As a response to several specific economic analysis questions asked by the staff of Massachusetts' Management and Information System for Occupational Education (MISOE), this paper focuses on concepts of costs and benefits required for cost-effectiveness analysis, as well as the methodology of cost-effectiveness analysis. Useful as background material for those unfamiliar with economics, this paper intends to clear up misconceptions surrounding cost-benefit analysis by a simple presentation of various conceptual issues. Topics covered include capital costs, average and marginal costs, cost sharing, and cost functions. Related documents are available in this issue as VT 018 600, VT 018 602, VT 018 606, and VT 018 810. (AG)
CONCEPTUAL ISSUES IN COST-IMPACT (BENEFIT) ANALYSIS

By

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Occasional Paper #9 is a response to several specific economic analysis questions of the MISOE resident staff. These questions were explored during several meetings of the MISOE resident staff and Dr. Jacob J. Kaufman, Director and Professor of Economics, Pennsylvania State University, Institute for Research on Human Resources. Dr. Kaufman included Dr. Elchanan Cohn of his staff in these discussions. Subsequent to these discussions, Kaufman and Cohn developed Occasional Paper #9 as a formal record of their suggestions to MISOE.

Since the questions of MISOE resident staff to Kaufman and Cohn were focused on technical aspects of cost-impact (benefit) analysis, the substance of this paper is designed to treat concepts of cost and benefits required for cost-impact analysis, as well as to deal with the methodology of cost-impact analysis.

An introduction to cost-impact analysis is included in Occasional Paper #9 as a basis for fitting the technical discussion of Parts II, III and IV together. This introduction was previously presented in Monograph I as a preliminary view of cost-impact analysis. Dr. Kaufman decided to repeat his earlier description in Occasional Paper #9 as background for the non-economist.

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Introduction

The purpose of cost-effectiveness or cost-benefit analyses is to introduce into the public sector the equivalent of the market constraints in the private sector. In the private sector a business firm has a useful barometer by which it can assess its performance during a given year, namely, the profit and loss statement. Simplistically, a profit and loss statement sets forth the revenues received from the sales of the products of the firm and the costs incurred in producing these products. The difference is profits (or losses). The firm is in a position of analyzing its revenues and costs and attempt to maximize the former and minimize the latter. One way of reducing costs or increasing revenues is to change the process of production so that profits can rise.

How can this approach be translated into the public sector—occupational education, for example? Educational administrators must seek proxies or surrogates for revenues. These "revenues" can be in terms of performance goals or impact goals. There is no problem of costs in the public sector which does not exist in the private sector, and the production process in the private sector is the equivalent of the educational process in occupational education.

The basic challenge to an analyst is to determine the proxies or surrogates for output in the public sector and attempt, if possible, to translate these output (performance or impact) measures into dollar terms. Certain measures—such as income—can be handled without great difficulty. Other measures of output can be translated into dollar terms indirectly or by comparing the results—in non-monetary terms—with the costs involved.
It should be recognized that these procedures are not simple--but reasonable estimates can be made.

The real problems in cost-effectiveness or cost-benefit analyses are a lack of understanding of the approach and a lack of understanding of the concepts.

It is the purpose of this occasional paper to set forth the theoretical concepts involving costs and benefits which underlie this approach. With such an understanding and with the elimination of ignorance or misconceptions, the groundwork can be laid for a system by which society is in a position to allocate its limited resources in the most effective manner so as to achieve its goals.
I. A Discussion of Cost-Impact Analysis

It is the purpose of this section to discuss the relationship of impact to costs (at times referred to as cost-benefit analysis) in terms of (a) its logic and meaning; (b) some of the misconceptions which prevail concerning this method; and (c) the methodological and data collection issues which arise.

Logic and Meaning of Cost-Benefit Analysis

Under a free enterprise economy most private wants are satisfied through the workings of the market mechanism. Under this system it is assumed that, as a result of consumer choice, goods and services will be produced to satisfy these private wants and that the limited resources of the economy will be allocated through the operations of the market in a manner which will yield the greatest output with a given amount of resources.

There are, on the other hand, certain needs and wants which are not or cannot be satisfied by the private sector. A second group of wants, described as social wants, are those which "must be satisfied by services that must be consumed in equal amounts by all." These services are such that some people can benefit from them even if they do not pay for them. And there is no reason to think that such persons would make voluntary payments. Governmental expenditures of this type might include expenditures for flood control, defense, sanitation, etc.
A third group of wants which could be provided by the private sector but, for a variety of reasons, are handled by the public sector because society considers them meritorious, may be referred to as "merit" wants. Included in this category are such items as low-cost housing and "free" education. In these instances the wants could be satisfied by the private sector but society apparently thinks that there are certain social benefits which flow from these activities and therefore society assumes the responsibility to satisfy these wants.

It is not the purpose of this section to discuss the pros and cons of whether the government should concern itself with these "merit" wants. But it is the purpose of this section to concern itself with the method by which it can be determined whether the provision of certain social and merit wants by the government are carried on efficiently, consistent with the objectives for which it has assumed the responsibility. And by efficiency is meant the attainment of a given objective at the lowest possible cost or the maximizing of a given goal at a given cost.

In the private sector of the economy the marketplace, in general, is the place where these evaluations take place. The inefficient firm may have to go out of business. The firm that does not produce goods and services which satisfy the needs of the consumers may not survive. But what tests for efficiency and survival do we have when the government provides the goods and services?

The only alternative to the market place for the purpose of testing the efficiency of production or the quality of the product is
by cost-benefit analysis. Such an analysis is nothing more than an attempt to establish the equivalent of a system of market principles for various types of government activities. It might be reasonable to assert that the method of analysis is not fully developed and that the data available are not adequate. Such charges, however, do not negate the necessity to develop appropriate tools and to obtain adequate data to judge a particular government activity.

The fact is that there is a tendency on the part of some educators to talk simply in terms of the "needs" of education. Their position is simple: the governmental agency should raise whatever funds are necessary to meet these "needs." On the other hand, there are some who assert that there is a fixed sum of money available for educators to spend on education. The fact is that one should not talk about education in terms of cost or needs alone. No cost can be justified without a reference to payoff. And the satisfaction of any need cannot be justified without reference to cost.

