DOCUMENT RESUME

ED 080 275 RC 007 208

AUTHOR	Kayser, Brian D.
TITLE	Educational Aspirations: Markov and Poisson Models. Rural Industrial Development Project Working Paper Number 14, August 1971.
INSTITUTION	Wisconsin Univ., Madison. Center of Applied Sociology.
SPONS AGENCY	National Inst. of Mental Health (DHEW), Bethesda, Md.
REPORT NO	NIMH-RG-1-R11-MH-19689
PUB DATE	Aug 71
NOTE	121p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	*Educational Attitudes; *High School Students; Industrialization; Mobility; *Models; *Rural Areas; Sociocultural Patterns; *Statistical Studies; Tables (Data)
IDENTIFIERS	*Illinois

ABSTRACT

The fit of educational aspirations of Illinois rural high school youths to 3 related one-parameter mathematical models was investigated. The models used were the continuous-time Markov chain model, the discrete-time Markov chain, and the Poisson distribution. The sample of 635 students responded to questionnaires from 1966 to 1969 as part of an ongoing research on rural industrialization in northern Illinois. The students were drawn from both the experimental and the control area of the study and were grouped as cohorts based on the year of graduation. Educational aspirations over time were treated as separate variables and were cross-tabulated. After the parameter values for each model were computed, data were tested for fit to the model. It was found that the data fit the discrete-time Markov model. The general pattern analysis carried out prior to the model test appeared to coincide w. .h the actual data analysis. (PS)

ERIC

FILMED FROM BEST AVAILABLE COPY

RECEIVED
SEP 11 1973
NMSU
E.R.I.C.

ED 080275

RURAL INDUSTRIAL DEVELOPMENT

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

PROJECT WORKING PAPER
Number 14
August, 1971

EDUCATIONAL ASPIRATIONS: MARKOV AND POISSON MODELS

BRIAN D. KAYSER

THE UNIVERSITY OF WISCONSIN

Supported by

U. S. PUBLIC HEALTH SERVICE
NIMH Research Grant 1–R11–MH 19689
Principal Investigators
Gene F. Summers and John P. Clark

CENTER OF APPLIED SOCIOLOGY
DEPARTMENT OF RURAL SOCIOLOGY
College of Agriculture and Life Sciences

RC007208

TABLE OF CONTENTS

Chapter | Page

Chapter Page

APPENDICES

LIST OF TABLES

Table		Page

Table		Page

CHAPTER I

THE PARADIGM

The Statement of Purpose

This research is an investigation into the fit of educational aspirations of rural community high school youth to three related one-parameter mathematical models. Although these data could possibly fit many such models, only three are to be studied. These three are the discrete-time Markov chain, the continuous-time Markov chain, and the Poisson distribution.

The use of these models rather than any others can be justified on ~~three~~ grounds. First, a previous study (and the only such study found) dealing with aspirations over time using the continuous-time Markov chain model found the data fit the model. Second, previous studies dealing with social and labor mobility (as well as other social processes) have used the model with some success. The use of the model to study aspirations represents a logical extension of the model to a prior phase in the process of mobility, since aspirations play an important role in determining educational attainments which in turn play a very important role in determining the occupational attainment. Occupational attainments are central in the

ERIC

study of mobility.[1] Third, all three models are related. The difference between the two Markov models is in their treatment of intervals of time between periods. The Poisson can be considered as a type of Markov chain as well as being one of the simplest continuous-time models for the study of change.

A Review of the Literature

The review of educational aspirational literature revealed only one study which used the Markov chain and panel data. This study was conducted by McDill and by Coleman using two-wave panel data for freshmen and seniors from nine high schools in Northern Illinois.[2] They used a continuous-time four-state Markov chain model in

---

[1]Recent studies indicate that the level of educational attainment is the most significant factor in determining the occupational attainment. Occupational attainments are the variables in the study of mobility where the typical unit of analysis is the father-son unit. Some of the more recent studies dealing with aspirations and attainments are B.K. Eckland, "Academic Ability, Higher Education, and Occupational Mobility," American Sociological Review 30 (1965), pp. 735-746, G.H. Elder, "Achievement Motivation and Intelligence in Occupational Mobility: a longitudinal Analysis," Sociometry 31 (1968), pp. 327-354, G.H. Elder, "Appearance and Education in Marriage Mobility," American Sociological Review 34 (1969), pp. 519-533, and W.H. Sewell, A.O. Haller and A. Portes, "The Educational and Early Occupational Attainment Process." American Sociological Review 34 (1969), pp. 82-92.

[2]E.L. McDill and J.S. Coleman, "High School Social Status, College Plans, and Interest in Academic Achievement: a Panel Analysis," American Sociological Review 28 (1963), pp. 905-918. Not only was this the only study found which combined both the model and the use of panel data, it also appeared to be the only study which viewed aspirations from the standpoint of a stochastic model at all.

their analysis of hig school social status, achievement valuation, and college plans. They found that the turnover rates for college plans were differentially affected by membership in the leading crowd and by valuation for scholastic achievement.

Much of the past research, nevertheless, has concentrated upon the descriptive rather than the predictive level. In addition, over-time data have seldom been used. Cross-sectional data are normally used. Any research with cross-sectional data which attempts to analyze causal linkages in order to describe the process of aspirations is severely hampered because of the problem in ordering the variables.

Without the benefit of time-series there have been e-forts to present models of aspirations by the inclusion of variables which have in past studies been found to differentiate among aspirational levels. These efforts have typically used the path analytical model which posits a set of regression equations for a given number of endogeneous variables. The analysis used in this research, nevertheless, is superior to this approach in its use of time-series data and its concentration on actual prediction of aspirations rather than on description of the causal linkages using a theoretical ordering of variables. Some of these studies using the path model technique will now be briefly mentioned.

Rehberg and Westby presented a provisional model using father's education, father's occupation, parental encouragement, family size, and educational aspirations.[3] They did not test this model, however. Testing the adequacy of four competing path models of aspirations, Duncan, Haller, and Portes included the student's and his best friend's intelligence, parental aspiration, family social status, and the occupational and educational aspirations.[4] In their study of the educational and early occupational process Sewell, Haller, and Portes presented a social psychological path model which included such variables as intelligence, high school rank in class, socio-economic status, significant others' influence, occupational aspirations and attainments, and educational aspirations and attainments.[5] Bayer using a national sample of students presented path models for both sexes dealing with marital plans, ability, and social status and their effect on educational aspirations.[6] In his study of schools and stratification Hauser presented models which included educational aspirations.[7] In his study of mobility Elder

---

[3]R.A. Rehberg and D. L. Westby, "Parental Encouragement, Occupation, Education and Family Size: Artifactual or Independent Determinants of Adolescent Educational Expectations?" Social Forces 45, 3 (1967), pp. 362-374.

[4]O.D. Duncan, A.O. Haller and A. Portes, "Peer Influences on Aspirations: a Re-interpretation," American Journal of Sociology 74 (1968), pp. 119-137.

[5]W.H. Sewell, A.O. Haller and A. Portes, "The Educational and Early Occupational Attainment Process," American Sociological Review 34, 1 (1969), pp. 82-92.

[6]A.E. Bayer, "Marriage Plans and Educational Aspirations," American Journal of Sociology 75, 2 (1969), pp. 239-244.

[7]R.M. Hauser, "Schools and the Stratification Process," American Journal of Sociology 74, 6 (1969), pp. 587-611.

presented three path models which included achievement drive rather than educational aspirations, intelligence, educational attainment, occupational status, and social class.[8] Rehberg, Schafer, and Sinclair analyzed two patterns of linkages in their path analytic study of social class, intelligence, mobility attitudes, and educational expectations.[9]

Of these studies only the Elder research used longitudinal data for adolescents over the high school period. Nonetheless, his study was limited to the analysis of 84 subjects who were sampled from 1932 to 1939 as youth and once more as adults. The McDill and Coleman study which was previously discussed sampled only at the freshman and senior years rather than during all the years in school.

Since the other studies used cross-sectional data in their analysis of the causal linkages in aspirations, the dangers of this method of data analysis will now be discussed.

The Problem of Cross-Sectional Data Analysis in the Study of Causal Linkages

The use of cross-sectional data to study causal inferences is of dubious value. In a study using regres-

---

[8]G.H. Elder, "Achievement Motivation and Intelligence in Occupational Mobility: a Longitudinal Analysis," Sociometry 31, 4 (1968), pp. 327-354.

[9]R.A. Rehberg, W.E. Schafer and J. Sinclair, "Toward a Temporal Sequence of Adolescent Achievement Variables," American Sociological Review 35, 1 (1970), pp. 34-48.

ERIC

sion analysis on cross-sectional and over-time data Maton investigated participation in high school and university education related to income per head, distance to the neighboring school, and pressure of demand for technically trained personnel.[10] He found that over-time predictions had higher accuracy than did cross-sectional estimates. He noted that a major objection in regressions based on cross-sectional data is that it tells how variables affect one another during a given time, but not over time, and that prediction is based on the assumption of no change in the relationship pattern. In the same vein one of the conclusions of the Rehberg, Schafer, and Sinclair article mentioned the problems involved. The authors' last conclusion was:

> Third, the drawing of causal inferences from cross-sectional, _ex post facto_ survey data is fraught with considerable risk, unless the temporal orderings are unequivocal, as with such variables as occupation of father and occupation of son. What is needed, then, are longitudinal data on pertinent adolescent achievement variables which will permit application of appropriate multivariate techniques of causal analysis. . . It is most probable then, that significant advances in our understanding of adolescent achievement await the application of more sophisticated methodological techniques to longitudinal data in order that we may construct theories congruent with the dynamic interdependence of the empirical circumstances.[11]

---

[10] J. Maton, "Regional Differences in Educational Participation: a Regression Analysis of Cross-sectional Data on Participation Coefficients," _Sociology of Education_ 39, 3 (1966), pp. 276-287.

[11] Rehberg, _et al._, _op. cit._, pp. 46-47.

It is when the temporal sequence of variables are of primary interest that one is interested in process. It is not until the variables are investigated over time that one can adequately study process. This would be the preferable method of investigating the educational aspirational process. One could develop a complex causal pattern where the variables at each time of observation are treated as separate variables. This would necessitate panel data with numerous waves of observation for a large sample. To this point, nonetheless, investigators have been willing to first discover the stable patterns of influence while risking debatable temporal orderings rather than waiting until over-time data can be collected and analyzed where the emphasis is upon aspirations as a stochastic process. Nevertheless, for the reasons given above the past efforts are inadequate.

Since past efforts in aspirational studies have been less than process minded and have been hampered by the use of cross-sectional data, interest in this paper has been focused on mobility literature. The study of mobility is intimately related to educational aspirations. This relationship will now be discussed.

Educational Aspirations and the Study of Mobility

The link between aspirations and mobility is the chief justification for extension of the Markov chain model to educational aspirational data. The importance of the level of aspirations is widely accepted, and a vast number of studies have been carried out to determine what influences aspirations.[12] In addition, the link between aspirations and attainments has been investigated. The relationship is clearly strong and positive.[13] It is

---

[12]Although I can quote no exact number of studies, the number is quite large. One study, for example, based their hypotheses on a review of over 200 previous studies. See Rehberg and Westby, op. cit., p. 362. For studies with extensive references see W.H. Sewell, A.O. Haller and M.A. Straus, "Social Status and Educational and Occupational Aspiration," American Sociological Review 22, 1 (1957), pp. 67-73, A.O. Haller and C.E. Butterworth, "Peer Influences on Levels of Occupational and Educational Aspiration," Social Forces 43 (1960), pp. 289-295, W.H. Sewell, "Community of Residence and College Plans," American Sociological Review 29 (1964), pp. 24-38, W.H. Sewell and J.M. Armer, "Neighborhood Context and College Plans," American Sociological Review 31, 2 (1966), pp. 159-168, W.H. Sewell and V.P. Shah, "Socioeconomic Status, Intelligence, and the Attainment of Higher Education," Sociology of Education 40 (1967), pp. 1-23, and W.H. Sewell and V.P. Shah, "Social Class, Parental Encouragement, and Educational Aspiration," American Journal of Sociology 73 (1968), pp. 559-572.

[13]Some of the studies which show this relationship are given below. In G.H. Elder, "Achievement Orientation and Career Patterns of Rural Youth," Sociology of Education 37, 1 (1963), pp. 30-58 census data on aspirations and college attendance were analyzed. Despite differential nonattenders from those who aspired to college, nationally of the 50 percent aspiring to college some 43 percent actually did attend. Elder also found in his study of occupational mobility that the path coefficient from aspiration to attainment was of moderate strength. Other studies which have found a strong relationship are Sewell, et al., op. cit., Sewell and Shah, loc. cit., C.N. Alexander and E.Q. Campbell, "Peer Influences on Adolescent Educational Aspirations and Attainments," American Sociological Review 29 (1964), pp. 568-575, E.G. Youmans, "Factors in Educational Attainment," Rural Sociology 24, 1 (1959), pp. 21-28, and W. Adams, "Financial and Non-Financial Factors Affecting Post-High School Plans and Evaluations, 1939-1965," in E.D. Goldfield (Ed.), American Statistical Association Proceedings of the Social Statistics Section, 1969 (Washington, D.C.: American Statistical Association, 1969), pp. 99-124.

attainment which is central to any study of mobility. The usual approach involves the study of change in attainments over time. This approach can apply equally as well to changes in aspirations over time. A look at an analysis of an early study of labor mobility will illustrate.

In his book on applications and theory of stochastic models of social processes Bartholomew reviews the Markov chain model as used in the study of labor mobility in Great Britain by Blumen, Kogan and McCarthy.[14] The model assumed a closed system where people change jobs at irregular intervals. The model was to account for those changes.

It was postulated that for any given individual there were "decision points" when he considered changing occupations. It was assumed that these changes were governed by time-homogeneous (that is stationary) transition probabilities. At the same time the number of decision points in any period of time will be a random variable m(T). The interval between decision points is considered to be a continuous rather than a discrete variable. The two considerations are the decision to change occupations (governed by the Markov process), and the stochastic process of decision points m(T).

---

[14] D.J. Bartholomew, Stochastic Models for Social Processes (New York: John Wiley, 1967), pp. 11-37. The work he presented of mobility was from I. Blumen, M. Kogan and P.J. McCarthy, The Industrial Mobility of Labor as a Probability Process (Ithaca, New York: Cornell University Press, 1955).

Since the number of decision points can seldom be recorded, the problem is to discover how much can be learned from the data from only changes in employment (which are normally recorded at discrete intervals of time). This represents little problem if one accepts the simplification that the states of the system (the occupational categories) at successive intervals of time (determined by the time of observations) can be treated as a Markov process. Bartholomew shows that a one-period transition matrix only defines a Markov chain if the decision points occur at either random or regular intervals.[15] If the Markov chain fails to fit the data then it is possible to modify the model to improve the fit.[16] Various modifications are possible, but regardless of most types of modifications the process when it reaches the stable state does not depend upon the nature of the process m(T). This essentially means that one can compute the limiting structure from the one-period transition matrix assuming it were from a Markov chain process.

This all means that regardless of the type of process

---

[15]This is due to the nature of the mathematics underlying the Markov chain process. Various stochastic processes can fulfill the necessary and sufficient conditions for the Markov chain. Bartholomew mentions two: the Poisson and the regular interval distribution. For a more detailed analysis see Ibid., pp. 27-37.

