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Theoretical Paper No. 40

LEVELS OF CONCEPT ATTAINMENT AND
THE RELATED COGNITIVE OPERATIONS

by

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and Dorothy A. Prayer

Report from the
Operations and Processes of
Learning Component of Program 1

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Statement of Focus

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programming for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints—financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programming model will lead to higher student achievement and self-direction in learning and in conduct and also to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.
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Abstract

This paper is a refinement of a model of conceptual learning and development presented initially as the Presidential Address to Division 15, Educational Psychology, of the American Psychological Association in September 1971, Washington, D.C. Thus it superseded the initial formulation in explicitness and detail.

This paper explains the nature of concepts with which the model deals. An overview of the model is presented which indicates that concepts are attained at four successive levels—concrete, identity, classificatory, and formal. It further explains how concepts attained at the classificatory or formal level may be used. The cognitive operations essential for attaining concepts at each of the successive levels are indicated. The role of language in the attainment of concepts is examined. The various internal and external conditions of concept learning are indicated. The research and theory on which the model is based are presented.
Introduction

The term "concept" is used in one context to indicate publicly defined entities that comprise a substantial part of the organized knowledge of the various disciplines which scholars have been amassing for generations. As such, concepts are the primary substance of education in the English language arts, mathematics, the sciences, and the social sciences at all levels of schooling, elementary through graduate. In another context, the term "concept" is used to indicate mental constructs of individuals which enable them to think about and relate instances and classes of things—objects, events, and processes—in the absence of actual instances of them.

A substantial amount of research has been done on concept learning during the past two decades. One type of research deals with the logical analysis of concepts within various subject matter fields; another type of research deals with how individuals learn concepts. Despite research of both types, at present there is no adequate description of the cognitive operations involved in the attainment of concepts at specific levels of mastery by individuals whose abilities change in predictable ways with age. We use the term "age" as a shorthand term for the combined effects of experience and maturation; age, not so, is not considered a determining factor of how well individuals can perform.

Much of this paper is given to describing a model of the cognitive operations in concept learning. The model was first reported in a highly condensed form by Klausmeier (1971). In this paper, the model is described more completely, but should still not be regarded as a psychological model as defined by Marx (1970). Marx limits the term "model" to "a conceptual framework or structure that has been successfully developed in one field and is now applied, primarily as a guide to research and thinking, in some other, usually less-well-developed field" (p. 11). And further, "When a model is used in the manner described, essentially as a guide to research, there is no intention that it (the model) be modified as a consequence of the results of that research..." (p. 11). The present model is not drawn from another field, but from the field of concept learning itself. It is intended to guide research and thinking regarding concept learning and concept development, but will probably be modified based on the results of that research. We refer to it as a model because it provides a framework for specifying and relating levels of concept attainment and use, and specifying the cognitive operations involved at the various levels. The levels of concept mastery and the operations at each level have been identified through logical analysis and through empirical research in laboratory and school settings carried out at the Wisconsin Research and Development Center and other research laboratories.

Our model is both similar to and different from four theories of concept learning generated by American experimental psychologists and reviewed by Bourne, Ekstrand, and Dominowski (1971): theory of associations (Bourne & Restle, 1959), theory of hypotheses (Levine, 1966; Trabasso & Bower, 1968), theory of mediation (Osgood, 1953), and theory of information processing (Hunt, 1962). Our model is most similar to Hunt's theory in that both incorporate information-processing constructs. Both our model and the preceding theories imply that all the concepts held by any individual are learned; they do not emerge simply with maturation. Thus our model, in agreement with American theories, specifies that the attainment of concepts is potentially explainable in terms of principles of learning. Our model differs from the four theories just mentioned in that it postulates different levels in the attainment of the same concept and specifies the cognitive operations involved in the attainment of a concept at each level. Many operations are postulated to be common to more than one level of
Our model is also intimately related to our view of concepts and the related experimentation with subjects ranging in age from three years to young adulthood. Because the methods used in studying concept learning are related to one's understanding of "concept," a brief discussion of the nature of concepts precedes the overview of the model.
II

The Nature of Concepts

Earlier we introduced the term "concept" to refer to the mental constructs that individuals and cultural public entities use to organize, classify, and make sense of the world. In this section, we will discuss some of the implications of this view of concepts.

In concert with these ideas, we define a concept as organized information about the properties of one or more things—objects, events, or processes—that enables an individual or a class of things to be differentiated from one another or related to other things or classes of things.

In connection with concepts as mental constructs, it is noted that each maturing individual attains concepts according to his unique learning experiences and maturational patterns. In turn, the concepts he attains are used in his thinking about the physical and social world. The role of concepts in explanations of thinking is stated well by Kagan (1966, p. 97):

"Conce...
Despite the importance of concepts, formal definitions of the word "concept" by the experts who study concepts vary so widely that communication is impeded across and even within disciplines (Flavell, 1972; Klausmeier & Harris, 1966). Definitions vary apparently for three reasons: the number of entities called concepts is very large; there are real differences in the nature of concepts both across and within disciplines and an individual's concept of the same thing or classes of things changes markedly with increasing maturational and learning.

