Reviewed was research on the development of the cognitive skills of language, concept formation, and arithmetic in children handicapped by blindness, mental retardation, or deafness. Research on the language skills of the blind included a rejection of sensory compensation, while research on language in the retarded was seen to focus on linguistic variables and reading ability. Included among the research on language development of the deaf was research which was reported to suggest the value of early sign language training for cognitive development and the author's research on written language comprehension by the deaf. Research on concept formation in the blind found deficiencies in concept formation among the blind, while concept problems in the retarded were found to be in the areas of language control and verbalization rather than perception. Research on concept development in the deaf showed conflicting findings on whether a concept deficiency exists once verbal aspects are removed. Little research on arithmetic skills in the blind was reported, but one finding of skill development in the retarded showed better computation skills than normal children of the same mental age. The author's research found that the mathematical performance of deaf children was usually slightly higher than that of normal hearing children. (L3)
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E. J. Crothers. Presentation orders for items from different categories. March 10, 1965


(Continued on Inside back cover)
A SURVEY OF COGNITION IN HANDICAPPED CHILDREN

by

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INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES

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A Survey of Cognition in Handicapped Children

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In this article I survey broadly the literature on cognition, with a special emphasis on the development of academic skills in handicapped children. Among such skills, I shall concentrate almost entirely on language and elementary mathematical skills. This concentration seems to need little justification, since these are the basic skills most important in training handicapped children for productive careers in society. It is also the set of skills most important for normal children. This does mean that to some extent I neglect the full range of psychological studies of concept formation in handicapped children in order to concentrate especially on language development and elementary mathematics. I am excluding the many studies on operant conditioning, reinforcement schedules, paired-associate learning and the like, especially in mentally retarded children. It is possible to make a case that these studies fall within the general area of cognition, but it is also reasonable to exclude them, and I have done so here. There have been a great many studies in the general area I am excluding, and the interested reader will find it easy to get into that literature from some of the survey references given below.
I have divided the article into three main parts, treating first problems of language and language development, second, concept formation and abstraction, and third, elementary mathematical skills. As might be expected, the literature on language development, for example, is larger by an order of magnitude than the literature on the development of mathematical skills. I have made some effort to locate studies dealing with mathematical skills, but it will be clear to the reader that additional studies of a substantial nature are needed in order to give a more complete picture of the problems and potentialities of developing mathematical skills in handicapped children.

In each of the three parts, I treat first the relatively small literature dealing with cognition in blind children. Second, I survey somewhat superficially the enormous literature on mentally retarded children. The psychological and educational literature on mental retardation is immense, with little hope for surveying it in this relatively brief article. The reader is referred especially to the Annual International Reviews of Research in Mental Retardation edited by Norman R. Ellis. Other single volumes reviewing the research in extensive form are, for example, Stevens and Heber (1964) and a book of considerable theoretical interest that I shall return to later, Estes (1970). I emphasize, however, that these references are only the top of the iceberg.

At the end of each part I turn to deaf children. My own research has been concerned with deaf children, and consequently, it is only here that I report any primary research from the Institute for Mathematical Studies in the Social Sciences at Stanford. Research on forms of handicap other than these three has not been covered, even though there are
substantial bodies of research available. Moreover, the analysis of research on the cognitive skills of the mentally retarded has mainly been restricted to studies dealing with educable mentally retarded children.

I have also restricted myself to cognition in handicapped children, because it seems most important to understand developmental processes and their absence in children. From a clear understanding of these we shall be able to predict cognitive abilities of handicapped adults, and it is really only in dealing with handicapped children as opposed to adults that we can hope to develop special education programs of long-range significance. Limitations of space and time have forced these various restrictions.

The general focus in this article is cognition, but because so much of the discussion is devoted to language skills, a few remarks on the relation of language to cognition seem appropriate. To begin with, it is important to note that an emphasis on cognition immediately narrows the interest in language skills. The development of phonology, a subject of great complexity and importance in its own right, is not deeply relevant to cognition. A similar case can be even made for the development of purely grammatical or syntactical skills. The relevance of language to cognition is, in semantical terms more than any others, the means by which language is used to convey information and meaning. As a consequence, what is said here about language and cognition will differ rather markedly in tone and emphasis from a purely linguistic account of language development in handicapped children.
1. Language Skills

1.1. Blind Children

Bean (1932) studied the language development of his son, who was blind until 18 months of age. He found that until the blindness was removed by an operation the child's vocabulary was composed primarily of words derived from senses other than the visual. After the operation the visual terms multiplied much more rapidly in his vocabulary than did terms referring to experience obtained through the other senses. Maxfield (1936) studied eight totally blind children over several observational periods. The children were young, ranging from 38 to 73 months. Not surprisingly, he found that even the youngest children had a significant percentage of visual terms in their spoken language. For example, one of the three subjects in the range from 38 to 42 months of age, who was totally blind, used visual terminology in 6 percent of his total responses. More surprising are the results of Cutsforth (1951), who investigated word associations in 26 congenitally blind children. He found that nearly one-half of their responses contained the names of visual qualities. Only about 7 percent referred to the qualities of taste or smell and approximately 3 percent to qualities of hearing. The remainder referred to abstract qualities not referring to particular sensory modalities. He concluded that the high percentage of visual responses was evidence that the children were developing language to meet social approval. An alternative hypothesis that would be interesting to investigate is that the saliency of visual terms in the language heard by the children is certainly much higher than that of terms referring to experience obtained through the other modalities. Some rather carefully
designed experiments would be necessary to disentangle these two ways of looking at the kind of results that Cutsforth reported.

Nolan (1960) obtained free and controlled associative responses to the stimulus words used by Cutsforth. He obtained a somewhat smaller number of visual responses, but concluded that the use of visual terms of blind children was not a significant problem for them.

Hayes (1938) studied 443 blind children (ages 10 to 23+), using Terman's English Group Vocabulary Test. The results indicated that among the blind inferiority in the understanding of words was about equal to their retardation in grade placement in the early grades.

In the research literature on blind children the use of terms referring to visual experience is often termed verbalism, because the use of such terms is not built on direct sensory experience of the students. A study by Harley (1963), carefully conducted with 40 children blind from birth, led to the conclusion that verbalism is not a significant problem. He found that chronological age, intelligence and experience were inversely related to verbalism, i.e., high occurrence of visual terms, and he found no significant relation between personal adjustment and verbalism.

The cognitive status of visual terms in the language of blind children cannot easily be determined from the studies reported. More detailed semantical analysis of their actual use of such terms is much to be desired. Some steps in this direction have been taken by Rathna (1963), who analyzes in some detail the visual terms used by blind persons in their spoken language.
Bateman (1965) studied the performance of partial-seeing children in comparison with normal-seeing children on the Illinois Test of Psycholinguistic Abilities (ITPA). Her subjects were 93 partial-seeing children in Grades 1 to 3. Their performance on each subtest was compared with the standardized group upon which the ITPA norms are established. In spite of an expectation of superior performance by the partial-seeing children on the Auditory Decoding Subtest, no difference in performance was found. On the Visual Decoding Subtest, the partial-seeing children, as would be expected, showed a clear and significant deficit. On the Auditory Vocal Association Subtest, differences in comparison according to chronological age were found, but when comparisons were based on mental age the slight deficit for the partial-seeing group was not significant. On the Visual Motor Association Subtest, the partial-seeing group was significantly below the sighted group as might be expected. On the Vocal Encoding Subtest, no significant differences between the groups were found. On the Motor Encoding Subtest, the partial-seeing children were significantly below the normative group. This deficit was perhaps the most significant and seems to point to a lack of knowledge of how objects are used, knowledge that is usually gained from visual experience. On the Auditory Vocal Automatic Subtest and the Auditory Vocal Sequential Subtest, no significant differences were found. On the other hand, again as would be expected, on the Visual Motor Sequential Subtest, significant deficits were found in the partial-seeing group.

A careful study following up on the question of whether blind children do compensate by developing superior auditory discrimination ability especially for spoken language has been conducted by Hare,
Hamill and Crandell (1970). This study also reviews the earlier studies. Using carefully selected samples of partial-seeing and seeing children, the investigators tested the following three hypotheses:

(i) Partial-seeing and normal-seeing children with similar mental ages and chronological ages do not differ significantly in sound discrimination ability.

(ii) Partial-seeing children who vary in degree of visual acuity do not differ in sound-discrimination ability.

(iii) Partial-seeing children show no significant differences in the relationship of sound-discrimination ability to chronological age, mental age and tactile kinesthetic ability.

