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ABSTRACT

A list of materials needed and step-by-step directions for constructing an abacus are given. Instructions are provided which tell how to use the abacus in teaching number combinations and in working addition, multiplication, subtraction, and division problems. (Related documents are SE 015 950 and SE 015 952.) (DT)

TEACHING COMBINATIONS with the ABACUS

Mathematics Series No. 6

PENNSYLVANIA CURRICULUM DEVELOPMENT PROGRAM

Commonwealth of Pennsylvania
DEPARTMENT OF PUBLIC INSTRUCTION

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MESSAGE FROM THE SUPERINTENDENT OF PUBLIC INSTRUCTION

Nothing is constant but change itself. Four significant changes have affected the elementary mathematics curriculum during the past several years. Mathematics for the younger child has been expanded beyond arithmetic to include simple intuitive geometry and algebra. The second change is more careful use and simplification of quantitative vocabulary.

Understanding the computational operations is the third change. The two important elements in learning these operations are meaning and skill. To dispel the rote-learning atmosphere, the four fundamental operations are developed in as meaningful a way as possible. Greater use of mathematical laboratory materials — the abacus, counters of all kinds, the number line, rods and models, plane and solid geometric figures and measuring instruments — help the child to visualize the problem and discover the algorithm for himself. Being involved in formulating the problem, picturing and thinking through to its solution enables the child to catch the spirit of mathematics as well as understand the content.

The fourth change is giving the responsibility for learning back to the child. He is becoming more actively involved in the learning process with more emphasis on the discovery approach.

For the most part, we do not wish to impose these changes upon the elementary schools. Rather we hope to develop materials which the teacher will find exciting to try with children so that a love of mathematics can develop in both teacher and learner.

J.R. Rackley

Superintendent of Public Instruction

First grade children use the abacus and the counting frame to understand operations on numbers.

CONSTRUCTION OF THE ABACUS

Materials Needed

1 piece of plywood, or hardboard, measuring 17¾" x 8½"

2 pieces of wood, 1" x 1" x 8½"

1 piece half-round, ½" x 8½"

4 pieces of wire, 16½" (may be cut from coat hangers)

9 yellow wooden beads with predrilled holes

10 red wooden beads with predrilled holes

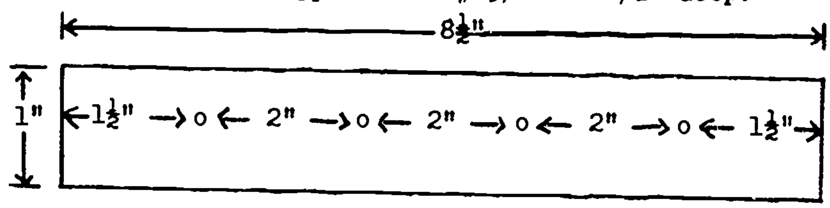
10 blue wooden beads with predrilled holes

11 green wooden beads with predrilled holes

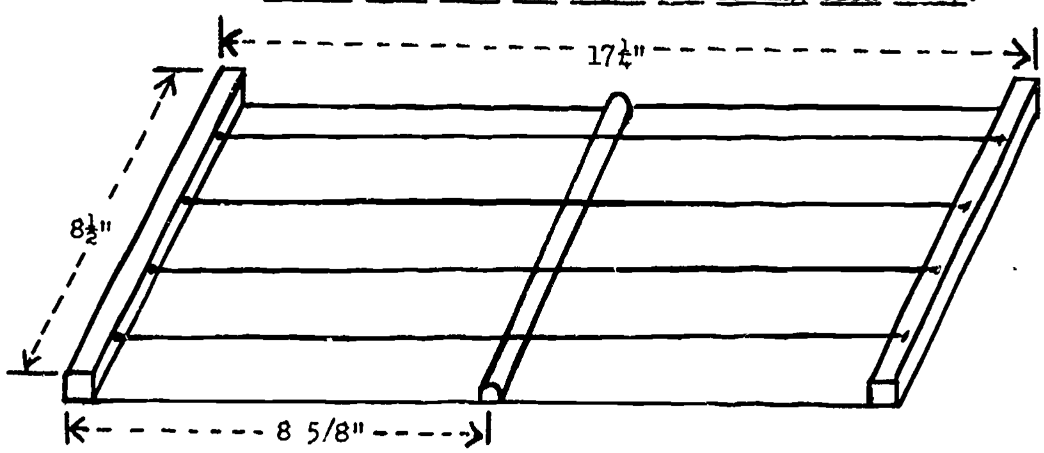
If the beads can be purchased only in natural color, dip the beads in paint to attain the desired colors.

Construction

Step 1 - Take the two pieces of 1" x 1" x 8½" and drill four holes in each one approximately 3/8" to 1/2" deep.