As a starting point for delineating a large programmatic research effort, Klausmeier, Davis, Ramsay, Frederick, and Davies (1965) formulated a conception of "concept" in terms of defining attributes and values which they identified as common to many concepts from various disciplines. Scholars at the Wisconsin R & D Center (Golub, Frederick, Nelson & Frayer, 1971; Romberg, Steltz & Frayer, 1971; Tabachnick, Weible, & Frayer, 1970; Velerker, Sorenson, & Frayer, 1971) and Markle and Tiemann (1969) have now demonstrated that analysis of a particular concept in terms of its defining and irrelevant attributes is useful in clarifying its meaning. Flavell (1970) has indicated that a formal definition of "concept" in terms of its defining attributes is useful in specifying what concepts are and are not and also in identifying the great variability among concepts. Therefore, we shall use this method of analysis in formally defining the word "concept".

The eight attributes of "concept" are learnability, usability, validity, generality, power, structure, instance numerosness, and instance perceptibility. These attributes are presumed to be applicable to any public concept, that is, to the societally accepted meaning of any word that stands for a concept. Experts in the various disciplines may either by consensual agreement or through empirical research relate each of the attributes to any particular concept. Take, for example, the concept of verb. Psychologists might reach consensus concerning the learnability and usability of verb on the basis of existing knowledge, but they would more likely have to experiment to ascertain the extent to which individuals of varying characteristics are able to attain a concept of verb. Similarly, linguists might reach consensus on the validity and generality of verb, or they might also carry out further research regarding these two attributes.

Learnability

Archer (1969) formally defined "concept" in terms of five attributes: identifiability, learnability, likability, transferability, and forgettability. The other attributes we have specified for public concepts imply that many concepts have already been identified and labeled and that new ones constantly are being formed and given labels, e.g., future shock. Therefore, identifiability and labelability are assumed by the nature of public concepts as defined. The attribute of learnability, as we perceive it, subsumes forgettability. Our attribute of usability corresponds roughly to that of transferability posited by Archer. The other six attributes, in addition to learnability and usability which we have specified, appear to be useful for differentiating concepts from other categories of learning outcomes, such as S-R associations, principles, and problem-solving techniques.

Learnability varies among concepts in the sense that some concepts are more readily learned by individuals who share similar cultural experiences and language than are others. For example, concepts that have readily perceptible instances, such as dog and tree, are more readily learned than are concepts without perceptible instances—atom and eternity, for example. The ease of learning a particular concept can be determined through research.

While public concepts vary in learnability, the level to which a particular concept as a mental construct is attained by a given individual also varies, increasing with further learning. For example, with more learning an individual's concept of plant comes closer to the concept held by the botanist.

The level of mastery of any public concept also varies among individuals of roughly equivalent maturational and experiential levels. For example, high school seniors vary widely in their mastery of the concept of valence. This variability among persons of roughly the same age and experience pertains to the other seven attributes as well as to usability; therefore, we shall not refer to it further.
Usability

Concepts vary in usability in the sense that some can be used more than others in understanding and forming principles and in solving problems. For example, the mathematical concepts of number and set are probably used more frequently in solving a variety of problems than are the concepts of proportion and ratio.

Concepts as mental constructs of the individual become more usable as they are attained at successively higher levels. Bruner, Goodnow, and Austin (1956) indicated that attaining a classificatory concept aids the individual by:

1. reducing the complexity of the organism's environment.
2. identifying the objects of the world about him.
3. reducing the necessity of constant learning.
4. providing direction for instrumental activity.
5. ordering and relating classes of events.

[Bruner et al., 1956, pp. 12-13]

As will be noted later in discussing the model of cognitive operations, the preceding general statements of Bruner et al. can be stated more precisely in a form in which they can be tested. Having a classificatory or formal concept enables the individual (1) to generalize to new instances and to discriminate noninstances of the concept, (2) to recognize other concepts in a taxonomy as supraclass, coordinate, or subordinate, (3) to recognize cause and effect, correlational, probability, and axiomatic relationships among concepts, and (4) to solve problems involving the concept. The maturing individual can use his concepts more effectively in the four ways just cited as he reaches successively higher levels of attainment.

Validity

A concept is valid to the extent that experts agree on its definition. Concepts comprising well-defined taxonomic systems such as those of the animal kingdom, the plant kingdom, and the table of chemical elements have greater validity than do many concepts in the behavioral sciences, e.g., intelligence, democracy, liberal, and group dynamics. Experts are in greater agreement concerning definitions of the first group of concepts than the second group.
by an individual may or may not occur with the judgment of scholars in the various disciplines, such as Bruner (1966), Piir (1966), and Novak (1966). It is recognized, however, that individuals do organize their concepts and that new information is more readily related to certain concepts they have already learned than to others.

**Structure**

Any public concept defined in terms of attributes has a structure, a relatedness of the defining attributes. Bourne (1970) described an internally consistent structure of concept attributes, which he called conceptual rules. The rules were derived from the calculus of propositions which generates a total of 16 possibilities for partitioning a stimulus population, using at most two stimulus dimensions. According to Bourne, ten of the possibilities are unique and nontrivial as related to concept structure and can be reduced to five pairs, each pair consisting of a primary and a complementary stimulus partition, or conceptual rule. The basis of the five pairs is that any instance which is positive under one rule is negative under its complement. The conceptual rules are described in Table I. The primary rules appear in the left columns and are labeled the affirmative, the conjunctive, the inclusive disjunctive, the conditional, and the biconditional conceptual rules.