[16]Modification of the model is done through relaxation of the assumptions underlying the model. As it will be pointed out later in the paper, modification of the basic model has been necessary and various assumptions have been relaxed in order to improve the fit of the model.

underlying the decision points, the change in occupation can be used to analyze the turnovers. Bartholomew maintains that if the Markov chain model adequately describes society then the future states of society will depend upon the transition probabilities, i.e., the parameters of the model. He therefore feels that the emphasis in the study of mobility should be on the transition matrices.

The paradigm used by Bartholomew is applicable to the process of educational aspirations. One can assume that changes in the level of aspirations (mobility) is governed by time-homogeneous transition probabilities. For each individual there are given "decision points" where he can change his level of aspirations. These intervals between points is treated as a random variable m(T). However, since data on the number of such points is not available, one is reduced to use of the actual changes (within the limits of measurement error) in the level of aspiration between discrete-interval waves of observations which are viewed as consequences of a Markov chain process based on either a discrete- or continuous-time model.

Direct application of the model may not be possible because of lack of fit. In the study above the authors had to modify the assumption of homogeneity of the population to improve the fit of the model. This modification became the Mover-Stayer Markov model. Other studies have also found modification necessary.

Hodge dropped the assumption of constant transition probabilities in his examination of intergenerational and intragenerational mobility.[17] He found that although prediction was not too poor, the model was insufficiently complex to characterize the full mobility process. White, in his recent article, concluded that any model using constant Markov chains cannot explain intragenerational mobility.[18] His modification of the Markov chain was similar to that of the Mover-Stayer model. Noting the lack of fit of the simple Markov chain model to mobility data, McGinnis proposed the Cornell Mobility model which assumed that cumulative inertia exists.[19] The longer one remains in any state in the system the greater the probability for remaining there. The increase in probability is assumed to be a monotonic function of time spent in the state. A more recent proposal for modification of the basic assumptions of the model is the Time-Stationary model of McFarland.[20] The assumption is that turnover rates for individuals are different but constant. Regrettably he offered no form of the distribution of the transition probabilities over individuals; Land, however,

---

[17] R.W. Hodge, "Occupational Mobility as a Probability Process," Demography 3, 1 (1966), pp. 19-34.

[18] H.C. White, "Movers and Stayers," American Journal of Sociology 76, 2 (1970), pp. 307-324.

[19] R. McGinnis, "A Stochastic Model of Social Mobility," American Sociological Review 33, 5 (1968), pp. 712-722.

[20] D.D. McFarland, "Intergenerational Social Mobility as a Markov Process: Including a Time-Stationary Markovian Model that Explains Observed Declines in Mobility Rates," American Sociological Review 35, 3 (1970), pp. 463-476.

showed that the assumption of a uniform distribution from the rate of zero to a rate of one led to the dropping of both the time and the transition rate parameters.[21]

Modification of the model can also be done by inclusion of independent variables into the model. This can be accomplished in any one of three methods. Goodman suggests that in developing a satisfactory model it is often important to divide the population into homogeneous units.[22] This would necessitate running separate analyses for each grouping. Coleman suggests the use of a multi-attribute system where independent variables are treated as partial determinants of the states of the system.[23] This necessitates the increase in the number of transitions as the number of variables increase. A third approach has been suggested by Spilerman.[24] This method uses regression equations to determine which variables are important for the turnover from state to state. His method does

---

[21] K.C. Land, "Some Exhaustible Poisson Process Models of Divorce by Marriage Cohort," Presented to the annual meeting of the American Sociological Association, Washington, D.C., August 1970.

[22] L.A. Goodman, "Statistical Methods for the Mover-Stayer Model," Journal of American Statistical Association 56 (1961), p. 867.

[23] J.S. Coleman, Introduction to Mathematical Sociology (New York: MacMillan-Free Press, 1964).

[24] S. Spilerman, "The Analysis of Mobility Processes by the Introduction of Independent Variables Into a Markov chain," a revised paper presented at the annual meeting of the American Sociological Association, Washington, D.C., August, 1970.

not suffer the loss of sample size due to separate analyses or the increase in the number of transitions as the number of variables are increased. This last method appears to offer, theoretically at least, the greatest opportunity to test the effects of various characteristics within the sample. It allows the integration of the cohort with the study of their differential characteristics which may affect the transition rates.

Although modification of the model has been found necessary, it is this modification which is one of the strengths of the approach. One knows the assumptions underlying the process as stipulated by the model. The model provides a logical basis upon which to start. In this manner one attempts to model the phenomenon under analysis as faithfully as the assumptions allow.

Up to this point the first two justifications for the use of the models have been presented in some depth. The last justification will now be discussed. It is that all three models are related to one another. In order to understand the interrelationship among the models one must first understand the basic mathematics underlying the models. First, the discrete-time Markov model will be discussed. The parallel continuous-time Markov model will then be presented. The Simple Poisson distribution will then be discussed followed by a discussion of the relationship between the Markov models and the Poisson model.

The Mathematics of the Models

The Discrete-Time Markov Chain

The discrete-time Markov chain is a stochastic process with a discrete-state space and a discrete-time domain.[25] It is characterized by a set of states, a probability distribution over the states at each time t, and a square matrix of transition probabilities. If there are only a finite number of states the process is called a chain.

If X_i is a random variable whose values represent any of n discrete states occupied by the system at time t (where t are discrete times), and if a finite number of states exist, then the sequence of observations of X_i over time is a Markov chain if the probability that X_i takes on any value depends on the previous value and on nothing else in the history of the variable.

For this study the states of the system are two, high and low aspirations. The general form of the table from which the transition probabilities are calculated is of the form:

		time t + n		
		High	Low	Total
time t	High	n_{11}	n_{12}	$n_{1 \cdot}$
	Low	n_{21}	n_{22}	$n_{2 \cdot}$

[25] See W. Feller, An Introduction to Probability Theory and Its Application, Volume 1 (2nd edition; New York: John Wiley, 1968), pp. 372 ff. for an extensive presentation of the theory of the Markov chain.

where the n_{ij}'s are the frequencies, and $n_{i.}$ represents the total frequency in the state i at time t.

The set of probabilities that at time t the system is in state i given that at time t - n the system was in state j is called the transition probabilities of the Markov chain. For the table above the probabilities are calculated by dividing the cell frequency by the row totals. The probabilities sum to one across the row. The form of the probabilities for a two-state time-hmogeneous (the probabilities depend only on the time differences between the states) Markov chain is:

$$P= \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix}$$

One feature of the Markov chain is the use of the higher transition probabilities P^n. Given the initial distribution of the states of the system the state of the system at any future time is given by multiplying the initial distribution by the one-year transition probability matrix raised to the nth power. The p_{ij}'s represent the sum of the probabilities of all possible paths from state i to j, and reflect the fact that the probability of the transition from state to state does not depend upon the manner in which the state is reached.

When a chain consists of only two states the matrix P^n can be determined by the equation:[26]

---

[26]*Ibid.*, p. 432.

$$(1)\quad P^n = \frac{1}{p_{12} + p_{21}} \begin{vmatrix} p_{21} & p_{12} \\ p_{21} & p_{12} \end{vmatrix} + \frac{(1-p_{21}-p_{12})^n}{p_{12} + p_{21}} \begin{vmatrix} p_{21} & -p_{21} \\ -p_{12} & p_{12} \end{vmatrix}$$

One feature of the model is that as time increases the influence of the initial distribution of states decreases such that a matrix is reached which contains identical rows. That is, the process reaches a state of equilibrium which is independent of the initial state of the system.

The Continuous-Time Markov Chain

If the time period between transitions for the discrete-time model is increased to an infinite number of periods the corresponding continuous-time process will be obtained.[27] For the continuous-time process the parameter is the transition intensity or rate, q_{ij}. This rate is not directly analogous to the transition probability in the discrete-time model. However, the rate operating over time affects the transition probability $p_{ij}(t)$ for the continuous-time process, since the probability of turnover in the continuous-time model is a function of the time interval. The q_{ij}'s are related to the $p_{ij}(t)$'s of the model in being the derivative of the $p_{ij}(t)$'s with respect to time as t approaches zero, except for the diagonal q_{ii} which is

---

[27] For a more detailed look at the interrelation of the two models see Coleman, op. cit., pp. 127-130.

the sum of all the rates of shift out of cell i. In the case of the two-state system the q_{ij} would equal the corresponding p_{ijt} times some value which is a function of the p_{ijt}'s. The equation for q_{21} is[28]

$$(2)\quad q_{21} = p_{21t}\frac{-\ln(1-p_{12t}-p_{21t})}{t(p_{12t} + p_{21t})}.$$

and similarly for q_{12}

$$(3)\quad q_{12} = p_{12t}\frac{-\ln(1-p_{12t}-p_{21t})}{t(p_{12t} + p_{21t})}$$

The sample provides direct estimates of the values in the above equations. These estimates are given by the general equation

$$(4)\quad p_{ij} = \frac{n_{ij}}{n_{i\cdot}}$$

These p_{ij}'s correspond to the transition probabilities for the discrete-time model if the estimates are taken from the same table with no change in the time interval. In most cases for the one-step transition the corresponding q_{ij} value will be higher than the p_{ij} term.[29] Beyond the two-state one-step stage the correspondence between the q_{ij}'s of the continuous-time model and the p_{ij}'s of

[29] As the sum of p_{12} and p_{21} goes from 0 to 1, the ratio of $-\ln(1-p_{12}-p_{21})$ to $p_{12} + p_{21}$ goes from slightly over 1 to around 14.6. As the sum increases from 1 to 1.27 the ratio decreases from 14.6 to 1. As the sum increases to 2 (the limit) the ratio decreases in value from 1 to 0.

the discrete-time model becomes complex, and any comparison between the models should be done by use of the $p_{ij}(t)$'s rather than the q_{ij}'s. In addition, if there are more than two states the procedure to calculate the rates is an iterative one.

Coleman used this model to bridge the gap between cross-sectional and time-series data. The model "is conceived in terms of processes operating over time, but it can equally mirror the state of aggregate equilibrium which our cross-sectional studies describe."[30] By aggregate equilibrium is meant that there are few shifts between categories, and near constancy in the marginal distributions. The state of aggregate equilibrium suggests that "random shocks" move individuals from one state to another. The q_{ij} is the size of that shock. The parameter can be thought of in these terms: for each person in state i at time 0, q_{ij} will migrate to the state j per time t. If the time unit is 1 (the one-step transition) the q_{ij} can be thought of as the proportion of the aggregate expected to move. For any other time unit q_{ij} must be translated back into a transition probability to be thought of as the proportion of the aggregate expected to be in any state at any given time.

This projection can be carried out when the rates are not arbitrarily restricted to any value as is the

[30]Coleman, _op_. _cit_., p. 132.

case in this analysis. In this case the simple projection from the transition matrix is appropriate. Projection forward in time for discrete intervals is done by use of the discrete-time transition probabilities, and taking the matrix obtained to the appropriate power. When the model is constrained one projects from the values of q_{ij}. The projection of the internal cell frequencies is similar to the projection for the q_{ij}'s because they are the limiting form of the p_{ij}'s. Projection must also be done from the q_{ij} values if the time intervals are not integral in nature.[31] For the two-state system the projection from the q_{ij}'s could be carried out treating them as transition probabilities. In this manner the projection would be the limiting form of the future states.

Projection long enough in the future would result in the stable state matrix. For the two-state system the equation for determining the expected proportion of individuals in state 1 at equilibrium is given by

$$(5)\quad p_1 = \frac{q_{21}}{q_{12} + q_{21}}$$

and similarly for those in state 2

$$(6)\quad p_2 = \frac{q_{12}}{q_{12} + q_{21}}$$

---

[31] Ibid., pp. 182-183.

The model provides two methods for determining the size of the "random shocks" which force movement from state to state over time. It is through these methods that the model integrates the cross-sectional and the over-time data. First, if there are only cross-sectional data available the relative size of the shocks, rather than the exact values, can be estimated by the following equation

$$(7) \qquad \frac{n_1}{n_2} = \frac{q_{21}}{q_{12}}$$

Second, the equations above for determining the values of q_{ij} provide exact estimates of the values, and constitute the other method using over-time data. Since the exact values take into account internal changes while the cross-sectional estimates provide only a gross indicant of the amount of turnover, the exact values are expected to be more accurate in their reflection of change.

<u>The Simple Poisson Distribution</u>

A Poisson variable is a random variable with possible outcomes 0, 1, 2,...which represent the number of occurrences of some event over a given length of time or region of space. The Poisson distribution is given by

$$(8) \qquad P(i) = \frac{e^{-\lambda}\,\lambda^{i}}{i!}$$

where λ is the parameter of the distribution. As the parameter increases the distribution becomes more and more symmetrical. For the Poisson distribution the sum of the P(i)'s equal one. The distribution function for the Poisson distribution is given by the equation

$$(9) \qquad F(i) = \sum_{t=0}^{i} \frac{e^{-\lambda} \lambda^{t}}{t!}$$

The mean and variance of the Poisson distribution are equal to the parameter, since the Poisson is a one-parameter model.

The Poisson distribution applies equally to both the one-way and the two-way process. The process is applicable to events (aspiring to college) which can occur to each of N elements (students) involved, each of whom has the same probability toward carrying out the event. That is, the events are independent.

When there is no zero observation the Poisson distribution (called a truncated Poisson) becomes

$$(10) \qquad P(i) = \frac{k^{i}}{i!\,(e^{k} - 1)}$$

where $k = \ln(n\Theta/\lambda)$ and where Θ and λ are the turnover rates for a two-way process.

When the above model is applied to individuals who have done some particular act (such as changing aspirational level) the equation for the process becomes for a group of size N

$$(11) \qquad P_i = \binom{N}{i} (1 - e^{-\lambda t})^i \; e^{-\lambda t(N-i)}$$

which gives a full probability distribution of movement from one state to another. If one is interested in the expected value for this distribution it is for state i

$$(12) \qquad E(i) = N(1 - e^{-\lambda t})$$

This result indicates that the probability distribution for the proportion shifting is a binomial distribution with the parameters $p = 1 - e^{-\lambda t}$ and N = total number of individuals.

<u>The Relationship Between the Markov and the Poisson Models</u>

In his book on probability theory Feller discusses the relationship between the Markov process and the Poisson distribution.[32] The key lies with the Chapman-Kolmogorov identity. Any solution to the identity is a Markov process which is time-homogeneous. The identity is

$$(13) \qquad P_{ik}(\Theta+t) = P_{ij}(\Theta)\; P_{jk}(t)$$

Feller considered various postulates which lead to systems of equations for the $P_{jk}(t)$ whose solutions are the transition probabilities of a Markov process which is uniquely determined by them and the initial state. This means being concerned with stochastic processes with countable

[32] Feller, *op. cit.*, pp. 444-478.

states depending upon a continuous time parameter. For a given j and t the transition probabilities $P_{jk}(t)$ define an ordinary discrete probability distribution which depends on the continuous time parameter t.

One family of distributions which is a time-homogeneous Markov process in the above sense is the Poisson process. However, the assumptions are weaker than assuming that the past history of the process does not influence the future development. The postulates for the Poisson process are:

> The process starts at epoch 0 from state E_0 (i) Direct transitions from state E_i are possible only to E_{j+1}. (ii) Whatever the state E_j at epoch t, the probability of a jump within the ensuing short time interval between t and t+h equals $\lambda h+0(h)$, while the probability of more than one jump is $0(h)$.[33]

The assumption is that time in any given state plays no role.