The null hypothesis was not rejected by the data for any of the three hypotheses, and the authors concluded that the "myth of sensory compensation" is thoroughly unsupported.

In this study, sound-discrimination ability was measured by Form A of the Irwin Sound-Discrimination Test, which consists of 30 items of word pairs. The pairs differed most by a single phoneme, and the subject was required to respond "same" or "different." The test is scored by counting the correct responses to the pairs. The partial-seeing children had a mean score of 19.7 with a standard deviation of 6.9, and the normal-seeing children had a mean score of 20.1 with a standard deviation of 6.2. It is clear without any statistical tests that these data do not represent samplings from significantly different populations and the null hypothesis is not rejected.

In considering the English comprehension of blind students, attention has been given especially in recent years to their ability to comprehend
rapid speech. The objective is to use rapid speech to increase input by two or three times the rate of Braille reading. Foulke, Amster, Nolan and Bixler (1962) measured the listening comprehension of 291 Braille readers of both sexes in the sixth, seventh and eighth grades of 11 residential schools for the blind. None of these students had previously been exposed to rapid or compressed speech. Materials were presented at rates of 175, 225, 275 and 325 words per minute. A 36-item multiple-choice test was conducted to measure comprehension. It was found that the comprehension level was satisfactory for the compressed speech. In particular, no loss of comprehension of either scientific or literary material was found in listening to compressed speech of up to 225 words per minute. The authors contrasted this to typical recording rates of 175 words per minute and the mean Braille reading rate for high-school blind students of about 90 words per minute. In the case of scientific materials they found that there was no significant loss of comprehension through 275 words per minute.

The studies on sensory compensation and rapid speech suggest that there is no easy road to educating blind children, when the normal mode of taking in information is so heavily dependent on printed texts. Many cognitive deficits of blind children are almost certainly due to this relatively simple fact of not living an alternative input channel that can match the rate of visual processing, and thus they are "information-poor," deprived in the quantitative sense of the amount of information transmitted to them.
1.2. Mentally Retarded Children

In spite of the great interest in language development, it is surprising that in the first five issues of the International Review of Research in Mental Retardation edited by Ellis not a single major article was devoted to language skills or language development of mentally retarded persons. However, some excellent reviews of language and language development in mentally retarded persons exist in the journal literature; especially noteworthy are the reviews by Blount (1968) and by Spreen (1965, 1966). The second article by Spreen deals with higher language functions and will be referred to in the discussion of abstraction and concept formation. These three articles provide extensive references to the literature, and I shall not duplicate their extensive bibliography here. These review articles do not cover the recent linguistically oriented work on language in retardates, and consequently I shall emphasize this newer literature.

Blount (1968) is particularly concerned with language in the more severely retarded, meaning by this persons with IQs below 50 and with a mental age range of 2 to approximately 8 years, and I want to mention briefly some of the more interesting studies he summarizes. Karlin and Strazzulla (1952), Iyle (1961b) and others find that the more severely retarded are delayed in their language development, but follow approximately the same sequence of development as do normal children. A natural comparison has been the language development of institutionalized and noninstitutionalized matched pairs. A number of studies have found better performance on the part of the noninstitutionalized children (Iyle, 1959, 1960a, 1960b, 1960c and 1961a; Schlanger, 1954). On the
other hand, Mueller and Weaver (1964) found opposite results. They found the ability of institutionalized, trainable mental retardates superior to that of day-school retardates of matched characteristics in terms of IQ, chronological age, sex and race. They used as their instrument the Illinois Test of Psycholinguistic Abilities.

A major study by Lenneberg, Nichols and Rosenberger (1964) examined over a period of three years the language development of Mongoloid children ranging in age from 3 to 22 years. The IQs of the children ranged from the 20s to the 70s. Their major findings were: IQ does not predict the stage of language development but chronological age does; a significant relation exists between motor development and the onset of language; although the rate is much slower, language development in Mongoloid children is similar to that in normal children; some Mongoloid children are able to process syntactically complex sentences. As might be expected, these authors used their results to defend the general proposition that language development is not closely related to intellectual ability, but rather it is more closely related to general biological processes of maturation. As with most general hypotheses of this kind, the data are not presented in a fashion that permits a sharp statistical evaluation or quantitative assessment of the degree to which the hypothesis is actually supported. For example, there are no statistical analyses of alternative hypotheses, and thus there is not even a rough idea of the statistical power of their data relative to their hypotheses.

In contrast to the study of the language of blind children, a number of highly specific linguistic studies of the language of retarded children are to be found in the literature. Lovell and Bradbury (1967) studied
160 children aged 8 to 15 inclusive. Their three hypotheses were:

(i) the ability of these children to inflect, derive and analyze compound words improves little between 8 and 15 years of age and is generally below that of normal first graders; (ii) there is a significant relationship between reading level and the ability to inflect lexicon words; (iii) there is a significant relationship between IQ and the ability to inflect nonsense words, but little relationship between reading attainment and the inflection of such words. The data confirmed all three hypotheses.

Graham and Graham (1971) studied the syntactic characteristics of the speech of nine retarded children with chronological ages ranging from 10 to 18 years and mental ages ranging from 3 years 6 months to 10 years. Their data supported the hypothesis that non-Mongoloid retardates develop language at a different rate but in approximately the same way as normal children.

Semmel, Barritt, Bennett and Perfetti (1968) undertook a grammatical analysis of word associations of educable mentally retarded and normal children. In studies of the language development of normal children it has been found that as they get older they tend to increasingly give associations to stimuli falling within the same grammatical form class as the stimulus. These investigators found the highest level of such form-class responses in the older normal children and the lowest incidence of such responses in the institutionalized retardates.

Cartwright (1968) studied the written language abilities of educable mentally retarded in comparison with normal children. His subjects were 80 12- through 15-year-old educable mentally retarded and 160 8- through 15-year-old normal children. Comparisons were made on the following
language measures: composition length, sentence length, type-token ratio, percentage of usage of different parts of speech, grammar and spelling. The normal children of the same age had significantly higher scores on all these measures. Younger normal children, aged 8 through 11, obtained significantly higher scores than the educable mentally retarded group on three of the measures, namely, type-token ratio, grammar and spelling. The absence of difference in sentence length is significant, considering the extent to which mean utterance length is currently used as a measure of language development by a number of psycholinguists.

One of the more extensive studies of the spoken vocabulary of retarded children has been made by Beier, Starkweather and Lambert (1969). They interviewed 30 retarded children and recorded 2700 words from each. The approximately 80,000 words of output were analyzed and compared with the output of normal children. They found differences in the word lists, but a large number of similarities in performance of the retarded and normal groups. They interpreted their overall findings as supporting the assumption that mentally retarded children suffer from a conceptual and organizational deficit in their language usage.

These various studies show that even if the sequence of language development is similar in normal and retarded children, most cognitive functions of language are less developed in retarded children. But it is not yet clear if the deficit is most pronounced in the primarily cognitive aspects of language. Much better and more detailed data on the impact of training would also be most desirable, for example, the rate of acquisition of new words, the rate of improvement in spoken and written grammar.
Reading. Several good studies exist on the particular deficiencies of mentally retarded children in reading. Dunn (1954), for example, compared 20 retarded boys with 30 normal students of comparable general mental age. He used a number of the standard battery of tests to measure reading achievement and found that the retarded group averaged one year below the normal group of comparable mental age. In reading errors, the retarded group had more faulty vowels and sound omissions. Also, they made less use of context clues. More significant, however, was the lack of differences between the two groups in frequency of faulty consonants or in word reversals. In addition, no significant differences were found between the groups on handedness, eye dominance or mixed lateral dominance. Because of a similar finding in other studies, it is worth noting that more personal-social maladjustments were found in the retarded group, on the basis of teacher ratings.

Ragland (1964) obtained results similar to Dunn’s. He was concerned to investigate more thoroughly why educable mentally retarded children lagged behind the reading achievement that would be predicted from their mental ages. Using the Illinois Test of Psycholinguistic Abilities, he found that the retarded readers scored significantly lower than nonretarded readers on the Auditory Vocal Automatic Subtest. In this literature this is called the automatic sequential level. The automatic sequential responses have been noted to be deficient in retardates in a number of studies using the Illinois test. Results rather similar to Ragland’s were also found by Kass (1962), in an unpublished doctoral dissertation. What is significant about Ragland’s conclusions, as well as those of other investigators with similar results, is that the reading
difficulties of retarded children seem to reside at the nonmeaningful automatic level of responding rather than at the meaningful level. These results seem rather surprising, for it would be natural to conjecture that the problem of understanding meaning would be the main source of difficulty. It would be desirable to have more detailed quantitative data on these matters under strictly defined learning conditions. It does suggest a very fruitful area of research.