Bourne (1970) summarized a series of experiments involving subjects' learning of the conjunctive, inclusive disjunctive, conditional, and biconditional rules. Sizable and positive intrarule and interrule transfer effects were observed, suggesting a tentative hierarchical model of the sophisticated subject's knowledge and skill based on the generative character of concepts. According to our logical analysis, most concepts involve any of four rules—affirmative, conjunctive, inclusive disjunctive, or conditional.

An individual may or may not be able to either identify or name the defining attributes of the concepts he holds. However, attaining the concept to the highest level of mastery requires recognition and naming of the defining attributes, as will be noted later in discussing the model. Bruner, Oliver, Greenfield *et al.* (1966), Nelson (1971), and Wiviot (1970) found that knowledge of the intrinsic attributes of concepts, which for many concepts are also the defining attributes, increased with age. Wide differences in the ability to identify and name the attributes were also observed by

**Instance Perceptibility**

Concepts vary with respect to the extent that instances of them can be sensed. For example, plant has many instances which can be manipulated, seen, and smelled, whereas eternity has no perceptible instances. Between these poles are concepts whose instances can be represented with varying degrees of accuracy by drawings or by other means. For example, one can generate a visual representation of a concept such as point but not an observable instance of it.

With increasing age, individuals are able to identify the less obvious attributes of instances they have experienced. Also, the various sensory modalities can be used in combination. According to Bruner (1964), the maturing child is successively able to interact with and represent the environment enactively, iconically, and symbolically, and to combine these modes of representation. As the child gets older, he can learn more through manipulating objects and seeing them. In addition, he can learn about them through symbolic, especially verbal, experiences.

**Instance Numerousness**

Most concepts have instances. The number of instances ranges from one to an infinite number: one instance—Earth's moon; a small number—the continents; a large number—drops of water; or an infinite number—integers. Certain concepts may have imaginary rather than actual instances, for example, plenity—exist only in the mind. Certain concepts may have imaginary rather than actual instances, for example, plenity—exist only in the mind.

Most individuals continue to encounter new instances, or pictorial or verbal representations of instances of the same concepts, with increasing age. But there is great variability among individuals with respect to the number of instances of the same concepts encountered and with respect to the nature of the instances encountered. Children living in desert regions do not encounter swamp lands; those who live in the lowlands along rivers, lakes, and oceans do not encounter deserts; some children living in our inner cities encounter no kind of land except the asphalt of the inner city.

For different individuals the particular instances of the same concept encountered may vary markedly. For example, most chil-
from encounters; instances of mother, father, four, love, and death. The instances encountered, however, are somewhat unique for each child and are associated with strong affect. Concepts such as these are more nearly non-conceptual mental constructs of the particular individual than are such concepts as plant or numeral that have many highly similar instances and that are not associated with much emotion.
<table>
<thead>
<tr>
<th>Primary rule</th>
<th>Complementary rule</th>
<th>Symbolic description</th>
<th>Verbal description</th>
<th>Symbolic description</th>
<th>Verbal description</th>
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<tr>
<td></td>
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<td>Name</td>
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<td></td>
<td></td>
<td><strong>Affirmative</strong></td>
<td>All red patterns are examples of the concept</td>
<td><strong>Negation</strong></td>
<td>All patterns which are not red are examples</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Conjunctive</strong></td>
<td>All patterns which are red and square are examples</td>
<td><strong>Alternative denial</strong></td>
<td>All patterns which are either not red or not square are examples</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Inclusive disjunctive</strong></td>
<td>All patterns which are red or square or both are examples</td>
<td><strong>Joint denial</strong></td>
<td>All patterns which are neither red nor square are examples</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Conditional</strong></td>
<td>If a pattern is red then it must be square to be an example</td>
<td><strong>Exclusion</strong></td>
<td>All patterns which are red and not square are examples</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Biconditional</strong></td>
<td>Red patterns are examples if and only if they are square</td>
<td><strong>Exclusive disjunctive</strong></td>
<td>All patterns which are red or square but not both are examples</td>
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</tbody>
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<tr>
<td>b</td>
<td>R and S stand for red and square (relevant attributes), respectively. Symbolic description using only three basic operations, ( \cap ), ( \cup ), and negation are given in brackets.</td>
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</table>
Figure 1 shows the structure of the model. Four successively higher levels in the attainment of the same concept are outlined. The four successive levels are **concrete**, **identity**, **classificatory**, and **formal**. A concept becomes increasingly usable and valid when attained by an individual at the successive levels.

To prevent confusion, we state here that the model in its totality describes the attainment of four levels of the same concept rather than four kinds of concepts. The four levels apply to the many concepts that can be defined in terms of attributes and which have actual perceptible instances or readily constructed representations of instances. We have already cited a few examples of this kind, which includes all the concepts comprising the plant kingdom and the animal kingdom. However, the operations at each level are intended to be applicable also to different kinds of concepts, some of which, because of their nature, are not attainable at all four levels. We can specify these kinds of concepts and the levels at which they may be attained.

