Feedback management in an individually-paced instruction system can be mathematically analyzed by the use of computer simulation models. Because of the student "down time" or waiting time associated with individualized instruction situations, reinforcement activities have been reduced to less than ideal levels. By proper time management the student is insured of making at least average continuous progress through the curriculum while receiving an acceptable level of reinforcement through the successful completion of tasks. This theoretical study used DYNAMO II programing on an IBM 360 computer to run several simulation models. In each case the pacing variable was found to be of primary importance to the success of the overall instructional system. (MC)
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THE DYNAMICS OF INSTRUCTION SYSTEM
FEEDBACK CONTROL IN INDIVIDUALLY-
PACED INSTRUCTION

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Abstract: Many contemporary developments in instruction use behavioral models of the learning process which demand repetitive application of a series of events to each student. These events — summarized by such terms as prescribe, instruct, evaluate, — require a set of decision rules or procedures which the teacher may use to minimize delays in student progress. To date, problems associated with the management of such systems have been dealt with primarily by prescription and not by analysis of relationships inherent in the instruction model. This paper illustrates the use of feedback control theory in the analysis of one such instruction system. The dynamics of that system are examined and several management practices are discussed and tested through simulation of the operation of the instruction system.
Behavioral Approaches to Instruction: Management Problems

The behavioral models of learning underlying many systems of individualized instruction used in schools today are represented by the set of procedures or steps whereby the student is moved through the subject matter. These steps are, in general, analogous to the events encountered in programmed instruction sequences and in the operant conditioning paradigm. As in programmed instruction, subject matter is divided into small units arrayed in a sequence defined by the internal order of the subject matter. These units or tasks are presented to the student requiring his response to questions or other stimuli and his work is evaluated following each task. If the student completes a task successfully, he is allowed to continue to the next task in sequence much as he would were his learning managed by a computer-controlled program.

In addition to the sequence of steps derived from programmed instruction, individualized instruction systems make use of reinforcement principles derived from operant conditioning. The above instruction model of prescribe, teach, student responds, evaluate, is augmented by some form of reinforcement following acceptable student responses. This reinforcement may be in several forms such as confirmation of correct responses, teacher praise or presentation of extrinsic reinforcers. The resulting steps in the instruction are diagrammed as follows:
FIGURE I
A Typical Individualized Instruction Sequence

Tasks In Progress

tasks completed

Tasks Being Evaluated By Teacher

remedial work completed

tasks needing remedial work

tasks ready for mastery test

Remedial Work On Tasks

Tasks Being Tested For Mastery

tasks needing remedial work

tasks to be re-done

re-cycling to next task in sequence
This sequence suggests several essential provisions which the teacher must make if the instruction of several students is to proceed smoothly. First, the tasks assigned students must be matched to their level of performance. The "operant level" of each child's academic behavior must be measured and charted over time to serve as a guide for determining the student's capacity to respond to instructional tasks. Second, evaluation of student performance must be based on measured task outcomes and must follow his behavior as closely as possible in time. Finally, the tutorial or remedial assistance given students with learning difficulties should be organized to bring the student back into the instruction "mainstream" with as little lost time as possible.

The sequence also points to classes of behavior which must be organized and managed in implementation of an instruction model of this type in the classroom. These behaviors relate to assignment of tasks, task performance, teacher help, and evaluation. In most applications, these activities involve some functional division of labor among instruction staff. Thus, teachers may take major responsibility for task assignment and assisting students while teacher's aides may carry out evaluation and reinforcement management functions. In any event, a finite amount of staff time is allocated across several instructional activities with the objective being smooth and continuous student progress.

Once the above activities are set into motion in a particular milieu, certain logistical problems arise which make the attainment of the student progress objective difficult, if not impossible. As children move through their work, they may encounter delays due to needs for help.
in completing tasks, unavailability of teacher assistance at given times and actual loss of task time while receiving assistance. After completing tasks, students may be delayed in waiting for the teacher or aide to evaluate their work and will, of course, forfeit the time required for actual evaluation. Reinforcement activities may produce no delays in student progress through the instructional events, but the assignment of new tasks following reinforcement may result in delays much like those associated with teacher assistance.

