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ABSTRACT

Advance organizers (AOs) at two levels of abstraction, each in the presence of objectives, were used to determine the effect of the degree of abstraction on the learning of mathematics concepts; the mathematical content used as the subject of instruction was the system of integers. One first-semester mathematics class of 23 elementary education majors and one second-semester mathematics class of 31 elementary education majors participated. Four videotapes covering topics of addition, multiplication, subtraction, and order of integers were developed. Class activities for each of the four tapes included reading pre-tape information which included AOs, viewing the tape, post-tape activities and discussions, and a homework assignment. The effects of the level of abstraction of the AOs on the learning of the students were measured by a portion of the final examination scheduled about two weeks after the instruction. An analysis showed no statistically significant results between the two levels of AOs. (DT)

ADVANCE ORGANIZERS AND OBJECTIVES IN TEACHING MATHEMATICS

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In the present study advance organizers (AOs) at two levels of abstraction, each in the presence of objectives, were used to determine the effect of the degree of abstraction on the learning of mathematics concepts. It should be noted that the kind of abstraction under consideration is mathematical abstraction. Such an approach is consistent with the defining characteristics of an AO (Ausubel, 1963, pp. 214-215) and exploits the perhaps critically important structural aspects of the content.

Although considerable attention has been given to the use of AOs in improving the effectiveness of instruction (e.g., Ausubel 1960, Weisberg 1970, Kuhn & Novak 1971, Ring & Novak 1971) the use of AOs in mathematics has provided generally inconclusive results (e.g., Scandura & Wells 1967, Romberg & Wilson 1969, Peterson 1970, Peterson, Lovett, Thomas & Bright 1973, Eastman 1972), with most studies reporting no significant differences. In part, this phenomenon may be attributable to the inherent structure of the content of mathematics, though there is virtually no empirical evidence either to support or to refute such an interpretation. Perhaps more important is that in most

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of the cited studies the instruction can be characterized as short-term programmed instruction on a topic not usually encountered by the subjects. The effects of AOs in ordinary classroom instruction on standard topics might be somewhat different than has been observed in more contrived situations.

The mathematical content used as the subject of the instruction was the system of integers. This topic was chosen because of the two natural abstractions that could be employed as AOs--mathematical field and mathematical system. All but one of the field axioms are satisfied in the integers, so this abstraction would seem to be useful in providing students with appropriate ideational anchorage for the concepts of integers. At the same time, however, the similarity of the field axioms to the properties of the integers might suggest that the concept of a mathematical field is neither sufficiently abstract nor inclusive to serve as an effective organizer. For this reason, a comparison of the relative effectiveness of field and mathematical system AOs was proposed, for a mathematical system is a more inclusive concept.

The integers in fact can be considered a mathematical system, since classification as a system is determined solely by the existence of a set of elements with two accompanying binary operations. There are in general no properties satisfied by the operations of all systems. Hence the concept of a system is more general than the concept of a field. The greater degree of generality might suggest that the concept of a system would provide more effective assistance in enhancing the learning of the concepts of the integers. The AOs are included in this report as Appendices A and B.

Two substudies were conducted to test the following hypothesis:

> There is no significant difference in the effects of AOs at different levels of abstraction in the learning of integer concepts.

Instructional Materials

The Es developed four videotapes for use on the closed circuit television system at Northern Illinois University. The tapes were each approximately twenty minutes in duration and covered the topics of addition, multiplication, subtraction, and order of integers. Each tape was accompanied by objectives, pre-tape activities, post-tape activities, and homework problems. The integers were presented via axiomatization, with properties illustrated through the number line, patterns of number sequences, and example problems. Major properties were verified formally through written proof of verbal arguments.

Subjects

Two mathematics classes of elementary education majors at Northern Illinois University (NIU) were used as subjects. One class was a section (N = 23) of first semester mathematics for elementary education majors, and the other was a section (N = 31) of second semester mathematics for elementary education majors. Only one semester of mathematics content is required for elementary certification at NIU, so the subjects in the second semester course had elected to enroll. A third class was originally included in the study, but during one of the playback sessions, the scheduled tape was not shown. Hence this class was dropped from the analysis.

Procedures

The AOs were printed on individual pages which were randomly ordered and distributed to the subjects immediately before the viewing of the first TV tape. Subjects were asked to write their names at the top of the page, to read the information, and to return the page to the instructor. The pre-tape exercises were completed, the first tape was played over the university closed-circuit system, and post-tape activities were completed and discussed. Discussion was led by the course instructor (one of the Es) assigned to the class. Questions about the AOs did not arise. Homework problems were assigned for out-of-class completion.

The remaining tapes were viewed during subsequent classes. In each instance, the following sequence of events was employed: complete pre-tape activities, view tape, complete post-tape activities and discussion, assign homework.

In substudy I, the Ss, students in the first semester course, were shown the first three tapes. In substudy II, the Ss, students in the second semester course, were shown all four tapes.

