DOCUMENT RESUME

ED 079 359 TM 002 958

AUTHOR	Jennrich, Robert I.
TITLE	Simplified Formulas for Standard Errors in Maximum Likelihood Factor Analysis.
INSTITUTION	Educational Testing Service, Princeton, N.J.
REPORT NO	ETS-RB-73-40
PUB DATE	May 73
NOTE	24p.
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Factor Analysis; *Matrices; *Orthogonal Rotation; Statistical Analysis; Technical Reports

ABSTRACT

Standard errors for maximum likelihood estimates of factor loadings are expressed in terms of the inverse of an augmented information matrix. This formulation arises naturally by viewing the problem as one in constrained maximum likelihood estimation. The constraints correspond to the form of rotation used. Results are given for canonical rotation and analytic rotations in the orthomax family. (Author)

RB-73-40

RESEARCH BULLETIN

SIMPLIFIED FORMULAS FOR STANDARD ERRORS IN MAXIMUM LIKELIHOOD FACTOR ANALYSIS

Robert I. Jennrich
University of California at Los Angeles
and
Educational Testing Service

This Bulletin is a draft for interoffice circulation. Corrections and suggestions for revision are solicited. The Bulletin should not be cited as a reference without the specific permission of the author. It is automatically superseded upon formal publication of the material.

Educational Testing Service
Princeton, New Jersey

May 1973

SIMPLIFIED FORMULAS FOR STANDARD ERRORS IN

MAXIMUM LIKELIHOOD FACTOR ANALYSIS

ABSTRACT

Standard errors for maximum likelihood estimates of factor loadings are expressed in terms of the inverse of an augmented information matrix. This formulation arises naturally by viewing the problem as one in constrained maximum likelihood estimation. The constraints correspond to the form of rotation used. Results are given for canonical rotation and analytic rotations in the orthomax family.

SIMPLIFIED FORMULAS FOR STANDARD ERRORS IN MAXIMUM LIKELIHOOD FACTOR ANALYSIS

1. INTRODUCTION

Formulas for the asymptotic standard errors of unrotated loading estimates which arise in factor analysis were given in an important paper by Lawley (1967). These were extended to analytically rotated loadings by Archer & Jennrich (1973) for the orthogonal case and by Jennrich (1973) for the oblique case. The results of the latter authors applied also to principal components analysis. Factor analysis is one of the most popular statistical methodologies. Literally hundreds of factor analysis programs produce thousands of estimates every day, but not a single standard error. In part this is due to the fact that the standard error formulas which are presently available are fairly complicated in form. It turns out, however, that considerably simpler formulas may be obtained in the maximum likelihood case by viewing the problem as one in constrained estimation. In a sense this should not be a surprise. One of the advantages of maximum likelihood estimation is summarized in the familiar formula

$$\operatorname{acov} \hat{\theta} = \mathcal{J}^{-1}(\theta) \qquad (1)$$

which states that the asymptotic covariance matrix for a maximum likelihood estimator of a parameter vector θ is simply the inverse of the population information matrix for that parameter vector. This result holds in a slightly altered form when θ is assumed to satisfy a set of functional constraints. The required alteration will be discussed in Section 2.

In maximum likelihood factor analysis (see, e.g., Anderson & Rubin, 1956 or Harman, 1967) estimation is based on a sample of size n from a multivariate normal population of score vectors. In the case of p scores and k orthogonal factors the covariance matrix of this population has the form

$$\Sigma = \Lambda\Lambda' + \psi \qquad (2)$$

where Λ is a p by k matrix of (natural) factor loadings λ_{ir} and ψ is a p by p diagonal matrix of unique variances ψ_i. Since Σ is unchanged when Λ is replaced by ΛT for any orthogonal T, Λ is not determined by Σ and hence cannot be consistently estimated. This indeterminacy is eliminated by employing a variety of rotation criteria. These may be viewed as constraints so that maximum likelihood factor analysis is in fact constrained maximum likelihood estimation.