The Poisson process viewed as a Markov process is based on the assumption of random occurrences of the same type of event. The primary concern is the total number Z(t) of events in an arbitrary length of time t. It is assumed that the influences on the process remain constant so that the probability of any event is the same for all time intervals of t duration, and is independent of the past history of the process, i.e., the process is a time-homogeneous Markov process. The basic probabilities are:

---

[33] *Ibid.*, p. 447.

(14) $\quad P_n(t) = P(Z(t)=n)$

$P_n(t)$ is the probability of the state E_n at time t or the transition probability from an arbitrary state E_j at an arbitrary epoch s to the state E_{j+n} at epoch s+t.

In a Poisson process it is assumed that the initial state begins at E_0 at time 0. However, the system may begin at any state with an arbitrary λ_n and if series $\Sigma\ \lambda_n^{-1}$ diverges (approaches infinity) then the system will have all the required properties. (The expected length of stay in any state j is equal to λ^{-1}.)

Feller also points out that both the time-homogeneous assumption and the transitions through neighboring states assumption can be dropped and the birth-and-death Poisson process satisfies the conditions of the Chapman-Kolmogorov identity, therefore representing the transition probabilities of a Markov process.

It should be noted at this time that the Markov chain and the Poisson distribution have been applied to various social processes. Some of these applications are presented in Appendix D.

Summary

In this chapter the theoretical justifications for the use of the models and the use of the cohort as the unit of analysis have been presented. It was noted that past research dealing with aspirations has not generally used either the logic of the study of process or over-time data. It is felt that only when over-time data are used with process models that the causal linkages involved can be discovered. Use of cross-sectional data alone is fraught with hazard. It is the cohort which offers the best possibility in the study of change.

The mathematical calculations underlying the models were presented. The parameter values of the continuous-time Markov chain present the limiting values of the corresponding discrete-time Markov chain. The Simple Poisson model also can be viewed as a Markov chain. The models provide a logical basis upon which to modify the assumptions underlying behavior in order to increase the goodness of fit. All of the models are one parameter models. This facilitates comparison of the models since goodness of fit tests are applicable only when the number of parameters in the models is the same.[34] This restriction represents no problem in this study. The techniques involved in testing the fit of the Markov chain as well as other methodological topics are covered in the next chapter.

---

[34]For this observation I am indebted to my advisor Professor Theodore R. Anderson.

CHAPTER II

METHODOLOGICAL CONSIDERATIONS

The Study

This analysis used data gathered from part of an ongoing research into the industrialization and urbanization of a rural area in Northern Illinois. The project began in 1965 in order to study longitudinally the impact of a large steel corporation moving into the Hennepin area of Putnam County. As part of the study a control area was also chosen. Both areas were rural in nature.

The Sample

High school students in both the experimental and control area were given questionnaires as part of the study of the social structure and behavior. The two schools were the Putnam County High School and the Watseka Community High School. The secondary school enrollments as of 1963 were 269 for Putnam County and were 2,267 for Iroquois County in the control area. The 1960 populations were 391 for the experimental community and 5,219 for the control community.[35]

The students were grouped by year of graduation across both schools. The level of aspirations within each school was essentially the same within each cohort.[36] Therefore, this procedure was not felt to affect the actual level within the cohorts. The gain by use of this procedure was an increased sample size for each cohort. For those for whom data were complete approximately 56 percent in the cohorts came from the Watseka school and 44 percent from the Putnam school.

The Cohorts[37]

There were five cohorts of students included in the study. These were the classes of 1968 to 1972.

---

[35]These figures were taken from G.F. Summers *et al.*, *Before Industrialization: a Rural Social System Base Study* (Champaign, Illinois: University of Illinois Agricultural Experiment Station Bulletin 736, 1969).

[36]The levels were in the range of the low .60's to the low .70's.

[37]Cohort analysis is the preferred method of analysis rather than strict longitudinal analysis. The cohort is a group of persons who have undergone a similar experience at approximately the same time. A cohort possesses a life history. Cohort data are normally grouped sequentially from differences of the time of the event (say, entrance into high school), and the interval since that event (the class of graduation). Cohort analysis is to be preferred because its emphasis is on the properties of populations rather than individuals. It is macrolongitudinal. See N. Ryder, "The Cohort as a Concept in the Study of Social Change," *American Sociological Review* 30 (1965), pp. 843-861, and N.B. Ryder, "Cohort Analysis," in D.L. Sills (Ed.), *International Encyclopedia of the Social Sciences*, Vol. 2 (New York: MacMillan Press, 1968), pp. 546-550.

Unfortunately all the cohorts were not followed from their freshman to their senior year.

The 1968 cohort were sampled in their junior and senior years in high school. Of the total group of students data were complete on aspirations for only about 71 percent. In this group there were 78 males and 53 females.

For the 1969 cohort the students filled out questionnaires their sophomore, junior, and senior years. Over the three waves of observation only 46 percent of the cohort had complete information regarding their level of aspirations for all three years. For those for whom data were complete there were 58 males and 66 females.

The 1970 cohort represented the only group of students followed from the freshman to the senior year. Of the total 48 percent had complete information on aspirations. There were 58 males and 69 females.

The freshman, sophomore, and junior years of the 1971 cohort were sampled. Forty-six percent of the

---

In addition, the use of the cohort is essential in the differentiation among the factors of age differences, age changes, and time of measurement in the study of change. A comparison of one cohort over time controls for the effects of life history but not for age changes or environmental effects. Age group comparison at any given time would control for time of measurement but not for age differences or for the effects of differing life experiences. Differences among life histories and environmental effects can be separated by the use of different cohorts of the same age over different times (time lag). It is this last method which appears to offer the best possibilities for the study of change. See K.W. Schaie, "Age Changes and Age Differences," The Gerontologist 7, 2, 1 (1967), pp. 128-122.

cohort had complete information. There were 44 males and 52 females.

For the 1972 cohort sampled in their freshman and sophomore years 75 percent of the data had complete information. In this group 67 were males and 100 were females.

Since the models do not control for missing data, the missing data were thrown out. Because this procedure would make the representativeness of the sample used somewhat questionable, the respondents and non-respondents within each cohort were compared on certain background variables which could affect the level of aspiration.

Nevertheless, in a recent article Williams and Wallows suggested that estimates from "identical persons" (the cohort) for whom all data were complete were the closest to the "true" values in the population.[38] Their article suggested that there can be systematic biases in panel data due to differential nonresponse as a result of the characteristics of the response probabilities then any observed change for the individuals in the sample would likely be real for them, but would not necessarily accurately represent the change for the population. Using contrived data they did find, however, that the use of identical persons who were re-interviewed

---

[38] W.H. Williams and C.L. Mallows, "Systematic Biases in Panel Surveys Due to Differential Nonresponse,". Journal of the American Statistical Association 65, 331 (1970), pp. 1338-1349.

provided closer estimates of the parameters than the use of all available persons or only those who had been interviewed once.

The Comparison of Respondents with Nonrespondents

The variables used for comparison by t-test were father's occupation, father's income, father's education, the youth's age, the youth's sex, and the school attended by the youth.[39] The data were taken from the first year's sampling within each cohort because the highest rate of response was at this time for these variables.

For those variables for each cohort for which there were significant differences the results are given in Table 1.

For the 1968 and 1969 cohort the respondents and

---

[39] The coding of these variables were as follows:

(1) School
- 0 Putnam County High School
- 1 Watseka Community High School

(2) Sex
- 0 Male
- 1 Female

(3) Father's Income
- 0 Under $2,500
- 1 $2,500 to $5,000
- 2 $5,000 to $7,500
- 3 $7,500 to $10,000
- 4 $10,000 to $12,500
- 5 $12,500 to $15,000
- 6 $15,000 to $20,000
- 7 $20,000 to $25,000
- 8 $25,000 to $30,000
- 9 $30,000 to higher

Father's occupation was coded by the U.S.B.C. occupation codes. Ungrouped data were used for the youth's age and the parent's level of education.

Table 1.--Differences Between Respondents and Nonrespondents on Selected Background Variables

Cohort	Variable	Means: Resp.	Means: Nonresp.	d.f.†	t-value
1968	Father's Occupation	408.5	575.2	161	-3.73**
1969	Father's Income	3.4	2.6	118	2.29*
	Student's Age	15.1	15.5	189	-4.60**
1970	School Attended	.51	.73	210	-3.36**
	Student's Age	14.1	14.3	214	-3.73**
1972	School Attended	.57	.72	100	-2.13*
	Student's Age	14.0	14.3	216	-4.51**
	Student's Sex	.56	.37	93	2.42*

* $p < .05$

** $p < .001$

† The differences in degrees of freedom are due to the dropping of the assumption of equal variances when necessary.

and nonrespondents differed significantly on social class variables. For the 1968 cohort the fathers of the respondents were on the average in higher occupational categories (the mean code was in the craftsman, foreman, and kindred category) than the fathers of the nonrespondents (the mean code was near the laborer category). For the 1969 cohort the fathers of the respondents were in a higher income bracket (the $7,500 to $10,000) than the fathers of the nonrespondents (the $5,000 to $7,500 range).

If the youth's responses on father's occupation and income can be considered as valid measures, it would seem that the exclusion of nonrespondents from both cohorts would result in an overestimation of the actual level of aspirations for the cohorts; past research indicates that higher social classes have higher aspirational levels. Nevertheless, the lack of differences for the other cohorts would indicate that this difference is not general, and the lack of difference on the other social class variables for the two cohorts would indicate that the differences are perhaps due to the youth's perceptions rather than actual class differences.

For the 1969, 1970, and 1972 cohorts the respondents were slightly younger than the nonrespondents (the actual difference was two to three months). This difference, although statistically significant, is unlikely to affect aspirational level. Unlike social class and other variables, there has been no evidence of direct age influences on aspirational level in past research.

ERIC

For the 1972 cohort among the nonrespondents there were a large majority of males. Since males typically have higher levels of aspiration, the exclusion of these from the cohort is likely to result in an underestimate of the actual level of aspirations for the cohort.

For the 1970 and 1972 cohorts the school attended affected the response. While for respondents about 51 to 57 percent of the students came from the Watseka school, for the nonrespondents about 73 percent came from this school. However, the affect of school attended on the level of aspirations appeared minimal for this study. Within each school for each cohort the level of aspiration was approximately equal. For both categories of aspiration the percentage by school was close to the actual distribution of students across schools.[40] Therefore,

---

[40]The relationship between school attended and the level of aspiration can be seen from the percentage tables below. Aspirations are given in the rows and the schools are in the columns. The figures for the two cohorts are essentially the same as those for the other cohorts.

1970 Cohort

	Putnam	Watseka
High	39	25
Low	61	75
Total	100%	100%

1970 Cohort

	Putnam	Watseka	Total
High	59	41	100%
Low	44	56	100%

1972 Cohort

	Putnam	Watseka
High	28	26
Low	72	74
Total	100%	100%

1972 Cohort

	Putnam	Watseka	Total
High	44	56	100%
Low	43	57	100%

the exclusion of nonrespondents based on school differences should not have affected the level of aspirations within the two cohorts.

In summary, there were no significant differences across all cohorts for any variable tested. The differences which were found were not felt to greatly affect aspirational level. For the 1968 and 1969 cohorts the observed levels of aspiration are probably somewhat higher than would have been obtained if there were no missing data. For the 1972 cohort the observed levels are probably somewhat lower than would have been obtained if the nonrespondents would have had complete data. For the other cohorts there were no differences found on the variables tested. The levels of aspiration found by use of only respondents are therefore felt to be adequately representative of the entire cohort.

Differences in levels of aspiration do not necessarily indicate differences in the process of aspirational change, however. Ir '·- models to be tested the levels are taken as the givens at the initial state of the system. The number of students who aspire to college does not affect the turnover rate. The level affects the actual number of students in the states of the system, but does not affect the distribution of individuals in the states.

The Variables

The level of educational aspiration obtained for each student at each wave of observation were treated as separate variables. For the first year of the study the student could check any of four responses in answer to the question of planned college attendance. These categories were 1) Yes, 2) Probably, 3) Probably Not, and 4) No. After the first year of the study the students could check any of six categores in response to the question of how far they planned to go in school. These categories were 1) less than high school, 2) finish high school, 3) vocational or technical school, 4) junior college, 5) four-year college or university, and 6) advanced degree.

Because the data were not comparable over the four waves of observation for use of the more detailed responses, the data were dichotomized into high and low categories. For the first year's responses the first two categories were coded as high aspirations. For the other years the last three responses were coded as high aspirations. In this manner use was made of all of the data, although at the cost of refinement in the level of aspiration. Nonetheless, it should be noted that most previous studies have dealt at this general level.

The Organization of the Data

Within each of the five cohorts for males and females the responses on educational aspirations were cross-tabulated with one anotner over successive periods of time.

These tables are given in Appendices A and B. For these as well as all other such tables the high category is in the first row and the first column. The rows represent the responses at time t and the columns represent the responses at time t+n. The cell frequency n_{ij} represents the number of students who at time t responded in the i category and who at time t+n responded in the j category. In this case, for example, the n_{11} cell represents the number of students who over a given period of time maintained high aspirations.

All of the tables in Appendix A are one-year (one-step) transition tables. The tables in Appendix B are for the two- and three-year (step) transitions. There are fewer tables in this appendix because only three cohorts were followed over a sufficient length of time to permit this cross-tabulation of responses.

The Statistical Method of Analysis Used in the Test of the Markov Models

There have been various methodological articles on the methods of testing the fit of empirical data to the Markov chain. Hoel in an early article presented a likelihood ratio test for the goodness of fit of the model when the transition probabilities were not known.[41] His test was a modification of an earlier test developed by

[41] P.G. Hoel, "A Test for Markov Chains," Biometrika 41 (1954), pp. 430-433.

Bartlett which required knowledge of the transition probabilities.[42] Later Anderson and Goodman presented a detailed mathematical presentation of the maximum likelihood estimates and their asymptotic distribution of arbitrary order when there are repeated observations of the data.[43] Goodman later suggested an alternative method to the maximum likelihood estimation method which he found, using data from Anderson's study of voting behavior,[44] provided a more detailed analysis of the data.[45] Goodman also presented a method for testing the persistence in a chain of events over a long period of time when the first-order Markov chain process fits.[46]

The alternative method suggested by Goodman is applicable to multi-wave data. The basic purpose of the method is to discover any regularities in the data which could help in the prediction of any future states of the system. The primary procedure is the use of the Chi-square test

---

[42] M.S. Bartlett, "The Frequency Goodness of Fit Test for Probability Chains," Proceedings of the Cambridge Philosophical Society 47 (1951), p. 86.

[43] T.W. Anderson and L.A. Goodman, "Statistical Inferences About Markov Chains," Annals of Mathematical Statistics 28 (1957), pp. 80-110.

[44] T.W. Anderson, "Probability Models for Analysing Time Changes in Attitudes," in P.F. Lazarsfeld (Ed.), Mathematical Thinking in the Social Sciences (Glencoe, Illinois: Free Press, 1954), pp. 17-66.

[45] L.A. Goodman, "Statistical Methods for Analyzing Processes of Change," American Journal of Sociology 68 (1962), pp. 57-78.

[46] L.A. Goodman, "The Analysis of Persistence in a Chain of Multiple Events," Biometrika 51 (1964), pp. 405-411.

of goodness of fit assuming a discrete-time Markov chain of nth order. That is, the basic technique is to use separate tables for each response category for each period (year in school) or each stratum (sex, cohort, and so on) and to test these for significant differences. By summing the Chi-square values and the degrees of freedom one can test the hypothesis of constant transition probabilities for the entire process rather than just for individual response categories.

