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ABSTRACT

Discussed in this booklet are curricular experiments; methods for teaching concepts, principles, and skills; guided discovery strategies; programed text materials; computer-based instruction; and culturally disadvantaged students. Pertinent research results are included in the discussion of each topic. (DT)

WHAT RESEARCH SAYS TO THE TEACHER

9

Teaching Secondary School Mathematics

Kenneth B. Henderson

Association of Classroom Teachers of the National Education Association

ers with concise, valid, and up-to-date summaries of educational research findings and their implications for teaching.

Each pamphlet in the series is designed to serve two prime functions: to suggest principles and practical procedures that may be applied directly by the classroom teacher and to provide a springboard for further study and use of research findings.

To serve the first purpose, authors of booklets in the series select from each field those research findings that promise to be of most help to the classroom teacher. However, research has not yet provided scientifically valid findings on many aspects of teaching. In such cases, the best that can be offered is expert opinion.

It is impossible, of course, to provide a complete summary of research in any field in 32 pages. To help teachers further explore research findings, selected references are listed at the end of each booklet in the series.

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In some cases a sequence of prescriptions, which in effect amounts to an algorithm, can be justified by a set of generalizations, and the teacher presents this algorithm in prescriptive language. Examples are the algorithm for reducing a common fraction to lower terms and that for synthetic division.

The implication for behavior of a prescription or algorithm is explicit; this is the pedagogical advantage of a prescription. In contrast, a slow learner may not know what to do after he has been taught the correlative generalization. Hence prescriptions are useful in teaching skills. However, excessive use of prescriptions, while producing students who are good manipulators, will not produce students who have much depth of understanding.

What seems desirable is a judicious blend of generalizations and prescriptions. In the present stage of the art of teaching, the proper blend is a matter of judgment, rather than a formula developed from research.

Moves in Teaching Principles

As in the case of teaching concepts, we can conceive of moves and strategies in teaching principles. When tapes of teachers

Mathematics

CONTENTS

2.4 Giving instances, provided the principle is a generalization, *or*

2.5 Demonstrating application provided the principle is a prescription.

3. The principle is justified, that is, the students are convinced that the principle, if a generalization, is true or, if a prescription, will result in the correct answer. If the principle is a generalization, the teacher—

3.1 Exhibits instances, each of which the students recognize as a true statement.

3.2 Challenges students to find a counterinstance, with their inability to find such taken as evidence that the generalization is true.

3.3 Actually proves the generalization if it is a theorem.

3.4 Tells how the generalization, if it is a theorem, can be proved but does not prove it. *Example:* "This theorem can be proved by induction." *or*

3.5 Points out that the generalization is an axiom provided, of course, it is taken as an axiom in the exposition.

If the principle is a prescription, the teacher—

3.6 Demonstrates that following the prescription attains the

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TEACHING SECONDARY SCHOOL MATHEMATICS

The findings of research on the teaching of mathematics are often contradictory. Many experiments are inconclusive; generalizations are often derived from an inadequate sampling or simply are not warranted. In view of the foregoing, many teachers conclude that the research findings have no implications for practice. Such teachers demand a sizable body of evidence before being willing to change their methods. They find much of the research in the teaching of mathematics not of such a nature as to cause them to question their existing beliefs.

Other teachers do not find the lack of conclusiveness of educational research so disconcerting. The very lack of conclusiveness places few constraints upon the teacher and leaves him free to test hypotheses for himself, to find out what works for him. The interpretation of the results of research and their implications for practice, therefore, depend to a large extent on the attitude of the person considering the findings.

CURRICULAR EXPERIMENTS

Probably teachers are more aware of revisions in the mathematics curriculum than of changes in any other discipline. The same can be said of laymen, whose children cope with the new and unfamiliar subject matter. On occasion, these curricular development projects are spoken of as experiments. This designation is appropriate provided one realizes both the distinctive and restricted nature of such experiments. They amount to the exercise of judgment in the selection, organization, and exposition of subject matter. The course resulting from this selection, organization, and exposition is tried out in various schools and classrooms, and the feedback afforded is used to revise and improve the course. The cycle is repeated until the authors or other decision makers are satisfied with the product.

One can cast curricular experiments into the form of sufficient or necessary conditions. By far the most common experiment demonstrates that if a certain topic (or topics) or a certain course is taught in a certain grade, e.g., seventh, ninth, or twelfth, certain effects are obtained. Such studies can be regarded as feasibility studies. Usually the only effects studied are those on students—how readily the students learn the items of subject matter, how interested they are in the topics, or how they fare in subsequent mathematics courses. Either directly or indirectly, the teachers' evaluation of the experimental subject matter is obtained. Where positive results are obtained, e.g., students do understand the subject matter, these experiments can be regarded as existence proofs; there exists such and such a body of subject matter which is learnable by students in such and such a grade.

There are some general conclusions which appear tenable from this kind of curricular research. Mathematics which emphasizes the properties of certain mathematical structures, e.g., the integers, the rationals, the reals, the complex numbers, convex sets under various transformations, and vectors, can be comprehended by secondary school students. The students can handle more precise language and greater rigor than was assumed in the past. Topics previously placed in more advanced courses can be learned in earlier courses.

The second kind of curricular experiment attempts to demonstrate a necessary condition. Such an experiment demonstrates that under certain conditions not teaching the experimental subject matter will have certain detrimental effects, such as difficulty in a subsequent course. Not much of this kind of curricular experimentation has been done on a scale which would facilitate decisions by teachers about what content to select from mathematics courses. Yet this kind of research is as necessay as the former kind. The former facilitates decisions about what can be included; the latter facilitates decisions about what can be excluded. And with the vast amount of learnable mathematics available for inclusion in the curriculum, help in deciding what to eliminate or not eliminate is of considerable value to the classroom teacher.