There are some concepts for which there is only one instance, such as [Earth's moon](https://en.wikipedia.org/wiki/Moon) and [Abraham Lincoln](https://en.wikipedia.org/wiki/Abraham_Lincoln), and some that have many identical instances, for example, [inch](https://en.wikipedia.org/wiki/Inch) and [pound](https://en.wikipedia.org/wiki/Pound). Related to Figure 1, such single-instance or identical-instance concepts which have defining attributes may be attained at the concrete, identity, and formal levels, but not at the classificatory level. By our definition of classificatory level, there must be at least two nominical instances that can be placed in the same class. Therefore, some concepts as specified in this paragraph cannot be attained at the classificatory level.

There are other concepts that are not useful for classifying instances and noninstances but that have defining attributes, for example, **force** and **space**. These might be learned at the concrete, identity, and formal levels, but not at the classificatory level.

Some concepts are of such low validity that there may not be agreement as to the defining attributes, for example, **beauty** and **morality**. Concepts such as these might be learned at the three lower levels but not at the formal level.

Finally, there are concepts with no perceptible instances, such as **infinity** and **atom**. These cannot be learned at the three lower levels but might be learned at the formal level.

Returning to the four levels given in Figure 1, we postulate that attaining a concept at the four successively higher levels is the normative pattern for large numbers of individuals under two conditions. First, the concept is of the kind for which there are actual perceptible instances or readily constructed representations; and second, the individual has experiences with the instances or representations starting in early childhood. Further, in order to proceed to the formal level, individuals must acquire labels for the concept and for its attributes.

Children have direct experiences during preschool years with many things and attain concepts of these things at the first two levels. They also attain many concepts at the classificatory level and learn the societally accepted names for the concepts and their attributes through formal and informal instruction.

Earlier we indicated that some individuals, due to environmental conditions, may not encounter actual instances of a concept; rather, they experience instances only in verbal or pictorial form. Thus, these individuals may attain a concept at either the classificatory or the formal level at the outset.

It should also be noted that the mature person, although capable of attaining a concept at the formal level, may attain it only at one of the lower levels and stop at that level because of the way in which the perceptible instances are encountered or other conditions of learning.

Figure 1 also shows the ways that concepts may be extended and used. Concepts acquired at the classificatory and formal levels...
Acquiring and remembering labels of concepts and attributes

**Concrete Level**
- Attending to things
- Discriminating one thing from other things
- Remembering the discriminated thing

**Identity Level**
- (Three prior operations and)
- Generalizing that two or more forms of the same thing are equivalent

**Classificatory Level**
- (Four prior operations and)
- Generalizing that two or more instances are equivalent in some way

**Formal Level**
- (Five prior operations and)
- Discriminating the defining and irrelevant attributes of concept
  - Hypothesizing the relevant attributes and/or rules
  - Remembering hypotheses
  - Evaluating hypotheses using positive and negative instances

**Concretizing**
- Generalizing to positive instances of the concept and classificatory instances
- Generalizing to other concepts of improved, inferior, or non-overlapping classificatory concept
- Generalizing concepts in different connotative and denotative relationships of the attaining concept with other concepts
- Using the concept in problem-solving situations

Figure 1. Cognitive operations in concept learning.
The fourth part of the model shows that acquiring and remembering the names of the concepts may occur at any of the four levels. The solid line indicates that acquiring the name of the concept and the names of attributes is essential to attaining concepts at the formal level. The broken lines indicate that an individual may acquire the name at once at the same time he first attains the concept at lower levels, but that this is not requisite. For example, a young child may attain a concept at all three lower levels but not know the concept name. The younger the child is upon attaining the concept, the less likely it is to have the name for it.
Having considered the overall features of the model, we may take up the operations in more detail, starting with those pertaining to the concrete level.

Concrete Level

Attainment of a concept at the concrete level is inferred when the individual cognizes an object that he has encountered on a prior occasion. It is appropriate to define some terms before proceeding further. We use the term "operations" much like Guilford (1967) does. Guilford defines the operations of cognition, memory, productive thinking, and evaluation formally and also operationally in terms of test performances. He states that cognition must be related to the products cognized and defines cognition formally as follows:

Cognition is awareness, immediate discovery or rediscovery, or recognition of information in various forms; comprehension or understanding .... The most general term, awareness, emphasizes having active information at the moment or in the present ... the term, recognition, is applied to knowing the same particular on a second encounter ... if cognition is practically instantaneous, call it recognition; if it comes with a slight delay, call it "immediate discovery!" [Guilford, 1967, pp. 203-204]

The first step in attaining a concept at the concrete level is attending to an object and representing it internally. Woodruff (1961) points out:

All learning begins with some form of personal contact with actual objects, events, or circumstances .... The individual gives attention to some object .... Through a light wave, or a sound wave, or some form of direct contact with a sensory organ in the body, an impression is picked up and lodged in the mind. [Woodruff, 1961, p. 66]

Gagné (1970) indicates that as the individual attends to an object he discriminates it from other objects. Woodruff (1961) calls the outcome of these attending and discriminating operations a concrete concept, a mental image of some real object experienced directly by the sense organs. The infant, for example, attends to a large red ball and a white plastic bottle, discriminates each one, maintains a mental image of each, and cognizes each of the objects when experienced later.

The discrimination of objects involves attending to distinctive features that serve to distinguish them one from another. Thus, very early the child learns to respond to gross differences in such features of objects as size, shape, color, and texture. As the child matures, he becomes capable of making finer discriminations involving these and other features.