The Test of the Hypotheses

The first general hypothesis which should be tested is that the transition probabilities relating to responses in any given period are independent of the responses in the preceding period. The basic assumption underlying the Markov model is the dependence of the present state on the state of the system during the last period; therefore, if this hypothesis were not rejected the Markov model would obviously be rejected as well. The hypothesis can be tested by use of the Chi-square test for separate tables if it is assumed that responses each period are independent, or by using a combined table if it is assumed that the transition probabilities for the periods are the same. In either case just scanning the tables should provide an adequate look at the acceptance or rejection of the hypothesis, since obvious lumping in the diagonals of the table would result in large Chi-square values, and rejection of the hypothesis.

ERIC

The next general hypothesis is that the transition probability matrices are the same for any given period. To test this hypothesis for each category of aspirations and for all categories over the freshman, sophomore, junior, and senior years one would arrange the data in the form of Table 2 and similarly for the given periods

Table 2. The Arrangement of the Data to Test the Hypothesis that the Transition Probability Matrices are the Same for the Given Periods.

Period	Response at time t	Response at time t + 1	
		High	Low
Freshman	High		
Sophomore	High		
Junior	High		
Freshman	Low		
Sophomore	Low		
Junior	Low		

of interest. As mentioned previously one tests the assumption that the transition probabilities for the periods in the table are the same by use of the Chi-square sum of both the Chi-square values for each separate category. If sufficiently small values of the statistic are obtained, the null hypothesis of no differences among the periods is not rejected either for any given category of response or for all categories of response. That is, one finds no evidence to reject the fit of the first-order Markov chain model to the data.

ERIC

One can also test the hypothesis that the process is a second-order (that is the responses at time t + 2 depend upon the responses at both t + 1 and t) rather than a first-order one. To test this hypothesis the data should be arranged as in Table 3. The test is

Table 3. The Arrangement of the Data to Test the Hypothesis that the Process is a Second-Order Rather than a First-Order One.

Time t	Time t + 1	Time t + 1 High	Low
High	High		
Low	High		
High	Low		
Low	Low		

carried out in a manner similar to that above. For each category as well as for the total process one can test the fit of a second-order model by examining the separate table and the sum of the table Chi-square values. Non-significant results indicate the hypothesis of a first-order process can not be rejected.

When data are stratified (by sex or by cohort) a test of the independence of the strata assuming a first-order process can be carried out by arranging the data in the form given in Table 4. The test of the hypothesis of independence is similar to the methods previously described. Small values of the Chi-square would indicate that the hypothesis of constant transition probabilities over the strata could not be rejected.

Table 4. The Arrangement of the Data to Test the Hypothesis that the Transition Probability Matrices Are the Same for the Given Strata.

Strata	Time t	Time t + 1 High	Time t + 1 Low
Strata 1	High		
Strata 2	High		
Strata 3	High		
Strata 1	Low		
Strata 2	Low		
Strata 3	Low		

The Hypotheses

The method outlined above leads to the following hypotheses. The level of 1 percent significance is accepted as appropriate to test these hypotheses.

(1) Responses in successive periods are independent for each cohort.

(2) The transition probability matrices are constant, assuming a first-order process, for each cohort.

(3) The process is a first-order rather than a second-order one.

(4) Males and females within each cohort are governed by the same process of change.

(5) Given the same process of change for males and females in each cohort, the cohorts when the total samples are used will have constant transition probability matrices.

(6) Given the veracity of (4) and (5) above, the data can be represented by one general transition probability matrix, i.e., the process is the same for all the data.

The Test of the Fit of the Simple Poisson Model

As it was previously mentioned, the Poisson distribution deals with phenomena in which events occur randomly in time or space. That is, given observation of an event from time t to t + n, the model predicts that the points of occurrence of the events occur randomly (independently uniformly) distributed from t to t + n. Two simple distributions characterize the Poisson process, the exponential and the Poisson distributions.[47]

The equation (12) previously given is used to calculate the value of the one parameter λ for the model. This equation is used in preference to the characteristic equation of the Poisson distribution given in equation (8). For equation (12) one needs to know only the proportion of individuals (E_i/N) in any given state i for any given time. One then uses this proportion in the equation and determines the exponential function which solves the equation.[48]

The test of the fit of the Simple Poisson model is extremely simple. One examines the λ values for each cohort over the various waves of observations. If the λ for all periods are all equal then the data fits the model, since the model is based on the assumption of

---

[47] See the following article for techniques of comparing the processes: A. Birnbaum, "Statistical Methods for Poisson Processes and Exponential Populations," Journal of the American Statistical Association 49 (1954), pp. 254-266.

[48] See K. Land, op. cit., for an example of this procedure.

the constancy of the parameter. That is, if the model is used in the manner where individuals (the N) change their aspiration level (the event) the λ (the turnover rates) should be constant over time. The null hypothesis is, therefore, that the parameters for each cohort over time will be equal.

If the hypothesis is rejected the Simple Poisson assumption of constant turnover rates is not accepted. Possible other patterns of changes in rates could then be explored.

There remains an essential consideration before the analysis can meaningfully be undertaken. One must distinguish between real change over time and measurement error. That is, one must determine the reliability of the instrument and the stability of the phenomenon under study. The preceding methods of analysis assume no measurement error. All change is assumed to be real change in the responses. If there were measurement error then any change in response could be due to any combination of real change or error. The representativeness of any conclusions regarding the process of change would be only as accurate as the assumption of no error.

The Problem of Reliability

In his book on mathematical sociology Coleman discussed the essentials of determining when change is true change when using panel data.

> It would appear at first that inferences about the effect of attribute A upon B by examining the transition rates estimated from over-time data would be very solid inferences. But when there is aggregate equilibrium, an alternative hypothesis is also tenable; there is simply unreliability of response at each interview or observation, and this unreliability appears as spurious "changes" of position between the two interviews. Such unreliability would show a spurious effect of any attribute or variable which was correlated with the dependent response. Obviously it is important to decide whether an apparent effect is actually more than unreliability. . . In brief, the comparison (among changes) is this: if the "changes" between 1 and 3 are no greater in number than those between 1 and 2 or between 2 and 3, then this is simply unreliability of response.[49]

The method of determining reliability and stability proposed by Heise[50] and elaborated upon by Wiley and Wiley provides a more elaborate approach to the problem of measurement error than that proposed by Coleman.[51] The Wiley and Wiley method offers certain advantages over the method proposed by Heise.[52]

Wiley and Wiley present a lag model for the determination of measurement error in panel data. Their measurement model is based on the assumption of constant

---

[49]J.S. Coleman, op. cit., p. 156.

[50]D.R. Heise, "Separating Reliability and Stability in Test-Retest Correlation," American Sociological Review 34 (1969), pp. 93-101.

[51]D.E. Wiley and J.A. Wiley, "The Estimation of Measurement Error in Panel Data," American Sociological Review 35, 1 (1970), pp. 112-117.

[52]D.R. Heise, "Comments on 'The Estimation of Measurement Error in Panel Data,'" American Sociological Review 35, 1 (1970), p. 117.

error variance rather than the assumption of stable reliability over time. They feel that while both assumptions are not valid in the general case, the former assumption is more reasonable because error variance will tend to remain stable whereas reliability will fluctuate. Given enough waves of observation it is also possible to test the assumption of homogeneous error variance.

For three waves of observation the model is just-identified. The six parameters of the model to be estimated (these are then used to determine the reliability and stability) are: a_{21} and a_{32}, the path regressions linking true scores, the error variance $V(\varepsilon)$, and the variances of the random shocks at each observation, $V(\Theta_1)$, $V(\Theta_2)$, and $V(\Theta_3)$. These parameters are estimated from the covariance matrix for the observed data.

If there are four waves of observation then the model is over-identified. This permits estimations based on the two variance values for the error term. The reliabilities and stabilities based on the two terms could be compared to determine the justification of the assumption of constant error variance.

The stability coefficient is the path coefficient of true scores over time. The path coefficient is the parameter of the model. A path coefficient is the amount of deviation in the variable for which another variable is directly responsible. Therefore, the stability coefficient of the lag model presented by Wiley and Wiley can be considered an alternative model for explaining change in aspirations over time. Its parameter is the stability of the phenomenon. This parameter can be considered comparable to the turnover probability or rate of the Markov models and the Poisson distribution. The disadvantages of this model are at least three waves of observations are required, the calculations are more complex, and the amount of information regarding the process is less than in the other models. An advantage of the model would be the parameter's use as an estimate of persistence in any category over time. That is, it would give evidence regarding the patterning of responses over time.

Since the Wiley and Wiley article did not use four-wave data, the equations were derived for use with four waves of data. The estimators of the parameters and their values used to determine the reliabilities and stabilities for the data given in Appendix B are given in Appendix C.

The stabilities are given in Table 5 and the reliabilities are given in Table 6. The error variance terms for the 1970 data were not identical in value, therefore the reliabilities and stabilities were determined using both values. The results indicate that the terms do produce different ranges of stability and reliability. The parameter of the model appears very sensitive to changes in the error variance value.

Table 5.--Stabilities for the 1969, 1970, and 1971 Cohorts

Cohort	Period	Stability
1969	Times 1 and 2	.995
	Times 2 and 3	.986
	Times 1 and 3	.981
1970†	Times 1 and 2	.624
	Times 1 and 3	.504
	Times 1 and 4	.376
	Times 2 and 3	.789
	Times 2 and 4	.452
	Times 3 and 4	.728
1970††	Times 1 and 2	.496
	Times 1 and 3	.399
	Times 1 and 4	.350
	Times 2 and 3	.808
	Times 2 and 4	.573
	Times 3 and 4	.710
1971	Times 1 and 2	.873
	Times 2 and 3	.924
	Times 1 and 3	.807

† The stabilities for these data are based on the .0596 error variance value.

†† The stabilities for these data are based on the .0423 error variance value.

For the 1969 cohort the stabilities and reliabilities were both very high. For the 1971 cohort the stabilities were in the .80 to .90 range, while the reliabilities ranged from the high .50's to the low .60's. For the 1970 cohort the stabilities varied from the middle .30's to the low .80's. The reliabilities ranged from the

Table 6.--Reliabilities for the 1969, 1970, and 1971 Cohorts

Cohort	Period	Reliability
1969	Time 1	.945
	Time 2	.942
	Time 3	.941
1970†	Time 1	.730
	Time 2	.757
	Time 3	.767
	Time 4	.745
1970††	Time 1	.808
	Time 2	.827
	Time 3	.836
	Time 4	.826
1971	Time 1	.579
	Time 2	.597
	Time 3	.605

†The reliabilities for these data are based on the .0596 error variance value.
††The reliabilities for these data are based on the .0423 error variance value.

low .70 to the low .80's.

Except for the 1971 cohort the level of reliability indicated the degree of measurement was fairly good while the stability of the phenomenon was low over the entire high school period. The low reliability for the 1971 cohort was due to the large error variance. The values obtained for all cohorts do indicate that the assumption of perfect measurement is a somewhat tenuous assumption except for the 1969 cohort. The assumption of all change being real change that is made in this analysis should be accepted as tenuous. Any further study made should account for measurement error in the analysis. For the analysis presented in the next chapter, however, the level of measurement reliability was accepted as satisfactory.

CHAPTER III

RESULTS OF THE ANALYSIS

Examination of the Pattern

It was previously mentioned that cohort analysis was the preferred method. The advantages were given as allowing the study of change where one could separate the effects of maturation, age differences, and environmental influences. Arrangement of the data as in Table 7 below facilitates this analysis. In the table the vertical lines indicate age differences, the horizontal lines indicate age changes, and the diagonal lines show the time-lag differences. These will now be discussed in some detail.

In Table 7 the mean percentages of students seeking to attend college are given for each cohort over each year. It is clearly evident that a majority hold high aspirations.[53] The pattern in the data can be revealed

[53]The level of aspirations is generally equal to that of Census data for 1965, cited by Adams. In 1965 a total of 60 percent of high school seniors planned on attending college. See W. Adams, "Financial and Non-Financial Factors Affecting Post-High School Plans and Eventuation, 1939-1965," in E.D. Goldfield (Ed.), American Statistical Association Proceedings of the Social Statistics Section, 1969 (Washington, D.C.: American Statistical Association, 1969), p. 108.

by examination of the lines.

Table 7. The Percentage of High Aspirations Over Time by Cohort.

Cohort	Time of Measurement 1	2	3	4
1968	66	56		
1969	68	66	72	
1970	68	58	58	66
1971		68	65	64
1972			73	64

Examination of the vertical lines indicates that there appears to be more variability in aspirational levels at time 2 and 3 than at times 1 and 4. These differences may be due to age differences or previous life histories of the cohorts. Examination of the horizontal lines indicates that eigher the change pattern (initial decrease and ultimate increase) are due to age differences or environmental effects. Examination of the diagonal lines indicates that for the freshman year (68-68-73) the level is relatively constant and any difference is due to differences among cohorts or environmental treatments. For the sophomore year (68-58-65-64) the level is again relatively constant (with the exception of the 1970 cohort). For the junior year (66-66-58-64) the level is again relatively constant except for the 1970 cohort. For the senior year (56-72-66) the level is not at all constant, although the level appears to increase moderately from the junior year except in the 1968 cohort.

The 1970 cohort appears to be the lowest in level of aspiration across age differences (at two times), age changes, and the time-lag differences. Since the level of aspirations for the cohort is not substantially different from the others either at time 1 or for other freshmen it would appear that the cohort is differentiated from the others by unique environmental effects. It would appear that the 1970 class from 1966 to 1968 were differentially affected by a depressing influence on their aspirational level.

For the analysis of age changes the cohorts over time are used (the horizontal lines). The pattern of an initial decrease from relatively high aspirations to a plateau for the sophomore and junior years to a slight increase at the senior year appears to be relatively consistent. Using the mean values the percentages follow the pattern of 70-64-64-65 which supports the statement above.

In the socio-cultural context of the school, a cohort maker, the influences of age change and environmental effect are compounded. Classes are treated as distinct groups, and could very well be subjected to differential treatment over time, although this treatment could be standardized for each class standing. It would appear the levels of aspiration are relatively constant over all cohorts during the sophomore and junior year. The general conclusion one can reach is that there is something in the freshman to sophomore and junior to senior transitions which affects the various cohorts in a like pattern but with a different magnitude.

One can theoretically postulate many such possible influences. The shock of adjusting to the new demands and freedoms would possibly cause a drop in the college desires of sophomore students who had marginally aspired to attend college at the start of the freshman year, while the pressure would be on college plans at the start of the senior year when it is time to think about college attendance. This pressure could force marginal students to plan on college.

The question remains, nonetheless, of the extent of the turnover each year within each cohort, and to what extent the turnover approaches the regularity suggested from the apparent pattern on the generalized level of aspirations. The examination of the generalized level has been of benefit in providing an overview of the process, and in suggesting that one cohort may be uniquely affected by environmental effects. The regularity of turnover as well as its extent cannot be determined by this method, but can be analyzed by testing the fit of the discrete-time Markov chain model, for the main questions are the establishment of any regularity in turnover and for which periods and for which groups the regularity exists.

The Test of the Discrete-Time Markov Chain

In this section the six hypotheses will be tested. They represent selective focusing upon the regularity of the process of aspirations over time and upon the applicability of the process to a more general population.