This means that one cannot discuss the need for, or the impact of, vocational education without relating them to costs. Nor can one talk about the costs of vocational education without relating them to impact. If private vocational schools survive it is reasonable to assume that these schools operate at a profit and that the private sector of the economy is willing to pay the price of tuition. It is not unreasonable to assume, further, that the buyers of the education find that it pays off. We can also assume that the profit motive will be a sufficient stimulant to the owner of the private vocational school to keep costs as low as possible.
But what controls do we have over the public education? What incentives are there for the public educator to keep his costs down? What evidence is there that public education is being provided efficiently? What evidence is there that the objectives are being achieved?

It is being suggested that these are legitimate questions to ask during a period in our society when there are many demands for the provision of social and merit goods by the government. And, even within education, there are many demands for different forms of education. This means that decisions must be made as to the allocation of resources among competing educational programs. The only appropriate method for making these decisions is on the basis of a cost-benefit analysis.

One aspect of cost-benefit analysis which should be stressed is that it is basically a "way of thinking." It tends, first, to force an administrator to think through his objectives. This does not mean that the objectives are easy to state. Too frequently they are expressed too broadly and do not reflect the "real" objectives. It is not enough, for example, to state that the schools educate for the so-called "whole man." We must be more specific. Nor can it be stated that, for example, vocational education is designed to place a youngster in a job. Is it a job related to his training? Is it a job solely in terms of an initial placement or are we concerned with the duration of the job? Is it simply the first job or a series of jobs? Is it a job that leads to promotion? Is it a job that is satisfying to the graduate?

Second, cost-benefit analysis, as a "way of thinking," tends to force an administrator to concentrate on costs as well as objectives.
The point need not be repeated that inputs, products, and impact are interrelated and must not be considered separately.

Third, cost-benefit analysis, as a "way of thinking," forces an administrator to think in terms of "alternatives," that is, to think in terms of alternative ways of achieving the same objective. To refer to the satisfying of wants in the private sector again, it should be noted that the pressures of competition tend to force private enterprise to seek other and better means of producing a good or a service. Similarly, the concentration of alternatives forces the educational administrator to seek other and better means for the education of youth. In this way we can get change and innovation in education. In fact, it is the failure to evaluate educational curricula that leads to stagnation. It is only through constant evaluation that we can obtain innovation.

The above comments are designed to indicate in a constructive manner the logic and meaning of cost-benefit analysis. Despite what appears to be a rather logical case for this type of analysis there is still considerable opposition to the technique. Such opposition reflects, first, certain misconceptions about the method. Second, educators have a different (and, at times, erroneous) view of education. And, finally, educators view evaluation as a threat to their institutions. Each of these points will be discussed briefly.

Misconceptions of Cost-Benefit Analysis

One of the most serious misconceptions about cost-benefit or cost-effectiveness analysis is that it is merely a subterfuge for seeking;
to conduct education on a "least-cost" basis. This is a complete misunderstanding of the notion of efficiency. To an economist efficiency means the achievement of a given objective with the least cost or the maximization of a given objective with a given cost. Efficiency combines both input and output.

A second misconception is that benefit is measured only in dollar terms, and that this is a form of crass materialism. Cost-benefit analysis recognizes that there are non-economic benefits which should be taken into account. Such non-economic benefits may include voting behavior, job satisfaction, cultural values, etc. However, it is essential that these objectives should be established on the basis of decisions of the community to determine whether it wants to spend its funds (and how much) for the explicitly stated objectives, economic or non-economic.

A third criticism usually advanced against cost-benefit analysis is that there are some things which are not quantifiable. Presumably, this means that there is no way in which one can determine whether or not a given objective has been attained. If this is so, what justification exists to continue expenditures for objectives which cannot be quantified? Why the assumption that non-quantifiable objectives are automatically good? Although certain objectives may be difficult to quantify, every effort should be made to develop "inferential" (or proxy) indexes. For example, the extent of "interest" of students in a curriculum might be inferred from an index of absenteeism. Psychologists can be of great assistance not only in the development of such indexes, but also in the creation of the necessary instruments designed to compute them.
A fourth criticism frequently mentioned is that the cost-benefit technique has not been fully developed and, therefore, should not be applied. The first part of the statement is correct, but the conclusion does not follow. The fact is that once a decision is made to spend more on, say, vocational education an implicit decision has been made that the benefits exceed the costs. Therefore, the issue is not whether cost-benefit analysis should be applied to vocational education. It is being done every day when an educational administrator decides to spend a dollar on vocational education rather than on another type of education. The only question is whether the vocational education administrator should be required to state explicitly the manner in which he arrived at the decision. When the process of decision-making is made explicit then others have an opportunity to judge the correctness of the process. It is only in this way that better decisions can be made on the allocation of limited resources for educational objectives. The rejection of an explicit cost-benefit analysis simply means a refusal to expose oneself to an evaluation of a decision-making process.

Fifth, there is a misconception that the cost-effectiveness analyst substitutes his judgement for that of the decision maker. The analyst may ask the administrator some pertinent (possibly impertinent) questions. In no instance, however, does he substitute his goals or values for those of the administrator. The analyst simply provides information—costs and benefits—of alternative lines of action designed to achieve the objectives as outlined by the administrator. The analyst simply assists the educational administrator in meeting the objectives of the community in the most efficient manner.
Finally, it is sometimes argued that cost-benefit analysis tends to ignore political considerations, or other constraints, which have been described earlier. Although the analyst ignores the political aspects of a program, it does not necessarily follow that the decision maker should ignore "politics." This type of analysis will, however, tend to reveal the cost of a political decision and may well tend to minimize the role of politics in the decision-making process.
II. Concepts of Costs

A. Opportunity Costs*

Perhaps the most important concept of cost in economic analysis is that of opportunity costs. According to this concept, all economic costs are opportunity costs**. In other words, the true cost of any activity is simply the foregone opportunity to undertake other activities.

When it is desired to measure the educational costs sustained by a pupil (or his family), it is necessary to consider not only direct expenditures but also foregone opportunities—such as parents' time devoted to helping the pupil with his schoolwork, transportation to and from school, and related activities. Obviously, the measurement of such costs is not easy.

