tr>
<td>e. $\triangle MST \sim \triangle MON$.</td>
<td>e.</td>
</tr>
<tr>
<td>f. $MS = MT$. $MO = MN$.</td>
<td>f.</td>
</tr>
<tr>
<td>g. $(MS)(MN) = (MT)(MO)$.</td>
<td>g.</td>
</tr>
</tbody>
</table>

19. In the given figure $RS \parallel AB$. Find $x$. $x - 1$ $x - 2$ $x + 2$ $x - 2$ $3x - 2$ $B$
1. An AC voltage of 120 volts is supplied to the primary coil of a transformer having 600 turns. What is the voltage across the terminals of the secondary coil of the transformer if it has (A) 1200 turns; (B) 300 turns; (C) 6 turns; (D) 12,000 turns.

2. A transformer to supply the various voltages required in a high fidelity amplifier has 300 turns in its primary coil which is connected to 120 volts. Calculate the number of turns required in the secondary coils designed to supply (A) 600 volts as a plate supply; (B) 400 volts for another plate supply; (C) 5 volts for a power tube supply; (D) and 6.3 volts for the regular tube supplies.

3. The normal alternating current supplied to a house has voltages of 110 volts and 220 volts. Devices that require potential differences (voltages) other than these often employ transformers. List four such devices that might be found in the home and tell whether they are step-up or step-down transformers and if possible what the required voltage is.

4. A transformer has 80 turns in its primary coil, and 560 turns in its secondary coil. The secondary circuit has 3.0 amperes at 9.0 x 10² volts. Calculate the voltage and current in the primary circuit.
5. A spot welding machine operates with a current of 75 amps, at 55 volts. It is supplied from a transformer with 550 volts in its primary. Calculate the current in the primary circuit.
It is a drawing to amps. How many amps are required from the generating station?

Suppose the only electricity you are using is an iron which is about 1150 watts. That means

\[ I = \frac{P}{V} \]

Do you see why some transformers weigh 700 tons?

Primary and secondary? \[ V = 500 \]

In your home, how many windings at each transformer lead into your home. How many windings at each transformer that there are 500 turns in the secondary windings that find the turns ratio for each transformer station.

Suppose there are 500 turns in the secondary windings that find the turns ratio for each transformer station.

\[ \frac{V}{I} = \frac{500}{10} = 50 \]

\[ V = 12,000 \text{v}, \text{the k stands for kilo or a thousand.} \]

12KV means 12,000V, the k stands for kilo or a thousand.
SAHUARITA HIGH SCHOOL
CAREER
CURRICULUM
PROJECT

COURSE TITLE: GEOMETRY
THIRD QUARTER
BY
JAMES MADEHEIM
S. I. C. S. PAK
(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

NUMBER: 370.1

CLUSTER: Transportation

AREA: Water Transportation - Piloting

TITLE: The Man in the Lighthouse
Rationale:

Navigators and other people trying to measure distances indirectly, use the terms "angle of elevation" and "angle of depression." This unit will help you to understand and to use these terms.

Behavioral Objectives:

Given an angle to measure with a protractor, you will measure the angle within one degree.

Given the measure of two angles of a triangle, you will find the measure of the third angle within one degree.

Given two parallel lines cut by a transversal, you will name eight pairs of equal angles.

Given the height of a pseudo-lighthouse, a theodolite to measure angles with, and a table of angles of depression, you will measure distances from the pseudo-lighthouse to various pseudo-boats to the nearest whole number of feet.
Information sources:

1. Read in "Patterns in Mathematics" textbook, Section 5-3, Angles, page 97 - 99.
2. Read in "Patterns..." textbook, top of page 100.
3. Read Data brief # 1 "The angle sum of a figure"
4. Read in "Patterns..." textbook, "Vertical Angles"
5. Read Data brief # 2 "Parallel lines"
6. Read Data brief # 3 "Complements and Supplements"
7. Read Data brief # 4 "Angles of depression and elevation."
8. Read Data brief # 5 "The man in the lighthouse."
Data Brief # 1

On a sheet of paper, draw three triangles and with your protractor measure each of the interior angles in each triangle.

Data Brief # 2

In the figure below, line $k$ is parallel to line $l$, and line $m$ is a transversal which intersects both line $k$ and line $l$. Parallel lines are lines that never meet no matter how far they are extended in each direction. The transversal forms many pairs of equal angles. They are listed below. "$\angle A$" is read "angle A"

$\angle A = \angle B$ They are corresponding angles
$\angle C = \angle D$ They are corresponding angles
$\angle E = \angle F$ They are corresponding angles
$\angle G = \angle H$ They are corresponding angles
$\angle E = \angle G$ They are alternating interior angles
$\angle C = \angle B$ They are alternating interior angles
$\angle E = \angle H$ They are vertical angles
$\angle C = \angle A$ They are vertical angles
$\angle G = \angle F$ They are vertical angles
$\angle B = \angle D$ They are vertical angles
A 20° angle and a 70° angle are complementary because their sum is 90°.

A 30° angle and a 60° angle are complementary.

A 43° angle and a 47° angle are complementary.

The angle that is the complement of 40° is 50° because

\[ 40° + 50° = 90° \]

The complement of 62° is 28°.

The complement of 89° is 1°.

A 30° angle and a 150° angle are supplementary because their sum is 180°.

A 40° angle and a 140° angle are supplementary.

A 92° angle and a 88° angle are supplementary.

The angle that is the supplement of 50° is 130° because

\[ 50° + 130° = 180° \]

The supplement of 80° is 100°.

The supplement of 110° is 70°.
In the drawing below, if the observer at point A wants to look at point B, he has to elevate his head from the horizontal position. His line of sight moves through the angle of elevation.

The observer at R has to depress his head from the horizontal position to see point Q. His line of sight moves through the angle of depression.

In the figure below the angle of depression is 14°

In the figure below the angle of elevation is 26°
A man in a lighthouse wants to know how far a boat is from land. The top of his lighthouse is 800 ft. above the level of the sea since it sits on top of a cliff. He sights through a theodolite at a boat at an angle of depression of 20°.

Look at your angle of depression tables and see if you understand why he knows the boat is 2198 feet from shore.

If his angle of depression were 32°, the boat would be 1280 ft. from shore.
### Table of angles of depression

<table>
<thead>
<tr>
<th>Angle of depression from lighthouse (degrees)</th>
<th>Distance from lighthouse (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45832.00</td>
</tr>
<tr>
<td>2</td>
<td>22900.04</td>
</tr>
<tr>
<td>3</td>
<td>15284.88</td>
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<td>4</td>
<td>11440.56</td>
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<td>5</td>
<td>9430.08</td>
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<td>23</td>
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<td>25</td>
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<td>26</td>
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<td>27</td>
<td>1504.56</td>
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<td>35</td>
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<td>37</td>
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<td>38</td>
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<tr>
<td>39</td>
<td>953.44</td>
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<tr>
<td>40</td>
<td>920.32</td>
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<tr>
<td>41</td>
<td>888.48</td>
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<td>42</td>
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<td>45</td>
<td>772.56</td>
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<td>54</td>
<td>560.16</td>
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<td>539.60</td>
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<td>56</td>
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<td>57</td>
<td>499.32</td>
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<td>480.72</td>
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<td>59</td>
<td>461.92</td>
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<td>425.36</td>
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<td>62</td>
<td>407.20</td>
</tr>
<tr>
<td>63</td>
<td>389.16</td>
</tr>
<tr>
<td>64</td>
<td>373.04</td>
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<tr>
<td>65</td>
<td>356.16</td>
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<td>66</td>
<td>339.60</td>
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<td>67</td>
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<td>69</td>
<td>291.20</td>
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<td>275.44</td>
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<td>224.36</td>
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<td>75</td>
<td>199.14</td>
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<td>76</td>
<td>184.72</td>
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<td>170.08</td>
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<td>79</td>
<td>141.04</td>
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<td>80</td>
<td>126.72</td>
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<td>81</td>
<td>112.60</td>
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<td>82</td>
<td>78.24</td>
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<td>64.08</td>
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<td>84</td>
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<td>55.32</td>
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<tr>
<td>87</td>
<td>27.12</td>
</tr>
<tr>
<td>88</td>
<td>14.00</td>
</tr>
</tbody>
</table>
Activity # 4

Measure each angle on these two pages and record the measurement in the interior of the angle.
With your protractor, draw the following angles on this sheet of paper:

1. 40°  2. 35°  3. 62°  4. 90°  5. 23°
6. 125°  7. 105°  8. 180°
\[ \angle A = \angle B = \angle C = 50^\circ \]

\[ \angle A = \angle B = 80^\circ \]

\[ \angle A = \angle C \]

\[ \angle A = \angle B = \angle C \]

\[ \angle A = \angle A = \angle C = 158^\circ \]
Activity # 4

Using the illustration below, do exercises # 16-20

In the "Patterns" testbook p. 108.

16.

17.

18.

19.

20.
If $\angle A = 60^\circ$ then

- $\angle B =$
- $\angle C =$
- $\angle D =$
- $\angle E =$
- $\angle F =$
- $\angle G =$
- $\angle H =$

If $\angle H = 140^\circ$ then

- $\angle C =$
- $\angle F =$
- $\angle B + \angle E =$
- $\angle C + \angle G =$

If $\angle D = 50^\circ$ then

- $\angle D + \angle F =$
- $\angle C + \angle G =$
- $\angle B + \angle E =$
- $\angle A + \angle H =$
- $\angle B + \angle G =$
Activity # 6

1. What is the complementary $\angle$ to 20°?
2. What is the complementary $\angle$ to 60°?
3. What is the supplementary $\angle$ to 120°?
4. What is the supplementary $\angle$ to 150°?
5. What is the complementary $\angle$ to 1°?
6. What is the complementary $\angle$ to 75°?
7. What is the complementary $\angle$ to 22 1/2°?
8. What is the complementary $\angle$ to 45°?
9. What is the supplementary $\angle$ to 30°?
10. What is the supplementary $\angle$ to 90°?
Activity 7

In each picture below name each angle of depression at point A, and each angle of elevation at point B.
Activity # 8

In the picture below the man in the lighthouse sights through his theodolite at different boats going by. For each angle of depression listed below tell what the distance of the boat is from the shore line.