Illustrative Curricular Experiments

Most curricular experiments have accepted the existing organization of the curriculum, *viz.*, algebra in the ninth grade, geometry in the tenth, a second year of algebra in the eleventh grade (In some schools in the eastern part of the United States, geometry and the second algebra course are interchanged.), and a semester of trigonometry and a semester of college algebra or newer integrated courses including analytic geometry and elementary functions in the twelfth grade.

The School Mathematics Study Group (SMSG) has largely accepted the conventional pattern and has sought to prepare upgraded texts for the existing courses. SMSG has developed alternatives, however. A second geometry course, which is an integration of synthetic and analytic geometry and also of plane and solid geometry, is available. For the twelfth grade a semester course on elementary functions and one on matrix algebra are provided.

The University of Illinois Committee on School Mathematics (UICSM) has departed somewhat from the conventional pattern. The scope and sequence of the units in algebra and the nature of the geometry course make these courses adaptable to the pattern of two consecutive years of algebra followed by a year of geometry. UICSM has developed a vector geometry course which is an alternative to the other geometry course. A text on transformational geometry presented from an inductive and intuitive point of view enables eighth-grade students to attain knowledge of some of the properties of geometric figures that are invariant under translations, rotations, reflections, and glide reflections. A course designed for culturally disadvantaged students based on the concept of fractions as stretchers or shrinkers appears successful for teaching such students mathematics they somehow have not acquired before.

The second kind of curricular experimentation does not necessarily accept the prevailing curricular pattern, i.e., division of the subject matter of mathematics into arithmetic, algebra, geometry, and analysis, and placement of these subjects in grades 7 through 12 in the sequence stated. Rather, the approach is to integrate some of each branch of mathematics into each year's course. A student who leaves this sequence of courses at some point then has some knowledge about each branch. The integration of the branches is attained by dealing with important general concepts, e.g., set, mapping, relation, and function. An experiment which seeks to assess the efficacy of this approach is the Secondary School Mathematics Curriculum Improvement Study (SSMCIS) centered at Teachers College, Columbia University, and directed by Howard F. Fehr.

Conclusions of Curricular Experiments

What can be concluded from the recent, extensive curricular research? Students learn the mathematics that is taught in the new courses; some learn more than others. The graduates of schools using the new courses do as well in college as graduates from older courses. They do as well on standardized tests, even though some of the standardized tests are not valid instruments for measuring achievement in the newer courses. Not unexpectedly, some students enjoy the upgraded courses; some do not.

Some of the early experimentation emphasized structure and deemphasized practice in transforming expressions. This was done in the belief that as understanding of the properties of relations and operations over a set of numbers, e.g., the rationals or reals, grows, skill in manipulations and the use of alogrithms will grow concomitantly. The findings indicate that this belief is not tenable. If students are not provided practice—*drill*, if one cares to use the term—they do not acquire speed and accuracy in the manipulation and use of algorithms.

A tenable conclusion is that some ambiguity and equivocation in the use of certain terms is not particularly detrimental to learning. For example, in the initial exuberance for meticulousness in distinguishing between numbers, numerals, and copies of numerals, some theoreticians contended that equivocation would confuse the students. The facts do not support this contention.

Finally, we need to bear in mind that the experiments which demonstrate that students *can* learn the mathematics taught do not demonstrate that this mathematics *should* be taught. That certain subject matter in mathematics is learnable is a necessary condition for its selection, but it is not a sufficient condition. There are other grounds and values which must influence the selection. These are either explicit or implicit in the minds of those—principally mathematicians—who make the selection. Classroom teachers, and laymen, too, will do well to insist that these grounds and values be subjected to continual appraisal and revision where necessary.

TEACHING CONCEPTS

We now pass to a consideration of the findings of research on an activity in which mathematics teachers spend most of their instructional time—the teaching of concepts.

It is difficult to distinguish between teaching a concept and teaching the meaning of a term or expression which designates the concept. A teacher does about the same thing when he teaches the concept of an ellipse, for example, as when he teaches what the term *ellipse* means. Hence it is not profitable pedagogically to distinguish between these two activities. A concept will be regarded as the meaning of a term.

General Methods

In general, there are three ways to teach a concept. Suppose a teacher wants to teach the concept of an ellipse. He can state the characteristics or properties of an ellipse. Or he can give examples and nonexamples of ellipses by drawing or showing representations or nonrepresentations. In both cases he is using the object language, that is, he is using the term *ellipse* to talk about ellipses—what their properties are or which particular objects are or are not ellipses. Finally, he can talk about the term *ellipse* itself; he can define the term. In this case he is using the metalanguage. To refer to these three ways of teaching a concept, we shall use the terms *characterization, exemplification,* and *definition* respectively. When a teacher is teaching a concept by stating the characteristics or properties an object must have to be referred to by the term designating the concept, he is teaching the concept by characterization. When he gives examples or nonexamples but does not tell why they are examples or nonexamples, he is teaching the concept by exemplification. When he stipulates how the term designating the concept is to be used, he is teaching the concept by definition.