A number of other studies on the reading difficulties of retardates are to be found in the literature, although I shall not attempt a wider review. One does come away from this literature with the impression that much more quantitative research should be undertaken in this area. Most of the studies use at the most relatively simple statistical tests; in many cases, even simple measures of this kind are missing. Detailed learning-theoretic studies with clear underlying theoretical assumptions about learning would seem to be called for in this significant area of training of retardates. The excellent studies of discrimination learning and paired-associate learning by retardates that lie outside the field of this review do not easily generalize to more complex problems like those of reading. However, the methodology of those studies, which is in many cases at an excellent level, needs to be brought to the study of teaching the retarded child to read. More is said about these matters in Section 2.

I have reported a number of different kinds of studies about the language development and language usage of retarded children. It is clear that we are still some distance from having a systematic and comprehensive theory of these phenomena. Perhaps the central issue
of a theoretical nature is whether the language development and usage of retarded children can be treated as qualitatively similar to that of normal children, but at a slower rate of development. This is the thesis of Lenneberg and other biologically oriented linguists. Psychologists concerned with the development of cognitive skills in conjunction with the development of language are probably inherently more skeptical of this thesis and have performed a number of studies to place it in doubt. As indicated by the conclusions of several of the studies summarized above (Lovell & Bradbury, 1967; Semmel et al., 1968; Cartwright, 1968; Beier et al., 1969), what is needed is a more precise definition of what is to be regarded as the central core of language development as opposed to the development of broad cognitive skills and knowledge.

It is also clear that although a number of studies have been performed on language training of retardates, much more is to be learned in this area. As yet, no extensive studies of language learning with an emphasis on the learning of syntax and semantics are available. It would be interesting to compare at a more abstract and systematic level the production grammar and semantics of retarded and normal children. The methodology for such studies is exemplified in the study of the speech of normal children in Smith (1972) and Suppes (1970, 1971). It is my judgment that this would be one of the most salient areas for future research of significance for the language training of retarded children.

1.3. Deaf Children

The problem of language deficits in deaf children has received more attention than any other cognitive or educational component of the competencies and skills of deaf children. Competence in a standard
natural language is the outstanding defect and problem of deaf persons. The magnitude of the defect in general varies directly with the magnitude of hearing loss and with the age of hearing impairment. These facts are well known, and I shall not review the data here. Studies of the language performance of deaf persons naturally fall into three parts: production and comprehension of spoken language, production and comprehension of written language, and production and comprehension of manual or sign language. Discussion of the hotly contested issue of whether deaf children should be taught manual communication or oral communication is given below.

Concerning the initial vocalizations of infants in the acquisition of spoken language, Lenneberg, Rebelsky and Nichols (1965) did not find significant differences between deaf and hearing infants during the first three months of life. The evidence seems to be that deaf children continue to develop a normal pattern of vocalizations (babbling, crying, cooing, etc.) until about six to nine months of age.

When we turn to older children, the number of studies on the spoken speech of deaf children is small. In their extensive survey of the language skills of reading and writing in deaf children, Cooper and Rosenstein (1966) indicated that they were able to find only a few studies concerning the spoken language of deaf children, and they excluded a survey for this reason. Six years later, at the writing of this article, the situation still seems to be true. There are a few studies of the spoken syntax of hard-of-hearing and deaf children, for example, Brannon and Murry (1963), but the number of studies is small, and the extent to which the studies
pursue the syntactic or semantic structure of the spoken speech is still unsatisfactory.

Brannon and Murry compared groups of hearing with hearing-impaired children in their oral as well as written responses to colored pictures. The responses were evaluated by use of Myklebust's Picture Story Language Test. As might be expected, they found that as the hearing loss increased, the ability to communicate orally decreased. More interesting is their finding that although the deaf were inferior in structural accuracy, they were not inferior in productivity. Also, corresponding to other findings in the literature, the deaf children began and ended their sentences with relatively few errors compared with the large number of errors occurring in the middle of sentences. Further, the inflectional patterns of English were not used extensively by the deaf; they tended to use kernel sentences more than did normal children.

The evidence of a paucity of studies of spoken language is reinforced by Quigley's (1966) excellent review of language research in countries other than the United States. He reported few studies of a research character dealing with spoken speech. He did mention Linder's (1962) study of the speech rates of deaf children and summarized Linder's finding that in spite of the considerably slower speech rate of the deaf children only the voice sounds were lengthened. In addition, the length of syllables shows less variation in the pronunciation of deaf children than in the pronunciation of adults with normal hearing. Klinghammer (1961) compared the recorded speech of ten normal, blind and deaf persons and had their speech judged by listeners in various areas. In comparison with the blind, the deaf did not do very well. Apparently the unusual features of deaf
speech to normal ears were an immediate source of difficulty for normal-hearing listeners. The other studies reported by Quigley are of a similar character, i.e., they are not linguistic in character, and it is only recently that we could anticipate really substantial linguistics studies of the spoken speech of deaf children. But, apart from those concerned with comparison of oral and manual methods of communication, which is discussed below, I have been unable to find any published studies.

The extensive studies of the written-language competence of deaf persons reported by Cooper and Rosenstein (1966), and since then by a number of other investigators, are too numerous to review in depth. A few general conclusions drawing on the summary of Cooper and Rosenstein are the following. First, deaf children have been found to be significantly retarded in their achievement test scores in terms of reading or writing. Their written language typically contains shorter and simpler sentences and displays a different distribution of the parts of speech from that of normal-hearing children. It is also true that the kinds of errors they exhibit are different from those found in normal-hearing children and their speech has qualities of rigidity and stereotyping not characteristic of the written language of normal-hearing children.

In the last few years Quigley and his associates at the University of Illinois have been extensively studying the written language of deaf children. For example, Marshall and Quigley (1970) analyzed (in terms of what are called in the literature) minimal terminal syntactic units or t units in order to measure comparatively the syntactic complexity of various samples of deaf speech; other measures of complexity were used as well. For instance, they used a subordination index, apparently
first introduced by Heider and Heider (1940), which is the ratio of verbs in subordinate clauses to the total number of verbs in sentences. Essentially this measure determines the extent to which subordinate clauses are employed in the construction of sentences. Marshall and Quigley found that growth in complexity of the written language of deaf children is due mainly to the use of increasingly complex noun phrases and only slightly due to more complex verb phrases.

As in the case of spoken speech, there do not seem to be any detailed empirical studies of the complete syntax of samples of deaf speech. By "complete syntax" I mean the construction of a generative grammar for large samples of such language. Some preliminary efforts to construct probabilistic generative grammars in the sense of Suppes (1970) for samples of written deaf language have been undertaken in our Institute by Dr. Robert Smith, but this work is as yet unpublished.

Sign language. In just the last few years there has been an intensive spurt of interest in the grammar of sign language, but interest in sign language has a history that extends back hundreds of years. Important studies during the past decade are those by Stokoe (1960, 1971), Stokoe, Casterline and Croneberg (1965) and McCall (1965). Detailed studies of the grammatical structure of sign language have appeared quite recently or are still in the process of being published. I mention here Battison (1971), who studied the relationship between signs and their reference but did not work out a complete semantics. Fant (1972) looked at the differences between American sign language and English from a syntactic standpoint. He characterized the syntax of sign language as resembling short, simple English sentences, but again an explicit generative grammar
has not been constructed. Schlesinger (1970) studied Israeli sign language and the relation of the syntax of that language to the existence of language universals. Quite recently Bellugi and Siple (1971) and Klima and Bellugi (1972) made psycholinguistic studies of the use of sign language, especially in the language development of young children. Still missing in this research is a formal generative grammar that encompasses a high percentage of the utterances in standard communication situations as well as any attempt at a systematic semantics. However, the absence of semantical analysis is generally true of the language studies reviewed here and is not peculiar to those concerned with sign language.

Schlesinger and Meadow (1971) studied the acquisition of sign language by four congenitally deaf children and concluded that the stages in the acquisition of sign language are about the same as the stages in the language acquisition of hearing children. Similar conclusions were reached by Bellugi (1970) and Tervoort and Verbeck (1967). I emphasize, however, that the data in these last studies on acquisition lack the kind of rigorous analysis characteristic of the best studies of handicapped children, for example, the studies of associative and discrimination learning in retarded children.

