A study on individual differences in problem solving approach and their relationships to various learning-related parameters was conducted with a random sample of 25 subjects enrolled in an introductory physics course utilizing instruction through audio-tutorial methods. The subjects received interviews consisting of four problems in energy conservation and simple harmonic motion to secure their intuitive and analytic approach rating results. Verbal and mathematical scholastic aptitude test scores, achievement scores, weekly time-consumption records in learning, and weekly and total learning efficiency scores were used to determine their relationships to data on rating. Statistical analyses showed the possibility of categorizing individual's problem solving approach using the intuitive and analytic dimensions as a basis. The individual with high intuitive and analytic ratings (Hi I - Hi A) is at a significant advantage, while the individual with Hi A and low intuitive ratings has low learning efficiency. Further research was recommended on the interactive effects of the analytic and intuitive dimensions on various instructional techniques. The desirability of gearing instructional regimes to individual differences was preferable. (CC)
THE IDENTIFICATION AND SIGNIFICANCE OF
INTUITIVE AND ANALYTIC PROBLEM SOLVING APPROACHES
AMONG COLLEGE PHYSICS STUDENTS

Martin N. Thorsland
Assistant Professor of Education
Brown University

Joseph D. Novak
Professor and Chairman - Science Education
Cornell University

Paper delivered at the National Association for Research in
One of the trends in science education today is toward individualization of instruction. New curricular programs are attempting to consider differential needs by producing instructional materials designed for individual use—the rationale being that such materials will cater to differences in individual learning styles, aptitudes, or whatever. But how individual differences are being considered in these efforts is not at all clear. Studies reported indicate that instructional decisions—decisions such as which activity to pursue and/or how much time to spend—usually rests with the learner, as it rightly should. While such studies could serve admirably to analyze how the learner functions in such a setting, it does little, if anything, to enable one to determine why the learner functions as he does in relation to any established theory of learning.

This study represents an effort at the identification of specific individual differences within one subject matter area and an analysis of their relationship to various learning-related parameters. Problem solving is considered basic for developing an understanding of the processes as well as the content of science. A better understanding of the role of problem solving abilities in the learning process would enable one to more fully comprehend the nature of learning. This study focuses on differences in problem solving approaches among college physics students using a theoretical framework derived from the ideas of David P. Ausubel.

**SETTING OF RESEARCH**

A group of seventy students enrolled in an introductory physics course received instruction via audio-tutorial methods. Initial problem solving interviews with several students had shown that students proceeded in the problem solving encounters in two distinct ways. One approach, termed analytic (A) (c.f. Bruner, 1960, and Nedelsky, 1965), was characterized by a step-by-step analysis of the problem, very explicit in nature. The analytical approach was often accompanied by the use of mathematical relationships and symbols.

A second approach, termed intuitive (I), was characterized by an implicit "feel" for the subject matter with little or no conscious awareness of the steps used in arriving at an answer (Bruner, 1960). Some students appeared to utilize both approaches successfully while others relied on only one approach in problem solving situations.
It was decided that these two problem solving approaches would be investigated in an attempt to relate these approaches to other learning parameters within the context of pedagogical theory.

**THEORETICAL LEARNING MODEL**

A theory of learning proposed by D. P. Ausubel (1968) was utilized. In this subsumption theory a differentiated cognitive structure is a prime determiner in learning subsequent related subject matter. The intuitive and analytic dimensions of problem solving behavior were related to cognitive functioning in the following manner. From the ideas of Ausubel, it is possible to obtain a representation of the organization of cognitive structure and related mechanisms involved. Figures 1 and 2 show diagramatic representations of a possible interpretation of the organization of concepts in cognitive structure for the intuitive and analytic individuals. The more inclusive, higher order, superordinate abstractions lie at the top of the structure. At the bottom are the less inclusive, more highly differentiated, subordinate concepts subsumed by the higher order superordinate concepts.

As learning progresses and higher order concepts are developed, it is inefficient and over-burdening for an individual to retain all of the low level concepts and ideas. Thus some (or all) are incorporated into the higher order abstractions and "meaningfully forgotten"-- a process that Ausubel refers to as obliterative subsumption. It is therefore possible for the individual to regenerate or reconstruct these subordinate concepts if necessary in the learning process.