26°
58°
72°
14°
45°
40°
53°

Suppose the man passing by in a boat talks to the lighthouse man by radio and tells him he sees the lighthouse through his theodolite at an angle of elevation of 36°. How far from the shore will the lighthouse man tell him he is?

Ask the teacher to take you outside to the pseudo-light-house and sight on some pseudo-boats.
Rationale:

If you have ever tried to measure the circumference of a circle, (the distance around the outside of a circle), you will understand why this is difficult to do. Around 2000 B.C. the Egyptians thought they had discovered a relationship between the circumference of a circle and its radius. But the relationship they discovered was not very accurate.

This package will help you discover and use the relationship between the radius of a circle and its circumference.

Behavioral Objectives:

Given the radius or diameter of a circle, you will be able to find the circumference of that circle, accurate to two decimal places.

Given the circumference of a circle, you will be able to find the radius and diameter of that circle, accurate to two decimal places.

Given the radius of the turning circle and the tread of a car, you will be able to find the distance an inner tire and an outer travel when making as tight a circle as possible, with accuracy to the nearest whole number of feet.
Information Sources:

1. Read Data brief # 1 "Inventing π"
2. Read data brief # 2 "Rolling radius and circumference."
3. Read Data brief # 3 "Revolutions of a tire."
4. Read Data brief # 4 "Turning radius."
5. Read Data brief # 5 "Turning radius and distance."

Data Brief # 1

Obtain a ruler, yard stick and a piece of string from the teacher and measure the circumference (C) and diameter (D) of each circle indicated in the table. Record your measurements in the table attached to activity # 1.

Data Brief # 2

Since a tire flattens out slightly as the weight of a car is put on it, its actual radius is larger than its rolling radius. (See picture below.)

Actual radius is 18 inches  
Rolling radius is 16 inches
The formula for finding the circumference of a circle is

$$\text{circumference} = 2 \times \text{radius} \times \pi,$$

or

$$c = 2\pi r$$

If the rolling radius of a tire is 16", the circumference of the tire (rolling circumference) is \(2 \times \pi \times 16 = 100.48"\)

This means that if the tire makes one revolution it travels 100.48". The circumference of a tire is the distance it travels during one revolution.

If the circumference of a tire is 120", then it travels 120" in one revolution. In two revolutions it travels 240".

The tires used on some of the trucks at the mines have a diameter of 8', which means their circumference is 25.12'. Therefore when the tire has made one complete revolution it has travelled a distance of 25.12'.

Data Brief # 3

If a tire has a circumference of 100" and it travels 100 " it makes one revolution.

If a tire's circumference is 100" and it travels 200", it makes two revolutions.

If \(C = 100\), and travel distance for this tire is 700" it makes 7 revolutions.

If \(C = 120"\) and it travels 600" it makes 5 revolutions.

Thus

$$\text{The number of revolutions a tire makes} = \frac{\text{Distance a tire rolls}}{\text{Rolling circumference}}$$
If a tire's circumference is 100" and it rolls 450", how many revolutions did it make?

\[
\text{revolutions} = \frac{450}{100} = 4.5 = 4\frac{1}{2}
\]

Data brief # 4

Below is a picture of the tire tracks of a car turning in as tight a circle as possible. The dashed lines are the inner tires track, the solid line is the outer tires track, and the dots represent the path of the center of the car.
The turning radius is the distance from the pivot point (which is the center of the circle) to the center of the car. The turning radius in the previous picture is distance B.

The tread of a car is the distance from the center of one rear wheel to the center of the other rear wheel. In the picture below the tread is 56".

The length of the turning circle is figured from the turning radius. The length of the turning circle is the circumference of the turning circle, figured from the turning radius.

As you can see in the above drawing, the inner tire makes a smaller circle than the turning circle, and the outer tire makes a larger circle than the turning circle.

The radius of the inner circle in the above picture is 15' - 28". The 28" is half the tread distance.

The radius of the outer circle is 15' plus 28". Again, because of the 56" tread.
Assuming that the standard tread of a car is 56", suppose a car has a turning radius of 16'. Then the radius of the circle made by the outer tires is 16' + 28". The radius of the circle made by the inner tires is 16' - 28"

If a car has a tread of 60" and its turning radius is 22', the radius of the circle made by the outer tires is 22' + 30", and the radius of the circle made by the inner tires is 22' - 30".

Data Brief # 5

If a car has a turning radius of 18' and a tread of 60", the radius of its turning circle is 18'. The circumference of its turning circle is $2 \times 3.14 \times 18 = 113.04'$. The circumference of the circle described by the outside tire is $2 \times 3.14 \times (18'30") = 2 \times 3.14 \times (20'6") = 2 \times 3.14 \times 20.5'$. Multiplying these numbers you get 128.74'.

The circumference of the circle described by the inside tire is $2 \times 3.14 \times (18' - 30") = 6.28 \times (15'6") = 97.34'$. If the tires on the car have a rolling radius of 12" or one foot, the circumference of the tire is 6.28'. Thus to find out how many revolutions the tire makes in travelling (The inside tire) use the formula discussed in Data Brief # 3 and divide 97.34 by 6.28 and you get 15.5 revolutions.

To find out how many revolutions the outer tire makes you divide 128.74 by 6.28 and you get 20.5 revolutions.
In each circle below measure the diameter and then measure the distance around the circle.
Record the diameter and circumference of each circle you measure in the appropriate places in the table below.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Circumference</th>
<th>Diameter</th>
<th>C/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle #1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle #2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle #3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle #4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discus platform at north end of football field</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top of waist basket</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom of waste basket</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cement base of lamp post in teacher’s parking lot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metal base of lamp post in teacher’s parking lot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car wheel in auto shop (not on a car)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle formed by holes of the speaker on the wall</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activity # 2

A tire has a rolling radius of 22″. What is its circumference?

<table>
<thead>
<tr>
<th>Rolling radius</th>
<th>Rolling diameter</th>
<th>Rolling circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>22″</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>18″</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>36″</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>40″</td>
<td>?</td>
</tr>
<tr>
<td>23″</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>44″</td>
<td>?</td>
</tr>
</tbody>
</table>

In the above table, fill in the appropriate measurement in place of each question mark.

Go out to the auto shop and get a tire. Measure its radius. Make it roll along a tape measure laid out on the ground and record the distance it travels for one complete revolution, here.

Now calculate the circumference of the tire from the radius you measured and record that figure here.

Do this experiment several times and then tell the teacher your results, if there are any.
Activity # 3

The circumference of a tire is 118". How far does it roll in one revolution? ___________

The circumference of a tire is 120". How far does it roll in two revolutions? ___________

A tire rolls 300" in three revolutions. What is its circumference? ___________

The circumference is 110". How far does it roll in 5 revolutions? ___________
in 7 revolutions? ___________
in 14 revolutions? ___________
in 300 revolutions? ___________

A tire has a rolling radius of 21".
What is its circumference? ___________
How far does it roll in one revolution? ___________
in two revolutions? ___________
in 100 revolutions? ___________

A tire has a radius of 24" (rolling radius). What is its diameter? ___________
What is its circumference? (rolling) ___________
How far does it roll in one revolution? ___________
in ten revolutions? ___________
in 25 revolutions? ___________
Activity #3

A tire rolls 400". Its rolling circumference is 100". How many revolutions did it make?

A tire rolls 600". Its rolling circumference is 120". How many revolutions did it make?

Complete the blank spaces in the table below.

<table>
<thead>
<tr>
<th># of revolutions</th>
<th>distance tire rolls</th>
<th>rolling circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>440&quot;</td>
<td>110&quot;</td>
</tr>
<tr>
<td>?</td>
<td>880&quot;</td>
<td>110&quot;</td>
</tr>
<tr>
<td>?</td>
<td>630&quot;</td>
<td>105&quot;</td>
</tr>
<tr>
<td>?</td>
<td>944&quot;</td>
<td>118&quot;</td>
</tr>
<tr>
<td>?</td>
<td>1373&quot;</td>
<td>114.5&quot;</td>
</tr>
<tr>
<td>26</td>
<td>?</td>
<td>111&quot;</td>
</tr>
<tr>
<td>21</td>
<td>?</td>
<td>114&quot;</td>
</tr>
<tr>
<td>98</td>
<td>?</td>
<td>122&quot;</td>
</tr>
<tr>
<td>400</td>
<td>?</td>
<td>113.5&quot;</td>
</tr>
<tr>
<td>20</td>
<td>360&quot;</td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>rolling radius</th>
<th>rolling circumference</th>
<th>distance tire rolls</th>
<th>number of revolutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>22&quot;</td>
<td>?</td>
<td>13816&quot;</td>
<td>?</td>
</tr>
<tr>
<td>20&quot;</td>
<td>?</td>
<td>879.2&quot;</td>
<td>?</td>
</tr>
<tr>
<td>26&quot;</td>
<td>?</td>
<td>816.40</td>
<td>?</td>
</tr>
<tr>
<td>23&quot;</td>
<td>?</td>
<td>1227.74</td>
<td>?</td>
</tr>
<tr>
<td>20&quot;</td>
<td>?</td>
<td>465&quot;</td>
<td>?</td>
</tr>
<tr>
<td>23&quot;</td>
<td>?</td>
<td>1000&quot;</td>
<td>?</td>
</tr>
<tr>
<td>4'</td>
<td>?</td>
<td>5280'</td>
<td>?</td>
</tr>
<tr>
<td>30&quot;</td>
<td>?</td>
<td>1266&quot;</td>
<td>?</td>
</tr>
</tbody>
</table>
Activity # 4

Fill in the spaces below where there is a question mark.

