DOCUMENT RESUME

ED 071 892		SE 015 529
TITLE	Project Physics Reader 3, The Triumph of Mechanics.	
INSTITUTION	Harvard Univ., Cambridge, Mass. Harvard Project Physics.	
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.	
BUREAU NO	BR-5-1038	
PUB DATE	68	
CONTRACT	OEC-5-10-058	
NOTE	261p.; Authorized Interim Version	
EDRS PRICE	MF-$0.65 HC-$9.87	
DESCRIPTORS	Energy; Heat; *Instructional Materials; Kinetic Molecular Theory; *Kinetics; *Physics; Science Materials; Secondary Grades; Secondary School Science; *Supplementary Reading Materials	
IDENTIFIERS	Harvard Project Physics	

ABSTRACT

As a supplement to Project Physics Unit 3, a collection of articles is presented in this reader for student browsing. Four excerpts are given under the following headings: On the kinetic theory of gases, Maxwell's Demon, Introduction to Waves, and Scientific Cranks. Five articles are included in terms of energy, barometers, randomness, fiddle family, and seven fallacies in science. The ten remaining book passages are related to silence production, steam engines, molecular theory of gases, disorder phenomena, statistical laws, arrow of time, James Clerk Maxwell's discoveries, wave concept, wave motion in acoustics, and musical instruments and scales. Illustrations for explanation purposes are also provided. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

Project Physics **Reader**

An Introduction to Physics

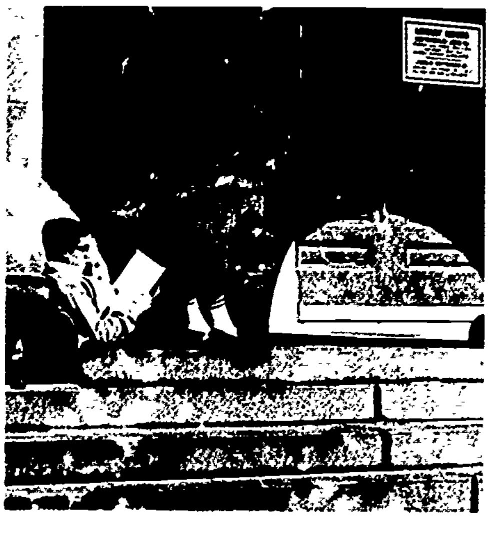
The Triumph of Mechanics

Project Physics **Reader**

An Introduction to Physics 3 The Triumph of Mechanics

Authorized Interim Version 1968-69

Distributed by Holt, Rinehart and Winston Inc.
New York • Toronto



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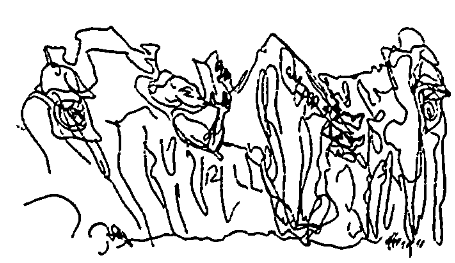
This is not a physics textbook. Rather, it is a physics reader, a collection of some of the best articles and book passages on physics. A few are on historic events in science, others contain some particularly memorable description of what physicists do; still others deal with philosophy of science, or with the impact of scientific thought on the imagination of the artist.

There are old and new classics, and also some little-known publications; many have been suggested for inclusion because some teacher or physicist remembered an article with particular fondness. The majority of articles is not drawn from scientific papers of historic importance themselves, because material from many of these is readily available, either as quotations in the Project Physics text or in special collections.



This collection is meant for your browsing. If you follow your own reading interests, chances are good that you will find here many pages that convey the joy these authors have in their work and the excitement of their ideas. If you want to follow up on interesting excerpts, the source list at the end of the reader will guide you for further reading.

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A fictional scientist tells of an apparatus for producing silence. Although the proposed scheme is improbable, the story has a charming plausibility.

1 Silence, Please

Arthur C. Clarke

1957

YOU COME upon the "White Hart" quite unexpectedly in one of these anonymous little lanes leading down from Fleet Street to the Embankment. It's no use *telling* you where it is: very few people who have set out in a determined effort to get there have ever actually arrived. For the first dozen visits a guide is essential: after that you'll probably be all right if you close your eyes and rely on instinct. Also—to be perfectly frank—we don't want any more customers, at least on *our* night. The place is already uncomfortably crowded. All that I'll say about its location is that it shakes occasionally with the vibration of newspaper presses, and that if you crane out of the window of the gent's room you can just see the Thames.

From the outside, is looks like any other pub—as indeed it is for five days of the week. The public and saloon bars are on the ground floor: there are the usual vistas of brown oak panelling and frosted glass, the bottles behind the bar, the handles of the beer engines . . . nothing out of the ordinary at all. Indeed, the only concession to the twentieth century is the juke box in the public bar. It was installed during the war in a laughable attempt to make G.I.'s feel at home, and one of the first things we did was to make sure there was no danger of its ever working again.

At this point I had better explain who "we" are. That is not as easy as I thought it was going to be when I started, for a complete catalogue of the "White Hart's" clients would probably be impossible and would certainly be excruciatingly tedious. So all I'll say at this point is that "we" fall into three main classes. First there are the journalists, writers and editors. The journalists, of course, gravitated here from Fleet Street. Those who couldn't make the grade fled elsewhere: the tougher ones remained. As for the writers, most of them heard about us from other writers, came here for copy, and got trapped.

Where there are writers, of course, there are sooner or later editors. If Drew, our landlord, got a percentage on the literary business done in his bar, he'd be a rich man. (We suspect he is a rich man, anyway.) One of our wits once remarked that it was a common sight to see half a dozen indignant authors arguing with a hard-faced editor in one corner of the "White Hart", while in another, half a dozen indignant editors argued with a hard-faced author.

So much for the literary side: you will have, I'd better warn you, ample opportunities for close-ups later. Now let us glance briefly at the scientists. How did *they* get in here?

Well, Birkbeck College is only across the road, and King's is just a few hundred yards along the Strand. That's doubtless part of the explanation, and again personal recommendation had a lot to do with it. Also, many of our scientists are writers, and not a few of our writers are scientists. Confusing, but we like it that way.

The third portion of our little microcosm consists of what may be loosely termed "interested laymen". They were attracted to the "White Hart" by the general brouhaha, and enjoyed the conversation and company so much that they now come along regularly every Wednesday—which is the day when we all get together. Sometimes they can't stand the pace and fall by the wayside, but there's always a fresh supply.

With such potent ingredients, it is hardly surprising that Wednesday at the "White Hart" is seldom dull. Not only have some remarkable stories been told there, but remarkable things have *happened* there. For example, there was the time when Professor ———, passing through on his way to Harwell, left behind a brief-case containing—well, we'd better not go into that, even though we did so at the time. And most interesting it was, too. . . . Any Russian agents will find me in the corner under the dartboard. I come high, but easy terms can be arranged.

Now that I've finally thought of the idea, it seems astonishing to me that none of my colleagues has ever got round to writing up these stories. Is it a question of being so close to the wood that they can't see the trees?

Or is it lack of incentive? No, the last explanation can hardly hold: several of them are quite as hard up as I am, and have complained with equal bitterness about Drew's "NO CREDIT" rule. My only fear, as I type these words on my old Remington Noiseless, is that John Christopher or George Whitley or John Beynon are already hard at work using up the best material. Such as, for instance, the story of the Fenton Silencer. . . .

I don't know when it began: one Wednesday is much like another and it's hard to tag dates on to them. Besides, people may spend a couple of months lost in the "White Hart" crowd before you first notice their existence. That had probably happened to Harry Purvis, because when I first came aware of him he already knew the names of most of the people in our crowd. Which is more than I do these days, now that I come to think of it.

But though I don't know *when*, I know exactly *how* it all started. Bert Huggins was the catalyst, or, to be more accurate, his voice was. Bert's voice would catalyse anything. When he indulges in a confidential whisper, it sounds like a sergeant major drilling an entire regiment. And when he lets himself go, conversation languishes elsewhere while we all wait for those cute little bones in the inner ear to resume their accustomed places.

He had just lost his temper with John Christopher (we all do this at some time or other) and the resulting detonation had disturbed the chess game in progress at the back of the saloon bar. As usual, the two players were surrounded by backseat drivers, and we all looked up with a start as Bert's blast whammed overhead. When the echoes died away, someone said: "I wish there was a way of shutting him up."

It was then that Harry Purvis replied: "There is, you know."

Not recognising the voice, I looked round. I saw a small, neatly-dressed man in the late thirties. He was smoking one of those carved German pipes that always makes me think of cuckoo clocks and the Black Forest. That was the only unconventional thing about him: otherwise he might have been a minor Treasury official all dressed up to go to a meeting of the Public Accounts Committee.

"I beg your pardon?" I said.

He took no notice, but made some delicate adjustments to his pipe. It was then that I noticed that it wasn't, as I'd thought at first glance, an elaborate piece of wood carving. It was something much more sophisticated—a contraption of metal and plastic like a small chemical engineering plant. There were even a couple of minute valves. My God, it *was* a chemical engineering plant. . . .

I don't goggle any more easily than the next man, but I made no attempt to hide my curiosity. He gave me a superior smile.

"All for the cause of science. It's an idea of the Biophysics Lab. They want to find out exactly what there is in tobacco smoke—hence these filters. You know the old argument—*does* smoking cause cancer of the tongue, and if so, how? The trouble is that it takes an awful lot of—er—distillate to identify some of the obscurer bye-products. So we have to do a lot of smoking."

"Doesn't it spoil the pleasure to have all this plumbing in the way?"

"I don't know. You see, I'm just a volunteer. I don't smoke."

"Oh," I said. For the moment, that seemed the only reply. Then I remembered how the conversation had started.

"You were saying," I continued with some feeling, for there was still a slight tintinus in my left ear, "that there was some way of shutting up Bert. We'd all like to hear it—if that isn't mixing metaphors somewhat."

"I was thinking," he replied, after a couple of experimental sucks and blows, "of the ill-fated Fenton Silencer. A sad story—yet, I feel, one with an interesting lesson for us all. And one day—who knows?—someone *may* perfect it and earn the blessings of the world."

Suck, bubble, bubble, *plop*. . . .

"Well, let's hear the story. When did it happen?"

He sighed.

"I'm almost sorry I mentioned it. Still, since you insist

—and, of course, on the understanding that it doesn't go beyond these walls."

"Er—of course."

"Well, Rupert Fenton was one of our lab assistants. A very bright youngster, with a good mechanical background, but, naturally, not very well up in theory. He was always making gadgets in his spare time. Usually the idea was good, but as he was shaky on fundamentals the things hardly ever worked. That didn't seem to discourage him: I think he fancied himself as a latter-day Edison, and imagined he could make his fortune from the radio tubes and other oddments lying around the lab. As his tinkering didn't interfere with his work, no-one objected: indeed, the physics demonstrators did their best to encourage him, because, after all, there is something refreshing about any form of enthusiasm. But no-one expected he'd ever get very far, because I don't suppose he could even integrate *e* to the *x*."

"Is such ignorance *possible?*" gasped someone.

"Maybe I exaggerate. Let's say *x e* to the *x*. Anyway, all his knowledge was entirely practical—rule of thumb, you know. Give him a wiring diagram, however complicated, and he could make the apparatus for you. But unless it was something *really* simple, like a television set, he wouldn't understand how it worked. The trouble was, he didn't realise his limitations. And that, as you'll see, was most unfortunate.

"I think he must have got the idea while watching the Honours Physics students doing some experiments in acoustics. I take it, of course, that you all understand the phenomenon of interference?"

"Naturally," I replied.

"Hey!" said one of the chess-players, who had given up trying to concentrate on the game (probably because he was losing.) "*I* don't."

Purvis looked at him as though seeing something that had no right to be around in a world that had invented penicillin.

"In that case," he said coldly, "I suppose I had better do some explaining." He waved aside our indignant protests. "No, I insist. It's precisely those who don't understand these things who need to be told about them. If someone had only explained the theory to poor Fenton while there was still time. . . ."

He looked down at the now thoroughly abashed chess-player.

"I do not know," he began, "if you have ever considered the nature of *sound*. Suffice to say that it consists of a series of waves moving through the air. Not, however, waves like those on the surface of the sea—oh dear no! *Those* waves are up and down movements. Sound waves consist of alternate compressions and rarefactions."

"Rare-what?"

"Rarefactions."

"Don't you mean 'rarefications'?"

"I do not. I doubt if such a word exists, and if it does, it shouldn't," retorted Purvis, with the *aplomb* of Sir Alan Herbert dropping a particularly revolting neologism into his killing-bottle. "Where was I? Explaining sound, of course. When we make any sort of noise, from the faintest whisper to that concussion that went past just now, a series of pressure changes moves through the air. Have you ever watched shunting engines at work on a siding? You see a perfect example of the same kind of thing. There's a long line of goods-wagons, all coupled together. One end gets a bang, the first two trucks move together—and then you can see the compression wave moving right along the line. Behind it the reverse thing happens—the rarefaction —I repeat, *rarefaction*—as the trucks separate again.

"Things are simple enough when there is only one source of sound—only one set of waves. But suppose you have two wave-patterns, moving in the same direction? That's when interference arises, and there are lots of pretty experiments in elementary physics to demonstrate it. All we need worry about here is the fact—which I think you will all agree is perfectly obvious—that if one could get two sets of waves *exactly* out of step, the total result would be precisely zero. The compression pulse of one sound wave would be on top of the rarefaction of another—net result—no change and hence no sound. To go back to my analogy of the line of wagons, it's as if you gave the last truck a jerk and a push simultaneously. Nothing at all would happen.

"Doubtless some of you will already see what I am driving at, and will appreciate the basic principle of the Fenton Silencer. Young Fenton, I imagine, argued in this manner. 'This world of ours,' he said to himself, 'is too full of noise. There would be a fortune for anyone who could invent a really perfect silencer. Now, what would that imply . . . ?'

"It didn't take him long to work out the answer: I told you he was a bright lad. There was really very little in his pilot model. It consisted of a microphone, a special amplifier, and a pair of loudspeakers. Any sound that happened to be about was picked up by the mike, amplified and *inverted* so that it was exactly out of phase with the original noise. Then it was pumped out of the speakers, the original wave and the new one cancelled out, and the net result was silence.

"Of course, there was rather more to it than that. There had to be an arrangement to make sure that the cancelling wave was just the right intensity—otherwise you might be worse off than when you started. But these are technical details that I won't bore you with. As many of you will recognise, it's a simple application of negative feed-back."

"Just a moment!" interrupted Eric Maine. Eric, I should mention, is an electronics expert and edits some television paper or other. He's also written a radio play about space-flight, but that's another story. "Just a moment! There's something wrong here. You *couldn't* get silence that way. It would be impossible to arrange the phase . . ."

Purvis jammed the pipe back in his mouth. For a moment there was an ominous bubbling and I thought of the first act of "Macbeth". Then he fixed Eric with a glare.

"Are you suggesting," he said frigidly, "that this story is untrue?"

"Ah—well, I won't go as far as that, but . . ." Eric's voice trailed away as if he had been silenced himself. He pulled an old envelope out of his pocket, together with an assortment of resistors and condensers that seemed to have got entangled in his handkerchief, and began to do some figuring. That was the last we heard from him for some time.

"As I was saying," continued Purvis calmly, *"that's* the way Fenton's Silencer worked. His first model wasn't very powerful, and it couldn't deal with very high or very low notes. The result was rather odd. When it was switched on, and someone tried to talk, you'd hear the two ends of the spectrum—a faint bat's squeak, and a kind of low rumble. But he soon got over that by using a more linear circuit (dammit, I can't help using *some* technicalities!) and in the later model he was able to produce complete silence over quite a large area. Not merely an ordinary room, but a full-sized hall. Yes. . . .

"Now Fenton was not one of these secretive inventors who won't tell anyone what they are trying to do, in case their ideas are stolen. He was all too willing to talk. He discussed his ideas with the staff and with the students, whenever he could get anyone to listen. It so happened that one of the first people to whom he demonstrated his improved Silencer was a young Arts student called—I think—Kendall, who was taking Physics as a subsidiary subject. Kendall was much impressed by the Silencer, as well he might be. But he was not thinking, as you may have imagined, about its commercial possibilities, or the boon it would bring to the outraged ears of suffering humanity. Oh dear no! He had quite other ideas.

"Please permit me a slight digression. At College we have a flourishing Musical Society, which in recent years has grown in numbers to such an extent that it can now tackle the less monumental symphonies. In the year of which I speak, it was embarking on a very ambitious enterprise. It was going to produce a new opera, a work by a talented young composer whose name it would not be fair to mention, since it is now well-known to you all. Let us call him Edward England. I've forgotten the title of the work, but it was one of these stark dramas of tragic love which, for some reason I've never been able to understand, are supposed to be less ridiculous with a musical accompaniment than without. No doubt a good deal depends on the music.

"I can still remember reading the synopsis while waiting for the curtain to go up, and to this day have never been able to decide whether the libretto was meant seriously or not. Let's see—the period was the late Victorian era, and the main characters were Sarah Stampe, the passionate postmistress, Walter Partridge, the saturnine gamekeeper, and the squire's son, whose name I forget. It's the old story of the eternal triangle, complicated by the villager's resentment of change—in this case, the new telegraph system, which the local crones predict will Do Things to the cows' milk and cause trouble at lambing time.

"Ignoring the frills, it's the usual drama of operatic jealousy. The squire's son doesn't want to marry into the Post Office, and the gamekeeper, maddened by his rejection, plots revenge. The tragedy rises to its dreadful climax when poor Sarah, strangled with parcel tape, is found hidden in a mail-bag in the Dead Letter Department. The villagers hang Partridge from the nearest telegraph pole, much to the annoyance of the linesmen. He was supposed to sing an aria while he was being hung: *that* is one thing I regret missing. The squire's son takes to drink, or the Colonies, or both: and that's that.

"I'm sure you're wondering where all this is leading: please bear with me for a moment longer. The fact is that while this synthetic jealousy was being rehearsed, the real thing was going on back-stage. Fenton's friend Kendall had been spurned by the young lady who was to play Sarah Stampe. I don't think he was a particularly vindictive person, but he saw an opportunity for a unique revenge. Let us be frank and admit that college life *does* breed a certain irresponsibility—and in identical circumstances, how many of *us* would have rejected the same chance?

"I see the dawning comprehension on your faces. But we, the audience, had no suspicion when the overture started on that memorable day. It was a most distinguished gathering: everyone was there, from the Chancellor downwards. Deans and professors were two a penny: I never did discover how so many people had been bullied into coming. Now that I come to think of it, I can't remember what I was doing there myself.

"The overture died away amid cheers, and, I must admit, occasional cat-calls from the more boisterous members of the audience. Perhaps I do them an injustice: they may have been the more musical ones.

"Then the curtain went up. The scene was the village square at Doddering Sloughleigh, *circa* 1860. Enter the heroine, reading the postcards in the morning's mail. She comes across a letter addressed to the young squire and promptly bursts into song.

"Sarah's opening aria wasn't quite as bad as the overture, but it was grim enough. Luckily, we were to hear only the first few bars. . . .

"Precisely. We need not worry about such details as how Kendall had talked the ingenuous Fenton into it—if, indeed, the inventor realised the use to which his device was being applied. All I need say is that it was a most convincing demonstration. There was a sudden, deadening blanket of silence, and Sarah Stampe just faded out like a TV programme when the sound is turned off. Everyone was frozen in their seats, while the singer's lips went on moving silently. Then she too realised what had happened. Her mouth opened in what would have been a piercing scream in any other circumstances, and she fled into the wings amid a shower of postcards.

"Thereafter, the chaos was unbelievable. For a few minutes everyone must have thought they had lost the sense of hearing, but soon they were able to tell from the behaviour of their companions that they were not alone in their deprivation. Someone in the Physics Department must have realised the truth fairly promptly, for soon little slips of paper were circulating among the V.I.P.'s in the front row. The Vice-Chancellor was rash enough to try and restore order by sign-language, waving frantically to the audience from the stage. By this time I was too sick with laughter to appreciate such fine details.

"There was nothing for it but to get out of the hall, which we all did as quickly as we could. I think Kendall had fled—he was so overcome by the effect of the gadget that he didn't stop to switch it off. He was afraid of staying around in case he was caught and lynched. As for Fenton—alas, we shall never know *his* side of the story. We can only reconstruct the subsequent events from the evidence that was left.

"As I picture it, he must have waited until the hall was empty, and then crept in to disconnect his apparatus. We heard the explosion all over the college."

"The *explosion?*" someone gasped.

"Of course. I shudder to think what a narrow escape we all had. Another dozen decibels, a few more phons—and it might have happened while the theatre was still packed. Regard it, if you like, as an example of the inscrutable workings of providence that only the inventor was caught in the explosion. Perhaps it was as well: at least he perished in the moment of achievement, and before the Dean could get at him."

"Stop moralising, man. What happened?"

"Well, I told you that Fenton was very weak on theory. If he'd gone into the mathematics of the Silencer he'd have found his mistake. The trouble is, you see, that one can't *destroy* energy. Not even when you cancel out one train of waves by another. All that happens then is that the energy you've neutralized accumulates *somewhere else*. It's rather like sweeping up all the dirt in a room—at the cost of an unsightly pile under the carpet.

"When you look into the theory of the thing, you'll find that Fenton's gadget wasn't a silencer so much as a *collector* of sound. All the time it was switched on, it was really absorbing sound energy. And at that concert, it was certainly going flat out. You'll understand what I mean if you've ever looked at one of Edward England's scores. On top of that, of course, there was all the noise the audience was making—or I should say was *trying* to make—during the resultant panic. The total amount of energy must have been terrific, and the poor Silencer had to keep on sucking it up. Where did it go? Well, I don't know the circuit details—probably into the condensers of the power pack. By the time Fenton started to tinker with it again, it was like a loaded bomb. The sound of his approaching footsteps was the last straw, and the overloaded apparatus could stand no more. It blew up."

For a moment no-one said a word, perhaps as a token of respect for the late Mr. Fenton. Then Eric Maine, who for the last ten minutes had been muttering in the corner over his calculations, pushed his way through the ring of listeners. He held a sheet of paper thrust aggressively in front of him.

"Hey!" he said. "I was right all the time. The thing couldn't work. The phase and amplitude relations. . . ."

Purvis waved him away.

"That's just what I've explained," he said patiently. "You should have been listening. Too bad that Fenton found out the hard way."

He glanced at his watch. For some reason, he now seemed in a hurry to leave.

"My goodness! Time's getting on. One of these days, remind me to tell you about the extraordinary thing we saw through the new proton microscope. That's an even more remarkable story."

He was half way through the door before anyone else could challenge him. Then George Whitley recovered his breath.

"Look here," he said in a perplexed voice. "How is it that we never heard about this business?"

Purvis paused on the threshold, his pipe now burbling briskly as it got into its stride once more. He glanced back over his shoulder.

"There was only one thing to do," he replied. "We didn't want a scandal—*de mortuis nil nisi bonum,* you know. Besides, in the circumstances, don't you think it was highly appropriate to—ah—*hush* the whole business up? And a very good night to you all."

The invention of the steam engine was a major factor in the early stages of the Industricl Revolution.

2 The Steam Engine Comes of Age

R. J. Forbes and E. J. Dijksterhuis

1963

THE steam engine, coke, iron, and steel are the four principal factors contributing to the acceleration of technology called the Industrial Revolution, which some claim to have begun about 1750 but which did not really gain momentum until about 1830. It started in Great Britain but the movement gradually spread to the Continent and to North America during the nineteenth century.

SCIENCE INSPIRES THE ENGINEER

During the Age of Projec the engineer had little help from the scientists, who were building the mathematical-mechanical picture of the Newtonian world and discussing the laws of nature. However, during the eighteenth century, the Age of Reason, when the principles of this new science had been formulated, the scientists turned to the study of problems of detail many of which were of direct help to the engineer. The latter was perhaps less interested in the new ideals of 'progress' and 'citizenship of the world' than in the new theory of heat, in applied mechanics and the strength of materials, or in new mathematical tools for their calculations. The older universities like Oxford and Cambridge contributed little to this collaboration. The pace was set by the younger ones such as the universities of Edinburgh and Glasgow, which produced such men as Hume, Roebuck, Kerr, and Black, who stimulated the new technology. The Royal Society, and also new centres like the Lunar Society and the Manchester Philosophical Society and the many similar societies on the Continent, contributed much to this new technology by studying and discussing the latest scientific theories and the arts. Here noblemen, bankers, and merchants met to hear the scientist, the inventor, and the engineer and to help to realize many of the projects which the latter put forward. They devoted much money to scientific investigations, to demonstrations and stimulated inventions by offering prizes for practical solutions of burning problems. They had the capital to promote the 'progress' which made Dr Johnson cry out: 'This age is running mad after innovation. All business of the world is to be done in a new way, men are to be hanged in a new way; Tyburn itself is not safe from the fury of innovation!' New institutions such as the Conservatoire des Arts et Métiers and the Royal Institution of Great Britain were founded to spread the new science and technology by lectures and demonstrations and the number of laymen attending these lectures was overwhelming.

ENGINEERS AND SKILLED LABOUR

The new professional engineers which the École des Ponts et Chaussées began to turn out were the descendants of the sappers and military engineers. However, the new technology also needed other types of engineers for which new schools such as the École Polytechnique and the École des Mines were founded. In Great Britain the State was less concerned with the education of the new master craftsmen. They were trained in practice: such famous workshops as that of Boulton and Watt in Soho, Birmingham, or those of Dobson and Barlow, Asa Lees, and Richard Roberts. Their success depended not only on good instruction but also on appropriate instruments and skilled labour.

The scientists of the eighteenth century had turned out many new instruments which were of great value to the engineer. They were no longer made individually by the research scientist, but by professional instrument makers in Cassel, Nuremberg, or London, and such university towns as Leiden, Paris, and Edinburgh. Their instruments became more efficient and precise as better materials became available such as good glass for lenses and more accurate methods for working metals.

Skilled labour was more difficult to create. The older generation of Boulton and Watt had to work with craftsmen such as smiths and carpenters, they had to re-educate them and create a new type of craftsmen, 'skilled labour'. The design of early machinery often reveals that it was built by the older type of craftsmen that belonged to the last days of the guild system. The new industrialists tried out several systems of apprenticeship in their machine shops during the eighteenth century until they finally solved this educational problem during the next century and created schools and courses for workmen for the new industries, qualified to design and to make well-specified engines and machine parts.

A factor that contributed greatly to this development was the rise of the science of applied mechanics and the methods of testing materials. The theories and laws which such men as Palladio, Derand, Hooke, Bernoulli, Euler, Coulomb, and Perronet formulated may have been imperfect but they showed the way to estimate the strength of materials so important in the construction of machinery. 's Gravesande and Van Musschenbroek were the first to design and demonstrate various machines for measuring tensile, breaking, and bending strengths of various materials early in the eighteenth century. Such instruments were gradually improved by Gauthey, Rondelet, and others. The elastic behaviour of beams, the strength of arches, and many other problems depended on such tests. Some scientists developed tests for certain types of materials, for instance for timber (Buffon), stone (Gauthey), or metals (Réaumur). Such knowledge was of prime importance to the development of the steam engine and other machinery which came from the machine shops.

MACHINE SHOPS

The engineers who led this Industrial Revolution had to create both the tools and the new workmen. Watt, himself a trained instrument maker, had to invent several new tools and machines and to train his workmen in foundries and machine shops. Hence his notebooks are full of new ideas and machines. He invented the copying press. His ingenious contemporaries Maudsley and Bramah were equally productive. Joseph Bramah was responsible for our modern water closet (1778) and the first successful patent lock (1784) which no one succeeded in opening with a skeleton key before Hobbs (1851), who spent fifty-one hours of labour on it.

The difficulty in finding suitable labour arose from the fact that the new machines were no longer single pieces created by one smith, but that series of such machines were built from standard parts which demanded much greater precision in manufacturing such parts. The steam engine parts had to be finished accurately to prevent the steam escaping between metal surfaces which slid over each other, especially as steam pressures were gradually increased to make these machines more efficient. Hence the importance of the new tools and finishing processes, such as the lathe and drilling, cutting and finishing machinery.

In 1797 Henry Maudsley invented the screw-cutting lathe. Lathes originally belonged to the carpenter's shop. Even before the eighteenth century they had been used to turn soft metals such as tin and lead. These lathes were now moved by means of treadles instead of a bow, though Leonardo da Vinci had already designed lathes with interchangeable sets of gear wheels to regulate the speed of the lathe. Maudsley applied similar ideas and introduced the slide rest. Brunel, Roberts, Fox, Witworth, and others perfected the modern lathe, which permitted moving the object horizontally and vertically, adjustment by screws, and automatic switching off when the operation was completed. The older machine lathes were first moved by hand, then by a steam engine, and finally by electric motors. Now the mass production of screws, bolts, nuts, and other standard parts became possible and machines were no longer separate pieces of work. They were assembled from mass-produced parts.

The tools of the machine shop were greatly improved during the nineteenth century, pulleys, axles, and handles being perfected. The new turret or capstan lathe had a round or hexagonal block rotating about its axis and holding in a hole in each side the cutting or planing tool needed. These tools could then at will be brought into contact with the metal to be finished, thus performing the work of six separate lathes in a much shorter time. The turret block was made to turn automatically (1857) and finally Hartness invented the flat turret lathe, replacing the block by a horizontal face plate which gave the lathe greater flexibility and allowed work at higher speeds. Such lathes ranged from the small types used by the watchmaker to those for processing large guns. This development was completed by the introduction of high-speed tool steels by Taylor and White about the beginning of our century, making the machine lathe a universal tool for the mass production of machine parts.

FACTORIES AND INDUSTRIAL REVOLUTION

This brought about a great change in the manufacturing process itself. No longer were most commodities now made in the private shops of craftsmen, but in larger workshops in which a water wheel or a steam engine moved an axle from which smaller machinery derived its power by means of gear wheels or belts, each machine only partly processing the metal or material. Hence the manufacturing process was split up into a series of operations, each of which was performed by a special piece of machinery instead of being worked by hand by one craftsman who mastered all the operations.

The modern factory arose only slowly. Even in 1800 the word 'factory' still denoted a shop, a warehouse, or a depot; the eighteenth century always spoke of 'mills' in many of which the prime mover still was a horse mill or tread mill. The textile factory law of 1844 was the first to speak of 'factories'.

It is obvious that the new factories demanded a large outlay of capital. The incessant local wars had impoverished central Europe and Italy and industry did not flourish there, so many German inventors left their country to seek their fortune in western Europe. State control of the 'manufactures' in France had not been a success. The French government had not created a new class of skilled labour along with the new engineers, and Napoleon's 'self-supporting French industry' was doomed to be a failure when overseas trade was re-established after his fall. Neither the Low Countries nor Scandinavia had the necessary capital and raw materials needed for the Industrial Revolution.

Only in eighteenth-century England did such a fortunate combination of factors exist, a flourishing overseas trade, a well-developed banking system, raw materials in the form of coal and iron ores, free trade and an industry-minded middle class willing to undertake the risks of introducing new machinery and recruiting the new skilled labour from the ranks of the farmers and immigrants from Ireland and Scotland.

Hence we find the first signs of the Industrial Revolution in Great Britain rather than in France, which, however, soon followed suit. Competition from Germany did not start until the middle of the nineteenth century, and from the United States not until the beginning of our century.

THE BEAM ENGINES

The prime mover of this new industry was the steam engine. The primitive machine that pumped water was transformed into a prime mover by the efforts of Newcomen and Watt. Thomas Newcomen (1663–1729) and John Calley built a machine in which steam of 100° C moved a piston in its cylinder by condensation (1705). This piston was connected with the end of a beam; the other end of which was attached to the rod of the pump or any other machine. Most of these engines were used to drain mines. John Smeaton (1724–92) studied the Newcomen engine and perfected it by measurement and calculation, changing its boiler and valves and turning it into the most popular steam engine up to 1800.

James Watt (1736–1819), trained as an instrument maker, heard the lectures of John Robison and Joseph Black at Edinburgh, where the new theory of heat was expounded and methods were discussed to measure the degree and the amount of heat, as well as the phenomena of evaporation and condensation. He perceived that a large amount of heat was wasted in the cylinder of the Newcomen engine, heating it by injection of steam and cooling it by injecting cold water to condense the steam. Hence he designed an engine in which the condensation took place in a separate condenser, which was connected with the cylinder by opening a valve at the correct moment, when the steam had forced the piston up (1763).

Watt tried to have his engine built at John Roebuck's Carron Iron Works in Scotland but did not find the skilled workmen there to make the parts. So he moved southwards and started work at the works of Matthew Boulton, who built Roebuck's share in Watt's patents (1774). At the nearby Bradley foundry of John Wilkinson, cylinders could be bored accurately and thus Watt produced his first large-scale engine in 1781. The power output of the Watt engine proved to be four times that of a Newcomen engine. It was soon used extensively to pump water in brine works, breweries, and distilleries. Boulton and Murdock helped to advertise and apply Watt's engines.

THE DOUBLE-ACTING ROTATIVE ENGINE

However, Watt was not yet satisfied with these results. His Patent of 1781 turned the steam engine into a universally efficient prime mover. The rod on the other arm of the beam was made to turn the up-and-down movement of the beam into a rotative one, by means of the 'sun and planet movement' of a set of gear wheels connecting the rod attached to the end of the beam with the axle on which the driving wheels and belts were fixed which moved the machines deriving their energy from this axle.

A further patent of 1782 made his earlier engine into a double-acting one, that is a steam engine in which steam was admitted alternately on each side of the piston. This succeeded only when Boulton and Watt had mastered the difficult task of casting and finishing larger and more accurate cylinders. Watt also had to improve the connexion of the beam and the piston rod by means of his extended three-bar system (1784) which he called the 'parallel movement'. He was also able to introduce a regulator which cut off the steam supply to the cylinder at the right moment and leaving the rest of the stroke to the expansion of the steam made better use of its energy.

In 1788 he designed his centrifugal governor which regulated the steam supply according to the load keeping constant the number of strokes of the piston per minute. Six years later he added the steam gauge or indicator to his engine, a miniature cylinder and piston, connected with the main cylinder. The small piston of this indicator was attached to a pen which could be made to indicate on a piece of paper the movements of the little piston and thus provide a control on the movements of the steam engine proper. William Murdock (1754–1839), by inventing the sliding valves and the means of preparing a paste to seal off the seams between the cast iron surface of the machine parts, contributed much to the success of these engines as proper packing was not yet available.

By 1800 some 500 Boulton and Watt engines were in operation, 160 of which pumped water back on to water wheels moving machinery. The others were mostly rotative engines moving other machinery and twenty-four produced blast air for iron furnaces, their average strength being 15–16 h.p.

THE MODERN HIGH-PRESSURE STEAM ENGINE

The period 1800–50 saw the evolution of the steam engine to the front rank of prime movers. This was achieved by building steam engines which could be moved by high-pressure steam of high temperature containing much more energy per pound than the steam of 100° C which moved the earlier Watt engines. This was only possible by perfecting the manufacture of the parts of the steam engine, by better designing, and by the more accurate finishing and fit of such parts.

Jabez Carter Hornblower built the first 'compound engine', in which the steam released from the first cylinder was left to expand further in a second one. These compound engines did away with the Watt condenser, but could not yet compete seriously until high pressure steam was applied. Richard Trevithick and Oliver Evans were the pioneers of the high-pressure engine, which meant more horse power per unit of weight of the steam engine. This again meant lighter engines and the possibility of using them for road and water traffic.

Nor were properly designed steam engines possible until the theory of heat had been further elaborated and the science of thermodynamics formulated, the theory of gases studied, and more evidence produced for the strength of metals and materials at high temperatures. Another important problem was the construction of boilers to produce the high-pressure steam. The ancient beehive-shaped boilers of Watt's generation could not withstand such pressures. Trevithick created the Cornish boiler (1812), a horizontal cylinder heated by an inner tube carrying the combustion gases through the boiler into the flue and adding to the fuel efficiency of the boilers. The Lancashire boiler, designed by William Fairbairn (1844), had two tubes and became a serious competitor of the Cornish boiler. Better grates for burning the coal fuel were designed such as the 'travelling grate stoker' of John Bodmer (1841), and more fuel was economized by heating the cold feed water of the boiler with flue gases in Green's economizer (1845). Then multitubular boilers were built in the course of the nineteenth century, most of which were vertical boilers, the best known of which was the Babcock and Wilcox tubular boiler (1876).

Further factors helping to improve the design of high-pressure steam engines were the invention of the direct-action steam pump by Henry Worthington (1841), the steam hoist (1830), and James Nasmyth's steam hammer (1839). In the meantime Cartwright (1797) and Barton (1797) had perfected metallic packing which ensure tight joints and prevented serious leakage.

Thus steam pressures rose from 3·5 atm in 1810 to about 5 or 6 atm in 1830, but these early high-pressure engines were still of the beam type. Then came the much more efficient rotation engines in which the piston rod was connected with the driving wheel by means of a crank. Though even the early American Corliss engine (1849) still clung to the beam design, John M'Naught (1845) and E. Cowper (1857) introduced modern rotative forms, which came to stay. Three-cylinder engines of this type were introduced by Brotherhood (1871) and Kirk (1874) and became very popular prime movers for steamships (1881).

Not until 1850 was the average output of the steam engines some 40 h.p., that is significantly more than the 15 h.p. windmill or water-wheel of the period. Again the steam engine was not bound to sites where water or wind were constantly available, it was a mobile prime mover which could be installed where needed, for instance in iron works situated near coal fields and iron ores. In 1700 Great Britain consumed some 3,000,000 tons of coal, mostly to heat its inhabitants. This amount had doubled by 1800 because of the introduction of the steam engine, and by 1850 it has risen to 60,000,000 tons owing to the steam engine and the use of coke in metallurgy. . .