In short, individualized instruction systems tend to generate waiting lines. These lines represent student 'down time' and a lack of productivity both individually and collectively. It seems intuitive and obvious that these problems can be solved by some mix of change in task time, reinforcement and evaluation practices, or in allocation of teacher and aide time; a more nearly optimum student behavior pattern may result. Unfortunately, intuition is unlikely to produce the desired solution and may enhance the conditions making for system problems. (Forrester, 1968, ch. 6) Clearly, a mode of analysis is needed which takes into account the interdependencies among instructional events and the fact that decisions made in present time may result in changes in student and staff behavior which may not be observed for several cycles of the instructional process.
Feedback Control in Instruction

The link between the learning of individual students and the over-all operation of an instruction system lies in the apportionment of time to student learning tasks. As Carroll (1963) notes, ... "the learner will succeed in learning a given task to the extent that he spends the amount of time that he needs to learn the task." p. 725.

Once a task has been assigned to a student, learning is determined by factors peculiar to the task on one hand and to the student on the other - both factors are, in turn, observable in the instruction system in terms of time.

Tasks are representative of varying levels of internal difficulty. This sets lower limits on the time needed to complete a given task. Presumably, each task has associated with it a probability distribution which expresses the likelihood that the task can be completed in a stated time period. Similarly, student capacity to complete tasks varies from one child to another. If task difficulty is held constant, some students are more likely to exhibit rapid task completion while others are slower to finish assigned work.

It is the function of the teacher and her assistants to insure that each child has a task assignment within the range of his capability. This is done by observing completion times and error rates. By minimizing error and holding to an average completion time, the teacher insures that each student in making at least average continuous progress through the curriculum and is, at the same time, receiving a level of reinforcement through successful completion of tasks.

The attainment of optimum allocation of student times to tasks is carried out in the instruction system by manipulation of staff behavior.
By moving staff activity from one function - such as evaluation - to another - such as teacher help - the teacher attempts to arrive at a controlled movement of students from one activity to another with minimum lost time. This is accomplished by a feedback of information to control teacher and aide behavior. Like other control problems, the teacher is using information about some feature of the instruction system to alter one or more of its dynamic aspects so that a system goal can be attained. To see how this is done, let us look more closely at the instruction process as a feedback system.

Individualized instruction is characterized by a flow of students from one class of activity to another. In the system described earlier, students may be working on tasks, waiting for or getting help, waiting for or receiving evaluation, or, finally, waiting for and receiving mastery testing. Each of these classes of activity are levels in the instruction system. They represent the descriptive categories a teacher or observer would use to picture the condition or state of the system if all movement were brought to rest. Levels also generate the data the teacher uses to manage the system. By observing the numbers of children in the various activities, the teacher infers needed change in system operation practices. These changes are brought about by control over movement from one level to another.

The student movements which increase and decrease the numbers of pupils in levels are called flows. Flows are expressed in numbers of students per minute or other time unit. Flows are central to system management for they are the points where teacher control is felt. For example, by controlling the size or difficulty of tasks assigned the teacher can influence the flow of students who complete tasks. This is represented in Figure II:
In Figure II, the dotted lines show information links. The means for controlling the number of tasks in progress is suggested by the flow of information through "task assign policy" to alter the flow of newly assigned tasks (ASGN). However, instructional activity is not completely under the control of the teacher. The triangle labelled "performance" in Figure II refers to a parameter of this simple system which is the statistical performance capabilities of students on tasks. The performance parameter controls the task completion rate (COMP) and has a restrictive effect on the smooth flow of assigned work to completion desired by the teacher. The resulting behavior of this system can be simulated by a simple mathematical procedure (Forrester, 1968).

To analyse the dynamics of models like that pictured in Figure II we need to express the human "physics" of instruction in a mathematical language. This is done by preparing difference equations which account for the flow of tasks through the assignment, in-progress at a given point in time (time k).
is given by the sum of tasks in progress at prior to time k (time j) plus the new flow of assignment and completion occurring between time j and time K. This intervening period is labelled (DT) and is equivalent in our analysis to the use of "dt" or "delta time" in formal mathematical notation. The resulting equation describing the condition of the TASK level is:

\[ \text{TASK}_k = \text{TASK}_j + \text{DT}(\text{ASGN}_{jk} - \text{COMP}_{jk}) \]