In this first table the tread of the car will always be 60" or 5'. The radius of the tires will be 12" or 1'.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Turning</th>
<th>outer tire turning</th>
<th>inner tire turning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>radius</td>
<td>24'</td>
<td>24' + 30&quot;</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>2.</td>
<td>radius</td>
<td>20'</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>3.</td>
<td>radius</td>
<td>15'</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>4.</td>
<td>radius</td>
<td>12'</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>5.</td>
<td>radius</td>
<td>18'</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>6.</td>
<td>radius</td>
<td>19'</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>circumference</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
In the series of problems below, assume the tread of a car is 56" or 4'8".

<table>
<thead>
<tr>
<th>Problem</th>
<th>Turning</th>
<th>Outer Tire Turning</th>
<th>Inner Tire Turning</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>radius</td>
<td>22'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>radius</td>
<td>16'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>radius</td>
<td>12'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>radius</td>
<td>10'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>radius</td>
<td>32'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>radius</td>
<td>15'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>radius</td>
<td>14'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>circumference</td>
<td></td>
</tr>
</tbody>
</table>
Fill in the empty spaces in this table.

<table>
<thead>
<tr>
<th></th>
<th>tire circumference</th>
<th>turning circumference</th>
<th>outer tires revolutions</th>
<th>inner tires revolutions</th>
<th>tire radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the table on the previous page are the data for the revolutions taken by the inner tires and by the outer tires of a car while it turns in a circle.

Explain to the teacher what would happen if the inner tire were attached directly to the outer tire and the inner tire turned exactly as it does on a car now.

Explain what would happen to the inner tire if the outer tire turned exactly as it does now and the two tires were fixed rigidly together.
S. I. C. S. PAK
(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

NUMBER: 370.5

SUBJECT: Marketing and Distribution

AREA: Advertising

TITLE: Hanging Up a Sign
Rationale.

At times a distance cannot be measured directly. For example:

How do you determine the length of the cable needed in the picture below before the sign is put up?

One method of finding the length of cable needed is by use of the Pythagorean theorem. This method of indirect measurement uses the fact that once you know two sides of a right triangle, you can determine the third side.

Behavioral Objectives:

Given any two sides of a right triangle, you will be able to find the third side with two decimal place accuracy.

Given the distance "a" and the distance "b" as in the picture below, representing two legs of a right triangle, and the use of the electronic calculator and square root tables, you will be able to find the length of the cable with an accuracy of one foot.
Data Brief #1

A right triangle has one angle that measures 90°.

In any right triangle, the length of one short side squared ($a^2$), plus the length of the other short side squared ($b^2$), equals the length of the longest side squared ($c^2$). $a^2 + b^2 = c^2$

\[
\begin{align*}
5^2 + 12^2 &= 13^2 \\
25 + 144 &= 169
\end{align*}
\]

\[
\begin{align*}
a &= 3, & b &= 4, & c &= ? \\
a^2 + b^2 &= c^2 \\
3^2 + 4^2 &= c^2 \\
9 + 16 &= c^2 \\
25 &= c^2 \\
5 &= c
\end{align*}
\]
Can the triangle below be a right triangle?

\[ 5^2 + 6^2 = 7^2 \]
\[ 25 + 36 = 49 \]
\[ 61 \neq 49 \quad \text{Therefore it is not a right triangle.} \]

Data Brief # 2

If the side "b" in the triangle below is 6' and the longest side "c" is 10', the length of the other short side is found by subtracting.

\[ a^2 + b^2 = c^2 \]
\[ a^2 + 6^2 = 10^2 \]
\[ a^2 + 36 = 100 \]
\[ a^2 = 100 - 36 \]
\[ a^2 = 64 \]
\[ a = 8 \]
Here is another example. If the longest side is 26', and one of the short sides is 24', how long is the other short side?

\[ 26^2 = 24^2 + x^2 \]
\[ 676 = 576 + x^2 \]
\[ 676 - 576 = x^2 \]
\[ 100 = x^2 \]
\[ 10 = x \]

In the triangle above you find the third side by setting up the following equations and solving for the unknown.

\[ 6^2 + 8^2 = x^2 \]
\[ 36 + 64 = x^2 \]
\[ 100 = x^2 \]
\[ 10 = x \]

In the triangle below you solve for the length of the unknown side in the following manner.

\[ 18^2 + x^2 = 30 \]
\[ 324 + x^2 = 900 \]
\[ x^2 = 900 - 324 \]
\[ x^2 = 676 \]
\[ x = \sqrt{676} \]
\[ x = 24 \]
Data Brief # 4

You are going to need your sheet of square root tables at this time. It is attached to this package.

Look in the column titled "No." and find "16". Look in the column titled "square root" directly across from the "16" and you should find the number "4". This means that the square root of 16 is 4; or written in math symbols

$$\sqrt{16} = 4$$

Find the square root of 3' ( $\sqrt{3} $ )
Do you see that

$$\sqrt{3} = 2.24$$
$$\sqrt{7} = 2.65$$
$$\sqrt{143} = 11.96$$

To find the length of the third side of the triangle below set up the problem in the following manner.

$$12^2 + 18^2 = x^2$$
$$144 + 324 = x^2$$
$$468 = x^2$$
$$\sqrt{468} = x$$
$$21.633 = x$$
Activity # 1

Each set of three numbers below represents the lengths of the three sides of a triangle. Indicate by writing "yes" or "no" after each triplet that, yes, the triangle is a right triangle, or no, the triangle is not a right triangle.

6,8,10  5,13,12  2,7,8
26,19,24  7,8,9  12,15,9
7,25,24  15,12,8  12,16,20
9,41,40  10,20,30  11,61,60
583,1344,1465  481,29,490  483,484,42
901 1260,1549  325,204,222  610,1189,1020
1889,1360,1311  1044,1165,517


Activity #2

In each pair of numbers below, the first is the length of the longest side of a right triangle and the second is the length of one of the shorter sides. Find the length of the third side.

<table>
<thead>
<tr>
<th>Pair A</th>
<th>Pair B</th>
<th>Pair C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 4</td>
<td>10, 6</td>
<td>15, 9</td>
</tr>
<tr>
<td>26, 24</td>
<td>15, 12</td>
<td>10, 8</td>
</tr>
<tr>
<td>20, 16</td>
<td>50, 30</td>
<td>500, 300</td>
</tr>
<tr>
<td>5000, 3000</td>
<td>100, 60</td>
<td>2500, 1500</td>
</tr>
<tr>
<td>35, 28</td>
<td>1409, 159</td>
<td>421, 420</td>
</tr>
<tr>
<td>1301, 51</td>
<td>269, 260</td>
<td>449, 351</td>
</tr>
<tr>
<td>365, 364</td>
<td>229, 221</td>
<td>281, 231</td>
</tr>
<tr>
<td>409, 391</td>
<td>221, 220</td>
<td>221, 171</td>
</tr>
<tr>
<td>1229, 1221</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activity #3

Find the third side of each right triangle. The answer in each case will be a whole number.
Activity # 3

A sign is to be hung as pictured. The cable support on the building is 30' above the sign and the hook on the sign is 16' from the building. How long a cable is needed to help support this sign?
Activity # 3

A sign company has to hang a sign as in the picture below. How long a piece of cable should he make up?

The cable \( x \) as in the picture below, helping to support the sign is cracking the window sill that it is attached to. A new cable will replace it and will be attached to the roof. The bar the sign is attached to is 16' below the roof and the end of the bar is 12' from the building. How long should the new cable be? (The dotted line sill be the new cable.)
Activity # 4

Look up the following values in the book of tables.

\[ \sqrt{36} \]
\[ \sqrt{144} \]
\[ \sqrt{151} \]
\[ \sqrt{324} \]
\[ \sqrt{729} \]
\[ \sqrt{961} \]
\[ \sqrt{3} \]
\[ \sqrt{17} \]
\[ \sqrt{18} \]
\[ \sqrt{22} \]

\[ 46^2 \]
\[ 57^2 \]
\[ 58^2 \]
\[ 99^2 \]
\[ 567^2 \]
\[ 824^2 \]
\[ 777^2 \]
\[ 666^2 \]
\[ 200^2 \]
Activity # 4

Find the third side of the triangle in each case

1. \( \sqrt{169 + 7^2 - 3^2} \)
2. \( \sqrt{10^2 + 10^2 - 7^2} \)
3. \( \sqrt{19^2 + 8^2 - 7^2} \)
4. \( \sqrt{11^2 + 6^2 - 7^2} \)
5. \( \sqrt{54^2 + 87^2 - 7^2} \)
Activity # 4

Find the length of the cable (c) for each problem below.

1. $a = 20, b = 12$
2. $a = 46, b = 10$
3. $a = 16, b = 11$
4. $a = 24, b = 19$
5. $a = 60, b = 11$
SICS-PAK

Series: Exploration

Number: 5

Cluster: Construction

Area: Land Development - Site Preparation

Title: Through a Mountain and Across a Canyon

By: Jim Madeheim
Math Teacher
Sahuarita High School
Rationale

One of the longer tunnels ever built goes through a mountain in France. Before the tunnel was started, the planners of the tunnel had to know how long it would be so that they could estimate its cost and appropriate the right amount of money. But you can't walk through a mountain so some other method of measuring had to be used. This package explains one such method.