Step. 2 - Fasten these two pieces to the 17¼" plywood frame, making sure that the holes are facing each other.



Step 3 - Nail the $\frac{1}{2}$" half-round to the center of the frame.

Step 4 - Give all wooden parts two coats of shellac or varnish. Sand all rough parts before applying finish coat.

r = red bead
y = yellow bead

b = blue bead
g = green bead

r y y y y y y y y y	
b r r r r r r r r r	
g b b b b b b b b b	
g g g g g g g g g g	

A. Place nine yellow beads and one red bead on wire. Insert wire in hole, then bend slightly to slip into opposite hole. This is the ones' or units' column.

B. Place nine red beads and one blue bead on wire and insert below the ones' column. This is the tens' column.

C. Place nine blue beads and one green bead on wire and insert below the tens' column. This is the hundreds' column.

D. Place ten green beads on wire and insert. This is the thousands' column.

If the materials are cut to proper size, children can assemble their own abacus (ab' a cus). Older children can do the whole project.

USING THE ABACUS IN THE CLASSROOM

Teaching of Combinations

1. Principles to be understood:

   Only like units may be combined (addition).

   Only like units may be separated from a group (subtraction).

2. Numerical operations are recorded on the abacus by placing the beads on one side of the center bar.

3. The abacus should always be cleared before beginning a new problem. The abacus is cleared by placing the beads on the opposite side of the center bar.

4. The abacus described in this publication can be used horizontally or vertically.

Introducing Combinations to Nine

Example: 6 ones + 3 ones

1. Pull 6 ones across center bar.

r y y y b r r r r r r r r r g r r r r r r r r r g g g g g g g g g g	y y y y y y

2. Pull 3 ones across center bar.

r b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y y y y y y

3. Have child count total: 9 ones. Repeat procedure using different combinations.

It is important to use the words ones, tens, hundreds, etc., so that the child understands the idea of combining like units.

Combinations That Equal Ten

These are the combinations: 5 + 5, 6 + 4, 4 + 6, 7 + 3, 3 + 7,

8 + 2, 2 + 8, 9 + 1, 1 + 9.

Start with 5 ones + 5 ones.

1. Pull 5 ones across center bar.

r y y y y b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y y

2. Pull 5 more ones across center bar.

b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	r y y y y y y y y y

3. There are a total of ten ones. Regroup 10 ones for 1 ten in tens' column by pushing back 10 ones and pulling forward 1 ten.

r y y y y y y y y y b r r r r r r r r g b b b b b b b b b g g g g g g g g g g	r

As the final step, teach that 10 ones and 1 ten have the same value; the idea of "equal to." Use the word regroup to mean the grouping of 10 ones as 1 ten.

Combinations Above Ten

Example: 7 ones + 6 ones

1. Pull 7 ones across center bar.

r y y	y y y y y y y
b r r r r r r r r r	
g b b b b b b b b b	
g g g g g g g g g g	

2. Pull 3 ones across center bar.

	r y y y y y y y y y
b r r r r r r r r r	
g b b b b b b b b b	
g g g g g g g g g g	

3. Regroup 1 ten for 10 ones, return 10 ones.

r y y y y y y y y y	
b r r r r r r r r	r
g b b b b b b b b b	
g g g g g g g g g g	

4. Pull 3 ones from ones' column.

r y y y y y y	y y y
b r r r r r r r r	r
g b b b b b b b b b	
g g g g g g g g g g	

This means that the combination is taught in this manner:

$$7 + 3 = 10$$

$$10 + 3 = 13$$

By placing the necessary emphasis on regrouping at ten, the children will learn that the combinations are not a difficult memory task. Children need to know that 7 + 6 is 13, but when in doubt, they may refer to the above method. This renaming of 6 as 3 + 3, and associating one of these 3's with 7, gives one group of ten and 3 ones.

ADDITION

Example:

$$
\begin{array}{r}
74 \\
87 \\
16 \\
+\ \underline{25}
\end{array}
$$

1. Pull 4 ones.

r y y y y y b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y

2. Pull 6 ones.

b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	r y y y y y y y y y

Regroup 1 ten for 10 ones.

r y y y y y y y y y b r r r r r r r r g b b b b b b b b b g g g g g g g g g g	r

Pull 1 one.

r y y y y y y y y b r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y r

(This gives the combination 7 + 4 = 11).

3. Pull 6 ones.

r y y	y y y y y y y
b r r r r r r r r	r
g b b b b b b b b b	
g g g g g g g g g g	

(This gives the combination 11 + 6 = 17).