When the decision-making unit is a school, a school district, or a state school system, the problems of identifying and measuring the costs of education are magnified. Again, the guiding principle should be opportunities foregone. If a given amount of resources were not to be spent on education, it could be used in other areas—e.g., for police and fire protection, better sanitation facilities, improved water and air quality, highway construction; or, perhaps, it could be returned to the citizens in the form of a tax cut or tax rebates.

Whenever cost data are desired, care should be taken to identify the unit for which costs are to be measured. If the cost of secondary

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*Adapted from Elchanan Cohn, Teh-wei Hu, and Jacob J. Kaufman, The Costs of Vocational and Nonvocational Programs, The Pennsylvania State University, Institute for Research on Human Resources, 1972

**See, for example, Fritz Machlup, The Production and Distribution of Knowledge in the United States (Princeton: Princeton University Press 1962), Chapter IV.
education to a school district is being considered, costs borne by other public bodies (local, state, and federal governments) as well as by private individuals—students, their parents, residents in the school districts (with or without school-age children) can be ignored. On the other hand, if the costs of secondary education borne by society are to be measured, consideration should be given not only to costs accruing to the school district but also to costs which are borne by other public and private bodies. For example, extra supplies purchased by the students should be calculated as well as the costs of added police and fire protection borne by the state government.

B. **Capital Costs**

Social (and private) capital costs are fundamentally no different in nature than social (and private) current costs, and, thus, what follows should not be construed as suggesting so. Capital costs can be broken down into four different elements:

a) Site acquisition costs;

b) Capital improvements to the site;

c) Physical plant and building costs; and,

d) Equipment costs.

There are serious problems involved in measuring capital costs to education. These problems stem from several physical and institutional factors. Two of the most important factors are: 1) the physical plant of the school usually has an economic life longer than the period of training for any given educational cohort; 2) the services of this capital stock are not easily valued in market terms.

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Four possible treatments for valuing this capital exist. First, one can argue that once the capital stock exists, especially the physical plant and buildings, it becomes specific to the educational process and thus has no alternative use. In this case, social capital costs would be zero in the short run, since no opportunity cost is involved in their use for a cohort of students which uses the capital after the decision was made to create the school. This is a tenuous assumption, though, for it is easy to discover alternative uses for such capital stock. Thus, the value of the educational physical plant is not zero in competing uses. But since it is not a perfect substitute for these competing uses, the market value of the competing uses does not exactly reflect the opportunity cost of using the non-renovated physical plant for educational purposes. If one went to the market to price the value of the non-renovated educational plant in terms of its potential value as a hospital simply by observing what the value of a hospital was, the value would be overstated. Thus, the value is not zero, but it is less than the apparent value of alternatives since, without renovation, it is not a perfect substitute. And, even with renovation, such factors as location, which cannot be changed, continue to exist and reduce the degree of substitutability, thus forcing one to adjust downward the opportunity costs implied by measures of values of foregone alternatives.

Second, historical costs of building construction and site acquisition can be used, but these historical costs are essentially irrelevant since they have no necessary bearing on the present opportunity costs involved in using the capital stock in question. They do not reveal what the current economic value of the capital resource is. Current economic value could be less than, equal to, or greater than historical cost.
Third, the use of replacement costs is a possibility in the attempt to measure capital costs. However, it is obvious that in many cases it would cost more to replace exactly a building than the building is currently worth in economic terms. The use of replacement costs would over-value the capital resource, given a rising price level and assuming no compensating technological change in construction technique.

Fourth, an estimate of current assessed valuation could be used to arrive at a measure of the capital costs. However, the valuation standard used becomes critical. In actual practice, the valuation standard amounts to a combination of historical costs adjusted by a price index of replacement cost so that this measure is no better than the replacement cost measure.

In short, it is not obvious what price resulting among these four choices should be attached to the capital inputs to get a measure of the opportunity costs. None of the above is correct in a pure theoretical sense.

The Capital Recovery Factor. Even if the true economic value of the capital resources in use has been measured, the problem still remains as to the measurement of the rate at which the given capital stock is used up over the course of the investment process when more than one cohort of students employs the capital stock. Two courses of action have been suggested for use. One is to attempt to measure an imputed rent and depreciation to the capital stock by making analogies with respect to what amount of rent (i.e., return on the capital investment) the capital item would yield if it were being employed in the private sector of the economy. Some notion of depreciation
is added to this. But such a technique is subject to a great deal of arbitrariness and uncertainty.

In order to get a measure of the rental opportunity cost it is necessary to go to the market place and attempt to identify capital resources which represent alternatives to the resources employed in the educational process. This will allow one to determine the value of foregone alternatives. But, again, any imputed rent based on market observations will most likely overstate the value of the capital resources which are already committed to education. Thus, a great deal of judgment is involved in adjusting the observed market prices so that they more closely reflect the true opportunity costs.*

An alternative technique for estimating the rate of capital use lies in employing the "capital recovery factor" (CRF). The application of this technique automatically accounts for both rent (interest) and depreciation.

The capital recovery factor is that factor which ". . . when multiplied by the present value of capital costs, is the level [average] end-of-year annual amount over the life of the project necessary to pay interest on and recover the capital costs in full." **


The formula is as follows:

\[ c = \frac{C_0 \cdot i (1 + i)^n}{(1 + i)^n - 1} \]

where \( c \) is the capital recovery factor (annual capital cost); \( C_0 \) is the present value of capital in use; \( i \) is the social opportunity cost rate of capital or investment funds; and \( n \) is the number of years over which benefits (of the capital in question) are returned, that is, the project life. In some respects, this technique is no less arbitrary than that which imputes rent and depreciation. Apart from the problem of establishing the present value of the capital in use, essentially arbitrary judgments must be made with respect to the values of \( n \) and \( i \).

Figure 1 describes how a hypothetical capital usage stream would appear for a school building built in 1917 with one wing added in 1937 and a major renovation occurring in 1957, given appropriate assumptions on the social opportunity cost of investment funds and the time period concerning the flow of benefits related to each capital item. (A major renovation is a renovation which increases the economic value of the capital item in question. At some point, arbitrary distinctions have to be made between what is renovation and what is maintenance.)