Behavioral objectives

Given any angle between 0 and 90° you will be able to look up its tangent in a trig table with 100% accuracy.

Given the tangent of an angle, you will be able to name the nearest whole angle with 1° accuracy.

Given a distance to measure that requires the use of the tangent formula, you will be able to find the distance wanted to the nearest whole number using the tangent formula, trig tables, and calculator.

Pre-Test

Tan 63° =
Tan 71° =
Tan 32° =
Tan x = 1.0000, x =
Tan x = .6605, x =
Tan x = .8201, x =
You need to find the distance from A to B. Draw a triangle that will allow you to use the tangent formula to find the distance AB. When you have drawn your triangle, ask the teacher for the measurements you will need to know.

Information sources:

Read Data Brief # 1 "Using the tangent table"
Read Data Brief # 2 "Further use of the trig tables"
Read Data Brief # 3 "Common fractions and decimals"
Read Data Brief # 4 "Using the tangent formula"
Read Data Brief # 5 "Using the theodolite"
Data Brief # 1

Get a trig table from the teacher.
Look in the column headed "angle" and find 20°. Look across from 20° in the column headed "tangent" and you should see ".3640"
This means the tangent of 20° is .3640 or in shorter form
\[ \tan 20° = .3640 \]

Do you see that the tangent of 45° is 1?
Do you see that \[ \tan 76° = 4.0108 \]
Do you see that \[ \tan 59° = 1.6643 \]
What is the tangent of 42°
You should have written down .9004.

Data Brief # 2

Question: The tangent of what angle is .8098?
Answer: 39° is the angle.

Question: \[ \tan x = .5774 \] What is x
Answer: \[ x = 30°, \] because \[ \tan 30° = .5774 \]
The symbol \( \theta \) is a Greek letter called "theta". Many times greek letters instead of our own letters are used to designate angles.
\[ \tan \theta = 2.9042, \theta = \] In the blank above you should have written 71°
Definition: \( \tan x = \frac{\text{length of side opposite angle } x}{\text{length of side adjacent to angle } x} \)

In the triangle above
\[
\tan x = \frac{a}{b}
\]

If \( a = 5' \) and \( b = 7' \), then \( \tan x = \frac{5}{7} = 0.7143 \)

Look in your trig tables and find the number in the tangent column closest to 0.7143.
Do you see that 0.7265 is closest?
This means that angle \( x = 36^\circ \) (approximately).

Find \( \tan x = 7/8 \), \( x = \) ________
\( \tan x = 15/11 \) \( x = \) ________

Your two answers should be 41° and 54°.
In the above triangle, the side 9" long is opposite the angle $\theta$. The side 12" long is adjacent to the angle $\theta$. And the side 15" long is the hypotenuse. The longest side is always the hypotenuse.

In the triangle below, the side 12' long is opposite angle $\theta$. The side 5' long is adjacent to the angle $\theta$. And the side 13' long is the hypotenuse.

In the triangle below, $\theta = 40^\circ$, the side adjacent to angle $\theta$ is 100' long and the side opposite angle $\theta$ is $x$. We want to find $x$.

\[
\tan \theta = \frac{\text{opp.}}{\text{adj.}}
\]

\[
\tan 40^\circ = \frac{x}{100}
\]

To find $x$ you must multiply $\tan 40^\circ$ times 100. $\tan 40^\circ = .8391$.

Thus .8391 times 100 = 83.91 feet. This is the length of the side opposite angle $\theta$. 
In the triangle at the left, suppose $a = 80$ and $\theta = 60^\circ$. Find side $b$.

\[
\tan \theta = \frac{b}{a}
\]

\[
\tan 60^\circ = \frac{b}{80}
\]

\[
1.7321 = \frac{b}{80}
\]

\[
1.7321 \times 80 = b
\]

\[
138.568 = b
\]

Data Brief # 5

For this work assignment you will accompany the teacher outside and will carry out the actual measurement of distances similar to the ones you have completed in this package at this time.
Activity #1

Tan 40° = 
Tan 38° = 
Tan 26° = 
Tan 67° = 
Tan 88° = 
Tan 12° = 
Tan 17° = 
Tan 31° = 
Tan 55° = 
Tan 0° = 
Tan 8° = 

Activity #2

Tan x = .6745,  x = 
Tan x = .9657  x = 
Tan x = .0875  x = 
Tan x = .5543  x = 
Tan x = 1.0355 x = 
Tan x = 1.8040 x = 
Tan x = .3640  x = 
Tan x = .4452  x = 
Tan x = 7.1154 x = 
Tan x = .9999  x = 
Tan x = .6000  x = 
Tan x = .7811  x = 
Tan x = .0700  x = 
Tan x = 2.144  x =
<table>
<thead>
<tr>
<th>Activity # 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tan x = 6/6</td>
</tr>
<tr>
<td>Tan x = 3/4</td>
</tr>
<tr>
<td>Tan x = 6/17</td>
</tr>
<tr>
<td>Tan x = 4/3</td>
</tr>
<tr>
<td>Tan x = 7/3</td>
</tr>
<tr>
<td>Tan x = 1/6</td>
</tr>
<tr>
<td>Tan x = 2/3</td>
</tr>
<tr>
<td>Tan x = 4/9</td>
</tr>
<tr>
<td>Tan x = 1/17</td>
</tr>
<tr>
<td>Tan x = 4/5</td>
</tr>
<tr>
<td>Tan x = 5</td>
</tr>
<tr>
<td>Tan x = 2</td>
</tr>
</tbody>
</table>
Activity # 4

1. \( \theta = 35^\circ \)

2. \( \theta = 20^\circ \)

3. \( \theta = 20^\circ \)

4. \( \theta = 40^\circ \)

5. \( \theta = 45^\circ \)

6. \( \theta = 30^\circ \)

7. \( \theta = 28^\circ \)

8. \( \theta = 20^\circ \)

9. \( \theta = 45^\circ \)

10. \( \theta = 40^\circ \)

11. \( \theta = 30^\circ \)

12. \( \theta = 28^\circ \)

13. \( \theta = 20^\circ \)

14. \( \theta = 45^\circ \)

15. \( \theta = 40^\circ \)

16. \( \theta = 30^\circ \)

17. \( \theta = 28^\circ \)

18. \( \theta = 20^\circ \)

19. \( \theta = 45^\circ \)

20. \( \theta = 40^\circ \)

21. \( \theta = 30^\circ \)

22. \( \theta = 28^\circ \)

23. \( \theta = 20^\circ \)

24. \( \theta = 45^\circ \)

25. \( \theta = 40^\circ \)

26. \( \theta = 30^\circ \)

27. \( \theta = 28^\circ \)

28. \( \theta = 20^\circ \)

29. \( \theta = 45^\circ \)

30. \( \theta = 40^\circ \)

31. \( \theta = 30^\circ \)

32. \( \theta = 28^\circ \)

33. \( \theta = 20^\circ \)

34. \( \theta = 45^\circ \)

35. \( \theta = 40^\circ \)

36. \( \theta = 30^\circ \)

37. \( \theta = 28^\circ \)

38. \( \theta = 20^\circ \)

39. \( \theta = 45^\circ \)

40. \( \theta = 40^\circ \)

41. \( \theta = 30^\circ \)

42. \( \theta = 28^\circ \)

43. \( \theta = 20^\circ \)

44. \( \theta = 45^\circ \)

45. \( \theta = 40^\circ \)

46. \( \theta = 30^\circ \)

47. \( \theta = 28^\circ \)

48. \( \theta = 20^\circ \)

49. \( \theta = 45^\circ \)

50. \( \theta = 40^\circ \)

51. \( \theta = 30^\circ \)

52. \( \theta = 28^\circ \)

53. \( \theta = 20^\circ \)

54. \( \theta = 45^\circ \)

55. \( \theta = 40^\circ \)

56. \( \theta = 30^\circ \)

57. \( \theta = 28^\circ \)

58. \( \theta = 20^\circ \)

59. \( \theta = 45^\circ \)

60. \( \theta = 40^\circ \)

61. \( \theta = 30^\circ \)

62. \( \theta = 28^\circ \)

63. \( \theta = 20^\circ \)
A and B are trees on opposite sides of a deep gorge. Angle A is a right angle. The distance from A to C is measured and found to be 300'. Angle $\theta$ is 38°. What is the distance across the gorge?
Above is a birds-eye view of a mountain and the surrounding terrain. The railroads want to put a tunnel through from point A to point B. Angle B is a right angle. The distance BC is measured at 1700'. Angle θ is 57°. Find the distance through the mountain.
The distance across this river in a canyon without wading through the alligator infested waters. You set up the triangle that will allow you to measure this distance and then ask the teacher for the necessary measurements that will let you find the width of the river.
A tunnel has to go through the mountain from point A to B and then a bridge spans the river canyon from point B to C. You set up the triangles that you will need to measure these distances and then ask the teacher for the necessary measurements that will let you find the distances wanted.
Post Test.

Take the pre-test.
### S. I. C. S PAK

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**SERIES**  
Exploration

**NUMBER**  
372.5

**CLUSTER**  
Construction

**AREA**  
Site Layout

**TITLE**  
Across a Lake, Across a River
Rationale:

At times it is not convenient to use the tangent formula because of the difficulty in measuring either of the legs of a right triangle. In the picture below, the distance AB is to be measured, but the distance BC crosses a river and is not easily measured. Therefore the distance AC must be used as a reference distance to find the distance AB. Since the only side of the triangle that is known is the hypotenuse and the side wanted is opposite the angle measured at C, the sine formula will be used instead of the tangent. (abbreviated Sin).