4. Pull 3 ones of the 5 ones.

	r y y y y y y y y y
b r r r r r r r r	r
g b b b b b b b b b	
g g g g g g g g g g	

Regroup 10 ones for 1 ten.

r y y y y y y y y y	
b r r r r r r r	r r
g b b b b b b b b b	
g g g g g g g g g g	

Pull 2 more ones.

r y y y y y y y	y y
b r r r r r r r	r r
g b b b b b b b b b	
g g g g g g g g g g	

(This gives the combination 17 + 5 = 22).

2 ones = 2
2 tens = 20
2 tens + 2 ones = 22

5. Be very careful that the child does not say seven.
The seven is either <u>seventy</u> or <u>seven tens</u>.

Pull 7 tens.

r y y y y y y y b g b b b b b b b b b g g g g g g g g g g	y y r r r r r r r r r

(adding 8 tens or eighty)

6. Pull 1 ten.

r y y y y y y y g b b b b b b b b b g g g g g g g g g g	y y b r r r r r r r r r

(2 tens + 7 tens + 1 ten = 10 tens)

Regroup one hundred for 10 tens.

r y y y y y y y b r r r r r r r r r g b b b b b b b b g g g g g g g g g g	y y b

Pull 7 tens.

r y y y y y y y b r r g b b b b b b b b g g g g g g g g g g	y y r r r r r r r b

Pull 1 ten. .

r y y y y y y y b r g b b b b b b b b g g g g g g g g g g	y y r r r r r r r r b

Pull 2 tens.

r y y y y y y y g b b b b b b b b g g g g g g g g g g	y y b r r r r r r r r r b

(8 tens + 2 tens = 10 tens)

Regroup 10 tens for 1 hundred.

r y y y y y y y b r r r r r r r r r g b b b b b b b g g g g g g g g g g	y y b b

2 hundreds + 0 tens + 2 ones
(200 + 0 + 2 = 202)

Children can also discover a second method which uses the entire addend at one time. However, this method should be used only after they understand subtraction.

1. Pull 7 tens and 4 ones.

r y y y y y b r r g b b b b b b b b b g g g g g g g g g g	y y y y r r r r r r r

2. Add 87 which would be 8 tens and 7 ones. However, there are only 3 tens and 6 ones. If the child understands regrouping he may say: pull 1 hundred and remove 13 which is 87.

Pull 1 hundred.

r y y y y y b r r g b b b b b b b b g g g g g g g g g g	y y y y r r r r r r r b

Remove 13 which is 1 ten and 3 ones.

r y y y y y y y y y	y
b r r r	r r r r r r
g b b b b b b b b	b
g g g g g g g g g g	

3. Pull 16 which is 1 ten and 6 ones.

r y y	y y y y y y y
b r r	r r r r r r r
g b b b b b b b b	b
g g g g g g g g g g	

4. Pull 25 which is 2 tens and 5 ones: however, the abacus shows only 3 ones. Here the child may say: add 3 tens which is 30 and remove 5 ones which is 25.

Pull 3 tens; rémove 5 ones.

r y y y y y y y	y y
	b r r r r r r r r r
g b b b b b b b b	b
g g g g g g g g g g	

5. Regroup 10 tens for 1 hundred.

r y y y y y y y	y y
b r r r r r r r r r	
g b b b b b b b	b b
g g g g g g g g g g	

Read 202 on the abacus.

MULTIPLICATION

Multiplication will be defined as a rapid form of the addition process. It may also be thought of as the addition of like addends. Therefore, those principles, which are true of addition, will also be true in multiplication.

Example: 76
x48

1. Eight times six ones is 48 or 4 tens and 8 ones.

r y	y y y y y y y y
b r r r r r	r r r r
g b b b b b b b b b	
g g g g g g g g g g	

2. Eight times 7 tens, or seventy, is 56 tens or 5 hundreds, 6 tens, and no ones.

r y	y y y y y y y y
	b r r r r r r r r r
g b b b b	b b b b b
g g g g g g g g g g	

3. Regroup the 10 tens as 1 hundred.

r y	y y y y y y y y
b r r r r r r r r r	
g b b b	b b b b b b
g g g g g g g g g g	

4. Our multiplier with the "4" is now forty because the 4 of the multiplier is in the tens' column. It is important that the child say: forty times.