The time period over which costs are measured in this illustration extends from fiscal year 1956 through fiscal year 1960. So, for example, average annual capital costs for physical plant in the example below would be 0a for fiscal year 1956 and 0b for fiscal year 1959.

Several problems exist with the use of this technique. The first is that the CRF does not necessarily indicate the amount of capital used in any given year. It only states the level annual amount
FIGURE 1

An Example of Cost Imputation Based on the Capital Recovery Factor
needed to recoup the principal and social opportunity cost, that is, interest, given the project life. The actual amount of capital used up in any given year could be the same, more, or less than this amount.

Related to this problem is the fact that more than one cohort of students may utilize a given capital item during the life of that item. For instance, if a capital item has an economic life of n years and it takes an educational cohort three years to complete its training (investment) process, then at least n-2 cohorts will make use of that capital stock. If a capital item is installed in an on-going educational project, given that it takes 3 years to train a cohort, three cohorts are always using that capital item. Here there is a joint cost problem. In any time period, how much of the capital use is due to each cohort?

Since there are no satisfactory solutions to these problems, the CRF may be used on the assumption that each cohort of students uses the capital stock at an equal rate during the investment (schooling) process.

Another method for the imputation of depreciation and implicit rent has been outlined by Schultz.* According to this method, the replacement value of school assets is first calculated. An interest rate (such as 5 percent) is chosen to calculate "implicit rent" (the foregone opportunity to rent the premises for noneducational uses). Further, depreciation is calculated by estimating (1) a given distribution of school

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asset values among land, buildings, and equipment and (2) the expected useful life of each class of assets. Using various sources, Schultz concluded that implicit rent and depreciation amounted to approximately 8 percent of the value of school assets.

A convenient source of information for the replacement value of assets is the insured value of school buildings and grounds. Capital costs can be computed by amortizing the insured values on the basis of estimates of expected lifetimes of assets.

C. **Average, and Marginal Costs**

Cost data compiled and analyzed by educational administrators and policy makers in various levels of government have long been confined (with some exceptions) to what economists call average costs. Yet, decisions made on the basis of average costs may be shown to lead at times to inefficient results, to the extent that average costs differ significantly from what economists call marginal (or incremental) costs.

**Definitions of Average and Marginal Costs.** Suppose that sufficient data exist so that the total costs of educational programs (the least possible costs necessary to sustain each program) may be calculated. For example, let the total costs of a program in a secondary curriculum be given as in column 3 of Table 1, where costs are given by section size (measured in terms of number of pupils). In Section A, with 10 pupils and $1,000 in total costs, per pupil cost is $100. To the extent that all pupils are alike, the figure of $100 is analogous to what economists define as average cost.

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Adapted from *The Costs of Vocational and Non-Vocational Programs*, op.cit.
TABLE 1
Hypothetical Cost Function for
a Secondary Educational Program

<table>
<thead>
<tr>
<th>Section</th>
<th>No. of Pupils</th>
<th>Total Cost (TC)</th>
<th>Average Cost(^a) (AC)</th>
<th>Marginal Cost (MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>$1,000</td>
<td>$100</td>
<td>100(^b)</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>1,500</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>1,750</td>
<td>58</td>
<td>25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>60</strong></td>
<td><strong>$4,250</strong></td>
<td><strong>$71</strong></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Average cost equals column 3 divided by column 2.

\(^b\) Derived on the assumption that total costs are zero when enrollment in this course is zero.

**DEFINITION:** Average costs are given by total costs divided by the number of units of output. In education, when all pupils may be assumed to be alike—or when characteristics of pupils are taken into account in some way (to be discussed later)—per pupil cost may be used as a surrogate for average cost.

But what is the cost of adding one more student to this program—or, for that matter, of adding another section of the program to the curriculum? If the data on the costs of this program are pooled, as shown in Table 1, it is found that, for the entire program, per pupil cost is $71. Can one therefore conclude that admitting one additional pupil will result in a cost increase equal to $71? Table 1 clearly indicates that this is not the case. If one individual is added to
Section A, extra costs would be at most $100. If he is added to Section B, extra costs would at most $50 (=[1,500 - $1,000]/10), while adding him to Section C would not be likely to result in an additional cost of more than $25 (=[1,750 - $1,500]/10). These figures are **marginal costs**. They are obtained by computing the additional cost per pupil of conducting a class of 10 pupils as opposed to no class; a class of 20 as opposed to a class of 10; and a class of 30 as opposed to one of 20.

**DEFINITION:** Marginal costs are derived by computing the change in total costs divided by the change in the number of units of output. In education, marginal costs may be obtained by calculating the change in total cost per change in the number of pupils.

It should be noted that the use of the number of pupils as a surrogate for the number of units of "output" is questionable, since the quality of education is likely to vary among pupils either within any one school or between schools. But average and marginal cost concepts with the number of pupils as a proxy for units of output may be used as long as average and marginal costs in education are clearly understood.

**The Marginal Costs of Vocational Education.** On the basis of the definition of the marginal costs of educational programs it is possible to offer a concise definition for the added costs of vocational education.

**DEFINITION:** The marginal costs of a vocational program are measured by calculating the difference between the marginal cost of the vocational program and the marginal cost of the alternative secondary academic program.
The choice of the alternative secondary academic program is of considerable significance. The reason for this is that marginal costs in secondary academic education are likely to vary by type of course taught. If the alternative to vocational education is English or social science, the added costs of vocational education are likely to be much greater than if the alternative is physics or chemistry.

These points can be illustrated with Table 2. Suppose that the vocational program requires a class size of 15 students or fewer, with total, average, and marginal costs as depicted in the table. It is also assumed that enrollment in a chemistry laboratory is limited (in this illustration) to a maximum of 15 students. On the other hand, enrollments in English and social science programs are allowed to reach capacity levels of 100 pupils per class. Furthermore, Table 2 assumes that marginal costs in the vocational program decrease slightly as enrollment expands (from 0 to 15); marginal costs are constant, at $100, in the chemistry laboratory (indicating that additional students require additional attention from teachers as well as their own equipment and supplies); and, finally, substantial economies are obtained by increasing English and social science classes (due to the fact that one teacher could handle 5, 10, 15, 20, or as many as 100 pupils, without educational "quality" being drastically affected; changes in the quality of instruction and the need for more spacious classrooms as the number of pupils increases are reflected in the increase in total costs from $250 to $500.