Behavioral Objectives:

Given any angle between 0° and 90°, you will be able to look up its sine exactly.

Given the sine of an angle, you will be able to find the angle with one degree accuracy.

Given a situation similar to the one pictured above, where the side opposite a given angle is to be found and the hypotenuse is known, you will be able to find the distance
asked for by using the sine formula, with whole number accuracy.

**Pre-Test:**

The sin 43° = ?

The sin 67° = ?

The sin x = .5000, x = ?

The sin x = .8660, x = ?

Referring to the picture on the preceding page of this package -

If the hypotenuse (AC) is 220 yards long, what is the distance AB?

**Information Sources:**

1. Read Data brief # 1 "The Sine Function and the trig tables."

2. Read Data brief # 2 "The Sine Ratio."

3. Read Data brief # 3 "Using the theodolite."

**Data Brief # 1**

Look at your trig tables and find out what the sin 30° is. You should get .5000.

Find the sin 42°. It is .6691
In the triangle below, side AB is opposite angle \( \theta \). And side AC is the longest side and thus is called the hypotenuse.

The sine formula states that

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}
\]

If \( \theta = 23^\circ \), and side AC = 80.

Then \( \sin 23^\circ = \frac{AB}{80} \)

This type of problem is solved by multiplying 80 \( \times \) (\( \sin 23^\circ \))

\( AB \approx 80 \times \sin 23^\circ \)

In the picture below \( \theta = 37^\circ \) \( AB = 230 \) \( AC = 230 \).

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin 37^\circ = \frac{AB}{230}
\]

\( AB = 230 \times \sin 37^\circ \)

\( AB = 138.41 \)
Data Brief # 3

Get a theodolite and go outside with the teacher to try some actual measurements using the sine formula.

Activity # 1

\[
\begin{align*}
\sin 43^\circ &= \sin 56^\circ = \\
\sin 72^\circ &= \sin 12^\circ = \\
\sin 30^\circ &= \sin 60^\circ = \\
\sin 88^\circ &= \\
\sin x &= .9945, \quad x = \\
\sin x &= .9135, \quad x = \\
\sin x &= .4384, \quad x = \\
\sin x &= .1045, \quad x = \\
\sin x &= 4/7 \quad x = \\
\sin x &= 5/11 \quad x = \\
\sin x &= 9/10 \quad x = 
\end{align*}
\]
AC = 225 yds
∠C = 22°
Find AB

BC = 300 yds
∠C = 41°
Find AB
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SERIES

EXPLORATION

NUMBER

372.6

CLUSTER

TRANSPORTATION

AREA

GENERAL AVIATION

TITLE

WHO'S GOING FASTEST
Rationale:

Just because a plane is flying at 400 mph doesn't mean he will travel a distance of 400 miles from one place on the earth to another in one hour. Maybe he's flying 400 mph straight up. This means his ground speed is different than his air speed.

This package will demonstrate a method for computing ground speed if the angle of elevation and air speed are known.

Behavioral Objectives:

Given the air speed and angle of elevation of an airplane you will be able to calculate its ground speed with whole number accuracy.

Pre-Test:

Find the ground speed of the airplane in the picture below.

\[ \text{Diagram} \]
An airplane is climbing at an angle of 16° at a speed of 375 mph. What is its ground speed?

Information Sources:

1. Read Data Brief #1 "Cosine and the trig tables"
2. Read Data Brief #2 "Triangles and Cosine"
3. Read Data Brief #3 "Using the theodolite"

Data Brief #1

Use your trig tables for looking up the cosine of an angle the same way you did for looking up sine and tangent.

Example:

\[
\begin{align*}
\cos 10° &= 0.9848 \\
\cos 60° &= 0.5000 \\
\cos 30° &= 0.8660 \\
\cos x &= 0.8192 \quad x = 35° \\
\cos x &= 0.7075 \quad x = 45°
\end{align*}
\]
The cosine formula says $\cos x = \frac{AD}{HYP}$.

Referring to the picture above, Line AC describes the path of an airplane climbing at an angle $x$. Line AC represents the speed of the airplane, not the distance it is flying. Thus, line AB represents its ground speed.

Given that the airplane is climbing at an angle of 25° at a speed of 200 mph, its ground speed is found in the following manner.

\[
\cos 25^\circ = \frac{AB}{200}
\]
\[
.9063 = \frac{AB}{200}
\]
\[
181 = AB
\]

The ground speed is 181 mph.

Data Brief # 3

Pick up a theodolite and a teacher and go outside for some actual measuring with Cosine.
\[ \cos 32^\circ = \]
\[ \cos 47^\circ = \]
\[ \cos 76^\circ = \]
\[ \cos 45^\circ = \]
\[ \cos 10^\circ = \]
\[ \cos 89^\circ = \]
\[ \cos 67^\circ = \]

\[ \cos x = 0.1564 \quad x = \]
\[ \cos x = 0.7193 \quad x = \]
\[ \cos x = 0.8910 \quad x = \]

\[ \cos x = \frac{6}{7} \quad x = \]
\[ \cos x = \frac{1}{4} \quad x = \]
\[ \cos x = \frac{7}{9} \quad x = \]
\[ \cos x = \frac{347}{456} \quad x = \]
\[ \cos x = \frac{23}{450} \quad x = \]
Activity 0 2.
Activity 2

Find the ground speed of the airplane in the above picture if the air speed is 350 mph, and the plane is climbing at an angle of 20°.

Find the ground speed of the airplane from the above picture.

An airplane is climbing at an angle of 29° at a speed (air speed) of 400 mph. What is its ground speed?

An airplane is climbing at an angle of 15° with an air speed of 275 mph. What is its ground speed?
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(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

NUMBER: 372.7

CLUSTER: Transportation

AREA: Data Handling

TITLE: WHERE ARE YOU
Rationale:

At times when you are on the ocean or in the air, it is helpful to know how far away different objects are. It is up to the navigator to decide which trig formula to use: sine, cosine, or tangent. In this package you are the navigator and you will pick which formula to use to get the information asked for.

Behavioral Objectives:

Given different situations involving indirect measurement by use of the sine, cosine, and tangent formulas, you will be able to pick which of the three formulas to use and find the distance with whole number accuracy.
Information Sources:

1. Read Data brief # 1 "Pick your formula - angles"
2. Read Data brief # 2 "Pick your formula - sides"
3. Read Data brief # 3 "Using the theodolite"
Data Brief # 1

In each problem on activity sheet # 1, you are to find the measure of the angle designated by \( x \), using the formula noted inside the triangle.

example:

\[
\cos x = \frac{\text{ADJ}}{\text{HYP}}
\]

\[
\cos x = \frac{1020}{1189}
\]

\[
\cos x = .8579
\]

Using the trig tables, \( x \approx 1^\circ \) (approximately)

Data Brief # 2

In the above triangle, the side opposite the known angle is the side wanted and the hypotenuse is known. For this set of data the Sine formula would be the easiest to use because

\[
\sin x = \frac{\text{OPP}}{\text{HYP}}
\]

\[
\sin 52^\circ = \frac{x}{330}
\]
In the above triangle, you are given the side adjacent to the known angle and you want the side opposite the known angle. Therefore the easiest way to solve for $x$ would be to use the tangent formula because

$$\tan \theta = \frac{opp}{adj}$$

$$\tan 35^\circ = \frac{x}{240}$$

$$0.7002 = \frac{x}{240}$$

Data Brief # 3

Pick up a tape measure, a theodolite and a teacher and go out side for some practical problems.
Activity § 1

Use the trig formula mentioned to find the angle \( x \).
Activity # 2

Name the trig formula you are going to use and show it to the teacher before solving for the unknown.
Because of the shallow water about island A, the ship must stay in a channel 1200' away from the island. The man in the lighthouse radios to the captain of the ship and tells him that he is 2092' from the lighthouse, just as he is passing directly opposite the island. What is the ship's distance from the island if the picture above represents the situation? Which trig formula???

Suppose that instead of telling the boat captain how far he was from the lighthouse, he told him how far the island was from the light house. Which trig formula would the ship's navigator use then?

How far is the island from the lighthouse?
The above picture describes the following situation:

A ship heading due west out of port sees a cloud of smoke on the shore line due north of the port, at an angle of 28°. He knows he is 16,000' from the shore (about 3 miles). How far up the coast is the fire?

Which trig formula?

Which trig formula would he use to find out how far the ship is from the fire?

How far is the ship from the fire?
Two airplanes are flying at an altitude of 10,000 feet. The navigator in plane A is looking through his theodolite and measures an angle of 21° just as the tower tells plane C that he is directly overhead.

What trig formula would you use to find the distance between the airplanes?

How far apart are the airplanes? __________

What trig formula would you use to find how far plane A is from the tower (line AB).

How far is plane A from the tower? __________
S. I. C. S. PAK
(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

NUMBER: 374.8

CLUSTER: Public Service

AREA: Public Records, Titles and Deeds

TITLE: The $25,000 Caper
Rationale:

Even a real estate broker or an escrow officer needs to know math, and not just simple arithmetic. This package leads up to the solving of a problem that actually came up in a Tucson real estate office. The solution to the problem was worth $25,000 to the owner of the real estate office.

But even after the solution was presented to him, the real estate man failed to capitalize on the information because he didn't understand the math involved which was the use of the Law of Cosines and the solving of a second degree equation.

Behavioral Objectives.