In order to complete a mathematical description of this system, we must also specify the ways the two flows (ASGN and COMP) are determined. For purposes of illustration, we assume that there is a constant assignment rate for new tasks equal to 10 tasks/day. This number is derived from a time sampling of actual teacher behavior in an elementary school where five teachers manage the instruction of some 180 students. Observation of learner performance in the same setting indicates that the time students spend on tasks is normally distributed with a mean of 7 days and a standard deviation of 4 days. To arrive at momentary performance rate in our system simulation, we sample from this distribution. The equations representing these system controls are:

\[ \text{TASK} = 100 \text{ (initial value)} \]
\[ \text{ASGN}_{k1} = 10 \text{ tasks/day} \]
\[ \text{COMP}_{k1} = \frac{\text{TASK}_k}{\text{REST}_k} \]
\[ \text{REST}_k = \text{FIFGE}(\text{PERF}_k, 7, \text{PERF}_k, 1) \]
\[ \text{PERF}_k = \text{NORMRN}(7,4) \]

The use of the term \text{REST} in the above equations, is as a dummy variable to restrict sampling from our normal distribution to positive values. The \text{NORMRN} statement is the appropriate instruction for computer sampling of the statistical distribution of student task performance times.
This simulation and others reported in this paper, was carried out using DYNAMO II on the IRM 360 computer (Pugh, 1970). The results of a typical simulation run on this simple model are shown in Figure III.

FIGURE III
Response Curve For TASK
Level, Constant Input
The analysis of the system dynamics pictured in Figure III suggest some important properties of the system with regard to its response to attempts at control. Control, we recall, is exercised in this system by adjustment of the assignment rate (ASGN). In this simulation, assignment rate is constant at ten tasks/day. The resulting behavior of the system - as represented in the number of tasks in progress (T in Figure III) - varies around the value of sixty (60) tasks. This result is, of course, dependent entirely on the sample values of student completion time (COMP) and is heavily influenced by an initial extreme value for completion time where dramatic reductions in the number of tasks in progress are experienced. However, following that extreme value, we see that assignment rate and completion rate are nearly in balance. To balance this system completely, it would be necessary to adjust assignment rate to student average task performance time which was observed to be 7 day/task.

To see how control adjustment might be made in the above case, let's examine the defining equation for COMP more closely. In the set of equations (1) we see that COMP is defined as:

\[ COMP_{kl} = \frac{\text{TASK}_k}{\text{REST}_k} \]

where \( \text{REST}_k \) is the sample value of task time suitably restricted to positive values. If we let \( \text{REST}_k = 7 \) days - the average value of our task time distribution - we can compute the average value of the COMP flow given a particular level of tasks. For example, if we presently have some 40 tasks in progress, our completion flow will be very nearly 6 tasks/day, as:

\[ COMP_{kl} = \frac{40 \text{ tasks}}{7 \text{ days}} = \frac{6}{\text{task/day}} \]

Thus, we could balance our simple system to continue with approximately 40 tasks in progress at any time by adjusting our assignment rate to approximately 6 tasks per day. We have essentially adjusted inflow to equal outflow in order to control this simple system.
Obviously, the above control example is far too simple in that it fails to account for subsequent events in the instruction cycle and for the flow of students through other stations and activities in the classroom. This can only be done by an expansion of our simple instruction model as illustrated in Figure IV.
The symbolism of Figure IV is chosen to represent the classroom activities being modeled.

ASGN = task assignment rate (teacher controlled)

TASK = tasks in progress

COMP = task completion rate (controlled by students)

EVAL = tasks being evaluated

REDO = tasks needing additional work (controlled by student errors in task work)

SPEC = special help request rate (controlled by student learning problems)

TEST = tasks ready for test rate (controlled by adequacy of student task performance)

EXAM = task being tested for mastery performance

PASS = task mastery rate (controlled by student mastery test performance)

FAIL = mastery test failure rate (controlled by student mastery test performance)

REMD = tasks receiving remedial assistance from staff

DONE = rate of completion of remedial work (controlled by student learning performance)

TOTL = running total completed by all students

The system equations which express the dynamic relationships in this instruction system are shown in Figure V.
FIGURE V
Simulation Equations
First Run