An example may serve to clarify these three ways of teaching a concept. Consider the concept of a rational number. If a teacher says that a rational number is the quotient of two integers a/b where $b \neq 0$, he is characterizing a rational number. So also would he if he said, "All the positive numbers—positive integers and fractions—and their opposites and zero make up the set of rational numbers." Or if he said, "Every terminating decimal is a rational number." The rational numbers can be characterized in different ways and with varying preciseness.

If the teacher said, "$+2$, -2, $+\frac{3}{4}$, $-\frac{3}{4}$, $+\frac{1}{3}$, $-\frac{1}{3}$, 0, $+22.8$, $+16.95$, and -0.013 are all rational numbers," he would be employing exemplification. The particular examples he chooses may be immaterial so long as he samples judiciously from among the significant subsets.

If he gives an explicit rule for using the term *rational number*, for example, "If a and b $\neq$ 0 are integers, the quotient a/b is called a *rational number*," he is defining the term *rational number*. As in the case of characterization, different definitions varying in form, substance, and precision are possible.

Moves in Teaching Concepts

Analysis of tapes of teachers teaching mathematics reveals various moves the teachers employ when teaching concepts by characterization.*

1. The teacher gives a single characteristic which may be—
   - 1.1 A sufficient condition. *Example:* "If all the sides of a polygon are congruent, the polygon is EQUILATERAL."
   - 1.2 A necessary condition. *Example:* "If a natural number is divisible by some natural number other than itself and 1, it is not a PRIME NUMBER." or
   - 1.3 A necessary and sufficient condition. *Example:* "A system of linear equations is a CONSISTENT SYSTEM if and only if its solution set is not the empty set."
2. The teacher classifies, in which he employs the subset (or superset) relation. *Example:* "A TRAPEZOID is a quadrilateral."
3. The teacher identifies. *Example:* "A PARALLELOGRAM is a quadrilateral whose opposite sides are parallel."
4. The teacher analyzes, in which he employs the partition relation and names one or more of the proper subsets of the set determined by the concept. *Example:* "Circles, ellipses, parabolas, and hyperbolas are CONIC SECTIONS."
5. The teacher employs analogy in which—
   - 5.1 The basis of the analogy is stated. *Example:* "PRECEDING is like being greater than in that both are order relations." *or*

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* In the examples that follow, the concept being taught appears in capital letters.

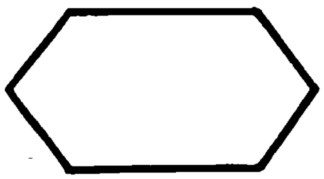
5.2 The basis of the analogy is not stated. *Example:* "A PARAMETER is like a variable."

6. The teacher employs differentiation in which—
   6.1 The basis of the difference is stated. *Example.* "CONGRUENCE is not the same as equality, in that the latter implies identity of the objects equated and the former does not." *or*
   6.2 The basis of the difference is not stated. *Example:* "A NUMERAL is not the same as the number it stands for."

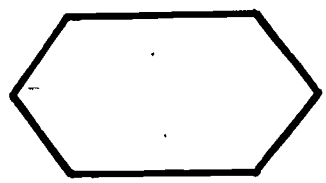
Upon reflection, it becomes apparent that some of these moves theoretically are more productive of precise concepts than others. The moves of necessary and sufficient condition and identification (which differ only in form) lead to precise concepts, assuming, of course, that the students understand the sentences used. The move of analysis, provided the complete partition is given, is a precise move. On the other hand, the moves of analogy and differentiation, particularly where the basis of the analogy or difference is not stated, are not conducive to precise concepts. However, these moves may be effective psychologically, particularly when they are initial moves in a sequence of moves.

The analysis of tapes of teachers teaching mathematics reveals that when they employ exemplification, the following moves can be identified:

1. The teacher gives one or more examples which—
   1.1 May not be accompanied by the reason why it or they are examples. *Example:* "The numbers 2, −4, and $\frac{1}{2}$ in $2x^2$, $-4bc$, and $\frac{1}{2}t$ are NUMERICAL COEFFICIENTS." *or*
   1.2 May be accompanied by the reason why it or they are examples. *Example:* "$\sqrt{2}$ is an IRRATIONAL NUMBER because it is not equal to the quotient of two integers."
2. The teacher gives one or more nonexamples which, as in the case of examples—
   2.1 May not be accompanied by the reason why it or they are not examples. *Example:* "The polygon at the right is not a REGULAR POLYGON." *or*



2.2 May be accompanied by the reason why it or they are not examples. *Example:* "The polygon at the right is not a REGULAR POLYGON because it is not equiangular."



3. The teacher gives a counterexample to correct a misconception a student has, as, for example, in the following dialogue:

   T: Who can tell us what a linear function is?
   S: It's a function whose graph is a subset of a straight line.
   T: Is the graph of $x = 6$ a straight line?
   S: Yes.
   T: Describe the position of the graph.
   S: It's a straight line parallel to the y-axis whose x-intercept is $+6$.
   T: Right. (Draws a representation of the graph on the chalkboard.) If $x = 6$, what is the value of y?
   S: It's undetermined.
   T: If $x = 6$, can $y = 1$?
   S: Yes.
   T: Can y equal -1, $+3$, $+10$?
   S: Yes; y can be any number.
   T: Then does $x = 6$, whose graph is a straight line, determine a function?
   S: Well, I guess not.
   T: Want to revise your concept of a linear function? What is a linear function?

4. The teacher specifies (lists) all the objects named by the term designating the concept. *Example:* "The UNITS MOST FREQUENTLY USED IN MEASURING LENGTH IN THE UNITED STATES are the inch, foot, yard, and mile."