The principle of conservation of energy was proposed simultaneously by many physicists, including von Mayer, Joule, von Helmholtz, and Thomson. This popularization appeared soon after the discovery. The author is perhaps better known as Lord Kelvin.

3 Energy

William Thomson and Peter G. Tait

1862

ENERGY.

BY PROFESSORS WILLIAM THOMSON AND P. G. TAIT.

THE non-scientific reader who may take up this article in the expectation of finding an exhortation to manly sports, or a life of continual activity, with corresponding censure of every form of sloth and sensual indulgence, will probably be inclined to throw it down when he finds that it is devoted to a question of physical science. But let him not judge too hastily. Rigorous and minute scientific investigation is repulsive to all but a few, and these specially trained, minds; but the principle on which we are about to offer a few remarks admits of being made, at all events in its elements, thoroughly popular. General theories, whether of Politics, International Law, or, as in the present case, of Natural Philosophy, are, indeed, by their very generality, capable of being clearly apprehended through the widest circle of intelligent readers, if properly presented; while special questions, such as Church-rates, and the Ballot, the Rights of Neutral Bottoms, or the Temperature of Space, require to be explained to each individual in a manner, and with precautions, suited to his individual bias or defect of apprehension.

Of late several attempts have been made, with various success, to impart to the great mass of the interested but unscientific public an idea of the ONE GREAT LAW of Physical Science, known as the *Conservation of Energy*, and it is on account of the defects, or rather errors, with which most of these attempts abound, that we have aimed primarily at preparing an article, which shall be at all events *accurate*, as far as human knowledge at present reaches. As to its intelligibility we cannot of course decide. But we take the precaution of inserting, in the form of notes, portions of the article which, though of very great importance, could only be made intelligible to the general reader by elaborate and tedious explanations.

Every one knows by experience what Force is. Our ideas are generally founded on the sensation of the effort required, say, to press or to move some mass of matter. In general, Force is defined as *that which produces, or tends to produce, motion.* Now, if no motion be produced, the force which may have been exerted is absolutely lost. Hence the inconvenience and error of the phrase, "Conservation of Force," which is very commonly applied to our present subject. Among the host of errors which are due to confounding Force with Energy, one of the most extraordinary was some time ago enunciated in a popular magazine in some such form as this, "The sum-total of the Forces in the Universe is Zero"—a statement meaningless if it be applied to Force in its literal sense, and untrue if it refer to Energy. This is one example of the errors we have undertaken to combat; another refers more to the history than to results of the principle. We were certainly amazed to find in a recent number of another popular magazine, and in an article specially intended for popular information, that one great branch of our present subject, which we had been accustomed to associate with the great name of Davy, was in reality discovered so lately as twenty years ago by a German physician. Such catering for the instruction of the public requires careful looking after; and we therefore propose to place on a proper basis the history of the discovery, and to enumerate and illustrate some of the principal truths already acquired to the theory of the Conservation of Energy. To do this in a popular form we shall commence with an examination of some cases of every-day occurrence, and gradually introduce the scientific terms when we feel that we have clearly made out the ideas for which they stand. Once introduced, they will be used freely, not so much for brevity as for definiteness.

When an eight-day clock has been wound up, it is thereby enabled to go for a week in spite of friction and the resistance which the air at every instant offers to the pendulum. It has got what in scientific language we call a supply of *Energy*. In this case the energy simply consists in the fact of a mass of lead being suspended some four feet or so above the bottom of the clock-case. The mere fact of its being in that position gives it a power of "doing work" which it would not possess if lying on the ground. This is called *Potential Energy*. It will evidently be just so much the greater as the weight is greater, and as the height through which it can fall is greater. Its amount is, therefore, proportional to the product of the weight and the height it has to fall, because such a product is doubled, as the energy is, by doubling either factor. Thus a weight of one pound with an available descent of forty feet, has the same amount of potential energy as ten pounds at four feet, eight pounds at five feet, or forty pounds at one foot. And we may easily see that the work required to lift the weight to its present position will be the same in all these cases, if we take for example such an illustration as the lifting of coals from a pit. Twice as much work is done (even in the popular signification of the phrase, "doing work") when two tons are raised as when one only has been so; and to raise a ton through forty fathoms requires twice as much work as to raise it through twenty. Hence it appears that work expended by an animal or machine in lifting a weight remains stored as potential energy in the relation between the earth and the lifted weight, and thus that energy so spent is not lost. But suppose the coals to be allowed to tumble down the pit again, what becomes of the energy? This question will give us an idea of the nature of the subject we are dealing with.

We have already adverted to the serious errors into which we are liable to fall from an incorrect use of the word *force*, but we may with advantage recur to the subject here. What becomes of the enormous force with which the earth continually attracts a mountain, or that with which the sun attracts the earth? Force is continually exercised in each of these cases, yet no progressive effect is produced on the mountain; and the changes which the velocity of the earth in its orbit undergoes, are, in the course of a year, as much in the way of *loss* as of *gain*. We do no work, however much we may fatigue ourselves, if we try to lift a ton from the ground. If we try to lift a hundredweight, we can raise it a few feet, and have then done work, and the work is expressible as so many pounds raised so many feet, and can therefore be stated as so many *foot-pounds*, each foot-pound being the work required to raise a pound weight a foot high. The true statement which meets all such cases is, *Energy is never lost*. But we must now return to our first illustration, to see how energy may be modified or transformed, and then we shall begin to understand how it is that no modification or transformation ever causes loss of energy.

There are two ways of raising a weight to a height: by a continuous application of force, as by a windlass, or by an almost instantaneous impulse, such as a blow from a cricket-bat, or the action of gunpowder. A 64 lb. shot, fired vertically from a gun loaded with an ordinary service charge of powder, would, if unresisted by the air, rise to about 35,000 feet, and if seized and secured at the highest point of its course, would possess there, in virtue of its position, a potential energy of 2,240,000 foot-pounds. When it left the gun it had none of this, but *it was moving at the rate of fifteen hundred feet per second*. It had KINETIC or (as it has sometimes been called) actual energy. We prefer the first term, which indicates motion as the form in which the energy is displayed. Kinetic energy depends on motion; and observation shows that its amount in each case is calculable from the mass which moves and the velocity with which it moves. And this being understood, it is easy, by considering a very simple case, to find how it so depends. For, if a stone be thrown up with a velocity of 32 feet per second, it will rise to a height of nearly 16 feet; if thrown with *double* velocity, or 64 feet per second, it will rise *four* times as high, or to about 63½ feet; if the velocity be *trebled*, it rises *nine* times as high, or to 143 feet, and so on. Hence, as we must measure the energy of a moving body by the height to which it will rise if its motion is directed vertically upwards, we find that we have to measure it by the *square* of the velocity. The recent tremendous performances of the 12-ton Armstrong gun form an admirable illustration of the same point, showing, as they do, that to penetrate a thick plate of iron mere *weight* of shot is comparatively unavailing—it must have great velocity; and in fact, with double the velocity we get at once four times the penetrating or destructive power. By such facts as these, we are led to measure kinetic energy by the square of the velocity with which a body moves. And there is particular advantage in taking as the exact expression, *one-half of the product of the moving mass and the square of its velocity in feet per second*, because this makes the unit of measurement agree with that adopted for potential energy. We may then express the relation between the forms of energy, in the case of a projectile unresisted by the air, by saying, *the sum of the potential and kinetic energies does not vary* during its flight. As it rises it gains potential energy, but its motion is slower, and thus kinetic energy is lost;—as it descends it continually loses potential energy, but gains velocity, and, therefore, kinetic energy. But what happens when it reaches the ground and comes to rest? Here it would appear to lose both its potential and kinetic energies. The first, indeed, is all gone just as the mass reaches the ground. To a superficial observer, the second might seem to be expended in bruising and displacing the bodies on which it impinges. But there is something more profound than this, as we shall presently see.

Meanwhile, as popular examples of the two kinds of energy, we may give such illustrations as a coiled spring, say the hair-spring of a watch when the balance-wheel is at one end of its range, a drawn bow, a head of water, compressed air,—all forms of potential energy; and the corresponding kinetic form in each case—the motion of the balance-wheel of the watch, the motion of an arrow, a jet of water, an air-gun bullet, and so on. But we need not dwell longer on this, as such matters abound in every-day experience.

To recur to the more mysterious transformations of energy, let us consider, as an excellent example, the case of motion of water in a basin. By stirring the water, originally at rest, we can easily give it a considerable velocity of rotation, in virtue of which it will, of course, possess considerable kinetic energy. Moreover, the level is disturbed; the water rises from the middle to the sides of the basin, and, in virtue of this, the centre of gravity of the whole is higher than when the water was at rest. It thus possesses potential energy also. If the stirring be discontinued, all visible motion ceases after a few minutes, and, the surface becoming level, the potential energy is lost. It seems as if the kinetic energy also is all lost in the ceasing of the visible motion. What remains in their place? Apparently the water has returned precisely to the state in which it was before the stirring commenced, and the work done in stirring has been thrown away. But this is not the case: the water is *warmer* than before the stirring, and warmer than during the time when it was moving. The energy which apparently disappeared really exists as *Heat*. We might multiply examples of this kind indefinitely, and in all we should be led to the inevitable question, *What becomes of energy apparently lost?* The answer is, *It ultimately becomes heat.* We say *ultimately*, because, as will afterwards be shown, energy apparently lost may take in succession various forms, all of which, however, finally become heat. Sensible heat is, in fact, *motion*, and is therefore a form of kinetic energy. This was surmised at least two centuries ago, for we find it stated with remarkable clearness in the writings of Locke and others. But it remained a conjecture, unsupported by scientific evidence, until the proof was furnished by Davy. The simple experiment of melting two pieces of ice by rubbing them together showed at once the impossibility of heat being a substance. But it is not to be imagined that for all this the pleasant fiction called *Caloric* was to be abandoned; and consequently, for upwards of forty years after Davy's proof of its non-existence, caloric was believed in, written about, and taught, all over the world.*

About the time of Davy's experiments, Rumford also was engaged on the subject, and by measuring the heat developed in boring a cannon, arrived at a very approximate answer to the question, "How much heat can be produced by the expenditure of so much work?" or, in other words, and with the modern phraseology, "*What is the Dynamical Equivalent of Heat?*"

The founder of the modern dynamical theory of heat, an extension immensely beyond anything previously surmised, is undoubtedly Joule. As early as 1840 we find him investigating the heat generated by electric currents, and in 1841 he published researches which contain the germ of the vast developments of dynamical science as applied to chemical action. In 1843 he published the results of a well planned and executed series of experiments, by which he ascertained that a pound of water is raised one degree Fahrenheit in temperature by 772 foot-pounds of mechanical work done upon it. In other words, if a pound of water fall from a height of 772 feet, and the kinetic energy thus acquired in the form of ordinary motion be entirely transformed into the kinetic energy of heat, the water will be one degree hotter than before its fall. Of course it is not in this way that the experiments of Joule were made, but it gives perhaps as clear an idea of his result as any other. The actual method which he first employed was to force water through small tubes. In later researches he arrived at the same numerical result (within [illegible] of difference), by stirring water by means of a paddle-wheel, driven by the descent of a weight. The number of foot-pounds of potential energy lost by the descending weight of course gave the value of the kinetic energy imparted to the water, and when the latter came to apparent rest, the heat produced was therefore the equivalent of either. These experiments, of course, required extreme precautions to prevent or to allow for loss of heat, etc.; but they agreed so well with each other in very varied experiments, that the definite transformation of work into heat was completely established, and the "dynamical equivalent of heat" determined with great accuracy. Various other methods of effecting the transformation of work into heat were also tried by Joule, and with a like result; such as using oil, or mercury, instead of water, in the paddle-wheel experiment; or, again, expending work in producing heat by friction of pieces of iron; or by turning a magneto-electric machine, and measuring the heat generated by the electric current so produced, etc.*

We can now see that when mechanical energy is commonly said to be lost, as by unavoidable friction in machinery, it is really only *changed* into a new form of energy—heat. Thus the savage who lights his fire by rubbing together pieces of dry wood, expends his muscular energy in pro-

* No one who knows the present state of science can ignore the fact that many of its most certain truths are still misunderstood, and their very opposites often taught, even by men who from their position or their notoriety are supposed by the public to be among the best informed.

* At the same time Joule published the full proof of the existence of relations of equivalence among the energies of chemical affinity, heat of combination or combustion, electrical currents from a galvanic battery or from a magneto-electric machine, engines worked by galvanism, and of all the varied and interchangeable manifestations of thermal action and mechanical energy which accompany them. These researches, and others (which soon followed) on the theory of animal heat and motion in relation to the heat of combustion of the food consumed, and the theory of the phenomena presented by shooting stars, which this naturalist based on true dynamical principles, have afforded to subsequent writers the chief groundwork for their speculations on the dynamical theory of heat.

ducing heat. By mere hammering, a skilful smith can heat a piece of iron to redness. In the old musket, the potential energy of the spring of the lock became, when the trigger was drawn, kinetic energy of the dog-head, and the latter was partly expended in generating the heat which ignited the steel sparks which inflamed the powder. Some of it may have been wasted in splitting the flint, and some in scratching the lid of the pan, some (as we shall see presently) certainly was wasted in producing the sound called the click of the lock.

Curiously enough, although similar coincidences are common, while Joule was pursuing and publishing his investigations, there appeared in Germany a paper by Mayer of Heilbronn. Its title is *Bemerkungen über die Kräfte der Unbelebten Natur*, and its date 1842. In this paper the results obtained by preceding naturalists are stated with precision—among them the fundamental one of Davy—new experiments are suggested, and a method for finding the dynamical equivalent of heat is propounded.* On the strength of this publication an attempt has been made to claim for Mayer the credit of being the first to establish in its generality the principle of the Conservation of Energy. It is true that "*La science n'a pas de patrie*," and it is highly creditable to British philosophers, that they have so liberally acted according to this maxim. But it is not to be imagined that on this account there should be no scientific patriotism, or that in our desire to do all justice to a foreigner, we should depreciate or suppress the claims of our own countrymen. And it especially startles us that the recent attempts to place Mayer in a position which he never claimed, and which had long before been taken by another, should have found support within the very walls wherein Davy propounded his transcendent discoveries.

Having thus considered the transformation of mechanical energy into heat, we must next deal with the converse process, or the production of mechanical energy from heat; a process to which the steam-engine owes its vast powers. But here we have no such general theorem as in the former case. Mechanical energy can always be changed into heat, but to obtain mechanical energy from heat it is necessary that we should have bodies of different temperatures; so that if all the matter in the universe were at one temperature it would be impossible, however great were that temperature, to convert any heat into work. This is a most important fact, because, as we shall presently see, it leads to the conclusion not that the energy in the universe can ever vary in amount, but that it is gradually becoming uniformly diffused heat, from which it can never afterwards be changed. However, granting that bodies of different temperatures are still procurable, heat in passing from the warmer to the colder body may (in part at least) be transformed into some other form of energy; and in the case of the steam-engine, that form is the mechanical effect produced by the expansion of water into vapour by heat; so that if the whole of the heat expended could be obtained as "work," we should have 772 foot-pounds for every portion of applied heat which was capable of raising the temperature of a pound of water through one degree of Fahrenheit's scale. In the best steam-engines, even with every modern improvement, only about one-tenth is actually so recovered. All such cases come under the following general proposition:—*When an engine does work in virtue of heat supplied to it, it emits heat from some part necessarily cooler than that where the heat is taken in; but the quantity so emitted is less than the quantity taken in, by an amount equivalent to the work done.* This is universally true, not only for artificial contrivances, such as the steam-engine, Stirling's air-engine, thermo-electric engines, etc., but for every action of dead matter in which the bodies concerned, if altered by change of temperature, of volume, of form, or of electric, magnetic, or chemical condition, are finally restored to their primitive state.

But whence do we get the heat which gives motion to the steam-engine, or, in other words, what was its potential form before it became heat? Here we answer at once, just as a stone falling to the earth changes its potential energy for kinetic, and finally for heat; so coal and the oxygen of the air, by virtue of their chemical affinity, have potential energy when uncombined, which is changed into its equivalent in heat as the combination takes place. Chemical affinity, then, is a form of potential, heat of combination or combustion the equivalent form of kinetic, energy. The heat thus obtained may be by various means, as the steam-engine or the air-engine, converted into mechanical energy. Or the combination may take place, as Joule has shown in one of his finest discoveries,

* Mayer's method is founded on the supposition that diminution of the volume of a body implies an evolution or generation of heat; and it involves essentially a false analogy between the natural fall of a body to the earth, and the condensation produced in an elastic fluid by the application of external force. The hypothesis on which he thus grounds a definite numerical estimate of the relation between the agencies here involved, is that the heat evolved when an elastic fluid is compressed and kept cool, is simply the dynamical equivalent of the work employed in compressing it. The experimental investigations of subsequent naturalists have shown that this hypothesis is altogether false, for the generality of fluids, especially liquids, and is at best only *approximately* true for air; whereas Mayer's statements imply its indiscriminate application to all bodies in nature, whether gaseous, liquid, or solid, and show no reason for choosing air for the application of the supposed principle to calculation, but that at the time he wrote air was the only body for which the requisite numerical data were known with any approximation to accuracy.

without generating its full equivalent of heat, and may be directed to spend a large part of its energy in producing electric currents, and through them raising weights. This is the case when zinc combines with oxygen in a galvanic battery. The heat of combination may then appear in the warming of wires through which the current passes. Or it may not appear at all, except in very small proportions, and an equivalent of mechanical work done may be had instead; as Joule and Scoresby found when, by their skilled appliance of mechanical and magnetic means, they prevented the chemical action from generating more than one-fourth of its heat, and got the remainder of its energy in the form of weights raised.

A remarkable result of electric development of the energy of chemical combination is, that through it the heat-equivalent may be made to appear at any time however long after, or in any place however distant from, the combustion. Thus if the weights raised by an electro-magnetic engine driven by a galvanic battery are allowed to fall, sooner or later they will generate in striking the ground the complement of heat till then wanting from the heat of combination of the "chemicals" which had been used. Or if a well insulated electric conductor were laid where the old Atlantic cable lies useless (for no other reason than that its insulation was never free from faults), the zinc fire might burn coolly at Valencia, and develop nine-tenths of its heat, or an equivalent of energy in mechanical work, in Newfoundland, wasting the remainder almost solely in the generation of heat by electricity escaping through the 2000 miles of gutta-percha cover. In every electric telegraph a portion (it may be and generally is only a small portion) of the energy of combination of oxygen, with the zinc if a battery is used, or with the operator's food if the magneto-inductive system is followed, actually appears first at the remote end of the wire as visible motions. Ultimately, through resistances to these motions, or subsidence of the sounds produced by the impacts of the needles, after they have told their tale, it becomes heat and is dissipated through space.

Many observations render it probable that an animal doing mechanical work does not allow the chemical combinations which go on between its food, more or less assimilated, and the oxygen inhaled in its breath, or otherwise introduced into its system directly or indirectly from our atmosphere, to generate their full equivalent of heat in its body as when resting, but directs a portion of their energy to be spent immediately in the muscular effort of pressing against external force. If this were the case, it would follow that dynamically the animal-engine is more like the electro-magnetic machine driven by the electric current from a galvanic battery, than a steam-engine or air-engine, which takes in all its energy in the form of heat from a fire. It seems even probable that it is actually through electric force that the energy of the food is placed at the disposal of that most inscrutable of finite, created, and subject agencies, a free will directing the motions of matter in a living animal. But whatever may be the true explanation of the means, it is, as regards the result, singularly noteworthy that the construction of the animal frame enables it to convert more of a given amount of potential energy into work than is procurable from the most perfect steam-engine.

The food of animals is, as we have just seen, by virtue of its chemical composition, and affinity (a true "attraction") for oxygen, a store of potential energy. Gunpowder or gun-cotton, by the arrangement of its constituents, is possessed of tremendous potential energy, which a single spark resolves into a kinetic form as heat, sound, and the kinetic energy of a cannon shŏt. For sound is a motion of air, air is matter, and thus sound is merely a form of kinetic energy. In a bayonet charge, then, the soldier's rations are the potential energy of war; in a cavalry charge, we have in addition that of the forage supplied to the horses; and when artillery or small arms are used, the potential energy of a mixture of nitre, sulphur, and charcoal is the tranquil antecedent of the terrible kinetic effects of noise and destruction.

But we now come to the grandest question of all, or at all events to a preliminary stage of it. Whence do we immediately derive all those stores of potential energy which we employ as fuel or as food? What produces the potential energy of a loaf or a beef-steak? What supplies the coal or the water-power, without which our factories must stop? The answer, going one stage back, is quite satisfactory. To the Sun we are indebted for water-power, coal, and animal and vegetable food. The sun's heat raises the water of seas and lakes as vapour in the air, to be precipitated as rain above its original level, and thus to form the store of potential energy known as a "head" of water. Kinetic energy, radiated from the sun, enables plants to separate carbon from oxygen, and so to become stores of potential energy which, as coal or vegetable food, may have been treasured for ages in the earth, or may be consumed annually as they are produced. And while the sheep and ox convert part of the potential energy of their grass or turnips into animal heat and energy, the rest, stored up as the potential energy of beef and mutton, becomes in its turn a source of human energy.

Now, to go yet a step back. Whence does the sun procure the energy which he thus so continually and so liberally distributes? To this question several answers have been given, one of which may be disposed of at once, and another will be found merely to shift, not to resolve the difficulty. The first of these supposes the sun to be the site of a great combustion or production of kinetic from potential energy by chemical combination. But it has been shown that, even supposing the mass of the sun to be made up (in the most effective proportions) of the combination of known bodies which would give the greatest potential energy, the whole could scarcely be adequate to produce 5000 years' radiation at the present rate; whereas there is abundant geological proof that the present state of things, if not a higher rate of distribution of energy from the sun, must have lasted already many hundreds of thousands of years. The second supposes the sun to be a white-hot liquid mass, but does not account for its heat. A third allows that the sun, all round his surface, if not throughout his mass, is most probably composed of melted matter, of a temperature not very many times greater than can actually be produced in our laboratories, but accounts for the original production, and the present maintenance of that state in spite of losses through radiation, by what is called the meteoric theory. A fourth, which is probably the true explanation, agrees with the third as to the origin of the sun's heat, but supposes the loss by radiation at present not to be compensated by fresh influx of meteoric matter. According to this theory, matter, when created, was diffused irregularly through infinite space, but was endowed with the attractive force of gravitation, by virtue of which it gradually became agglomerated into masses of various sizes, and retaining various amounts of kinetic energy in the shape of actual motion, which still appear in the orbital and axial revolutions, not only of the bodies composing the solar system, but of those in stellar systems also. The temperature produced by collisions, etc., would not only be in general higher for the larger bodies, but they would, of course, take longer to cool; and hence, our earth, though probably in bygone ages a little sun, remains but a slight amount of its original heat, at least in its superficial strata, while the sun still shines with brilliance perhaps little impaired. Supplies of energy are, no doubt, yet received continually by the sun, on its casual meeting with masses traversing space, or the falling in of others revolving about it, just as, on an exceedingly small scale, the earth occasionally gets a slight increase of kinetic energy by the impact of a shooting star or aerolite. In this sense it is easily calculable that the direct fall of the earth to the sun would supply the latter with energy equivalent to ninety-five years' loss at the present rate. But it is not probable that the sun receives in this way more than a very small proportion of the heat which he emits by radiation. He must therefore at present be in the condition of a heated body cooling. But being certainly liquid for a great depth all round his surface, if not throughout, the superficial parts must sink by becoming heavier as they contract through cooling. The currents thus produced, bringing fresh portions from below to the surface, and keeping all the liquid thoroughly stirred up, must distribute the loss of heat very equably throughout the whole liquid mass, and so prevent the surface from cooling quickly, as it certainly would do if the superficial stratum were solid. So vast is the capacity of such a mass for heat, when under the influence of the enormous pressure produced in the interior by mutual gravitation of the parts, that if the sun is liquid to his centre, he may emit, as it has been estimated, from seven to seven thousand years' heat at the present rate before his average temperature can go down by one degree Fahrenheit.

This view of the possible origin of energy at creation is excessively instructive. Created simply as difference of position of attracting masses, the potential energy of gravitation was the original form of all the energy in the universe; and as we have seen that all energy tends ultimately to become heat, which cannot be transformed without a new creative act into any other modification, we must conclude that when all the chemical and gravitation energies of the universe have taken their final kinetic form, the result will be an arrangement of matter possessing no realizable potential energy, but uniformly hot—an undistinguishable mixture of all that is now definite and separate—chaos and darkness as "*in the beginning*." But before this consummation can be attained, in the matter of our solar system, there must be tremendous throes and convulsions, destroying every now existing form. As surely as the weights of a clock run down to their lowest position, from which they can never rise again, unless fresh energy is communicated to them from some source not yet exhausted, so surely must planet after planet creep in, age by age, towards the sun. When each comes within a few hundred thousand miles of his surface, if he is still incandescent, it must be melted and driven into vapour by radiant heat. Nor, if he has crusted over and become dark and cool externally, can the doomed planet escape its fiery end. If it does not become incandescent like a shooting-star by friction in its passage through his atmosphere, its first graze on his solid surface must produce a stupendous flash of light and heat. It may be at once, or it may be after two or three bounds, like a cannon-shot ricochetting on a surface of earth or water, the whole mass must be crushed, melted, and evaporated by a crash generating in a moment some thousands of times as much heat as a coal of the same size could produce by burning.

Thus we have the sober scientific certainty that heavens and earth shall "wax old as doth a garment;" and that this slow progress must gradually, by natural agencies which we see going on under fixed laws, bring about circumstances in which "the elements shall melt with fervent heat." With such views forced upon us by the contemplation of dynamical energy and its laws of transformation in dead matter, dark indeed would be the prospects of the human race if unillumined by that light which reveals "new heavens and a new earth."

We have not made in the foregoing pages any but the slightest allusions to the remaining known forms of energy, such as light, electric motion, etc. Nor have we examined into the nature and effects of the so-called vital force. All that we need at present say of them is, that, as far as experiment has yet taught us, nothing known with regard to them can modify the preceding conclusions. For, as we may show in a future paper, light, electric motion, and all other forms of energy, ultimately become heat, and, therefore, though the progress of energy through these various stages may modify the course of events, it cannot in the least affect their inevitable termination.

A physicist and educator here tells a parable to illustrate the inadequacies he sees in the present system of teaching.

4 The Barometer Story

Alexander Calandra

1964

SOME time ago, I received a call from a colleague who asked if I would be the referee on the grading of an examination question. It seemed that he was about to give a student a zero for his answer to a physics question, while the student claimed he should receive a perfect score and would do so if the system were not set up against the student. The instructor and the student agreed to submit this to an impartial arbiter, and I was selected.

The Barometer Problem

I went to my colleague's office and read the examination question, which was, "Show how it is possible to determine the height of a tall building with the aid of a barometer."

The student's answer was, "Take the barometer to the top of the building, attach a long rope to it, lower the barometer to the street, and then bring it up, measuring the length of the rope. The length of the rope is the height of the building."

Now, this is a very interesting answer, but should the student get credit for it? I pointed out that the student really had a strong case for full credit, since he had answered the question completely and correctly. On the other hand, if full credit were given, it could well contribute to a high grade for the student in his physics course. A high grade is supposed to certify that the student knows some physics, but the answer to the question did not confirm this. With this in mind, I suggested that the student have another try at answering the question. I was not surprised that my colleague agreed to this, but I was surprised that the student did.

Acting in terms of the agreement, I gave the student six minutes to answer the question, with the warning that the answer should show some knowledge of physics. At the end of five minutes, he had not written anything. I asked if he wished to give up, since I had another class to take care of, but he said no, he was not giving up. He had many answers to this problem; he was just thinking of the best one. I excused myself for interrupting him, and asked him to please go on. In the next minute, he dashed off his answer, which was:

"Take the barometer to the top of the building and lean over the edge of the roof. Drop the barometer, timing its fall with a stopwatch. Then, using the formula $S = \frac{1}{2} at^2$, calculate the height of the building."

At this point, I asked my colleague if he would give up. He conceded and I gave the student almost full credit. In leaving my colleague's office, I recalled that the student had said he had other answers to the problem, so I asked him what they were. "Oh, yes," said the student. "There are many ways of getting the height of a tall building with the aid of a barometer. For example, you could take the barometer out on a sunny day and measure the height of the barometer, the length of its shadow, and the length of the shadow of the building, and by the use of simple proportion, determine the height of the building."

"Fine," I said. "And the others?"

"Yes," said the student. "There is a very basic measurement method that you will like. In this method, you take the barometer and begin to walk up the stairs. As you climb the stairs, you mark off the length of the barometer along the wall. You then count the number of marks, and this will give you the height of the building in barometer units. A very direct method.

"Of course, if you want a more sophisticated method, you can tie the barometer to the end of a string, swing it as a pendulum, and determine the value of 'g' at the street level and at the top of the building. From the difference between the two values of 'g,' the height of the building can, in principle, be calculated."

Finally he concluded, "If you don't limit me to physics solutions to this problem, there are many other answers, such as taking the barometer to the basement and knocking on the superintendent's door. When the superintendent answers, you speak to him as follows: 'Dear Mr. Superintendent, here I have a very fine barometer. If you will tell me the height of this building, I will give you this barometer.'"

At this point, I asked the student if he really didn't know the answer to the problem. He admitted that he did, but that he was so fed up with college instructors trying to teach him how to think and to use critical thinking, instead of showing him the structure of the subject matter, that he decided to take off on what he regarded mostly as a sham.

The kinetic theory of gases is a marvelous structure of interconnecting assumption, prediction, and experiment. This chapter supplements and reinforces the discussion of kinetic theory in the text of Unit 3.

5 The Great Molecular Theory of Gases

Eric M. Rogers

1960

Newton's theory of universal gravitation was a world-wide success. His book, the *Principia*, ran into three editions in his lifetime and popular studies of it were the fashion in the courts of Europe. Voltaire wrote an exposition of the *Principia* for the general reader; books were even published on "Newton's Theory expounded to Ladies." Newton's theory impressed educated people not only as a brilliant ordering of celestial Nature but as a model for other grand explanations yet to come. We consider Newton's theory a *good* one because it is simple and productive and links together many different phenomena, giving a general feeling of understanding. The theory is simple because its basic assumptions are a few clear statements. This simplicity is not spoiled by the fact that some of the deductions need difficult mathematics. The success of Newton's planetary theory led to attempts at more theories similarly based on the laws of motion. For example, gases seem simple in behavior. Could not some theory of gases be constructed, to account for Boyle's Law by "predicting" it, and to make other predictions and increase our general understanding?

Such attempts led to a great molecular theory of gases. As in most great inventions the essential discovery is a single idea which seems simple enough once it is thought of: the idea that gas pressure is due to bombardment by tiny moving particles, the "molecules" of gas. Gases have simple common properties. They always fill their container and exert a uniform pressure all over its top, bottom, and sides, unlike solids and liquids. At constant temperature, PRESSURE · VOLUME remains constant, however the gas is compressed or expanded. Heating a gas increases its pressure or volume or both—and the rate of increase with temperature is the same for all gases ("Charles' Law"). Gases move easily, diffuse among each other and seep through porous walls.

Could these properties be "explained" in terms of some mechanical picture? Newton's contemporaries revived the Greek philosophers' idea of matter being made of "fiery atoms" in constant motion. Now, with a good system of mechanics they could treat such a picture realistically and ask what "atoms" would do. The most striking general property that a theory should explain was Boyle's Law.

Boyle's Law

In 1661 Boyle announced his discovery, "not without delight and satisfaction" that the pressures and volumes of air are "in reciprocal proportions." That was his way of saying: PRESSURE $\propto$ 1/VOLUME or PRESSURE · VOLUME remains constant, when air is compressed. It was well known that air expands when heated, so the restriction "at constant temperature" was obviously necessary for this simple law. This was Boyle's discovery of the "spring of the air"—a spring of variable strength compared with solid Hooke's Law springs.

In laboratory you should try a "Boyle's-Law experiment" with a sample of dry air, not to "discover" a law that you already know, but as a problem in precision, "your skill against nature." You

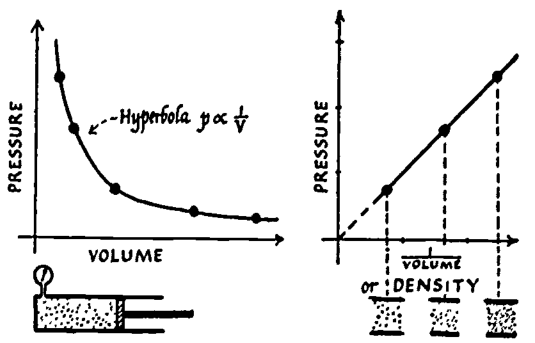


Fig. 25-1. Boyle's Law

will be limited to a small range of pressures (say ½ atmosphere to 2 atm.) and your accuracy may be sabotaged by the room temperature changing or by a slight taper in the glass tube that contains the sample.[1] If you plot your measurements on a graph showing PRESSURE *vs.* VOLUME you will find they mark a hyperbola—but that is too difficult a curve to recognize for sure and claim as verification of Boyle's Law.[2] Then plot PRESSURE *vs.* 1/VOLUME and look for a straight line through the origin.

Boyle's measurements were fairly rough and extended only from a fraction of an atmosphere to about 4 atm. If you make precise measurements with air you will find that pV changes by only a few tenths of 1% at most, over that range. Your graph of p *vs.* $1/V$ will show your experimental points very close to a straight line through the origin. Since MASS/VOLUME is density and MASS is constant, values of $1/V$ represent DENSITY, and Boyle's Law says

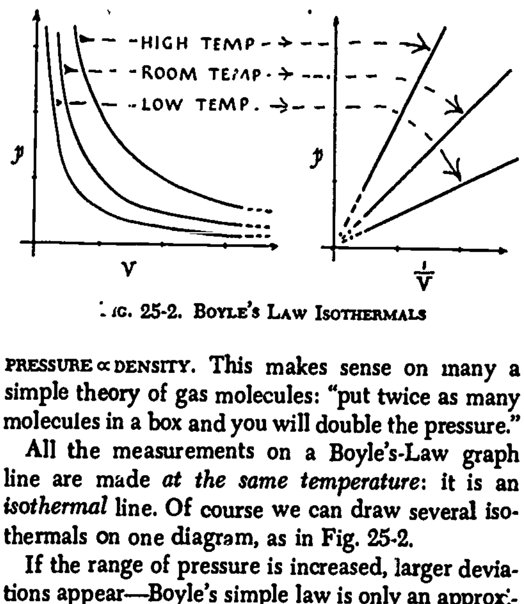


FIG. 25-2. BOYLE'S LAW ISOTHERMALS

PRESSURE $\propto$ DENSITY. This makes sense on many a simple theory of gas molecules: "put twice as many molecules in a box and you will double the pressure."

All the measurements on a Boyle's-Law graph line are made *at the same temperature*: it is an *isothermal* line. Of course we can draw several isothermals on one diagram, as in Fig. 25-2.

If the range of pressure is increased, larger deviations appear—Boyle's simple law is only an approximate account of real gas behavior. It fits well at low pressures but not at high pressures when the sample is crowded to high density. Fig. 25-3 shows the experimental facts for larger pressures, up to 3000 atmospheres. (For graphs of CO_2's behavior, including liquefaction, see Ch. 30.)

Theory

Boyle tried to guess at a mechanism underlying his experimental law. As a good chemist, he pictured tiny atomic particles as the responsible agents. He suggested that gas particles might be springy, like little balls of curly wool piled together, resisting compression. Newton placed gas particles farther apart, and calculated a law of repulsion-force to account for Boyle's Law. D. Bernoulli published a bombardment theory, without special force-laws, that predicted Boyle's Law. He pointed out that *moving* particles would produce pressure by bombarding the container; and he suggested that heating air must make its particles move faster. This was the real beginning of our present theory. He made a brave attempt, but his account was incomplete. A century later, in the 1840's, Joule and others set forth a successful "kinetic theory of gases," on this simple basic view:

> *A gas consists of small elastic particles in rapid motion: and the pressure on the walls is simply the effect of bombardment.*

Joule showed that this would "explain" Boyle's Law, and that it would yield important information about the gas particles themselves. This was soon polished by mathematicians and physicists into a large, powerful theory, capable of enriching our understanding.

In modern theories, we call the moving particles *molecules*, a name borrowed from chemistry, where it means the smallest particle of a substance that exists freely. Split a molecule and you have separate atoms, which may have quite different properties from the original substance. A molecule of water, H_2O, split into atoms yields two hydrogen atoms and one oxygen atom, quite different from the particles or molecules of water. Left alone, these separated atoms gang up in pairs, H_2, O_2—molecules of hydrogen and oxygen gas. In kinetic theory, we deal with the complete molecules, and assume they are not broken up by collisions. And we assume the molecules exert no forces on each other except during collisions; and then, when they are very close, they exert strong repulsive forces for a very short time: in fact that is all a collision *is*.

You yourself have the necessary tools for constructing a molecular theory of gases. Try it. Assume

[1] Even modern glass tubing is slightly tapered, unless made uniform by an expensive process; so when experiments "to verify Boyle's Law" show deviations from pV = constant they are usually exhibiting tube-taper rather than misbehavior of air. If the air sample is replaced by certain other gases such as CO_2, or by some organic vapor, real deviations from Boyle's Law become obvious and interesting. See Ch. 30.

[2] The only safe shapes of graphs for testing a law, or finding one, are straight lines and circles.