The tables in Appendix A will be used as well as those in Appendix B.

The Hypothesis of Independence of Responses

The first hypothesis was that responses over periods are independent. This was easily rejected. Simple examination of the raw data in the tables in Appendix A reveals a marked lumping on the diagonals which indicates that the responses over time are obviously dependent.

The Hypothesis of Constant Transition Matrices

The second hypothesis was the constancy of the transition probability matrices within each cohort assuming the process was of the first order (that is, dependent only upon the previous state). This hypothesis was tested for both males and females by cohort over time. At least three waves of observations are needed to test this hypothesis, however, since three are needed to obtain at least two matrices of probabilities. Therefore, only the 1969 to 1971 cohorts were used in this analysis.

None of the Chi squares was significant at the 1 percent level. These values are given in Table 8. Since none was significant, the hypothesis of constancy was not rejected. Therefore, the hypothesis of the fit of the Markov process model for each sex within

Table 8. The Test of the Hypothesis of Constant Transition Probabilities.*

Cohort	Transitions	Sex	d.f.	High	Low	Total
1969	So-Jr-Sr	Male	1	.13	2.80	2.93
		Female	1	.88	.42	1.30
1970	Fr-So-Jr-Sr	Male	2	4.27	.42	4.69
		Female	2	1.42	.77	2.1[illegible]
1971	Fr-So-Jr	Male	1	.86	.23	1.09
		Female	1	.05	.36	.41

*The values were corrected for contingency where necessary.

each of the tested cohorts was not rejected. This means that aspirations act as random variables for which the future state depends only upon the past state and nothing else in the history. The rates are not time-dependent and are constant for the aggregate of both males and females within each cohort. The test of similarity over cohorts would seem to be the next logical test. This, however, must await the tests of the order of the process and the similarity of the processes for males and females within each cohort.

The Hypothesis of the Order of the Process

This hypothesis was that the process is a first-order rather than a second-order one. That is, the response depends only upon the last response rather than on the last two responses. This test as the previous one required the use of only the 1969 to 1971 cohorts. Unfortunately, this hypothesis could not be adequately tested because of the inadequate sample size obtained by further breakdown of the tables. Since the Chi square test could not be used because of small cell sizes, the Fisher Exact Test was used. The results, given in Table 9, are therefore offered only as suggestive. The values obtained suggest the process appeared to be first-order.

Table 9. The Test of the Order of the Process.

Class	Transitions	Sex	d.f.	High	Low
1969	So-Jr-Sr	Male	1	.97	.97
		Female	1	.05	.70
1970	Fr-So-Jr	Male	1	.85	.27
		Female	1	.03	.41
1970	So-Jr-Sr	Male	1	.91*	.48
		Female	1	1.83*	.76
1971	Fr-So-Jr	Male	1	.01	.88
		Female	1	.93	.24

*These are Chi square values.

The patterning of responses appeared in the high category and only for females in the 1969 and 1970 classes (the probabilities were .05 and .03) and for males in the class of 1971 (the probability was .01). No such patterning was evident in the low category. Patterning suggests that high aspirations more than low aspirations depended upon the constancy of high aspirations in the past; however, this appeared largely restricted to females. Examination of the three tables revealed that the turnover given high aspirations for the past two years was extremely low (around the .10 level) whereas for the low category the probability of either a high or low level of aspiration remained roughly equal.

The female must apparently have a long history of high aspirations in order to continue college plans. This indicates that the pressures which tend to lower the level of aspirations do have an effect. Females typically have lower college attendance rates than males as well as lower aspirations.[54] Nevertheless, the

---

[54]Using a sample of 127,125 college freshmen in 1961 Werts found that achievement and socioeconomic status interacted with sex in determining attendance rates. His figure of 60 percent male attendance was close to the national figure of 58 percent. He concluded that low achievers who went to college were typically male while high achievers were equally male and female. In low SES families it was the boy who went to college. See C.W. Werts, "A Comparison of Male Versus Female College Attendance Probabilities," Sociology of Education 41 (1968), pp. 103-110. Developing a computer model to measure enrollment Froomkin and Pfeferman of the Office of Education used Census data and found a sudden rise in the rate for enrollment in college for all income groups beginning in 1960. Except for the highest income and highest ability groups the males had a higher probability of college attendance in the first year following graduation. In the highest groups the probability for attendance was similar for both sexes. See J. Froomkin and M. Pfeferman, "A Computer Model to Measure the Requirements for Student Aid in Higher Education," in E.D. Goldfield (Ed.), American Statistical Association Proceedings of the Social Statistics Section, 1969 (Washington,D.C.: American Statistical Association, 1969), pp. 125-137.

Examination of the percentages aspiring to college for males and females for the tables in Appendix A reveals a general grend for higher aspiration levels for males than females.

process of turnover appears to be the same, although the exact rates may not apply to both sexes.

The Hypothesis of No Sex Differences

The fourth hypothesis was that the same process of change with the same rates characterizes both males and females. That is, treating males and females as separate populations is unnecessary.

In Table 10 the results of the test are given. None of the Chi square values was significant at the 1 percent level. Therefore, the null hypothesis of no

Table 10. The Test of Sex Differences for the Process of Change.

Class	Transition	d.f.	High	Low	Total
1968	Fr-So	1	.06	.02	.08
1969	So-Jr	1	1.05	.89	1.94
	Jr-Sr	1	.18	2.32	2.50
1970	Fr-So	1	.02	.01	.03
	So-Jr	1	.00	.12	.12
	Jr-Sr	1	.62	.05	.67
1971	Fr-So	1	.00	.20	.20
	So-Jr	1	1.07	.36	1.43
1972	Fr-So	1	.25	.13	.38

sex differences in the actual rates of turnover cannot be not rejected. At first this finding may be surprising. Females do have lower levels of aspirations. Socio-economic status, ability, and other variables interact with sex in determining the level of aspirations. These would seem to indicate different processes. In actuality they do not. The only difference between the males and females found is the leve and not the process of aspirational change over time. The turnover remains the same for both groups given an initial level. The determination of that level is another question. However, eventually even this becomes meaningless, since the process over a long enough period of time reaches a stable state where the proportion of students in each state becomes independent of the initial distribution. This concept will be discussed somewhat later.

The Hypothesis of Constant Matrices

Once again the tables were tested for constancy of the transition probability matrices for the cohorts. This time, however, the total sample was used with the pooling of males and females. This test is somewhat redundant, but it represents checking the inferences one could draw from the previous hypothesis, i.e., no differences between males and females would mean that the total sample would also be represented by constant transition probability matrices.

As expected the hypothesis was not rejected. The values are given in Table 11. This test does pave the way to the final test of the applicability of the Markov model. This is the hypothesis that the nine cohorts could be represented by the same process of change with the same rates.

Table 11. The Test of Constant Transition Probabilities for the Total Sample for Each Cohort.

Class	Transitions	d.f.	High	Low	Total
1969	So-Jr-Sr	2	1.34	.13	1.47
1970	Fr-So-Jr-Sr	4	4.15	1.34	5.29
1971	Fr-So-Jr	2	.43	.06	.49

The Hypothesis of the General Transition Matrix

The last hypothesis was a test of the greatest extension of the fit of the model where all the data were lumped into one homogeneous population. The Chi square was based on each period for each cohort over each category of response. With 8 degrees of freedom the values were 15.36 for the high category and 6.71 for the low category. Neither of these was significant at the 1 percent level. The null hypothesis of no differences among cohorts for the process could therefore not be rejected. That is, the hypothesis that a general matrix of transition probabilities which applied to all the data could not be rejected.

The General Transition Matrix

The data from the one-year transition period tables were combined into one table. This table was then used to calculate the turnover probabilities for the various transition periods for all of the data. These matrices are given in Table 12. The P^n vector is the stable state prediction.

Table 12. The One-, Two-, and Three-State Transition Matrices and the Stable State Vector for the Discrete-Time Markov Model.*

$$P^1 = \begin{vmatrix} .821 & .179 \\ .239 & .761 \end{vmatrix}$$

$$P^2 = \begin{vmatrix} .717 & .283 \\ .378 & .622 \end{vmatrix}$$

$$P^3 = \begin{vmatrix} .656 & .344 \\ .459 & .541 \end{vmatrix}$$

$$P^n = [.572 \quad .428]$$

*The system approaches equilibrium at n=12.

Examination of the matrices reveals that the turnover rate from high to low aspirations wa consistently lower than the opposite turnover. In addition, the difference between these rates increased over time. Over time the turnover from low to high aspirations increased at a faster rate. The model predicts that eventually regardless of the initial distribution of aspirations about 57 percent of the population would desire college attendance. This, however interesting, is academic since the length of time in high school is restricted to three or four years. The model does tell us that over the first year 18 percent of those with high aspirations will lower them and 24 percent of those with low aspirations originally will plan on college. After two years the corresponding percentages will increase from 18 percent to 28 percent and from 24 percent to 38 percent. From the freshman to senior year the proportions will have gone from 18 percent to nearly double the figure at 34 percent for the high to low transition and will have gone from 24 percent to nearly 46 percent for turnover from low aspirations to high aspirations. If high school continued for twelve years or so the corresponding rates would reach 43 percent and 57 percent respectively.

The Continuous-Time Markov Chain

The Two-State Model

The data from Appendix B were used to calculate the transition rates for the two-state system. Since the q_{ij}'s represent the limiting forms of the p_{ij}'s they are expected to be higher in value than the discrete-time transition probabilities. In Table 13 the transition rates for each cohort for each turnover are given. The transition rates based on the general transition table, and the projections from it, are given in Table 14. As can be seen when these matrices are compared to the corresponding discrete-time matrices, the values are higher except in the case of the stable state matrix. Examination of the rates in Table 13 for each cohort over time offers information about the rate consistency which the discrete-time model did not provide. The transition rate drops

Table 13. Transition Rates for each Cohort using the Continuous-Time Markov Model.

Class	Transition	Transition Rates High-Low	Low-High
1968	Jr-Sr	.236	.133
1969	So-Jr	.182	.318
	So-Jr	.106	.314
1970	Fr-So	.355	.338
	Fr-Jr	.218	.220
	Fr-Sr	.145	.212
1971	Fr-So	.251	.393
	Fr-Jr	.129	.226
1972	Fr-So	.236	.222

after the first year approximately 40 percent for both turnovers. For the high to low transition the rate continues to drop after the second year (based only on the 1970 cohort) while the rate appears to remain somewhat level for the low to high transition over the same period.

It was previously mentioned that the model could be used to bridge the gap between cross-sectional and over-time data, where the row marginals provided estimates of the relative size of the random shocks which move individuals from state to state, while the over-time q_{ij}'s are the

Table 14. The One-, Two-, and Three-Step Transition Matrices and the Stable State Vector for the Two-State Continuous-Time Markov Model.*

$$P^1 = \begin{vmatrix} .768 & .232 \\ .310 & .690 \end{vmatrix}$$

$$P^2 = \begin{vmatrix} .662 & .338 \\ .452 & .548 \end{vmatrix}$$

$$P^3 = \begin{vmatrix} .613 & .387 \\ .517 & .483 \end{vmatrix}$$

$$P^n = [.572 \quad .428]$$

*The system approaches equilibrium at n=12.

actual values for these shocks. For these data these two estimates were carried out and compared. The cross-sectional estimates consistently estimated higher turnover rates for the low to high transitions, while the over-time values showed these ratios were actually lower. In addition the cross-sectional estimates tended to increase in the direction of higher aspirations. Therefore, internal changes were important.

In the section on the theory of the continuous-time model it was noted that for unconstrained q_{ij}'s projection from the discrete-time transition probabilities can be done if the time interval is integral in nature. In the case of the two-state system the q_{ij}'s are not arbitrarily restricted in value since transitions from low to high or high to low states are possible. Projection from the q_{ij}'s could be carried out and since the q_{ij}'s represent the limiting form of the p_{ij}'s the resulting projections could be considered as the limiting form of the projected values. Unless the limits were radically different from the p_{ij}'s any difference would be somewhat trivial in nature, although this difference would increase over time, since the raising of the matrix to the appropriate power would increase the difference. This is indeed what occured when the predictions were compared in Table 15.

Table 15. Comparison Between Discrete-Time and Continuous-Time Predictions and the Actual Frequencies for Each Cohort.

Cohort	Period	Actual Frequency High	Actual Frequency Low	Predicted Frequencies: Discrete Model High	Discrete Model Low	Continuous Model High	Continuous Model Low
1968	Jr-Sr	74	57	81	50	80	51
1969	So-Jr	82	42	79	45	77	47
	Jr-Sr	89	35	75	49	74	50
1970	Fr-So	74	53	80	47	79	48
	Fr-Jr	73	54	77	50	76	51
	Fr-Sr	79	48	75	52	74	53
1971	Fr-So	62	34	61	35	60	36
	Fr-Jr	63	33	58	38	57	39
1972	Fr-So	107	60	111	56	108	59

The difference in the predicted number of persons in the two states for the two models over all the cohorts over time was relatively small, and was in the range of from one to three cases.

The advantage of the continuous-time two-state Markov model was in providing information regarding the transition rates within each cohort over time. It also provided values for the limit of the transition probabilities. Nonetheless, the full strength of the model is not realized when the system is restricted to only two states. The analysis carried out so far was based on dichotomous data in order to make full use of the data available. The model was also used with multi-category data.

The Five-State Model

From 1967 to 1969 multi-category data were available for aspirations. For this analysis the categories of 1) finish high school, 2) technical or business school, 3) junior college, 4) four-year college or university, and 5) advanced degree were utilized (the category of "less than high school" was so rarely used it was thrown out). There were six one-step transition tables for the 1969 to 1972 cohorts. These were combined into one transition table, given in Table 16. This table was then used to generate the transition rates.

Since there were five states there were twenty independent transition rates. Therefore, five rates were set to zero. These were the q_{15}, q_{25}, q_{35}, q_{41}, and the q_{51} transitions. They were chosen because they represent the most extreme transition possible, and therefore would be the more likely to be zero in the actual population. Since some transitions are restricted

Table 16. The Raw Data Transition Table for the Five-State Continuous-Time Markov Model.

		(1)	(2)	(3)	(4)	(5)	TOTAL
H.S.	(1)	74	24	19	5	1	123
Tech.	(2)	23	66	30	11	2	132
J.C.	(3)	17	20	65	23	5	130
Univ.	(4)	13	16	24	133	39	225
Adv.	(5)	1	7	5	40	77	137
TOTAL		128	133	143	212	124	747

in value, and since there are more than two states, the calculations were carried out by a modified version of Coleman's computer program.[55]

The independent transition rates for the data were the following:

$q_{22}=.326$ $q_{21}=.280$ $q_{31}=.202$ $q_{42}=.097$ $q_{52}=.064$

$q_{13}=.218$ $q_{23}=.402$ $q_{32}=.253$ $q_{43}=.165$ $q_{53}=.015$

$q_{14}=.020$ $q_{24}=.092$ $q_{34}=.294$ $q_{45}=.284$ $q_{54}=.473$

These rates were then used to calculate the transition probabilities. These probabilities for the one-step, two-step, three-step, and the stable matrix vector are given in Table 17. The one-step transition was then used to regenerate the data to test for the accuracy of fit. Although the model tended to underestimate the diagonal values the degree of fit to the actual data

[55]See Coleman, op. cit., pp. 177-188.