Manual vs. oral. As already noted, the really intense controversy in the language of the deaf has been over the relative advantages of manual versus oral communication, not only in schools but starting from the earliest age of the child. While it is not appropriate in this review article to take a position on this controversy, I do want to refer to some of the studies, especially the more recent ones. Much of the
controversy has been marked by strong expressions of opinion rather than by skillful and objective experimentation and analysis of results. It is hard to think of an area in which really careful and extended experimentation would be of more use, for there is a long tradition of support of each position. Until recently, the oral position was probably the dominant one, but in the last few years there has been an increasing interest in and respect for what has been achieved by manual methods beginning with the very young child.

The studies I review here draw upon the recent report by Bonvillian and Charrow (1972). Alterman (1970) reviewed the two positions and found no basis for the claim that oral skills are necessary for adjustment to hearing society, that usage of the sign language makes learning standard natural language more difficult, and that early exposure of the deaf child to parental spoken speech is beneficial. All in all, his arguments make a case for early manual training. Tervoort and Verbeck (1967) found no correlation between early manual training and progress in speech training. Montgomery (1966) found that learning sign language does not negatively affect speech or speech reading skills. Hester (1963) found that manual finger-spelling deaf students were superior to an oral group of deaf children on standardized achievement tests. Stevenson (1964) examined the educational achievement of children who had learned sign language versus an orally taught group and found the manual group superior in 90 percent of the matched pairs. Stuckless and Birch (1966) compared 105 deaf children who were taught sign language and whose parents were deaf with 337 matched deaf children who were taught orally and whose parents were normal-hearing adults. They
found the manual group was better in speech reading, reading and somewhat better in psychosocial adjustment. They found no differences between the groups in speech. Meadow (1968) reached similar conclusions and also found that the manual group did somewhat better in elementary mathematics learning.

To avoid the possible confounding in the Stuckless and Birch study of having deaf parents in one group and hearing parents in the other, Vernon and Koh (1970) studied subjects with a family history of genetic deafness. The manual and oral groups were matched for IQ, sex and age and were examined on the variables of educational achievement, communication skill and psychological adjustment. The investigators found that the use of early manual communication produced better overall educational achievement, including better performance in reading skills and written language. Similar conclusions from groups somewhat differently selected were also obtained in a later study by Vernon and Koh (1971).

The findings just cited, together with those cited above about the apparent parallel between the acquisition of sign language and the acquisition of English, do seem to call for a thorough reevaluation of the oral position in the language training of deaf children.

The studies reviewed by Bonvillian and Charrow and cited here obviously favor the manual approach. The results of some of these studies are impressive, but there are also some impressive gaps. We do not have, for example, detailed learning studies comparing language acquisition rates for the two methods, and the evidence of substantial success using either method with average deaf children is still unsatisfactory.
That a positive correlation exists between deafness and other disabilities is well known. However, the acquisition of language by deaf children, who exhibit additional handicaps such as brain damage causing language disorders and motor disorders, has not been examined here. A good review of the literature on these matters may be found in Withrow (1966).

Language comprehension. As has already been indicated, the most salient missing aspect of the analyses of the language of either deaf or retarded children is the absence of serious attention to the semantics of their language and the identification of effects in semantics, either in terms of comprehension or production. The problems of identifying difficulties of comprehension may be approached at many different levels of detail. The most satisfactory would offer a full systematic semantics.

At this point I would like to give an example of some research conducted in the Institute on the written language comprehension of deaf students. This example applies the kind of regression methods we have used extensively for the analysis of relative difficulty of exercises in elementary mathematics (Suppes, Hyman & Jerman, 1967; Suppes, Jerman & Brian, 1968; Suppes & Morningstar, 1972). The regression models considered were developed and tested by Mrs. Jamesine Friend, who was Coordinator of the project in computer-assisted instruction for deaf students in the Institute from 1968 to 1971. This example deals with the analysis of difficulties deaf students encounter in reading and following written directions. The directions occur at the beginning of the computer-assisted instruction course "Language Arts for the Deaf," which was delivered to deaf students in residential schools and also to deaf
students in day classes using teletype terminals connected by telephone lines to the Institute's computer at Stanford. Some examples of the directions are the following. I show in capital letters the question and the example to which the question must be applied.

Example 1 (from Directions Lesson 1):
// WHICH IS THE FIRST WORD?
SOME DOGS ARE FRIENDLY.

Example 2 (from Directions Lesson 2):
// WHICH WORD COMES AFTER "VERY"?
MY TYPEWRITER IS VERY BIG AND HEAVY.

Example 3 (from Directions Lesson 9):
// WHICH LETTER COMES BEFORE "E"?
SILVER

Example 4 (from Directions Lesson 16):
// TYPE THE LAST TWO LETTERS.
MILLION

Example 5 (from Directions Lesson 25):
// TYPE THE NUMBER BELOW 4.

2 7
6 4
8 3

A number of structural features in these exercises affect their difficulty. In this kind of analysis we identify the structural features independent of any response data from the students, so that typical structural features are syntax, number of words, number of characters, and so forth. Variables
of this kind have been used as structural features to predict the relative difficulty of arithmetic word problems (Jerman, 1971; Loftus & Suppes, 1972; Suppes, Loftus & Jerman, 1969). Mrs. Friend identified 14 such variables in the context of the language arts exercises on following directions. The variables she tested are the following.

Variable $X_1$: 0 if the direction is imperative. 
1 if interrogative.

Variable $X_2$: 0 if the direction is a simple sentence or a transform of a simple sentence. 
1 if compound.

Variable $X_3$: Number of key words in direction. ("Key words" distinguish one direction from another within the same lesson. In Example 1 above, there is only one key word, "FIRST," whereas in Example 5, there are two key words, "LAST" and "TWO.")

Variable $X_4$: 0 if the position cue is named (as in WHICH LETTER COMES BEFORE "E"). 
1 if the position cue is described (as in WHICH LETTER COMES BEFORE THE LAST LETTER?).

Variable $X_5$: Number of words in the instruction.

Variable $X_6$: 0 if direction does not contain "above," "below," "under," "before" or "after." 
1 if it contains "above," "below" or "under." 
2 if it contains "before" or "after."

Variable $X_7$: Lesson number.

Variable $X_8$: Ordinal position of the exercise within the lesson.
Variable $X_9$: 0 if preceding exercise involved the same task, 1 if otherwise.

Variable $X_{10}$: Number of elements (words, letters, numbers) in the stimulus display.

Variable $X_{11}$: 0 if there are no critical distractors, i.e., distractors that would be correct responses if the direction from the preceding exercise were used, 1 if otherwise.

Variable $X_{12}$: Length of correct response (in characters).

Variable $X_{13}$: Number of distractors preceding the correct response.

Variable $X_{14}$: Number of characters in the stimulus display (spaces not included).

These 14 variables were applied to predict the mean probability of a correct response to each of 125 exercises in lesson pretests for a sample of some 300 students. To be explicit, the regression equation is first transformed because in an ordinary additive regression probability is not necessarily preserved, and we can get predictions of negative probabilities or probabilities greater than one. We have therefore customarily used the transformation

$$z_i = \frac{\log (1 - p_i)}{p_i}.$$

The regression equation then assumes the following form in terms of the dependent variable $z_i$

$$z_i = \sum a_i X_i + a_0.$$
The results of the stepwise linear regression are shown in Table 1. Nine of the variables account for 44 percent of the variance and the remaining five contribute little. (The square of the multiple correlation ($R^2$) is a measure of the percentage of variance accounted for by the model.) The most powerful variable is $X_6$, which deals with the inclusion or exclusion of certain prepositions. The relative difficulty deaf students have with prepositions is well known and familiar in the literature. The second most important variable is $X_{13}$, which deals with the number of distractors preceding the correct response. This variable corresponds closely to a serial position variable for the correct response. The other variables entering during the first nine steps of the regression, namely, variables $X_7$, $X_9$, $X_{10}$, $X_{14}$, $X_2$, $X_8$ and $X_4$, each contribute something, but do not make the dramatic contribution of variables $X_6$ and $X_{13}$.