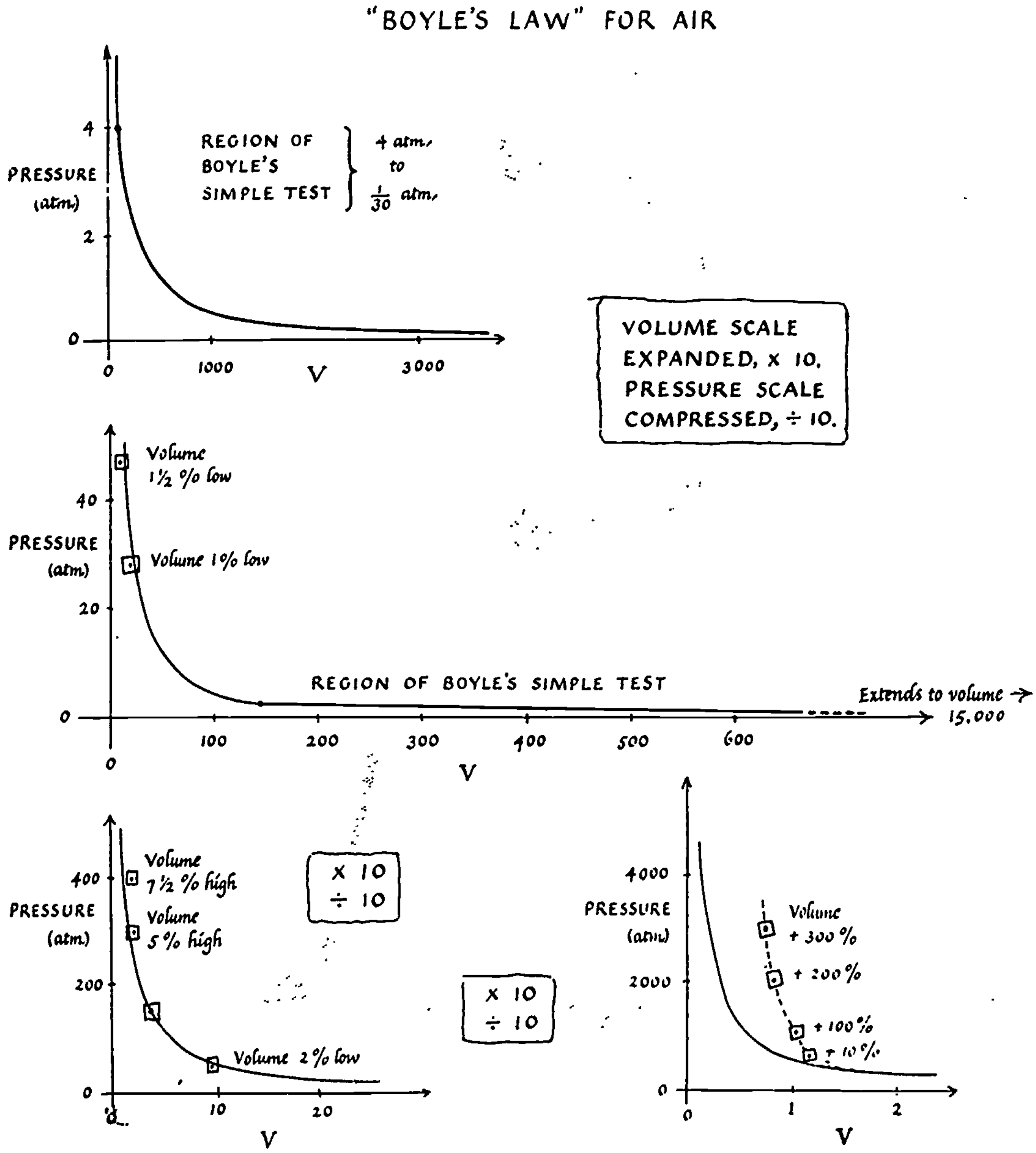


FIG. 25-3. DEVIATIONS FROM BOYLE'S LAW FOR AIR AT ROOM TEMPERATURE
The curve shows the PRESSURE:VOLUME relationship for an ideal gas obeying Boyle's Law.
The points show the behavior of air, indistinguishable from the curve at low pressures.

that gas pressure is due to molecules bouncing elastically on the containing walls. Carry out the first stages by working through Problems 1 and 2. They start with a bouncing ball and graduate to many bouncing molecules, to emerge with a prediction of the behavior of gases. After you have tried the problems, return to the discussion of details.

"PROBLEM SHEETS"

The problems here are intended to be answered on typewritten copies of these sheets. Work through the problems on the enlarged copies, filling in the blanks, (———), that are left for answers.

SPECIAL PROBLEMS ON MOLECULAR THEORY NAME________________

Introduction

These problems will help you to build a great molecular theory.

The success of Newton's planetary theory soon led to attempts at building more theories similarly based on Newton's Laws of Motion, again using a few clear assumptions. Theories of gases were tried with gas pictured as a cloud of many tiny molecules bouncing about very fast. Assuming a few simple properties for these molecules (including the basic assumption that they exist!) and assuming that Newton's Laws of Motion applied to molecules, scientists were able to deduce (predict) Boyle's Law and so many other properties of gases that this theory too has come to be regarded as a good one.

As in most theories, having made our assumptions we need to do some calculations to arrive at deductions. To make these calculations easier, a series of problems about bouncing balls is given you below. These lead on to a calculation about molecules which will in fact make valuable predictions. The main calculation may seem hard at first, perhaps just because it is about mysterious molecules, but if you will carry it through once and then leave it you will find it makes good sense the next time.

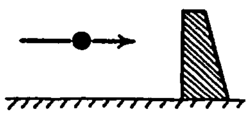
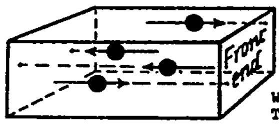


FIG. 25-4.



THIS IS THE FRONT END OF BOX ON WHICH WE CALCULATE THE PRESSURE

Box containing balls or molecules which move to-and-fro between front end and other end, making pressure by their impact.

PROBLEM 1.

I(a) EXCHANGE OF MOMENTUM

A ball of mass 2 kilograms moving 12 meters/sec hits a massive wall head-on and stops dead.

(i) The ball's momentum before impact is . . . ________ . ________ units

(ii) The ball's momentum after impact is. . . . ________ . ________ units

(iii) .·. change of momentum suffered by ball is ________ . ________ units

(iv) If Newton's Law III, which summarizes universally observed behavior of colliding bodies, is correct and applies to this case, we can say that change of momentum suffered by the wall (and whatever it is attached to) must be ________ ________ units

SPECIAL PROBLEMS ON MOLECULAR THEORY SHEET 2

PROBLEM 1. (continued)

I(b) FORCE ON WALL DUE TO STREAM OF BALLS

Now suppose the wall is hit by a stream of such balls, each of mass 2 kg and speed 12 meters/sec. 1000 such balls hit the wall head-on in the course of 10 seconds and stop dead. What is the push on the wall?

The total momentum-change suffered by the wall (in the course of that 10-sec period) is ________ . ________ units

(NOTE: Actually the changes of momentum occur in bumps, one bump when each ball hits the wall, but you can still calculate the total change, and you can then use that to calculate the average force, averaging the sudden bump-forces over the whole 10 seconds. To find the size of the very short-lived bump forces, you would need to know, and use, the very short time taken by each ball to lose its momentum, that is, the duation of a single bump. This time is not given you, so you can calculate only the averaged-out value of the force.)

The average force on the wall, during the 10-sec time, due to all 1000 balls losing momentum, is given by applying $F \cdot t = \Delta(mv)$ to the whole collection of 1000 balls.

.·. Average force, F, on wall must be. . . . . ________ . ________ units

NOTE that since the relation $F \cdot t = \Delta(mv)$ is a form of Newton's Law II, all forces used in it must be in "absolute" units, as in $F = m \cdot a$, e.g., newtons.

II. FORCE ON WALL DUE TO STREAM OF ELASTIC BALLS

As in I(b) suppose that 1000 balls, each of mass 2 kg, hit a massive wall head-on in the course of 10 seconds; but this time they arrive with speed 12 meters/sec and bounce straight back with equal speed, 12 meters/sec.

(i) Each ball's momentum before impact is. . . ________ . ________ units

(ii) Each ball's momentum after impact is . . . ________ . ________ units

(Remember momentum is a vector. Use + and - signs.)

(iii) Change of momentum of one ball is. . . . . ________ . ________ units

(iv) .·. change of momentum suffered by wall is ________ . ________ units

NOTE: ANSWER IS NOT ZERO

(v) If, during 10 seconds, 1000 balls strike the wall and rebound thus, total change of momentum suffered by wall is________ . ________ units

(vi) .·. Average force on wall during the 10-second period is . . ________ . ________ units

(vii) If the 1000 balls hit a patch of wall 2 meters high by 3 meters wide, average PRESSURE (= FORCE/AREA) on that patch is ________ . ________ units

PROBLEM 1. (continued)

III. MOTION INSIDE A BOX. Before changing from bouncing balls to bouncing molecules, we must put the moving things inside a closed box. Suppose we have an oblong box, 4 meters long from end to end, with only one ball in it moving to-and-fro from end to end with speed 12 meters/sec. The ball hits each end head-on and rebounds with speed 12 meters/sec to the other end. Now the same ball will hit the front end of the box many times in ten seconds. Instead of using the number of balls hitting the wall, we must calculate and use the number of hits made by this one ball. To find force on one end, we use the hits on that end only.

(i) Between successive hits on the front end of the box the ball travels one "round trip." It travels the whole length of the box from the front end to the other end and back to the front end. So it travels ________ meters

(ii) With its speed of 12 meters/sec, the total distance the ball travels in 10 seconds is . . . . ________ meters

(iii) ∴ the number of round trips the ball makes in 10 seconds is . . . ________ round trips

(iv) ∴ in 10 seconds the number of hits the ball makes on the front end is . . . ________

(So if we want to have 1000 hits on the front end wall in 10 sec, as in II above, we need more than one ball in the box. In fact we need about ______ balls all moving to-and-fro between the ends.)

(v) At each hit on the front end, the single ball suffers a change of momentum . . . . ________ ________ units

(vi) ∴ in 10 secs the ball makes ______ hits on front end of box, suffering a momentum-change of ________ ________ units at each hit.

∴ the total momentum-change suffered by front-end of box in 10 seconds is . . . . ________ ________ units

∴ average force on front end during the 10-second period is. . . . . . ________ ________ units

(PRESSURE is given by FORCE/AREA; and if you knew the area of the front end you could calculate the (average) pressure on it caused by the repeated impacts of the ball. In this case of a single large ball the pressure is not a sensible thing to calculate; but you can make the calculation for molecules and thus predict the pressure of a gas.)

PROBLEM 1. (continued)

We now apply a similar calculation to gas molecules in a box. Later we shall repeat it with algebra (Problem 2.)

IV. GAS MOLECULE IN A BOX. A metal box, 4 meters long, with ends 3 meters by 2 meters, contains one gas molecule which moves to-and-fro along the length with speed 500 meters/sec, bouncing back elastically at each end. The molecule approaches one end with speed 500 meters/sec, hits it, and bounces back with speed 500 meters/sec, travels to the other end, hits that and rebounds moving forward 500 meters/sec. The mass of the molecule is approximately 5×10^{-26} kilograms.*

(i) The molecule's change of momentum when it hits the first end and rebounds is. . . . ________ ________ units

(ii) In 10 seconds the molecule travels total distance. . . ________ meters

(iii) Between successive impacts on the first end, molecule has to travel to other end and back, so travels a distance. . . . . . . ________ meters

(iv) ∴ in 10 seconds, molecule can make ________ trips to-and-fro, and so can make this number of impacts on the first end.

(v) ∴ in 10 seconds, the molecule makes ________ impacts on the first end of the box, suffering a change of momentum at each impact of. . . . . ________ ________ units

(vi) ∴ total change of momentum suffered by first end-wall of box in 10 seconds is . . . ________ ________ units

(vii) ∴ average FORCE, during 10 second period, on first end of box is . . . . ________ ________ units

(viii) PRESSURE is FORCE/AREA. The end wall has area 3 x 2 sq. meters.

∴ average PRESSURE on end wall is. . . . . ________ ________ units

*NOTE: Simple chemical measurements suggest that the oxygen and nitrogen molecules (of air) are roughly 30 times as massive as a hydrogen atom. Difficult physical measurements tell us that a hydrogen atom has mass 1.67×10^{-27} kilograms. So the molecular mass suggested here, 5×10^{-26} kilograms, is a fair value for air.

PROBLEM 1. (continued)

V. MANY GAS MOLECULES IN A BOX

(i) Now suppose that this box contains 6×10^{26} molecules (= 600,000,000,000,000,000,000,000,000). That is roughly the actual number in such a box if filled with air at atmospheric pressure.* In reality these would be moving about in all directions at random; but to simplify the calculation pretend they are sorted out into three regimented groups, one lot moving up-and-down, one lot to-and-fro along the length, and one lot moving to-and-fro across the width. Symmetry considerations suggest we should have the molecules equally divided among the three groups (FIG. 25-5). The pressure on an END of the box will be solely due to impacts of molecules moving to-and-fro along the length. We now proceed to calculate that pressure, assuming there are only one-third of the molecules involved; that is, 2×10^{26} or 200,000,000,000,000,000,000,000,000 molecules, moving 500 meters/sec along the 4-meter length of the box, hitting the end, rebounding 500 meters/sec, hitting the other end, rebounding, and so on.

Using result of IV above, we predict that:

average pressure on end of box will be . . . . ______________ ______________ units

(The data are roughly right for ordinary air in a room. What value for atmospheric pressure, in the same units, is given by direct measurement with a barometer?) . . . . ______________ ______________ units

How does your result calculated above for the molecules compare with atmospheric pressure measured in lab? . . . . . . ______________

(ii) Now suppose the box is gently squashed end-ways, so that its length is reduced to 2 meters (i.e., half the original length) without changing the number of molecules or their speeds, or the size of the end-wall.

Average pressure on end of box will be . . . ______________ ______________ units

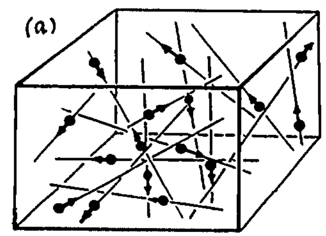
NOTE: There is very little change in the arithmetic from (i) to (ii). Check through to get the new answer.

(iii) Comment on the answer to V(ii).

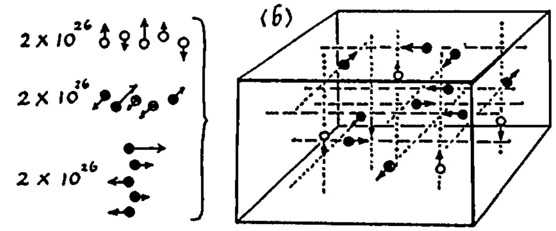
VI. OPTIONAL. On a separate sheet, repeat the calculation of average pressure, V(i), with algebra. Take a box of length a meters, width b meters, and height c meters, containing a TOTAL of N molecules moving with average speed v meters/sec. (i) Calculate pressure p. (ii) Calculate the product (PRESSURE) x (VOLUME), by multiplying p by abc.

*NOTE: Chemists often deal with a "mole" or "gram-molecule" of gas. A mole of any gas occupies 22.4 liters at atmospheric pressure and the temperature of melting ice. At room temperature (and one atmosphere) a mole occupies about 24 liters. Here we have chosen 1000 moles, a "kilo-mole" or "kilogram-molecule," which would occupy about 24,000 liters or 24 cu. meters at room temperature (about 20 °C.)

BOX CONTAINING 6×10^{26} MOLECULES IN RANDOM MOTION



Instead of random directions of motion, as in (a), pretend there are three regimented groups as in (b), each group of 2×10^{26} having full velocity but moving parallel to one edge of the box.



Then assume that the pressure on end face is due to impacts by one group, 2×10^{26} molecules, moving to-and-fro parallel to the length of the box, as in (c).

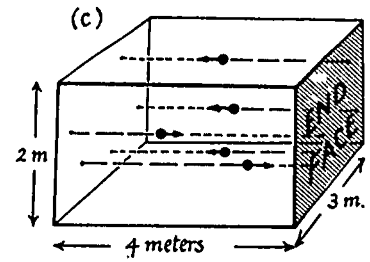


FIG. 25-5.

SPECIAL PROBLEMS ON MOLECULAR THEORY NAME ____________

PROBLEM 2. KINETIC THEORY WITH ALGEBRA SHEET 6

(This treats many gas molecules in a box with algebra. It should be tried after Problem 1 has been answered and corrected.)

Suppose that the box contains N molecules (N in the whole box, not N molecules in each cubic meter as in some texts). Suppose that the box has length a meters and ends of dimensions b meters by c meters.

In the course of their random motion with many collisions the molecules will exchange momentum and will not all keep the same velocity. However, if the temperature is kept constant, we believe their velocities will range around a fixed average velocity, which we call v meters/sec. To calculate the pressure on one end of the box we deal only with molecular impacts on that end. So to simplify the problem we pretend that the N molecules are regimented in three equal groups, one lot moving up-and-down, one lot to-and-fro across the width, and one lot moving forwards-and-backwards along the length. For the pressure on one end we then consider the last lot only. Symmetry-considerations suggest we should imagine the molecules equally divided among the three groups. Making these assumptions, answer the questions below, using m kilograms for the mass of one molecule.

(i) When one molecule hits the front end head-on and rebounds, its change of momentum is ____________

(ii) Between successive impacts on the front end a molecule travels to the other end and back: a total distance ____________ meters

(iii) In a total time t seconds, a molecule moving with velocity v meters/sec travels a total distance ____________ meters

(iv) ∴ in t seconds, a molecule can make ____________ round trips and so can make this number of impacts on front end.

(v) ∴ in t seconds, a molecule makes ____________ impacts on front end of box, suffering at each impact a change of momentum ____________

(vi) ∴ total change of momentum, due to impacts of one molecule, suffered by front end in t seconds is ____________

(vii) But there are N molecules in the box, of which ____________ are in the group moving forward and backward between the ends.

∴ the total change of momentum, due to impacts of all molecules concerned, suffered by front end in t seconds is ____________

SPECIAL PROBLEMS ON MOLECULAR THEORY SHEET 7

PROBLEM 2. (continued)

(viii) But, $F \cdot t = \Delta(\text{MOMENTUM})$ ∴ $F = \Delta(\text{MOMENTUM})/t$,

and in this case the average FORCE, during this period of t seconds, on the front end of the box is* . . . . ____________

(ix) PRESSURE = FORCE/AREA, and the area of the end face is ____________

∴ average PRESSURE on end of box is . . . . ____________

(x) The volume of the box is . . . . ____________ cu.meters

∴ the product (PRESSURE)·(VOLUME) = ____________

But m is the mass of one molecule, and there are N molecules, so the total mass of gas in the box, M kilograms = ____________ kilograms

Substituting M into the algebra above, we have

(PRESSURE)·(VOLUME) = ____________

(xi) Providing we use a closed box or other apparatus allowing no leakage of gas, then N is constant. Suppose we keep temperature constant; then other experiments in physics suggest that the average velocity v remains constant. Then in this case when the volume is changed, the result of (x) above suggests that ____________

(xii) If we measure the volume of a sample of gas, say in a globe, and find its mass (by weighing the globe full of gas and then evacuated), and measure the pressure of the sample with a barometer, then the result of (x) above enables us to calculate a very important piece of information, the value of ____________, which is the ____________ of the molecules.

(xiii) UNITS TO BE USED. In making the calculation of (xii) above, if the volume is in cu. meters the mass should be in . . . . ____________

and the pressure should be in. . . . . ____________

(xiv) We have already derived two useful things from our molecular theory, a behavior-suggestion in (xi) and a very interesting measurement in (xii), and more results will emerge; but we must pay for them by the assumptions that go into the machine. List on a separate sheet as many assumptions as you can,
(a) of general physical laws assumed to apply to molecules,
(b) of special properties, of behavior, size, etc., assumed for molecules.

*Here, t is the time during which the average force would have to act to produce this momentum-change. Therefore t IS the time t seconds for which we have calculated the total momentum-change.

Difficulties of the Simple Theory

The relation you worked out in Problem 2 seems to predict a steady pressure and Boyle's-Law behavior, from molecular chaos. How can a rain of molecules hitting a wall make a steady pressure? Only if the collisions come in such rapid succession that their bumps seem to smooth out into a constant force. For that the

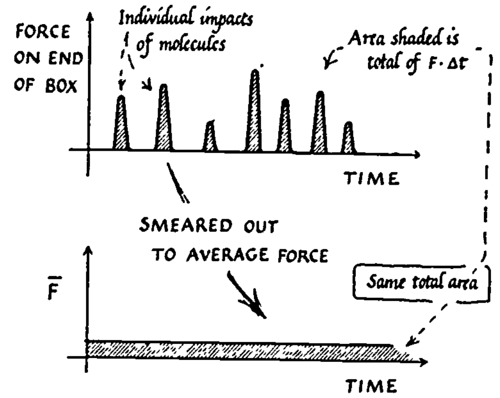


FIG. 25-6. SMOOTHING OUT IMPACTS

molecules of a gas must be exceedingly numerous, and very small. If they are small any solid pressure-gauge or container wall will be enormously massive compared with a single gas molecule, so that, as impacts bring it momentum, it will smooth them out to the steady pressure we observe. (What would you expect if the container wall were as light as a few molecules?)

The problem pretended that molecules travel straight from end to end and never collide with each other en route. They certainly do collide—though we cannot say how often without further information. How will that affect the prediction?

★ PROBLEM 3. COLLISIONS IN SIMPLE THEORY

(a) Show that it does not matter, in the simple derivation of Problems 1 and 2, whether molecules collide or not. (Consider two molecules moving to and fro from end to end, just missing each other as they cross. Then suppose they collide head-on and rebound. Why will their contribution to the pressure be unchanged? Explain with a diagram.)

(b) What special assumption about molecules is required for (a)?

(c) Suppose the molecules swelled up and become very bulky (but kept the same speed, mass, etc.), would the effect of mutual collisions be an increase of pressure (for the same volume etc.) or a decrease or what? (Note: "bulky" means large in size, not necessarily large in mass.)

(d) Give a clear reason for your answer to (c).

Molecular Chaos

Molecules hitting each other, and the walls, at random—some head on, some obliquely, some glancing—cannot all keep the same speed v. One will gain in a collision, and another lose, so that the gas is a chaos of molecules with random motions whose speeds (changing at every collision) cover a wide range. Yet they must preserve some constancy, because a gas exerts a steady pressure.

In the prediction $p \cdot V = (\frac{1}{3})[N\, m \overline{v^2}]$, we do not have all N molecules moving with the same speed, each contributing $m\overline{v^2}$ inside the brackets. Instead we have molecule #1 with its speed v_1, molecule #2 with $v_2, \ldots,$ molecule N with speed v_N. Then

$$\begin{aligned} p \cdot V &= (\tfrac{1}{3})\,[m\,v_1^2 + m\,v_2^2 + \ldots + m v_N^2] \\ &= (\tfrac{1}{3})\,[m\,(v_1^2 + v_2^2 + \ldots + v_N^2)\,] \\ &= (\tfrac{1}{3})\,[m\,(N \cdot \text{AVERAGE } v^2)\,] \quad \text{See note 3.} \end{aligned}$$

The v^2 in our prediction must therefore be an average v^2, so that we write a bar over it to show it is an average value. Our theoretical prediction now runs:

$$\text{PRESSURE} \cdot \text{VOLUME} = \tfrac{1}{3} N \cdot m \cdot \overline{v^2}.$$

We know that if we keep a gas in a closed bottle its pressure does not jump up and down as time goes on; its pressure and volume stay constant. Therefore in spite of all the changes in collisions, the molecular $\overline{v^2}$ stays constant. Already our theory helps us to picture some order—constant $\overline{v^2}$—among molecular chaos.

A More Elegant Derivation

To most scientists the regimentation that leads to the factor ⅓ is too artificial a trick. Here is a more elegant method that treats the molecules' random velocities honestly with simple statistics. Suppose molecule #1 is moving in a slanting direction in the box, with velocity v_1. (See Fig. 25-7.) Resolve this vector v_1 into three

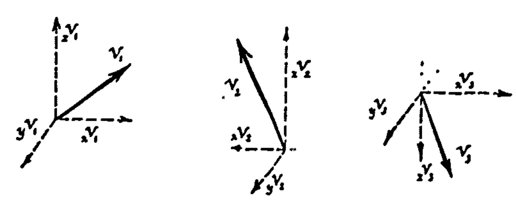


FIG. 25-7. ALTERNATIVE TREATMENT OF GAS MOLECULE MOTION
(More professional, less artificial.)
In this we keep the random velocities, avoiding regimentation, but split each velocity v into three components, ${}_xv$, ${}_yv$, ${}_zv$, parallel to the sides of the box. Then we deal with ${}_xv^2$ in calculating the pressure and arrive at the same result. Sketches show three molecules with velocities split into components.

components along directions x, y, z, parallel to the edges of the box. Then v_1 is the resultant of ${}_xv_1$ along x and ${}_yv_1$ along y and ${}_zv_1$ along z; and since these are mutually perpendicular, we have, by the three-dimensional form

[3] Because AVERAGE v^2 = (sum of all the v^2 values)/(number of v^2 values) $= (v_1^2 + v_2^2 + \ldots + v_N^2)/(N)$
$\therefore (v_1^2 + v_2^2 + \ldots + v_N^2) = N \cdot$ (AVERAGE v^2) or $N \cdot \overline{v^2}$
This $\overline{v_2}$ is called the "mean square velocity." To obtain it, take the speed of each molecule, at an instant, square it, add all the squares, and divide by the number of molecules. Or, choose one molecule and average its v^2 over a long time—say a billion collisions.

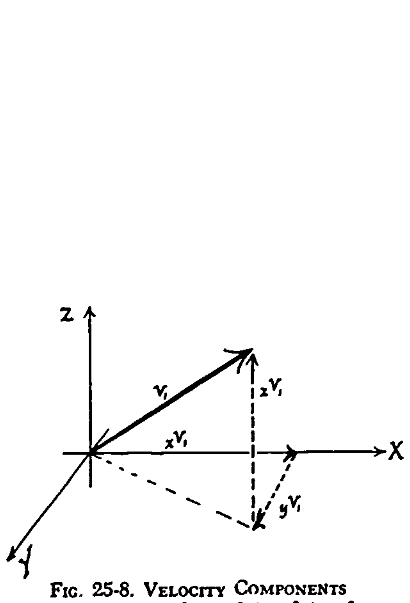


FIG. 25-8. VELOCITY COMPONENTS
PYTHAGORAS: $v_1^2 = {}_xv_1^2 + {}_yv_1^2 + {}_zv_1^2$

of Pythagoras' theorem: $v_1^2 = {}_xv_1^2 + {}_yv_1^2 + {}_zv_1^2$
And for molecule #2 $\quad v_2^2 = {}_xv_2^2 + {}_yv_2^2 + {}_zv_2^2$
And for molecule #3 $\quad v_3^2 = {}_xv_3^2 + {}_yv_3^2 + {}_zv_3^2$
and so on $\quad . \; . \; . \; . \; . \; . \; . \; .$
And for molecule #N $\quad v_N^2 = {}_xv_N^2 + {}_yv_N^2 + {}_zv_N^2$

Add all these equations:

$$\begin{aligned}(v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2) &= ({}_xv_1^2 + {}_xv_2^2 + {}_xv_3^2 + \ldots + {}_xv_N^2)\\ &\quad + ({}_yv_1^2 + {}_yv_2^2 + {}_yv_3^2 + \ldots + {}_yv_N^2)\\ &\quad + ({}_zv_1^2 + {}_zv_2^2 + {}_zv_3^2 + \ldots + {}_zv_N^2)\end{aligned}$$

Divide by the number of molecules, N, to get *average* values:

$$\therefore \quad \overline{v^2} = \overline{{}_xv^2} + \overline{{}_yv^2} + \overline{{}_zv^2}$$

Appealing to symmetry, and ignoring the small bias given by gravity, we claim that the three averages on the right are equal—the random motions of a statistically large number of molecules should have the same distribution of velocities in any direction.

$$\therefore \quad \overline{{}_xv^2} = \overline{{}_yv^2} = \overline{{}_zv^2}$$

$$\therefore \quad \overline{v^2} = 3\overline{{}_xv^2}$$

To predict the pressure on the end of the box we proceed as in Problem 2, but we use v_x for a molecule's velocity along the length of the box. (That is the velocity we need, because ${}_yv$ and ${}_zv$ do not help the motion from end to end and are not involved in the change of momentum at each end.) Then the contribution of molecule #1 to PRESSURE · VOLUME is $m \cdot {}_xv_1^2$ and the contribution of all N molecules is

$$m\,({}_xv_1^2 + {}_xv_2^2 + \ldots + {}_xv_N^2) \text{ or } m \cdot N \cdot \overline{{}_xv^2};$$

and by the argument above this is $m \cdot N \cdot (\overline{v^2}/3)$

$$\therefore \quad \text{PRESSURE} \cdot \text{VOLUME} = (\tfrac{1}{3})\, N \cdot m \cdot \overline{v^2}$$

(If you adopt this derivation, you should carry through the algebra of number of hits in t secs, etc., as in Problem 2.)

Molecular Theory's Predictions

Thinking about molecular collisions and using Newton's Laws gave the $(\frac{1}{3})\, N \cdot m \cdot \overline{v^2}$ prediction:

$$\text{PRESSURE} \cdot \text{VOLUME} = (\tfrac{1}{3})\, N \cdot m \cdot \overline{v^2}$$

This looks like a prediction of Boyle's Law. The fraction ($\frac{1}{3}$) is a constant number; N, the number of molecules, is constant, unless they leak out or split up; m, the mass of a molecule, is constant. Then *if* the average speed remains unchanged, $(\frac{1}{3})\, N \cdot m \cdot \overline{v^2}$ remains constant and therefore $p \cdot V$ should remain constant, as Boyle found it does. But *does* the speed of molecules remain fixed? At this stage, you have no guarantee. For the moment, anticipate later discussion and *assume* that molecular motion is connected with the heat-content of a gas, and that *at constant temperature gas molecules keep a constant average speed*, the same speed however much the gas is compressed or rarefied.[4] Later you will receive clear reasons for believing this. If you accept it now, you have predicted that:

The product $p \cdot V$ is constant for a gas at constant temperature.

You can see the prediction in simplest form by considering changes of DENSITY instead of VOLUME: just put twice as many molecules in the same box, and the pressure will be doubled.

A marvelous prediction of Boyle's Law? Hardly marvelous: we had to pour in many assumptions—with a careful eye on the desired result, we could scarcely help choosing wisely. A theory that gathers assumptions and predicts only one already-known law—and that under a further assumption regarding temperature—would not be worth keeping. But our new theory is just beginning: it is also helpful in "explaining" evaporation, diffusion, gas friction; it predicts effects of sudden compression; it makes vacuum-pumps easier to design and understand. And it leads to measurements that give validity to its own assumptions. Before discussing the development, we ask a basic question, "Are there really any such things as molecules?"

Are there really molecules?

> "That's the worst of circumstantial evidence. The prosecuting attorney has at his command all the facilities of organized investigation. He uncovers facts. He selects only those which, in his opinion, are significant. Once he's come to the conclusion the defendant is guilty, the only facts he considers significant are those which point to the guilt of the defendant. That's why circumstantial evidence is such a liar. Facts themselves are meaningless. It's only the interpretation we give those facts which counts."
>
> "Perry Mason"—Erle Stanley Gardner[*]

[4] Actually, compressing a gas warms it, but we believe that when it cools back to its original temperature its molecules, though still crowded close, return to the same average speed as before compression.

[*] *The Case of the Perjured Parrot*, Copyright 1939, by Erle Stanley Gardner.

A century ago, molecules seemed useful: a helpful concept that made the regularities of chemical combinations easy to understand and provided a good start for a simple theory of gases. But did they really exist? There was only circumstantial evidence that made the idea plausible. Many scientists were skeptical, and at least one great chemist maintained his right to disbelieve in molecules and atoms even until the beginning of this century. Yet one piece of experimental evidence appeared quite early, about 1827: the Brownian motion.

The Brownian Motion

The Scottish botanist Robert Brown (1773-1858) made an amazing discovery: he practically saw molecular motion. Looking through his microscope at small specks of solid suspended in water, he saw them dancing with an incessant jigging motion. The microscopic dance made the specks look alive, but it never stopped day after day. Heating made the dance more furious, but on cooling it returned to its original scale. We now know that any solid specks in any fluid will show such a dance, the smaller the speck the faster the dance, a random motion with no rhyme or reason. Brown was in fact watching the effects of water molecules jostling the solid specks. The specks were being pushed around like an elephant in the midst of a football game.

Watch this "Brownian motion" for yourself. Look at small specks of soot in water ("India ink") with a high-magnification microscope. More easily, look at smoke in air with a low-power microscope. Fill a small black box with smoke from a cigarette or a dying match, and illuminate it with strong white light from the side. The smoke scatters bluish-white light in all directions, some of it upward into the microscope. The microscope shows the smoke as a crowd of tiny specks of white ash which dance about with an entirely irregular motion.[5] (See Fig. 30-3 for an example)

Watching the ash specks, you can see why Brown at first thought he saw living things moving, but you can well imagine the motion to be due to chance bombardment by air molecules. Nowadays we not only think it may be that; we are sure it is, because we can calculate the effects of such bombardment and check them with observation. If air molecules were infinitely small and infinitely numerous, they would bombard a big speck symmetrically from all sides and there would be no Brownian motion to see. At the other extreme, if there were only a few very big molecules of surrounding air, the ash speck would make great violent jumps when it did get hit. From what we see, we infer something between these extremes; there must be many molecules in the box, hitting the ash speck from all sides, many times a second. In a short time, many hundreds of molecules hit the ash speck from every direction; and occasionally a few hundreds more hit one side of it than the other and drive it noticeably in one direction. A big jump is rare, but several tiny random motions in the same general direction may pile up into a visible shift.[6] Detailed watching and calculation from later knowledge tell us that what we see under the microscope are those gross resultant shifts; but, though the individual movements are too small to see, we can still estimate their speed by cataloguing the gross staggers and analysing them statistically.

You can see for yourself that smaller specks dance faster. Now carry out an imaginary extrapolation to smaller and smaller specks. Then what motion would you expect to see with specks as small as molecules if you could see them? But can we see molecules?

Seeing molecules?

Could we actually see a molecule? That would indeed be convincing—we feel sure that what we *see* is real, despite many an optical illusion. All through the last century's questioning of molecules, scientists agreed that seeing one is hopeless—not just unlikely but impossible, for a sound physical reason. Seeing uses light, which consists of waves of very short wavelength, only a few thousand Angström Units[7] from crest to crest. We see by using these waves to form an image:

with the naked eye we can see the shape of a pin's head, a millimeter across, or 10,000,000 AU

with a magnifying glass we examine a fine hair, 1,000,000 AU thick

with a low-power microscope we see a speck of smoke ash, 100,000 AU

with a high-power microscope, we see bacteria, from 10,000 down to 1000 AU

but there the sequence stops. It must stop because the wavelength of visible light sets a limit there. Waves can make clear patterns of obstacles that are larger

[5] There may also be general drifting motions—convection currents—but these are easily distinguished. An ash speck in focus shows as a small sharp wisp of white, often oblong; but when it drifts or dances away out of focus the microscope shows it as a fuzzy round blob, just as camera pictures show distant street lights out of focus.

[6] Imagine an observer with poor sight tracing the motion of an active guest at a crowded party. He might fail to see the guest's detailed motion of small steps here and there, and yet after a while he would notice that the guest had wandered a considerable distance.

[7] 1 Angström Unit, 1 AU, is 10^{-10} meter.

than their wavelength, or even about their wavelength in size. For example, ocean waves sweeping past an island show a clear "shadow" of calm beyond. But waves treat smaller obstacles quite differently. Ocean waves meeting a small wooden post show no calm behind. They just lollop around the post and join up beyond it as if there were no post there. A blind man paddling along a stormy seashore could infer the presence of an island nearby, but would never know about a small post just offshore from him.[8] Light waves range in wavelength from 7000 AU for red to 4000 for violet. An excursion into the short-wave ultraviolet, with photographic film instead of an eye, is brought to a stop by absorption before wavelength 1000 AU: lenses, specimen, even the air itself, are "black" for extreme ultraviolet light. X-rays, with shorter wavelength still, can pass through matter and show grey shadows, but they practically cannot be focused by lenses. So, although X-rays have the much shorter wavelength that could pry into much finer structures, they give us only unmagnified shadow pictures. Therefore the limit imposed by light's wavelength seemed impassable. Bacteria down to 1000 AU could be seen, but virus particles, ten times smaller, must remain invisible. And molecules, ten times smaller still, must be far beyond hope. Yet viruses, responsible for many diseases, are of intense medical interest—we now think they may mark the borderline between living organisms and plain chemical molecules. And many basic questions of chemistry might be answered by seeing molecules.

The invisibility of molecules was unwelcome, but seemed inescapable. Then, early in this century, X-rays offered indirect information. The well-ordered atoms and molecules of crystals can scatter X-rays into regular patterns, just as woven cloth can "diffract" light into regular patterns—look at a distant lamp at night through a fine handkerchief or an umbrella. X-ray patterns revealed both the arrangement of atoms in crystals and the spacing of their layers. Such measurements confirmed the oil-film estimates of molecular size. More recently, these X-ray diffraction-splash pictures have sketched the general *shape* of some big molecules—really only details of crystal structure, but still a good hint of molecular shape. Then when physicists still cried "no hope" the electron microscope was invented. Streams of electrons, instead of light-waves, pass through the tiny object under examination, and are focused by electric or magnetic fields to form a greatly magnified image on a photographic film. Electrons are incomparably smaller agents than light-waves,[9] so small that even "molecules" can be delineated. Then we can "see" virus particles and even big molecules in what seem to be reliable photographs with huge magnifications. These new glimpses of molecular structure agree well with the speculative pictures drawn by chemists arguing very cleverly from chemical behavior.