While the decomposition of Σ given in (2) is the simplest, the decomposition most frequently encountered in practice involves standardized loadings. The standardized loadings are given by

$$\alpha_{ir} = \lambda_{jr}/\sigma_i \quad , \qquad 1 \leq i \leq p,\ 1 \leq r \leq k \qquad (3)$$

where the λ_{jr} are the natural loadings from (2) and the $\sigma_i = (\sigma_{ii})^{\frac{1}{2}}$ are the score standard deviations. In terms of the standardized loadings, the population correlation matrix P has a decomposition

$$P = AA' + \Gamma \qquad (4)$$

which is of the same form as that given in (2) for the population covariance matrix. Here A is a p by k matrix of standardized loadings α_{ir} and Γ is a p by p diagonal matrix of standardized unique variances. Because the diagonal of P is the p by p identity matrix I, it follows easily from (4) that

$$P = I + \text{ndg } AA' \tag{5}$$

where ndg AA' denotes the nondiagonal part of AA'. If X is the p by p diagonal matrix with diagonal elements σ_i, then

$$\Sigma = XPX \tag{6}$$

and using (5),

$$\Sigma = X(I + \text{ndg } AA')X \quad . \tag{7}$$

This gives a parameterization of Σ in terms of standardized loadings and score standard deviations. It will be used in Section 6 to find standard errors for standardized loading estimates.

2. STANDARD ERRORS FOR CONSTRAINED MAXIMUM LIKELIHOOD ESTIMATORS

As observed earlier, rotation criteria impose constraints on factor loadings. Here we consider a general result on the distribution of constrained maximum likelihood estimators to be used in the following sections. Let $\hat{\theta}$ be a constrained maximum likelihood estimator of a parameter vector $\theta = (\theta_1, \ldots, \theta_q)$ which is assumed to satisfy constraints

$$g_j(\Theta) = 0 \quad ; \qquad j = 1,\ldots,r \quad . \tag{8}$$

Let $\mathcal{J}(\Theta)$ be the population information matrix and let $dg/d\Theta$ denote the q by r matrix of partial derivatives $\partial g_j/\partial\Theta_i$. If

$$\begin{pmatrix} \mathcal{J}(\Theta) & \frac{dg}{d\Theta} \\ \frac{dg}{d\Theta}^T & 0 \end{pmatrix}^{-1} = \begin{pmatrix} \mathcal{J}^{\sim}(\Theta) & * \\ * & * \end{pmatrix} , \tag{9}$$

i.e., if $\mathcal{J}^{\sim}(\Theta)$ is the q by q matrix in the upper left-hand corner of the inverted augmented information matrix on the left, then

$$\text{acov } \hat{\Theta} = \mathcal{J}^{\sim}(\Theta) \quad . \tag{10}$$

This result may be found in Silvey (1971, p. 81). Sufficient regularity conditions are that $\mathcal{J}(\Theta)$ and $dg/d\Theta$ exist and are continuous in a neighborhood of the population value of Θ , that the indicated inverse exists and that $\hat{\Theta}$ is consistent. As expressed in (10) the result here is similar in form to that in the unconstrained case. It is easy to show that $\mathcal{J}(\Theta)$ is a pseudo-inverse of $\mathcal{J}^{\sim}(\Theta)$. In general it will not be a Moore-Penrose inverse so that in general $\mathcal{J}^{\sim}(\Theta)$ is not a pseudo-inverse of $\mathcal{J}(\Theta)$. The motivation for the notation is that $\mathcal{J}^{\sim}(\Theta)$ is obtained from the inversion of an augmented information matrix.

The information matrix $\mathcal{J}(\Theta)$ may, and in our applications will, be singular. The constraints, however, must be sufficient to identify the parameter and in particular sufficient to make the augmented information matrix invertible.

3. THE INFORMATION MATRIX FOR A NORMAL POPULATION

The multivariate normal population density for a score vector z is

$$f(z) = (2\pi)^{-p/2} |\Sigma|^{-\frac{1}{2}} \exp(-\tfrac{1}{2}(z - \mu)'\Sigma^{-1}(z - \mu)) \tag{11}$$

where $|\Sigma|$ denotes the determinant of Σ and μ denotes the mean of z. Since we are interested in the factor analytic structure of Σ and since the distribution of the maximum likelihood estimate of this structure does not depend on μ we may assume without loss of generality that $\mu = 0$. With this assumption the population information for any pair of parameters α and β is

$$\mathcal{J}(\alpha,\beta) = E\left(\frac{\partial}{\partial\alpha}\log f\right)\left(\frac{\partial}{\partial\beta}\log f\right) \tag{12}$$

$$= \tfrac{1}{2}\,\mathrm{tr}\left(\Sigma^{-1}\frac{\partial\Sigma}{\partial\alpha}\Sigma^{-1}\frac{\partial\Sigma}{\partial\beta}\right)$$

(see, e.g., Jennrich, 1970). Here $\frac{\partial\Sigma}{\partial\alpha}$ denotes the p by p matrix of partial derivatives $\partial\sigma_{ij}/\partial\alpha$ of the components σ_{ij} of Σ with respect to an arbitrary parameter α.