As in the case of characterization moves, some exemplification moves are more effective in teaching a concept than others. Theoretically the mere giving of examples is not sufficient to teach a precise concept. But in practice, it is found that students seem to induce the proper set of sufficient conditions, and the concept they attain is satisfactory for practical purposes. Accompanying examples with the reason why they are examples, of course, makes the sufficient condition explicit. Such a move would be used by a teacher if he felt the students might not induce the correct sufficient condition.

To be sure, exclusive use of nonexamples is futile. Exhibiting polygons which are not regular polygons will never teach a student what a regular polygon is. Nonexamples are most effectively used in conjunction with examples to make apparent a necessary condition. In this case, an observation analogous to that made about the use of examples can be made. Giving a nonexample without the reason why it is a nonexample leaves the burden of induction (discovery) on the student; supplying the reason makes the necessary condition explicit. The teacher's judgment must determine whether or not the reason should be supplied or elicited by questioning.

A specification move necessarily results in a precise concept, provided the student can remember all the members in the set determined by the concept. If a teacher teaches the concept of truth value by saying, "There are two truth values: true and false," the concept is precise, since the complete denotation of the term is given.

We now turn to definitional moves, as identified by analyses of tapes and textbooks. Modern logicians regard definition as an operation performed on terms; we define terms but not objects which are not terms. For example, we define the term *rectangle*, but describe or characterize a rectangle. It follows that when defining we have to talk about the term being defined—indicate what it is to mean or other expressions to which it is to be synonymous. (To show clearly that we are talking about a term rather than about what the term denotes, we use the conventions of either setting the term in Italics or enclosing it in quotation marks.) Thus the following are definitions:

By *vector* we mean an ordered n-tuple of real numbers.

An equilateral parallelogram is called a *rhombus*.

A polynomial is called a *binomial* if and only if it has just two terms.

Because of the difference in logic, it is useful to distinguish between two kinds of definition: stipulated and reported definitions. (Some writers speak of these respectively as nominal and lexical definitions.) Both definitions may have the same form; it is their use which serves to distinguish them. A stipulated definition is used to introduce into the language a new term, usually a shorter term, as a replacement for a longer term. For example:

> We shall denote a quadrilateral having a pair of parallel sides by the term *trapezoid*.

is clearly stipulative; the definition introduces the short expression *trapezoid* and stipulates that it means the same as the longer expression, *quadrilateral having a pair of parallel sides.* Either expression can be replaced by the other in any sentence. Conceivably, there can be an expression other than *trapezoid* to denote a quadrilateral having a pair of parallel sides.

It follows that a stipulated definition is a directive and does not have a truth value; one cannot say "true" or "false" or "correct" or "incorrect" to a stipulated definition. Logically, one could propose a different definition, e.g., "trapezoid" could be used to refer to a quadrilateral having only one pair of parallel sides, and no one could say either definition was right or wrong.

A reported definition is a report of how, in fact, a term is used. Hence a reported definition has a truth value. If it is a correct report, it is true; if it is an incorrect report, it is false. It follows that reported definitions logically can be items in a true-false test; stipulated definitions cannot.

Tapes of classroom teaching reveal that teachers use definitional moves in teaching concepts only infrequently; they eschew the metalanguage and utilize the object language. Textbooks, however, frequently use definitional moves; the more formal and rigorous the exposition, the more definitional moves are used. About the only use of a stipulated definitional move found from observations of classroom teaching occurs when a teacher encourages his students to invent a name to refer to some mathematical object they have identified.

Strategies in Teaching Concepts

As might be expected, it is found that when teachers teach concepts they do not restrict themselves to one move; they use a sequence of moves. Let us call a sequence of moves a *strategy*. A frequently used strategy is an identification move or its metalinguistic correlate, a definition, followed by one or more exemplification moves. The reverse sequence, examples followed by an identification, is also used. One can conceive of many strategies by listing all the permutations of the moves.

So far not enough research has been done to indicate under which conditions various strategies are effective. One might conjecture that the intellectual aptitude of the student, what he may already know about the concept, the nature of the concept, its significance in the structure of other mathematical concepts, and the level of performance the teacher wants to develop are factors which a teacher would consider in choosing among the strategies available. For slow learning students, one might expect a longer sequence with stress on exemplification moves. For fast learners, one might expect shorter sequences with more recourse to characterization moves. If the concept is important, i.e., it will be used to build many other concepts, the teacher probably will select a strategy which will ensure a precise concept. Finally, other things being equal, the teacher may want to use different strategies if for no other reason than to provide variety in his teaching.

TEACHING PRINCIPLES

The term *principle* is usually used to denote a generalization other than an existential generalization, or a prescription. For example, we speak of the distributive principle of multiplication over addition and the principle of mathematical induction, both of which are generalizations; all their variables are universally quantified. We also speak of the principle of the density of the rationals. Although one of the variables in this generalization is existentially quantified, the others are universally quantified.

From a true generalization a teacher can, if he wishes, formulate a prescription for how a student should proceed to attain the desired result. For example, from the generalization,

> For all real numbers x and $y \neq 0$ and each positive rational number n, $(x/y)^n = x^n/y^n$,

a teacher can formulate the prescription:

> To raise a common fraction to a power, raise both the numerator and the denominator to the power.

to assume that it must be true, otherwise the teacher or textbook would not present it. Similarly, teachers infrequently explicitly justify a prescription or algorithm. However, if an instantial move (move 2.4) or a demonstration move (move 2.5) is used to clarify the principle, one can argue that such moves simultaneously clarify and justify the principle.