Table 17. The One-, Two-, and Three-Step Transition Matrices and the Stable State Vector for the Five-State Continuous-Time Markov Model.*

$$P^1 = \begin{vmatrix} .610 & .190 & .154 & .042 & .004 \\ .171 & .516 & .218 & .085 & .011 \\ .129 & .249 & .527 & .171 & .024 \\ .019 & .071 & .104 & .637 & .170 \\ .009 & .051 & .039 & .285 & .616 \end{vmatrix}$$

$$P^2 = \begin{vmatrix} .425 & .240 & .221 & .096 & .018 \\ .222 & .338 & .262 & .145 & .032 \\ .175 & .193 & .349 & .224 & .059 \\ .051 & .109 & .146 & .478 & .216 \\ .030 & .085 & .086 & .369 & .429 \end{vmatrix}$$

$$P^3 = \begin{vmatrix} .330 & .245 & .245 & .143 & .037 \\ .230 & .268 & .262 & .185 & .055 \\ .190 & .204 & .278 & .243 & .085 \\ .080 & .133 & .167 & .403 & .219 \\ .055 & .110 & .124 & .381 & .330 \end{vmatrix}$$

$$P^n = [.172 \quad .190 \quad .217 \quad .279 \quad .144]$$

*P^n approaches equilibrium at n=25.

appeared to be good.[56]

Using the five-state model permits comparison of twelve transition rates rather than two for the two-state model. The use of more categories permits more detailed analysis of the process of change in the level of aspiration.

The transition rate for the high school to technical or business school was .326 and was higher than the rate for the change to junior college aspirations of .218. The rate for a jump to university plans was very small at .020. Apparently attendance at a four-year college or university was seen as unrealistic for those who have previously though of finishing only high school.

For those who were planning on technical or business school the rate for upward aspirations was higher than for lowering aspirations. The rate of transition for this group compared with the high school group was still higher for the higher levels of aspiration.

For those who planned on attending junior college

---

[56] The regenerated data were the following:

		(1)	(2)	(3)	(4)	(5)
H.S.	(1)	75	23	19	5	1
Tech.	(2)	23	68	29	11	1
J.C.	(3)	17	19	68	22	3
Univ.	(4)	4	16	23	143	38
Adv.	(5)	1	7	5	39	84
TOTAL		120	134	145	221	128

the transitions upward and downward were in the same range. The rate was .202 for turnover to high school plans, .253 for technical or business school plans, and .294 for university plans. It would appear that while the rate for the university was the strongest, the two transitions to high school and technical or business school would dominate. This is consistent wit.ı the theory that junior colleges are "cooling out" functionaries in society where some transfer to the university but where most drop out or go no further.

For those who plan attendance at a four-year college or university the turnover rates for a decrease in the level of aspirations were low. For the junior college turnover the rate was .165 while for the technical or business school transition it was .097. The rate for the upward transition for the advanced degree plans was a higher .284. These rates indicate that for this group the lowering of the general level of aspirations was not the tendency.

The same may be said for the advanced degree group. For here the transition to junior college attendance was even lower than that to technical or business school. These rates were .015 and .064 respectively. The turnover to university plans only was a high .473.

Examination of Table 17 reveals that over the four years of high school movement among levels of aspiration was very probable. For all levels of aspiration the probability for change in the exact level was more probable than retaining the same level.

For the entire P^3 matrix there appeared to be a general trend for a higher probability for movement one step up in level than down one step. For those levels which could increase the probabilities for one step up in aspirational level were in the .20 range. For those levels in which movement downward was possible the probabilities for such movement were higher for the extreme levels of aspiration (.381 for the advanced level and .230 for the technical school level) than for the middle range levels (.204 for the junior college and .167 for the university levels).

If as with the dichotomous categories high aspirations are those from junior college upward, then the five levels of aspiration showed a predictable increase in level of high aspirations: 42 percent wanting to attend college for the high school group, 50 percent for the technical school group, 61 percent for the junior college group, 79 percent for the university group, and 83 percent for the advanced group.

The model predicts that regardless of the initial distribution the final distribution would result in 17 percent planning only on finishing high school, 19 percent intending on technical or business school, 22 percent hoping to go to junior college, 28 percent wanting to attend a four-year college or university, and finally 14 percent who wanted an advanced degree. The model predicts that at least 65 percent of the population would desire attendance at a college at least at the level of the junior college. This figure is close to that of 57 percent obtained by use of the two-state model. The disparity is probably due to the differences in the samples used.

In summary, the model's regenerated data were close to the actual data for the five-state system. Since more transitions were available for examination, the preceding analysis was more detailed and informative than the analysis using only dichotomous categories for aspirations. This analysis revealed that turnover within the general level of noncollege or college aspirations was very probable, while the turnover between these levels was less common. In addition, the probability of turnover from low to high aspirations was higher than in the other direction.

The results of the test of the fit of the data to the Poisson model will now be discussed.

The Test of the Simple Poisson and the Cumulative Inertia Poisson

The Simple Poisson

The cumulative percentages of persons in each of the low and high aspiration categories over time for each cohort were derived from the data in Appendix B. These percentages were then used in equation (12) to calculate the model's parameter of turnover λ. The results of these calculations are given in Table 18. As is seen from the table, the values are not constant over time. Therefore, the data do not appear to fit the Simple Poisson model.

Table 18. Turnover [illegible] the Poisson Model.

Cohort	Transitio[illegible]	[illegible]gh-Low	Low-High
1968	Jr-Sr	.[illegible]0	.118
1969	So-Jr		.288
	So-Sr	[illegible]	.278
1970	Fr-So	.300	.280
	Fr-Jr	.172	.173
	Fr-Sr	.104	.165
1971	Fr-So	.204	.343
	Fr-Jr	.102	.195
1972	Fr-So	.209	.196

Since the data do not fit the model, modification of the model might improve the fit. Since the rates decrease somewhat precipitously at first, and then decrease at a slower rate, the decline in value would be exponential. Therefore, the Poisson was modified using the Cumulative Inertia model which assumes that the longer the residence in a state, the greater the probability of remaining there.

The Cumulative Inertia Poisson

The equation for the proportion of persons in state i for the Poisson process for this model is

$$(15) \quad P_{it} = \binom{N}{i}(1 - e^{-a(0)H(t)})^{i} \quad e^{-a(0)H(t)(N-i)}$$

with an expected value of

$$(16) \quad E(i) = N(1 - e^{-a(0)H(t)})$$

where the form of the function $H(t) = 1 - e^{-bt}/b$. Therefore, the model is a two parameter model where a(0) and b must be estimated from the data. If the model fits the data the model should be able to predict the cumulative percent in each category for each cohort with relatively good accuracy. Projecting the time-homogeneous process governed by $a(0)H(t)/t$ over the time period t is equivalent to projecting the time-inhomogeneous process governed by a(0) over the time period $1 - e^{-bt}/b$. Therefore, one can iteratively calculate b and use b to estimate a(0). Large values of b should indicate relatively rapid decreases in the percentage differences over time. Large values of a would indicate relatively large initial percentages at the first observation.

The iterative equation used to estimate b is given by

$$(17) \quad b = (-1/t)\ln(1-(R(s,t)(1 - e^{-bs})(t/s)))$$

where R(s,t) is the ratio of the transition intensities at time s to time t. These are obtained from the data. Parameter a(0) is estimated from the data by use of the fact that $a(0)\ (1 - e^{-bt})/bt$ should equal the estimated transition intensities for each cohort.

The calculations were carried out using data from the 1969 to 1971 cohorts. The results are given in Table 19. Both the model and the data appear to be inadequate.

Table 19. Estimated Parameters for the Cumulative Inertia Poisson Model.*

Cohort	Transition	a(0)	b	a(0)h(t) 1	2	3
1969	High-Low	---	---			
	Low-High	.31	.08	.30	.57	
1970	High-Low	1.32	2.98	.42	.44	.44
	Low-High	.63	.67	.46	.69	.81
1971	High-Low	---	---			
	Low-High	1.40	2.00	.60	.69	

*The blank entries indicate lack of convergence in the iterative equation for the data.

For the 1969 and 1971 data the high to low transition parameters could not be obtained due to lack of convergence in the equation. For both the high-low transitions for these data the absolute number of persons who were in each state remained the same over the transition periods.[57]

In conclusion, it appears that the a parameter

[57]Examination of the b parameter values in the Land article revealed that the values based on small numbers of observations were inaccurate estimates of the b values obtained by use of data from the longest wave of observation. If the nine-year b value is taken as the correct one, the values obtained by use of the one- to eight-year transitions were the following: .5158, .3768, .3164, .2884, .2678, .2489, .2357, and .2279. It is noted parenthetically that the b values given in his paper for Table 3 are wrong. They are somewhat high.

values do not echo the initial percentage in the categories well, nor do the b values reflect decreases in the percentage differences with time accurately. Although the rates do decline over time the rate appears not to be adequately described as exponential. Definitive answers to the actual decline function must await better data.

Up to this point the results of the analysis of the fit of the discrete-time Markov chain, the continuous-time Markov chain, and the Simple and Cumulative Inercia Poisson models have been represented. At this time the results using these models will be briefly compared.

A Comparison of the Models

The primary emphasis in this section will be the comparison of the parameter values for the two models which were presented, the Markov chain and the Poisson. The two-state continuous-time Markov chain and the Simple Poisson models will be used.

Comparing Table 13 with Table 17 shows that the corresponding parameter value for both models are practically equal to one another.

In Table 20 the mean transition rates for the Poisson model are given. The mean rates for the total data for the Poisson model are .171 for the high to low turnover and .226 for the other. These values are very close to the turnover values calculated from the discrete-time model of .179 and .239 respectively. The limiting values of these rates as determined by use of the continuous-time model are .232 and .310 respectively.

Table 20. The Mean Transition Rates for the Poisson Model.

Period	Transition	Rate
One-Year	High-Low	.218
	Low-High	.235
Two-Year	High-Low	.117
	Low-High	.215
Three-Year	High-Low	.104
	Low-High	.165
Total	High-Low	.171
	Low-High	.226

Summary

Examination of the general pattern of aspirations for the students revealed the general trend of change from an initially high level to a somewhat lower level for the sophomore and junior years followed by an increase in the senior year.

The data were not inconsistent with the discrete-time Markov chain model. The hypothesis of a general process for all students regardless of sex or cohort would not be rejected.

Although the two-state continuous-time model did offer more information than the discrete-time model, it was shown that the five-state Markov chain model provided a more detailed analysis. Had there been consistency in the question asking the student about his level of aspiration over all the years of the study the five-state model would have been used throughout the analysis.

The hypothesis that aspirations act as though they were random variables where the future state of the system depends only upon the previous state could not be rejected on the basis of the evidence presented. Turnover within the same general level of aspirations (college or noncollege) was predicted over the four years of high school. The evidence suggested that the rate of low to high turnover in aspirations was greater than the rate from high to low. The process appeared to be one of upward mobility in level of aspiration.

The data were found to not fit either the Simple or the Cumulative Inertia Poisson models. The transition rates were not constant over time nor did they appear to decline in an exponential manner.

The mean rates for the Simple Poisson model were very close to those of the discrete-time Markov chain model. Since the two-state continuous-time Markov parameter values were the limiting form of the discrete-time values, these values were close to the rates for both the Poisson and the discrete-time model.

CHAPTER IV

SUMMARY AND CONCLUSION

Summary

The mathematical models which have been employed in the study of social mobility have rarely been used in the study of the educational or occupational aspirations of youth. One research which used a continuous-time Markov chain model found the data fit the model. It is the power of the models that the same reasoning used in the study of mobility can be also applied to the educational aspirations of high school students.

The aspirations of high school students can be thought of as being governed by a time-homogeneous process where the decision points are randomly distributed, or at least differentially distributed among the various students. The process is therefore considered to be a stochastic process which is governed by the assumptions underlying either a Markov chain or a Poisson distribution. The Markov chain and Poisson processes are related to one another. The basic assumption in the Markov chain process is that the future distribution depends only upon the present distribution and the transition matrix, while the basic assumption underlying the

Poisson process is that the transition rates are constant throughout time. Both assume the variables are randomly distributed, and both can be modified by similar methods to try to improve the fit of the models to the empirical data. Both models have been used with varying success in the social sciences.

Having discussed the basic models and their relationsh) to one another, the problem of reliability was brought forth as a central consideration in the study of change. The reliabilities of the measurement were determined in order to determine if analysis of the data would be valuable. The level of reliability was accepted as being relatively good, and were in the .80 to .90 range with the exception of one cohort where it dropped to the .60 range.

In addition to measurement error the problem of response bias was discussed. Although the use of cohort analysis where only those individuals are used who are in a the interviews can lead to error due to lack of generalizability to the entire population, the estimates based on these data appear to provide the closest estimates of the "true" population value.

The sample were 635 high school students of both sexes who were sampled from 1966 to 1969 in part of an ongoing research on the industrialization of rural area in Northern Illinois. The students were drawn from both the experimental and the control area of the study, and were grouped as cohorts based on the year of graduation.

Missing data were thrown out. The percentage of data which did not contain missing data varied from a low of 16 percent for the 1969 and 1971 cohorts to the high of 75 percent for the class of 1972.

Educational aspirations over time were treated as separate variables and were cross-tabulated with one another. Aspirations were dichotomized into high aspirations (college attendance planned) and low aspirations (no college plans).

After the parameter values for each model were computed the data was tested for fit to the model. The data fit the discrete-time Markov model. The model predicted a one-year probability of 18 percent turnover from the high to low aspirations, and a corresponding probability of 24 percent turnover from the low to high aspirations. Although the emphasis was on the first-year transition in the analysis, these rates apply to any one-year transition. The corresponding yearly rates predicted by the model were for the high to low transition .18, .28, and .34 while the rates were .24, .38, and .46 for the low to high transition. The suggestion of the patterning of responses for females desiring college attendance was presented, although the data were less than adequate.

The continuous-time model values were higher than the discrete-time parameters. The transition rates dropped over time within each cohort over each category of aspiration. It was shown that cross-sectional analysis overestimated the size of the "random shock." Use of over-time data, as was previously suggested, provided better insight into the process.

The Simple Poisson model was rejected because of the decrease in turnover rates over time. The modification of the model was also rejected because of lack of fit and because of lack of adequate data.

Comparison of the results of the three models tested revealed that the mean rates are very close in the case of the Poisson and the discrete-time Markov chain model, and reasonably close to the continuous-time Markov chain model. The rates of this model represent the limit of the parallel discrete-time model. The rates for the high to low transition over the years were .23, .34, and .39 while the corresponding rates were .31, .45, and .52 for the low to high categories. Both Markov models predicted a stable state where 57 percent of the youth would eventually desire college attendance regardless of the initial distribution.

The general pattern analysis carried out prior to the test of the models appeared to coincide relatively well with the results of the actual data analysis. Examining the gross percentage levels suggested a regularity within each cohort and possibly over cohorts which was shown to be correct.