Regression models of the kind just described are by no means a final answer to the theoretical problems of language production or recognition on the part of deaf students. They do provide a good first entry into the detailed study of comprehension. From the standpoint of constructing curriculum they can be especially useful in providing a practical technique for creating items of a given desired level of difficulty, for new items—questions or exercises—can be written such that they have specified values of the structural variables, and thus a predicted probability correct for a given reference population of students.
TABLE 1
Step-wise Linear Regression for 125 Exercises on Following Directions

<table>
<thead>
<tr>
<th>Step number</th>
<th>Variable number</th>
<th>Multiple R</th>
<th>Multiple ( R^2 )</th>
<th>Increase in ( R^2 )</th>
<th>F value for del. Last regression coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0.37960</td>
<td>0.14410</td>
<td>0.14410</td>
<td>20.7108</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0.56830</td>
<td>0.32319</td>
<td>0.17910</td>
<td>32.2826</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>0.59690</td>
<td>0.35629</td>
<td>0.03310</td>
<td>6.2309</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0.61200</td>
<td>0.37454</td>
<td>0.01825</td>
<td>3.4883</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.61880</td>
<td>0.38291</td>
<td>0.00837</td>
<td>1.6261</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>0.62590</td>
<td>0.39175</td>
<td>0.00884</td>
<td>1.7131</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.62880</td>
<td>0.39539</td>
<td>0.00364</td>
<td>0.6907</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0.65500</td>
<td>0.42903</td>
<td>0.03364</td>
<td>6.8437</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>0.66430</td>
<td>0.44129</td>
<td>0.01227</td>
<td>2.5144</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>0.66930</td>
<td>0.44796</td>
<td>0.00667</td>
<td>1.3822</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0.67070</td>
<td>0.44984</td>
<td>0.00188</td>
<td>0.3970</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>0.67130</td>
<td>0.45064</td>
<td>0.00081</td>
<td>0.1502</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>0.67160</td>
<td>0.45105</td>
<td>0.00040</td>
<td>0.0735</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>0.67190</td>
<td>0.45145</td>
<td>0.00040</td>
<td>0.0820</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>0.67190</td>
<td>0.45145</td>
<td>0.00040</td>
<td>0.00952</td>
</tr>
</tbody>
</table>
Perhaps the most important feature of regression models is that they give an estimate of magnitudes of effect and not just a significant relationship between a given variable and the responses of students. From the standpoint of practical applications, a central weakness of many of the studies reviewed in this chapter is that they have been concerned to establish a statistically significant relationship between two variables rather than to estimate the magnitude of an effect. The greater power of an estimate of magnitude of effect is evident and is especially important for any practical applications. When large samples of students are used, ordinarily a statistically significant relationship can often be obtained, even if the actual effect of one variable on another is small. In the designing of educational programs, especially the detailed articulation of remedial programs for handicapped students, methods that aim at main effects and have substantial consequences for learning of the students are of prime importance. For purposes of identifying such methods, regression models are more useful than the usual F tests and t tests.

2. Concept Formation and Abstraction

In this section I try to emphasize some of the critical theoretical issues, for in many respects the quality of the empirical studies on concept formation in handicapped children has exceeded the quality of the theoretical analysis of the results. I emphasize in the discussion of retarded children the use of mathematical models to estimate individual learning parameters, and in the discussion of deaf children the issue of verbal versus nonverbal learning and mastery of concepts.
2.1. Blind Children

Zweibelson and Barg (1967) reviewed some of the earlier literature and studied the concrete, functional and abstract levels of concept formation in blind children in comparison with sighted children. The sample was small (eight in each group), but carefully selected. The primary instrument of measurement was the Wechsler Intelligence Scale for Children. Using nonparametric tests because of the smallness of the sample, the investigators tested the hypothesis that the blind children would use as many abstract concepts as the sighted children, and they rejected it at the .05 level. The authors point out that their findings are in agreement with those of Hayes (1941, 1950), who found that blind children tend to obtain lower scores than sighted children on reasoning tasks, and those of Rubin (1964), who found that deficiencies in concept formation in the congenitally blind tend to persist into adulthood. A detailed explanation of the source of these related deficits is not to be found in the literature and is not obvious.

Juurmaa (1967) studied the cognitive ability structure of 228 blind persons by testing verbal comprehension, mental arithmetic, spatial ability, arithmetic reasoning, and memory. The results of factor analysis showed that the differentiation of mental abilities was not hindered by blindness as such. The analysis differentiated in a fairly standard fashion the various mental abilities. A significant finding on the memory tests was that a larger portion of the variance of test performances of the blind (in comparison with sighted persons) was due to the memory for meaningless rather than meaningful word pairs.
Domino (1968) used his nonverbal measure of 44 problems, each consisting of a series of dominoes, in finding a principle of progression to study the intelligence of totally blind adults. The subjects were 30 male adults of chronological age ranging from 20 to 46 all totally blind from birth. As hypothesized by Domino, the test proved to be quite difficult for the blind subjects. The mean of 17.97 was lower than the means obtained by fifth- (18.68) and sixth- (20.02) grade students in a previous study by Gough and Domino (1963). Domino pointed out, however, that it was difficult to decide whether the results were due to retarded mental development on the part of the blind or to greater difficulty of the test forms when presented in tactile as opposed to visual form. The care with which this study was conducted and the data were analyzed points to the difficulties of making inferences about the relative difficulty of concept formation tasks for blind individuals, when the concept task for almost all normal subjects makes extensive use of visual cues. Unfortunately, the extensive literature on concept formation in psychology in the past 10 years has contained few tasks that are not defined primarily in terms of visual cues. A useful area of research would be to study concept formation more extensively, using cues from nonvisual modalities in comparison of blind and sighted persons. For example, many classical experiments on concept formation or identification of geometrical shapes and sizes could be replicated almost without structural changes by using the tactile rather than the visual modality.

It is a familiar story, and I shall not attempt to review the extensive literature, that handicaps are positively correlated. It
is difficult to determine the extent to which a cognitive deficit exhibited in a study may be due to sensory deprivation alone in the case of either blind or deaf children. Useful results are reported in the following study.

Cohen (1966) reported a study of 57 out of 66 children followed from birth in the Chicago metropolitan area. The significant point to report here is the high correlation with other handicaps in the case of those children who were under 1500 grams at birth. Cohen reported that 85 percent of the blindness within the group was caused by retrolental fibroplasia, which is primarily the result of overoxygenization of premature infants. (This is a common cause of blindness among newborn infants in this country.) He found that the significant relationship is that of mental retardation with blindness in those children who were under 1500 grams at birth. In particular, about 50 percent of those who were totally blind or had only light perception and who weighed under 1500 grams at birth had IQs below 70. None of the full-term children in the sample was so impaired in terms of mental retardation.

Cohen also gave the Wechsler Intelligence Scale for Children, and he found a lower than average performance for the whole test on the comprehension parts and a higher than average performance on the sub-tests dealing with digit memory. A more detailed analysis of the comprehension items would be desirable to identify more precisely what cognitive deficiencies accounted for the reduction in scores. In Cohen's study, as in others of like nature dealing with the use of standard test estimates, little attention is paid to the structural features of individual items that might be used to deepen the analysis of cognitive deficits.
Tillman (1967) did report extensive analysis of variance results for the Wechsler Intelligence Scale for Children for 167 blind children ages 8 to 12. The results showed that main effects of sex and age are not significant. The main effect of subtests (information, comprehension, arithmetic, similarities, vocabulary and digit span) was significant at the .001 level.

In another article, Tillman and Bashaw (1968) reported a multivariate analysis of the Wechsler Intelligence Scale for Children, comparing blind and sighted children. On the basis of their results, which will not be reported in detail, the authors questioned the validity of this test, especially the verbal sections, when used with blind children without modification.

2.2. Retarded Children

There are a large number of relevant papers in the psychological literature on concept formation and abstraction in retarded persons. I shall try to review only some of the more recent studies and to emphasize at the end some of the theoretical issues that seem to need attention.

An excellent review of the relative efficiency of concept usage by retarded and nonretarded children is to be found in Zigler and Balla (1971); they reviewed eight major studies, which by and large equated the mental age of the retarded and nonretarded subjects. A couple of
the studies reported more than one experiment. The 19 experiments, whose results are summarized, include the tasks of selecting three pictures that illustrate a concept from a set of seven pictures, verbalizing a concept common to the three pictures, associative clustering, defining all words in an experiment, sorting cards in terms of some concept, and selecting four pictures that illustrate a concept from a set of seven using different types of concepts (perceptual, use and human). The performance of the normal and retarded subjects was about the same in 12 of the experiments, and that of the nonretarded subjects was better in the remaining 7.

Similar results are reported in Blake and Williams (1968). Retarded, normal and superior groups of students were compared on their attainment of concepts by deduction, induction-discovery and induction-demonstration. When mental age was held constant, the groups did not differ in level of concept attainment. Also, for all three groups, deduction was the most effective, while the two inductive methods were about equal in effectiveness.