The highly intuitive individual, it is conjectured, would possess the superordinate ideas and higher level subsumers necessary to enable him to move across the upper levels of Figure 1 with frequent referrals to (and from) subordinate concepts. The highly analytic individual, it is conjectured, would be very effective at regenerating the lower level, subordinate ideas and would therefore move primarily from the subordinate to the superordinate concepts as shown in Figure 2. The analytic dimension is thus treated as an aspect of information processing ability similar to that set forth previously in a model of concept formation (Novak, 1965). This particular analytic aspect of information processing would play a
very important role in such mathematically oriented and highly-structured subjects as physics.

The intuitive dimension can be regarded as closely related to the availability and usage of relevant superordinate ideas in Ausubel's terms or to the store of related information in Novak's model. The highly intuitive individual would possess a general "feel" for the subject matter and would have the major ideas and structures of the discipline available for problem solving encounters. Thus, as Bruner points out, intuitive thinking "tends to involve maneuvers based seemingly on an implicit perception of the whole problem" (Bruner, 1960).

Based on the preceding conception of cognitive functioning it is possible to arrive at several hypotheses concerning the relationship between the analytic and intuitive dimension and various learning related parameters.

**Hypothesis I:** The analytic dimension is more highly related to scholastic ability than is the intuitive dimension.

**Hypothesis II:** (A)-Students rated high intuitive will achieve at a significantly higher level than those rated low intuitive.

(B)-Students rated high analytic will achieve at a significantly higher level than those rated low analytic.

**Hypothesis III:** (A)-Students rated high intuitive will spend less time in learning than those rated low intuitive.

(B)-Students rated high analytic will spend less time in learning than those rated low analytic.

**Hypothesis IV:** (A)-Students rated high intuitive will be more efficient in learning than those rated low intuitive.

(B)-Students rated high analytic will be more efficient in learning than those rated low analytic.

**EXPERIMENTAL DESIGN**

Twenty-five subjects were randomly selected from the seventy A-T students. Each of the 25 participated in a problem solving interview session which was taped for subsequent analysis. From the analysis of the interviews, intuitive and analytic (I and A) ratings were assigned to each subject. Four groups of subjects were established according to extremes of the I and A ratings. These groups were then compared on various learning-related parameters.
I-A Interviews and Analysis:

In the interviews the subjects were given four problems selected to conform to the following criteria- (1) wording of the problem must not clue the subject toward using a specific approach, (2) problems needed to be within the scope of the material already presented but not just a reformulation of previously encountered problems, (3) problems must be at low enough difficulty such that an intuitive and/or analytic approach is feasible. The topics of energy conservation and simple harmonic motion were chosen for the problems as this material was discussed fairly extensively in the audio-tape lessons previously.

Subjects were presented with a problem and instructed to solve and explain what he was doing in the process. For each of the four problems an intuitive (I) and analytic (A) rating from zero to five points was assigned, resulting in a combined rating for each student of 0-20 for each of the I and A dimensions.

Correlations between ratings on individual problems and the total rating were all significant beyond the .01 level. Also intercorrelations between the analytic ratings for the various problems were also significant beyond the .01 level indicating that the analytic dimension was very consistently manifested. Intercorrelations between the intuitive ratings for the problems were not as consistent (significant at approximately the .05 level) indicating that the intuitive ratings assigned varied somewhat more than the analytic ratings from problem to problem.

Subjects were placed on an I-A grid according to their total ratings as shown in Figure 3. It can be seen that a reasonably random pattern was obtained. Subjects were widely scattered on the basis of the two ratings. The correlation between I and A ratings for the 25 students was -0.08, indicating no significant relation between the two ratings.

INSTRUMENT RELIABILITY

In order to determine the reliability of the interview tape analysis method, three science educators familiar with the work independently assigned I and A ratings to four subjects randomly selected-- one from each of the four "areas" of Figure 3. The results of these ratings were used to rank the four subjects and the results
are shown in Table 1.

**TABLE 1: Rankings of Intuitive and Analytic Ratings for Each Judge**

<table>
<thead>
<tr>
<th>INTUITIVE</th>
<th>Subject</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Judge One</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Judge Two</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Judge Three</td>
<td>1.5*</td>
<td>3.5</td>
<td>1.5</td>
<td>3.5</td>
<td></td>
</tr>
</tbody>
</table>

**Coefficient of Concordance, \( W = 0.86 \) (p< .01, Kendall, 1955)**

<table>
<thead>
<tr>
<th>ANALYTIC</th>
<th>Subject</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author</td>
<td>4</td>
<td>1.5*</td>
<td>1.5</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Judge One</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Judge Two</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Judge Three</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**Coefficient of Concordance, \( W = 0.87 \) (p< .01; Kendall, 1955)**

When ties occurred in ratings given, corresponding rankings were assigned equal values as appropriate.