Given such data, it is quite clear that the added costs of vocational education vary with (1) the number of pupils attending vocational classes and (2) the types of academic programs that the
### TABLE 2
Hypothetical Cost Functions for Selected Secondary Academic and Vocational Programs

<table>
<thead>
<tr>
<th>Program</th>
<th>No. of Pupils</th>
<th>Total Cost</th>
<th>Average Cost</th>
<th>Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vocational</td>
<td>5</td>
<td>$1,000</td>
<td>$200</td>
<td>$200&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1,800</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>2,550</td>
<td>170</td>
<td>150</td>
</tr>
<tr>
<td>Chemistry Laboratory</td>
<td>5</td>
<td>$500</td>
<td>$100</td>
<td>$100&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1,000</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1,500</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>English or Social Science</td>
<td>5</td>
<td>$250</td>
<td>$50</td>
<td>$50&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>275</td>
<td>27.5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>295</td>
<td>19.7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>350</td>
<td>11.7</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>500</td>
<td>5</td>
<td>2.8</td>
</tr>
</tbody>
</table>

<sup>a</sup>Derived on the assumption that total costs are zero when enrollment in this course is zero.

vocational students give up. If a vocational program is substituted for a chemistry laboratory, and if enrollment in the vocational program and the chemistry laboratory are initially 14 and 15, respectively, the cost of removing the fifteenth student from chemistry into the vocational program is given by $150 - $100 = $50. Compare this figure to the difference in average cost: $170 - $100 = $70. If the overall per pupil cost in the academic program were compared to the per pupil cost of the
vocational program (a comparison many would make), it is likely that the “added” costs of vocational education would appear to be far greater (considering that chemistry laboratories are probably atypically expensive).

The extra cost of adding one student to the vocational program will be higher, in this case, the smaller the program’s initial enrollment. For example, if a student were to switch from the chemistry laboratory to the vocational program—which had only nine pupils—the extra costs would be approximately $60. If enrollment in the vocational program was less than five before the student changed, the extra costs could approach $100.

Obviously, if the alternative to attending a vocational program were attendance in an English or a social science class, the extra costs of adding a student to the vocational program might vary from $100 to $173.20, depending on initial enrollments in the two programs. If two vocational programs are selected (assuming both have the same cost functions as illustrated in Table 2), a combination of alternatives might be applicable (such as one English class and one chemistry laboratory). The essential points, however, are that (1) the alternatives to vocational enrollment should be considered and (2) average and marginal costs of each program must be computed.

Many problems are encountered when these conceptual ideas are utilized in actual cost studies. For example, in computing the total cost for a given program, how could one impute maintenance costs, administrative costs, and fixed charges to this program? Further, how could one obtain total cost schedules, even such simple ones as illustrated in Tables 1 and 2, and how can one know which programs
are replaced by the vocational program without going through a detailed survey of students and teachers in each school?

D. Short-Run and Long-Run Costs*

The discussion to this point has been cast mainly in what may be termed "short-run" analysis. The questions were posed in terms of one individual switching from one program to another, and the physical size of the school in question was presumed to be fixed. In the long run, however, construction of new facilities may take place, and long-run cost functions may be substantially different from short-run cost functions.

Long-Run Cost Functions. Long-run decision-making processes involve such questions as: Should one wing be added to the building? Should new schools be built? What should be the optimal size for each of the schools in the district (size being measured by enrollment capacity)? To answer such questions, the school administrator and his advisers must be aware of the long-run cost function facing the schools.

In Tables 1 and 2 it was implicitly assumed that the short-run cost functions for the respective programs would apply to any and all sections of similar enrollment, regardless of whether there were one section or several. Such an assumption is acceptable in the short run since, by definition, additional classroom space could not be made available during the decision period; the number of sections for each course could not vary significantly. (One could still shift students from one program to the other; but new machine shops, chemistry laboratories, and so forth could not be constructed.) In the long run, however, it may be possible to vary the number of classrooms and

laboratories and, as a result, to change the average and marginal cost schedules for each program.

**Economies of Scale.** When long-run cost functions are such that lower per unit costs are obtained when school size (or the number of classrooms) increases, economies of scale are said to exist. For a range of school sizes, economies of scale are realized by the school such that expansion of school size is expected to result in reduced per unit costs. If the per unit cost function is U-shaped, a point is reached when scale economies are entirely exhausted, and where diseconomies take over, such that larger school sizes result in increased per unit costs. Several studies of elementary and secondary education costs have found economies of scale at some range of school sizes, indicating a plausible discrepancy between short- and long-run costs of education.*

**Optimum School Size.** An optimum school (or program) size may be calculated from a U-shaped long-run average cost curve. In some cases the optimum point is meaningless, as it reflects enrollments that are not likely to be reached in the foreseeable future. Then the optimum is given by the largest possible enrollment which could be contained in a single school (or program). Consolidation of school districts or of separate schools within a district may be suggested.

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A unique optimum exists only when the long-run average cost curve is U-shaped. It is possible that the long-run average cost curve declines in a given range of school (or program) sizes, but then approaches a constant level. Statistical support of such a phenomenon has been shown in recent studies on the costs of secondary education in Iowa and Michigan.

E. Statistical Cost Functions

Two kinds of general specifications for cost functions will be presented.

First, we assume that the quality of training under each program is homogeneous across all schools. Therefore, the statistical cost function for each program is:

\( \text{Cost} = f(\text{total number of student hours}) \)

Equation (1) specifies implicit relations between the dependent (costs) and independent (student hours) variables. The explicit functional form chosen may be a linear relationship or a nonlinear relationship (such as the addition of a squared term to the function).