An earlier study by Braun (1963) is worth mentioning because of the finding of a statistically reliable correlation between reading comprehension and concept formation. He found, furthermore, that the relation between concept formation and reading comprehension was significantly stronger than the relationship between IQ and reading comprehension.
The task he used in his experiments required the subjects to identify a concept represented in each of a series of cards.

A study of Hermelin and O'Connor (1958) supports the somewhat surprising results on language that in many cases retardates show deficiencies not at the level of meaningfulness, but at the level of automatic-sequential performance. They found that 20 institutionalized children with mean IQs of 40.7 did better in a concept task utilizing classification and quantity concepts than they did in a rote learning series. Explicitly, the subjects were presented with a series of pictures and were rewarded upon selection of the correct picture. In the rote memory series the pictures simply consisted of random items. The third series utilized pictures containing items of class and quantity, and it was in the latter series that performance was better.

Elam (1962) utilized 216 subjects: 72 normal subjects at the junior high school level who were slightly above average, 72 normal fourth- and fifth-grade students, and 72 retarded students with an IQ range between 50 and 80. These were compared on similarity-difference problems under a variety of stimulus-response and reinforcement conditions. Elam reported that aside from their lower performances, the retarded subjects reacted to the experimental variables in much the same way as the normal subjects did.

A recent study of Elount (1970) found no significant difference between retarded and normal subjects on a concept-usage task made up from familiar items. The task required choosing the three of five pictures that went together, as well as giving a verbal label for the exemplified concept. The only superior aspect of the nonretarded subjects' performance was in their verbal labeling of the concept. Jones (1971)
studied the feasibility of educable mentally handicapped children learning simple schemata exemplified in stimulus patterns on checkerboards. While the results were positive, they were not compared with a control group of normal subjects.

As some of the studies just mentioned indicate, it is especially in the areas of language control and verbalization that retarded persons show the greatest difficulties. Milgram and Furth (1962), following on Furth's earlier work with deaf children, showed that retarded children perform more poorly in the discovery and application of a language-relevant concept, but perform as well as normal children in solving problems that depend only on perceptual rather than verbal modes of solution.

Similar results were obtained by Milgram (1966). To compare normal and retarded children, subjects were shown 18 sets of seven cards picturing common objects, three of which belonged to a conceptual class by function, material, situation or shape. They were asked which three "go together." In Task II the three correct cards in each set were readministered and subjects were asked to say in what way "these three go together." There was no significant difference between normal and retarded children on Task I. There was a significant difference on Task II, which required a verbalization of the relevant concept. Stephens (1968) has studied the types of errors retarded children make in attempting verbal labels in order to get a better understanding of what their difficulties seem to be, or, to put it another way, to identify more precisely the linguistic deficiencies of retarded children in concept tasks. His findings indicated that a higher percentage of errors by retarded children, in comparison with those of normal children, are
either no response at all, or responses that are enumerative rather than conceptual in character.

In a comparative study of learning and problem solving in retarded and normal children, Miller, Hale and Stevenson (1968) found that when the two groups were equated for mental age, no significant differences were found in paired-associate and discrimination learning, but the retarded children did markedly poorer than the normal children on tasks involving the concept of conservation, the concept of probability, verbal memory and anagrams. As the authors point out, the study provides further evidence of the difficulty retarded children face with complex tasks involving verbal processes.

Cawley (1970) studied verbal problem solving among educable mentally retarded children with differing IQs. As might be expected, children with higher IQs outperformed children with lower IQs, but the problems dealing with existential quantification, superordinate set identification and the inclusion of extraneous information were difficult for all the subjects and provide further evidence of the central difficulty of verbal processing for retarded children.

A widely accepted generalization is that retarded children equated in mental age with normal children have greater difficulty with abstraction, and a number of experimental studies with reasonable controls support this generalization. I shall not review that literature here but refer to some of the better-known studies: Halpin, 1958; Jones and Spreen, 1967; Kerstvedt, Stacey and Reynolds, 1954; Prothro, 1943; Rosenberg, 1963. What is important, however, is to emphasize that the differences between normal and retarded students, especially those
equated for mental age, cannot simply be assigned in terms of abstractness or complexity, as the studies reviewed above about language indicate. If a single generalization were to be made, it would be that verbal performance rather than abstraction as such is the critical deficiency of retarded persons.

As has already been indicated, it is beyond the scope of this article to cover the extensive literature on learning in retarded children; however, the excellent review of these matters by Estes (1970) raises a number of issues pertinent to cognition as well. (An excellent older review of the research on learning in mentally retarded children is Denny, 1964.) Estes devotes a number of pages to reviewing the Zeaman and House (1963) two-stage attentional model for discrimination learning, which is applicable to concept identification and, if not in principle at least in practice, to some concept-formation tasks. The Zeaman and House work is almost unique in being one of the few cases in which a theoretically detailed set of assumptions has been applied to problems of concept formation or identification in retarded children, for example, in color-form discriminations. The two stages in their model represent an attentional process and a learning process.

What is surprising and almost paradoxical in the theory is that the main differences in learning for subjects of different mental ages are reflected in the initial attentional process, which primarily consists of learning to attend to the correct or relevant dimensions of a problem. Very small differences are reflected in the learning of the appropriate associations once the proper dimensions are attended to. In one analysis, for example, groups of children with mean mental ages of
2 years 4 months and 4 years 6 months, respectively, were compared. The curve for the higher group rose steeply from chance to nearly 100 percent correct responses over about 40 trials. The curve for the lower group differed only in that it hovered around the chance level of 50 percent correct, responding with no obvious trend for about 180 trials before beginning to rise. Then, like the curve for the higher group, the trend rose steeply to virtually 100 percent correct responses over about 40 trials.

Backward or Vincent learning curves were used in this study to detect learning trends (the theoretical reasons for using such curves are set forth in detail in Suppes & Ginsberg, 1963). As Estes points out, it is hard to accept that the only differences in learning of retarded children can be identified simply as the probability of attending to the correct dimension. Since the attentional function is a probabilistic function and sums to one, this would mean that if the theory were pushed relentlessly, on some dimensions the performance of retarded children should be better than that of normal children, because they must have a higher probability of attending to these dimensions.

In principle individual parameters can be estimated in the model, but in practice this has not been done. In fact, I have been unable to identify any studies of concept formation or identification in retarded students, or even for groups of subjects stratified according to mental age, that actually work out models in sufficient detail to estimate in standard statistical fashion learning parameters for individual subjects. In view of the extensive work that has been devoted in mathematical psychology to the development of such models over the past two
decades, it would seem especially desirable to push the detailed analysis of data by the application of such models and the identification of various phases of learning at a more abstract level in terms of the estimation of parameters. It would also be interesting to then regress the estimated parameters for individual subjects or stratified groups of subjects on variables of mental age, chronological age and other features of overall performance.

I conclude this subsection with a sketch of the kind of quantitative model I would advocate applying initially to concept-formation experiments with retarded children. The experiment with normal first graders on the concepts of equivalence and identity of sets reported in Suppes (1965) is fit fairly well by a one-element learning model. The assumption of the model is that each concept corresponds to a single stimulus pattern that is conditioned on an all-or-none basis to the correct response. By assuming a beta distribution for individual differences in the conditioning parameter $c$, more exact and quantitative comparisons between normal and retarded children could be made by estimating such beta distributions for the two populations. It would be anticipated that in many studies the mean for the beta distribution of the retarded children would be significantly lower than that for the normal children, but the overlap in the two distributions, as well as in the scatter plots of the individual estimated parameters, would provide information to deepen our summary view on the differences and similarities of the two populations with respect to different conceptual tasks. As I have emphasized before, the estimated magnitude of the difference in the two distributions, not the mere existence of a difference, is what is needed, both for deeper
theoretical developments and also for consideration of practical problems of providing retarded children a differentiated, special school curriculum. Concept-formation experiments with normal children but feasible for retarded children and relevant to the school mathematics curriculum are reported in Suppes (1965) and Suppes and Ginsberg (1963).

2.3. Deaf Children

Excellent reviews of the literature on concept formation in deaf children have been provided by Furth (1964, 1966, 1971). In the most recent of these reviews (Furth, 1971), 39 studies are listed and summarized. In view of the up-to-date character of this review and its accessibility, I shall not review this literature, but rather, shall comment on some of the issues raised by Furth and others.