Recently, still sharper methods have been developed. At the end of this book you will see a picture of the individual atoms of metal in a needle point. Why not show that now? Because, like so much in atomic physics, the method needs a sophisticated knowledge of assumptions as well as techniques before you can decide in what sense the photograph tells the truth. Going still deeper, very-high-energy electrons are now being used to probe the structure of atomic nuclei, yielding indirect shadow pictures of them.

In the last 100 years, molecules have graduated from being tiny uncounted agents in a speculative theory to being so real that we even expect to "see" their shape. Most of the things we know about them—speed, number, mass, size—were obtained a century ago with the help of kinetic theory. *The theory promoted the measurements, then the measurements gave validity to the theory.* We shall now leave dreams of seeing molecules, and study what we can measure by simple experiments.

Measuring the Speed of Molecules

Returning to our prediction that:

$$\text{PRESSURE} \cdot \text{VOLUME} = (1/3)\ N \cdot m \cdot \overline{v^2}$$

We can use this if we trust it, to estimate the actual speed of the molecules. N is the number of molecules and m is the mass of one molecule so *Nm is the total mass M of all the molecules* in the box of gas. Then we can rewrite our prediction:

$$\text{PRESSURE} \cdot \text{VOLUME} = (1/3) \cdot M \cdot \overline{v^2}$$

where M is the total mass of gas. We can weigh a big sample of gas with measured volume at known pressure and substitute our measurements in the relation above to find the value of $\overline{v^2}$ and thus the value of the average speed.

Fig. 25-9 shows the necessary measurements. Using the ordinary air of the room, we measure its pressure by a mercury barometer. (Barometer height and the measured density of mercury and the measured value of the Earth's gravitational field strength, 9.8 newtons per kilogram, will give the pressure in absolute units, newtons per square meter.)[10] We weigh the air which fills a flask. For this, we weigh the flask first full of air at atmospheric pressure and second after a vacuum pump has taken out nearly all the air. Then we open the flask under water and let water enter to replace the air pumped

[8] Tiny obstacles do produce a small scattered ripple, but this tells nothing about their shape. Bluish light scattered by very fine smoke simply indicates there are very tiny specks there, but does not say whether they are round or sharp-pointed or oblong. The still more bluish light of the sky is sunlight scattered by air molecules.

[9] Electrons speeding through the electron microscope behave as if they too have a wavelength, but far shorter than the wavelength of light. So they offer new possibilities of "vision," whether you regard them as minute bullets smaller than atoms, or as ultra-short wave patterns. A technology of "electron optics" has developed, with "lenses" for electron microscopes and for television tubes (which are electron projection-lanterns).

[10] Since we made our kinetic theory prediction with the help of Newton's Law II, the predicted *force* must be in absolute units, newtons; and the predicted *pressure* must be in newtons per square meter.

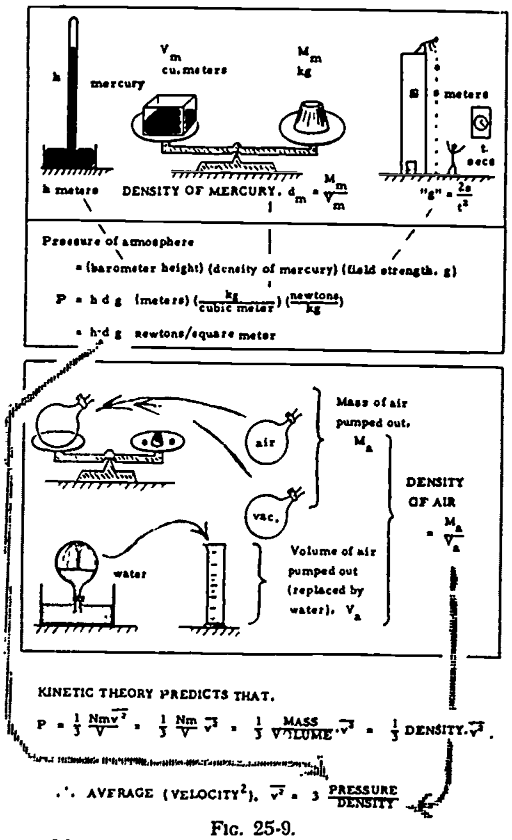


FIG. 25-9.
MEASURING MOLECULE VELOCITIES INDIRECTLY, BUT SIMPLY, ASSUMING KINETIC THEORY.

out. Measuring the volume of water that enters the flask tells us the volume of air which has a known mass. Inserting these measurements in the predicted relation we calculate $\overline{v^2}$ and thence its square root $\sqrt{(\overline{v^2})}$ which we may call the "average speed," $\bar{v}$ (or more strictly the "root mean square," or R.M.S. speed). You should see these measurements made and calculate the velocity, as in the following problem.

★ PROBLEM 4. SPEED OF OXYGEN MOLECULES

Experiment shows that 32 kg of oxygen occupy 24 cubic meters at atmospheric pressure, at room temperature.
(a) Calculate the density, *MASS/VOLUME*, of oxygen.
(b) Using the relation given by kinetic theory, calculate the *mean square velocity*, $\overline{v^2}$, of the molecules.
(c) Take the square root and find an "*average*" velocity, in meters/sec.
(d) Also express this very roughly in miles/hour. (Take 1 kilometer to be 5/8 mile)

Air molecules moving ¼ mile a second! Here is theory being fruitful and validating its own assumption, as theory should. We assumed that gases consist of molecules that are moving, probably moving fast; and our theory now tells us how fast, with the help of simple gross measurements. Yet theory cannot prove its own prediction is true—the result can only be true to the assumptions that went in. So we need experimental tests. If the theory passes one or two tests, we may trust its further predictions.

Speed of Molecules: experimental evidence

We have rough hints from the speed of sound and from the Brownian motion.

PROBLEM 5. SPEED OF SOUND

We believe that sound is carried by waves of compression and rarefaction, with the changes of crowding and motion handed on from molecule to molecule at collisions. If air does consist of moving molecules far apart, what can you say about molecular speed, given that the measured speed of sound in air is 340 meters/sec (≈ 1100 ft/sec)?

PROBLEM 6. BROWNIAN MOTION

Looking at smoke under a microscope you will see large specks of ash jigging quite fast; small specks jig faster still.
(a) There may be specks too small to see. What motion would you expect them to have?
(b) Regarding a single air molecule as an even smaller "ash speck," what can you state about its motion?

The two problems above merely suggest general guesses. Here is a demonstration that shows that gas molecules move very fast. Liquid bromine is released at the bottom of a tall glass tube.* The

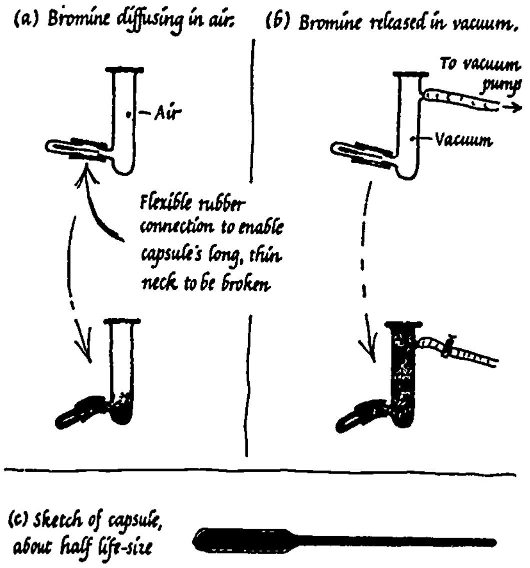


FIG. 25-10. MOTION OF BROMINE MOLECULES: DEMONSTRATION OF MOLECULAR SPEED.

* The bromine is inserted as liquid bromine in a small glass capsule with a long nose that can be broken easily.

liquid evaporates immediately to a brown vapor or "gas," which slowly spreads throughout the tube. The experiment is repeated in a tube from which all air has been pumped out. Now the brown gas moves very fast when released. (In air, its molecules still move fast, but their net progress is slow because of many collisions with air molecules.)

Direct Measurement

The real test must be a direct measurement. Molecular speeds have been measured by several experimenters. Here is a typical experiment, done by Zartman. He let a stream of molecules shoot through a slit in the side of a cylindrical drum that could be spun rapidly. The molecules were of bismuth metal, boiled off molten liquid in a tiny oven in a vacuum. A series of barriers with slits selected a narrow stream to hit the drum. Then each time the slit in the drum came around, it admitted a small flock of moving molecules. With the drum at rest, the molecules travelled across to the opposite wall inside the drum and made a mark on a receiving film opposite the slit. With the drum spinning, the film was carried around an appreciable distance while the molecules were travelling across to it, and the mark on it was shifted to a new position. The molecules' velocity could be calculated from the shift of the mark and the drum's diameter and spin-speed. When the recording film was taken out of the drum it showed a sharp central mark of deposited metal but the mark made while it spun was smeared out into a blur showing that the molecular velocities had not all been the same but were spread over a considerable range. Gas molecules have random motion with frequent collisions and we must expect to find a great variety of velocities at any instant. It is the *average* velocity, or rather the root-mean-square average, $\sqrt{(\overline{v^2})}$, that is involved in kinetic theory prediction. The probable distribution of velocities, clustering round that average, can be predicted by extending simple kinetic theory with the help of the mathematical statistics of chance. In Zartman's experiment, we expect the beam of hot vapor molecules to have the same chance distribution of velocities with its peak at an average value characteristic of the temperature. Measurements of the actual darkening of the recording film showed just such a distribution and gave an average that agreed well with the value predicted by simple theory (see sketch of graph in Fig. 25-12).[11]

ZARTMAN'S EXPERIMENT

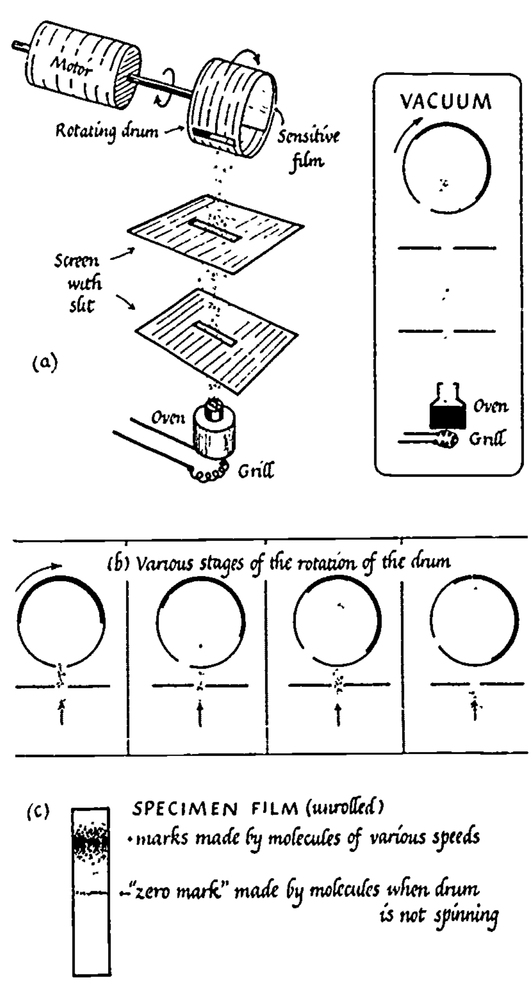


FIG. 25-11. MEASURING MOLECULE VELOCITIES DIRECTLY
(a) Sketch of Zartman's experiment.
(b) These sketches show various stages of the rotation of the drum.
(c) Specimen film (unrolled).

Molecular Speeds in Other Gases. Diffusion

Weighing a bottle of hydrogen or helium at atmospheric pressure and room temperature shows these gases are much less dense than air; and carbon dioxide is much more dense. Then our predic-

[11] Zartman's method is not limited to this measurement. One method of separating uranium 235 used spinning slits, though the uranium atoms were electrically charged and were given high speeds by electric fields. And mechanical "chopper" systems are used to sort out moving neutrons. Such choppers operate like traffic lights set for some constant speed. The simplest prototype of Zartman's experiment is the scheme shown in Fig. 8-8 for measuring the speed of a rifle bullet.

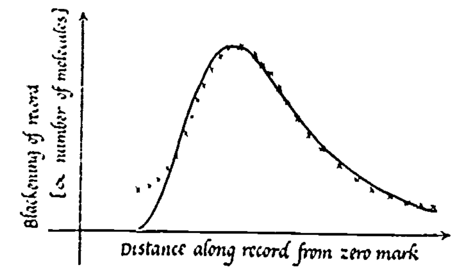


FIG. 25-12. RESULTS OF ZARTMAN'S EXPERIMENT
The curve, drawn by a grayness-measuring-machine, shows the experimental results. The crosses show values predicted by kinetic theory with simple statistics.

tion $pV = (1/3)\, M\, \overline{v^2}$ tells us that hydrogen and helium molecules move faster than air molecules (at the same temperature), and carbon dioxide molecules slower. Here are actual values:

Gas	Measurements at Room Temperature and Atmospheric Pressure: *Volume*	*Mass*
hydrogen	24 cu. meters	2.0 kilograms
helium	24 " "	4.0 kg
carbon dioxide	24 " "	44.0 kg
oxygen	24 " "	32.0 kg
nitrogen	24 " "	28.0 kg
air (1/5 oxygen 4/5 nitrogen)	24 " "	28.8 kg

★ PROBLEM 7. SPEEDS

(i) If oxygen molecules move about 1/4 mile/sec at room temperature, how fast do hydrogen molecules move?
(ii) How does the average speed of helium molecules compare with that of hydrogen molecules at the same temperature? (Give the ratio of "average" speeds.)
(iii) How does the speed of carbon dioxide molecules compare with that of air molecules at the same temperature? (Give the ratio of "average" speeds.)

PROBLEM 8

Making a risky guess,* say whether you would expect the speed of sound in helium to be the same as in air, or bigger or smaller. Test your guess by blowing an organ pipe first with air, then with helium (or with carbon dioxide). Or breathe in helium and then talk, using your mouth and nose cavities as miniature echoing organ pipes. A change in the speed of sound changes the time taken by sound waves to bounce up and down the pipe, and thus changes the frequency at which sound pulses emerge from the mouth. And that changes the musical note of the vowel sounds, which rises to higher pitch at higher frequency.

* It is obviously risky, since we are not considering the mechanism of sound transmission in detail. In fact there is an unexpected factor, which is different for helium: the ease with which the gas heats up as sound-compressions pass through. This momentary rise of temperature makes sound compressions travel faster. The effect is more pronounced in helium than in air, making the speed of sound 8% bigger than simple comparison with air suggests. Kinetic theory can predict this effect of specific heat, telling us that helium must have a smaller heat capacity, for a good atomic-molecular reason.

PROBLEM 9

How would you expect the speed of sound in air to change when the pressure is changed without any change of temperature? (Try this question with the following data, for air at room temperature: 28.8 kg of air occupy 24 cubic meters at 1 atmosphere pressure; at 2 atmospheres they occupy 12 cubic meters.)

Diffusion

If molecules of different gases have such different speeds, one gas should outstrip another when they diffuse through long narrow pipes. The pipes must be very long and very narrow so that gas seeps through by the wandering of individual molecules and not in a wholesale rush. The pores of unglazed pottery make suitable "pipes" for this. See Fig. 25-13a, b. The white jar J has fine pores that run right through its walls. If it is filled with compressed gas and closed with a stopper S, the gas will slowly leak out through the pores into the atmosphere, as you would expect. But if the pressure is the same (atmospheric) inside and out you would not expect any leakage even if there are different gases inside and outside. Yet there are changes, showing the effects of different molecular speeds. The demonstrations sketched start with air inside the jar and another gas, also at atmospheric pressure, outside. You see the effects of hydrogen molecules whizzing into the jar faster than air can move out; or of air moving out faster than CO_2 molecules crawl in. These are just qualitative demonstrations of "diffusion," but they suggest a process for separating mixed gases. Put a mixture of hydrogen and CO_2 inside the jar; then, whether there is air or vacuum outside, the hydrogen will diffuse out faster than the CO_2, and by repeating the process in several stages

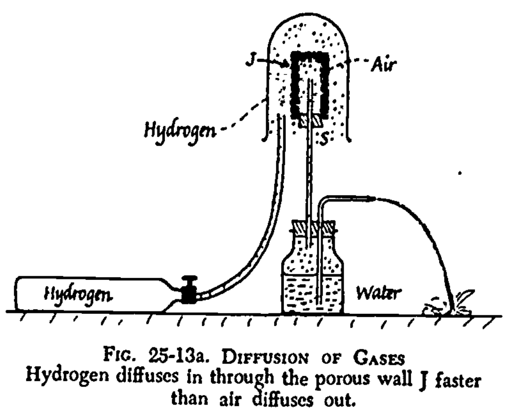


FIG. 25-13a. DIFFUSION OF GASES
Hydrogen diffuses in through the porous wall J faster than air diffuses out.

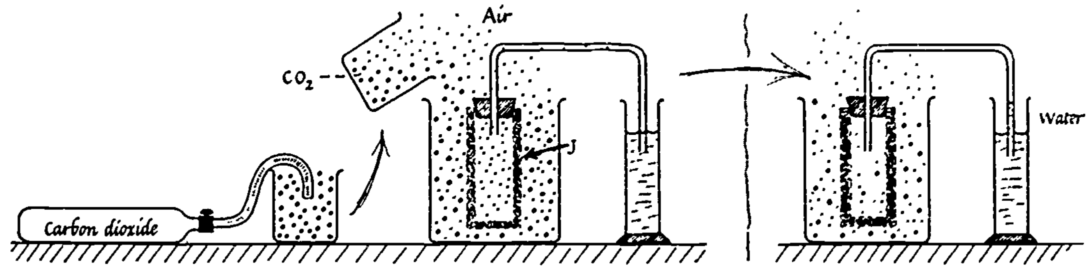


FIG. 25-13b. DIFFUSION OF GASES
Carbon dioxide diffuses in through the porous wall, J, slower than air diffuses out.

you could obtain almost pure hydrogen. This is a *physical* method of separation depending on a difference of molecular speeds that goes with a difference of molecular masses (see Fig. 25-14). It does not require a difference of *chemical* properties; so it can be used to separate "isotopes," those twin-brothers that are chemically identical but differ slightly in atomic masses. When isotopes were first discovered, one neon gas 10% denser than the other, some atoms of lead heavier than the rest, they were interesting curiosities, worth trying to separate just to show. Diffusion of the natural neon mixture from the atmosphere proved the possibility But now with two uranium isotopes hopelessly mixed as they come from the mines, one easily fissionable, the other not, the separation of the rare fissionable kind is a matter of prime importance. Gas diffusion is now used for this on an enormous scale. See Problem 11, and Figs. 25-15, 16 and 17. Also see Chs. 30 and 43.

Temperature

Heating a gas increases p or V or both. With a rise of temperature there is always an increase of pV, and therefore of $(1/3)\, N\, m\, \overline{v^2}$. Therefore making a gas hotter increases v^2, makes its molecules move faster. This suggests some effects of temperature.

★ PROBLEM 10

(a) Would you expect the speed of sound to be greater, less, or the same in air at higher temperature? Explain.
(b) Would you expect diffusion of gases to proceed faster, slower, or at the same rate, at higher temperature? Explain.

Kinetic Theory To Be Continued

We cannot give more precise answers to such questions until we know more about heat and temperature and energy. Then we can extract more predictions concerning gas friction, heat conduction, specific heats; and we shall find a way of

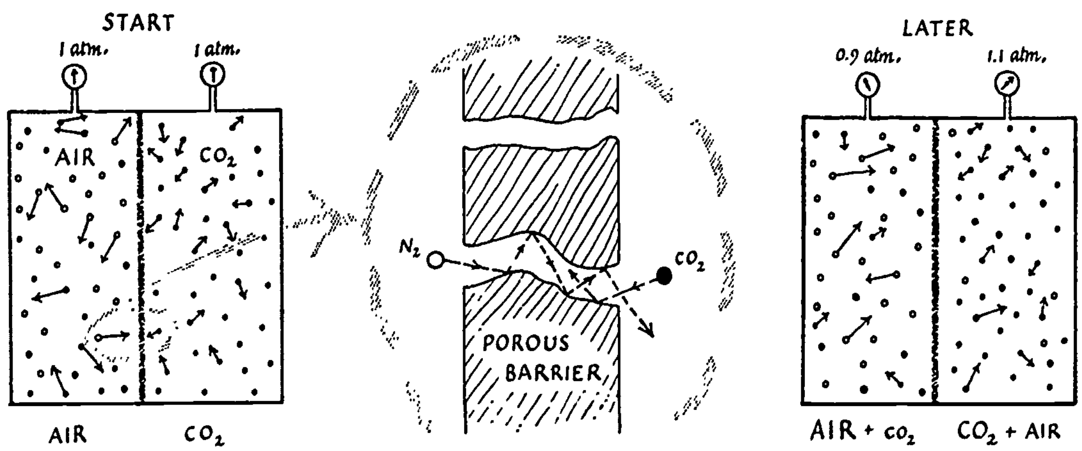


FIG. 25-14. UNEQUAL DIFFUSION OF GASES
Air and carbon dioxide, each originally at atmospheric pressure, are separated by a porous barrier.
At the start, with equal volumes at the same pressure, the two populations have equal numbers of molecules.
On the average, air molecules stagger through the pores faster than CO_2 molecules.
Then the populations are no longer equal so the pressures are unequal.

FIG. 25-15. SEPARATION OF URANIUM ISOTOPES BY DIFFUSION OF UF_6 THROUGH A POROUS BARRIER
Gas molecules hit the barrier, and the walls of its pores, many times—net result: a few get through.

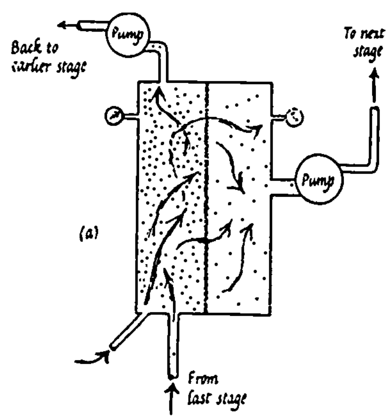


FIG. 25-16a. SEPARATION OF URANIUM ISOTOPES BY DIFFUSION OF UF_6 THROUGH A POROUS BARRIER.

measuring the mass of a single molecule, so that we can count the myriad molecules in a sample of gas. We shall return to kinetic theory after a study of energy. Meanwhile, it is kinetic theory that leads us towards energy by asking a question:

What is mv^2?

The expression $(\frac{1}{3})\, N\, m\, v^2$ is very important in the study of all gases. Apart from the fraction $(\frac{1}{3})$ it is

THE NUMBER OF MOLECULES · (mv^2 for one molecule)

What is mv^2 for a moving molecule? It *is* just the mass multiplied by the square of the speed; but what kind of thing does it measure? What are its properties? Is it an important member of the series: $m \quad mv \quad mv^2 \quad \ldots\ldots$? We know m, mass, and treat

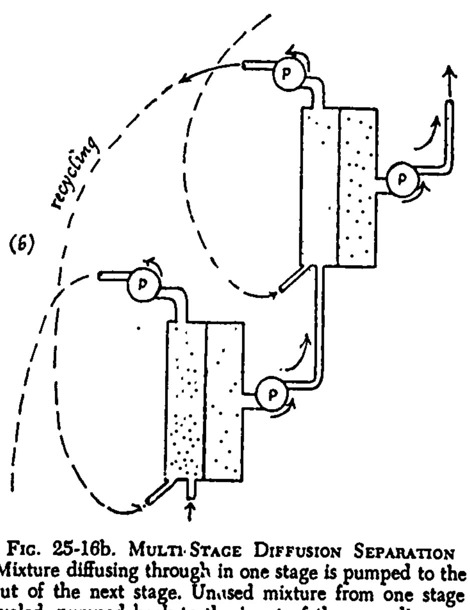


FIG. 25-16b. MULTI-STAGE DIFFUSION SEPARATION
Mixture diffusing through in one stage is pumped to the input of the next stage. Unused mixture from one stage is recycled, pumped back to the input of the preceding stage.

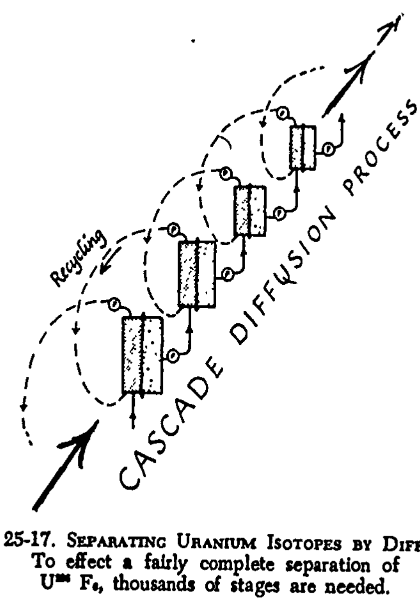


FIG. 25-17. SEPARATING URANIUM ISOTOPES BY DIFFUSION
To effect a fairly complete separation of $U^{235}F_6$, thousands of stages are needed.

it as a constant thing whose total is universally conserved. We know mv, momentum, and trust it as a vector that is universally conserved. Is mv^2 equally useful? Its structure is $mv \cdot v$ or $Ft \cdot v$ or

FORCE · TIME · DISTANCE/TIME.

Then mv^2 is of the form FORCE · DISTANCE. Is that product useful? To push with a force along some distance needs an engine that uses fuel. Fuel . . . money . . . energy. We shall find that mv^2 which appears in our theory of gases needs only a constant factor (½) to make it an expression of "energy."

PROBLEMS FOR CHAPTER 25

★ 1. DERIVING MOLECULAR PRESSURE

Work through the question sheets of Problem 1 shown earlier in this chapter. These lead up to the use of Newton's mechanics in a molecular picture of gases.

★ 2. KINETIC THEORY WITH ALGEBRA

Work through the question sheets of Problem 2.

Problems 3-10 are in the text of this chapter.

★ 11. URANIUM SEPARATION (For more professional version, see Problem 3 in Ch. 30)

Chemical experiments and arguments show that oxygen molecules contain two atoms so we write them O_2; hydrogen molecules have two atoms, written H_2; and the dense vapor of uranium flouride has structure UF_6.

Chemical experiments tell us that the *relative* masses of single atoms of O, H, F, and U are 16, 1, 19, 238. Chemical evidence and a brilliant guess (Avogadro's) led to the belief that a standard volume of any gas at one atmosphere and room temperature contains *the same number of molecules* whatever the gas (the same for O_2, H_2, or UF_6). Kinetic theory endorses this guess strongly (see Ch. 30).

(a) Looking back to your calculations in Problem 7 you will see that changing from O_2 to H_2 changes the mass of a molecule in the proportion 32 to 2. For the same temperature what change would you expect in the $\overline{v^2}$ and therefore what change in the average velocity? (That is, how fast are hydrogen molecules moving at room temperature compared with oxygen ones? Give a ratio showing the *proportion* of the new speed to the old. Note you do not have to repeat all the arithmetic, just consider the one factor that changes.)

(b) Repeat (a) for the change from oxygen to uranium fluoride vapor. Do rough arithmetic to find approximate numerical value.

(c) Actually there are several kinds of uranium atom The common one has mass 238 (relative to oxygen 16) but a rare one (0.7% of the mixture got from rocks) which is in fact the one that undergoes fission, has mass 235. One of the (very slow) ways of separating this valuable rare uranium from the common one is by converting the mixture to fluoride and letting the fluoride vapor diffuse through a porous wall Because the fluoride of U^{235} has a different molecular speed the mixture emerging after diffusing through has different proportions.

(i) Does it become richer or poorer in U^{235}?

(ii) Give reasons for your answer to (i).

(iii) Estimate the percentage difference between average speeds of $\left[U^{235}_{1}F_6\right]$ and $\left[U^{238}_{1}F_6\right]$ molecules.

(Note: As discussed in Ch. 11, a change of x % in some measured quantity Q makes a change of about $\frac{1}{2}$ x % in $\sqrt{Q}$.)

12. Figs. 25-13a and 25-13b show two diffusion demonstrations. Describe what happens and interpret the experiments.

★ 13. MOLECULAR VIEW OF COMPRESSING GAS

(a) When an elastic ball hits a massive wall head-on it rebounds with much the same *speed* as its original speed. The same happens when a ball hits a massive bat which is held firmly. However, *if the bat is moving towards the ball*, the ball rebounds with a different speed. Does it move faster or slower?

(b) (Optional, hard: requires careful thought.) When the bat is moving towards the ball is the time of the elastic impact longer, shorter, or the same as when the bat is stationary? (*Hint*: If elastic . . . . S.H.M. . . .)

(c) When a gas in a cylinder is suddenly compressed by the pushing in of a piston, its temperature rises. Guess at an explanation of this in terms of the kinetic theory of gases, with the help of (a) above.

(d) Suppose a compressed gas, as in (c), is allowed to push a piston out, and expand. What would you expect to observe?

★ 14. MOLECULAR SIZE AND TRAVEL

A closed box contains a large number of gas molecules at fixed temperature. Suppose the molecules magically became more bulky by swelling up to greater volume, without any increase in number or speed, without any change of *mass*, and without any change in the volume of the box.

(a) How would this affect the *average distance apart* of the molecules, center to center (great increase, decrease, or little change)?

(b) Give a reason for your answer to (a).

(c) How would this affect the *average distance travelled by a molecule between one collision and the next* (the "mean free path")?

(d) Give a reason for your answer to (c).

Abandoning a mechanical view of studying the behavior of each individual gas molecule, Maxwell adopts a statistical view and considers the average and distribution for velocity and energy.

6 On the Kinetic Theory of Gases

James Clerk Maxwell

1872

A gaseous body is supposed to consist of a great number of molecules moving with great velocity. During the greater part of their course these molecules are not acted on by any sensible force, and therefore move in straight lines with uniform velocity. When two molecules come within a certain distance of each other, a mutual action takes place between them, which may be compared to the collision of two billiard balls. Each molecule has its course changed, and starts on a new path. I have concluded from some experiments of my own that the collision between two hard spherical balls is not an accurate representation of what takes place during the encounter of two molecules. A better representation of such an encounter will be obtained by supposing the molecules to act on one another in a more gradual manner, so that the action between them goes on for a finite time, during which the centres of the molecules first approach each other and then separate.

We shall refer to this mutual action as an Encounter between two molecules, and we shall call the course of a molecule between one encounter and another the Free Path of the molecule. In ordinary gases the free motion of a molecule takes up much more time than that occupied by an encounter. As the density of the gas increases, the free path diminishes, and in liquids no part of the course of a molecule can be spoken of as its free path.

In an encounter between two molecules we know that, since the force of the impact acts between the two bodies, the motion of the centre of gravity of the two molecules remains the same after the encounter as it was before. We also know by the principle of the conservation of energy that the velocity of each molecule relatively to the centre of gravity remains the same in magnitude, and is only changed in direction.

Let us next suppose a number of molecules in motion contained in a vessel whose sides are such that if any energy is communicated to the vessel by the encounters of molecules against its sides, the vessel communicates as much energy to other molecules during their encounters with it, so as to preserve the total energy of the enclosed system. The first thing we must notice about this moving system is that even if all the molecules have the same velocity originally, their encounters will produce an inequality of velocity, and that this distribution of velocity will go on continually. Every molecule will then change both its direction and its velocity at every encounter; and, as we are not supposed to keep a record of the exact particulars of every encounter, these changes of motion must appear to us very irregular if we follow the course of a single molecule. If, however, we adopt a statistical view of the system, and distribute the molecules into groups, according to the velocity with which at a given instant they happen to be moving, we shall observe a regularity of a new kind in the proportions of the whole number of molecules which fall into each of these groups.

And here I wish to point out that, in adopting this statistical method of considering the average number of groups of molecules selected according to their velocities, we have abandoned the strict kinetic method of tracing the exact circumstances of each individual molecule in all its encounters. It is therefore possible that we may arrive at results which, though they fairly represent the facts as long as we are supposed to deal with a gas in mass, would cease to be applicable if our faculties and instruments were so sharpened that we could detect and lay hold of each mole cule and trace it through all its course.

For the same reason, a theory of the effects of education deduced from a study of the returns of registrars, in which no names of individuals are given, might be found not to be applicable to the experience of a schoolmaster who is able to trace the progress of each individual pupil.

The distribution of the molecules according to their velocities is found to be of exactly the same mathematical form as the distribution of observations according to the magnitude of their errors, as described in the theory of errors of observation. The distribution of bullet-holes in a target according to their distances from the point aimed at is found to be of the same form, provided a great many shots are fired by persons of the same degree of skill.

We have already met with the same form in the case of heat diffused from a hot stratum by conduction. Whenever in physical phenomena some cause exists over which we have no control, and which produces a scattering of the particles of matter, a deviation of observations from the truth, or a diffusion of velocity or of heat, mathematical expressions of this exponential form are sure to make their appearance.

It appears then that of the molecules composing the system some are moving very slowly, a very few are moving with enormous velocities, and the greater number with intermediate velocities. To compare one such system with another, the best method is to take the mean of the squares of all the velocities. This quantity is called the Mean Square of the velocity. The square root of this quantity is called the Velocity of Mean Square.

Completely random motion, such as the thermal motion of molecules, might seem to be out of the realm of lawfulness. But on the contrary, just because the motion is completely disorderly, it is subject to statistical laws.

7 The Law of Disorder

George Gamow

1947

IF YOU pour a glass of water and look at it, you will see a clear uniform fluid with no trace of any internal structure or motion in it whatsoever (provided, of course, you do not shake the glass). We know, however, that the uniformity of water is only apparent and that if the water is magnified a few million times, there will be revealed a strongly expressed granular structure formed by a large number of separate molecules closely packed together.

Under the same magnification it is also apparent that the water is far from still, and that its molecules are in a state of violent agitation moving around and pushing one another as though they were people in a highly excited crowd. This irregular motion of water molecules, or the molecules of any other material substance, is known as *heat (or thermal) motion*, for the simple reason that it is responsible for the phenomenon of heat. For, although molecular motion as well as molecules themselves are not directly discernible to the human eye, it is molecular motion that produces a certain irritation in the nervous fibers of the human organism and produces the sensation that we call heat. For those organisms that are much smaller than human beings, such as, for example, small bacteria suspended in a water drop, the effect of thermal motion is much more pronounced, and these poor creatures are incessantly kicked, pushed, and tossed around by the restless molecules that attack them from all sides and give them no rest (Figure 77). This amusing phenomenon, known as *Brownian motion*, named after the English botanist Robert Brown, who first noticed it more than a century ago in a study of tiny plant spores, is of quite general nature and can be observed in the study of any kind of sufficiently small particles suspended in any kind of liquid, or of microscopic particles of smoke and dust floating in the air.

If we heat the liquid the wild dance of tiny particles suspended in it becomes more violent; with cooling the intensity of the motion noticeably subsides. This leaves no doubt that we are actually watching here the effect of the hidden thermal motion of matter, and that what we usually call temperature is nothing else but a measurement of the degree of molecular agitation. By studying the dependence of Brownian motion on temperature, it was found that at the temperature of $-273°$ C or $-459°$ F,

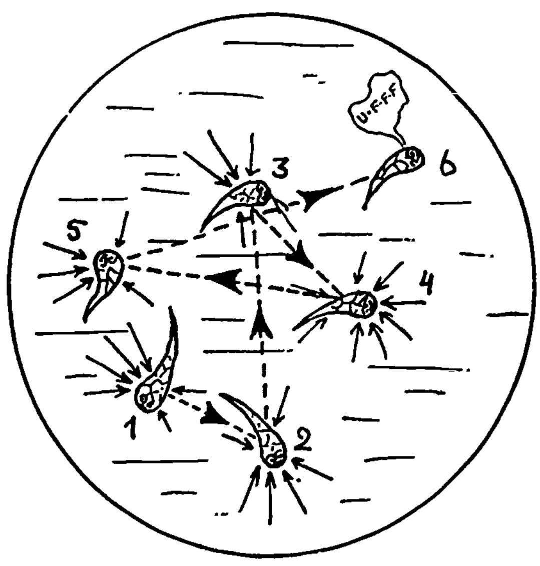


FIGURE 77

Six consecutive positions of a bacterium which is being tossed around by molecular impacts (physically correct; bacteriologically not quite so).

thermal agitation of matter completely ceases, and all its molecules come to rest. This apparently is the lowest temperature and it has received the name of *absolute zero*. It would be an absurdity to speak about still lower temperatures since apparently there is no motion slower than absolute rest!

Near the absolute zero temperature the molecules of any substance have so little energy that the cohesive forces acting upon them cement them together into one solid block, and all they can do is only quiver slightly in their frozen state. When the temperature rises the quivering becomes more and more intense, and at a certain stage our molecules obtain some freedom of motion and are able to slide by one another. The rigidity of the frozen substance disappears, and it becomes a fluid. The temperature at which the melting process takes place depends on the strength of the cohesive forces acting upon the molecules. In some materials such as hydrogen, or a mixture of nitrogen and oxygen which form atmospheric air, the cohesion of molecules is very weak, and the thermal agitation breaks up the frozen state at comparatively low temperatures. Thus hydrogen exists in the frozen state only at temperatures below 14° abs (i.e., below −259° C), whereas solid oxygen and nitrogen melt at 55° abs and 64° abs, respectively (i.e. −218° C and −209° C). In other substances the cohesion between molecules is stronger and they remain solid up to higher temperatures: thus pure alcohol remains frozen up to −130° C, whereas frozen water (ice) melts only at 0° C. Other substances remain solid up to much higher temperatures; a piece of lead will melt only at +327° C, iron at +1535° C, and the rare metal known as osmium remains solid up to the temperature of +2700° C. Although in the solid state of matter the molecules are strongly bound to their places, it does not mean at all that they are not affected by thermal agitation. Indeed, according to the fundamental law of heat motion, the amount of energy in every molecule is the same for all substances, solid, liquid, or gaseous at a given temperature, and the difference lies only in the fact that whereas in some cases this energy suffices to tear off the molecules from their fixed positions and let them travel around, in other cases they can only quiver on the same spot as angry dogs restricted by short chains.