Given a problem to solve that requires the use of the Law of Cosines, you will be able to set up the equation and solve the problem with 3 decimal place accuracy, with the use of the calculator and trig tables.

Information Sources:

Read Data brief #1 "The Law of Cosines"
Read Data brief #2 "The Quadratic Formula"
Read Data brief #3 "For $25,000, what is x"
Read Data brief #4 "The theodolite and the Law of Cosines"
The Pythagorean theorem states that in any right triangle, as in the picture to the right

\[ a^2 + b^2 = c^2 \]

There is also a "Pythagorean theorem" for any triangle, not just right triangles. It is called the "Law of Cosines".

Using the triangle at the right as an example, this law states that:

\[ c^2 = a^2 + b^2 - 2ab\cos C \]

Suppose that angle \( C = 90^\circ \). Then solve the above equation for \( c \) if \( a = 3 \), \( b = 4 \). (Remember that \( \cos 90^\circ = 0 \).)

You should get \( c = 5 \). This illustrates that the Pythagorean theorem can be developed from the Law of Cosines.

If you have to find side \( b \) in a triangle the Law of Cosines states that

\[ b^2 = a^2 + c^2 - 2ac\cos B \]

If you want to find side \( a \) the law states that

\[ a^2 = c^2 + b^2 - 2bc\cos A. \]
Data Brief # 2

The quadratic formula allows you to solve any second degree equation by plugging numbers into the formula and gringding out the answer.

The formula states that if \( ax^2 + bx + c = 0 \) then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\( a \) is the coefficient of the \( x^2 \) term, \( b \) is the coefficient of the \( x \) term and \( c \) is the constant.

Given the equation \( 4x^2 + 6x - 7 = 0 \) to solve,

\( a = 4 \)
\( b = 6 \)
\( c = -7 \)

Given the equation \( x^2 + 3x = 13 \) to solve,

\( a = 1 \)
\( b = 3 \)
\( c = -13 \)

This is not a mistake, \( c = -13 \). To put this equation in the proper form to be solved by the quadratic formula the \( -13 \) has to be brought over to the other side of the equal sign.

At this point you should understand that the \( a, b, c \) in the quadratic formula have nothing to do with the \( a, b, c \) in the Law of Cosines.
In the triangle at the right, if the drawing is accurate find side $c$ if $a = 4$, $b = 3$ and $A = 62^\circ$.

Since you are given angle $A$ you have to use the Law of Cosines with an $a$ by itself on one side of the equal sign as illustrated below.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$16 = 9 + c^2 - 2(3)(c)\cos 62^\circ$$

$$0 = -7 + c^2 - 6(.4695)$$

$$0 = -7 + c^2 - 2.817c$$

Since we want to solve this equation for $c$, we will change the letter $c$ to an $x$. Thus,

$$0 = -7 + x^2 - 2.817x$$

$$0 = x^2 - 2.817x - 7$$

Thus $a = 1$

$b = -2.817$

$c = -7$
The following page of this package has a drawing of the situation. As you read this page look at the picture and you will understand the problem better.

A curved road cuts across a piece of property. The curved road forms part of the circumference of a circle, and the radius of that circle is known: 1604.86 ft. This radius is measured from the center of the road to the center of the circle. The road is 150 ft. wide altogether. The edges of the road and the center line form three concentric circles. The property line is cut by the three concentric circles just as a transversal would be cut by three parallel lines an equal distance apart. The problem is: do the three concentric circles cut the property line into two equal parts? The property line does not go through the (imaginary) center of the circles, but is fairly close to it. In the drawing accompanying this problem, the drawing of the curve of the road and the width of the road are not drawn to scale. What you have to find is

Does \( x = 97 \) ft.

Your first job is to prove to a non-mathematician who had high school geometry 40 years ago, that either the boundary line is cut into equal parts or it isn't, by the circles. Then prove your answer by finding the length of \( x \).

The company that originally drew up the deed for this property assumed that \( x \) was 97 ft. The real estate man felt that if it was shorter then his property along the side of the road had been encroached on and he could probably get an extra $25,000 from the county highway department because of this.
Data Brief #4

With a theodolite and a measuring tape, you will accompany the teacher outside to actually use the law of Cosines.
Activity # 1

Using the Law of cosines find the information requested below.

Find $a$ if $b = 6, c = 4, A = 60^\circ$

Find $c$ if $b = 3, a = 5, C = 40^\circ$

Find $b$ if $a = 10, c = 1, B = 30^\circ$

Find $c$ if $b = 2, a = 6, c = 20^\circ$

Find $a$ if $b = 2, c = \sqrt{3}, A = 39^\circ$

Find $c$ if $b = \sqrt{2}, a = 7, C = 45^\circ$

For the two problems below, set them both up and show them to the teacher BEFORE doing any arithmetic.

Find $c$ if $a = 2, b = 4, A = 60^\circ$

Find $c$ if $a = \sqrt{2}, b = \sqrt{3}, A = 60^\circ$

1. A surveyor at $C$ sights two points $A$ and $B$ on opposite sides of a lake. If $C$ is 5000 ft. from $A$ and 7500 ft. from $B$, and angle $ACB$ measures $30^\circ$, how wide is the lake?
A railroad company wants to tunnel thru a mountain from point A to point B. To make an accurate estimate as to cost, they have to know how far it is from A to B. The only practical place to set up a theodolite is at point C.

AC measures 1359 yards
BC measures 1925 yards
Angle C = 72°

What is the distance from A to B?

Explain how you would do this problem if the hill at point D blocked your sight of point A, and you can find no other place to put the theodolite.
Activity # 2

Solve the following equations for all possible values of $x$ using the quadratic formula where necessary.

1. $x^2 + 7x + 12 = 0$
2. $x^2 + 9x + 14 = 0$
3. $x^2 + 3x - 70 = 0$
4. $x^2 - 6x + 8 = 0$
5. $x^2 - 10x + 9 = 0$
6. $x^2 - 11x - 26 = 0$
7. $x^2 - x - 42 = 0$
8. $x^2 - 3x + 1 = 0$
9. $x^2 - 6x - 11 = 0$
10. $x^2 - 3x - 8 = 0$
11. $x^2 + 2x - 7 = 0$
12. $x^2 - 3x - 11 = 0$
13. $x^2 + 4x + 1 = 0$
14. $x^2 + 9x = 22$
15. $x^2 = 7x - 10$
16. $-x^2 + 13x = -30$
17. $x^2 - 21x = -20$
18. $x^2 + 8x = -15$
19. $x^2 - 6x = 0$
20. $x^2 = 11x$
21. $3x^2 - 5 = 4$
22. $x^2 - 14 = -5x$
23. $11 + x^2 = -10x$
24. $3 - x = x^2$
ACTIVITY # 2

Use the Law of Cosines to solve for the unknown in the following problems. The picture is fairly accurate.

1. $A = 52^\circ$   $b = 6.3$   $a = 8.4$   Find $c$

2. $A = 48^\circ$   $a = 11$   $c = 9.1$   Find $b$

3. $B = 80^\circ$   $a = 9$   $c = 7$   Find $b$

4. $C = 65^\circ$   $a = 4$   $b = 9$   Find $c$

Use the triangle below for the following problem.

5. $B = 35^\circ$   $a = 14$   $b = 10$   Find $c$

The drawing is accurate.
Find x. Use triangles ABC and ACD.
SAHUARITA HIGH SCHOOL
CAREER
CURRICULUM
PROJECT

COURSE TITLE: GEOMETRY
FOURTH QUARTER
BY
JAMES MADEHEIM
<table>
<thead>
<tr>
<th>Unit 380</th>
<th>Miscellaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-unit 382</td>
<td>Proofs involving circles</td>
</tr>
<tr>
<td>Sub-unit 384</td>
<td>Constructions</td>
</tr>
<tr>
<td>Sub-unit 386</td>
<td>Loci</td>
</tr>
<tr>
<td>Sub-unit 388</td>
<td>Co-ordinate Geometry</td>
</tr>
</tbody>
</table>

| Unit 390 | Math and the Steel Industry |
Career Resources.

"A job with a future in the steel industry" by Robert O. Davis.
Sub-unit 392

Given a statement to prove - within the framework of this unit, you must be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- Equal central angles have equal arcs
- An inscribed angle measures one-half its arc
- A diameter perpendicular to a chord bisects it

Given a test covering this material, you should be able to answer 60% of the questions accurately.

Materials: Geometry textbook

Read p. 321-325
Do oral ex. p. 325 with the teacher
Do written ex. o. 326 #1-7, 9-11, 13, 15

Read p. 323-324
Do oral ex. p. 329 with teacher
Do written ex. o. 333 #1-12, odd 10, 21

Read p. 330-33

Take test
QUESTIONS

1. \( \overline{AC} \) is a diameter, \( \overline{OB} \) is a radius, and \( \overline{BC} \) is a chord of \( \odot O \). Without introducing any new labels, name
   a. A minor arc.
   b. A major arc.
   c. A semicircle.
   d. A central angle.
   e. An inscribed angle.

2. Points \( X, Y, Z, \) and \( W \) lie on a circle \( O \).
   a. If \( \angle XOY = \angle ZOW \), then \( \overline{XY} \rightarrow \overline{ZW} \).