The coefficient of concordance values indicate that the observers showed a high level of agreement in rankings for both the intuitive and analytic dimensions.

**OTHER MEASURING INSTRUMENTS**

In addition to the intuitive and analytic ratings, for each of the 25 subjects in the sample the following data was also obtained.

a) scholastic aptitude test scores, verbal and math (SATV, SATM)

b) achievement on major course exams and weekly quizzes

c) weekly time spent in learning as recorded in the A-T center

d) weekly and total learning efficiency scores (learning efficiency defined as achievement divided by associated learning time)

**RESULTS**

Due to the preliminary, exploratory nature of this study, the probability levels (p levels) stated will result from two-tailed tests even though hypotheses are stated directionally. Based on this "conservative" approach, interpretation of the evidence on the basis of the significance levels stated is left to the reader.

**Hypothesis I**: The analytic dimension is more highly related to scholastic ability than is the intuitive dimension.
Figures 4 and 5 and Tables 2 and 3 show the comparison of SATV and SATM respectively for the four groups. The data indicates that no significant interaction occurred between I and A ratings on either SATV or SATM. The high and low intuitive students did not differ significantly on either SATV or SATM scores. However, the students rated as high analytic did differ on both SATV and SATM scores.

**TABLE 2: ANOVA-- Intuitive-Analytic Analysis of SAT Verbal (SATV as Dependant Variable).**

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitive Rating (I)</td>
<td>1</td>
<td>7.493</td>
<td>.001</td>
<td>.975</td>
</tr>
<tr>
<td>Analytic Rating (A)</td>
<td>1</td>
<td>49507.676</td>
<td>6.538</td>
<td>.018</td>
</tr>
<tr>
<td>IA</td>
<td>1</td>
<td>3017.146</td>
<td>.398</td>
<td>.535</td>
</tr>
</tbody>
</table>

**TABLE 3: ANOVA-- Intuitive-Analytic Analysis with SAT Math as Dependant Variable.**

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitive Rating (I)</td>
<td>1</td>
<td>555.583</td>
<td>.136</td>
<td>.716</td>
</tr>
<tr>
<td>Analytic Rating (A)</td>
<td>1</td>
<td>21665.168</td>
<td>5.301</td>
<td>.032</td>
</tr>
<tr>
<td>IA</td>
<td>1</td>
<td>3589.991</td>
<td>.878</td>
<td>.359</td>
</tr>
</tbody>
</table>

Non-significant correlations ($r = -.22$) between I ratings and SAT scores, as well as significant ($p < .05$) correlations between A ratings and SATV and SATM scores ($r = .46$) further substantiate the relation of SAT scores and analytic ratings.

**Hypothesis II:** (A)-Students rated high intuitive will achieve at a significantly higher level than those rated low I.

(B)-Students rated high analytic will achieve at a significantly higher level than those rated low A.

Data from all achievement measures resulted in very similar results. Consequently only data from exam One will be presented.

Data shown in Table 4 and Figure 6 lend support to hypotheses II-A and II-B. The high intuitive students did achieve at a higher level than the low intuitive students ($p < .048$). High analytic students in turn achieved at a higher level than the low A students ($p < .028$). The individual who was both highly intuitive and highly analytic is at a great advantage over all other groups.

Thus, hypotheses II-A and II-B are accepted.
TABLE 4: ANOVA-- Intuitive-Analytic Analysis with Exam One as Dependent Variable.

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitive Rating</td>
<td>1</td>
<td>1442.663</td>
<td>4.427</td>
<td>.048</td>
</tr>
<tr>
<td>Analytic Rating</td>
<td>1</td>
<td>1808.355</td>
<td>5.550</td>
<td>.028</td>
</tr>
<tr>
<td>IA</td>
<td>1</td>
<td>364.653</td>
<td>1.119</td>
<td>.302</td>
</tr>
</tbody>
</table>

Hypothesis III: (A)-Students rated high intuitive will spend less time in learning than those rated low intuitive. (B)-Students rated high analytic will spend less time in learning than those rated low analytic.

Data is presented in Table 5 and Figure 7 showing comparison of I-A groups on A-T center learning times. As shown in Figure 7, the mean A-T time for the low intuitive students is greater than that for the high intuitive students. Table 5 shows the difference to be reasonably significant (p < .155). Thus some measure of support is indicated for hypothesis III-A.