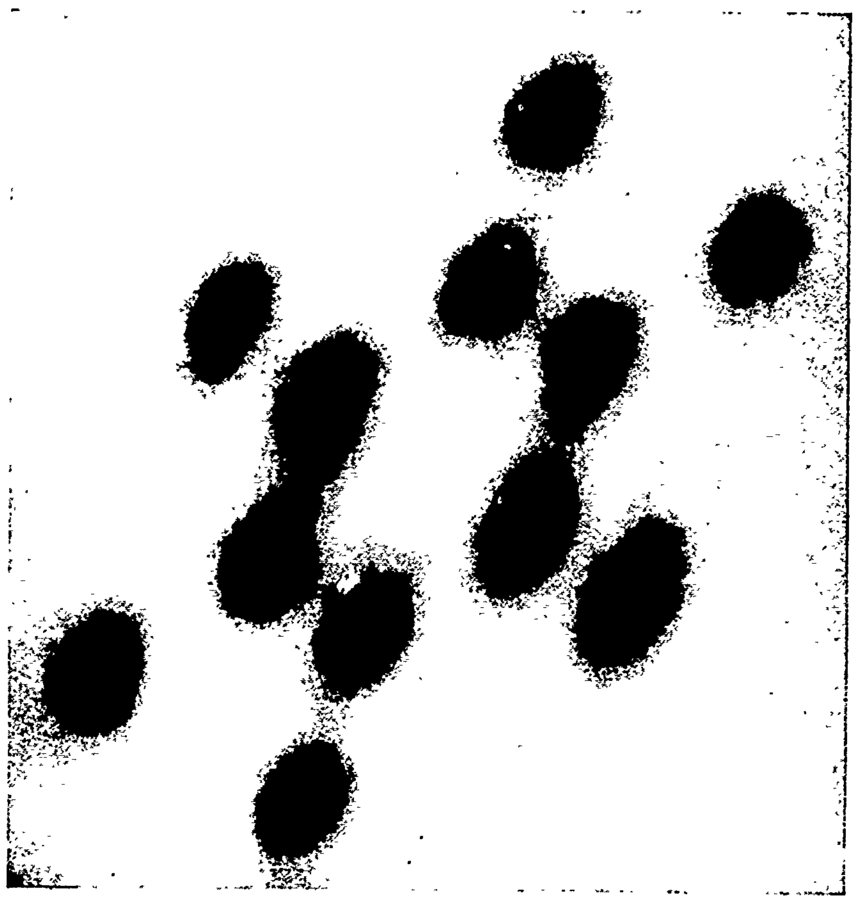
This thermal quivering or vibration of molecules forming a solid body can be easily observed in the X-ray photographs described in the previous chapter. We have seen indeed that, since taking a picture of molecules in a crystal lattice requires a considerable time, it is essential that they should not move away from their fixed positions during the exposure. But a constant quivering around the fixed position is not conducive to good photography, and results in a somewhat blurred picture. This oxygen and nitrogen at $-183°$ C and $-196°$ C, alcohol at $+78°$ C, lead at $+1620°$ C, iron at $+3000°$ C and osmium only above $+5300°$ C.[1]

The breaking up of the beautiful crystalline structure of solid bodies forces the molecules first to crawl around one another like a pack of worms, and then to fly apart as though they were a flock of frightened birds. But this latter phenomenon still does not represent the limit of the destructive power of increasing thermal motion. If the temperature rises still farther the very existence of the molecules is threatened, since the ever increasing violence of intermolecular collisions is capable of breaking them up into separate atoms. This *thermal dissociation*, as it is called, depends on the relative strength of the molecules subjected to it. The molecules of some organic substances will break up into separate atoms or atomic groups at temperatures as low as a few hundred degrees. Other more sturdily built molecules, such as those of water, will require a temperature of over a thousand degrees to be destroyed. But when the temperature rises to several thousand degrees no molecules will be left and the matter will be a gaseous mixture of pure chemical elements.

This is the situation on the surface of our sun where the temperature ranges up to 6000° C. On the other hand, in the comparatively cooler atmospheres of the red stars,[2] some of the molecules are still present, a fact that has been demonstrated by the methods of spectral analysis.

The violence of thermal collisions at high temperatures not only breaks up the molecules into their constituent atoms, but also damages the atoms themselves by chipping off their outer electrons. This *thermal ionization* becomes more and more pronounced when the temperature rises into tens and hundreds of thousands of degrees, and reaches completion at a few million degrees above zero. At these tremendously hot temperatures, which are high above everything that we can produce in our laboratories but which are common in the interiors of stars and in particular inside our sun, the atoms as such cease to exist. All electronic shells are completely stripped off, and the matter

[1] All values given for atmospheric pressure.
[2] See Chapter XI.



(Courtesy of Dr. M. L. Huggins, Eastman Kodak Laboratory.)

PLATE I

Photograph of Hexamethylbenzene molecule magnified 175,000,000 times.

becomes a mixture of bare nuclei and free electrons rushing wildly through space and colliding with one another with tremendous force. However, in spite of the complete wreckage of atomic bodies, the matter still retains its fundamental chemical

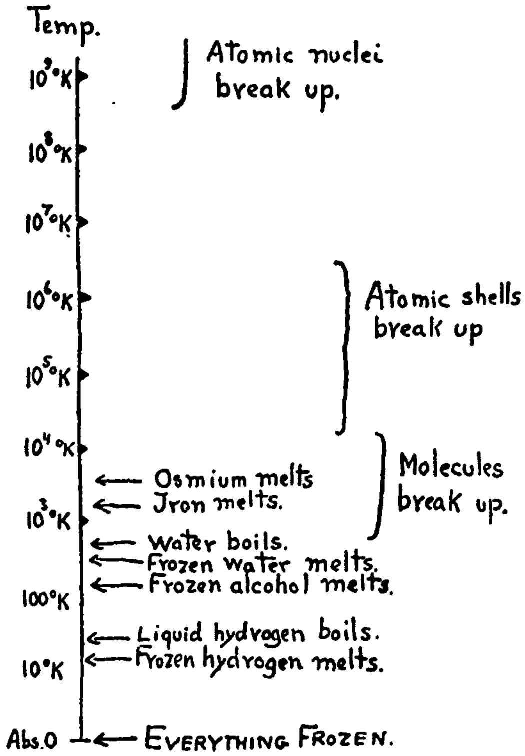


FIGURE 79

The destructive effect of temperature.

characteristics, inasmuch as atomic nuclei remain intact. If the temperature drops, the nuclei will recapture their electrons and the integrity of atoms will be reestablished.

In order to attain complete thermal dissociation of matter, that is to break up the nuclei themselves into the separate nucleons (protons and neutrons) the temperature must go up to at least several billion degrees. Even inside the hottest stars we do not find such high temperatures, though it seems very likely that temperatures of that magnitude did exist several billion years ago when our universe was still young. We shall return to this exciting question in the last chapter of this book.

Thus we see that the effect of thermal agitation is to destroy step by step the elaborate architecture of matter based on the law of quantum, and to turn this magnificent building into a mess of widely moving particles rushing around and colliding with one another without any apparent law or regularity.

2. *HOW CAN ONE DESCRIBE DISORDERLY MOTION?*

It would be, however, a grave mistake to think that because of the irregularity of thermal motion it must remain outside the scope of any possible physical description. Indeed the fact itself that thermal motion is *completely irregular* makes it subject to a new kind of law, the *Law of Disorder* better known as the *Law of Statistical Behavior.* In order to understand the above statement let us turn our attention to the famous problem of a "Drunkard's Walk." Suppose we watch a drunkard who has been leaning against a lamp post in the middle of a large paved city square (nobody knows how or when he got there) and then has suddenly decided to go nowhere in particular. Thus off he goes, making a few steps in one direction, then some more steps in another, and so on and so on, changing his course every few steps in an entirely unpredictable way (Figure 80). How far will be our drunkard from the lamp post after he has executed, say, a hundred phases of his irregular zigzag journey? One would at first think that, because of the unpredictability of each turn, there is no way of answering this question. If, however, we consider the problem a little more attentively we will find that, although we really cannot tell where the drunkard will be at the end of his walk, we can answer the question about his *most probable* distance from the lamp post after a given large number of turns. In order to approach this problem in a vigorous mathematical way let us draw on the pavement two co-ordinate axes with the origin in the lamp post; the X-axis coming toward us and the Y-axis to the right. Let R be the distance of the drunkard from the lamp post after the total of N zigzags (14 in Figure 80). If now X_N and Y_N are the projections of the N^{th} leg of the track on the corresponding axis, the Pythagorean theorem gives us apparently:

$$R^2 = (X_1 + X_2 + X_3 \cdots + X_N)^2 + (Y_1 + Y_2 + Y_3 + \cdots Y_N)^2$$

where X's and Y's are positive or negative depending on whether our drunkard was moving to or from the post in this particular

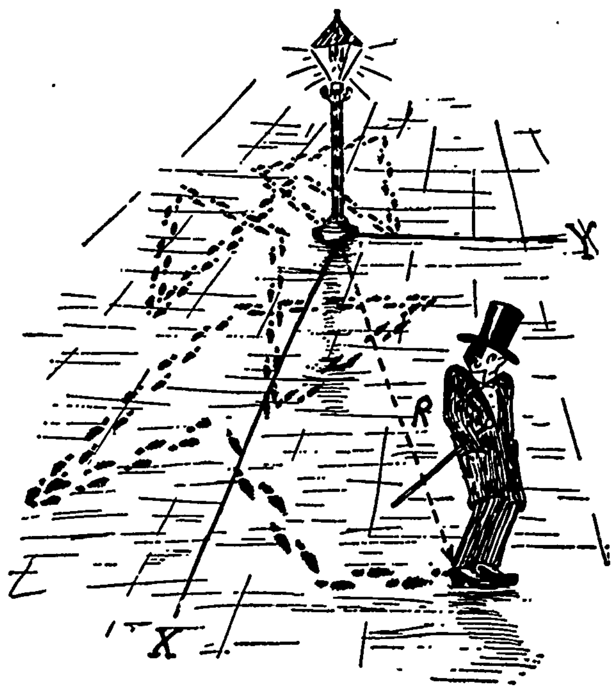


FIGURE 80
Drunkard's walk.

phase of his walk. Notice that since his motion is *completely disorderly*, there will be about as many positive values of X's and Y's as there are negative. In calculating the value of the square of the terms in parentheses according to the elementary rules of algebra, we have to multiply each term in the bracket by itself and by each of all other terms.

Thus:

$$(X_1+X_2+X_3+\cdots X_N)^2$$
$$=(X_1+X_2+X_3+\cdots X_N)\ \ (X_1+X_2+X_3+\cdots X_N)$$
$$=X_1^2+X_1X_2+X_1X_3+\cdots X_2^2+X_1X_2+\cdots X_N^2$$

This long sum will contain the square of all X's ($X_1^2, X_2^2 \cdots X_N^2$), and the so-called "mixed products" like X_1X_2, X_2X_3, etc.

So far it is simple arithmetic, but now comes the statistical point based on the disorderliness of the drunkard's walk. Since he was moving entirely at random and would just as likely make a step toward the post as away from it, the values of X's have a fifty-fifty chance of being either positive or negative. Consequently in looking through the "mixed products" you are likely to find always the pairs that have the same numerical value but opposite signs thus canceling each other, and the larger the total number of turns, the more likely it is that such a compensation takes place. What will be left are only the squares of X's, since the square is always positive. Thus the whole thing can be written as $X_1^2+X_2^2+\cdots\cdot X_N^3=N\ X^2$ where X is the average length of the projection of a zigzag link on the X-axis.

In the same way we find that the second bracket containing Y's can be reduced to: NY^2, Y being the average projection of the link on the Y-axis. It must be again repeated here that what we have just done is not strictly an algebraic operation, but is based on the statistical argument concerning the mutual cancellation of "mixed products" because of the random nature of the pass. For the most probable distance of our drunkard from the lamp post we get now simply:

$$R^2=N\ (X^2+Y^2)$$

or

$$R=\sqrt{N}\cdot\sqrt{X^2+Y^2}$$

But the average projections of the link on both axes is simply a 45° projection, so that $\sqrt{X^2+Y^2}$ right is (again because of the Pythagorean theorem) simply equal to the average length of the link. Denoting it by 1 we get:

$$R=1\cdot\sqrt{N}$$

In plain words our result means: *the most probable distance of* diffusion is a rather slow process; when you put a lump of sugar into your cup of tea you had better stir it rather than wait until the sugar molecules have been spread throughout by their own motion.

Just to give another example of the process of diffusion, which is one of the most important processes in molecular physics, let us consider the way in which heat is propagated through an iron poker, one end of which you put into the fireplace. From your own experience you know that it takes quite a long time until the other end of the poker becomes uncomfortably hot, but you probably do not know that the heat is carried along the metal stick by the process of diffusion of electrons. Yes, an ordinary iron poker is actually stuffed with electrons, and so is any metallic object. The difference between a metal, and other materials, as for example glass, is that the atoms of the former lose some of their outer electrons, which roam all through the metallic lattice, being involved in irregular thermal motion, in very much the same way as the particles of ordinary gas.

The surface forces on the outer boundaries of a piece of metal prevent these electrons from getting out,[3] but in their motion inside the material they are almost perfectly free. If an electric force is applied to a metal wire, the free unattached electrons will rush headlong in the direction of the force producing the phenomenon of electric current. The nonmetals on the other hand are usually good insulators because all their electrons are bound to be atoms and thus cannot move freely.

When one end of a metal bar is placed in the fire, the thermal motion of free electrons in this part of the metal is considerably increased, and the fast-moving electrons begin to diffuse into the other regions carrying with them the extra energy of heat. The process is quite similar to the diffusion of dye molecules through water, except that instead of having two different kinds of particles (water molecules and dye molecules) we have here the *diffusion of hot electron gas into the region occupied by cold electron gas.* The drunkard's walk law applies here, however, just

[3] When we bring a metal wire to a high temperature, the thermal motion of electrons in its inside becomes more violent and some of them come out through the surface. This is the phenomenon used in electron tubes and familiar to all radio amateurs.

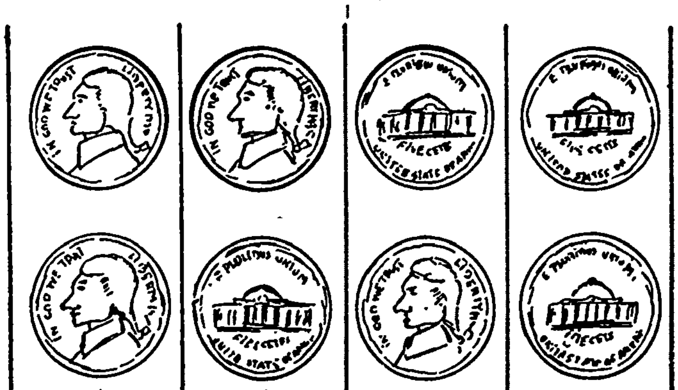
as well and the distances through which the heat propagates along a metal bar increase as the square roots of corresponding times.

As our last example of diffusion we shall take an entirely different case of cosmic importance. As we shall learn in the following chapters the energy of our sun is produced deep in its interior by the alchemic transformation of chemical elements. This energy is liberated in the form of intensive radiation, and the "particles of light," or the light quanta begin their long journey through the body of the sun towards its surface. Since light moves at a speed of 300,000 km per second, and the radius of the sun is only 700,000 km it would take a light quantum only slightly over two seconds to come out provided it moved without any deviations from a straight line. However, this is far from being the case; on their way out the light quanta undergo innumerable collisions with the atoms and electrons in the material of the sun. The free pass of a light quantum in solar matter is about a centimeter (much longer than a free pass of a molecule!) and since the radius of the sun is 70,000,000,000 cm, our light quantum must make $(7 \cdot 10^{10})^2$ or $5 \cdot 10^{21}$ drunkard's steps to reach the surface. Since each step requires $\frac{1}{3 \cdot 10^{10}}$ or $3 \cdot 10^{-9}$ sec, the entire time of travel is $3 \cdot 10^{-9} \times 5 \cdot 10^{21} = 1.5 \cdot 10^{13}$ sec or about 200,000 yr! Here again we see how slow the process of diffusion is. It takes light 2000 centuries to travel from the center of the sun to its surface, whereas after coming into empty intraplanetary space and traveling along a straight line it covers the entire distance from the sun to the earth in only eight minutes!

3. *COUNTING PROBABILITIES*

This case of diffusion represents only one simple example of the application of the statistical law of probability to the problem of molecular motion. Before we go farther with that discussion, and make the attempt to understand the all-important *Law of Entropy*, which rules the thermal behavior of every material body, be it a tiny droplet of some liquid or the giant universe of stars, we have first to learn more about the ways in which the probability of different simple or complicated events can be calculated.

By far the simplest problem of probability calculus arises when you toss a coin. Everybody knows that in this case (without cheating) there are equal chances to get heads or tails. One usually says that there is a *fifty-fifty chance* for heads or tails, but it is more customary in mathematics to say that the chances are *half and half.* If you add the chances of getting heads and getting tails you get $\frac{1}{2}+\frac{1}{2}=1$. Unity in the theory of probability means a certainty; you are in fact quite certain that in tossing a



1st. 2d. 3d. 4th

FIGURE 83

Four possible combinations in tossing two coins.

coin you get either heads or tails, unless it rolls under the sofa and vanishes tracelessly.

Suppose now you drop the coin twice in succession or, what is the same, you drop 2 coins simultaneously. It is easy to see that you have here 4 different possibilities shown in Figure 83.

In the first case you get heads twice, in the last case tails twice, whereas the two intermediate cases lead to the same result since it does not matter to you in which order (or in which coin) heads or tails appear. Thus you say that the chances of getting heads twice are 1 out of 4 or $\frac{1}{4}$ the chances of getting tails twice are also $\frac{1}{4}$, whereas the chances of heads once and tails once are 2 out of 4 or $\frac{1}{2}$. Here again $\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$ meaning that you

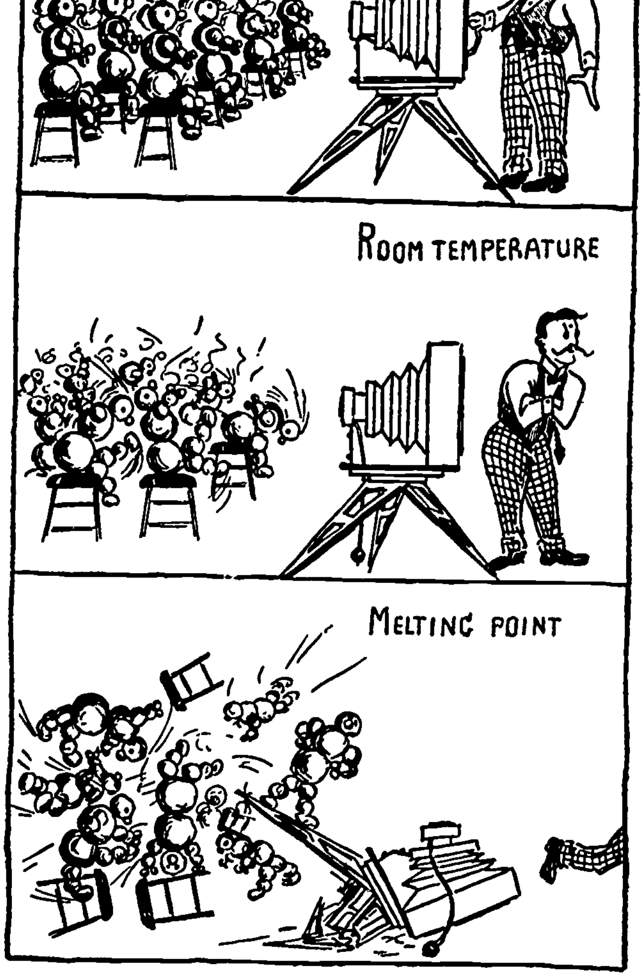


Figure 78

are certain to get one of the 3 possible combinations. Let us see now what happens if we toss the coin 3 times. There are altogether 8 possibilities summarized in the following table:

First tossing	h	h	h	h	t	t	t	t
Second	h	h	t	t	h	h	t	t
Third	h	t	h	t	h	t	h	t
	I	II	II	III	II	III	III	IV

If you inspect this table you find that there is 1 chance out of 8 of getting heads three times, and the same of getting tails three times. The remaining possibilities are equally divided between heads twice and tails once, or heads once and tails twice, with the probability three eighths for each event.

Our table of different possibilities is growing rather rapidly, but let us take one more step by tossing 4 times. Now we have the following 16 possibilities:

First tossing	h	h	h	h	h	h	h	h	t	t	t	t	t	t	t	t
Second	h	h	h	h	t	t	t	t	h	h	h	h	t	t	•	t
Third	h	h	t	t	h	h	t	t	h	h	t	t	h	h	t	t
Fourth	h	t	h	t	h	t	h	t	h	t	h	t	h	t	h	t
	I	II	II	III	II	III	III	IV	II	III	III	IV	III	IV	IV	V

Here we have $\frac{1}{16}$ for the probability of heads four times, and exactly the same for tails four times. The mixed cases of heads three times and tails once or tails three times and heads once substance.

After solid material melts, the molecules still remain together, since the thermal agitation, though strong enough to dislocate them from the fixed position in the crystalline lattice, is not yet sufficient to take them completely apart. At still higher temperatures, however, the cohesive forces are not able to hold the molecules together any more and they fly apart in all directions unless prevented from doing so by the surrounding walls. When this happens, of course, the result is matter in a gaseous state. As in the melting of a solid, the evaporation of liquids takes place at different temperatures for different materials, and the substances with a weaker internal cohesion will turn into vapor at lower temperatures than those in which cohesive forces are stronger. In this case the process also depends rather essentially on the pressure under which the liquid is kept, since the outside pressure evidently helps the cohesive forces to keep the molecules together. Thus, as everybody knows, water in a tightly closed kettle boils at a lower temperature than will water in an open one. On the other hand, on the top of high mountains, where atmospheric pressure is considerably less, water will boil well below 100° C. It may be mentioned here that by measuring the temperature at which water will boil, one can calculate atmospheric pressure and consequently the distance above sea level of a given location.

But do not follow the example of Mark Twain who, according to his story, once decided to put an aneroid barometer into a boiling kettle of pea soup. This will not give you any idea of the elevation, and the copper oxide will make the soup taste bad.

The higher the melting point of a substance, the higher is its boiling point. Thus liquid hydrogen boils at −253° C, liquid

the probability of getting heads three or four times in succession is the product of probabilities of getting it separately in each tossing ($\frac{1}{8}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$; $\frac{1}{16}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$). Thus if somebody asks you what the chances are of getting heads each time in ten tossings you can easily give the answer by multiplying $\frac{1}{2}$ by $\frac{1}{2}$ ten times. The result will be .00098, indicating that the chances are very low indeed: about one chance out of a thousand! Here we have the rule of "multiplication of probabilities," which states that *if you want several different things, you may determine the mathematical probability of getting them by multiplying the mathematical probabilities of getting the several individual ones.* If there are many things you want, and each of them is not particularly probable, the chances that you get them *all* are discouragingly low!

There is also another rule, that of the "addition of probabilities," which states that *if you want only one of several things (no matter which one), the mathematical probability of getting it is the sum of mathematical probabilities of getting individual items on your list.*

This can be easily illustrated in the example of getting an equal division between heads and tails in tossing a coin twice. What you actually want here is *either* "heads once, tails twice" or "tails twice, heads once." The probability of each of the above combinations is $\frac{1}{4}$, and the probability of getting either one of them is $\frac{1}{4}$ plus $\frac{1}{4}$ or $\frac{1}{2}$. Thus: If you want "that, *and* that, *and* that . . ."

three, four, ten, and a hundred tossings. You see that with the increasing number of tossings the probability curve becomes sharper and sharper and the maximum at fifty-fifty ratio of heads and tails becomes more and more pronounced.

Thus whereas for 2 or 3, or even 4 tosses, the chances to have heads each time or tails each time are still quite appreciable, in 10 tosses even 90 per cent of heads or tails is very improbable.

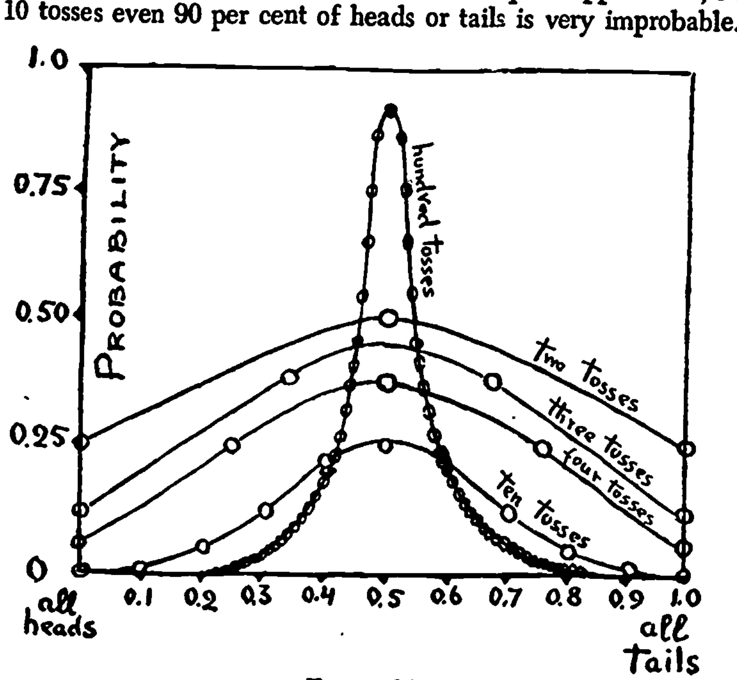


FIGURE 84
Relative number of tails and heads.

For a still larger number of tosses, say 100 or 1000, the probability curve becomes as sharp as a needle, and the chances of getting even a small deviation from fifty-fifty distribution becomes practically nil.

Let us now use the simple rules of probability calculus that we have just learned in order to judge the relative probabilities of various combinations of five playing cards which one encounters in the well-known game of poker.

In case you do not know, each player in this game is dealt 5 cards and the one who gets the highest combination takes the bank. We shall omit here the additional complications arising from the possibility of exchanging some of your cards with the hope of getting better ones, and the psychological strategy of bluffing your opponents into submission by making them believe that you have much better cards than you actually have. Although this bluffing actually is the heart of the game, and once led the famous Danish physicist Niels Bohr to propose an entirely new type of game in which no cards are used, and the players simply bluff one another by talking about the imaginary combinations they have, it lies entirely outside the domain of probability calculus, being a purely psychological matter.

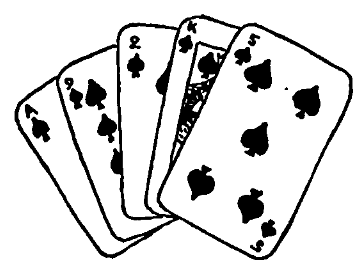


FIGURE 85
A flush (of spades).

In order to get some exercise in probability calculus, let us calculate the probabilities of some of the combinations in the game of poker. One of these combinations is called a "flush" and represents 5 cards all of the same suit (Figure 85).

If you want to get a flush it is immaterial what the first card you get is, and one has only to calculate the chances that the other four will be of the same suit. There are altogether 52 cards in the pack, 13 cards of each suit,[4] so that after you get your first card, there remain in the pack 12 cards of the same suit. Thus the chances that your second card will be of the proper suit are 12/51. Similarly the chances that the third, fourth, and fifth cards

[4] We omit here the complications arising from the presence of the "joker," an extra card which can be substituted for any other card according to the desire of the player.

pairs, with coinciding birthdays. As a matter of fact, there are more chances that there is such a coincidence than that there is not.

You can verify that fact by making a birthday list including about 24 persons, or more simply, by comparing the birth dates of 24 persons whose names appear consecutively on any pages of some such reference book as "Who's Who in America," opened at random. Or the probabilities can be ascertained by using the simple rules of probability calculus with which we have become acquainted in the problems of coin tossing and poker.

Suppose we try first to calculate the chances that in a company of twenty-four persons everyone has a different birth date. Let us ask the first person in the group what is his birth date; of course this can be any of the 365 days of the year. Now, what is the chance that the birth date of the second person we approach is *different* from that of the first? Since this (second) person could have been born on any day of the year, there is one chance out of 365 that his birth date coincides with that of the first one, and 364 chances out of 365 (i.e., the probability of 364/365) that it does not. Similarly, the probability that the third person has a birth date different from that of either the first or second is 363/365, since two days of the year have been excluded. The probabilities that the next persons we ask have different birth dates from the ones we have approached before are then: 362/365, 361/365, 360/365 and so on up to the last person for whom the probability is $\frac{(365-23)}{365}$ or $\frac{342}{365}$.

Since we are trying to learn what the probability is that one of these coincidences of birth dates exists, we have to multiply all the above fractions, thus obtaining for the probability of all the persons having different birth dates the value:

$$\frac{364}{365}\times\frac{363}{365}\times\frac{362}{365}\times\cdots\frac{342}{365}.$$

One can arrive at the product in a few minutes by using certain methods of higher mathematics, but if you don't know them you can do it the hard way by direct multiplication,[5] which would not take so very much time. The result is 0.46, indicating

[5] Use a logarithmic table or slide rule if you can!

that the probability that there will be no coinciding birthdays is slightly less than one half. In other words there are only 46 chances in 100 that no two of your two dozen friends will have birthdays on the same day, and 54 chances in 100 that two or more will. Thus if you have 25 or more friends, and have never been invited to two birthday parties on the same date you may conclude with a high degree of probability that either most of your friends do not organize their birthday parties, or that they do not invite you to them!

The problem of coincident birthdays represents a very fine example of how a common-sense judgment concerning the probabilities of complex events can be entirely wrong. The author has put this question to a great many people, including many prominent scientists, and in all cases except one[6] was offered bets ranging from 2 to 1 to 15 to 1 that no such coincidence will occur. If he had accepted all these bets he would be a rich man by now!

It cannot be repeated too often that if we calculate the probabilities of different events according to the given rules and pick out the most probable of them, we are not at all sure that this is exactly what is going to happen. Unless the number of tests we are making runs into thousands, millions or still better into billions, the predicted results are only "likely" and not at all "certain." This slackening of the laws of probability when dealing with a comparatively small number of tests limits, for example, the usefulness of statistical analysis for deciphering various codes and cryptograms which are limited only to comparatively short notes. Let us examine, for example, the famous case described by Edgar Allan Poe in his well-known story "The Gold Bug." He tells us about a certain Mr. Legrand who, strolling along a deserted beach in South Carolina, picked up a piece of parchment half buried in the wet sand. When subjected to the warmth of the fire burning gaily in Mr. Legrand's beach hut, the parchment revealed some mysterious signs written in ink which was invisible when cold, but which turned red and was quite legible when heated. There was a picture of a skull, suggesting that the docu-

[6] This exception was, of course, a Hungarian mathematician (see the beginning of the first chapter of this book).

yards. If he had not turned, but had gone straight, he would be a hundred yards away—which shows that it is definitely advantageous to be sober when taking a walk.

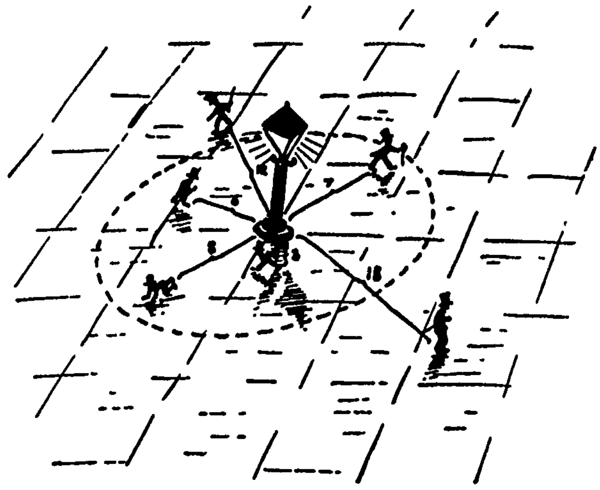


FIGURE 81

Statistical distribution of six walking drunkards around the lamp post.

The statistical nature of the above example is revealed by the fact that we refer here only to the *most probable* distance and not to the exact distance in each individual case. In the case of an individual drunkard it may happen, though this is not very probable, that he does not make any turns at all and thus goes far away from the lamp post along the straight line. It may also happen, that he turns each time by, say, 180 degrees thus returning to the lamp post after every second turn. But if a large number of drunkards all start from the same lamp post walking in different zigzag paths and not interfering with one another

ment was written by a pirate, the head of a goat, proving beyond any doubt that the pirate was none other than the famous Captain Kidd, and several lines of typographical signs apparently indicating the whereabouts of a hidden treasure (see Figure 87).

We take it on the authority of Edgar Allan Poe that the pirates of the seventeenth century were acquainted with such typographical signs as semicolons and quotation marks, and such others as: ‡, +, and ¶.

Being in need of money, Mr. Legrand used all his mental powers in an attempt to decipher the mysterious cryptogram and

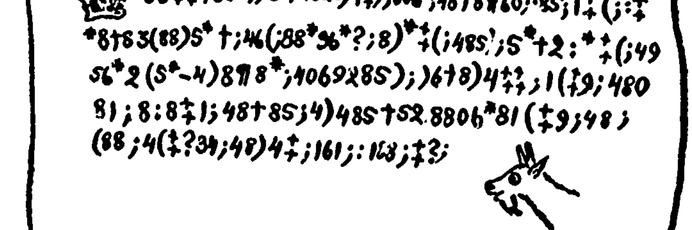


FIGURE 87

Captain Kidd's Message.

and the larger the number of turns they make in their disorderly walk, the more accurate is the rule.

Now substitute for the drunkards some microscopic bodies such as plant spores or bacteria suspended in liquid, and you will have exactly the picture that the botanist Brown saw in his microscope. True the spores and bacteria are not drunk, but, as we have said above, they are being incessantly kicked in all possible directions by the surrounding molecules involved in thermal motion, and are therefore forced to follow exactly the same irregular zigzag trajectories as a person who has completely lost his sense of direction under the influence of alcohol.

If you look through a microscope at the Brownian motion of a large number of small particles suspended in a drop of water, you will concentrate your attention on a certain group of them that are at the moment concentrated in a given small region (near the "lamp post"). You will notice that in the course of time they become gradually dispersed all over the field of vision, and that their average distance from the origin increases in proportion to the square root of the time interval as required by the mathematical law by which we calculated the distance of the drunkard's walk.

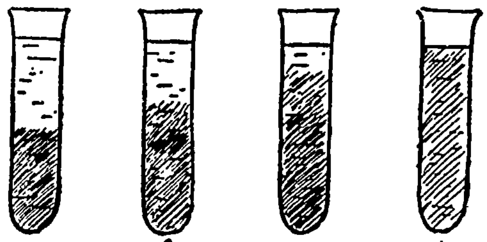
The same law of motion pertains, of course, to each separate molecule in our drop of water; but you cannot see separate molecules, and even if you could, you wouldn't be able to distinguish between them. To make such motion visible one must use two different kinds of molecules distinguishable for example by their different colors. Thus we can fill one half of a chemical test tube with a water solution of potassium permanganate, which will give to the water a beautiful purple tint. If we now pour on the top of it some clear fresh water, being careful not to mix up the two layers, we shall notice that the color gradually penetrates the clear water. If you wait sufficiently long you will find that all the

probable and not at all certain. In fact if the secret message had been "You will find a lot of gold and coins in an iron box in woods two thousand yards south from an old hut on Bird Island's north tip" it would not have contained a single "e"! But the laws of chance were favorable to Mr. Legrand, and his guess was really correct.

Having met with success in the first step, Mr. Legrand became overconfident and proceeded in the same way by picking up the letters in the order of the probability of their occurrence. In the following table we give the symbols appearing in Captain Kidd's message in the order of their relative frequency of use:

Of the character 8 there are	33	e	e
;	26	a	t
4	19	o	h
‡	16	i	o
(	16	d	r
*	13	h	n
5	12	n	a
6	11	r	i
†	8	s	d
1	8	t	
0	6		

Since in water the molecules are packed rather tightly (in contrast to the arrangement of those in a gas) the average free path of each molecule between two successive collisions is very short, being only about one hundred millionths of an inch. Since on the other hand the molecules at room temperature move with the speed of about one tenth of a mile per second, it takes only one million-millionth part of a second for a molecule to go from one collision to another. Thus in the course of a single second



a b c d

FIGURE 82

each dye molecule will be engaged in about a million million consecutive collisions and will change its direction of motion as many times. The average distance covered during the first second will be one hundred millionth of an inch (the length of free path) times the square root of a million millions. This gives the average diffusion speed of only one hundredth of an inch per second; a rather slow progress considering that if it were not deflected by collisions, the same molecule would be a tenth of a mile away! If you wait 100 sec, the molecule will have struggled through 10 times ($\sqrt{100}=10$) as great distance, and in 10,000 sec, that is, in about 3 hr, the diffusion will have carried the coloring 100 times farther ($\sqrt{10000}=100$), that is, about 1 in. away. Yes,

English language. Therefore it was logical to assume that the signs listed in the broad column to the left stood for the letters listed opposite them in the first narrow column to the right. But using this arrangement we find that the beginning of Captain Kidd's message reads: *ngiisgunddrhaoecr* . . .