Conclusions[58]

The extension of the Markov model to an earlier time in the process of mobility appears to be justified. As was the case with the McDill and Coleman study the data fit the continuous-time Markov chain model. Unlike much of the previous work dealing with mobility, modification of the model was not necessary in order to get fit. The model is therefore felt to offer enough justification for further study related to this field. The model offers superior capacity in its ability to predict

[58]Certain limitations of this study should be held in mind. First, there is the problem of the representativeness of the sample. Since the students were sampled in a purposive manner there is no guarantee that the students adequately represent the rural community student. Nonetheless, one is encouraged by the adequacy of the fit of the data to the model by McDill and Coleman for students from the same general geographical area using a more heterogeneous sample of schools. Another problem was the high frequency of missing data within each cohort. Because it is essential to have complete data to use with these models, the test of the difference between respondents and nonrespondents should be carried out. The model does suggest, however, that nothing in the life history of the response makes any difference in determining the future state. Therefore, even if there were significant differences, there would still be the question of these differences relating to the process rather than the level of aspirations. Finally, there is the question of the reliability of the measurement. With the models it is assumed that there is perfect measurement, and that all changes in response states are actual changes and not fluctuations due to measurement error. Unless the level of measurement is very good, the unreliability of the measurement will invalidate the results. In this case the model would predict change due to error rather than change due to actual change in the response. Methods of coping with this problem would be triangulation of methods, or perhaps the use of only those data for which the level of measurement has been shown to be very high (high in this case would be in the .90 range).

future states of the system which are not possible in the usual cross-sectional study procedures. The next step would be, besides the necessary replications, using larger samples over the entire career in high school instead of only one cohort to study differential transition rates based on the model. One would next seek to introduce other characteristics into the model by use of attributes. This approach would permit integration of the past descriptive studies with this type of analysis. Past studies have used attributes (or cohort characteristics) which have differentiated among levels of aspiration. These attributes are important influences which cut across cohorts, and do not negate the necessity to study the cohort over time. They do suggest important intervening influences which operate in the process.

A method superior to the use of the concept of "state" used by McDill and Coleman in studying attributes which affect transition intensities and superior to the use of calculating separate matrices for homogeneous groupings of the population was suggested by Spilerman. His method uses regression analysis procedures where the attributes of the population are tested for their significance in any transition from one state to another. By its use of dummy variables it thereby allows the use of all the population in the analysis while still permitting determination of the individual transition probability matrices. In addition it permits the study of change due to either structural changes of the rules governing transitions and changes in the distribution of attributes over time. Included in the method is the use of individual transition matrices to project forward into time. This method also allows for the use of a general homogeneous population since it would be reflected in the regression equation where none of the beta weights for the attributes (such as social class, intelligence, and so on) would be significant at a given level.

By use of this procedure the results of previous research on educational aspirations can be integrated into the Markov chain model. It is superior to the method used by McDill and Coleman since the number of transitions do not increase with the increase in the number of attributes studied.

APPENDIX A

The one-year transition tables for males and females for the 1968 to 1972 cohorts.

Table 21. The One-Year Transition Tables for the 1968 Cohort for Males and Females.

Junior-Senior

	Males				Females		
	High	Low	Total		High	Low	Total
High	45	11	56	High	24	6	30
Low	2	20	22	Low	3	20	23
Total	47	31	78	Total	27	26	53

Table 22. The One-Year Transition Tables for the 1969 Cohort for Males and Females.

Sophomore-Junior Transition

	Males				Females		
	High	Low	Total		High	Low	Total
High	39	4	43	High	33	8	41
Low	2	13	15	Low	8	17	25
Total	41	17	58	Total	41	25	66

Junior-Senior Transition

	Males				Females		
	High	Low	Total		High	Low	Total
High	39	2	41	High	37	4	41
Low	8	9	17	Low	5	20	25
Total	47	11	58	Total	42	24	66

Table 23. The One-Year Transition Tables for the 1970 Cohort for Males and Females.

Freshman-Sophomore Transition

Males	High	Low	Total	Females	High	Low	Total
High	29	11	40	High	35	11	46
Low	5	13	18	Low	5	18	23
Total	34	24	58	Total	40	29	69

Sophomore-Junior Transition

Males	High	Low	Total	Females	High	Low	Total
High	28	6	34	High	34	6	40
Low	6	18	24	Low	5	24	29
Total	34	24	58	Total	39	30	69

Junior-Senior Transition

Males	High	Low	Total	Females	High	Low	Total
High	31	3	34	High	32	7	39
Low	8	16	24	Low	8	22	30
Total	39	19	58	Total	40	29	69

ERIC

Table 24. The One-Year Transition Tables for the 1971 Cohort for Males and Females.

Freshman-Sophomore Transition

Males	High	Low	Total	Females	High	Low	Total
High	25	5	30	High	33	8	41
Low	3	11	14	Low	8	17	25
Total	28	16	44	Total	41	25	66

Sophomore-Junior Transition

Males	High	Low	Total	Females	High	Low	Total
High	23	5	28	High	30	14	44
Low	6	10	16	Low	4	14	18
Total	29	15	44	Total	34	28	62

Table 25. The One-Year Transition Tables for the 1972 Cohort for Males and Females.

Freshman-Sophomore Transition

Males	High	Low	Total	Females	High	Low	Total
High	38	7	45	High	61	16	77
Low	4	18	22	Low	4	19	23
Total	42	25	67	Total	65	35	100

APPENDIX B

The one-, two-, and three-year transition tables for the 1968 to 1972 cohorts.

Table 26. The One-Year Transition Table for the 1968 Cohort.

Junior-Senior Transition

	High	Low	Total
High	69	17	86
Low	5	40	45
Total	74	57	131

Table 27. The One- and Two-Year Tables for the 1969 Cohort.

Sophomore-Junior Transition

	High	Low	Total
High	72	12	84
Low	10	30	40
Total	82	32	124

Sophomore-Senior Transition

	High	Low	Total
High	72	12	84
Low	17	23	40
Total	89	35	124

Table 28. The One-, Two-, and Three-Year Transition Tables for the 1970 Cohort.

Freshman-Sophomore Transition

	High	Low	Total
High	64	22	86
Low	10	31	41
Total	74	53	127

Freshman-Junior Transition

	High	Low	Total
High	61	25	86
Low	12	29	41
Total	73	54	127

Freshman-Senior Transition

	High	Low	Total
High	63	23	86
Low	16	25	41
Total	79	48	127

ERIC

Table 29. The One- and Two-Year Transition Tables for the 1971 Cohort.

Freshman-Sophomore Transition

	High	Low	Total
High	53	12	65
Low	9	22	31
Total	62	34	96

Freshman-Junior Transition

	High	Low	Total
High	53	12	65
Low	10	21	31
Total	63	33	96

Table 30. The One-Year Transition Table for the 1971 Cohort.

Freshman-Sophomore Transition

	High	Low	Total
High	99	23	122
Low	8	37	45
Total	107	60	167

APPENDIX C

The parameters, parameter values, and equations for the reliability and stability for four waves of observations.

Table 31. Population Covariance Matrix for Lag-1 Model Assuming Constant Error Variance for Four Waves of Observations.

$$V(X_1) = V(\Theta_1) + V(\varepsilon)$$

$$C_{12} = a_{21}V(\Theta_1)$$

$$V(X_2) = a_{21}^2V(\Theta_1) + V(\Theta_2) + V(\varepsilon)$$

$$C_{13} = a_{21}a_{32}V(\Theta_1)$$

$$C_{14} = a_{21}a_{32}a_{43}V(\Theta_1)$$

$$C_{23} = a_{32}(a_{21}^2V(\Theta_1) + V(\Theta_2))$$

$$C_{24} = a_{43}a_{32}(a_{21}^2V(\Theta_1) + V(\Theta_2))$$

$$V(X_3) = a_{32}^2(a_{21}^2V(\Theta_1) + V(\Theta_2)) + V(\Theta_3) + V(\varepsilon)$$

$$C_{34} = a_{43}(a_{32}^2(a_{21}^2V(\Theta_1) + V(\Theta_2)) + V(\Theta_3))$$

$$V(X_4) = a_{43}^2(a_{32}^2(a_{21}^2V(\Theta_1) + V(\Theta_2)) + V(\Theta_3)) + V(\Theta_4) + V(\varepsilon)$$

Table 32. Parameters and Estimators for the Lag-1 Model for Four Waves of Observations.*

a_{32}	$\hat{C}_{13}/\hat{C}_{12}$
a_{43}	$\hat{C}_{14}/\hat{C}_{13}$
$V'(\varepsilon)$	$\hat{V}(X_2) - (\hat{C}_{23}/\hat{a}_{32})$
$V''(\varepsilon)$	$\hat{V}(X_3) - (\hat{C}_{34}/\hat{a}_{43})$
$V'(\Theta_1)$	$\hat{V}(X_1) - \hat{V}'(\varepsilon)$
$V''(\Theta_1)$	$\hat{V}(X_1) - \hat{V}''(\varepsilon)$
a'_{21}	$\hat{C}_{12}/\hat{V}'(\Theta_1)$
a''_{21}	$\hat{C}_{12}/\hat{V}''(\Theta_1)$
$V'(\Theta_2)$	$\hat{V}(X_2) - (\hat{a}_{21}\hat{C}_{12} + \hat{V}'(\varepsilon))$
$V''(\Theta_2)$	$\hat{V}(X_2) - (\hat{a}_{21}\hat{C}_{12} + \hat{V}''(\varepsilon))$
$V'(\Theta_3)$	$\hat{V}(X_3) - (\hat{a}_{32}\hat{C}_{23} + \hat{V}'(\varepsilon))$
$V''(\Theta_3)$	$\hat{V}(X_3) - (\hat{a}_{32}\hat{C}_{23} + \hat{V}''(\varepsilon))$
$V'(\Theta_4)$	$V(X_4) - (a_{43}C_{34} + V'(\varepsilon))$
$V''(\Theta_4)$	$V(X_4) - (a_{43}C_{34} + V''(\varepsilon))$

*The ' values and the " values are based on separate error variance values.

Table 33. Parameter Values for the Data for the Lag-1 Model.*

Parameter	1969	Cohort 1970	1971
a_{32}	.9486	.8328	.8809
a_{43}		.6842	
$V(\varepsilon)$	.0456	.0596'	.0931
		.0423"	
$V(\Theta_1)$	.7367	.1608'	.1279
		.1780"	
a_{21}	.9456	.6857'	.8228
		.6915"	
$V(\Theta_2)$	.0853	.1099'	.0329
		.1172"	
$V(\Theta_3)$	.0509	.0581'	.0207
		.0754"	
$V(\Theta_4)$		.0819'	
		.0992"	

*The ' values are based on the .0596 error variance value and the " values are based on the .0423 value.

Table 34. Stability Coefficient Equations for Lag-1 Model for Four Waves of Observations.

$$\gamma_{12} = a_{21} \frac{\sqrt{V(\Theta_1)}}{\sqrt{a_{21}^2 V(\Theta_1) + V(\Theta_2)}}$$

$$\gamma_{23} = a_{32} \frac{\sqrt{a_{21}^2 V(\Theta_1) + V(\Theta_2)}}{\sqrt{a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2) + V(\Theta_3)}}$$

$$\gamma_{13} = a_{21} a_{32} \frac{\sqrt{V(\Theta_1)}}{\sqrt{a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2)) + V(\Theta_3)}}$$

$$\gamma_{14} = a_{21} a_{32} a_{43} \frac{\sqrt{V(\Theta_1)}}{\sqrt{a_{43}^2 (a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2)) + V(\Theta_3) + V(\Theta_4)}}$$

$$\gamma_{34} = a_{43} \frac{\sqrt{a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2)) + V(\Theta_3)}}{\sqrt{a_{43} (a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2)) + V(\Theta_3)) + V(\Theta_4)}}$$

$$\gamma_{24} = a_{32} a_{43} \frac{\sqrt{a_{21}^2 V(\Theta_1) + V(\Theta_2)}}{\sqrt{a_{43}^2 (a_{32}^2 (a_{21}^2 V(\Theta_1) + V(\Theta_2))) + V(\Theta_3)) + V(\Theta_4)}}$$

APPENDIX D

The applications of the Markov Chain and the Poisson Distribution models to various social processes.

ERIC

APPENDIX D

The applications of the Markov Chain and the Poisson Distribution models to various social processes.

The Markov Chain

The application of the Markov chain to social processes is a fairly recent phenomenon. Coleman in his book on mathematical sociological models presented various applications of the Markov chain.[1] Bartholomew reviewed the model's use in the study of social mobility, labor mobility, graded social systems, and so on.[2] Both of these works presented both the theory and the applications of the models.

Matras used the model to study differential fertility, occupational change, and the change in occupational distribution.[3] His article presented the model in extensive mathematical terms, and he reviewed briefly some of the measures of the variables before presenting a method of measuring their interrelations.

Anderson used the model to predict voter behavior.[4] He found two patterns of voting behavior; one was prior to the convention and the other was after it. Reanalysis of the same data revealed, however, that the processes

---

[1]J.S. Coleman, Introduction to Mathematical Sociology (New York: MacMillan-Free Press, 1964).

[2]D.J. Bartholomew, Stochastic Models for Social Processes (New York: John Wiley, 1967).

[3]J. Matras, "Differential Fertility, Intergenerational Occupational Mobility, and Change in the Occupational Distribution: Some Elementary Interrelationships," Population Studies 15 (1961), pp. 187-197.

[4]T.W. Anderson, "Probability Models for Analyzing Time Changes in Attitudes," in Paul F. Lazarsfeld (Ed.), Mathematical Thinking in the Social Sciences (Glencoe, Illinois: Free Press, 1954), pp. 17-66.

did not significantly differ from one another except for one category of response.[5]

Savage and Deutsch used the basic model to investigate import-export data.[6] The authors concluded that the model was sufficiently accurate. It is also of interest to note that the model was presented in terms of "transaction flows" where the model develops a matrix of expected data from assumptions of "complete indifference" among the actors (countries), and measures differences from this baseline. The method removes size effects and permits inferences about clustering among actors. The method used was found by Goodman to need modification. He presented the necessary modifications and suggested alternative and somewhat superior methods.

The model has also been applied to marketing behavior. Lipstein presented the model and modifications of the model as a method of relating consumer behavior in a marketplace under advertising stimulation.[8] Kotler reviewed seven models of consumer behavior and felt that

---

[5] L.A. Goodman, "Statistical Methods for Analyzing Processes of Change," American Journal of Sociology 68 (1962), pp. 57-78.

[6] I.R. Savage and K.W. Deutsch, "A Statistical Model of the Gross Analysis of Transaction Flows," Econometrika 28 (1960), pp. 551-572.

[7] L.A. Goodman, "Statistical Methods for the Preliminary Analysis of Transaction Flows," Econometrika 31 (1963), pp. 197-208, and L.A. Goodman, "A Short Computer Program for the Analysis of Transaction Flows," Behavioral Sciences 9 (1964), pp. 176-186.

[8] B. Lipstein, "A Mathematical Model of Consumer Behavior," Journal of Marketing Research 2 (1965), pp. 259-265.

a total behavior model was needed. The essential models were Markovian.[9] In his review of the application of the Markov model to conumber behavior Ehrenberg found what he felt were no successful applications of the model to the data. He felt that many of the assumptions of the model are not mirrored by reality.[10]

Other diverse applications have been made. Weiss used the model to study the duration and magnitude of wars, i.e., a two-dimensional Markov chain.[11] The model did generate empirical distributions of 315 wars. However, Horvath suggested that the simple Markov model was unnecessary since a stochastic model which required fewer assumptions could fit the data.[12] Horvath found that the Weibull distribution, based on the theory of extremes in a given sample, could fit not only wars but also strikes. In a somewhat novel use of the model, Brownsberger used the model in the dilemma of clinical versus statistical assessment of psychotherapy.[13] He

---

[9]P. Kotler, "Mathematical Models of Individual Buyer Behavior," Behavioral Science 17 (1968), pp. 274-287.