On the other hand, the high analytic students spent more time in learning in the A-T center (p < .069). Thus hypothesis III-B is not supported. This reversal of expected results occurred primarily in the low intuitive groups and not in the high intuitive groups.

Hypothesis IV: (A)-Students rated high intuitive will be more efficient in learning than those rated low intuitive. (B)-Students rated high analytic will be more efficient in learning than those rated low analytic.

Data on A-T center learning efficiency is shown in Figure 8. Associated ANOVA data is presented in Table 6. It can be seen that the high intuitive students were somewhat more efficient in learning (p < .126) than the low intuitive students. This lends some support to hypothesis IV-A. Also the high analytic students were slightly more efficient than the low A students. Probably their very high achievement levels more than compensated for their greater learning.
TABLE 6: ANOVA--Intuitive-Analytic Analysis with A-T Center Learning Efficiency as Dependant Variable.

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitive Rating (I)</td>
<td>1</td>
<td>21514.363</td>
<td>2.533</td>
<td>.126</td>
</tr>
<tr>
<td>Analytic Rating (A)</td>
<td>1</td>
<td>5213.660</td>
<td>.614</td>
<td>.442</td>
</tr>
<tr>
<td>IA</td>
<td>1</td>
<td>27254.301</td>
<td>3.209</td>
<td>.088</td>
</tr>
</tbody>
</table>

Thus limited support is indicated for hypothesis IV-B.

As seen by the interaction term (IA), the interaction variance was reasonably large (p< .088). This interaction between I and A ratings on efficiency is due mainly to the low efficiency of the low I-high A group and the extremely high efficiency of the high I-high A group.

Changes in Learning Efficiency for I-A Groups

When the learning efficiencies for successive weeks are plotted versus time for the four groups of students, the results shown in Figures 9 (a-d) and Table 7 were obtained. As can be seen from the

TABLE 7: Correlation Between A-T Learning Efficiency and Weeks for Groups in Intuitive-Analytic Sample.

<table>
<thead>
<tr>
<th>Group</th>
<th>Correlation Between Weeks and Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lo I- Lo A</td>
<td>-.127</td>
</tr>
<tr>
<td>Lo I- Hi A</td>
<td>.769</td>
</tr>
<tr>
<td>Hi I- Lo A</td>
<td>.724</td>
</tr>
<tr>
<td>Hi I- Hi A</td>
<td>.904*</td>
</tr>
</tbody>
</table>

* p < .05 (d.o.f. = 3)

figures, different patterns resulted from the data for the different groups. Figure 9 (a) shows that in the Lo I- Lo A group the efficiencies jumped around as weeks progressed, ending with a slightly less efficient trend. The correlation between weeks and the corresponding efficiencies for this group, as shown in Table 7, was -.127--indicating that as time progressed efficiency was about constant (or decreased slightly) on the average.

At the other extreme for the Hi I- Hi A group shown in Figure 9 (d), the efficiency pattern is one of gradual but consistent increase. The corresponding correlation between weeks and efficiencies is .904, significant beyond the .05 level. The patterns for the Lo I- Hi A and
Hi I- Lo A groups are somewhat similar to each other—showing an increasing trend in efficiencies but with much variation. Correlations between weeks and efficiencies for these groups were high (.769 and .724) but were not significant at the .05 level.

**DISCUSSION OF RESULTS**

The interview technique for determining an individual's analytic or intuitive tendencies in problem solving gave consistent and reliable results. This suggests that not only are the problem solving approaches real but that they are identifiable and categorizable. The greater consistency of the analytic ratings compared to the intuitive ratings is expected according to the theoretical framework in which the intuitive dimension is related to the availability of relevant higher order concepts and the analytic dimension is more closely related to an information processing ability.

The close relationship of the analytic and not the intuitive dimension to scholastic aptitude (hypothesis I) is consistent as well with the view of the analytic dimension as an information processing ability as opposed to the intuitive dimension being a manifestation of the existence and utilization of over-all superordinate concepts. In other words, when separated into high and low analytic groups, the subjects differed on some ability to process information bits similar to the skills measured by aptitude tests. It was hypothesized that the high and low intuitive separation, on the other hand, was on the basis of the availability of global subject matter concepts—which is not necessarily dependent on general scholastic ability.