No sense at all!

What happened? Was the old pirate so tricky as to use special words that do not contain letters that follow the same rules of frequency as those in the words normally used in the English language? Not at all; it is simply that the text of the message is not long enough for good statistical sampling and the most probable distribution of letters does not occur. Had Captain Kidd hidden his treasure in such an elaborate way that the instructions for its recovery occupied a couple of pages, or, still better an entire volume, Mr. Legrand would have had a much better chance to solve the riddle by applying the rules of frequency.

If you drop a coin 100 times you may be pretty sure that it will fall with the head up about 50 times, but in only 4 drops you may have heads three times and tails once or vice versa. To make a rule of it, the larger the number of trials, the more accurately the laws of probability operate.

Since the simple method of statistical analysis failed because of an insufficient number of letters in the cryptogram, Mr. Legrand had to use an analysis based on the detailed structure of different words in the English language. First of all he strength-degrees and thirteen minutes northeast by north. Main branch seventh limb east side. Shoot from the left eye of the death's head. A bee-line from the tree through the shot fifty feet out."

The correct meaning of the different characters as finally deciphered by Mr. Legrand is shown in the second column of the table on page 217, and you see that they do not correspond exactly to the distribution that might reasonably be expected on the basis of the laws of probability. It is, of course, because the text is too short and therefore does not furnish an ample opportunity

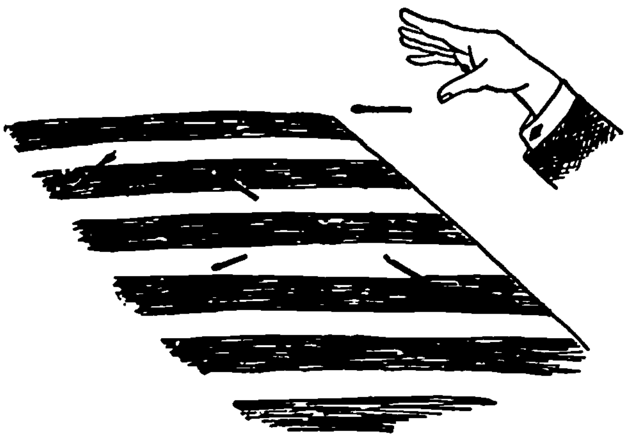


FIGURE 88

for the laws of probability to operate. But even in this small "statistical sample" we can notice the tendency for the letters to arrange themselves in the order required by the theory of probability, a tendency that would become almost an unbreakable rule if the number of letters in the message were much larger.

There seems to be only one example (excepting the fact that insurance companies do not break up) in which the predictions of the theory of probability have actually been checked by a very large number of trials. This is a famous problem of the American flag and a box of kitchen matches.

To tackle this particular problem of probability you will need an American flag, that is, the part of it consisting of red and white stripes; if no flag is available just take a large piece of paper and draw on it a number of parallel and equidistant lines. Then you need a box of matches—any kind of matches, provided they are shorter than the width of the stripes. Next you will need a Greek pi, which is not something to eat, but just a letter of the Greek alphabet equivalent to our "p." It looks like this: π. In addition to being a letter of the Greek alphabet, it is used to

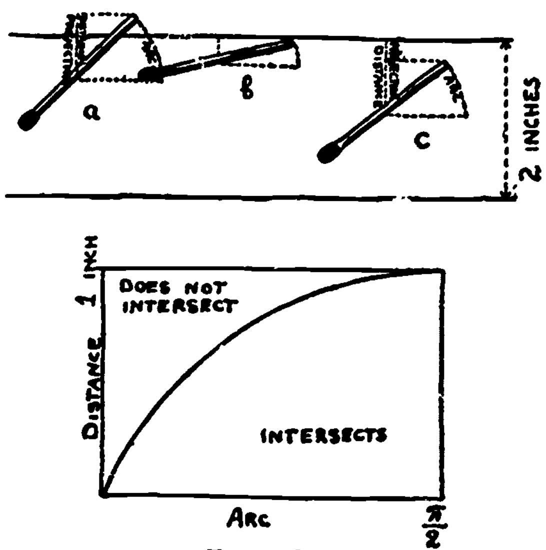


FIGURE 89

signify the ratio of the circumference of a circle to its diameter. You may know that numerically it equals 3.1415926535 . . . (many more digits are known, but we shall not need them all.)

Now spread the flag on a table, toss a match in the air and watch it fall on the flag (Figure 88). It may fall in such a way that it all remains within one stripe, or it may fall across the boundary between two stripes. What are the chances that one or another will take place?

Following our procedure in ascertaining other probabilities, we must first count the number of cases that correspond to one or another possibility.

But how can you count all the possibilities when it is clear that a match can fall on a flag in an infinite number of different ways?

Let us examine the question a little more closely. The position of the fallen match in respect to the stripe on which it falls can be characterized by the distance of the middle of the match from the nearest boundary line, and by the angle that the match forms with the direction of the stripes in Figure 89. We give three typical examples of fallen matches, assuming, for the sake of simplicity, that the length of the match equals the width of the stripe, each being, say, two inches. If the center of the match is rather close to the boundary line, and the angle is rather large (as in case *a*) the match will intersect the line. If, on the contrary, the angle is small (as in case *b*) or the distance is large (as in case *c*) the match will remain within the boundaries of one stripe. More exactly we may say that the match will intersect the line if the projection of the half-of-the-match on the vertical direction is larger than the half width of the stripe (as in case *a*), and that no intersection will take place if the opposite is true (as in case *b*). The above statement is represented graphically on the diagram in the lower part of the picture. We plot on the horizontal axis (abscissa) the angle of the fallen match as given by the length of the corresponding arc of radius 1. On the vertical axis (ordinate) we plot the length of the projection of the half-match length on the vertical direction; in trigonometry this length is known as the *sinus* corresponding to the given arc. It is clear that the sinus is zero when the arc is zero since in that case the match occupies a horizontal position. When the arc is $\frac{1}{2}\pi$, which corresponds to a straight angle,[7] the sinus is equal to unity, since the match occupies a vertical position and thus coincides with its projection. For intermediate values of the arc the sinus is given by the familiar mathematical wavy curve known as sinusoid. (In Figure 89 we have only one quarter of a complete wave in the interval between 0 and $\pi/2$.)

[7] The circumference of a circle with the radius 1 is π times its diameter or 2π. Thus the length of one quadrant of a circle is $2\pi/4$ or $\pi/2$.

Having constructed this diagram we can use it with convenience for estimating the chances that the fallen match will or will not cross the line. In fact, as we have seen above (look again at the three examples in the upper part of Figure 89) the match will cross the boundary line of a stripe if the distance of the center of the match from the boundary line is less than the corresponding projection, that is, less than the sinus of the arc. That means that in plotting that distance and that arc in our diagram we get a point *below* the sinus line. On the contrary the match that falls entirely within the boundaries of a stripe will give a point *above* the sinus line.

Thus, according to our rules for calculating probabilities, the chances of intersection will stand in the same ratio to the chances of nonintersection as the area below the curve does to the area above it; or the probabilities of the two events may be calculated by dividing the two areas by the entire area of the rectangle. It can be proved mathematically (*cf.* Chapter II) that the area of the sinusoid presented in our diagram equals exactly 1. Since the total area of the rectangle is $\frac{\pi}{2} \times 1 = \frac{\pi}{2}$ we find the probability that the match will fall across the boundary (for matches equal in length to the stripe width) is: $\frac{1}{\pi/2} = \frac{2}{\pi}$.

The interesting fact that π pops up here where it might be least expected was first observed by the eighteenth century scientist Count Buffon, and so the match-and-stripes problem now bears his name.

An actual experiment was carried out by a diligent Italian mathematician, Lazzerini, who made 3408 match tosses and observed that 2169 of them intersected the boundary line. The exact record of this experiment, checked with the Buffon formula, substitutes for π a value of $\frac{2+3408}{2169}$ or 3.1415929, differing from the exact mathematical value only in the seventh decimal place!

This represents, of course, a most amusing proof of the validity of the probability laws, but not more amusing than the determination of a number "2" by tossing a coin several thousand times and dividing the total number of tosses by the number of times heads come up. Sure enough you get in this case: 2.000000 . . . with just as small an error as in Lazzerini's determination of π.

4. *THE "MYSTERIOUS" ENTROPY*

From the above examples of probability calculus, all of them pertaining to ordinary life, we have learned that predictions of that sort, being often disappointing when small numbers are involved, become better and better when we go to really large numbers. This makes these laws particularly applicable to the description of the almost innumerable quantities of atoms or molecules that form even the smallest piece of matter we can conveniently handle. Thus, whereas the statistical law of Drunkard's Walk can give us only approximate results when applied to a half-dozen drunkards who make perhaps two dozen turns each, its application to billions of dye molecules undergoing billions of collisions every second leads to the most rigorous physical law of diffusion. We can also say that the dye that was originally dissolved in only one half of the water in the test tube tends through the process of diffusion to spread uniformly through the entire liquid, because, such uniform distribution is *more probable* than the original one.

For exactly the same reason the room in which you sit reading this book is filled uniformly by air from wall to wall and from floor to ceiling, and it never even occurs to you that the air in the room can unexpectedly collect itself in a far corner, leaving you to suffocate in your chair. However, *this horrifying event is not at all physically impossible, but only highly improbable.*

To clarify the situation, let us consider a room divided into two equal halves by an imaginary vertical plane, and ask ourselves about the most probable distribution of air molecules between the two parts. The problem is of course identical with the coin-tossing problem discussed in the previous chapter. If we pick up one single molecule it has equal chances of being in the right or in the left half of the room, in exactly the same way as the tossed coin can fall on the table with heads or tails up.

The second, the third, and all the other molecules also have equal chances of being in the right or in the left part of the room regardless of where the others are.[8] Thus the problem of distributing molecules between the two halves of the room is equivalent to the problem of heads-and-tails distribution in a large number of tosses, and as you have seen from Figure 84, the fifty-fifty distribution is in this case by far the most probable one. We also see from that figure that with the increasing number of tosses (the number of air molecules in our case) the probability at 50 per cent becomes greater and greater, turning practically into a certainty when this number becomes very large. Since in the average-size room there are about 10^{27} molecules,[9] the probability that all of them collect simultaneously in, let us say, the right part of the room is:

$$(1/2)^{10^{27}} \cong 10^{-3 \cdot 10^{26}}$$

i.e., 1 out of $10^{3 \cdot 10^{26}}$.

On the other hand, since the molecules of air moving at the speed of about 0.5 km per second require only 0.01 sec to move from one end of the room to the other, their distribution in the room will be reshuffled 100 times each second. Consequently the waiting time for the right combination is $10^{299,999,999,999,999,999,999,999,998}$ sec as compared with only 10^{17} sec representing the total age of the universe! Thus you may go on quietly reading your book without being afraid of being suffocated by chance.

To take another example, let us consider a glass of water standing on the table. We know that the molecules of water, being involved in the irregular thermal motion, are moving at high speed in all possible directions, being, however, prevented from flying apart by the cohesive forces between them.

Since the direction of motion of each separate molecule is

[8] In fact, owing to large distances between separate molecules of the gas, the space is not at all crowded and the presence of a large number of molecules in a given volume does not at all prevent the entrance of new molecules.

[9] A room 10 ft by 15 ft, with a 9 ft ceiling has a volume of 1350 cu ft, or $5 \cdot 10^7$ cu cm, thus containing $5 \cdot 10^4$ g of air. Since the average mass of air molecules is $3 \cdot 1.66 \times 10^{-24} = 5 \times 10^{-23}$ g, the total number of molecules is $5 \cdot 10^4 / 5 \cdot 10^{-23} = 10^{27}$. ($\cong$ means: approximately equal to.)

governed entirely by the law of chance, we may consider the possibility that at a certain moment the velocities of one half of the molecules, namely those in the upper part of the glass, will all be directed upward, whereas the other half, in the lower part of the glass, will move downwards.[10] In such a case, the cohesive forces acting along the horizontal plane dividing two groups of molecules will not be able to oppose their "unified desire for parting," and we shall observe the unusual physical phenomenon of half the water from the glass being spontaneously shot up with the speed of a bullet toward the ceiling!

Another possibility is that the total energy of thermal motion of water molecules will be concentrated by chance in those located in the upper part of the glass, in which case the water near the bottom suddenly freezes, whereas its upper layers begin to boil violently. Why have you never seen such things happen? Not because they are absolutely impossible, but only because they are extremely improbable. In fact, if you try to calculate the probability that molecular velocities, originally distributed at random in all directions, will by pure chance assume the distribution described above, you arrive at a figure that is just about as small as the probability that the molecules of air will collect in one corner. In a similar way, the chance that, because of mutual collisions, some of the molecules will lose most of their kinetic energy, while the other part gets a considerable excess of it, is also negligibly small. Here again the distribution of velocities that corresponds to the usually observed case is the one that possesses the largest probability.

If now we start with a case that does not correspond to the most probable arrangement of molecular positions or velocities, by letting out some gas in one corner of the room, or by pouring some hot water on top of the cold, a sequence of physical changes will take place that will bring our system from this less probable to a most probable state. The gas will diffuse through the room until it fills it up uniformly, and the heat from the top of the glass will flow toward the bottom until all the water as-

[10] We must consider this half-and-half distribution, since the possibility that *all* molecules move in the same direction is ruled out by the mechanical law of the conservation of momentum.

But please do not think that in 500 hands you are sure to get a flush. You may get none, or you may get two. This is only probability calculus, and it may happen that you will be dealt many more than 500 hands without getting the desired combination, or on the contrary that you may be dealt a flush the very first time you have the cards in your hands. All that the theory of prob-

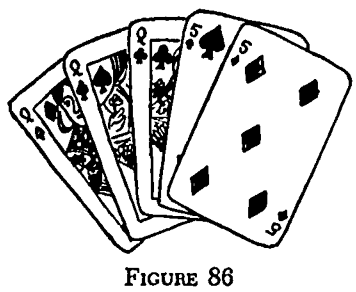


FIGURE 86
Full house.

ability can tell you is that you will *probably* be dealt 1 flush in 500 hands. You may also learn, by following the same methods of calculation, that in playing 30,000,000 games you will probably get 5 aces (including the joker) about ten times.

Another combination in poker, which is even rarer and therefore more valuable, is the so-called "full hand," more popularly called "full house." A full house consists of a "pair" and "three of a kind" (that is, 2 cards of the same value in 2 suits, and 3 cards of the same value in 3 suits—as, for example, the 2 fives and 3 queens shown in Figure 86).

If you want to get a full house, it is immaterial which 2 cards you get first, but when you get them you must have 2 of the remaining 3 cards match one of them, and the other match the

sumes an equal temperature. Thus we may say that *all physical processes depending on the irregular motion of molecules go in the direction of increasing probability*, and *the state of equilibrium, when nothing more happens, corresponds to the maximum of probability*. Since, as we have seen from the example of the air in the room, the probabilities of various molecular distributions are often expressed by inconveniently small numbers (as $10^{-3\cdot10^{26}}$ for the air collecting in one half of the room), it is customary to refer to their logarithms instead. This quantity is known by the name of *entropy*, and plays a prominent role in all questions connected with the irregular thermal motion of matter. The foregoing statement concerning the probability changes in physical processes can be now rewritten in the form: *Any spontaneous changes in a physical system occur in the direction of increasing entropy, and the final state of equilibrium corresponds to the maximum possible value of the entropy.*

This is the famous *Law of Entropy*, also known as the Second Law of Thermodynamics (the First Law being the Law of Conservation of Energy), and as you see there is nothing in it to frighten you.

The Law of Entropy can also be called the *Law of Increasing Disorder* since, as we have seen in all the examples given above, the entropy reaches its maximum when the position and velocities of molecules are distributed completely at random so that any attempt to introduce some order in their motion would lead to is 4/48. Thus the total probability of a full house is:

$$\frac{6}{50}\times\frac{5}{49}\times\frac{4}{48}=\frac{120}{117600}$$

or about one half of the probability of the flush.

In a similar way one can calculate the probabilities of other combinations as, for example, a "straight" (a sequence of cards), and also take into account the changes in probability introduced by the presence of the joker and the possibility of exchanging the originally dealt cards.

By such calculations one finds that the sequence of seniority used in poker does really correspond to the order of mathematical probabilities. It is not known by the author whether such an arrangement was proposed by some mathematician of the old times, or was established purely empirically by millions of players risking their money in fashionable gambling salons and little dark haunts all over the world. If the latter was the case, we must admit that we have here a pretty good statistical study of the relative probabilities of complicated events!

Another interesting example of probability calculation, an example that leads to a quite unexpected answer, is the problem of "Coinciding Birthdays." Try to remember whether you have ever been invited to two different birthday parties on the same day. You will probably say that the chances of such double invitations are very small since you have only about 24 friends who are likely to invite you, and there are 365 days in the year on which their birthdays may fall. Thus, with so many possible dates to choose from, there must be very little chance that any 2 of your 24 friends will have to cut their birthday cakes on the same day.

However, unbelievable as it may sound, your judgment here is quite wrong. The truth is that there is a rather high probability that in a company of 24 people there are a pair, or even several

(for example, through friction), the heat energy can never go completely into mechanical motion. This rules out the possibility of the so-called "perpetual motion motor of the second kind,"[11] which would extract the heat from the material bodies at normal temperature, thus cooling them down and utilizing for doing mechanical work the energy so obtained. For example, it is impossible to build a steamship in the boiler of which steam is generated not by burning coal but by extracting the heat from the ocean water, which is first pumped into the engine room, and then thrown back overboard in the form of ice cubes after the heat is extracted from it.

But how then do the ordinary steam-engines turn the heat into motion without violating the Law of Entropy? The trick is made possible by the fact that in the steam engine *only a part of the heat liberated by burning fuel is actually turned into energy,* another larger part being thrown out into the air in the form of exhaust steam, or absorbed by the specially arranged steam coolers. In this case we have two opposite changes of entropy in our system: (1) the increase of entropy corresponding to the transformation of a part of the heat into mechanical energy of the pistons, and (2) the decrease of entropy resulting from the flow of another part of the heat from the hot-water boilers into the coolers. The Law of Entropy requires only that *the total amount* of entropy of the system increase, and this can be easily arranged by making the second factor larger than the first. The *molecules we can bring some order in one region, if we do not mind the fact that this will make the motion in other parts still more disorderly.* And in many practical cases, as in all kinds of heat engines, we do not mind it.

5. *STATISTICAL FLUCTUATION*

The discussion of the previous section must have made it clear to you that the Law of Entropy and all its consequences is based entirely on the fact that in large-scale physics we are always dealing with an immensely large number of separate molecules, so that any prediction based on probability considerations becomes almost an absolute certainty. However, this kind of prediction becomes considerably less certain when we consider very small amounts of matter.

Thus, for example, if instead of considering the air filling a large room, as in the previous example, we take a much smaller volume of gas, say a cube measuring one hundredth of a micron[12] each way, the situation will look entirely different. In fact, since the volume of our cube is 10^{-18} cu cm it will contain only $\frac{10^{-18}\cdot 10^{-3}}{3\cdot 10^{-23}}$ 30 molecules, and the chance that all of them will collect in one half of the original volume is $(\frac{1}{2})^{30}=10^{-10}$.

On the other hand, because of the much smaller size of the cube, the molecules will be reshuffled at the rate of $5\cdot 10^{9}$ times per second (velocity of 0.5 km per second and the distance of only 10^{-6} cm) so that about once every second we shall find that one half of the cube is empty. It goes without saying that the cases when only a certain fraction of molecules become concentrated at one end of our small cube occur considerably more often. Thus for example the distribution in which 20 molecules are at one end and 10 molecules at the other (i.e only 10 extra molecules collected at one end) will occur with the frequency of $(\frac{1}{2})^{10}\times 5\cdot 10^{10}=10^{-3}\times 5\times 10^{10}=5\times 10^{7}$, that is, 50,000,000 times per second.

Thus, on a small scale, the distribution of molecules in the air is

[12] One micron, usually denoted by Greek letter *Mu* (μ), is 0.0001 cm.

far from being uniform. If we could use sufficient magnification, we should notice the small concentration of molecules being instantaneously formed at various points of the gas, only to be dissolved again, and be replaced by other similar concentrations appearing at other points. This effect is known as *fluctuation of density* and plays an important role in many physical phenomena. Thus, for example, when the rays of the sun pass through the atmosphere these inhomogeneities cause the scattering of blue rays of the spectrum, giving to the sky its familiar color and making the sun look redder than it actually is. This effect of reddening is especially pronounced during the sunset, when the sun rays must pass through the thicker layer of air. Were these fluctuations of density not present the sky would always look completely black and the stars could be seen during the day.

Similar, though less pronounced, fluctuations of density and pressure also take place in ordinary liquids, and another way of describing the cause of Brownian motion is by saying that the tiny particles suspended in the water are pushed to and fro because of rapidly varying changes of pressure acting on their opposite sides. When the liquid is heated until it is close to its boiling point, the fluctuations of density become more pronounced and cause a slight opalescence.

We can ask ourselves now whether the Law of Entropy applies to such small objects as those to which the statistical fluctuations become of primary importance. Certainly a bacterium, which through all its life is tossed around by molecular impacts, will sneer at the statement that heat cannot go over into mechanical motion! But it would be more correct to say in this case that the Law of Entropy loses its sense, rather than to say that it is violated. In fact all that this law says is that molecular motion cannot be transformed completely into the motion of large objects containing immense numbers of separate molecules. For a bacterium, which is not *much* larger than the molecules themselves, the difference between the thermal and mechanical motion has practically disappeared, and it would consider the molecular collisions tossing it around in the same way as we would consider the kicks we get from our fellow citizens in an excited crowd.

If we were bacteria, we should be able to build a perpetual motion motor of the second kind by simply tying ourselves to a flying wheel, but then we should not have the brains to use it to our advantage. Thus there is actually no reason for being sorry that we are not bacteria!

The "law of averages" applies to all randomly moving objects whether in kinetic theory or in city traffic. This story from The New Yorker magazine raises in fictional form the question of the meaning of a statistical law.

8 The Law

Robert M. Coates

1947

THE first intimation that things were getting out of hand came one early-fall evening in the late nineteen-forties. What happened, simply, was that between seven and nine o'clock on that evening the Triborough Bridge had the heaviest concentration of outbound traffic in its entire history.

This was odd, for it was a weekday evening (to be precise, a Wednesday), and though the weather was agreeably mild and clear, with a moon that was close enough to being full to lure a certain number of motorists out of the city, these facts alone were not enough to explain the phenomenon. No other bridge or main highway was affected, and though the two preceding nights had been equally balmy and moonlit, on both of these the bridge traffic had run close to normal.

The bridge personnel, at any rate, was caught entirely unprepared. A main artery of traffic, like the Triborough, operates under fairly predictable conditions. Motor travel, like most other large-scale human activities, obeys the Law of Averages—that great, ancient rule that states that the actions of people in the mass will always follow consistent patterns—and on the basis of past experience it had always been possible to foretell, almost to the last digit, the number of cars that would cross the bridge at any given hour of the day or night. In this case, though, all rules were broken.

The hours from seven till nearly midnight are normally quiet ones on the bridge. But on that night it was as if all the motorists in the city, or at any rate a staggering proportion of them, had conspired together to upset tradition. Beginning almost exactly at seven o'clock, cars poured onto the bridge in such numbers and with such rapidity that the staff at the toll booths was overwhelmed almost from the start. It was soon apparent that this was no momentary congestion, and as it became more and more obvious that the traffic jam promised to be one of truly monumental proportions, added details of police were rushed to the scene to help handle it.

Cars streamed in from all directions—from the Bronx approach and the Manhattan one, from 125th Street and the East River Drive. (At the peak of the crush, about eight-fifteen, observers on the bridge reported that the drive was a solid line of car headlights as far south as the bend at Eighty-ninth Street, while the congestion crosstown in Manhattan disrupted traffic as far west as Amsterdam Avenue.) And perhaps the most confusing thing about the whole manifestation was that there seemed to be no reason for it.

Now and then, as the harried tollbooth attendants made change for the seemingly endless stream of cars, they would question the occupants, and it soon became clear that the very participants in the monstrous tieup were as ignorant of its cause as anyone else was. A report made by Sergeant Alfonse O'Toole, who commanded the detail in charge of the Bronx approach, is typical. "I kept askin' them," he said, " 'Is there night football somewhere that we don't know about? Is it the races you're goin' to?' But the funny thing was half the time they'd be askin' *me*. 'What's the crowd for, Mac?' they would say. And I'd just look at them. There was one guy I mind, in a Ford convertible with a girl in the seat beside him, and when he asked me, I said to him, 'Hell, you're *in* the crowd, ain't you?' I said. 'What brings *you* here?' And the dummy just looked at me. 'Me?' he says. 'I just come out for a drive in the moonlight. But if I'd known there'd be a crowd like this . . .' he says. And then he asks me, 'Is there any place I can turn around and get out of this?' " As the *Herald Tribune* summed things up in its story next morning, it "just looked as if everybody in Manhattan who owned a motorcar had decided to drive out on Long Island that evening."

THE incident was unusual enough to make all the front pages next morning, and because of this, many similar events, which might otherwise have gone unnoticed, received attention. The proprietor of the Aramis Theatre, on Eighth Avenue, reported that on several nights in the recent past his auditorium had been practically empty, while on others it had been jammed to suffocation. Lunchroom owners noted that increasingly their patrons were developing a habit of making runs on specific items; one day it would be the roast shoulder of veal with pan gravy that was ordered almost exclusively, while the next everyone would be taking the Vienna loaf, and the roast veal went begging. A man who ran a small notions store in Bayside revealed that over a period of four days two hundred and seventy-four successive customers had entered his shop and asked for a spool of pink thread.

These were news items that would ordinarily have gone into the papers as fillers or in the sections reserved for oddities. Now, however, they seemed to have a more serious significance. It was apparent at last that something decidedly strange was happening to people's habits, and it was as unsettling as those occasional moments on excursion boats when the passengers are moved, all at once, to rush to one side or the other of the vessel. It was not till one day in December when, almost incredibly, the Twentieth Century Limited left New York for Chicago with just three passengers aboard that business leaders discovered how disastrous the new trend could be, too.

Until then, the New York Central, for instance, could operate confidently on the assumption that although there might be several thousand men in New York who had business relations in Chicago, on any single day no more—and no less—than some hundreds of them would have occasion to go there. The play producer could be sure that his patronage would sort itself out and that roughly as many persons would want to see the performance on Thursday as there had been on Tuesday or Wednesday. Now they couldn't be sure of anything. The Law of Averages had gone by the board, and if the effect on business promised to be catastrophic, it was also singularly unnerving for the general customer.

The lady starting downtown for a day of shopping, for example, could never be sure whether she would find Macy's department store a seething mob of other shoppers or a wilderness of empty, echoing aisles and unoccupied salesgirls. And the uncertainty produced a strange sort of jitteriness in the individual when faced with any impulse to action. "Shall we do it or shan't we?" people kept asking themselves, knowing that if they did do it, it might turn out that thousands of other individuals had decided similarly; knowing, too, that if they *didn't*, they might miss the one glorious chance of all chances to have Jones Beach, say, practically to themselves. Business languished, and a sort of desperate uncertainty rode everyone.

At this juncture, it was inevitable that Congress should be called on for action. In fact, Congress called on itself, and it must be said that it rose nobly to the occasion. A committee was appointed, drawn from both Houses and headed by Senator J. Wing Slooper (R.), of Indiana, and though after considerable investigation the committee was forced reluctantly to conclude that there was no evidence of Communist instigation, the unconscious subversiveness of the people's present conduct was obvious at a glance. The problem was what to do about it. You can't indict a whole nation, particularly on such vague grounds as these were. But, as Senator Slooper boldly pointed out, "You can control it," and in the end a system of reëducation and reform was decided upon, designed to lead people back to—again we quote Senator Slooper—"the basic regularities, the homely averageness of the American way of life."

In the course of the committee's investigations, it had been discovered, to everyone's dismay, that the Law of Averages had never been incorporated into the body of federal jurisprudence, and though the upholders of States' Rights rebelled violently, the oversight was at once corrected, both by Constitutional amendment and by a law—the Hills-Slooper Act—implementing it. According to the Act, people were *required* to be average, and, as the simplest way of assuring it, they were divided alphabetically and their permissible activities catalogued accordingly. Thus, by the plan, a person whose name began with "G," "N," or "U," for example, could attend the theatre only on Tuesdays, and he could go to baseball games only on Thursdays, whereas his visits to a haberdashery were confined to the hours between ten o'clock and noon on Mondays.

The law, of course, had its disadvantages. It had a crippling effect on theatre parties, among other social functions, and the cost of enforcing it was unbelievably heavy. In the end, too, so many amendments had to be added to it—such as the one permitting gentlemen to take their fiancées (if accredited) along with them to various events and functions no matter what letter the said fiancées' names began with—that the courts were frequently at a loss to interpret it when confronted with violations.

In its way, though, the law did serve its purpose, for it did induce—rather mechanically, it is true, but still adequately—a return to that average existence that Senator Slooper desired. All, indeed, would have been well if a year or so later disquieting reports had not begun to seep in from the backwoods. It seemed that there, in what had hitherto been considered to be marginal areas, a strange wave of prosperity was making itself felt. Tennessee mountaineers were buying Packard convertibles, and Sears, Roebuck reported that in the Ozarks their sales of luxury items had gone up nine hundred per cent. In the scrub sections of Vermont, men who formerly had barely been able to scratch a living from their rock-strewn acres were now sending their daughters to Europe and ordering expensive cigars from New York. It appeared that the Law of Diminishing Returns was going haywire, too. —Robert M. Coates

How can a viewer distinguish whether a film is being run forward or backward? The direction of increasing disorder helps to fix the direction of the arrow of time.

9 The Arrow of Time

Jacob Bronowski

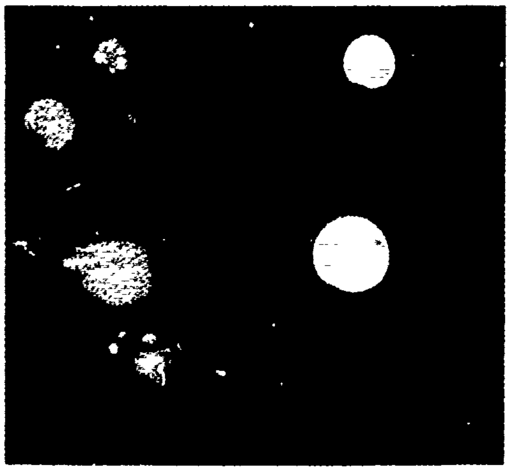
1964

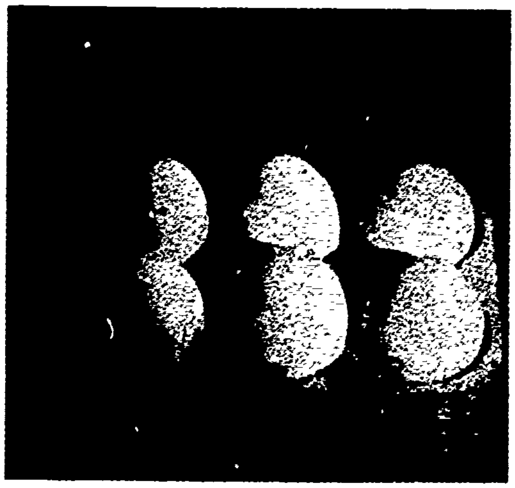
This chapter and those that follow deal with time. In particular, this chapter looks at the direction of time. Why does time go one way only? Why cannot we turn time backw.rds? Why are we not able to travel in time, back and forth?

The idea of time travel has fascinated men. Even folklore contains legends about travel in time. And science fiction, from *The Time Machine* onwards, has been pre-occupied with this theme. Plainly, men feel themselves to be imprisoned in the single direction of time. They would like to move about in time as freely as they can move in space.

And time is in some way like space. Like space, time is not a thing but is a relation between things. The essence of space is that it describes an order among things—higher or lower, in front or behind, to left or to right. The essence of time also is that it describes an order—earlier or later. Yet we cannot move things in time as we can in space. Time must therefore describe some fundamental process in nature which we do not control.

It is not easy to discuss time without bringing in some way of measuring it—a clock of one sort or another. Yet if all the clocks in the world stopped, and if we all lost all inner sense of time, we could still tell earlier from later. The essential nature of time does not depend on clocks. That is the point of this chapter, and we will begin by illustrating it





from very simple and common experiences.

The three pairs of pictures point the way. They help to show what it is that enables us to tell earlier from later without a clock. In each pair, the pictures are arranged at random, and not necessarily in the sequence of time. Yet in all except the first pair, it is easy to arrange the pictures; the sequence in time is obvious. Only the first pair does not betray its time sequence. What is the difference between the first pair of pictures and the other two pairs?

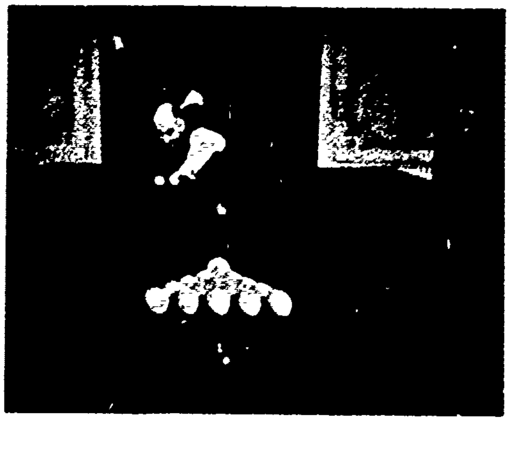
We get a clue to the difference when we study the arrangement of the things in each picture. In the first pair, we cannot really distinguish one arrangement from another; they are equally tidy and orderly. The two pictures of the first pair show a shot at billiards. The billiard balls are as well arranged after the shot as before; there is no obvious difference between the arrangements.

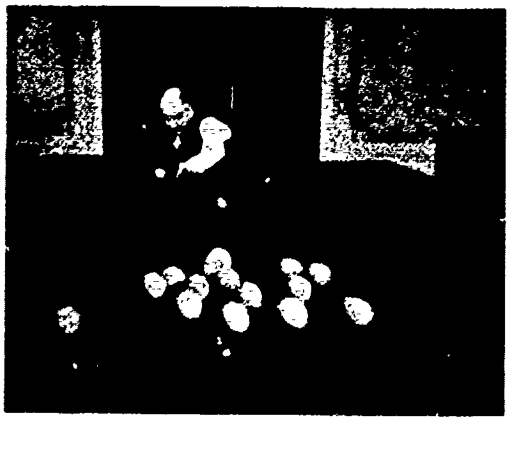
The situation is different in the other two pairs. A broken egg is an entirely different arrangement from a whole egg. A snooker pyramid is quite different from a jumble of balls.

And not only are the arrangements here different. Once they are different, it is quite clear which arrangement comes before the other. Whole eggs come before broken ones. The snooker pyramid comes before the spread of the balls.

In each case, the earlier arrangement is more ordered than the later. Time produces disorder; that is the lesson of these pictures. And it is also the lesson of this chapter. The arrow of time is loss of order.

In a game of snooker, we know quite well that the highly ordered arrangement of the balls at the beginning of the game comes before the disordered arrangement at the end of the first shot. Indeed, the first shot is called 'breaking the pyramid'; and breaking is a destructive action—it destroys order. It is just not conceivable that fifteen balls would gather themselves up into a pyramid, however skilful the player. The universe does not suddenly create order out of disorder.





These pictures show the same thing again. When a spot of powdered dye is put on the surface of water, it spreads out and gradually dissolves. Dye would never come out of solution and stream together by itself to gather in a spot on the surface. Again time is breaking down order and making disorder. It disperses the dye randomly through the water.

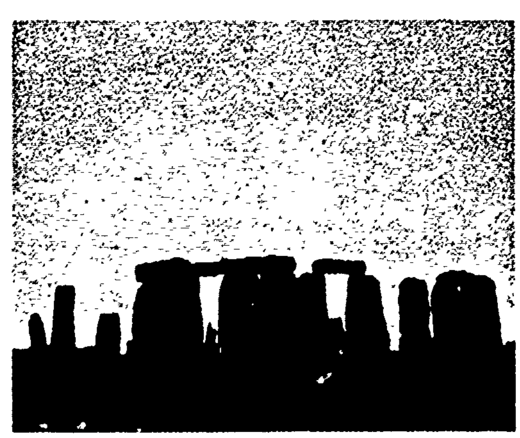
We know at once that the stones in the picture below were shaped and erected a very long time ago. Their rough, weathered surfaces bear the mark of time. It is still possible to reconstruct the once orderly arrangement of the stones of Stonehenge. But the once orderly surface of each stone cannot be recovered. Atom by atom, the smooth surface has been carried away, and is lost to chaos.

And here finally is the most interesting of all the pictures in which time betrays itself. In these shots from an old film the heroine has been tied to the rails—a splendid tradition of silent films. A train is approaching, but of course it stops just in time. The role of the heroine would seem to call for strong nerves as well as dramatic ability, if she has to trust the engine driver to stop the locomotive exactly where he is told. However, the last few yards of the approach are in fact done by a trick. The locomotive is *started* close to the heroine and is backed away: and the film is then run backwards.

There is only one thing that gives this trick away. When the film is run backwards, the smoke visibly goes into the funnel instead of coming out of it. We know that in reality, smoke behaves like the spreading dye: it becomes more disorderly, the further it gets from the funnel. So when we see disorder coming before order, we realise that something is wrong. Smoke does not of itself collect together and stream down a funnel.