4. NATURAL LOADINGS WITH CANONICAL ROTATION

This case is considered first because it is the simplest and may most readily be compared to previously published results. Canonical rotation means that the factors are orthogonal and the loadings satisfy the constraint:

$$\Lambda'\psi^{-1}\Lambda \text{ is diagonal} \quad . \tag{13}$$

As its name implies this is the rotation which arises naturally in canonical factor analysis (Rao, 1955).

Differentiating Σ with respect to the natural loadings λ_{ir} and unique variances ψ_i gives

$$\frac{\partial\Sigma}{\partial\lambda_{ir}} = \frac{\partial\Lambda}{\partial\lambda_{ir}}\Lambda' + \Lambda\frac{\partial\Lambda'}{\partial\lambda_{ir}} = J_{ir}\Lambda' + \Lambda J'_{ir}$$

$$\frac{\partial\Sigma}{\partial\psi_i} = K_{ii}$$

where J_{ir} is a p by k unit matrix with a one in row i and column r and zeros elsewhere. Similarly K_{ii} is a p by p unit matrix with a one in the i -th diagonal position and zeros elsewhere. Inserting these derivatives in (12) and simplifying one finds that the information matrix relative to the parameters λ_{ir} and ψ_i is given by the pleasingly simple formulas:

$$\begin{aligned} \mathcal{J}(\lambda_{ir},\lambda_{js}) &= \sigma^{ij}(\Lambda'\Sigma^{-1}\Lambda)_{rs} + (\Sigma^{-1}\Lambda)_{is}\,(\Sigma^{-1}\Lambda)_{jr} \\ \mathcal{J}(\lambda_{ir},\psi_j) &= \sigma^{ij}(\Sigma^{-1}\Lambda)_{jr} \qquad\qquad (14) \\ \mathcal{J}(\psi_i,\psi_j) &= \tfrac{1}{2}(\sigma^{ij})^2 \end{aligned}$$

where $1 \leq i,j \leq p$ and $1 \leq r,s \leq k$. Here $(\ \cdot\ )_{rs}$ denotes the element in row r and column s of the matrix inside the parentheses and σ^{ij} denotes $(\Sigma^{-1})_{ij}$.

Using (13) the constraint functions associated with canonical rotation are:

$$g_{uv}(\Lambda,\psi) = (\Lambda'\psi^{-1}\Lambda)_{uv} \quad , \quad 1 \leq u < v \leq k \quad . \qquad (15)$$

These have derivatives:

$$\frac{\partial g_{uv}}{\partial \lambda_{ir}} = (\delta_{ru}\lambda_{iv} + \delta_{rv}\lambda_{iu})\psi_i^{-1}$$

$$\frac{\partial g_{uv}}{\partial \psi_i} = -\lambda_{iu}\lambda_{iv}\psi_i^{-2} \tag{16}$$

for $1 \leq i \leq p$, $1 \leq r \leq k$, and $1 \leq u < v \leq k$. Here δ_{ru} denotes the Kronecker delta.

From the general result (10) the asymptotic covariance matrix for the maximum likelihood estimators of the λ_{ir} and ψ_j constrained by canonical rotation is the square matrix of order $p(k + 1)$ in the upper left-hand corner of the inverse of the augmented information matrix:

$$\begin{pmatrix} \mathcal{J}(\Lambda,\Lambda) & \mathcal{J}(\Lambda,\psi) & \frac{\partial g}{\partial \Lambda} \\ \mathcal{J}(\psi,\Lambda) & \mathcal{J}(\psi,\psi) & \frac{\partial g}{\partial \psi} \\ \frac{\partial g^T}{\partial \Lambda} & \frac{\partial g^T}{\partial \psi} & 0 \end{pmatrix} . \tag{17}$$

To specify this matrix uniquely one must specify some order for the parameters and constraints. For example, he may use

$$\lambda_{ir} \quad , \qquad 1 \leq i < p \quad , \quad 1 \leq r \leq k \tag{18}$$

ordered lexicographically on i and r followed by

$$\psi_j \quad , \quad 1 \leq j \leq p \tag{19}$$

in natural order and finally by

$$g_{uv} \quad , \qquad 1 \leq u < v \leq k \tag{20}$$

in lexicographic order on u and v .