The attainment of a concept at the concrete level thus requires attending to the distinctive features of an object and forming a mental image which represents the object as a unique bundle of features. The concept at this level may or may not be associated with the concept label, depending on whether the label has been learned and remembered, and whether it has been associated with the concept.

The preceding analysis of the operations in attaining concepts at the concrete level is sufficiently comprehensive to include motoric experiencing of objects. That is, an object may be manipulated physically and represented enactively as well as explored visually and
reerereere30,0_, to use Bruner's (1964) terminology. The model postulates that attending, discriminating, and remembering are involved in sensorimotor experiencing as well as in the visual perception of objects.

Identity Level

Attainment of a concept at the identity level is inferred when the individual cognizes an object as the same one previously encountered when observed from a different perspective or sensed in a different modality. For example, when the child makes the same response to the family poodle when seen from straight ahead, from the side, and from various angles, he has attained the concept of poodle at the identity level. Whereas concept attainment at the concrete level involves simply discriminating an object from other objects, attainment at the identity level involves both discriminating various forms of the same object from other objects and also generalizing the forms as equivalent. Generalizing is the new operation postulated to emerge as a result of learning and maturation and to make attainment at the identity level possible.

As noted earlier, there are some valid and powerful concepts, such as English alphabet, for which there is only one instance but which can be represented in different ways, i.e., aurally and in printed form. These concepts are typically learned at the concrete and identity levels but not at the classificatory level. Therefore, as shown in Figure 1 by the arrow going directly from identity to formal level, individuals proceed directly from the identity to the formal level with this kind of concept.

Bruner, Goodnow, and Austin (1956) have pointed out that identity responses occur very early in life and that the capacity to recognize identity may be innate and merely extended to new events through learning. Vernon (1970), however, believes that infants have to learn by experience that objects and events in the environment are permanent even though they may change their appearance from time to time as their distance and orientation change. Whether identity responses are innate or learned, the capacity to recognize identity, indeed the expectation of the continuity of objects and events in the environment, is well developed in adult perception.

Recognition of object identity is central to Piaget's formulations. According to Elkind (1969), Piaget's conception of concept emphasizes the variability that occurs within things, that is, the changes in state, form, and appearance that can occur to any entity. Piaget postulated the principles of identity and conservation, identity being concerned with maintaining the likeness or sameness of the same thing in thought and conservation being concerned with maintaining the likeness or sameness of the same thing in experience. An individual's concept of dog, for example, presumes that an individual dog will retain its "dogness" both in the internal representation and in the direct experience of the individual with the dog. Without this permanence both in the mental construct and in the actual instance of the specific dog, the individual's criteria for recognizing a dog, or dogs, would shift from moment to moment.

Elkind pointed out that American psychologists have tended to ignore this within-instance variability of concepts and have emphasized the discrimination response aspect of concept attainment by which positive instances are cognized and discriminated from noninstances. Elkind (1969) summarized the two points of view thus:

From the discriminative response point of view, the major function of a concept is the recognition or classification of examples. The Piagetian conception, however, assumes that a major function of the concept is the differentiation between the apparent and the real. This discrimination, in turn, can be reduced to the differentiation of between-things and within-things types of variability. Here again, a comprehensive conception of a concept must include both functions because, in fact, every concept does serve both purposes. [p. 187]

The present model proposes that a concept is attained at the identity level temporarily before it is attained at the classificatory level. Stated differently, the individual must be able to cognize various forms of the same object as equivalent before he is able to generalize that two or more different objects belong to the same class.

Classificatory Level

The lowest level of mastery at the classificatory level is inferred when the individual responds to at least two different instances of the same class as equivalent even though he may not be able to describe the basis for his response. For example, when the child treats the family's toy poodle and the neighbor's miniature poodle as poodles although he may not be able to name the defining attributes of...
If the child has learned the concept and can discriminate the defining attributes of the concept, he has attained a concept at the classificatory level.

Generalizing that at least two different instances are equivalent in some way is the lower limit of this level of concept learning; the individual is still at the classificatory level of concept learning when he can correctly classify a larger number of instances as examples and nonexamples, but cannot accurately describe the basis for his grouping in terms of the defining attributes. Henley (cited in Deese, 1967), like many other researchers, has observed this phenomenon. Many of her subjects were able to sort cards correctly into examples and nonexamples of the concepts being learned—things with serrated edges and things that hold things together—yet gave totally erroneous definitions of the concepts.

Formal Level

A concept at the formal level involving classification is inferred when the individual can give the name of the concept, can name its intrinsic or societally accepted defining attributes, can accurately designate instances as belonging and not belonging to the set, and can state the basis for their inclusion or exclusion in terms of the defining attributes. For example, the maturing child demonstrates a concept of dog at the formal level if when shown dogs, foxes, and wolves of various sizes and colors, he properly designates the dogs as such, calls them "dogs," and names the attributes that differentiate the dogs from the foxes and wolves. The distinctive aspect of this level of concept mastery is the learner's ability to specify and name the defining attributes and to differentiate among newly encountered instances and noninstances on the basis of the presence or absence of the defining attributes.