If the statement move is not the first move but appears, if it appears at all, near the end of the sequence, with instantial moves or demonstration moves as the initial moves, we may regard the strategy as a *guided discovery strategy*. Typically, if the principle is a generalization, the teacher presents instances of the generalization (The students do not know what the generalization in question is.) and guides the students by questions or suggestions into inducing (discovering) the generalization. In other words, if the principle is a prescription or algorithm, the teacher demonstrates how several problems are solved and seeks to educe the statement of the principle or algorithm (move 1.3). Application moves then follow.

GUIDED DISCOVERY STRATEGIES

Enough research has been done on guided discovery to warrant special consideration. Moreover, there are few topics in pedagogical theory of teaching mathematics for which greater enthusiasm in speeches and in writing exists than for guided discovery. Enthusiasm for guided discovery is even greater in mathematics than in the natural sciences. Moreover, this enthusiasm has been maintained for 35 years or more. Belief in the efficacy of the discovery method of teaching is for some theoreticians so profound that they appear not to want to be confused by facts.

What is the argument for the effectiveness of teaching by guided discovery? It is argued that the student is involved in the learning and, therefore, manifests greater attention and thought. The thrill of discovery is used to explain the retention and transferability of the knowledge learned. In learning by discovery, the student improves his ability to solve problems. Being intrinsically motivated, the thrill of discovery sustains the motivation and students are less disposed to badger the teacher concerning the practicability of the knowledge discovered. By learning the heuristics of discovery, a student can continue his education after he has left the guidance of a teacher.

Finally, it is claimed that with guided discovery based on induction, it is not possible to go too fast for the student. When a student fails to make the discovery; that is, does not see the pattern admist the differences, the teacher needs to supply more examples or instances or make the examples or instances more alike. In the conventional method of exposition, as many teachers know, it is easy to go too fast for the students.

There are, however, arguments against the method of guided discovery. It is a fact that many children are greatly threatened by having to exert the necessary cognitive effort to make a discovery. They often give up and wait for the discovery to be verbalized either by the teacher or by another student. Discovery is time-consuming and hence may not be an efficient way of teaching. Further, the argument maintains that one must learn to comprehend much of his culture, as well as learn to discover new knowledge and solve problems. To accentuate the latter at the expense of the former is to be carried away by the charisma of only one particular style of learning. So goes the argument against guided discovery.

What does research have to say about teaching and learning by guided discovery? It may serve to quote from three scholars who have made intensive studies of the research bearing on these topics. Wittrock (21, p. 33)* says, "Many strong claims for learning by discovery are made in educational psychology. But almost none of these claims has been empirically sustantiated or even clearly tested in an experiment." Cronbach (3, p. 76) has drawn a similar conclusion: "In spite of the confident endorsements we read in popular discourses of learning by discovery, there is precious little substantiated knowledge about what advantages it offers, under what conditions the advantages accrue." Perhaps it is on the basis of such conclusions and others of a similar vein that Glaser (7, p. 23) reaches the conclusion that "Since we know so little about it, one can say anything and enjoy his own speculations without the constraints of knowledge."

* Numbers in parentheses identify Selected References listed on pages 31-32.

Suggestions for Using Guided Discovery

But is there no guidance to be obtained from the existing research? No, this is not the case. One can safely conclude that for a particular teacher—a skillful practitioner—guided discovery is an effective way of teaching that subject matter which lends itself to this approach. In the hands of a poor practitioner it is likely to be no more effective than is the expository method in the hands of a poor practitioner of that method.

Certainly guided discovery can be as mechanical as expository teaching. For example, suppose the students understand the concepts of a variable, an open sentence, a closed sentence, and logical quantifiers (whether phrased in English or in abbreviated notation) and are able to replace a constant by a variable and to append quantifiers to an open sentence. Then guided discovery can be quite mechanical. The students are simply taught to replace the constants in a true statement by variables and quantify them over such a domain as to make the quantified statement true. Such a procedure requires very little thinking and the "discovered" generalization is readily formed.

In the hands of a poor practitioner, students can be called upon to make conjectures and verbalize their discoveries before they are ready to do this. Moreover, closure can be secured on the discovery before a significant number of students in the class have made the discovery. In such a case, for the students who have not made the discovery the teaching reverts to telling, the telling being done by whoever verbalizes the discovery.

Using guided discovery on occasions and where appropriate has the advantage of providing variety in instruction. Discovering is exciting for many students, and a change to this method probably would be welcomed.

Theoretically, guided discovery is appropriate for some items of knowledge, but not for others. A generalization not definitional in nature can be taught by guided discovery. The teacher knows what generalization he wants to teach, hence he can present a sequence of instances of the generalization sufficiently varied as to help students identify the domain over which the generalization can be made. By using the move of counterexample to correct any false generalizations the student may have reached, the teacher can guide the students, or most of them at least, to induce the generalization of which each of the true statements the students have considered is an instance.

Logically, it is not possible to discover a stipulated definition. Such definitions are created, not discovered. However, consideration of definitions can lead students to discover how people use language since definitions reveal how people do, in fact, use language. In short, reported definitions can be discovered; stipulated definitions cannot.

Although it is possible to teach concepts by discovery, such practice is inefficient. A concept can be taught quite satisfactorily by exposition in such a way that students gain a clear comprehension of it and do so more expeditiously than by the method of guided discovery.