One thing remains to clear up in these examples. We began with an example in which earlier and later were equally well ordered. The example was a shot at billiards. The planets in their orbits would be another example, in which there would be nothing to say which arrangement comes first.

Then does time stand still in billiards and planetary motion? No, time is still doing its work of disorder. We may not see the effects at once, but they are there. For billiard balls and planets gradually lose material from their surface, just like the stones of Stonehenge. Time destroys their orderly shape too. A billiard ball is not quite the same after a shot







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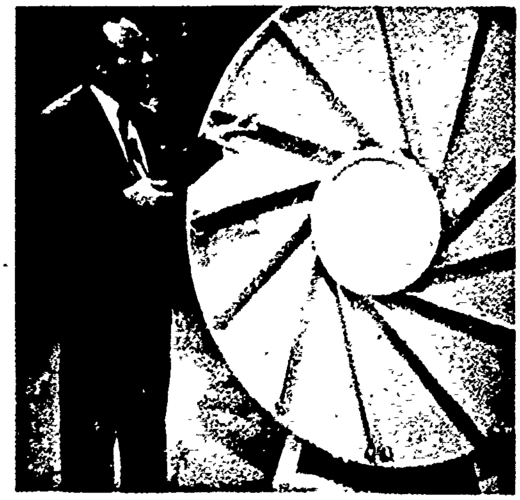
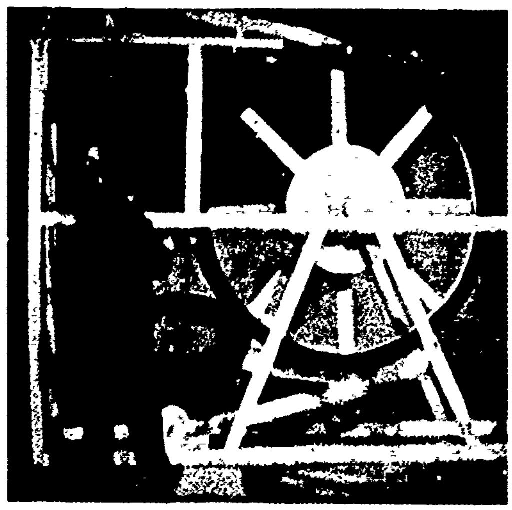
as before it. A planet is not quite the same in each successive orbit. And the changes are in the direction of disorder. Atoms are lost from ordered structures and return to chaos. The direction of time is from order to disorder.



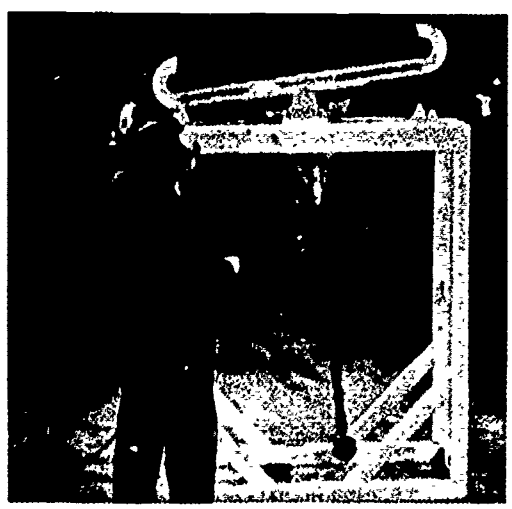
That is one reason why perpetual motion machines are impossible. Time cannot be brought to a standstill. We cannot freeze the arrangement of the atoms, even in a tiny corner of the universe. And that is what we should have to do to make a perpetual motion machine. The machine would have to remain the same, atom for atom, for all time. Time would have to stand still for it.

For example, take the first of these three machines from a famous book of Perpetual Motion Machines. It is meant to be kept going by balls in each sector, which roll from the centre to the rim and back again as the wheel turns. Of course it does not work. There is friction in the bearing of the wheel, and more friction between the balls and the tracks they run on. Every movement rubs off a few atoms. The bearings wear, the balls lose their smooth roundness. Time does not stand still.



The second machine is more complicated and sillier. It is designed to work like a waterwheel with little balls instead of water. At the bottom the balls roll out of their compartments down the chute, and on to a moving belt which is to lift them to the top again. That is how the machine is meant to keep going. In fact, when we built it, it came to a stop every few minutes.

The pendulum arrangement in the third picture also comes from the book of Perpetual Motion Machines. A ball runs backwards and forwards in the trough on top to keep it going. There are also elastic strings at each end for good measure. This machine at least works for short bursts. But as a perpetual motion machine, it has the same defects as the others. Nothing can be done to get rid of friction; and where there is friction, there must be wear.



This last point is usually put a little differently. Every machine has friction. It has to be supplied with energy to overcome the friction. And this energy cannot be recovered. In fact, this energy is lost in heat, and in wear—that is, in moving atoms out of their order, and in losing them. That is another way of putting the same reasoning, and shows equally (in different language) why a perpetual motion machine cannot work.

Before we put these fanciful monsters out of mind, it is worth seeing how beautifully a fine machine can be made. It cannot conquer the disorder of time, it cannot get rid of friction, but it can keep them to a minimum. So here on this page are two splendid clocks which make no pretence to do the impossible, yet which go as far as it is possible to go by means of exact and intelligent craftsmanship.

These clocks are not intended to be perpetual motion machines. Each has an outside source of energy to keep it going. In the clock at the top, it is ordinary clockwork which tips the platform whenever the ball has completed a run. The clock below is more tricky: it has no clockwork spring, and instead is driven by temperature differences in the air. But even if there was someone to wind one clock, and suitable air conditions for the other, they could not run for ever. They would wear out. That is, their ordered structure would slowly become more disordered until they stopped. The clock with no spring would run for several hundred years, but it could not run for ever.

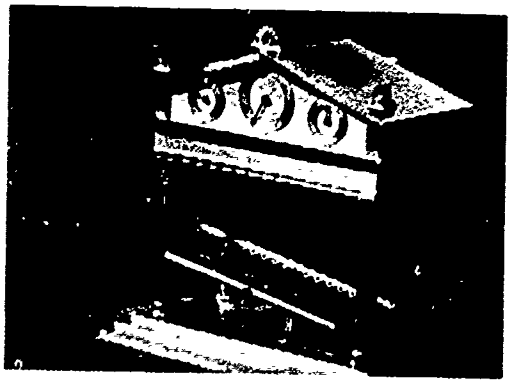
To summarise: the direction of time in the universe is marked by increasing disorder. Even without clocks and without an inner sense of time, we could tell later and earlier. 'Later' is characterised by the greater disorder, by the growing randomness of the universe.

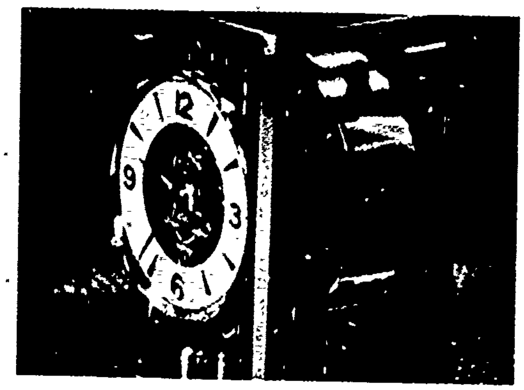
We ought to be clear what these descriptive phrases mean. Order is a very special arrangement; and disorder means the loss of what makes it special. When we say that the universe is becoming more disordered, more random, we mean that the special arrangements in this place or that are being evened out. The peaks are made lower, the holes are filled in. The extremes disappear, and all parts sink more and more towards a level average. Disorder and randomness are not wild states; they are simply states which have no special arrangement, and in which everything is therefore near the average.

Even in disorder, of course, things move and deviate round their average. But they deviate by chance, and chance then takes them back to the average. It is only in exceptional cases that a deviation becomes fixed, and perpetuates itself. These exceptions are fascinating and important, and we now turn to them.

The movement towards randomness, we repeat, is not uniform. It is statistical, a general trend. And (as we saw in Chapter 8) the units that make up a general trend do not all flow in the same direction. Here and there, in the midst of the flow towards an average of chaos, there are places where the flow is reversed for a time. The most remarkable of these reversals is life. Life as it were is running against time. Life is the very opposite of randomness.

How this can come about can be shown by an analogy. The flow of time is like an endless shuffling of a pack of cards. A typical hand dealt after long





shuffling will be random—say four diamonds, a couple of spades, four clubs, and three hearts. This is the sort of hand a bridge player expects to pick up several times in an evening. Yet every now and then a bridge player picks up a freak hand. For example, from time to time a player picks up all thirteen spades. And this does not mean that the pack was not properly shuffled. A hand of thirteen spades can arise by chance, and does; the odds against it are high, but they are not astronomic. Life started with a chance accident of this kind. The odds against it were high, but they were not astronomic.

The special thing about life is that it is self-perpetuating. The freak hand, instead of disappearing in the next shuffle, reproduces itself. Once the thirteen spades of life are dealt, they keep their order, and they impose it on the pack from then on. This is what distinguishes life from other freaks, other deviations from the average.

There are other happenings in the universe that run against the flow of time for a while. The formation of a star from the interstellar dust is such a happening. When a star is formed, the dust that forms it becomes less random; its order is increased, not decreased. But stars do not reproduce themselves. Once the star is formed, the accident is over. The flow towards disorder starts again. The deviation begins to ebb back towards the average.

Life is a deviation of a special kind; it is a self-reproducing accident. Once its highly ordered arrangement occurs, once the thirteen spades happen to be dealt in one hand, it repeats itself. The order was reached by chance, but it then survives because it is able to perpetuate itself, and to impose itself on other matter.

It is rare to find in *dead* matter behaviour of this kind which illustrates the way in which *life* imposes its order. An analogy of a kind, however, is found in the growth of crystals. When a supercooled solution is ready to form crystals, it needs something to start it off. Now we introduce the outside accident, the freak hand at bridge. That is, we introduce a tiny crystal that we have made, and we drop it in. At once the crystal starts to grow and to impose its own shape round it.

In this analogy, the first crystal is a seed, like the seed of life. Without it, the supercooled solution would remain dead, unchanged for hours or even days. And like the seed of life, the first crystal imposes its order all round it. It reproduces itself many times over.

Nearly five hundred years ago, Leonardo da Vinci described time as the destroyer of all things. So we have seen it in this chapter. It is the nature of time to destroy things, to turn order into disorder. This indeed gives time its single direction—its arrow.

But the arrow of time is only statistical. The general trend is towards an average chaos; yet there are deviations which move in the opposite direction. Life is the most important deviation of this kind. It is able to reproduce itself, and so to *perpetuate the order which began by accident*. Life runs against the disorder of time.

The biography of this great Scottish physicist, renowned both for kinetic theory and for his mathematical formulation of the laws of electricity [illegible]etism, is presented in two parts. The sec[illegible]his selection is in Reader 4.

10 James Clerk Maxwell

James R. Newman

1955

JAMES CLERK MAXWELL was the greatest theoretical physicist of the nineteenth century. His discoveries opened a new epoch of science, and much of what distinguishes our world from his is due to his work. Because his ideas found perfect expression in mathematical symbolism, and also because his most spectacular triumph — the prophecy of the existence of electromagnetic waves — was the fruit of theoretical rather than experimental researches, he is often cited as the supreme example of a scientist who builds his systems entirely with pencil and paper. This notion is false. He was not, it is true, primarily an experimentalist. He had not the magical touch of Faraday, of whom Helmholtz once observed after a visit to his laboratory that "a few wires and some old bits of wood and iron seem to serve him for the greatest discoveries." Nonetheless he combined a profound physical intuition with a formidable mathematical capacity to produce results "partaking of both natures." On the one hand, Maxwell never lost sight of the phenomena to be explained, nor permitted himself, as he said, to be drawn aside from the subject in pursuit of "analytical subtleties"; on the other hand, the use of mathematical methods conferred freedom on his inquiries and enabled him to gain physical insights without committing himself to a physical theory. This blending of the concrete and the abstract was the characteristic of almost all his researches.

Maxwell was born at Edinburgh on November 13, 1831, the same year Faraday announced his famous discovery of electromagnetic induction. He was descended of the Clerks of Penicuick in Midlothian, an old Scots family distinguished no less for their individuality, "verging on eccentricity," than for their talents. His forbears included eminent lawyers, judges, politicians, mining speculators, merchants, poets, musicians, and also the author (John Clerk) of a thick book on naval tactics, whose naval experience appears to have been confined entirely to sailing mimic men of war on the fishponds at Penicuick. The name Maxwell was assumed by James's father, John Clerk, on inheriting the small estate of Middlebie from his grandfather Sir George Clerk Maxwell.

At Glenlair, a two-day carriage ride from Edinburgh and "very much in the wilds," in a house built by his father shortly after he married, Maxwell passed his infancy and early boyhood. It was a happy time. He was an only son (a sister, born earlier, died in infancy) in a close-knit, comfortably-off family. John Clerk Maxwell had been called to the Scottish bar but took little interest in the grubby pursuits of an advocate. Instead the laird managed his small estates, took part in county affairs and gave loving attention to the education of his son. He was a warm and rather simple man with a nice sense of humor and a penchant for doing things with what he called "judiciosity"; his main characteristic, according to Maxwell's

James Clerk Maxwell.
(The Bettmann Archive)

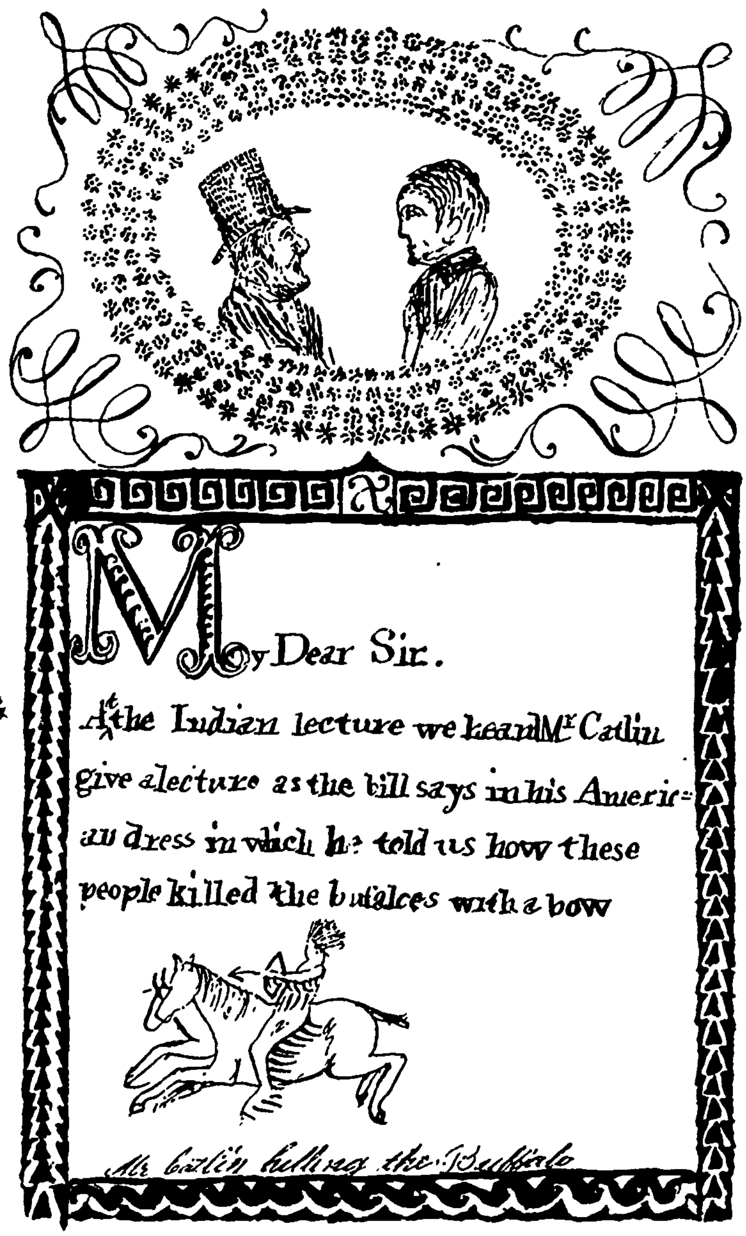
biographer Lewis Campbell,* was a "persistent practical interest in all useful purposes." Maxwell's mother, Frances Cay, who came of a well-known Northumbrian family, is described as having a "sanguine, active temperament."

Jamesie, as he was called, was a nearsighted, lively, affectionate little boy, as persistently inquisitive as his father and as fascinated by mechanical contrivances. To discover of anything "how it doos" was his constant aim. "What's the go of that?" he would ask, and if the answer did not satisfy him he would add, "But what's the *particular* go of that?" His first creation was a set of figures for a "wheel of life," a scientific toy that produced the illusion of continuous movement; he was fond of making things with his hands, and in later life knew how to design models embodying the most complex motions and other physical processes.

When Maxwell was nine, his mother died of cancer, the same disease that was to kill him forty years later. Her death drew father and son even more closely together, and many intimate glimpses of Maxwell in his younger years emerge from the candid and affectionate letters he wrote to his father from the time he entered school until he graduated from Cambridge.

Maxwell was admitted to Edinburgh Academy as a day student when he was ten years old. His early school experiences were painful. The master, a dryish Scotsman whose reputation derived from a book titled *Account of the Irregular Greek Verbs* and from the fact that he was a good disciplinarian, expected his students to be orderly, well-grounded in the usual subjects and unoriginal. Maxwell was deficient in all these departments. He created something of a sensation because of his clothes, which had been designed by his strong-

* The standard biography (London, 1882) is by Lewis Campbell and William Garnett. Campbell wrote the first part, which portrays Maxwell's life; Garnett the second part, dealing with Maxwell's contributions to science. A shorter biography, especially valuable for the scientific exposition, is by the mathematician R. T. Glazebrook (*James Clerk Maxwell and Modern Physics*, London, 1901). In this essay, material in quotation marks, otherwise unattributed, is from Campbell and Garnett.



My Dear Sir.

At the Indian lecture we heard Mr Catlin give alecture as the bill says in his Americ= an dress in which he told us how these people killed the bufaloes with a bow

Mr Catlin killing the Buffalo

Illuminated letter was written by Maxwell to his father in 1843, when the younger Maxwell was 11. The letter refers to a lecture by the American frontier artist, George Catlin. (Scientific American) minded father and included such items as "hygienic" square-toed shoes and a lace-frilled tunic. The boys nicknamed him "Dafty" and mussed him up, but he was a stubborn child and in time won the respect of his classmates even if he continued to puzzle them. There was a gradual awakening of mathematical interests. He wrote his father that he had made a "tetra hedron, a dodeca hedron, and two more hedrons that I don't know the wright names for," that he enjoyed playing with the "boies," that he attended a performance of some "Virginian minstrels," that he was composing Latin verse and making a list of the Kings of Israel and Judah. Also, he sent him the riddle of the simpleton who "wishing to swim was nearly drowned. As soon as he got out he swore that he would never touch water till he had learned to swim." In his fourteenth year he won the Academy's mathematical medal and wrote a paper on a mechanical method, using pins and thread, of constructing perfect oval curves. Another prodigious little boy, René Descartes, had anticipated Maxwell in this field, but Maxwell's contributions were completely independent and original. It was a wonderful day for father and son when they heard "Jas's" paper on ovals read before the Royal Society of Edinburgh by Professor James Forbes: "Met," Mr. Maxwell wrote of the event in his diary, "with very great attention and approbation generally."

After six years at the Academy, Maxwell entered the University of Edinburgh. He was sixteen, a restless, enigmatic, brilliantly talented adolescent who wrote not very good but strangely prophetic verse about the destiny of matter and energy:

> When earth and sun are frozen clods,
> When all its energy degraded
> Matter to aether shall have faded

His friend and biographer Campbell records that James was completely neat in his person "though with a rooted objection to the vanities of starch and gloves," and that he had a "pious horror of destroying anything — even a scrap of writing paper." He had a quaint humor, read voraciously and passed much time in mathematical speculations and in chemical, magnetic and optical experiments. "When at table he often seemed abstracted from what was going on, being absorbed in observing the effects of refracted light in the finger glasses, or in trying some experiment with his eyes — seeing around a corner, making invisible stereoscopes, and the like. Miss Cay [his aunt] used to call his attention by crying, 'Jamesie, you're in a prop!' [an abbreviation for mathematical proposition]." He was by now a regular visitor at the meetings of the Edinburgh Royal Society, and two of his papers, on "Rolling Curves" and on the "Equilibrium of Elastic Solids," were published in the *Transactions*. The papers were read before the Society by others "for it was not thought proper for a boy in a round jacket to mount the rostrum there." During vacations at Glenlair he was tremendously active and enjoyed reporting his multifarious doings in long letters to friends. A typical communication, when Maxwell was seventeen, tells Campbell of building an "electro-magnetic machine," taking off an hour to read Poisson's papers on electricity and magnetism ("as I am pleased with him today"), swimming and engaging in "aquatic experiments," making a centrifugal pump, reading Herodotus, designing regular geometric figures, working on an electric telegraph, recording thermometer and barometer readings, embedding a beetle in wax to see if it was a good conductor of electricity ("not at all cruel, because I slew him in boiling water in which he never kicked"), taking the dogs out, picking fruit, doing "violent exercise" and solving props. Many of his letters exhibit his metaphysical leanings, especially an intense interest in moral philosophy. This bent of his thought, while showing no particular originality, reflects his social sympathy, his Christian earnestness, the not uncommon nineteenth-century mixture of rationalism and simple faith. It was a period when men still shared the eighteenth-century belief that questions of wisdom, happiness and virtue could be studied as one studies optics and mechanics.

In 1850 Maxwell quit the University of Edinburgh for Cambridge. After a term at Peterhouse College he migrated to Trinity where the opportunity seemed better of obtaining ultimately a mathematical fellowship. In his second year he became a private pupil of William Hopkins, considered the ablest mathematics coach of his time. It was Hopkins's job to prepare his pupils for the stiff competitive examinations, the mathematical tripos, in which the attainment of high place insured academic preferment. Hopkins was not easily impressed; the brightest students begged to join his group, and the famous physicists George Stokes and William Thomson (later Lord Kelvin) had been among his pupils. But from the beginning he recognized the talents of the black-haired young Scotsman, describing him as "the most extraordinary man I have ever met," and adding that "it appears impossible for [him] to think incorrectly on physical subjects." Maxwell worked hard as an undergraduate, attending the lectures of Stokes and others and faithfully doing what he called "old Hop's props." He joined fully in social and intellectual activities and was made one of the Apostles, a club limited to twelve members, which for many years included the outstanding young men at Cambridge. A contemporary describes him as "the most genial and amusing of companions, the propounder of many a strange theory, the composer of many a poetic *jeu d'esprit*." Not the least strange of his theories related to finding an effective economy of work and sleep. He would sleep from 5 in the afternoon to 9:30, read very hard from 10 to 2, exercise by running along the corridors and up and down stairs from 2 to 2:30 A.M. and sleep again from 2:30 to 7. The occupants of the rooms along his track were not pleased, but Maxwell persisted in his bizarre experiments. Less disturbing were his investigations of the process by which a cat lands always on her feet. He demonstrated that a cat could right herself even when dropped upside down on a table or bed from about two inches. A complete record of these valuable researches is unfortunately not available.

A severe illness, referred to as a "sort of brain fever," seized Maxwell in the summer of 1853. For weeks he was totally disabled and he felt effects of his illness long afterward. Despite the abundance of details about his life, it is hard to get to the man underneath. From his letters one gleans evidence of deep inner struggles and anxieties, and the attack of "brain fever" was undoubtedly an emotional crisis; but its causes remain obscure. All that is known is that his illness strengthened Maxwell's religious conviction — a deep, earnest piety, leaning to Scottish Calvinism yet never completely identified with any particular system or sect. "I have no nose for heresy," he used to say.

In January, 1854, with a rug wrapped round his feet and legs (as his father had advised) to mitigate the perishing cold in the Cambridge Senate House where the elders met and examinations were given, he took the tripos. His head was warm enough. He finished second wrangler, behind the noted mathematician, Edward Routh. (In another competitive ordeal, for the "Smith's Prize," where the subjects were more advanced, Maxwell and Routh tied for first.)

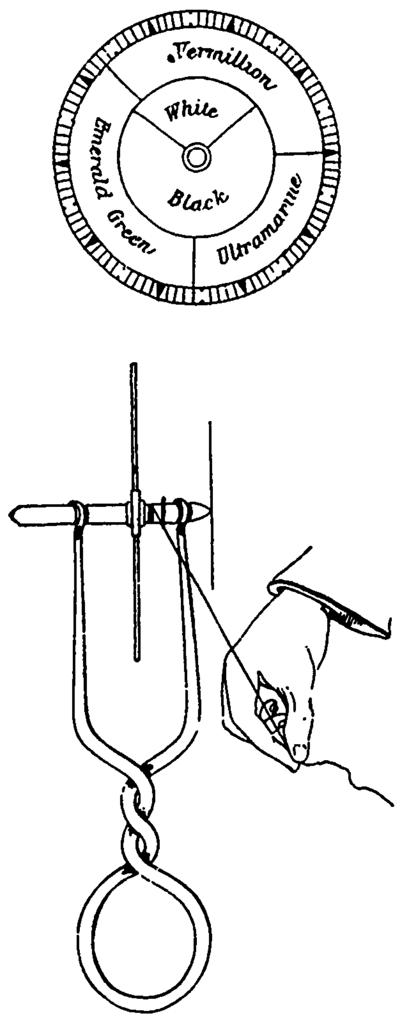
After getting his degree, Maxwell stayed on for a while at Trinity, taking private pupils, reading Berkeley's *Theory of Vision*, which he greatly admired, and Mill's *Logic*, which he admired less: ("I take him slowly . . . I do not think him the last of his kind"), and doing experiments on the effects produced by mixing colors. His apparatus consisted of a top, which he had designed himself, and colored paper discs that could be slipped one over the other and arranged round the top's axis so that any given portion of each color could be exposed. When the top was spun rapidly, the sectors of the different colors became indistinguishable and the whole appeared of one uniform tint. He was able to show that suitable combinations of three primary colors — red, green and blue — produced "to a very near degree of approximation" almost every color of the spectrum. In each case the required combination could be quantitatively determined by measuring the sizes of the exposed sectors of the primary-color discs. Thus, for example, 66.6 parts of red and 33.4 parts of green gave the same chromatic effect as 29.1 parts of yellow and 24.1 parts of blue. In general, color composition could be expressed by an equation of the form

$$xX = aA + bB + cC$$

— shorthand for the statement that x parts of X can be matched by a parts of A, b parts of B and c parts of C. This symbolism worked out very prettily, for "if the sign of one of the quantities, a, b, or c was negative, it simply meant that that color had to be combined with X to match the other two."* The problem of color perception drew Maxwell's attention on and off for several years, and enlarged his scientific reputation. The work was one phase of his passionate interest in optics, a subject to which he made many contributions ranging from papers on geometrical optics to the invention of an ophthalmoscope and studies in the "Art of Squinting." Hermann von Helmholtz was of course the great leader in the field of color sensation, but Maxwell's work was independent and of high merit and in 1860 won him the Rumford Medal of the Royal Society.

These investigations, however, for all their importance, cannot be counted the most significant activity of the two post-graduate years at Trinity. For during this same period he was reading with intense absorption Faraday's *Experimental Researches*, and the effect of this great record on his mind is scarcely to be overestimated. He had, as he wrote his father, been "working away at Electricity again, and [I] have been working my way into the views of heavy German writers. It

* Glazebrook, *op. cit.*, pp. 101-102. See also Maxwell's paper, "Experiments on Colour, as perceived by the Eye, with remarks on Colour-Blindness," *Transactions of the Royal Society of Edinburgh*, vol. XXI, part II; collected in *The Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven, Cambridge, 1890.



Color wheel is depicted in Maxwell's essay "Experiments in Colour, as perceived by the Eye, with remarks on Colour-Blindness." The wheel is shown at the top. The apparatus for rotating it is at the bottom.
(Scientific American)

takes a long time to reduce to order all the notions one gets from these men, but I hope to see my way through the subject, and arrive at something intelligible in the way of a theory." Faraday's wonderful mechanical analogies suited Maxwell perfectly; they were what he needed to stimulate his own conjectures. Like Faraday, he thought more easily in images than abstractions: the models came first, the mathematics later. A Cambridge contemporary said that in their student days, whenever the subject admitted of it, Maxwell "had recourse to diagrams, though the rest [of the class] might solve the question more easily by a train of analysis." It was his aim, he wrote, to take Faraday's ideas and to show how "the connexion of the very different orders of phenomena which he had discovered may be clearly placed before the mathematical mind."* Before the year 1855 was out, Maxwell had published his first major contribution to electrical science, the beautiful paper "On Faraday's Lines of Force," to which I shall return when considering his over-all achievements in the field.

Trinity elected Maxwell to a fellowship in 1855, and he began to lecture in hydrostatics and optics. But his father's health, unsettled for some time, now deteriorated further, and it was partly to avoid their being separated that he became a candidate for the chair of natural philosophy at Marischal College, Aberdeen. In 1856 his appointment was announced; his father, however, had died a few days before, an irreparable personal loss to Maxwell. They had been as close as father and son could be. They confided in each other, understood each other and were in certain admirable traits much alike.

The four years at Aberdeen were years of preparation as well as achievement. Management of his estate, the design of

* The following quotation from the preface to Maxwell's *Treatise on Electricity and Magnetism* (Cambridge, 1873) gives Maxwell's views of Faraday in his own words: "Before I began the study of electricity I resolved to read no mathematics on the subject till I had first read through Faraday's *Experimental Researches in Electricity*. I was aware that there was supposed to be a difference between Faraday's way of conceiving phenomena and that of the mathematicians so that neither he nor they were satisfied with each other's language. I had also the conviction that this discrepancy did not arise from either party being wrong. . . . As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms, and these compared with those of the professed mathematicians."

a new "compendious" color machine, and the reading of metaphysics drew on his time. The teaching load was rather light, a circumstance not unduly distressing to Maxwell. He took his duties seriously, prepared lectures and demonstration experiments very carefully, but it cannot be said he was a great teacher. At Cambridge, where he had picked students, his lectures were well attended, but with classes that were, in his own words, "not bright," he found it difficult to hit a suitable pace. He was unable himself to heed the advice he once gave a friend whose duty it was to preach to a country congregation: "Why don't you give it to them thinner?"* Electrical studies occupied him both during term and in vacation at Glenlair. "I have proved," he wrote in a semijocular vein to his friend C. J. Monro, "that if there be nine coefficients of magnetic induction, perpetual motion will set in, and a small crystalline sphere will inevitably destroy the universe by increasing all velocities till the friction brings all nature into a state of incandescence. . . ."

Then suddenly the work on electricity was interrupted by a task that engrossed him for almost two years. In competition for the Adams prize of the University of Cambridge (named in honor of the discoverer of Neptune), Maxwell prepared a brilliant essay on the subject set by the electors: "The Structure of Saturn's Rings."

Beginning with Galileo, the leading astronomers had observed and attempted to explain the nature of the several concentric dark and bright rings encircling the planet Saturn. The great Huygens had studied the problem. as had the Frenchman, Jean Dominique Cassini, Sir William Herschel and his son John, Laplace, and the Harvard mathematician and astronomer Benjamin Peirce. The main question at the time Maxwell entered the competition concerned the stability of the ring system: Were the rings solid? Were they fluid?

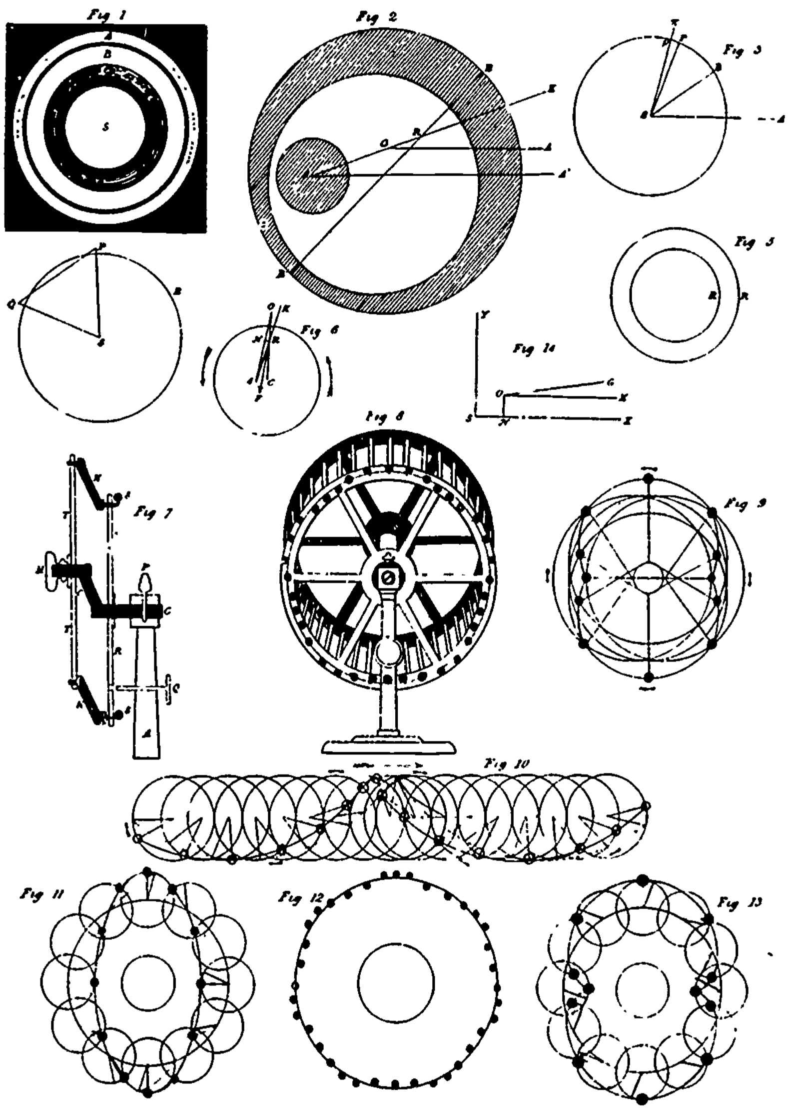
* Occasionally he enjoyed mystifying his students, but at Aberdeen, where, he wrote Campbell, "No jokes of any kind are understood," he did not permit himself such innocent enjoyments.

Did they consist of masses of matter "not mutually coherent"? The problem was to demonstrate which type of structure adequately explained the motion and permanence of the rings.

Maxwell's sixty-eight-page essay was a mixture of common sense, subtle mathematical reasoning and profound insight into the principles of mechanics.* There was no point, he said at the outset, in imagining that the motion of the rings was the result of forces unfamiliar to us. We must assume that gravitation is the regulating principle and reason accordingly. The hypothesis that the rings are solid and uniform he quickly demonstrated to be untenable; indeed Laplace had already shown that an arrangement of this kind would be so precarious that even a slight displacement of the center of the ring from the center of the planet "would originate a motion which would never be checked, and would inevitably precipitate the ring upon the planet. . . ."

Suppose the rings were not uniform, but loaded or thickened on the circumference — a hypothesis for which there appeared to be observational evidence. A mechanically stable system along these lines was theoretically possible; yet here too, as Maxwell proved mathematically, the delicate adjustment and distribution of mass required could not survive the most minor perturbations. What of the fluid hypothesis? To be sure, in this circumstance the rings would not collide with the planet. On the other hand, by the principles of fluid motion it can be proved that waves would be set up in the moving rings. Using methods devised by the French mathematician Joseph Fourier for studying heat conduction, by means of which complex wave motions can be resolved into their simple harmonic, sine-cosine elements, Maxwell succeeded in demonstrating that the waves of one ring will force waves in another and that, in due time, since the wave amplitudes will increase

* A summary of the work was published in the *Proceedings of the Royal Society of Edinburgh*, vol. IV; this summary and the essay "On the Stability of the Motion of Saturn's Rings" appear in the *Scientific Papers* (*op. cit.*).



Mechanical model is depicted here in Figures 7 and 8 of this page from Maxwell's essay "On the Stability of the Motion of Saturn's Rings." In this essay, Maxwell demonstrated that the rings were neither liquid nor solid, but composed of particles. (Scientific American) indefinitely, the rings will break up into drops. Thus the continuous-fluid ring is no better a solution of the problem than the solid one.

The third possibility remained, that the rings consist of disconnected particles, either solid or liquid, but necessarily independent. Drawing on the mathematical theory of rings, Maxwell proved that such an arrangement is fairly stable and its disintegration very slow; that the particles may be disposed in a series of narrow rings or may move through each other irregularly. He called this solution his "dusky ring, which is something like the state of the air supposing the siege of Sebastopol conducted from a forest of guns 100 miles one way, and 30,000 miles from the other, and the shot never to stop, but go spinning away around a circle, radius 170,000 miles. . . ."

Besides the mathematical demonstration, Maxwell devised an elegantly ingenious model to exhibit the motions of the satellites in a disturbed ring, "for the edification of sensible image-worshippers." His essay — which Sir George Airy, the Astronomer Royal, described as one of the most remarkable applications of mathematics he had ever seen — won the prize and established him as a leader among mathematical physicists.

In 1859 Maxwell read before the British Association his paper "Illustrations of the Dynamical Theory of Gases."* This marked his entry into a branch of physics that he enriched almost as much as he did the science of electricity. Two circumstances excited his interest in the kinetic theory of gases. The first was the research on Saturn, when he encountered the mathematical problem of handling the irregular motions of the particles in the rings — irregular but resulting nonetheless in apparent regularity and uniformity — a problem analogous to that of the behavior of the particles of gas. The second was the publication by the German physicist Rudolf Clausius

* *Philosophical Magazine*, January and July, 1860; also Maxwell's *Scientific Papers, op. cit.*

of two famous memoirs: on the heat produced by molecular motion and on the average length of the path a gas molecule travels before colliding with a neighbor.