As noted in Figure 1, the labels for the concept and the defining attributes may be learned at any of the three lower levels but are not essential at those levels. Similarly, the discrimination of the defining attributes may occur prior to the formal level but is not essential. Thus, discrimination of things on their global and diffuse stimulus properties which is essential at the concrete level changes to discrimination of more specific and abstract properties at the identity and classificatory levels. However, at the formal level, the individual must be able to discriminate the defining attributes from irrelevant attributes and label the defining attributes. We hypothesize that individuals who can do the discriminating and labeling of the defining attributes, in comparison with those who cannot, will also identify concept examples with fewer errors of overgeneralization and undergeneralization and will also be able to use the concept more effectively in the three other ways specified in Figure 1.

The operations involved in concept learning at the formal level are also shown in Figure 1. The first operation given at the formal level is that of discriminating the attributes. For some concepts with obvious attributes such as color and form, the discriminations may have occurred at earlier levels. However, both discrimination and labeling of the attributes are essential at the formal level. This is true whether the individual infers the concept by hypothesizing and evaluating relevant attributes or by cognizing the attributes common to positive instances as shown in Figure 1.

Individuals differ in their ability to analyze stimulus configurations into abstract dimensions or attributes. There is evidence (Gibson, 1969) that this ability develops with age. Retarded children may have difficulty with simple concept learning tasks because of difficulty in learning to select out and attend to specific dimensions (Zeaman & House, 1963). Even among children of adequate intelligence there are those who characteristically analyze the stimulus field and apply labels to attributes while others tend to categorize on the basis of a relatively undifferentiated stimulus (Kagan, Moss & Sigel, 1963).

Orienting instructions may be given to make explicit the attributes of the stimuli (Klausmeier & Meinke, 1968). These instructions facilitate the learning of concepts at the formal level by assuring that the learner knows all of the attributes which may be relevant to the concept.

In connection with language and concept attainment, we recognize that deaf individuals and others who lack normal speech development may attain concepts at the formal level. By our definition, the individual must know the defining attributes of the concept and must be able to communicate this knowledge. Verbalizing is normally used in this kind of communication, but symbolic communication—for example, sign language—may also be employed. Thus, speech, per se, is not necessary for the attainment of concepts at the formal level, but there must be some means for symbolizing and communicating the concept in the absence of examples.

Having discriminated and named the attributes, an individual may infer the formal level of a concept in either of the two ways shown in Figure 1. One way involves formulating and evaluating hypotheses and the other
Acquiring Appropriate Labels

The importance of language in concept learning is widely acknowledged by American and Russian psychologists (Bruner, 1964; Vygotsky, 1962). Having the labels of concepts enables the individual to think in symbols rather than in images and also to attain other concepts through language experiences in the absence of perceptible instances. Carroll (1964), as noted earlier, has outlined the close relationships among

Essential to the hypothesis-testing approach are the operations of remembering and evaluating hypotheses. There is support (Levine, 1963; Williams, 1971) for the idea that the subject formulates and remembers a population of hypotheses, remembers the hypotheses that were rejected, and also remembers the last one accepted as correct. Bruner, Goodnow, and Austin (1956) indicate that an individual determines whether or not his hypothesized concept is valid by recourse to an ultimate criterion, test by consistency, test by consensus, or test by affective congruence.

Inherent in all four procedures is establishing a criterion for judging the correctness of an hypothesis. In the present model, the validity of an individual's concept may be assessed in terms of how nearly it corresponds to experts' agreement concerning the concept.

The operations involved in the hypothesis-testing approach to inferring concepts appear to characterize individuals who cognize the information available to them in laboratory and classroom settings from both positive and negative instances. These individuals apparently reason like this: Instance 1 has land surrounded by water. It is a member of the class. Instance 2 has land but is not surrounded by water. It is not a member of the class. Therefore, lands surrounded by water belong to the class and lands not surrounded by water do not. Surrounded by water is a defining attribute of the concept. This individual has attained a partial and possibly complete definition of the concept based on experiences with only one positive and one negative instance.

Another way of inferring the concept is by noting the commonalities in examples of the concept. The commonality approach is used more often than the hypothesizing approach by children because they are either incapable of carrying out the hypothesizing and evaluating operations or for other reasons pursue the commonality strategy (Tagatz, 1967). In this connection, the commonality approach is entirely appropriate for use when only positive instances of the concept are available. Thus, it is probably employed in situations where the individual is given only positive instances or verbal descriptions of positive instances.

The importance of language in concept learning is widely acknowledged by American and Russian psychologists (Bruner, 1964; Vygotsky, 1962). Having the labels of concepts enables the individual to think in symbols rather than in images and also to attain other concepts through language experiences in the absence of perceptible instances. Carroll (1964), as noted earlier, has outlined the close relationships among
cor cepts, meanings, and words. However, our purpose here is not to deal with the relationships between language and concept learning, but to show at what points labels may be learned and associated with the various levels of concepts.