As was pointed out earlier in this section, guided discovery is effective and appropriate for some students. Students who see relationships readily, are able to abstract, and have facility with language should be able to learn readily by discovery. Students who are not able in these intellectual processes will have difficulty with guided discovery and may experience no more satisfaction with this method of instruction than with an expository method. Students who are insecure, have short attention spans, and are not persistent have been known to give up easily and simply to waste time until someone else verbalizes the discovery.

Cronbach (3) has offered the conjecture that some experience in discovering principles will provide a student new insight into the nature of knowledge and how new knowledge is established. It will also alter his conception of how a person copes with solving problems. He continues his conjecture by opining that these objectives may be realized by devoting only part of the instructional time to inductive procedures.

Expanding on Cronbach's conjecture, theoretically an insightful teaching procedure would be to use guided discovery to enable students to form conjectures. If these conjectures can be proved, they become theorems. If they are disproved, say by finding a counterexample, it might become possible to restrict the domain over which the generalization is made, thereby obtaining a new conjecture. This conjecture then is subject to proof. Such a procedure should provide considerable insight as to how a mathematician operates and how, in part, mathematics is extended and developed.

Most teachers are aware that skill in questioning enhances the effectiveness of guided discovery. Socrates is the putative progenitor of the method of questioning whereby the learner is enabled to discover whether a statement is true or false by following the sequence of questions put to him by a teacher. One might extend this technique and speak of it as *dialectical dialogue*. It is dialectical because the questions are based on a logical approach. This is not to imply that the questioner ignores the individual—what he knows, how he reacts, and what he seems able to comprehend. But the pattern of enabling the student to realize whether a statement is true or false necessarily has a logical dimension. Certainly skill in dialectical dialogue is learnable; we have many examples of teachers who become increasingly proficient in its use. Whether or not it is teachable is questionable. It is doubtful that we have sufficient knowledge about this technique that we can by teaching such knowledge enable a teacher to build such skill. Presently, the skill or art is developed by practice and by receiving suggestions and practitioner's maxims from a supervisor.

Of these practitioner's maxims, there seem to be a few which are useful to a teacher who seeks to teach by guided discovery. One is to make sure that students have a clear understanding of their objective—the task they are to accomplish. This provides focus and enables them to see the relation between the various tasks in the sequence they consider.

Another valuable maxim is to make sure that students test their conjectures for themselves. If a teacher is not careful, he may by facial expression or tone of voice imply that the conjecture is, true, if it is true, or false, if it is false. The students then tune to the instructor, rather than to the data which may be available to them.

The final maxim concerns pacing. If a student appears to be having trouble either in forming a conjecture or testing his conjecture, it is often helpful for the teacher to pose simpler instances, such that the analogy is more readily seen. This may be done by restricting the domain over which the generalization is to hold. It may also be done by having the student form a table of data so that comparison and contrast are facilitated and the analogy more readily apprehended.

TEACHING MATHEMATICAL SKILLS

Teachers recognize that not only should students learn concepts and principles, but they should also learn to compute and manipulate algebraic symbols. The concepts and principles provide the understanding which is the cognitive basis for the skills of computation and manipulation.

Under the impact of the psychology of operant conditioning, it was believed that drill was the sufficient condition for developing skill. Students were given a set of prescriptions which they memorized and practiced. Little attempt, if any, was made to show how the prescriptions were justified by theorems or postulates. The students learned that if they followed the prescriptions they got the right answer.

When the drill approach was found wanting (The students forgot how to perform the operations once the drill ceased.), the point of view shifted to a belief that understanding must precede skills. Hence students were taught the "why" before they were taught the "how." In some cases the faith in understanding was so strong that insufficient practice was provided the students.

Research evidence does not support the belief that for students to develop and retain a skill they must be taught the mathematical justification of an algorithm before they practice the algorithm. It does not even support the belief that students must understand an algorithm, in the sense of being able to explain its mathematical basis, for them to develop skill in using it, provided they continue to use it. We find, for example, some elementary teachers who can divide one rational number by another with speed and accuracy without knowing the mathematical basis for the algorithm they use.

But if an operation is not continually used, it appears that understanding the operation aids in its retention. For example, after a long period of not performing the operation of finding square roots the student who understands the concept of a square root of a positive rational number is more likely to be able to find a rational approximation of a square root than the student who once learned the conventional algorithm and developed skill in using it but never understood it. With the understanding that if y is a positive number and $xx = y$, x is a square root of y, he can guess at a factor of y, divide y by that factor and see whether the quotient is the same as x. If not, guided by the difference between the two factors he can guess at a second approximation, repeat the process, and ultimately find an approximation of the square root to whatever accuracy is desired. Similarly, if a student knows what the notation $(x + y)(2x - 5)$ means and knows the distributive property of multiplication over addition, he can compute the product even if he has forgotten an elegant algorithm he was once taught.

Suggestions for Teaching Skills

In teaching skills, there appear to be some principles which can be justified by evidence. One is that whereas practice—drill —is a necessary condition in learning a skill, it is not a sufficient condition. One can practice wrong moves as well as right moves. There is nothing magic about practice; the practice must be in accordance with certain conditions.

Practice will be most effective when the student wants to improve. This principle is an instance of the more general principle of motivation. The student must be convinced that the end is worth the means; that the skill he seeks to develop is worth the drill he must accept. Teachers can strengthen the desire to acquire a skill by pointing out the usefulness of the skill.

Assuming that a teacher allocates a certain amount of time for practicing an algorithm, the practice time will be most effective if it is expended in periods spaced out over a week or more than if it is expended all at once. The shorter periods sustain interest and avoid fatigue and boredom. Some theorists even speculate that there is latent learning during the intervals when the algorithm is not being practiced.