3. What is the measure of an angle determined by the hands of a clock at 10 o'clock?

4. Points \( A, X, \) and \( B \) lie on a circle as shown. \( \overline{AX} \) is a tangent. Let point \( X \) approach point \( A \) as a limit.
   a. Ray \( \overrightarrow{AX} \) approaches \( \overrightarrow{A} \) as a limit.
   b. The measure of \( \overline{BX} \) approaches \( \overrightarrow{a} \) as a limit.
   c. The measure of \( \angle BAX \) approaches \( \overrightarrow{b} \) as a limit.
   d. Since \( \angle BAX = \frac{1}{2} \overline{BX}, \angle BAK = \overrightarrow{c} \).
QUESTIONS

5. Points $R$, $S$, $T$, $Q$, and $U$ lie on $\bigcirc O$. $\overline{RZ}$ is a tangent, and $SU$ is a diameter.
   $\overline{RS} = 80^\circ$, $\overline{ST} = 44^\circ$, $\overline{OU} = 32^\circ$.

   a. $\angle 1 = \_\_\_^\circ$
   b. $\angle 2 = \_\_\_^\circ$
   c. $\angle 3 = \_\_\_^\circ$
   d. $\angle 4 = \_\_\_^\circ$
   e. $\angle 5 = \_\_\_^\circ$
   f. $\angle 6 = \_\_\_^\circ$
   g. $\angle 7 = \_\_\_^\circ$
   h. $\angle 8 = \_\_\_^\circ$
   i. $\angle 9 = \_\_\_^\circ$
   j. $\angle 10 = \_\_\_^\circ$

6. $\overline{AC}$ and $\overline{BD}$ are chords of the circle shown.
   a. If $\angle ATB = 72^\circ$ and $\overline{AB} = 65^\circ$, $\overline{CD} = \_\_\_^\circ$.
   b. If $\angle CTD > 69^\circ$ and $\overline{CD} < 82^\circ$, then $\overline{AB}$ __.

6. Complete the demonstration of Case 1 of the theorem about the measures of inscribed angles.

   Given: $\angle RSV$ is inscribed in $\bigcirc O$; $\overline{SV}$ is a diameter.
   To Prove: $\angle RSV = \frac{1}{2} \angle ROV$.

   STATEMENT
   a. Draw $\overline{OR}$.
   b. $\angle R + \angle S = \angle ROV$.
   c. $\overline{OR} = \overline{OS}$.
   d. $\angle R = \angle S$.
   e. $\angle S + \angle S = \angle ROV$.
   f. $\angle S = \frac{1}{2} \angle ROV$.

   REASON
   a. ..............................................................
   b. ..............................................................
   c. ..............................................................
   d. ..............................................................
   e. ..............................................................
   f. ..............................................................
7. If chord $RS$ of a circle is not a diameter, but chord $QT$ is a diameter, and $QT \perp RS$, what can you conclude about $\triangle QRS$?

8. Points $A, B, C, D, E, F, G$ lie on a circle in the order named and divide the circle into seven equal arcs. $AC, DF, CE,$ and $BF$ are drawn.
   a. How are $AC$ and $DF$ related?
   b. How are $CE$ and $BF$ related?

9. $PT$ is a tangent and $PR$ is a secant of the circle. As secant $PX$ rotates about point $P$ so that points $X$ and $Y$ approach each other,
   a. Point $X$ and point $Y$ both approach + as a limit.
   b. $PX$ and $PY$ both approach _ as a limit.
   c. $PX$ and $PY$ both approach _ as a limit.
   d. Since $PR \cdot PS = PX \cdot PY$, $PR \cdot PS = (?)^2$.

10. Given: $\odot O$ with diameter $TX$ and chord $RS$; $TX \perp RS$.
    To Prove: $RM = SM$; $RX = SX$.

   The proof begins with the statement "Draw $OR$ and $OS." Write the remaining statements—but not any reasons—of the proof.
Sub-unit 384

Given a simple construction do do, you will be able to do it correctly and you will be able to state which postulate or theorem justifies this construction. Your proficiency in these skills will be judged by the work handed in.

Do constructions 1-12

Do w/s 384.2
1. Draw an acute triangle and bisect each of its angles.
2. Draw two supplementary adjacent angles. Bisect each angle and demonstrate to the teacher that a right angle has been formed.
3. Construct an angle with a measure of 45°.
4. Draw an obtuse triangle and construct its medians.
5. Draw a triangle and construct the perpendicular bisector of each side.
6. Circumscribe a circle around the right triangle below.

7. Trisect an angle. You pick the angle.
Sub-unit 386

Given an accurate description of a set of points, you should be able to name the figure they represent or draw a picture of them. Your proficiency will be judged by the work you hand in.

Materials: Geometry textbook

Read p. 375-7
   Do oral ex. with the teacher p. 377
   Do written ex. p. 377 # 1-9, 15, 23

Read p. 379-80
   Do oral ex. p. 381 with the teacher
   Do written ex. p. 381 # 1, 3, 6

Read p. 382
   Do oral ex. with the teacher
   Do written ex. p. 383 # 1, 5, 10, 14
Introduction to the Fourth Quarter.

This quarter contains proofs involving circles, geometrical constructions, loci, and co-ordinate geometry.

The subjects covered in this unit are useful in all types of construction and the final unit deals with their use in the steel industry.
Objectives.

During this quarter you will increase your knowledge of careers in every cluster.

You will increase your awareness of skills necessary for every job area in the steel industry, from garbage man to engineer.

Given a statement to prove — within the framework of this unit, and units 340, 330, and 350 — you will be able to form a rigorous proof of this statement using the following phrases in your reasoning:

- equal central angles have equal arcs
- an inscribed angle measures 1/2 its arc
- a diameter perpendicular to a chord bisects it

Given a test covering this material, you will be able to answer 60% of the questions accurately.

Given a simple construction to do, you will be able to do it correctly and you will be able to state which postulate or theorem justifies this construction. Your proficiency in these skills will be judged by the work handed in.

Given a description of a set of points, you will be able to name the figure they represent or draw a picture of them. Your proficiency will be judged by the work handed in.
When you have completed unit 388 you will be able to:

- graph a quadratic equation
- find the distance between any two points on a graph
- recognize the equation of a circle
- find the midpoint of any line segment
- find the slope of any line
- recognize from their equations if two lines are parallel or perpendicular
- write equations of straight lines given two points that satisfy the equation

Given a test covering this material, you will be able to answer 60% of the questions accurately.

When you have finished unit 390, you will be able to name at least a dozen steel industry jobs that call for a background in math.
You should be able to:

1. Given any two points you should be able to write the equation of the straight line containing these points.

2. Given a test covering this material you should be able to answer 60% of the questions accurately.

Materials:
- Geometry Textbook

Assignment:
- Read p. 399
- Do W/S 388.2
- Do Oral Ex. p. 400 with the Teacher
- Do written ex. p. 406 #1-7
Do Oral Ex. with the Teacher
Do Written Ex. p. 408 #1-6, 13, 19-22
Lecture for p. 410
Do Oral with the teacher p. 411
Do Written Ex. p. 411 #1,2,5,6
Do W/S 388.6
Do W/S 888.8

Quest Read p. 412-414. Do Written Exam
Read p. 419-422 Do Written Exam

Take Test 388
SUB-UNIT 340

When you have completed this unit you should be able to:

- Graph a quadratic equation
- Find the distance between any two points on a graph
- Recognize the equation of a circle
- Find the midpoint of any line segment
- Find the slope of any line or parabola (at any point)
- Recognize from their equations if two lines are parallel or perpendicular
- Write simple equations of straight lines given two points that satisfy the equation

Given a test covering this material you should be able to answer 60% of the questions accurately.

Given any two points you should be able to write the equation of the straight line containing these points.

Materials: Geometry textbook.
- Do W/S 362-2, 388-2
- Read p. 399
- Do oral ex. p. 400 with the teacher
- Do W/S 362-2, 388-4
- Read p. 404-5
- Do oral ex. p. 405 with the teacher
- Do written ex. p. 406 #1-7
DO ORAL EX. WITH THE TEACHER
DO WRITTEN EX. P. 408 # 1-6, 13, 19-22

LECTURE FOR P. 419
DO ORAL WITH THE TEACHER P. 411
DO WRITTEN EX. P. 411 # 1, 2, 5, 6

DO w/s 386-C

DO w/s 588, 8

QUEST READ P. 412-414
READ P. 419-422

TAKE TEST 288
Graph the pairs of equations below.
Find their points of intersection.
Find the distance between these two points.

A \[ y = x^2 + 5 \]
B \[ y = 3x + 3 \]

C \[ y = x^2 - 3 \]
D \[ y = x - 1 \]

E \[ y = 4x^2 + 3 \]
F \[ y = 8 \]

G \[ y = x^2 + 3x - 5 \]
H \[ y = -2x - 3 \]

What is the slope of equation A at $x = 3$

B at $x = -456$
C at $x = -4$
D at $x = 0$
E at $x = 16$
F at $x = 12, 3/4$
G at $x = -2$
G at $x = 3$
G at $x = -1$
H at $x = 4$
TEST 3ON P. 2

Pick pairs of equations that are parallel or perpendicular. You may use graph paper if you wish.

<table>
<thead>
<tr>
<th></th>
<th>Parallel</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( y = 2x - 4 )</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( y = 3x - 11 )</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>( 3y = 4x + 5 )</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>( x = -y )</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>( 2y = -5x + 1 )</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>( 2y = 6x + 1 )</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>( y = -1 )</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>( 3y = 4x - 7 )</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>( y = 7 )</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>( 8y = -6x + 13 )</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>( y = -5x + 3 )</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>( y = x )</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>( y = 17x - 14 )</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>( y = -4x )</td>
<td></td>
</tr>
</tbody>
</table>

Write the equation for the straight line that goes thru each set of points below.

\((2,5)\) (6,15)
\((-2,-1)\) (6,3)
\((3,-5)\) (1,-1) (4,-7)
USE GRAPH PAPER

Plot each equation below on one graph. Your axes should cover the whole side of the graph paper. Label each graph.

\[ y = x \quad y = x + 5 \quad y = x - 6 \]
\[ y = x + 2 \quad y = x + 8 \quad y = x - 10 \]

Using induction, write below what effect the constant term has on the graph.