L TASK.K = TASK.J + DT*(ASGN.JK + REDU.JK - COMP.JK)
N TASK = 190
R ASGN.KL = 24
R RECQ.KL = DELAY3(EVAL.K, .04)*.15
R COMP.KL = TASK.K/REST.K
A REST.K = IFGE(PERF.K, 7, PERF.K, 1)
A PERF.K = NORMRN(7, 3)
L EVAL.K = EVAL.J + DT*(COMP.JK + DCNE.JK - TEST.JK - REDC.JK - SPEC.J)
N EVAL = 0
R DGCL.KL = REMD.K/RHEL.K
A RHEL.K = IFGE(HELP.K, 2, HELP.K, 1)
A HELP.K = NORMRN(2, 4)
R TEST.KL = DELAY3(EVAL.K, .06)*.90
R SPEC.KL = DELAY3(EVAL.K, .04)*.05
A REPD.K = REMD.J + DT*(SPEC.JK - FAIL.JK - DCNE.JK)
N REMD = 0
L EXAM.K = EXAM.J + DT*(TEST.JK - FAIL.JK - PASS.JK)
N EXAM = 0
R FAIL.KL = DELAY3(EXAM.K, .4)*.10
R PASS.KL = DELAY3(EXAM.K, .4)*.90
L TOTL.K = TOTL.J + DT*iPASS.JK)
N TOTL = 0
S SPEC CT = .1/LENGTH = 160/PRTPER = 2/PLTPER = 2
PRINT TASK, EVAL, REMD, EXAM, TOTL(3, 2)
PLDT TASK = 1/REAL = R, EXAM = X, EVAL = E/TOTL = L
RUN IMPACT MODEL

EXponent OVERFLOW IN EON FOR SPEC. 353 AT TIME = 8.7
In this set of equations, FIFGE and NORMRN serve the limiting and sampling functions discussed earlier. DELAY 3 is a third-order delay or "pipeline" delay which produces an output change equal to alterations in input after a given delay time (Forrester, 1968, p. 8-23). SPEC, PRINT, PLOT and RUN are computer instructions for carrying out the simulation and tabulating results.

Note, also, that a second probability sampling has been introduced into the set of system equations. The outflow from system remedial activities (DONE) is controlled by sampling from a normal probability distribution of mean 2 days and standard deviation of .4 days. As is true for other system parameters, these values are determined by time sampling of actual student performance on remedial work in the experimental classroom. In several cases we have also added "dummy" variables to assist in controlling the range of variation of certain system variables - this is the case in the use of HELP in the above set of equations.

If we begin our simulation with all 190 students in the TASK activity, and assign new tasks as the rate of 24 tasks/day, we produce the system dynamics graphed in Figure VI.

Two significant observations can be drawn from the dynamics shown in Figure VI. First the system quickly establishes a number of students in the remedial activity (REMD) and keeps this number relatively constant during the entire 180 day simulation period. This behavior corresponds to observations made by the authors in several individualized instruction systems. Some students, due to their incapacity to progress at some minimum rate are always in a remedial "loop" in the instruction cycle. This is generally an activity which allows the teacher to prompt desired behavior and, in effect, to "carry" students along with little active student engagement with
FIGURE VI
Complete Instruction System
First Simulation Output
Second, the total number of tasks being processed in the system is increased during the 180 day simulation period. Beginning with 190 tasks (or one task per student) in progress at the beginning of the simulation, we see a steady increase so that the total tasks in progress exceeds 250 tasks later in the simulation. This is due to the constant assignment rate of 24 tasks/day and to the fact that the completion rate is unable to keep pace with this input of work. The result is that the system is overdriven and some students in the experimental classroom have more than one task in progress at a time.

The latter point illustrates the importance of pacing controls in those instruction systems based on individual progress through curriculum materials. Unless pacing is closely geared to student performance capability, those systems will quickly load students with task materials and increase the probability that the resulting pressure will lead to student failure in task performance (Siegel and Wolf, 1963). Overdriving the instruction system also runs counter to the research base underlying individualized instruction. Student performance is optimized only by arranging for individual student mastery of each task prior to assignment of additional work. Under overdriven conditions, this basic condition cannot be met.

Clearly, these features of the system ought to be corrected in the interests of attainment of system objectives. An attempt is made at this in Simulation II. Here we alter the pacing rule to conform to our notions of appropriate individualized instruction practices. We now assign, not a standard daily rate, but only tasks for those students who have successfully completed mastery testing.
This closes the system and insures that only one task will be assigned to each student. We express this new policy in the following set of system equations in the statement for ASCN and its associated "dummy" variable DIFF.

Equations (4) are the complete set of computer instructions used for our second try at making our instruction system perform up to specifications. Note that the equations defining ASGN and DIFF accomplish our new instruction management policy goal. First, the PIFGE limit on ASGN insures that this flow will always be either positive or zero. Second, the equation for DIFF guarantees that the new task added to the TASK level will correspond directly to children in a way that our first simulation fails to allow. DIFF is a statement for the difference between the number of tasks (children) in all of the system levels and the total number of children in the system (190). Thus, if all students are currently engaged in system activities, no new tasks are assigned. Only when fewer than 190 tasks (students) are present in the total of all the levels, do we find a positive value for ASCN.