The results of comparison of achievement (hypothesis II) is consonant with the aforementioned view of the analytic and intuitive approaches. The result that the individual who is high on both dimensions achieves far above all others suggests that if information in cognitive structure is available and so organized as to enable much freedom of movement at all levels of Figures 1 and 2, the individual is certainly at an advantage. In Ausubelian terms, the desirability of a highly differentiated cognitive structure is shown.

The results on learning time and learning efficiency (hypotheses III and IV), while not providing conclusive information on the role of these variables, do suggest that these parameters are indeed impor-
tant and should be considered more fully in future investigations of concept learning. It does appear that the individual who functions both highly intuitive and highly analytic in problem solving is at an advantage in both learning time spent and resulting learning efficiency.

The comparison of changes in learning efficiencies for the I-A groups led to some interesting results. In terms of the intuitive-analytic scheme, the Hi I- Hi A individuals possess the most highly differentiated cognitive structures--structures necessary to gain information in an increasingly efficient manner which the data seem to suggest. Also the Lo I- Lo A group would be the least likely to be successful in this regard--as the data suggests. Therefore the data obtained on changes in learning efficiency seem to be consistent with an Ausubelian interpretation of learning--in the context of the model of intuitive and analytic problem solving approaches so outlined.

CONCLUSIONS

The following general conclusions can be drawn from this study.

- It is possible to identify consistent and reliable individual differences in problem solving approach and to categorize an individual's preferred mode of attack using as a basis the intuitive and analytic dimensions established. A large variability exists within the population on the A and I dimensions, but individuals appear to be reasonably stable in their approach from problem to problem, particularly on the analytic dimension.

- One of the crucial variables relating to the approach an individual uses in problem solving is the degree of differentiation of his cognitive structure and the concomitant availability of subsuming concepts. The individual who possesses the global, superordinate concepts in a discipline (Hi I) and also has the ability to reconstruct lower level concepts when and if needed (Hi A) is at a significant advantage in terms of achievement and learning efficiency.

- The individual who possesses the ability to regenerate subordinate concepts (Hi A) but lacks the overall subsuming concepts (Lo I) finds it necessary to spend large amounts of learning time resulting in low efficiency.

- There is some evidence to suggest a facilitating effect of a highly differentiated cognitive structure on new learning. The learning efficiency steadily increased in the Hi I- Hi A group. Increases in
efficiency for the other group was not as pronounced.

It is hoped that this research will serve to suggest further investigations of a similar nature. This study does seem to point to the desirability of gearing instructional regimes to specific individual differences—i.e., individualization of instruction is indicated. Studies designed to test the interactive effects of the analytic and intuitive dimensions and various instructional techniques could serve to further an understanding of their role in the learning process.

Since in this study it was possible only to functionally link the intuitive and analytic dimensions with various cognitive-related variables, the establishment of any causal links in this regard would considerably clarify the mechanisms involved. The first step though must be to describe how an individual functions. Then, and only then, will we be able to supply a substantive answer to why he functions as he does.

REFERENCES


High Intuitive Individual Moves Freely From One Superordinate Concept to Another with Frequent Referral Primarily TO (and less frequently FROM) Subordinate Exemplars.


High Analytic Individual Moves Primarily Within Subordinate Concepts and To Superordinate Concepts, with Referral Back to Subordinate Concepts, Thus Expanding the Superordinate Concepts. (Very little, if any, exchange between Superordinate Concepts.)


FIGURE 3: Intuitive-Analytic Rating Grid (Each Point Represents One Student)
Dotted Lines Show Assignment to Groups

FIGURE 4: Interaction of Intuitive and Analytic Ratings on SATV
FIGURE 5: Interaction of Intuitive and Analytic Ratings on SATM

FIGURE 6: Interaction of Intuitive and Analytic Ratings on Preliminary Exam One

FIGURE 7: Interaction of Intuitive and Analytic Ratings on Average A-T Learning Time

FIGURE 8: Interaction of Intuitive and Analytic Ratings on A-T Efficiency Average
Efficiency

Error bars show one standard deviation.

FIGURE 9(a): Changes in Weekly Learning Efficiency for Low Intuitive - Low Analytic Group

FIGURE 9(b): Changes in Weekly Learning Efficiency for Low Intuitive - High Analytic Group

FIGURE 9(c): Changes in Weekly Learning Efficiency for High Intuitive - Low Analytic Group

FIGURE 9(d): Changes in Weekly Learning Efficiency for High Intuitive - High Analytic Group

* Error bars show one standard deviation.