Secondly, the homogeneity assumption referred to above may not be entirely realistic. The quality and cost of a program may vary with a school's location, the type of community, the type of schools, and other factors. To test and/or control for structural differences among schools the following variables are introduced:

1. locational factors (region, population, type of community);
2. socio-economic conditions which might affect costs (e.g., number of "poverty"

*Adapted in part from The Costs of Vocational and Nonvocational Programs, Op.cit.*
or minority students); (3) type of school (comprehensive secondary, secondary vocational, etc.); (4) quality or price of school inputs (e.g., average teacher salaries, price per unit of equipment, etc.); and (5) measures of instructional quality—given by output and/or impact proxies. The general specification of this, more complex, description of the cost function is given by:

\[
(2) \quad \text{Cost} = f (\text{enrollment, locational factors, socio-economic conditions, type of school, quality or price of school inputs, and instructional quality})
\]

**Shape of Cost Function:** In line with accepted economic theory, a short-run production function is likely to look something like the graph in Figure 2 (where factors other than enrollment are properly controlled).

\[\text{Cost}\]

\[\text{TC}\]

\[\text{Fixed Costs}\]

\[\text{enrollment}\]

**FIGURE 2**

Short-Run Cost Function
Algebraically, such a function may be represented by a cubic function:

\[(2) \quad TC = a + bE + cE^2 + dE^3\]

Where \(TC\) = total costs, \(E\) = enrollment, and \(a, b, c,\) and \(d\) are constants (\(a\) being fixed costs, i.e., \(TC\) when \(E = 0\)).

Since average cost (\(AC\)) is defined by \(TC/E\), we have from (2):

\[(3) \quad AC = \frac{TC}{E} = \frac{a}{E} + b + cE + dE^2\]

The \(AC\) function in (3) is likely to display a U-shaped curve.

Marginal cost (\(MC\)) is the change in \(TC\) associated with a unit change in \(E\). Mathematically:

\[(4) \quad MC = \frac{dTC}{dE} = b + 2cE + 3dE^2\]

From Equations (2)-(4) we can calculate \(TC, AC\) and \(MC\) for a stipulated level of \(E\).

Since, in most cases, cost and enrollment data encompass a relatively small range of all possible enrollment-cost combinations, it is quite likely that a linear function for \(TC\) will be a very good approximation of reality. Then we have:

\[(5) \quad TC = a + bE\]

and therefore,

\[(6) \quad AC = \frac{a}{E} + b\]
and

\[(7) \quad MC = b\]

AC will approximate a rectangular hyperbola whereas \(MC\) will be constant throughout (for all levels of \(E\)) and be equal to \(b\).

F. Cost-Sharing:

The problem of joint costs has been alluded to earlier. When vocational education shares facilities with adult education or MDTA programs, there is both a potential problem of joint costs and a conceptual issue of opportunity costs. Would the sites, buildings and equipment be the same had there been no adult education? If the answer is yes, then capital costs of adult education are limited to extra wear and tear (necessitating more frequent replacement of equipment or facilities) and some repair and maintenance costs. On the other hand, if investment in facilities and equipment is likely to be greater because it is anticipated that the same facilities would be used in adult education or other nonstudent programs, it would be legitimate to assign to adult education, etc., these extra costs (in addition to depreciation, repair and maintenance costs as discussed above).

Since the problem of joint costs exists in any event, one might once again assume that all users of the facilities utilize the capital stock at an equal rate during the investment process. One possibility for carrying out such a scheme would be to assign costs on the basis of hours per person instructed (students or adults).
G. Social vs. Private Costs

The concept of opportunity costs suggests a distinction between social and private costs. What is a cost to an individual may not be a cost to society, and vice versa. For example, since educational institutions are exempt from paying various taxes, they can purchase resources at a favorable rate when compared with other economic agents. This advantage is not costless to society; it is generally costless to the individual. Other examples are educational costs covered by government subsidies (which are social, but not private) costs, or pollution from industrial shops at a vocational school (which are costs to the wider community but not necessarily to the school).
III. Concepts of Benefits

There is presented below a list of impact variables which probably encompass most of the impact goals that various levels of government are striving to meet. In addition to the mere listing of such variables, subheadings for each major variable are provided, when appropriate. The subheadings, in some cases, permit a rather broad choice of proxies; in other cases, specific proxies are suggested.

1. Employment:

(a) reduction in overall unemployment rate
(b) reduction in teenage unemployment
(c) reduction in minority-group unemployment
(d) reduction in severity of "shortages" in certain skill-categories (plumbers?)
(e) relatedness of employment to training

Presumably, different government agencies might be interested largely in different aspects of employment. As suggested in Table 3, goals a - d are likely to be considered by the legislature, whereas persons in lower government hierarchies may be interested only in goals a and e.

2. Wages:

(a) hourly rate
(b) annual earnings.
(c) seasonally in job
(d) overtime work and rate
(e) fringe benefits
### TABLE 3
Impact Variables and Hierarchy in State Government

<table>
<thead>
<tr>
<th>Hierarchy</th>
<th>Employment</th>
<th>Wages</th>
<th>Civic Activities</th>
<th>Employer Benefits</th>
<th>Inventions</th>
<th>Crime</th>
<th>Attitudes</th>
<th>Self-improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Legislature</td>
<td>a, b, c, d</td>
<td>b</td>
<td>a, b</td>
<td>?</td>
<td>?</td>
<td>a, b</td>
<td>a, c</td>
<td>b</td>
</tr>
<tr>
<td>(2) Commissioner of Education</td>
<td>b, e</td>
<td>b, a, e</td>
<td>a, b</td>
<td>?</td>
<td>a</td>
<td>a, b</td>
<td>a, b, c</td>
<td>a, b</td>
</tr>
<tr>
<td>(3) Director of Occupational Education</td>
<td>e</td>
<td>a, e</td>
<td>?</td>
<td>?</td>
<td>a</td>
<td>?</td>
<td>a, b</td>
<td>?</td>
</tr>
<tr>
<td>(4) School Principal</td>
<td>e</td>
<td>a, e</td>
<td>?</td>
<td>?</td>
<td>a</td>
<td>?</td>
<td>a, b</td>
<td>?</td>
</tr>
<tr>
<td>(5) Teacher</td>
<td>e</td>
<td>a, e</td>
<td>?</td>
<td>?</td>
<td>a</td>
<td>?</td>
<td>a, b</td>
<td>?</td>
</tr>
</tbody>
</table>

Note: ? = not clear a priori.
The hourly rate is generally taken to represent returns to one's skill and productivity. Annual earnings are probably a function of skill, too, but are affected by such elements as overtime work, amount of non-paid vacations that individuals elect (choice of leisure time), and so on. However, when hourly earnings are considered, the seasonality of the job and extent of fringe benefits should also be considered. Table 3 gives our judgment regarding the importance of the various elements to different government hierarchies.