Table 1 gives maximum likelihood estimates for a matrix of canonically rotated factor loadings obtained by Lawley & Maxwell (1971, p. 43).

Insert Table 1 about here

Table 2 gives the corresponding asymptotic standard errors computed by Lawley & Maxwell (1971, p. 63) assuming a sample size $n = 211$.

Insert Table 2 about here

Table 3 contains the same standard errors using the formulas derived here.

Insert Table 3 about here

Differences between the results in Tables 2 and 3 may be traced to a slight problem in Lawley's formulas (Jennrich & Thayer, 1973). In terms of the presentation by Lawley & Maxwell (1971) this may be corrected by inserting a θ_r immediately in front of the summation sign in equation (5.27). When this is done the values obtained agree to within one digit in the last decimal place with the results presented in Table 3.

5. NATURAL LOADINGS WITH ANALYTIC ROTATION

In maximum likelihood factor analysis canonically rotated loadings are commonly referred to as unrotated loadings because these are the initial loadings produced by most estimation algorithms. Usually one is interested in other rotations. In the orthogonal case these include predominantly quartimax, varimax, and equamax all of which are members of the orthomax family (Harman, 1960, p. 334). As in canonical rotation, the loadings obtained from analytic rotation satisfy constraints which are associated with the form of rotation used. Archer & Jennrich (1973) have given a general method to generate constraints from arbitrary orthogonal rotation criteria. For the quartimax family they give constraint functions g_{uv} whose derivatives with respect to the loadings λ_{ir} are given by:

$$\frac{\partial g_{uv}}{\partial \lambda_{ir}} = \delta_{ur} a_{iuv} - \delta_{vr} a_{ivu} \tag{21}$$

for $1 \leq i \leq p$, $1 \leq r \leq k$, and $1 \leq u < v \leq k$ where

$$a_{iuv} = \lambda_{iv}^3 - 3\lambda_{iu}^2 \lambda_{iv} - \frac{\gamma}{p} [\lambda_{iv}((\Lambda'\Lambda)_{vv} - (\Lambda'\Lambda)_{uu}) - 2\lambda_{iu}(\Lambda'\Lambda)_{uv}] \tag{22}$$

for $1 \leq i \leq p$ and $1 \leq u,v \leq k$. Here $\gamma = 0,1,k/2$ corresponds to quartimax, varimax, and equamax rotation respectively. In the case of quartimax rotation and in fact in the case of analytic rotation generally, the constraints do not involve the unique variances ψ_i. Thus

$$\frac{\partial g_{uv}}{\partial \psi_i} = 0 \quad , \qquad 1 \le i \le p \quad , \quad 1 \le u < v \le m \quad . \tag{23}$$

Since only the constraint functions have been changed, the formulas for the asymptotic standard errors in the case of natural ' ' with orthomax rotation are precisely the same as those in the previous section except that the constraint derivatives are now defined by equations (22) and (23) instead of (16). The augmented information matrix here has the form:

$$\begin{pmatrix} \mathcal{J}(\Lambda,\Lambda) & \mathcal{J}(\Lambda,\psi) & \frac{\partial g}{\partial \Lambda} \\ \mathcal{J}(\psi,\Lambda) & \mathcal{J}(\psi,\psi) & 0 \\ \frac{\partial g}{\partial \Lambda}^T & 0 & 0 \end{pmatrix} . \tag{24}$$

Table 4 gives a varimax rotation of the loadings given in Table 1 and Table 5 contains the corresponding standard errors obtained by using

---

Insert Tables 4 and 5 about here

---

the formulas of this section. These standard errors were also computed using Lawley's formulas modified as observed in the previous section together with the results of Archer & Jennrich (1973). The values obtained agreed exactly (when rounded to three decimal places) with those presented in Table 5.