Figure 1 indicates that a concept label may be associated with an instance of the concept at any of the four levels—concrete, identity, classificatory, or formal. American children who have somewhat similar experiences and instruction regarding certain concepts might manifest a sequence like this: A young child first encounters a dog. The child's mother points to the dog and says "dog." The child then says "dog," and associates the name with his concrete concept of the dog. Next, the child develops the concept of the same dog at the identity level through experiencing it in different locations and situations. His mother repeats the name at various times in the presence of the dog; the child says the word repeatedly. The word "dog" now comes to represent the child's concept of the dog at the identity level. Subsequently, the child encounters other dogs and observes that they, too, are called "dogs." He generalizes the different dogs as equivalent in some way and associates the name "dog" with whatever similarities he has noted. The word thus comes to represent his class of things called "dogs." At the formal level, the more mature child discriminates and learns societally accepted attributes of the class of things called "dogs" and also learns the names of the attributes. Now the child's concept of dog approaches or becomes identical to the societally accepted definition of the word "dog." As Carroll (1964) pointed out, the concepts held by individuals and the meanings of the words representing the concepts are the same for mature individuals who share similar cultural experiences and the same language.
Concept Extension and Utilization

The individual who has formed a concept may extend and use it as shown in Figure 1. Concepts learned at the classificatory and formal levels can be used in generalizing to new instances, cognizing supraordinate-subordinate relations, cognizing predictive and axiomatic relations among concepts, and generalizing to problem-solving situations. Ausubel (1963) and Gagne (1970) have theorized concerning the use and extension of attained concepts; however, very little empirical research has been done. In this regard, Ausubel formulated the constructs of cognitive structure, advance organizer, correlative subsumption, and derivative subsumption to show how previously attained and newly encountered concepts are related, while Gagne has indicated that attained concepts are prerequisite for the learning of rules. Because of the paucity of theory and research, we are able to offer only tentative suggestions regarding the extension and use of knowledge about an already formed concept in the following section.

Generalizing to New Instances and Discriminating Noninstances

The attainment of concepts at the classificatory and formal levels reduces the need for additional learning and relearning, primarily because the individual is able to generalize to new instances of a concept and to discriminate noninstances. Having a concept also provides the individual with expectations which help him deal effectively with new instances of it. Once he identifies a plant as poison ivy, he may treat it gingerly. One test of concept attainment in our experiments is the individual's ability to properly categorize instances not previously encountered as instances or noninstances of the particular concept. We find that both school children and college-age students generalize to new instances readily.

Further, the use of instances and noninstances in instructional materials to teach concepts can be manipulated so that errors of overgeneralization and undergeneralization can be predicted (Feldman, 1972; Swanson, 1972). Not only does having a concept enable the learner to recognize new instances and act appropriately toward them, but direct and verbal experiences with the new instances possibly increase the validity and power of the concept for the individual, as these attributes were defined earlier. For example, the Canadian visiting Kenya during January, when it is summer there, may attain more valid and powerful concepts of flower and plant. Similarly, by being told that a whale is a mammal, an individual comes to realize that some mammals can live in the water as well as on land. Hence, his concept of mammal has increased validity.

Cognizing Supraordinate-Subordinate Relationships

Besides generalizing to new instances, individuals can also use concepts attained at the formal level, and possibly at the classificatory level, in cognizing coordinate, supraordinate, and subordinate relationships among classes of things. The lowest level of cognizing these relationships is inferred when the individual, according to verbal instructions, puts instances of hierarchically-arranged concepts in their proper groups. For example, an individual upon request puts all instances of red and blue equilateral triangles in a grouping of equilateral triangles, all instances of equilateral triangles and of right triangles in a grouping of triangles, and all instances of triangles and of rectangles in a grouping of polygons. Further, he justifies each group formed on the basis of the defining attributes of the group. For example, he states that equilateral triangles include...
all the triangles that have three equal sides, triangles include all the figures that have three sides, and polygons include all the forms of figures that have three or more sides. More precise terminology might be required, such as "an equilateral triangle is a plane closed figure with three sides of equal length."

Possible higher levels of attaining the supraordinate-coordinate-subordinate relationships include what Kofsky (1966) designates as relationships involving inclusion and exclusion. Again, merely being able to group a few instances properly according to verbal instructions is not a sufficient test of understanding these relationships; an adequate justification for the actions is required. According to Kofsky (1966), knowledge concerning supraordinate-subordinate relationships increases with age.

The understanding of supraordinate-subordinate relationships increases the validity and usability of the individual's concepts. For example, knowing the attributes of acid and also that vinegar is an acid leads to the inference that vinegar has the attributes of all acids, as well as the attributes peculiar to vinegar. Thus, all of the things known about acids—for example, how they react with bases—are true for vinegar also. In this way, learning that acid is a concept superordinate to vinegar increases the validity and usability of the concept of vinegar for the individual.

Cognizing Other Relationships

In the model, statements of relations between or among concepts involving cause and effect, correlation, probability, and other lawful relations such as contained in axioms are treated as different from relations of inclusion and exclusion involving supraordinate and subordinate concepts. These first three kinds of relationships are referred to by Marx (1970) as laws and by Gagné (1966, 1970) as principles or rules. Mathematicians particularly state lawful relations or "givens" in axiomatic statements.

Bruner, Goodnow, and Austin (1956) have pointed out that understanding lawful relationships between or among concepts permits the relating of classes of things instead of individual things. In this connection, Gagné (1970) cites the example of the rule "round things roll," and indicates that understanding the relationship among the concepts incorporated in the rule enables the individual to predict what will happen to all round things under certain circumstances. Or, consider the more complex relationship "When two substances at different temperatures come into contact, the temperatures of the substances tend to equalize." This relationship permits us to infer what will happen in such diverse situations as putting ice cubes in warm soda pop or being lost in a snowstorm.