Now, let's see if this change brings us closer to our design objectives. Study the simulation output in Figure VIII (page 19).
FIGURE VII
Simulation Equations
Second Run

L  TASK.K = TASK.J + DT*(ASGN.JK + REDO.JK - COMP.JK)
N  TASK = 190
R  ASGN.KL = FIFGE(FIFF.E, K, 0, FIFF.E, K, 0)
A  DIFF.K = 190 - TASK.K - EVAL.K - EXAM.K - REDO.K
R  COMP.KL = TASK.K / REST.K
A  REST.K = FIFGE(PERF.K, 7, PERF.K, 1)
L  EVAL.K = EVAL.J + DT*(COMP.JK + DONE.JK - TEST.JK - REDO.JK - SPEC.KK)
N  EVAL = 0
R  REDO.KL = EVAL.K * 0.15
R  TEST.KL = EVAL.K * 0.80
R  SPEC.KL = EVAL.K * 0.05
R  DONE.KL = REDO.KK / HELP.K
A  HELP.K = NORMRN(7, 4)
L  REMO.K = REMO.J + DT*(SPEC.KK + FAIL.KK - DONE.KK)
N  REMO = 0
L  EXAM.K = EXAM.J + DT*(TEST.KK - FAIL.KK - PASS.KK)
N  EXAM = 0
R  PASS.KL = DELAY3(EXAM.KK, 4) * 0.90
R  FAIL.KL = DELAY3(EXAM.KK, 4) * 10
L  TOTL.K = TOTL.J + DT*(PASS.KK)
N  TOTL = 0
SPEC  DT = 0.1 / LENGTH = 180 / PRITPER = 2 / PLTPER = 2
PRINT  TASK, EVAL, REMO, EXAM, TOTL (3, 2)
PLOT  TASK = T / REMO = R, EXAM = X, EVAL = E / TOTL = L
RUN  IMPACT MODEL
FIGURE VIII
Complete Instruction System
Second Run
This change in assignment policy brings the output of the simulation within reasonable values. By controlling the total number of tasks such that there is only one task per student at a given time, we have avoided the overdriven condition observed earlier. If we make a spot check of the total number of tasks in progress at any given day in Figure II, we will observe that the total is always 190 - which corresponds to the total number of students in the simulated system. Also, the loading of the several instructional activities are relatively uniform over time. Despite continuous variation in each of the levels, there are no unwarranted excursions in any level. We can therefore say that the system is under control for all practical purposes.

However, the simulation does not permit attainment of the earlier pacing goal. Note that the total number of tasks attained in the 180 day simulation period is approximately 3000. This is a good bit less than the 4000+ tasks the system had to complete if every student were to master a year's worth of work in the traditional sense of uniform pacing. The reason the system is unable to realize this goal is that the pattern of student task completion is unequal to the rate necessary. This means that any attempt at more complete control of the system has to center attention on the rate of student performance and reduce the mean time associated with task completion. In fact, some way must be found to reduce mean completion time to the order of 3-4 days/task is the goal of 4000+ tasks completed is to be attained by the end of the 180 day period. This requirement has been verified by trying alternate normal distributions with several mean values (Miller, 1972).
Conclusion

It may be helpful for the reader to think of the previous example as an instance of educational engineering. What we have done is, in fact, quite similar to the engineer's approach to a system control problem. We have shown that a working instruction system can be modelled and, further, that such a model can be analysed to determine the utility of certain control policies. In the example illustrated, the results of simulated control policies were shared with teachers working in the experimental classroom and were the stimulus for change in pacing practices. An instruction design has been "engineered" so that its management practices are in accord with our best estimate of the human "physics" of learning.

Control problems like these peculiar to instruction are found in large numbers throughout human organizations. By treating them as engineering problems we are likely to take into account multiple causation as expressed in the interdependencies among a large set of organizational variables. The general framework for this approach to organization management has been developed by the authors (Ammentorp and Foster, 1973). By building on the organization engineering perspective (Forrester, 1961), a large number of complex educational problems can be analysed to determine the likely effects of proposed management practices and changes in educational policies.
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