Plot each equation below on one graph. Use a whole side of the graph paper. Label each graph.

\[ y = x^2 + 2 \quad y = x^2 + 5 \quad y = x^2 - 2 \]
\[ y = x^2 - 7 \quad y = x^2 \quad y = x^2 + 8 \]

Use induction again and make a comment about the affect the exponent has on the graph, and also the affect of the constant.
SAME DIRECTIONS AS ON THE PREVIOUS PAGE.

\[ y = 2x^2 + 3 \quad y = \frac{\alpha^2}{\beta} + 3 \quad y = 3x^2 \]
\[ y = \frac{x^2}{3} \quad y = 4x^2 - 5 \quad y = \frac{x^2}{4} - 5 \]

USE SOME INDUCTION ON THE COEFFICIENT OF THE \( x^2 \) TERM.

SAME DIRECTIONS.

\[ y = 2x^2 + 4x \quad y = x^2 + 2x \quad y = \frac{\alpha^2}{\beta} + x \]
\[ y = \frac{x^2}{\gamma} + \delta \quad y = \frac{x^2}{\gamma} + \delta \]

USE INDUCTION ON THE AFFECT OF THE LINEAR TERM IN THESE EQUATIONS.

GRAPH THE EQUATIONS BELOW. USE FOUR SETS OF AXIS TO ONE SIDE OF THE GRAPH PAPER. ONE GRAPH FOR EACH SET OF AXES,

\[ y = x^2 + 3x + 2 \quad y = x^2 + 7x + 12 \]
\[ y = 2x^2 + 14x + 24 \quad y = 3x^2 + x - 2 \]
\[ y = 2x^2 + 5x - 3 \quad y = x^2 + 2x + 4 \]

FOR EACH GRAPH, IF \( y = 0 \), THEN WHAT IS THE VALUE FOR \( x \)?
WITHOUT GRAPHING ANY OF THE EQUATIONS BELOW, LABEL EACH ONE AS EITHER SYMMETRIC TO THE X-AXIS OR THE Y-AXIS OR BOTH OR NEITHER.

\[ Y = x^2 \]
\[ Y = x^2 + 3 \]
\[ Y = x - 4 \]
\[ x = y^2 - 2 \]
\[ y = -x^2 \]
\[ x^2 + y^2 = 25 \]
\[ x = y^2 + 4x - 3 \]
\[ y = x^2 - 7x + 3 \]
\[ y = \frac{\sqrt{x^2}}{5} - 6x + 3 \]
\[ y = \frac{\sqrt{y^2}}{3} - 17.5 \geq 7 \]
\[ x = 3y^2 - 4 \]
\[ 3x^2 + 4y^2 = 15 \]
\[ y = x^3 + 3x^2 - 5x + 1 \]
**Find the slope of each equation below**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x + 4$</td>
<td>1</td>
</tr>
<tr>
<td>$y = x - 5$</td>
<td>1</td>
</tr>
<tr>
<td>$y = 2x - 3$</td>
<td>2</td>
</tr>
<tr>
<td>$y = 5x - 1$</td>
<td>5</td>
</tr>
<tr>
<td>$y = 4x - 2$</td>
<td>?</td>
</tr>
<tr>
<td>$y = 7x + 3,456$</td>
<td>?</td>
</tr>
<tr>
<td>$y = x - 1$</td>
<td>?</td>
</tr>
<tr>
<td>$y = x$</td>
<td>?</td>
</tr>
<tr>
<td>$3y = 6x - 7$ (IT IS NOT 6)</td>
<td>?</td>
</tr>
<tr>
<td>$y = \frac{2}{3}x + 3$</td>
<td>?</td>
</tr>
<tr>
<td>$y = \frac{3}{4}x + \frac{1}{2}$</td>
<td>?</td>
</tr>
<tr>
<td>$2y = 3x - 8$ (IT IS NOT 3)</td>
<td>?</td>
</tr>
<tr>
<td>$y = -2x - 4$</td>
<td>?</td>
</tr>
<tr>
<td>$y = -x - 6$</td>
<td>?</td>
</tr>
<tr>
<td>$y = 2$</td>
<td>?</td>
</tr>
<tr>
<td>$x = 6 - y$</td>
<td>-1</td>
</tr>
<tr>
<td>$x = 3 + 2y$</td>
<td>?</td>
</tr>
<tr>
<td>$8 = 3x - y$</td>
<td>?</td>
</tr>
<tr>
<td>$-4y = 6x + 1$</td>
<td>?</td>
</tr>
<tr>
<td>$x = 4$</td>
<td>?</td>
</tr>
<tr>
<td>$2x - y = 4$</td>
<td>?</td>
</tr>
<tr>
<td>$4y + 2x - 7 = 0$</td>
<td>?</td>
</tr>
<tr>
<td>$2y - 8x + 3 = 0$</td>
<td>?</td>
</tr>
<tr>
<td>$3x + 4y = 2$</td>
<td>?</td>
</tr>
<tr>
<td>$y = \frac{6}{4} - 3$</td>
<td>?</td>
</tr>
</tbody>
</table>
**Theorem:** Two nonvertical lines are parallel if and only if they have equal slopes.

For each pair of equations below graph them together on one set of axes.

\[ y = 2x - 1 \quad y = x - 3 \quad y = 3x + 5 \]
\[ y = 2x + 2 \quad y = x + 4 \quad 2y = 6x - 2 \]

Write yes or no after each pair of equations below, meaning that they are parallel or they aren't.

\[ y = x - 7 \quad y = 3x + 4 \quad y = 1x - 2 \]
\[ y = x + 4 \quad y = 3x - 6 \quad y = 2x - 1 \]
\[ y = 2x + 3 \quad 2y = 3x + 7 \quad y = \frac{2}{3}x - 1 \]
\[ 2y = 4x - 1 \quad 2y = 2x + 6.5 \quad 3y = 2x + 1 \]
\[ 3y = 2x - 4 \quad 4y = 10x + 6 \quad 3x + 5y = 7 \]
\[ 8y = 4x - 1 \quad 2y = 5x - 4 \quad 5y = 3x - 2 \]
\[ y = -x + 3 \quad 3x + 3y = 0 \]

\[ 4x - 3y - 2 = 0 \]

\[ \frac{y}{3} = \frac{x}{4} + \frac{2}{4} \]
THEOREM: Two nonvertical lines are perpendicular if and only if the slope of one line is the negative reciprocal of the slope of the other line.

Graph each set of equations below on a separate set of axes.

\[
\begin{align*}
Y &= x \\
Y &= -x \\
Y &= 2x - 3 \\
Y &= -\frac{1}{2}x + 1 \\
Y &= -3x - 1 \\
2Y &= -6x + 3 \\
3Y &= x - 2
\end{align*}
\]

Tell if each set of equations below are parallel, perpendicular or neither. (Do not graph)

\[
\begin{align*}
2Y &= -4x - 6 \\
Y &= -\frac{x}{2} + 1 \\
Y &= -\frac{5}{3}x + 1 \\
5Y &= 3x + 10 \\
8x + 4y &= 7 \\
Y &= -2x - 1 \\
5x + 2y &= 11 \\
3x - 6y &= 4 \\
2y + 3x &= 5 \\
3y - 2x &= 4
\end{align*}
\]
Plot each equation below on one graph. Use a whole sheet of paper. Label each graph.

\[ y = -x^2 \]
\[ y = -x^2 + 2x - 11 \]
\[ y = -x^2 + 6 \]
\[ -y = x^2 \]
\[ -y = -x^2 + 3 \]

Use some induction on the negative signs.

Plot the equation below. Use values for \( x \)
Use integral values for \( x \) between and including 3, 5 to -5
\[ y = x^3 - 9x + 2 \]

Try some induction on all equations with an exponent of 3.
(2,4)  (5,10)
The equation for the line that goes thru the two above points is
\[ y = 2x \]

(3,12)  (-2,-8)
The equation for the line that goes thru the two above points is
\[ y = 4x \]

(1,5)  (3,13)
The equation for the above two points is
\[ y = 4x + 1 \]

Try to find the equations that go thru the following sets of points.

\[
\begin{array}{ccccccc}
3, & -6 & 4, & -8 \\
-2, & 17 & 4, & -20 \\
7, & 15 & 2, & 5 & 8, & 17 \\
0, & 0 & -3, & -3 & & \\
2, & 4 & 3, & 4 & 5, & 4 & -2, & 4 & -1, & 4 \\
0, & 0 & 1, & 3 & 3, & 9 & & \\
0, & 0 & -2, & 5 & & \\
3, & a & 3, & 7 & & \\
8, & 5 & 10, & 6 & & \\
3, & 1 & 12, & 8 & & \\
\end{array}
\]
When you have finished this unit, you should have a better understanding of why a math background is necessary for finding a job, in some industries.

Materials: "A job with a future in the Steel Industry" by Robert O. Davis.

Blueprints are scale drawings. Reasoning and analyzing are thinking skills such as you covered in the beginning of this year. Using the materials mentioned above, how many jobs can you find that mention the need for these skills.

List some of the jobs.

What definite math skill does a pipefitter need?

Which of the skills listed below is mentioned most often?

- Reasoning
- Analyzing
- Read prints
- Judgement
- Interpret drawings
- Electronic skills
- Mechanical skills

Compare the jobs that require math skills to the jobs that don't. What do you notice about salaries?

Do any of the jobs mentioned appeal to you? Why?

What other industries might have jobs that require the same skills as mentioned for the steel industry?