[10]A.S.C. Ehrenberg, "An Appraisal of Markov Brand-Switching Models," Journal of Marketing Research 2 (1965), pp. 347-363.

[11]H.K. Weiss, "Stochastic Models for the Duration and Magnitude of a 'Deadly Quarrel,'" Operations Research 11 (1963), pp. 101-121.

[12]W.J. Horvath, "A Statistical Model for the Duration of Wars and Strikes," Behavioral Science 17 (1968), pp. 18-28.

[13]C.N. Brownsberger, "Clinical Versus Statistical Assessment of Psychotherapy: a Mathematical Model of the Dilemma," Behavioral Science 10, 4 (1965), pp. 421-428.

used a five-year course of treatment simulation where pairs of patient groups were treated to treatment probabilities and to natural variation probabilities. Interestingly the effects of treatment against those of natural variation were insignificant. The method is suggested as a means to assess therapy. Sheps and Perrin used the Markov Renewal process (that is where the time intervals between changes in state are treated as random variables as was mentioned earlier in this paper) to analyze the interval between marriage and the first birth or between subsequent births.[14] It was suggested that the model offered many advantages in the study of fertility because of its flexibility. A study of migration by Morrison used the model and found that propensity to move decreases as a function of duration in residence.[15]

The Poisson Model

The Poisson model has also found application in social processes. Coleman presents some of these applications. In a preliminary study of free-forming group James found evidence for the fit of a Poisson model.[16]

---

[14] M.C. Sheps and E.B. Perrin, "The Distribution of Birth Intervals Under a Class of Stochastic Fertility Models," Population Studies 17, 3 (1964), pp. 321-331.

[15] P.A. Morrison, "Duration of Residence and Prospective Migration: the Evaluation of a Stochastic Model," Demography 4, 2 (1967), pp. 553-560.

[16] J. James, "A Preliminary Study of the Size Determinant in Small Group Interaction," American Sociological Review 16 (1951), pp. 474-477.

In further study James found that the negative binomial rather than the Poisson fit the data better.[17] Later Coleman and James suggested that the Truncated Poisson model fit the acquisition and loss of group members, and found the model fit the data.[18] White later found fault with the article because a wide range of models all predict the truncated Poisson equation.[19] Goodman found that both Coleman and James and White conclusions needed modification. He found that the truncated Poisson formula used would not describe equilibrium size distributions except under special conditions (the emphasis on the prior articles was on the stable state) and suggested another model, the Emigration-Immigration, provided better fit.[20]

The model has been applied in other fields. In his study of population growth and migration de Cani used the model.[21] He analyzed three models of increasing complexity

---

[17] J. James, "The Distribution of Free-Forming Small Group Size," American Sociological Review 18 (1953), pp. 569-570.

[18] J.S. Coleman and J. James, "The equilibrium Size Distribution of Free-Forming Groups," Sociometry 24 (1961), pp. 36-45.

[19] H.C. White, "Chance Models of Systems of Causal Groups," Sociometry 25 (1962), pp. 153-172.

[20] L.A. Goodman, "Mathematical Methods for the Study of Systems of Groups," American Journal of Sociology 70, 2 (1964), pp. 170-192.

[21] J.S. de Cani, "On the Construction of Stochastic Models of Population Growth and Migration," Journal of Regional Science 3, 2 (1961), pp. 1-13.

and suggested that the results of use of the models could be of value. In his study of brand choice Frank simulated consumer behavior (Monte Carlo approach) using the assumption of spurious contagion.[22] He found the model fit relatively poorly. Mellinger _et al_. found the model fit the accident repeatedness of children.[23] Land used data from Ferriss[24] of divorce rates for marriage cohorts and applied the model to them.[25] Land applied both the Cornell Mobility and the Mover-Stayer models to the data. The latter model fit the data with the most accuracy.

---

[22] R.E. Frank, "Brand Choice as a Probability Process," _Journal of Business_ 35 (1962), pp. 43-56.

[23] G.D. Mellinger _et al_., "A Mathematical Model with Applications to a Study of Accident Proneness Among Children," _Journal of American Statistical Association_ 60 (1965), pp. 1046-1059.

[24] A.L. Ferriss, "An Indicator of Marriage Dissolution by Marriage Cohort," _Social Forces_ 48 (1970), pp. 356-365.

[25] K.C. Land, "Some Exhaustible Poisson Process Models of Divorce by Marriage Cohort," A paper presented to the annual meeting of the American Sociological Association, Washington, D.C., August 1970.

BIBLIOGRAPHY

Adams, W. "Financial and Non-Financial Factors Affecting Post-High School Plans and Evaluations, 1939-1965." In E.D. Goldfield (Ed.), American Statistical Association Proceedings of the Social Statistics Section, 1969. Washington, D.C.: American Statistical Association, 1969. Pp. 99-124.

Alexander, C.N. and E.Q. Campbell. "Peer Influences on Adolescent Educational Aspirations and Attainments." American Sociological Review 29 (1964), pp. 568-575.

Anderson, T.W. "Probability Models for Analyzing Time Changes in Attitudes." In P.F. Lazarsfeld (Ed.), Mathematical Thinking in the Social Sciences. Glencoe, Illinois: Free Press, 1954. Pp. 17-66.

Anderson, T.W. and L.A. Goodman. "Statistical Inferences About Markov Chains." Annals of Mathematical Statistics 28 (1957), pp. 80-110.

Bartholomew, D.J. Stochastic Models for Social Processes. New York: John Wiley, 1967.

Bartlett, M.S. "The Frequency Goodness of Fit Test for Probability Chains." Proceedings of the Cambridge Philosophical Society 47 (1951), p. 86.

Bayer, A.E. "Marriage Plans and Educational Aspirations." American Journal of Sociology 75,2 (1969), pp. 239-244.

Birnbaum, A. "Statistical Methods for Poisson Processes and Exponential Populations." Journal of the American Statistical Association 49 (1954), pp. 254-266.

Blumen, I., M. Kogan, and P.McCarthy. The Industrial Mobility of Labor as a Probability Process. Ithaca, N.Y.: Cornell University Press, 1955.

Brownsberger, C.N. "Clinical Versus Statistical Assessment of Psychotherapy: a Mathematical Model of a Dilemma." Behavioral Science 10,4 (1965), pp. 421-428.

Coleman, J.S. Introduction to Mathematical Sociology. New York: MacMillan, 1964.

Coleman, J.S. and J. James. "The Equilibrium Size Distribution of Freely-Forming Groups." Sociometry 24 (1961), pp. 36-45.

de Cani, J.S. "On the Construction of Stochastic Models of Population Growth and Migration." Journal of Regional Science 3,2 (1961), pp. 1-13.

Duncan, O.D., A.O. Haller, and A. Portes. "Peer Influences on Aspirations: a Reinterpretation." American Journal of Sociology 74 (1968), pp. 119-137.

Eckland, B.K. "Academic Ability, Higher Education, and Occupational Mobility." American Sociological Review 30,5 (1965), pp. 735-746.

Ehrenberg, A.S.C. "An Appraisal of Markov Brand-Switching Models." Journal of Marketing Research 2 (1965), pp. 347-363.

Elder, G.H. "Achievement Orientation and Career Patterns of Rural Youth." Sociology of Education 37,1 (1963), pp. 30-58.

__________. "Achievement Motivation and Intelligence in Occupational Mobility: a Longitudinal Analysis." Sociometry 31,4 (1968), pp. 327-354.

__________. "Appearance and Education in Marriage Mobility." American Sociological Review 34,4 (1969), pp. 519-533.

Feller, W. An Introduction to Probability Theory and Its Application, Volume 1 (2nd Edition). New York: John Wiley, 1968.

Ferriss, A.L. "An Indicator of Marriage Dissolution by Marriage Cohort." Social Forces 48 (1970), pp. 356-365.

Frank, R.E. "Brand Choice as a Probability Process." Journal of Business 35 (1962), pp. 43-56.

Froomkin, J. and M. Pfeferman. "A Computer Model to Measure the Requirements for Student Aid in Higher Education." In E.D. Goldfield (Ed.), American Statistical Association Proceedings of the Social Statistics Section, 1969. Washington, D.C.: American Statistical Association, 1969. Pp. 125-137.

Goodman, L.A. "Statistical Methods for the Mover-Stayer Model." Journal of the American Statistical Association 56 (1961), pp. 841-868.

__________. "Statistical Methods for Analyzing Processes of Change." American Journal of Sociology 68 (1962), pp. 57-78.

__________. "Statistical Methods for the Preliminary Analysis of Transaction Flows." Econometrika 31 (1963), pp. 197-208.

__________. "A Short Computer Program for the Analysis of Transaction Flows." Behavioral Science 9 (1964), pp. 176-186.

__________. "Mathematical Methods for the Study of Systems of Groups." American Journal of Sociology 70,4 (1964), pp. 170-192.

__________. "The Analysis of Persistence in a Chain of Multiple Events." Biometrika 51 (1964), pp. 405-411.

Haller, A.O. and C.E. Butterworth. "Peer Influences on Levels of Occupational and Educational Aspirations." Social Forces 43 (1960), pp. 289-295.

Hauser, R.M. "Schools and the Stratification Process." American Journal of Sociology 74,6 (1969), pp. 587-611.

Heise, D.R. "Separating Reliability and Stability in Test-Retest Correlation." American Sociological Review 34,1 (1969), pp. 93-101.

__________. "Comment on 'The Estimation of Measurement Error in Panel Data.'" American Sociological Review 35,1 (1970), p. 117.

Herricott, R.E. "Some Social Determinants of Educational Aspiration." Harvard Educational Review 33,2 (1967), pp. 157-177.

Hodge, R.W. "Occupational Mobility as a Probability Process." Demography 3,1 (1966), pp. 19-34.

Hoel, P.G. "A Test for Markov Chains." Biometrika 41 (1954), pp. 430-433.

Horvath, W.J. "A Statistical Model for the Duration of Wars and Strikes." Behavioral Science 17,4 (1968), pp. 18-28.

James, J. "A Preliminary Study of the Size Determinant in Small Group Interaction." American Sociological Review 16 (1951), pp. 474-477.

________. "The Distribution of Free-Forming Small Group Size." American Sociological Review 18 (1953), pp. 569-570.

Kotler, P. "Mathematical Models of Individual Buyer Behavior." Behavioral Science 17,4 (1968), pp. 274-287.

Land, K.C. "Some Exhaustible Poisson Models of Divorce by Marriage Cohort." A Paper Presented at the Annual Meeting of the American Sociological Association, Washington, D.C., August, 1970.

Lipstein, B. "A Mathematical Model of Consumer Behavior." Journal of Marketing Research 2 (1965), pp. 259-265.

Matras, J. "Differential Fertility, Intergenerational Occupational Mobility, and Change in the Occupational Distribution: Some Elementary Interrelationships." Population Studies 15 (1961), pp. 187-197.

McDill, E.L. and J.S. Coleman. "High School Status, College Plans, and Interest in Academic Achievement: a Panel Analysis." American Sociological Review 28, 6 (1963), pp. 905-918.

McFarland, D.D. "Intergenerational Social Mobility as a Markov Process: Including a Time-Stationary Markovian Model That Explains Observed Declines in Mobility Rates." American Sociological Review 35,3 (1970), pp. 463-476.

McGinnis, R. "A Stochastic Model of Social Mobility." American Sociological Review 33,5 (1968), pp. 712-722.

Mellinger, G.D., et al. "A Mathematical Model with Applications to a Study of Accident Repeatedness Among Children." Journal of the American Statistical Association 60 (1965), pp. 1046-1059.

Morrison, P.A. "Duration of Residence and Prospective Migration: the Evaluation of a Stochastic Model." Demography 4,2 (1967), pp. 553-560.

Rehberg, R.A. and D.L. Westby. "Parental Encouragement, Occupation, Education and Family Size: Artifactual or Independent Determinants of Adolescent Educational Expectations?" Social Forces 45,3 (1967), pp. 362-374.

Rehberg, R.A., W.E. Schafer and J. Sinclair. "Toward a Temporal Sequence of Adolescent Achievement Variables." American Sociological Review 35,1 (1970), pp. 34-48.

Ryder, N.B. "The Cohort as a Concept in the Study of Social Change." American Sociological Review 30 (1965), pp. 843-861.

__________. "Cohort Analysis." In D.L. Sills (Ed.), International Encyclopedia of the Social Sciences, Volume 2. New York: MacMillan, 1968.

Savage, I.R. and K.W. Deutsch. "A Statistical Model of the Gross Analysis of Transaction Flows." Econometrika 28 (1960), pp. 551-572.

Schaie, K.W. "Age Changes and Age Differences." The Gerontologist 7,2 (1967), pp. 128-132.

Schoenfieldt, L.F. "Education After High School." Sociology of Education 41,4 (1968), pp. 351-369.

Sewell, W.H. "Community of Residence and College Plans." American Sociological Review 29 (1964), pp. 24-38.

Sewell, W.H. and J.M. Armer. "Neighborhood Context and College Plans." American Sociological Review 31,2 (1966), pp. 159-168.

Sewell, W.H. and V.P. Shah. "Socioeconomic Status, Intelligence, and the Attainment of Higher Education." Sociology of Education 40 (1967), pp. 1-23.

__________. "Social Class, Parental Encouragement, and Educational Aspirations." American Journal of Sociology 73 (1968), pp. 559-572.

__________. "Parents' Education and Children's Educational Aspirations and Achievements." American Sociological Review 33,2 (1968), pp. 191-209.

Sewell, W.H., A.O. Haller and A. Portes. "The Educational and Early Occupational Attainment Process." American Sociological Review 34,1 (1969), pp. 82-92.

Sewell, W.H., A.O. Haller and M.A. Straus. "Social Status and Educational and Occupational Aspirations." American Sociological Review 22,1 (1957), pp. 67-73.

Sheps, M.C. and E.B. Perrin. "The Distribution of Birth Intervals Under a Class of Stochastic Fertility Models." Population Studies 17,3 (1964), pp. 321-331.

Spilerman, S. "The Analysis of Mobility Processes by the Introduction of Independent Variables into a Markov Chain.' A Revised Draft of a Paper Presented at the Annual Meeting of the American Sociological Association, Washington, D.C., August, 1970.

Summers, G.F., et al. Before Industrialization: a Rural Social System Base Study. Champaign,Illinois: University of Illinois Agricultural Experiment Station Bulletin 736, 1969.

Weiss, H.K. "Stochastic Models for the Duration and Magnitude of a 'Deadly Quarrel.'" Operations Research 11 (1963), pp. 101-121.

Werts, C.E. "A Comparison of Male Versus Female College Attendance Probabilities." Sociology of Education 41 (1968), pp. 103-110.

White, H.C. "Chance Models of Systems of Casual Groups." Sociometry 25 (1962), pp. 153-172.

__________. "Stayers and Movers." American Journal of Sociology 76,2 (1970), pp. 307-324.

Wiley, D.E. and J.A. Wiley. "The Estimation of Measurement Error in Panel Data." American Sociological Review 35,1 (1970), pp. 112-117.

Williams, W.H. and C.L. Mallows. "Systematic Biases in Panel Surveys Due to Differential Nonresponse." Journal of the American Statistical Association 65, 331 (1970), pp. 1338-1349.

Youmans, E.G. "Factors in Educational Attainment." Rural Sociology 24,1 (1959), pp. 21-28.