Practice is more effective when the student is kept aware of his progress and improvement. This principle is an instance of the more general principle that learning is more effective when the student has immediate knowledge of the results of his actions. Knowledge of improvement is itself a motivating factor as well as a basis for self-diagnosis and self-remediation.

In keeping with the principle of individualization of instruction, practice should be differentiated in terms of the needs manifested by different students. Some students need more practice at a time than others; some need review more than others. A minimum amount of practice is essential; thereafter for any student the teacher's judgment should dictate the kind, amount, and frequency of practice.

Practice should be subject to the observation and criticism of the teacher. When a teacher assigns practice exercises, he should not expect as much improvement if he does not observe at least some of the exercise sessions as if he observes the students at work and offers suggestions and praise for improvement where appropriate.

The issue of whether or not students should be permitted to use "crutches," for example, writing a numeral at the top of a column when "carrying" in adding, or writing the decomposition of digits when "borrowing" in subtracting, seems to reflect a value judgment a teacher has to make for himself. Some teachers feels that "crutches" are inelegant and should be discouraged. Others see nothing wrong with a student using any "crutch" he finds helpful. Perhaps there is a tenable middle position. The primary objective is accuracy. If a student cannot attain accuracy without using a "crutch," he should be permitted to use it. His span of attention is insufficient to enable him to work otherwise. Elegance is less important than accuracy and absence of frustration.

PROGRAMED TEXT MATERIALS

We now turn to another topic on which, like guided discovery, there has been much research, namely, learning by means of programed texts. Programed text materials have had an interesting history. Scarcely any pedagogical phenomenon has experienced so much initial enthusiasm and subsequent waning of this enthusiasm in such a short period of time as programed text materials. There continue to remain, however, enthusiastic proponents who would have us believe that programed text materials (and, more generally, programed instruction) are the solution to many of the problems of our present educational processes. Those regarding programed text materials with a jaundiced eye see this pedagogical medium as providing some assistance but having several concomitant undesirable aspects.

Since the initial enthusiasm for programed text materials, there have been several variations on the initial programs developed. These various adaptations have been employed for programing the acquisition of knowledge in most academic subjects, mathematics being the most common. There are several reasons for the popularity of programed text materials in mathematics. For one reason, the concepts in mathematics are precise and the generalizations hold over a well-defined domain. For another reason, there is considerably more notation in mathematics than in any other subject, and the learning of notation can be easily taught by programed text materials. As a third reason, the deductive nature of mathematics makes sequencing of the frames in programed text materials easier than for other subjects. Finally, examples of concepts and instances of generalizations are readily obtained and the implications of theory for the solution of problems is clearer than for any other subject.

Findings from Research

What conclusions can be drawn from the research on the use of programed texts? One conclusion is that students do learn from programed texts. Some students learn more than others. However, there is no conclusive evidence that students either learn significantly more knowledge, acquire it with greater efficiency, or reach more depth in understanding.

Programed texts can produce understanding as well as knowledge. However, knowledge and understanding at higher intellectual levels are not particularly suitable for programed learning. For example, although programed materials can enable the student to understand a particular proof, they are not as successful in teaching him how to construct a proof. Programed text materials are not very successful in teaching the intellectual skills of analysis, synthesis, and evaluation.

Programed text materials are effective for some students but not for others. For highly motivated students and students who learn quickly programed texts can be useful. For students who are somewhat bored with the conventional style of teaching, the novelty effect of programed text may sustain interest at least for a period of time. Particularly is this the case for students who have experienced lack of success in the conventional style of teaching; a change to a different mode may be therapeutic.

Programed text materials can be used to supplement the conventional instruction guided by a textbook. If a student does not understand the presentation made either by the teacher or by the conventional text, he can turn to a programed text for further clarification. The careful pacing of programed texts may make it easier for some students to follow the exposition.

It has been pointed out that programed texts can be effectively used by students who transfer into a course and find themselves out of step with what is being taught. A programed text will enable them to catch up with less direction on the part of the teacher than if a conventional text is used. How able the student will be to solve problems using the knowledge he acquires is another matter, however.

Programed texts can also be used to supplement the curriculum. By using a programed text, an able student might be able to complete a course in analytic geometry or even a course in calculus with a minimum of direction by the teacher. In a small high school where a four-year program in mathematics is not possible because of the small number of students who might take the third and fourth years, programed texts can be used to enable the few students who wish to take the third and fourth courses to do so. Of course, the same comment can be made for the conventional textbook. However, it is probably the case that a student who attempts to study trigonometry, advanced algebra, or elementary functions independently by using a conventional text would require more assistance from a teacher than a student who would attempt these courses by means of programed texts. On the other hand, students who complete such courses probably will not be so adept at solving problems as students who complete conventional courses; typically, programed texts do not stress problem solving.

Another claim for programed text materials is that their use increases the efficiency of instruction. One study has shown that students can learn the same amount of knowledge taught in the conventional way with a savings of time up to 30 percent when programed text materials are used. Moreover, the individualization which is permitted by programed text, so it is claimed, enables the teacher to make better use of his time in working with individual students than in the conventional way classes in mathematics are taught.

We now turn to disadvantages of programed texts. One disadvantage is their high expense. Such expense is justified because of the vast number of hours required to write the program and to try it out and revise it. There is the danger that once a board of education has invested a sizable amount of money in programed text materials, there may be considerable reluctance to abandon their use. The board may feel that it must use them for a long period of time to get its money's worth out of them. A reluctance to change texts may result in an inflexible curriculum which does not keep abreast of current changes.