3. Civic Activities

(a) voting behavior—change in % of voting in given age brackets

(b) participation in civic and communal affairs: membership in civic organizations (League of Women Voters, etc.), boards of education, political activities, etc.

4. Benefits to Employer and Co-Workers:

(a) reaction of employer to workers' productivity and morale

(b) reaction of co-workers

Proxies to measure these variables have been developed in a recent Institute study. See M. V. Lewis, E. Cohn, and others, Recruiting, Placing and Retaining the Hard-to-Employ (IRHR, 1971), especially Chapters 6 and 9.

5. (a) Inventions, patents, copyrights, etc.
6. Crime

   (a) reduction in juvenile crime
   (b) reduction in the general crime rate

7. Individual Attitude

   (a) attitude toward work
   (b) feeling about one's position in life
   (c) attitudes toward society

8. Self-improvement

   (a) self (informal) study: books, magazines, etc.
       at home
   (b) formal study: adult and continuing education, etc.
IV. Cost-Impact Analyses

When impacts (or outputs) are both quantifiable and may be converted into dollars, one may utilize the familiar benefit-cost analysis. When monetization is not practicable (but quantification of impacts in some other manner still possible), cost-effectiveness analysis may be performed. These will be discussed in turn.

A. Benefit-Cost Analysis:*  

Three criteria for public expenditure decisions have been widely used in recent years: (1) the net present value rule; (2) the internal rate of return rule; and (3) the benefit-cost ratio rule. These shall be examined in turn.

The net present value rule (PV rule) would lead to the selection of "all projects where the present value of benefits exceeds the present value of costs."* If \( b_t \) and \( c_t \) denote annual benefits and costs, respectively, and if the rate of discount is \( r \), the PV rule implies that we select all projects for which

\[
\sum_{t=1}^{T} b_t (1 + r)^{-t} > \sum_{t=1}^{T} c_t (1 + r)^{-t}
\]

When it is desired to rank projects in order of their contribution to economic well-being, the PV rule implies that projects with higher net present values will be selected first. However, when the decision-makers are constrained by a budgetary limitation, projects must be ranked on the basis of their contribution per dollar of investment.

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*Adapted from Elchahan Cohn, Public Expenditure Analysis, unpublished manuscript.

That is, we compute the present value of benefits as of period 1—rather than the present, which is period 0. Let \( s_t = b_t - c_t \). Then the present value as of time 1 (\( PV_1 \)) is given by:

\[
PV_1 = s_1 + \sum_{t=2}^{T} s_t (1 + r)^{-(t-1)}
\]

Thus, when a budget constraint must be taken into account, we compute for each project \( PV_1/c_0 \) (where \( c_0 \) is the "fund input" for the current period), and "the rule is successively to adopt projects with the highest values of this ratio until the fixed budget is exhausted or until the alternative use of funds elsewhere becomes more desirable than further investments" (Hirshleifer et al., 1960, p. 161).

The internal rate of return (IROR) rule would lead to the selection of "all projects where the internal rate of return exceeds the chosen rate of discount" (Prest and Turvey, 1965, p. 703). If the IROR is denoted by \( R \), we obtain \( R \) by solving the following equation:

\[
\sum_{t=1}^{T} (b_t - c_t) (1 + R)^{-t} = 0
\]

That is, \( R \) is given by the rate of discount (\( r \)) such that the present value of net benefits is zero. In general, this rule is equivalent to the PV rule. However, the ranking of projects, using the IROR rule, may lead to different results than the PV rule (Hirshleifer et al., 1960, p. 167). For the ranking of projects, using the IROR rule, we select those projects having the higher IROR's.
Finally, the benefit-cost ratio rule (B/C rule) calls for the selection of "all projects where the ratio of the present value of benefits to the present value of costs exceeds unity" (Prest and Turvey, 1965, p. 703). Symbolically, we select projects for which

$$\frac{\sum_{t=1}^{T} b_t (1 + r)^{-t}}{\sum_{t=1}^{T} c_t (1 + r)^{-t}} > 1$$

Again, this rule will select the same projects as would the PV rule. However, ranking projects according to the value of the B/C ratio could, in some cases, lead to the selection of a different set of projects than would result if the PV rule were used.

Example: Assume that the benefits of investment in schooling could be measured by the difference between the expected lifetime income of an individual with a given level of schooling and that of a similar individual with one unit less of education. Using census cross-sectional data for 1949, Hansen obtained lifetime income estimates by educational levels.* For this example, we consider only investment in 4 years of high school, compared to no investment in high school, and 4 years of college, compared to no college. The additional lifetime earnings associated with each of these investments are given in rows 1 and 4, respectively, of Table 5. Hansen also provides estimates of the costs of education of 1949 (see table 4), from which we have computed the present value of costs of 4 years of high school and 4 years of college, respectively, in rows 2 and 5 of Table 5. The respective net present values of benefits (present value of benefits less present value of costs) are given in rows 3 and 6.

---

The present value computations are given for 5 different rates of discount: 0, 3, 6, 8, and 10 percent. The data in the table illustrate the effect of discounting: the large benefits of higher education (nearly $100,000) that are obtained with no discounting (i.e., when the rate of discount is zero), are reduced to slightly over $2,000 when a discount rate of 10 percent is applied. Moreover, when net benefits are considered, investment in higher education appears to result in a small loss to society, when the applicable rate of discount is 10 percent.

The data of Table 5 are used to derive a ranking of the two investments, as illustrated in Table 6. The table provides two interesting conclusions. First, the ranking is sensitive to the rate of discount chosen, when the net present value rule is applied. Second, the internal rate of return rule yields a different ranking of investments than does the net present value rule for rates of discount of 8 percent or less.