The procedures were designed to be as consistent as possible with the normal classroom procedures and philosophy of the instructors. The procedures included opportunity for student-teacher interaction, which had been encouraged in both classes prior to the start of the experiment. TV tapes had not been used in either class, though many students at NIU receive instruction via TV in at least one class during their first two years.

In each case, the effects of the AOs on the learning of the students were measured by a portion of the final examination, scheduled about two weeks after the instruction. In substudy I, ten questions were used as the criterion measure; and in substudy II, five questions were used as the criterion measure. Test questions are included in this report as Appendices C and D.

Results

The means and standard deviations of the test scores of the subjects are reported in Table 1.

INSERT TABLE 1 ABOUT HERE

The sample variances in substudy II were found to be significantly different ($F = 4.08$, $df = 11/18$, $p < .01$), so the distributions of scores were compared by computing Kolmogorov-Smirnov statistics. (Siegel, 1956, pp. 127-136) In order to facilitate interpretation of the results, the distributions of scores in substudy I were compared in the same way. The distributions of scores are presented in Table 2.

INSERT TABLE 2 ABOUT HERE

Since the sample sizes were small, the Kolmogorov-Smirnov conservative approximation to chi square, $\chi^2 = \frac{4D^2 N_1 N_2}{N_1 + N_2}$, $df = 2$, was used, for which D = absolute value of maximum deviation between cumulative percentage distribution. (Siegel, 1956, p. 134) For substudy I, $D = \frac{11}{14} - \frac{5}{9} = .23$ and $\chi^2 = 1.16$; and for substudy II, $D = \frac{3}{12} - \frac{0}{19} = .25$ and $\chi^2 = 1.84$. Neither χ^2 value was statistically significant at the .05 level.

Discussion

The failure to obtain statistically significant results might in part be attributed to the classroom setting of the study. That is, the subjects may have shared the information contained in the AOs. Too, the backgrounds of the subjects with respect to their knowledge of the integers was not measured. Extensive prior knowledge of the content would tend to obscure the effects of the treatments. On the other hand, completely effective instruction would also obscure the effects of the AOs. On the basis of the data of Table 1, however, this explanation is discounted.

Other possible causes of lack of differential results may be gleaned from an examination of the AOs themselves. Numerous questions concerning the role of the AOs remain unanswered. Were the AOs of sufficient length relative to the total instructional time to have an effect? Did the subjects learn any of the content of the AOs? What effect does learning the content of the AO have on learning new material? More generally, can material which has been previously learned serve effectively as an AO for new material?

The experimenters feel that the major flaws of the study lie in the uncontrolled previous knowledge of the subjects and the probable ineffectiveness of the short AOs. It is interesting to note, however, first that in both substudies the group that received the field axioms AO displayed greater variability in their scores than the group that received the mathematical systems AO; and second, as has been explained earlier, that the content of the field axioms AO was more nearly at the same level of abstraction than the mathematical systems AO.

Future studies are planned which wi attempt to examine in more detail the uses of AOs in mathematics instruction. The present experiment adds information to the study of AOs primarily through the questions which are suggested.

TABLE I

Means and Standard Deviations of Test Scores

Type of AC	N	Mean	Std. Dev.
	Substudy I		
Field Axioms	9	6.89[a]	2.51
Mathematical Systems	14	6.21	2.16
	Substudy II		
Field Axioms	12	3.50[b]	2.12
Mathematical Systems	19	4.05	1.05

[a]Total possible score is 10.

[b]Total possible score is 5.

TABLE 2

Cumulative Frequency Distributions of Scores

Type of AO	Test Scores										
	0	1	2	3	4	5	6	7	8	9	10
Substudy I											
Field Axioms	0	0	0	1	1	3	4	5	7	7	9
Mathematical Systems	0	0	1	1	3	4	8	11	12	13	14
Substudy II											
Field Axioms	1	3	3	4	7	12					
Mathematical Systems	0	0	2	5	11	19					

APPENDIX A

Advance Organizer: Mathematical Field

During the next few days you will learn about the integers. The set of integers together with two operations that will be explained later is almost a mathematical field. The concept of mathematical field is very important in many branches of mathematics.

In general a set of elements and two binary operations, such as + and ×, defined on pairs of these elements form a field when the following properties are satisfied:

+ and × are commutative operations

+ and × are associative operations

+ and × have distinct identity elements

every element has an additive inverse

every element except the additive identity has a multiplicative inverse

× is distributive over +

Some of these properties are probably familiar to you, but you might not recognize all of them. As you learn about the integers you may find it helpful to recall these properties.

APPENDIX B

Advance Organizer: Mathematical System

A very important concept in mathematics is mathematical system. A mathematical system is a set of elements with one or more binary operations defined for pairs of these elements. You have already encountered the system of whole numbers. The set was

$$W = \{0,1,2,3,4,\ldots\}$$

and the binary operations were + and $\times$. To be completely correct we should write $(W,+,\times)$ when we talk about the system of whole numbers. Actually we often use the short-hand notation W in place of the complete notation $(W,+,\times)$.