Programed texts pose somewhat of a problem for poor readers because greater reliance is placed on obtaining meaning from the printed word. In the conventional style of teaching, much understanding can be obtained by listening to the teacher's oral presentation. For students who enjoy discussion—the give and take of differing opinions—programed texts can be stifling.

As a further negative aspect, the sheer logistics of handling vast amounts of text materials is sobering. For example, the SMSG ninth-gr[illegible]bra course in Crowder requires 2,357 pages organized in six volumes. Storing such quantities of texts, passing them out at the beginning of an instruction period, and collecting them at the end, when it is unlikely that many students are at the same point in the course or even in the same course, is a bit overwhelming. Should the time come when every teacher has the services of teacher aides, much of this kind of nonprofessional work can be transferred to such individuals. However, until that day arrives, one wonders whether teachers will most profitably spend their time keeping records and moving instructional materials about.

In conclusion, one is tempted to conjecture that the solution is not to abandon the conventional textbook, but rather to show the teacher how to use it effectively and also to teach the student to use it effectively. Research toward this objective is conspicuous by its absence.

COMPUTER-BASED INSTRUCTION

Teaching which is performed and monitored by a computer, usually referred to as computer-based instruction, is an offshoot of programed instruction. In a typical design, the student communicates with the computer by means of an electric typewriter, and the computer, in turn, communicates with the student by means of a cathode ray tube similar to the viewing screen of a television set. The student types information or directions on the electric typewriter. The computer is programed to process and interpret this information or directions and respond by sentences, words, or other symbols which appear on the viewing screen before the student.

Findings of Studies

As in the case of studies of programed texts, studies of computer-based teaching indicate that many students learn as well by this mode of instruction as by conventional classroom instruction. Moreover, the efficiency of instruction, that is, the amount of learning in a given period of time, seems to be about the same. As might be expected, students enjoy the variation in style of teaching that occasional use of computer-based instruction provides. But they tire of it if it is used exclusively and long for the discussion that often characterizes group instruction.

How well students will improve in the higher mental processes, e.g., reasoning, valuing, and solving problems, as the result of computer-based instruction is as open to question as in the case of programed texts. But the theoretical possibilities for improving these mental processes is great, and it should surprise few people if before long we find that computers and programs are developed which do this very thing.

Of great value in computer-based instruction is the feedback about student responses and progress which are provided the teacher. Of course, in conventional group instruction feedback is also present, but it cannot be so comprehensive, systematic, and easily recorded. Hence it is not of such value for adapting the teaching to each student. Studies repeatedly make mention of how this information is used to alter sequences of items, phrasing of items, response time, the number of errors on a particular item, and the modification of remediation loops.

Finally, the expense of computer-based instruction is continually being reduced. More sophisticated computers are being developed which will service more students studying different courses at different points in the course. These will reduce the cost per student hour. One prediction is that by 1975 the cost will be 10 cents per student hour which will approach the cost of conventional group instruction. The future for computer-based instruction is both promising and exciting.

THE CULTURALLY DISADVANTAGED

We now turn to a special group of learners to whom attention increasingly is being paid. The terms *culturally disadvantaged student* and *culturally deprived student* are used to denote students in economically and culturally impoverished areas of large cities and certain geographical areas. There is more concern for the educational welfare of the children in these areas than there is knowledge about how to teach and control them. This is because until only recently, when the concern has manifested itself in the availability of federal funds to support research, there has been little systematic research done on teaching these children. Gordon (8, pp. 421-22) points out that "the literature is replete with discussions of what a teacher should be and do, but very few of the suggestions or conclusions are supported by research evidence." In spite of this conclusion, with which anyone who reads the literature will concur, the suggestions represent conjectures which a teacher can try.

As far as he subject matter of mathematics is concerned, certainly priority should be given to the mathematics the citizen needs to know to count, estimate, measure, compare, and compute—the arithmetic of the normal elementary grades. Whether this is taught to 6- to 12-year-olds or to 15- to 18-year-olds should not be crucial when such knowledge is so important. The social utility of the knowledge and skills probably will be given more weight than its academic utility—usefulness in portraying mathematical structure. Certainly the principle of readiness must be used.

The context in which mathematics is taught appears to be particularly crucial for the culturally disadvantaged, since they do not handle abstractions easily. Familiar and concrete situations, e.g., buying in a store, gambling, racing pigeons, sports, and music, have been used with some success. It is difficult to place mathematics teaching in meaningful contexts. Perhaps getting the students to tell what they do will reveal some contexts which a teacher can utilize.

Reportedly, role-playing has been used with success, e.g., "Pretend you don't understand how to do the problem. What questions would you ask?" It gets the students to talk rather than listen to the teacher talk. Also, when a student is playing a role, he is somewhat freed from his normal insecurity and defensiveness and is more disposed to attempt problems and tasks which fear of failure would lead him not to attempt.

CONCLUSION

In the final analysis, the justification of research is the extent to which it enables practitioners to make wise choices. The conclusions based on some of the research in the teaching of mathematics which have been stated above may serve the classroom teacher to this end. But the teacher should not ignore the possibility of informal research he may do on his own and in his own situation. In fact, such research may be of more value to the teacher just because it is restricted to the context in which he operates and focuses on the very problems he faces. There certainly is a place for such research; it should be both encouraged and fostered.

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