In all cases, being able to understand and use a lawful relationship depends on knowing the concepts that are related. Only then can the principle or axiom be applied to the appropriate phenomena.

Generalizing to Problem-Solving Situations

Woodruff (1967) discusses the rule of concepts in higher-level mental activities, including problem solving. Also, Gagné (1970) indicates that one way in which concepts are used in solving problems is by the application of principles to the problem-solving situations. For example, principles underlying the concepts of pressure, volume, gravity, and distance can be utilized to determine the height of a mountain using a barometer.
VI
Conditions of Concept Learning

Klausmeier et al. (1965) outlined the variables to be taken into account in their long-term programmatic research on concept learning and instruction. Three classes of variables identified were organismic, task, and instructional conditions. The more explicit subsets of interest for the model are age, or internal conditions, task variables associated with the nature of concepts, and instructional conditions.

Age and Concept Mastery

The two age variables of primary concern in the present model are (1) the ability of the individual to carry out the cognitive operations at each of the successive levels, and (2) the ability to carry out each operation on more highly differentiated and abstract properties of concept instances as required at successively higher levels. Also, being able to speak and comprehend words is critical in attaining any concept at the formal level. These and other abilities may be treated as age variables from a developmental point of view and as internal conditions of learning from a learning point of view.

The preceding variables may be reviewed briefly as they are related to the successive levels of concept attainment. The attending, discriminating, remembering, and generalizing operations requisite for attaining a concept at the concrete and identity levels are present in very young children. In order to attain a concept at the classificatory level the child must be able to carry out all these operations on more highly differentiated properties of the instances and must be able, in addition, to generalize that two different instances of the same set are equivalent in some way. Because verbal definition of the concept is required for attainment of a concept at the formal level, the child must have sufficient language competence to formulate or comprehend such definitions. Further, when concepts are attained at the formal level through verbal instruction, as is often the case, the child must understand the meanings of the words used. Thus, acquisition of language competence and specific terminology as well as the ability to carry out certain cognitive operations are essential for learning a concept at the formal level.

Concept Variables and Concept Mastery

The earlier discussion concerning concept attributes imply that variables associated with concepts, attributes of concepts, and instances of concepts are key manipulable stimulus variables in laboratory experiments and also in printed textual materials. In connection with learnability, for example, it was pointed out that some concepts are more difficult to learn than others. Also, the structure of the concept in terms of how the attributes are joined determines the demand upon the various cognitive operations of the learner. For example, even when a child can discriminate and label the attributes of instances, he may not be able to correctly infer the concept if a disjunctive rule is involved. This is because the strategy of cognizing commonalities among positive instances is inappropriate for attaining disjunctive concepts. A more sophisticated hypothesis-testing approach using information from negative instances is required which may not, as pointed out earlier, be fully developed in young children. These and other variables associated with concepts and those dealing with instructional conditions may be grouped as external conditions of learning.

Instructional Conditions and Concept Mastery

Earlier we indicated agreement with Bruner et al. (1966) whose cross-cultural
studies led to the conclusion that a location was not only important in determining the particular concepts learned by individuals, but also that it determined the forms of identifying the employee's location. In controlled laboratory experiments we have found that seemingly small changes in instructions to the experimental subjects made large differences in the results obtained. Further, in control experiments in school settings, Trayer (1970), Pellicer (1972), and Swanson (1972) found significant differences as a result of varying the instructions in printed lessons. One set of variables can be manipulated in the printed instructional materials that provide the location and its context can be varied within the lesson. For example, a content set can be established with lesson material while the remainder of the instruction is constant, or vice versa. In lesson formats a content set can be established, presentation order and other instructional characteristics that affect concept attainment. In research to each of the four levels of concept attainment and the uses of attained concepts is identified by the models.
Increasingly, behavioral scientists and designers of instructional materials are defining concept formally in terms of the attributes of that category of learning products called concepts. A concept may be defined as ordered information about the properties of one or more things—objects, events, or processes—that enables a particular thing or class of things to be differentiated from, and also related to, other things or classes of things. Thus, a concept is properly treated both as a mental construct of the individual and also as a societally accepted meaning of a word standing for the concept. As a societally accepted word meaning, a concept has eight attributes: learnability, usability, validity, generality, power, structure, instance numerousness, and instance perceptibility.

According to the model of cognitive operations described in this paper, the same concept is learned at four successively higher levels: concrete, identity, classificatory, and formal. The operations involved at the successively higher levels include attending, discriminating, and remembering at the concrete level; the preceding three operations and generalizing that two or more forms of the same thing are equivalent at the identity level; the preceding operations and generalizing that two or more instances of the same set are equivalent in some way at the classificatory level; and the preceding operations, other higher level operations, and the use of language or other symbols at the formal level.

A concept attained at the classificatory or formal level may be used in generalizing to positive instances and discriminating non-instances; cognizing other concepts as superordinate, coordinate, or subordinate; cognizing cause-and-effect, correlational, and other relationships of the attained concept with other concepts; and using the concept in problem solving.

The model implies various internal conditions of learning that are essential for learning a concept at the successively higher levels; it also implies external conditions that are facilitative at each level. The model thus provides a possible framework for further research on conceptual learning and development and also for instruction. In this paper, research bearing directly on the model was reviewed; research dealing with internal and external conditions of learning was not.
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