6. STANDARDIZED LOADINGS WITH ANALYTIC ROTATION

As observed earlier this is probably the most important case. We are interested here in parameterizing Σ in terms of the standardized loadings α_{ir} and the score standard deviations σ_j as displayed in (7). Since the maximum likelihood estimates of the α_{ir} are invariant under changes of scale in the score vector population, we may assume without loss of generality that each $\sigma_i = 1$ for the purpose of computing standard errors for the standardized loading estimates. Using (12) and being careful to employ this assumption only after differentiation, the population information matrix is found to be given by:

$$\begin{aligned}
\mathcal{J}(\alpha_{ir},\alpha_{js}) &= \rho^{ij}(A'P^{-1}A)_{rs} + (P^{-1}A)_{is}(P^{-1}A)_{jr} \\
&\quad - 2\rho^{ij}\alpha_{ir}(P^{-1}A)_{is} - 2\rho^{ij}\alpha_{js}(P^{-1}A)_{jr} \\
&\quad + 2(\rho^{ij})^2\alpha_{ir}\alpha_{js} \\
\mathcal{J}(\alpha_{ir},\sigma_j) &= \rho^{ij}\alpha_{jr} + \delta_{ij}(P^{-1}A)_{jr} - 2\delta_{ij}\rho^{ii}\alpha_{ir} \\
\mathcal{J}(\sigma_i,\sigma_j) &= \delta_{ij} + \rho_{ij}\rho^{ij}
\end{aligned} \tag{25}$$

where $1 \leq i,j \leq p$, $1 \leq r,s \leq k$, and ρ_{ij} and ρ^{ij} denote components of the matrices P and P^{-1} respectively. These formulas are a little but not a great deal more complicated than the corresponding formulas (14) in Section 4.

In the case of orthomax rotation, the constraints are those of the previous section applied to A instead of Λ. It follows from (10) that the asymptotic covariance matrix for the maximum likelihood estimates of the α_{ir} and c_j (assuming the latter have true value one) is the square matrix of order $p(k+1)$ in the upper left-hand corner of the inverse of the augmented information matrix:

$$\begin{pmatrix} \mathcal{J}(A,A) & \mathcal{J}(A,X) & \frac{\partial g}{\partial A} \\ \mathcal{J}(X,A) & \mathcal{J}(X,X) & 0 \\ \frac{\partial g}{\partial A}^T & 0 & 0 \end{pmatrix} . \qquad (26)$$

Table 6 contains standard errors for the varimax rotated loadings

---

Insert Table 6 about here

---

given in Table 4 using the results of this section to correct for standardization. We note in passing that when corrected for standardization every loading except one has a smaller asymptotic standard error.

As in the previous section, Lawley's modified formulas together with the results of Archer & Jennrich were used to check the standard errors in Table 6. The results agreed to within one digit in the last decimal place presented.

7. DISCUSSION

We have seen that relatively simple formulas result when asymptotic standard errors for analytically rotated factor loadings are obtained by inverting an augmented information matrix. From a practical point of view further simplification results from the fact that the formulas may be easily implemented as an independent computer module with simple input. For example, in the case of standardized loadings with orthomax rotation the required input consists of the rotated standardized loadings matrix A , the orthomax parameter γ , and the sample size n . Thus it is easy to modify an arbitrary maximum likelihood factor analysis program to produce asymptotic standard errors.

As always, however, there are trade-offs. The information matrix method is not as computationally efficient as that based on the formulas of Lawley. It requires the inversion of a large matrix whose order is equal to the number of parameters plus the number of constraints. In the example there were 27 loadings, 9 unique variances (or score standard deviations), plus 3 constraints making a 39 by 39 matrix. Such a matrix, while large, is about the same size as that which is inverted in a linear regression problem with the same number of parameters so the cost per standard error is no worse than in linear regression. This is perhaps only minor comfort since factor analysis typically involves considerably more parameters than regression analysis. It should be observed that within limits matrix inversion is not a terribly expensive operation. A medium speed computer such as the IBM 360/65 will invert the 39 by 39 matrix in our example in well under a second. One might also worry about precision problems resulting from the need to invert a large matrix. We know very little about the precision of this method compared to that based on Lawley's results. We observe only that numerically accurate inversion is not difficult simply because a matrix is large (consider the identity matrix), nor is a method based on matrix inversion necessarily less accurate than one which involves primarily matrix multiplication. Fortunately it is not difficult to monitor the precision of a matrix inversion. While we have reason to believe the results in our examples are accurate to about 5 significant digits, the whole area of standard errors in factor analysis seems to be developing too rapidly to invest a great deal of effort in problems of numerical precision at this time.

The results derived here apply to maximum likelihood factor analysis. Unlike the results of Archer & Jennrich (1973) they are not easily adapted to other methods such as principal components analysis. The results here are also limited in that the oblique case has not been included. This is because in the oblique case with standardized loadings the information matrix becomes too messy to be included in a discourse whose title asserts simplification.