During the next few days you will learn about another mathematical system--the system of integers. The set of integers contains the set of whole numbers as a subset, and the operations on the integers will still be called addition and multiplication. The old ideas of these operations, however, will have to be expanded.

APPENDIX C

Test Questions, Substudy I

Questions 1 through 3 refer to steps in this proof that

For all a, b in $\mathbb{Z}$, $a({}^-b) = {}^-(ab)$

Proof:
1) $ab + a({}^-b) = a(b + {}^-b)$
2) $ab + a({}^-b) = a \cdot 0$
3) $ab + a({}^-b) = 0$
4) $ab + a({}^-b) = ab + {}^-(ab)$
5) so, $a({}^-b) = {}^-(ab)$

1. The justification for step 1 is

   A. associativity of $+$ in $\mathbb{Z}$
   B. distributivity of $\times$ over $+$ in $\mathbb{Z}$
   C. the additive inverse property in $\mathbb{Z}$
   D. additive cancellation in $\mathbb{Z}$
   E. not listed here

2. The reason that step 4 follows from step 3 is

   A. associativity of $+$ in $\mathbb{Z}$
   B. distributivity of $\times$ over $+$ in $\mathbb{Z}$
   C. the additive inverse property in $\mathbb{Z}$
   D. additive cancellation in $\mathbb{Z}$
   E. not listed here

3. The reason that step 5 follows from step 4 is

   A. associativity of $+$ in $\mathbb{Z}$
   B. distributivity of $\times$ over $+$ in $\mathbb{Z}$
   C. the additive inverse property in $\mathbb{Z}$
   D. additive cancellation in $\mathbb{Z}$
   E. not listed here

4. Let m, n be integers. Then by definition $m - n$

A. is defined if and only if $m \geq n$
B. equals $p \in \mathbb{Z}$ for which $n + p = m$
C. equals $d \in \mathbb{Z}$ for which $m + d = n$
D. is that element of $\mathbb{Z}$ that makes $\underline{m}$ equal to $\underline{n}$

5. The set of <u>negative</u> integers is closed with respect to

A. addition and subtraction, but not multiplication
B. addition and multiplication, but not subtraction
C. addition, but neither subtraction nor multiplication
D. subtraction, but neither addition nor multiplication
E. none of A, B, C, D

6. The solution set in $\mathbb{Z}$ of ${}^{-}4n - 6 = {}^{-}14$ is

A. $\emptyset$
B. $\{5\}$
C. $\{-5\}$
D. $\{-2\}$
E. not listed here

7. Which one of the following is <u>FALSE?</u>
For all integers p and q,

A. ${}^{-}0 \cdot (p \cdot {}^{-}q) = 0$
B. ${}^{-}({}^{-}p - q) = p + q$
C. ${}^{-}(p + {}^{-}q) = {}^{-}p + q$
D. ${}^{-}({}^{-}p \cdot {}^{-}q) = pq$
E. All of A, B, C, D are false

8. The solution set in $\mathbb{Z}$ of

$$n(n - 6)(2n + 5)({}^{-}3n - 6) = 0$$

is

A. $\emptyset$
B. $\{0, {}^{-}2\frac{1}{2}, 2, 6\}$
C. $\{0, {}^{-}2\frac{1}{2}, {}^{-}2, {}^{-}6\}$
D. $\{0, {}^{-}2, 6\}$
E. not listed here

9. Let n be a negative integer and p be a positive integer. Consider

I. $n + p$ is always a positive integer.
II. $p - n$ is always a positive integer.
III. $n \cdot p$ is always a negative integer.
IV. $^{-}(n \cdot n)$ is always a negative integer.

Which of the following is true?

A. Only III is true.
B. Only III and IV are true.
C. Only I and III are true.
D. Only II, III, and IV are true.
E. None of A, B, C, D is true.

10. The solution set in $\mathbb{Z}$ of

$$(n - {}^{-}51) - ({}^{-}51 - 3n) = 2n - ({}^{-}38 - {}^{-}56)$$

is

A. $\emptyset$
B. $\{30\}$
C. $\{{}^{-}21\}$
D. $\{21\}$
E. not listed here

APPENDIX D

Test Questions, Substudy II

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1. If a and b are integers, then $(-a)(-b)$ is

   A. always positive
   B. always negative
   C. never negative
   D. may be positive, zero, or negative depending on a and b

2. Which of the following closure properties is not a property of the system of integers?

   A. addition
   B. subtraction
   C. division
   D. multiplication

3. Which of the following is <u>not</u> a group?

   A. The integers under addition
   B. The rigid motions under composition
   C. The set of reflections under composition
   D. The set of translations under composition

4. If a, b, c are integers, which of the following is true?

   A. $a - (b + c) = (a - b) + c$
   B. $a - (b - c) = (a - b) + c$
   C. $a - (b - c) = (a + b) - c$
   D. $a - (b + c) = (a + b) - c$

5. If a, b, c are integers, $a \geq b$, $c < 0$, then which of the following is true?

   A. $ac \geq bc$
   B. $ac > bc$
   C. $ac < bc$
   D. $ac \leq bc$

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