40 times 6 is 240

r y	y y y y y y y y
b r r r r r	r r r r
g b	b b b b b b b b
g g g g g g g g g g	

Now the child can see why the 4 of the second partial product is "in" one space to the left. There is only one way to write the number 240.

```
   76
x  48
   48)  1st partial product            8 x 6 = 48
  560)                                 8 x 70 = 560
  608
  240                                  40 x 6 = 240
```

5. 40 times 70 = 2800. There are not enough hundreds on the abacus so we add 3 thousands and subtract 2 hundred which is 2800.

r y	y y y y y y y y
b r r r r r	r r r r
g b b b	b b b b b b
g g g g g g g	g g g

3 thousands + 6 hundreds + 4 tens + 8 ones
(3000 + 600 + 40 + 8 = 3648)

```
   76
x  48
   48 )   1st partial product
  560 )
  608
  240 )   2nd partial product
 2800 )
 3648     product
```

DEMONSTRATING SUBTRACTION COMBINATIONS

Example: 16
– <u>8</u>

1. Set up one ten and 6 ones on the abacus by pulling the beads to the right.

r y y y b r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y y y r

2. The child sees he cannot subtract 8 ones from 6 ones. However, he can subtract 8 ones from 1 ten by removing 1 ten and pulling 2 ones.

r y b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y y y y y

SUBTRACTION

Subtraction is the inverse of addition. The abacus shows that subtraction "undoes" the "putting together" idea of addition - the idea of "taking away."

Example: 6432
– <u>4683</u>

1. Place the minuend on the abacus by pulling to the right 6 thousands, 4 hundreds, 3 tens, and 2 ones.

r y y y y y y y b r r r r r r g b b b b b g g g g	y y r r r b b b b g g g g g g

2. Subtract 3 ones by removing 1 ten and pulling 7 ones.

r b r r r r r r r g b b b b b g g g g	y y y y y y y y y r r b b b b g g g g g g

3. Subtract 8 tens by removing 1 hundred and pulling 2 tens.

r b r r r r r g b b b b b b g g g g	y y y y y y y y y y r r r r b b b g g g g g g

4. Subtract 6 hundreds by removing 1 thousand and pulling 4 hundreds.

r b r r r r r g b b g g g g g	y y y y y y y y y r r r r b b b b b b b g g g g g

5. Remove 4 thousands.

r b r r r r r g b b g g g g g g g g g	y y y y y y y y y r r r r b b b b b b b g

1 thousand + 7 hundreds + 4 tens + 9 ones
(1000 + 700 + 40 + 9 = 1749)

DIVISION

Division is the inverse of multiplication. Therefore, divisi[illegible] may be thought of as the repeated subtraction of like subtrahends.

Example: 26) 8974

r y y y y y y y	y y y y
b r r	r r r r r r r
g	b b b b b b b b b
g g	g g g g g g g g

1. In determining the trial quotient the thinking could be as follows:

$$26 \times 1000 = 26{,}000$$

$$26 \times 100 = 2{,}600$$

Therefore, the best quotient will be more than 100 but less than 1000. Try 300.

```
26 ) 8974 |
     7800 | 300
```

300 x 26 is the same as 3 x 26 x 100 = 7800

300 sets of 26 are removed.

r y y y y y	y y y y
b r r	r r r r r r r
g b b b b b b b b	b
g g g g g g g g g	g

8974 - 7800 = 1174

2. Thinking:

$$26 \times 100 = 2{,}600$$

$$26 \times 10 = 260$$

Therefore, the second trial quotient is more than 10 but less than 100. If 40 is tried as multiplier, then 4 x 26 x 10 = 1040.

40 sets of 26 are removed.

r y y y y y b r r r r r r g b b b b b b b b g g g g g g g g g g	y y y y r r r b

1170 - 1040 = 130

```
26) 8974 |
    7800 |  300
    1174 |
    1040 |   40
     134
```

3. Thinking:

26 x 10 = 260

26 x 1 = 26

Therefore, the trial quotient will be more than 1 but less than 10. Try 5.

26 x 5 = 130

55 sets of 26 are removed and 4 ones remain.

r y y y y y y y y y b r r r r r r r r r g b b b b b b b b b g g g g g g g g g g	y y y y

134 - 130 = 4

```
26 ) 8974 |
     7800 |  300
     1170 |
     1040 |   40
      134 |
      130 |    5
        4    345
```

8974 ÷ 26 = 345 and remainder 4.

Instruction with this abacus should suggest how a simple counting device can be used in problems involving the four fundamental operations of arithmetic. Most children will find this brief experience with the abacus a decided aid to an understanding of mathematics. Some will want to do more work with this computer. The accompanying bibliography will be helpful in showing further possibilities of this ancient calculating device. Pupils will also be interested in learning that the abacus in its most frequently used form is different from the device described in this paper.

The Chinese abacus has two five-unit beads above a bar and five unit beads below the bar. The Japanese abacus has one five-unit bead above a bar and four unit beads below the bar. Each of these is capable of faster manipulation than the abacus described in this publication and requires more mental agility. The study of other types of abaci is recommended for the teacher or student who is interested in learning more about this ancient but still useful digital computer.

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