The fundamental issue raised by Furth and many of the investigators whose experiments he summarized is the question of whether deaf children show a deficit in concept formation once verbal aspects of the task are removed. Put another way, in experiments that require no verbal comprehension are there significant differences in performance between deaf and normal children? Even more than in the case of concept formation or identification by retarded children, Furth has presented persuasive evidence from a number of experiments that there are often not significant differences. As he admits, however, the situation is not simple, and some contrary evidence can be cited. The important issue, however, is the role of language in concept formation. Here, it seems to me, Furth does not really make a strong theoretical point, because his analysis is concerned entirely with command of a standard natural language. As he points out, in letter recognition tasks and others,
the processes deaf children use are not clear. Process-oriented approaches to cognitive skills seem to argue strongly that some sort of language is being used internally, even if the language is not that of the society in which the children live.

Apart from the issue of the necessity of an internal processing language, two other remarks may be made about Furth's position. The first is that it would be interesting to see what the performance of deaf children who understand sign language would be if sign language were used to provide equivalent verbal instructions, or in the case of responses, to provide a medium for response by the child. There are of course some difficult problems of methodology. If comparison with normal children is desired, as in most cases it is, then comparability of the two media of communication is needed to judge whether a communication deficit exists. The methodological problem is rather similar to the study of concept formation in blind children when concepts are transferred from the visual to some other sensory modality.

The second remark concerns Furth's discussion of logical reasoning and the claim from some of his own experiments that deaf children exhibit capacities that show only small deficits at most. The point is that the experiments on logical reasoning are all extremely elementary. More complex kinds of inference, even of the kind that can be given young normal children (ages 6 and 7 years, for example), are difficult to test outside a verbal context. For example, in Suppes (1965), data on the intuitive inference capacities of young children are cited for the classical forms of inference running from modus ponendo ponens to quantificational logic using universal and existential quantifiers and two-place
predicates. The experimental items are all verbal in form, and it would not be possible to give an exact parallel in nonverbal form.

When we turn to still more complex material requiring logical inference, the situation is even more completely and more thoroughly imbedded in a verbal context. I mention, for example, recent studies of the kinds of mathematical proofs given by college students in introductory logic courses (Kane, 1972; Moloney, 1972; Goldberg & Suppes, 1972). Here again, more sophisticated forms of reasoning can scarcely be investigated in a nonverbal context. It seems to me that the real test will be not successful efforts to transform more sophisticated forms of inference into nonverbal contexts, because this seems prima facie impossible, but rather to test the ability to communicate and handle such inferences in sign language. These more developed forms of inference are not primarily auditory in nature but visual; for example, there is very little development of mathematical proofs in purely auditory fashion.

Additional studies in support of Furth's thesis can also be mentioned. Vernon (1967) surveyed 33 research studies and came to the following three conclusions: there is no close relationship between verbal language and cognitive thought processes, verbal language does not serve as a mediating symbolic system of thought, and there is no relationship between concept formation and the level of verbal language development.

Competence in abstraction of deaf persons has in many studies been found closely linked to verbal functioning. For example, Cléron (1953) found the deaf deficient in nonverbal abstract functioning as determined by a sorting test, and he concluded that the source of the deficiency was the result of language retardation. On the other hand, Rosenstein
(1959) and Kates, Yudin and Tiffany (1962) found no significant difference between deaf and hearing children in their ability to abstract or generalize when the language requirements of the experiment were within the capacity of the deaf child. Stachyra (1967) found similar results in a study of 123 deaf pupils and a control group of 100 normal children in Lublin, Poland. Using picture tests of the kind described earlier, he concluded that the ability to abstract a concept from concrete objects or pictures does depend on the development of verbal skills.

Although I only cite a few of the studies here, the literature is large and the controversy is far from settled. From an educational standpoint the critical issue is one of discovering the best means of facilitating the learning of concepts and abstractions by hearing-impaired children. To what extent this can be done by extensive development of manual communication as more abstract and systematic areas of knowledge are reached is as yet not clear. We badly need to understand better how successful we can be at teaching manual communication, with subsequent transfer to the use of a written natural language of a conventional sort. So far as I have been able to determine, the appropriate research studies do not exist.

3. Arithmetic Skills

3.1. Blind Children

The one extensive study of arithmetic achievement of blind children identified in the literature (Nolan, 1959) studied the differences in achievement in computation among several schools for the blind. The conclusions were interesting in the following respect. Nolan found
that the problems in achievement did not seem to stem directly from problems of mental ability, but rather they varied so much from one school to another that they had to be accounted for in terms of social and other environmental variables.

I have not been able to find any detailed studies analyzing the specific difficulties blind children encounter in arithmetic.

3.2. Retarded Children

There are a number of studies dealing with the performance in elementary mathematics, and especially arithmetic, of retarded children. In terms of achievement on standardized arithmetic tests, Cruickshank (1946a, 1946b, 1948a, 1948b), Dunn (1954), Jones (1920), and Merrill (1924) found that retarded and normal children do not differ much in arithmetic computation, but Cruickshank and Dunn found significant differences in the results of arithmetic reasoning tests. Cruickshank looked in more detail at the differences between the two groups and found that normal children score better than the retarded children on most types of arithmetic skills involving either reasoning, abstraction, exclusion of extraneous information, or using verbal information.

Klausmeier and Check (1962) studied retention and transfer in arithmetic. The problems they dealt with concerned mainly the computing or "compilation" of a specific amount of money with the fewest number of coins. The average and above-average children used paper and pencil, but the retarded children were permitted to use actual coins. They found that when the retarded children were given an appropriate representation of the problem, in this case by means of actual coins, the normal and retarded groups were able to retain and transfer arithmetic problem-solving abilities without
significant differences between the groups for periods of either 5 minutes or 7 weeks. In a related study, Klausmeier and Feldhusen (1959) examined arithmetic learning and retention as related to school instruction for low-, average- and high-intelligence students. Although original acquisition scores were different for the three groups, the retention scores were not significantly different. This lack of significance also held for a related transfer condition in the task.

An excellent study to mention, in order to guard against too simple generalizations about the arithmetic skills of retarded children, is Finley (1962). Fifty-four mentally retarded children in special classes with IQs ranging from 50 to 75, mean chronological age of 13 years 7 months, were compared with normal subjects of equivalent mental age; the normal subjects had IQs ranging from 90 to 110 with a mean chronological age of 8 years 9 months and were in regular classes. Three 20-item tests were prepared and administered in weekly intervals in the following sequence: concrete, pictorial and symbolic representation. Three hypotheses were tested:

(i) Arithmetic achievement of retarded children is independent of the context in which the problem is presented;

(ii) Arithmetic achievement of normal children is independent of the context;

(iii) There is no difference between the arithmetic achievement of retarded and normal children of the same mental age in instruments of like context.

Hypotheses (i) and (ii) were rejected; significant differences were found for both retarded and normal children. For the retarded children
the concrete test items tended to be more difficult than either the pictorial or symbolic items, although the difference did not reach statistical significance. For normal subjects the pictorial items were significantly easier than the other two kinds. Hypothesis (iii) was accepted in the concrete and pictorial forms of the test, but was rejected for the symbolic form. The real surprise is that the retarded children performed significantly better than normal children of the same mental age on this test of computational skills. A possible explanation offered by Finley is that the curriculum of the retarded children was different, and because of their age their years of exposure to arithmetic were considerably greater.

Some studies have tried to apply Piaget's developmental sequences to the development of number concepts in retarded children and adults. Such studies are still in a preliminary state and would seem to require more extensive and detailed data analysis. An example of work in this area is Woodward (1961), who considered one-to-one correspondence and equivalency of sets, as well as seriation and conservation of continuous quantity. She found that the performance of retarded adults whose chronological age was 19 and retarded children whose chronological age was 12.9 was at about a level similar to an average normal child of from 4 to 7 years.

Considering the practical value of educable retarded persons learning elementary arithmetic skills, it is evident that more extensive and detailed research is needed on the problems and potentialities of teaching them arithmetic. The extensive use of computer facilities as described in the next section in the teaching of arithmetic to deaf children could also be exploited to advantage in the teaching of arithmetic to retarded
children. Such settings would provide not only practical opportunities for intensive teaching, but also opportunities for understanding in a much deeper way the actual course of learning of arithmetic skills in retarded children. The detailed regression models and still more specific automaton models tested in Suppes, Jerman and Brian (1968) and Suppes and Morningstar (1972) seem suitable for application.

3.3. Deaf Children

I have been able to find no detailed studies dealing with the mathematical abilities of deaf students beyond the skills of arithmetic. Various reports show that deaf students have a grade-placement deficit on arithmetic achievement scores (computation, concepts and applications) relative to their chronological age, and data show that their rate of progress in any given year of school is usually below the average for normal children.