Perhaps the most important use of the results derived here is that of verifying results derived elsewhere. They have already proved useful in uncovering a problem in Lawley's results and verifying those of Archer & Jennrich.

The author is indebted to Allen Yates and Thomas Stroud for their detailed review and suggestions for improvement of this manuscript and to Karl Jöreskog for providing a much needed reference.

REFERENCES

ANDERSON, T. W. & RUBIN, H. (1956). Statistical inference in factor analysis. Proc. Third Berkeley Symp. Math. Statist. Probability 5, 111-150.

ARCHER, C. C. & JENNRICH, R. I. (1973). Standard errors for rotated factor loadings. Research Bulletin 73-17. Princeton, N.J.: Educational Testing Service.

HARMAN, H. H. (1960, 1967). Modern factor analysis (1st and 2nd eds.). Chicago: University of Chicago Press.

JENNRICH, R. I. (1970). An asymptotic X^2 test for the equality of two correlation matrices. J. Am. statist. Ass. 65, 904-912.

JENNRICH, R. I. (1973). Standard errors for obliquely rotated factor loadings. Research Bulletin 73-28. Princeton, N.J.: Educational Testing Service.

JENNRICH, R. I. & THAYER, D. T. (1973). A note on Lawley's formulas for standard errors in maximum likelihood factor analysis. Research Bulletin 73-31. Princeton, N.J.: Educational Testing Service.

LAWLEY, D. N. (1967). Some new results in maximum likelihood factor analysis. Proc. Roy. Soc. Edinburgh A67, 256-264.

LAWLEY, D. N. & MAXWELL, A. E. (1971). Factor analysis as a statistical method. New York: American Elsevier.

RAO, C. R. (1955). Estimation and tests of significance in factor analysis. Psychometrika 20, 93-111.

SILVEY, D. S. (1971). Statistical inference. Baltimore: Penguin Books.

Table 1.--Canonically Rotated Maximum Likelihood Loadings

	Factor			
Variate	I	II	III	Communality
1	.664	.321	.074	.550
2	.689	.247	-.193	.573
3	.493	.302	-.222	.383
4	.837	-.292	-.035	.788
5	.705	-.315	-.153	.619
6	.819	-.377	.105	.823
7	.661	.396	-.078	.600
8	.458	.296	.491	.538
9	.766	.427	-.012	.769

Table 2.--Standard Errors for the Loadings in Table 1 Using Lawley's (1967) Formulas

Variate	Factor I	Factor II	Factor III
1	.066	.058	.076
2	.064	.061	.068
3	.070	.071	.083
4	.060	.046	.045
5	.065	.057	.072
6	.064	.046	.037
7	.068	.057	.066
8	.073	.093	.142
9	.066	.047	.055

Table 3.--Standard Errors for the Loadings in Table 1 Using the Inverse of the Augmented Information Matrix

	Factor		
Variate	I	II	III
1	.068	.066	.091
2	.066	.076	.069
3	.072	.081	.084
4	.061	.068	.054
5	.067	.073	.081
6	.069	.049	.036
7	.071	.068	.080
8	.076	.103	.124
9	.070	.058	.078

Table 4.--Varimax Rotation of the Loadings in Table 1

	Factor			
Variate	I	II	III	Communality
1	.605	.306	.300	.549
2	.656	.376	.036	.573
3	.589	.190	-.022	.384
4	.298	.832	.081	.787
5	.243	.746	-.065	.620
6	.178	.870	.187	.824
7	.709	.258	.175	.600
8	.323	.158	.640	.538
9	.772	.319	.269	.769

Table 5.--Standard Errors for the Loadings in Table 4

Ignoring Standardization

	Factor		
Variate	I	II	III
1	.085	.059	.113
2	.067	.060	.128
3	.071	.063	.130
4	.049	.060	.047
5	.052	.063	.093
6	.048	.061	.048
7	.072	.055	.120
8	.132	.058	.200
9	.081	.052	.121

Table 6.--Standard Errors for the Loadings in Table 4

Corrected for Standardization

	Factor		
Variate	I	II	III
1	.066	.052	.103
2	.048	.052	.120
3	.056	.060	.124
4	.041	.029	.070
5	.049	.037	.092
6	.038	.032	.044
7	.048	.050	.109
8	.124	.055	.190
9	.055	.044	.109