TABLE 4

Average Annual per Student Costs: U.S., 1949

<table>
<thead>
<tr>
<th>Age</th>
<th>School Level</th>
<th>School Costs (2)</th>
<th>Other Costs (3)</th>
<th>Total Costs (4)</th>
<th>Tuition and Fees (5)</th>
<th>Other Costs (6)</th>
<th>Total Costs (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-13</td>
<td>Elementary</td>
<td>$201</td>
<td>$201</td>
<td>$402</td>
<td>31</td>
<td>31</td>
<td>34</td>
</tr>
<tr>
<td>14-17</td>
<td>High School</td>
<td>354</td>
<td>31</td>
<td>385</td>
<td>31</td>
<td>31</td>
<td>62</td>
</tr>
<tr>
<td>18-21</td>
<td>College</td>
<td>801</td>
<td>142</td>
<td>943</td>
<td>245</td>
<td>142</td>
<td>387</td>
</tr>
</tbody>
</table>

Source: W. Lee Hansen (1963), Table 2.
TABLE 5
Calculation of Net Present Values of Investment in High School and College Education, 1949 Data

<table>
<thead>
<tr>
<th>Rate of Discount (per cent)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. 4 Years of High School</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Present Value of Benefits</td>
<td>$46,038</td>
<td>$18,156</td>
<td>$6,488</td>
<td>$3,601</td>
<td>$1,949</td>
</tr>
<tr>
<td>2. Present Value of Costs</td>
<td>$1,540</td>
<td>$1,474</td>
<td>$1,414</td>
<td>$1,377</td>
<td>$1,342</td>
</tr>
<tr>
<td>Internal rate of return = 11.4 percent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **II. 4 Years of College** |     |     |     |     |     |
| 4. Present Value of Benefits | $95,430 | $31,273 | $10,764 | $5,121 | $2,186 |
| 5. Present Value of Costs | $3,772 | $3,208 | $2,744 | $2,479 | $2,255 |
| Internal rate of return = 9.9 percent |     |     |     |     |     |

Source: Calculated from Hansen (1963), Tables 2 and 6

TABLE 6
A Comparison of the Net Present Value Rule and the Internal Rate of Return Rule: Ranking of Educational Investments

<table>
<thead>
<tr>
<th>Rank</th>
<th>Net Present Value Rule</th>
<th>Rate of Discount (percent)</th>
<th>Internal rate of Return Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>1.</td>
<td></td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

Source: Table 5
Note: C = four years of college; H = four years of high school
The implications of this example are clear. First, the absolute and relative value of benefits associated with a given project could be extremely sensitive to the rate of discount chosen. Second, different rules may yield different rankings of investments. (In this example, the benefit/cost ratio rule yields rankings identical to those given by the internal rate of return rule for each of the discount rates employed here)

B. Cost-Effectiveness Analysis

In the foregoing it has been assumed that program effects could be measured in monetary terms. In a wide range of circumstances, however, program effects (outputs) are not given in monetary form—either because dollar imputation is extremely difficult (or expensive), or because it is not believed that such an imputation will have more than a negligible effect on the decision process.

When a single index of effectiveness is developed (and dollars could be one basis for such an index), the attractiveness of each program relative to that of any alternative could, in some cases, be determined. For example, in Figure 3 costs are measured on the vertical axis and effectiveness on the horizontal axis. Each of the single points a, b, and c represents a point estimate of the cost and expected effectiveness of programs A, B, and C, respectively. It is seen that program B dominates A—i.e., is more "cost-effective"—since B produces higher expected effectiveness and costs less than A. Program C also dominates A, but it is not immediately clear whether B is more or less attractive than C since, although the latter produces a higher level of output, it also costs more.

FIGURE 3

Cost-Effectiveness Alternatives

Suppose that the effectiveness and cost scales are linear (that is, a point on the axis twice as far from the origin as any other point is twice as effective). Then, by drawing rays from the origin to each point in Figure 3, and by computing the slopes of such rays, we obtain the marginal cost associated with a small increase in effectiveness associated with each project. If it is assumed that projects are subject to "constant returns to scale"—i.e., when all inputs are varied by a certain proportion, output will vary by the same proportion—the relative attractiveness of project B with respect to C could be studied. In Figure 3 the slope of the line Oc is larger than that of Ob. This implies that additional effectiveness could be obtained at a lesser cost by extending project B to point $b^1$ in Figure 3 rather than by employing project C. Again, this is true only to the extent that additional output may be obtained through the extension of project B at the same extra cost as obtained initially. Furthermore, sufficient resources to achieve this level of efficiency must be available.
Uncertainty. Consider the possibility that points a, b, and c are estimated subject to a given degree of uncertainty: each project might be more or less effective and more or less costly. In some cases we might be able to specify the potential boundaries for such uncertainty (i.e., we specify by how much effectiveness may be over or underestimated). For example, areas of uncertainty around points a, b, and c have been drawn in Figure 3. For some points within the uncertainty areas, project B is superior to C (being more effective and less costly). The existing literature, however, does not seem to contain any empirical work in which the areas of uncertainty have been defined.

An interesting theoretical consideration of cost-effectiveness analysis in relation to community health systems is considered by Packer (1968). A number of cost-effectiveness studies by the U. S. Department of Health, Education, and Welfare concerning human investment programs were undertaken in recent years. Among them are the study of maternal and child health care (1966), and Delivery of Health Services to the Poor (1967). In both cases, program effectiveness is given by a number of outputs, not linked by an index, so that a simple analysis as in Figure 3 could not be made. Of course, when a given project dominates another for all program outputs, there is no question as to which project is more "cost-effective." But when different programs are associated with different sets of outputs, or when one program is more cost-effective with respect to one output but less cost-effective with respect to another output—all in comparison to some alternatives—it is not possible to determine which program is "better."
A study by the Stanford Research Institute (1965), concerning rural mental health programs in California, employed two output indices—and unfortunately both proxies were input variables. Cost-effectiveness was determined on the basis of charts such as Figure 3 (with no areas of uncertainty considered) for each of the outputs.∗

*Bibliographic references to these studies are given in Hu et al., A Cost-Effectiveness Study of Child Health and Welfare Programs. (IRHR, 1971)

See also Neil M. Singer, Public Microeconomics, Boston: Little, Brown & Co., 1972, especially Parts Four and Five.