Apart from data on achievement tests, I have been able to find few, if any, studies providing a detailed profile of arithmetic skills in deaf children. For this reason, I have decided to devote this section to reporting some of the extensive data on the arithmetic performance of deaf children we have collected in our Institute at Stanford over the past several years. As far as I can determine, the data I report here, which are being analyzed with Lindsay L. Flannery and will be published in detail elsewhere, constitute the largest body of data on specific arithmetical skills of deaf children yet analyzed.

Data from our various drill-and-practice programs in arithmetic have been collected in the context of extensive curriculum development in computer-assisted instruction at the Institute. This development
includes continuous revisions running from 1964 to the present, with the result that an increasingly individualized curriculum has evolved. The data cited were collected for the strands program, which presents an individualized lesson to each student depending upon his level of achievement in each of 14 basic strands or skills. Movement of an individual student upward in a strand from one class of exercises to the next depends only upon his level of performance. In a curriculum organized in this fashion we thus have an unparalleled opportunity to compare in some detail the performance of deaf and normal-hearing students, because each student is advanced to the next step in a given skill only after he has exhibited mastery at the level on which he is currently working.

The 14 strands on which the curriculum is based are shown in Table 2. Within each strand, exercises of a homogeneous type are grouped into equivalence classes; for example, all horizontal addition exercises with a sum between 0 and 5 constitute one equivalence class. Each strand contains either five or ten classes per half year, with each class being labeled in terms of a grade-placement equivalent. As can be seen from the list of strands in Table 2, the standard core curriculum in arithmetic is covered by these strands.

In addition to the identification of the strands and equivalence classes of exercises within a strand, a decision is made about how much emphasis should be given to each strand at each grade level. To determine this, the curriculum was divided into 12 parts corresponding to half-year intervals and a probability distribution was determined for
### TABLE 2
Content and Duration of Each Strand

<table>
<thead>
<tr>
<th>Strand</th>
<th>Content</th>
<th>Grade range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Counting and place value</td>
<td>1.0-7.0</td>
</tr>
<tr>
<td>2</td>
<td>Vertical addition</td>
<td>1.0-6.0</td>
</tr>
<tr>
<td>3</td>
<td>Horizontal addition</td>
<td>1.0-3.5</td>
</tr>
<tr>
<td>4</td>
<td>Vertical subtraction</td>
<td>1.5-6.0</td>
</tr>
<tr>
<td>5</td>
<td>Horizontal subtraction</td>
<td>1.0-3.5</td>
</tr>
<tr>
<td>6</td>
<td>Equations</td>
<td>1.5-7.0</td>
</tr>
<tr>
<td>7</td>
<td>Horizontal multiplication</td>
<td>2.5-5.5</td>
</tr>
<tr>
<td>8</td>
<td>Vertical multiplication</td>
<td>3.5-7.0</td>
</tr>
<tr>
<td>9</td>
<td>Fractions</td>
<td>3.5-7.0</td>
</tr>
<tr>
<td>10</td>
<td>Division</td>
<td>3.5-7.0</td>
</tr>
<tr>
<td>11</td>
<td>Large numbers and units of measure: time, money, linear measure, dozen,</td>
<td>1.5-7.0</td>
</tr>
<tr>
<td></td>
<td>liquid measure, weight, Roman numerals, metric measure</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Decimals</td>
<td>3.0-7.0</td>
</tr>
<tr>
<td>13</td>
<td>Commutative, associative and distributive laws</td>
<td>3.0-7.0</td>
</tr>
<tr>
<td>14</td>
<td>Negative numbers</td>
<td></td>
</tr>
</tbody>
</table>
the proportion of exercises in each strand for each half year. The determination of the probability distribution was based upon a prior analysis of three standard textbook series, with subsequent smoothing and adjustments of the empirical distribution thus derived. A more detailed account of the curriculum of the strands structure and the particular way in which the individual student moves through the structure is given in Suppes and Morningstar (1970).

The data for this curriculum are drawn from the school year 1970-71 when the program was used by approximately 1500 hearing and 800 deaf students across the United States. The various schools were all linked to the Institute's computer at Stanford by phone line. The exercises were presented in the schools to students seated at teletype terminals, and the data represent entirely responses input on a teletype keyboard. About half the normal-hearing children were drawn from an economically depressed district. A high percentage of the students in this district are black.

The basic data are the mean percentage correct for each of the equivalence classes of the strands curriculum as described above for both deaf and normal-hearing students. It is important to emphasize that before a student could reach a given equivalence class on a given strand he had to master the previous equivalence class leading up to it, independent of his grade placement or chronological age. In a genuine sense, therefore, we were able on a very broad basis to compare the performance of deaf and normal-hearing students with a common basis of preparation and previous performance. Moreover, numerous predictive studies of achievement suggest that this equating of past
achievement is more important than equating of IQ. In other words, a regression equation with achievement as the dependent variable and previous achievement and IQ as independent variables will almost always have a larger positive coefficient for previous achievement than for IQ. Detailed results of this kind may be found in Suppes and Morningstar (1972, Chapter 9).

Two conclusions, founded on answers to several hundred thousand exercises, emerge from this massive data analysis. The first is that objective features of the curriculum, for example, whether a vertical addition problem has a carry or not, dominate the ease or difficulty of exercises in much the same way for both deaf and normal-hearing children. Although the massive tabulation of data to demonstrate this is omitted, two typical graphs of proportion of correct responses for the equivalence classes in two strands, the fraction strand and the strand concerned with the commutative, associative and distributive laws of arithmetic, are shown in Figures 1 and 2. The relatively close match between the curves for deaf and normal-hearing children is equaled by corresponding results for the other 12 strands (detailed quantitative data are given in Suppes & Flannery, 1972). The significant point is that the algorithmic fraction strand and the conceptual strand concerned with the laws of arithmetic show quite similar features.

This leads to the second conclusion, which is more surprising than the first: the performance of the deaf children is almost always slightly better than that of the normal-hearing children. More exactly, of the 781 equivalence classes, summing across all grades and strands for which we have data, the mean percentage correct of the deaf students was higher than that of the
Fig. 1. Comparison of performance of deaf and hearing students on the fractions strand.
Fig. 2. Comparison of performance of deaf and hearing student in the strand dealing with the laws of arithmetic.
normal-hearing students for 673 classes, and the same to two decimals for 22 classes. These massive data support the thesis that the cognitive performance of deaf children is as good as that of normal-hearing children, when the cognitive task does not directly involve in a central way verbal skills. From an educational standpoint, the data suggest that with proper organization of teaching effort, we should be able to obtain results in arithmetic as good for deaf children as we do for average to slightly below-average normal-hearing children.

4. Concluding Remarks

From this survey of cognition in handicapped children several broad conclusions emerge. First of all, language problems are central to the education of handicapped children and to their becoming productive members of the society. At the same time, it is clear that a great deal still needs to be learned about the source of their language difficulties, and how these difficulties can be met. Extensive experimentation and theoretical analysis seem called for in terms both of language comprehension and language production. It is surprising to find how inadequate the detailed information is about the grammatical structure of productions by any of the three main groups of handicapped children, and it is also surprising that a detailed semantical theory of their problems of communication is as yet scarcely developed. On the other hand, adequate theoretical tools for systematic analysis of either production or comprehension grammar and semantics have only become available in the last few years. Hopefully we may look forward to significant developments on these matters in the next decade.
Another conclusion is that we need to transfer the excellent methodology developed for the study of learning, especially in retarded children, of discrimination and simple association paradigms to more complex tasks and to blind and deaf children as well. There now exists in general psychology a wealth of quantitative and mathematical models of learning, several of which have been applied to complex concept-formation tasks. In view of the importance of understanding in detail the learning problems of these children, it is hoped that the tools developed in general psychology will be applied to their special problems. In fact, I see no reason not to urge that detailed mathematical models be applied to subject-matter learning and performance, especially to the curriculum of basic skills of language, mathematics and reading. The increasingly widespread availability of computer facilities for on-line computer-assisted instruction makes such studies considerably more feasible than in the past.

Finally, I would like to emphasize a point made earlier, namely, that in future experimentation we need to give more attention to estimating the magnitudes of effect of various training procedures and less attention to establishing the existence of a statistically significant difference. Only from knowing the magnitudes of effect as opposed to the mere fact of the existence of differences can we make wise practical judgments about embarking on new and possibly costly training programs.

The study of cognitive processes in handicapped children is an opportunity both for important theoretical work and for direct application of significant theoretical results to practical problems of education. As this survey should make clear, a great deal has already been done, but it is fair to say that the most important work lies ahead of us.
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Footnote

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