Maxwell's predecessors in this field — Daniel Bernoulli, James Joule, Clausius, among others — had been successful in explaining many of the properties of gases, such as pressure, temperature, and density, on the hypothesis that a gas is composed of swiftly moving particles. However, in order to simplify the mathematical analysis of the behavior of enormous aggregates of particles, it was thought necessary to make an altogether implausible auxiliary assumption, namely, that all the particles of a gas moved at the same speed. The gifted British physicist J. J. Waterson alone rejected this assumption, in a manuscript communicated to the Royal Society in 1845: he argued cogently that various collisions among the molecules must produce different velocities and that the gas temperature is proportional to the square of the velocities of all the molecules. But his manuscript lay forgotten for half a century in the archives of the Society.

Maxwell, without knowledge of Waterson's work, arrived at the same conclusions. He realized that further progress in the science of gases was not to be cheaply won. If the subject was to be developed on "strict mechanical principles" — and for him this rigorous procedure was essential — it was necessary, he said, not only to concede what was in any case obvious, that the particles as a result of collisions have different speeds, but to incorporate this fact into the mathematical formulation of the laws of motion of the particles.

Now, to describe how two spheres behave on colliding is hard enough; Maxwell analyzed this event, but only as a prelude to the examination of an enormously more complex phenomenon — the behavior of an "indefinite number of small, hard and perfectly elastic spheres acting on one another only during impact."* The reason for this mathematical investiga-

* "Illustrations of the Dynamical Theory of Gases," *op. cit.*

tion was clear. For as he pointed out, if the properties of this assemblage are found to correspond to those of molecular assemblages of gases, "an important physical analogy will be established, which may lead to more accurate knowledge of the properties of matter."

The mathematical methods were to hand but had hitherto not been applied to the problem. Since the many molecules cannot be treated individually, Maxwell introduced the statistical method for dealing with the assemblage. This marked a great forward step in the study of gases. A fundamental Maxwellian innovation was to regard the molecules as falling into groups, each group moving within a certain range of velocity. The groups lose members and gain them, but group population is apt to remain pretty steady. Of course the groups differ in size; the largest, as Maxwell concluded, possesses the most probable velocity, the smaller groups the less probable. In other words, the velocities of the molecules in a gas can be conceived as distributed in a pattern — the famous bell-shaped frequency curve discovered by Gauss, which applies to so many phenomena from observational errors and distribution of shots on a target to groupings of men based on height and weight, and the longevity of electric light bulbs. Thus while the velocity of an individual molecule might elude description, the velocity of a crowd of molecules would not. Because this method afforded knowledge not only of the velocity of a body of gas as a whole, but also of the groups of differing velocities composing it, Maxwell was now able to derive a precise formula for gas pressure. Curiously enough this expression did not differ from that based on the assumption that the velocity of all the molecules is the same, but at last the right conclusions had been won by correct reasoning. Moreover the generality and elegance of Maxwell's mathematical methods led to the extension of their use into almost every branch of physics.

Maxwell went on, in this same paper, to consider another factor that needed to be determined, namely, the average number of collisions of each molecule per unit of time, and its mean free path (i.e., how far it travels, on the average, between collisions). These data were essential to accurate formulations of the laws of gases. He reasoned that the most direct method of computing the path depended upon the viscosity of the gas. This is the internal friction that occurs when (in Maxwell's words) "different strata of gas slide upon one another with different velocities and thus act upon one another with a tangential force tending to prevent this sliding, and similar in its results to the friction between two solid surfaces sliding over each other in the same way." According to Maxwell's hypothesis, the viscosity can be explained as a statistical consequence of innumerable collisions between the molecules and the resulting exchange of momentum. A very pretty illustration by the Scotch physicist Balfour Stewart helps to an understanding of what is involved. Imagine two trains running with uniform speed in opposite directions on parallel tracks close together. Suppose the passengers start to jump across from one train to the other. Each passenger carries with him a momentum opposite to that of the train onto which he jumps; the result is that the velocity of both trains is slowed just as if there were friction between them. A similar process, said Maxwell, accounts for the apparent viscosity of gases.

Having explained this phenomenon, Maxwell was now able to show its relationship to the mean free path of the molecules. Imagine two layers of molecules sliding past each other. If a molecule passing from one layer to the other travels only a short distance before colliding with another molecule, the two particles do not exchange much momentum, because near the boundary or interface the friction and difference of velocity between the two layers is small. But if the molecule penetrates deep into the other layer before a collision, the friction and velocity differential will be greater; hence the exchange of momentum between the colliding particles will be greater. This amounts to saying that in any gas with high viscosity the molecules must have a long mean free path.

Maxwell deduced further the paradoxical and fundamental fact that the viscosity of gas is independent of its density. The reason is that a particle entering a dense — i.e., highly crowded — gas will not travel far before colliding with another particle; but penetration on the average will be deeper when the gas entered is only thinly populated, because the chance of a collision is smaller. On the other hand, there will be more collisions in a dense than in a less dense gas. On balance, then, the momentum conveyed across each unit area per second remains the same regardless of density, and so the coefficient of viscosity is not altered by varying the density.

These results, coupled with others arrived at in the same paper, made it possible for Maxwell to picture a mechanical model of phenomena and relationships hitherto imperfectly understood. The various properties of a gas — diffusion, viscosity, heat conduction — could now be explained in precise quantitative terms. All are shown to be connected with the motion of crowds of particles "carrying with them their momenta and their energy," traveling certain distances, colliding, changing their motion, resuming their travels, and so on. Altogether it was a scientific achievement of the first rank. The reasoning has since been criticized on the ground, for example, that molecules do not possess the tiny-billiard-ball properties Maxwell ascribed to them; that they are neither hard, nor perfectly elastic; that their interaction is not confined to the actual moment of impact. Yet despite the inadequacies of the model and the errors of reasoning, the results that, as Sir James Jeans has said, "ought to have been hopelessly wrong," turned out to be exactly right, and the formula tying the relationships together is in use to this day, known as Maxwell's law.*

* "Maxwell, by a train of argument which seems to bear no relation at all to molecules, or to the dynamics of their movements, or to logic, or even to ordinary common sense, reached a formula which, according to all precedents and all the rules of scientific philosophy ought to have been hopelessly wrong. In actual fact it was subsequently shown to be exactly right. . . ." (James Jeans, "Clerk Maxwell's Method," in *James Clerk Maxwell, A Commemoration Volume, 1831-1931*, New York, 1931.)

This is perhaps a suitable place to add a few lines about Maxwell's later work in the theory of gases. Clausius, Max Planck tells us, was not profoundly impressed by the law of distribution of velocities, but the German physicist Ludwig Boltzmann at once recognized its significance. He set to work refining and generalizing Maxwell's proof and succeeded, among other results, in showing that "not only does the Maxwell distribution [of velocities] remain stationary, once it is attained, but that it is the only possible equilibrium state, since any system will eventually attain it, whatever its initial state."* This final equilibrium state, as both men realized, is the thermodynamic condition of maximum entropy — the most disordered state, in which the least amount of energy is available for useful work. But since this condition is in the long run also the most probable, purely from the mathematical standpoint, one of the great links had been forged in modern science between the statistical law of averages and the kinetic theory of matter.

The concept of entropy led Maxwell to one of the celebrated images of modern science, namely, that of the sorting demon. Statistical laws, such as the kinetic theory of gases, are good enough in their way, and, at any rate, are the best man can arrive at, considering his limited powers of observations and understanding. Increasing entropy, in other words, is the explanation we are driven to — and indeed our fate in physical reality — because we are not very bright. But a demon more favorably endowed could sort out the slow- and fast-moving particles of a gas, thereby changing disorder into order and converting unavailable into available energy. Maxwell imagined one of these small, sharp fellows "in charge of a frictionless, sliding door in a wall separating two compartments of a vessel filled with gas. When a fast-moving molecule moves from left to right the demon opens the door, when a slow moving molecule approaches, he (or she) closes the door. The

* Max Planck, "Maxwell's Influence on Theoretical Physics in Germany," in James Jeans, *ibid.*

fast-moving molecules accumulate in the right-hand compartment, and slow ones in the left. The gas in the first compartment grows hot and that in the second cold." Thus the demon would thwart the second law of thermodynamics. Living organisms, it has been suggested, achieve an analogous success; as Erwin Schrödinger has phrased it, they suck negative entropy from the environment in the food they eat and the air they breathe.

Maxwell and Boltzmann, working independently and in a friendly rivalry, at first made notable progress in explaining the behavior of gases by statistical mechanics. After a time, however, formidable difficulties arose, which neither investigator was able to overcome. For example, they were unable to write accurate theoretical formulas for the specific heats of certain gases (the quantity of heat required to impart a unit increase in temperature to a unit mass of the gas at constant pressure and volume).* The existing mathematical techniques simply did not reach — and a profound transformation of ideas had to take place before physics could rise to — a new level of understanding. Quantum theory — the far-reaching

* In order to resolve discrepancies between theory and experiment, as to the viscosity of gases and its relationship to absolute temperature, Maxwell suggested a new model of gas behavior, in which the molecules are no longer considered as elastic spheres of definite radius but as more or less undefined bodies repelling one another inversely as the fifth power of the distance between the centers of gravity. By this trick he hoped to explain observed properties of gases and to bypass mathematical obstacles connected with computing the velocity of a gas not in a steady state. For, whereas in the case of hard elastic bodies molecular collisions are a discontinuous process (each molecule retaining its velocity until the moment of impact) and the computation of the distribution of velocities is essential in solving questions of viscosity, if the molecular interaction is by repulsive force, acting very weakly when the molecules are far away from each other and strongly when they approach closely, each collision may be conceived as a rapid but *continuous* transition from the initial to the final velocity, and the computation both of relative velocities of the colliding molecules and of the velocity distribution of the gas as a whole can be dispensed with. In his famous memoir *On the Dynamical Theory of Gases*, which appeared in 1866, Maxwell gave a beautiful mathematical account of the properties of such a system. The memoir inspired Boltzmann to a Wagnerian rapture. He compared Maxwell's theory to a musical drama: "At first are developed majestically the

system of thought revolving about Planck's universal constant, h — was needed to deal with the phenomena broached by Maxwell and Boltzmann.* The behavior of microscopic particles eluded description by classical methods, classical concepts of mass, energy and the like; a finer mesh of imagination alone would serve in the small world of the atom. But neither quantum theory, nor relativity, nor the other modes of thought constituting the twentieth-century revolution in physics would have been possible had it not been for the brilliant labors of these natural philosophers in applying statistical methods to the study of gases.

Variations of the Velocities, then from one side enter the Equations of State, from the other the Equations of Motion in a Central Field; ever higher swoops the chaos of Formulae; suddenly are heard the four words: 'Put n = 5'. The evil spirit V (the relative velocity of two molecules) vanishes and the dominating figure in the bass is suddenly silent; that which had seemed insuperable being overcome as if by a magic stroke . . . result after result is given by the pliant formula till, as unexpected climax, comes the Heat Equilibrium of a heavy gas; the curtain then drops."

Unfortunately, however, the descent of the curtain did not, as Boltzmann had supposed, mark a happy ending. For as James Jeans points out, "Maxwell's belief that the viscosity of an actual gas varied directly as the absolute temperature proved to have been based on faulty arithmetic, and the conclusions he drew from his belief were vitiated by faulty algebra." [Jeans, *op. cit.*] It was, says Jeans, "a very human failing, which many of us will welcome as a bond of union between ourselves and a really great mathematician" — even though the results were disastrous.

* Explanation of the discrepancies they found had to await the development of quantum theory, which showed that the spin and vibration of molecules were restricted to certain values.

Throughout history, there have been men who endeavored to design machines that produce energy from nothing. All their efforts have been thwarted by the law of conservation of energy. But why not a machine that extracts unlimited energy by cooling its surroundings? Unless Maxwell's Demon intervenes, this machine is highly improbable.

11 Maxwell's Demon

George Gamow

1965

'I've read something somewhere about such hypothetical machines—perpetual motion machines, I believe they are called,' said Mr Tompkins. 'If I remember correctly, machines planned to run without fuel are considered impossible because one cannot manufacture energy out of nothing. Anyway, such machines have no connection with gambling.'

'You are quite right, my boy,' agreed the professor, pleased that his son-in-law knew something at least about physics. 'This kind of perpetual motion, "perpetual motion machines of the first type" as they are called, cannot exist because they would be contrary to the law of the Conservation of Energy. However the fuel-less machines I have in mind are of a rather different type and are usually known as "perpetual motion machines of the second type". They are not designed to create energy out of nothing, but to extract energy from surrounding heat reservoirs in the earth, sea or air. For instance, you can imagine a steamship in whose boilers steam was gotten up, not by burning coal but by extracting heat from the surrounding water. In fact, if it were possible to force heat to flow away from cold toward greater heat, instead of the other way round, one could construct a system for pumping in sea-water, depriving it of its heat content, and disposing of the residue blocks of ice overboard. When a gallon of cold water freezes into ice, it gives off enough heat to raise another gallon of cold water almost to the boiling point. By pumping through several gallons of sea-water per minute, one could easily collect enough heat to run a good-sized engine. For all practical purposes, such a perpetual motion machine of the second type would be just as good as the kind designed to create energy out of nothing. With engines like this to do the work, everyone in the world could live as carefree an existence as a man with an unbeatable roulette system. Unfortunately they are equally impossible as they both violate the laws of probability in the same way.'

'I admit that trying to extract heat out of sea-water to raise steam in a ship's boilers is a crazy idea,' said Mr Tompkins. 'However, I fail to see any connexion between that problem and the laws of chance. Surely, you are not suggesting that dice and roulette wheels should be used as moving parts in these fuel-less machines. Or are you?'

'Of course not!' laughed the professor. 'At least I don't believe even the craziest perpetual motion inventor has made that suggestion yet. The point is that heat processes themselves are very similar in their nature to games of dice, and to hope that heat will flow from the colder body into the hotter one is like hoping that money will flow from the casino's bank into your pocket.'

'You mean that the bank is cold and my pocket hot?' asked Mr Tompkins, by now completely befuddled.

'In a way, yes,' answered the professor. 'If you hadn't missed my lecture last week, you would know that heat is nothing but the rapid irregular movement of innumerable particles, known as atoms and molecules, of which all material bodies are constituted. The more violent this molecular motion is, the warmer the body appears to us. As this molecular motion is quite irregular, it is subject to the laws of chance, and it is easy to show that the most probable state of a system made up of a large number of particles will correspond to a more or less uniform distribution among all of them of the total available energy. If one part of the material body is heated, that is if the molecules in this region begin to move faster, one would expect that, through a large number of accidental collisions, this excess energy would soon be distributed evenly among all the remaining particles. However, as the collisions are purely accidental, there is also the possibility that, merely by chance, a certain group of particles may collect the larger part of the available energy at the expense of the others. This spontaneous concentration of thermal energy in one particular part of the body would correspond to the flow of heat against the temperature gradient, and is not excluded in principle. However, if one tries to calculate the relative probability of such a spontaneous heat concentration occurring, one gets such small numerical values that the phenomenon can be labelled as practically impossible.'

'Oh, I see it now,' said Mr Tompkins. 'You mean that these perpetual motion machines of the second kind might work once in a while but that the chances of that happening are as slight as they are of throwing a seven a hundred times in a row in a dice game.'

'The odds are much smaller than that,' said the professor. 'In fact, the probabilities of gambling successfully against nature are so slight that it is difficult to find words to describe them. For instance, I can work out the chances of all the air in this room collecting spontaneously under the table, leaving an absolute vacuum everywhere else. The number of dice you would throw at one time would be equivalent to the number of air molecules in the room, so I must know how many there are. One cubic centimetre of air at atmospheric pressure, I remember, contains a number of molecules described by a figure of twenty digits, so the air molecules in the whole room must total a number with some twenty-seven digits. The space under the table is about one per cent of the volume of the room, and the chances of any given molecule being under the table and not somewhere else are, therefore, one in a hundred. So, to work out the chances of all of them being under the table at once, I must multiply one hundredth by one hundredth and so on, for each molecule in the room. My result will be a decimal beginning with fifty-four noughts.'

'Phew... !' sighed Mr Tompkins, 'I certainly wouldn't bet on those odds! But doesn't all this mean that deviations from equipartition are simply impossible?'

'Yes,' agreed the professor. 'You can take it as a fact that we won't suffocate because all the air is under the table, and for that matter that the liquid won't start boiling by itself in your high-ball glass. But if you consider much smaller areas, containing much smaller numbers of our dice-molecules, deviations from statistical distribution become much more probable. In this very room, for instance, air molecules habitually group themselves somewhat more densely at certain points, giving rise to minute inhomogeneities, called statistical fluctuations of density. When the sun's light passes through terrestrial atmosphere, such inhomogeneities cause the scattering of the blue rays of spectrum, and give to the sky its familiar colour. Were these fluctuations of density not present, the sky would always be quite black, and the stars would be clearly visible in full daylight. Also the slightly opalescent light liquids get when they are raised close to the boiling point is explained by these same fluctuations of density produced by the irregularity of molecular motion. But, on a large scale, such fluctuations are so extremely improbable that we would watch for billions of years without seeing one.'

'But there is still a chance of the unusual happening right now in this very room,' insisted Mr Tompkins. 'Isn't there?'

'Yes, of course there is, and it would be unreasonable to insist that a bowl of soup couldn't spill itself all over the table cloth because half of its molecules had accidentally received thermal velocities in the same direction.'

'Why that very thing happened only yesterday,' chimed in Maud, taking an interest now she had finished her magazine. 'The soup spilled and the maid said she hadn't even touched the table.'

The professor chuckled. 'In this particular case,' he said, 'I suspect the maid, rather than Maxwell's Demon, was to blame.'

'Maxwell's Demon?' repeated Mr Tompkins, surprised. 'I should think scientists would be the last people to get notions about demons and such.'

'Well, we don't take him very seriously,' said the professor. 'CLERK MAXWELL, the famous physicist, was responsible for introducing the notion of such a statistical demon simply as a figure of speech. He used this notion to illustrate discussions on the phenomena of heat. Maxwell's Demon is supposed to be rather a fast fellow, and capable of changing the direction of every single molecule in any way you prescribe. If there really were such a demon, heat could be made to flow against temperature, and the fundamental law of thermodynamics, the *principle of increasing entropy*, wouldn't be worth a nickel.'

'Entropy?' repeated Mr Tompkins. 'I've heard that word before. One of my colleagues once gave a party, and after a few drinks, some chemistry students he'd invited started singing—

'*In*creases, *de*creases
*De*creases, *in*creases
What the hell do we care
What entropy does?'

to the tune of "Ach du lieber Augustine". What is entropy anyway?'

'It's not difficult to explain. "Entropy" is simply a term used to describe the degree of disorder of molecular motion in any given physical body or system of bodies. The numerous irregular collisions between the molecules tend always to increase the entropy, as an absolute disorder is the most probable state of any statistical ensemble. However, if Maxwell's Demon could be put to work, he would soon put some order into the movement of the molecules the way a good sheep dog rounds up and steers a flock of sheep, and the entropy would begin to decrease. I should also tell you that according to the so-called H-theorem Ludwig Boltzmann introduced to science....'

Apparently forgetting he was talking to a man who knew practically nothing about physics and not to a class of advanced students, the professor rambled on, using such monstrous terms as 'generalized parameters' and 'quasi-ergodic systems', thinking he was making the fundamental laws of thermodynamics and their relation to Gibbs' form of statistical mechanics crystal clear. Mr Tompkins was used to his father-in-law talking over his head, so he sipped his Scotch and soda philosophically and tried to look intelligent. But all these highlights of statistical physics were definitely too much for Maud, curled up in her chair and struggling to keep her eyes open. To throw off her drowsiness she decided tc and see how dinner was getting along.

'Does madam desire something?' inquired a tall, elegantly dressed butler, bowing as she came into the dining room.

'No, just go on with your work,' she said, wondering why on earth he was there. It seemed particularly odd as they had never had a butler and certainly could not afford one. The man was tall and lean with an olive skin, long, pointed nose, and greenish eyes which seemed to burn with a strange, intense glow. Shivers ran up and down Maud's spine when she noticed the two symmetrical lumps half hidden by the black hair above his forehead.

'Either I'm dreaming,' she thought, 'or this is Mephistopheles himself, straight out of grand opera.'

'Did my husband hire you?' she asked aloud, just for something to say.

'Not exactly,' answered the strange butler, giving a last artistic touch to the dinner table. 'As a matter of fact, I came here of my own accord to show your distinguished father I am not the myth he believes me to be. Allow me to introduce myself. I am Maxwell's Demon.'

'Oh!' breathed Maud with relief, 'Then you probably aren't wicked, like other demons, and have no intention of hurting anybody.'

'Of course not,' said the Demon with a broad smile, 'but I like o play practical jokes and I'm about to play one on your father.'

'What are you going to do?' asked Maud, still not quite eassured.

'Just show him that, if I choose, the law of increasing entropy can be broken. And to convince you it can be done, I would appreciate the honour of your company. It is not at all dangerous, I assure you.'

At these words, Maud felt the strong grip of the Demon's hand on her elbow, and everything around her suddenly went crazy. All the familiar objects in her dining room began to grow with

'Is this what hell looks like?'

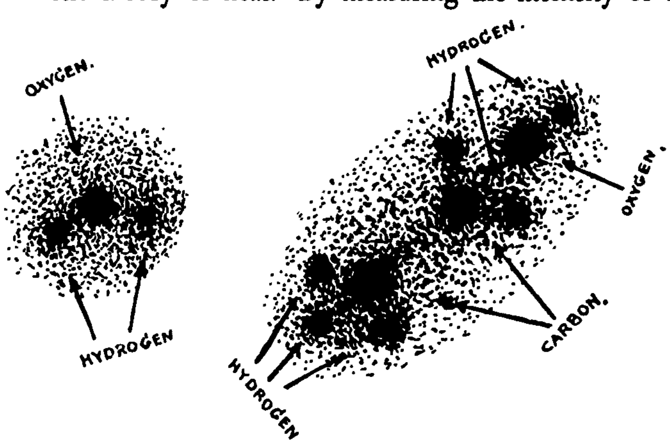
terrific speed, and she got a last glimpse of the back of a chair covering the whole horizon. When things finally quieted down, she found herself floating in the air supported by her companion. Foggy-looking spheres, about the size of tennis balls, were whizzing by in all directions, but Maxwell's Demon cleverly kept them from colliding with any of the dangerous looking things. Looking down, Maud saw what looked like a fishing boat, heaped to the gunwales with quivering, glistening fish. They were not fish, however, but a countless number of foggy balls, very like those flying past them in the air. The Demon led her closer until she seemed surrounded by a sea of coarse gruel which was moving and working in a patternless way. Balls were boiling to the surface and others seemed to be sucked down. Occasionally one would come to the surface with such speed it would tear off into space, or one of the balls flying through the air would dive into the gruel and disappear under thousands of other balls. Looking at the gruel more closely, Maud discovered that the balls were really of two different kinds. If most looked like tennis balls, the larger and more elongated ones were shaped more like American footballs. All of them were semi-transparent and seemed to have a complicated internal structure which Maud could not make out.

'Where are we?' gasped Maud. 'Is this what hell looks like?'

'No,' smiled the Demon, 'Nothing as fantastic as that. We are simply taking a close look at a very small portion of the liquid surface of the highball which is succeeding in keeping your husband awake while your father expounds quasi-ergodic systems. All these balls are molecules. The smaller round ones are water molecules and the larger, longer ones are molecules of alcohol. If you care to work out the proportion between their number, you can find out just how strong a drink your husband poured himself.'

'*Very* interesting,' said Maud, as sternly as she dared. 'But what are those things over there that look like a couple of whales playing in the water. They couldn't be atomic whales, or could they?'

The demon looked where Maud pointed. 'No, they are hardly whales,' he said. 'As a matter of fact, they are a couple of very fine fragments of burned barley, the ingredient which gives whisky its particular flavour and colour. Each fragment is made up of millions and millions of complex organic molecules and is comparatively large and heavy. You see them bouncing around because of the action of impacts they receive from the water and alcohol molecules animated by thermal motion. It was the study of such intermediate-sized particles, small enough to be influenced by molecular motion but still large enough to be seen through a strong microscope, which gave scientists their first direct proof of the kinetic theory of heat. By measuring the intensity of the



tarantella-like dance executed by such minute particles suspended in liquids, their Brownian motion as it is usually called, physicists were able to get direct information on the energy of molecular motion.'

Again the Demon guided her through the air until they came to an enormous wall made of numberless water molecules fitted neatly and closely together like bricks.

'How very impressive!' cried Maud. 'That's just the background I've been looking for for a portrait I'm painting. What is this beautiful building, anyway?'

'Why, this is part of an ice crystal, one of many in the ice cube in your husband's glass,' said the Demon. 'And now, if you will excuse me, it is time for me to start my practical joke on the old, self-assured professor.'

So saying, Maxwell's Demon left Maud perched on the edge of the ice crystal, like an unhappy mountain climber, and set about his work. Armed with an instrument like a tennis racquet, he was swatting the molecules around him. Darting here and there, he was always in time to swat any stubborn molecule which persisted in going in the wrong direction. In spite of the apparent danger of her position, Maud could not help admiring his wonderful speed and accuracy, and found herself cheering with excitement whenever he succeeded in deflecting a particularly fast and difficult molecule. Compared with the exhibition she was witnessing, champion tennis players she had seen looked like hopeless duffers. In a few minutes, the results of the Demon's work were quite apparent. Now, although one part of the liquid surface was covered by very slowly moving, quiet molecules, the part directly under her feet was more furiously agitated than ever. The number of molecules escaping from the surface in the process of evaporation was increasing rapidly. They were now escaping in groups of thousands together, tearing through the surface as giant bubbles. Then a cloud of steam covered Maud's whole field of vision and she could get only occasional glimpses of the whizzing racquet or the tail of the Demon's dress suit among the masses of maddened molecules. Finally the molecules in her ice crystal perch gave way and she fell into the heavy clouds of vapour beneath....

When the clouds cleared, Maud found herself sitting in the same chair she was sitting in before she went into the dining room.

'Holy entropy!' her father shouted, staring bewildered at Mr Tompkins' highball. 'It's boiling!'

The liquid in the glass was covered with violently bursting bubbles, and a thin cloud of steam was rising slowly toward the ceiling. It was particularly odd, however, that the drink was boiling only in a comparatively small area around the ice cube. The rest of the drink was still quite cold.

'Think of it!' went on the professor in an awed, trembling voice. 'Here I was telling you about statistical fluctuations in the law of entropy when we actually see one! By some incredible chance, possibly for the first time since the earth began, the faster molecules have all grouped themselves accidentally on one part of the surface of the water and the water has begun to boil by itself!

'Holy entropy! It's boiling!'

In the billions of years to come, we will still, probably, be the only people who ever had the chance to observe this extraordinary phenomenon.' He watched the drink, which was now slowly cooling down. 'What a stroke of luck!' he breathed happily.

Maud smiled but said nothing. She did not care to argue with her father, but this time she felt sure she knew better than he.

The use of random elements is common today not only in science, but also in music, art, and literature. One influence was the success of kinetic theory in the nineteenth century.

12 Randomness and The Twentieth Century

Alfred M. Bork

1967

■ As I write this I have in front of me a book that may be unfamiliar to many. It is entitled *One Million Random Digits with 1,000 Normal Deviates* and was produced by the Rand Corporation in 1955. As the title suggests, each page contains digits—numbers from 1 to 9—arranged as nearly as possible in a completely random fashion. An electronic roulette wheel generated the numbers in this book, and afterwards the numbers were made even more random by shuffling and other methods. There is a careful mathematical definition of randomness, and associated with it are many tests that one can apply. These numbers were shuffled until they satisfied the tests.

I want to use this book as a beginning theme for this paper. The production of such a book is entirely of the twentieth century. It could not have been produced in any other era. I do not mean to stress that the mechanism for doing it was not available, although that is also true. What is of more interest is that before the twentieth-century no one would even have thought of the *possibility* of producing a book like this; no one would have seen any use for it. A rational nineteenth-century man would have thought it the height of folly to produce a book containing only random numbers. Yet such a book is important, even though it is not on any of the usual lists of one hundred great books.

That this book *is* strictly of the twentieth century is in itself of importance. I claim that it indicates a cardinal feature of our century: randomness, a feature permeating many different and apparently unrelated aspects of our culture. I do not claim that randomness is the only feature which characterizes and separates twentieth-century thought from earlier thought, or even that it is dominant, but I *will* argue, admittedly on a speculative basis, that it is an important aspect of the twentieth century.

Before I leave the book referred to above, you may be curious to know why a collection of random numbers is of any use. The Rand Corporation, a government-financed organization, is not likely to spend its money on pursuits having no possible application. The principal use today of a table of random numbers is in a calculational method commonly used on large digital computers. Because of its use of random numbers, it is called the Monte Carlo method, and it was developed primarily by Fermi, von Neumann, and Ulam at the end of the Second World War. The basic idea of the Monte Carlo method is to replace an exact problem which cannot be solved with a probabilistic one which can be approximated. Another area where a table of random numbers is of importance is in designing experiments, particularly those involving sampling. If one wants, for example, to investigate certain properties of wheat grown in a field, then one wants thoroughly random samplings of wheat; if all the samples came from one corner of the field, the properties found might be peculiar to that corner rather than to the whole field. Random sampling is critical in a wide variety of situations.

Actually, few computer calculations today use a table of random numbers; rather, a procedure suggested during the early days of computer development by John von Neumann is usually followed. Von Neumann's idea was to have the computer generate its own random numbers. In a sense numbers generated in this way are not "random," but they can be made to satisfy the same exacting tests applied to the Rand Table; randomness is a matter of degree. It is more generally convenient to let the computer produce random numbers than to store in the computer memory a table such as the Rand Table. Individual computer centers often have their own methods for generating random numbers.

I shall not give any careful definition of randomness, but shall rely on intuitive ideas of the term. A formal careful definition would be at odds with our purposes, since, as A. O. Lovejoy noted in *The Great Chain of Being*, it is the vagueness of the terms which allows them to have a life of their own in a number of different areas. The careful reader will notice the shifting meanings of the word "random," and of related words, in our material.

However, it may be useful to note some of the different ideas connected with randomness. D. M. Mackay, for example, distinguishes between "(a) the notion of *well-shuffledness* or impartiality of distribution; (b) the notion of *irrelevance* or absence of correlation; (c) the notion of '*I don't care*'; and (d) the notion of *chaos*"[1] Although this is not a complete, mutually exclusive classification—the editor of the volume in which it appears objects to it—the classification indicates the range of meaning that "random" has even in well-structured areas like information theory.

Let us, then, review the evidence of randomness in several areas of twentieth-century work, and then speculate on why this concept has become so pervasive, as compared with the limited use of randomness in the nineteenth century.

I begin with the evidence for randomness in twentieth-century physics. There is no need to search far, for the concept helps to separate our physics from the Newtonian physics of the last few centuries. Several events early in this century made randomness prominent in physics. The first was the explanation of Brownian motion. Brownian movement, the microscopically observed motion of small suspended particles in a liquid, had been known since the early 1800's. A variety of explanations had been proposed, all unsatisfactory. But Albert Einstein showed, in one of his three famous papers of 1905, that Brownian motion could be understood in terms of kinetic theory:

> . . . it will be shown that according to the molecular-kinetic theory of heat, bodies of microscopically visible size, suspended in a liquid, will perform movements of such magnitude that they can be easily observed

[1]Donald M. Mackay, "Theoretical Models of Space Perception—Appendix," in "Aspects of the Theory of Artificial Intelligence," *The Proceedings of the First International Symposium of Biosimulation*, edited by C. A. Muses (Plenium Press, New York, 1962), p. 240.

in a microscope on account of the molecular motions of heat. It is possible that the movements to be discussed here are identical with the so-called "Brownian molecular motion." . . . if the movement discussed here can actually be observed . . . then classical thermodynamics can no longer be looked on as applicable with precision to bodies even of dimensions distinguishable in a microscope. . . . On the other hand [if] the prediction of this movement proves to be incorrect, weighty argument would be provided against the molecular-kinetic theory of heat.[2]

It is the randomness of the process, often described as a "random walk," which is the characteristic feature of Brownian motion.

But an even more direct experimental situation focused attention on randomness. During the last years of the nineteenth century, physicists suddenly found many new and strange "rays" or "radiations," including those from radioactive substances. A series of experimental studies on alpha-rays from radioactive elements led Rutherford to say in 1912 that "The agreement between theory and experiment is excellent and indicates that the alpha particles are emitted at random and the variations accord with the laws of probability."[3] These radiations were associated with the core of the atom, the nucleus, so randomness was present in the heart of matter.

One of the two principal physical theories developed in the past forty years is the theory of atomic structure, quantum mechanics, developed during the period from 1926 to 1930. Wave mechanics, the form of quantum mechanics suggested by the Austrian physicist Erwin Schrödinger, predicted in its original form only the allowable energy levels and hence the spectroscopic lines for an atom of some particular element. Later, Max Born and Werner Heisenberg gave quantum theory a more extensive interpretation, today called the "Copenhagen Interpretation," which relinquishes the possibility of predicting *exactly* the outcome of an individual measurement of an atomic (or molecular) system. Instead, statistical predictions tell what, on the average, will happen if the same measurement is performed on a large number of identically prepared systems. Identical

[2]Albert Einstein, *Investigations on the Theory of Brownian Movement*, edited by R. Fürth, translated by A. A. Cowper (E. P. Dutton, New York).

[3]E. Rutherford, *Radioactive Substances and their Radiations* (Cambridge University Press, Cambridge, 1913), p. 191.

measurements on identically prepared systems, in this view, do not always give the same result. Statistical ideas had been used in the nineteenth-century physics, but then it was always assumed that the basic laws were completely deterministic. Statistical calculations were made when one lacked complete information or because of the complexity of the system involved. In the statistical interpretation of quantum mechanics I have just described, however, randomness is not accepted purely for calculational purposes. It is a fundamental aspect of the basic physical laws themselves. Although some physicists have resisted this randomness in atomic physics, it is very commonly maintained. A famous principle in contemporary quantum mechanics, the "uncertainty principle," is closely related to this statistical view of the laws governing atomic systems.

These examples illustrate randomness in physics; now we proceed to other areas. Randomness in art is particularly easy to discuss because it has been so consistently and tenaciously used. My first example is from graphic design. For hundreds of years books and other publications have been "justified" in the margins in order to have flush right margins in addition to flush left margins. This is done by hyphenation and by adding small spaces between letters and words. But recently there is a tendency toward books that are not "justified"; the right margins end just where they naturally end, with no attempt to make them even. This is a conscious design choice. Its effect in books with two columns of print is to randomize partially the white space between columns of print, instead of maintaining the usual constant width white strip.

In the fine arts, the random component of assemblages, such as those of Jean Tinguely, often lies in the use of "junk" in their composition. The automobile junkyard has proved to be a particularly fruitful source of material, and there is something of a random selection there. Random modes of organization, such as the scrap-metal press, have also been used.

In art, as elsewhere, one can sometimes distinguish two kinds of randomness, one involving the creative technique and another exploiting the aesthetic effects of randomness. We see examples of this second type, called "accident as a compositional principle" by Rudolf Arnheim, in three woodcuts by Jean Arp, entitled "Placed According to the Laws of Chance." We would perhaps not have understood the artist's intent if we did not have the titles. Arp, like other contemporary artists, has returned repeatedly to the exploration of such random arrangements. As James Thrall Soby says, "There can be no doubt that the occasional miracles of accident have particular meaning for him. . . . One assumes that he considers spontaneity a primary asset of art."[4]

An area which has been particularly responsive to the exploration of randomness for aesthetic purposes is "op art." Again the titles often identify this concept, as in "Random Field" by Wen-Yin Tsai.

Perhaps more common, however, is the former aspect, an artistic technique by which the artist intentionally employs some random element. The contemporary school of action painting is an example. Jackson Pollock often would place his canvas on the ground and walk above it allowing the paint to fall several feet from his brush to the canvas. Soby describes it as follows: "Pollock's detractors call his current painting the 'drip' or 'spatter' school, and it is true that he often spreads large canvases on the floor and at them flings or dribbles raw pigments of various colors."[5] With this method he did not have complete control of just where an individual bit of paint fell—this depended in a complicated way on the position of the brush, the velocity of the brush, and the consistency of the paint. Thus this technique had explicit chance elements, and its results have been compared to Brownian motion.

Similarly, J. R. Rierce, in *Symbols, Signals, and Noise,* discussing random elements in art, gives some examples of computer-generated art. He emphasizes the interplay of "both randomness and order" in art, using the kaliedoscope as an example.

I will comment even more briefly on music. In Percy Granger's "Random Round" each instrument has a given theme to play; the entrances are in sequence, but each player decides for himself just when he will enter. Thus each performance is a unique event, involving random choices. The most famous example of random musical composition is the work of John Cage. One of his best known works involves a group of radios on a stage, each

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[4]James Thrall Soby, *Arp* (Museum of Modern Art, New York, 1958).

[5]James Thrall Soby, "Jackson Pollock," in *The New Art in America* (Frederick Praeger, Inc., Greenwich, Conn., 1957).

with a person manipulating the controls. They work independently, each altering things as he wishes, and the particular performance is further heavily dependent on what programs happen to be playing on the local radio stations at the time of the performance. There is no question that Cage furnishes the most extreme example of exploitation of techniques with a chance component.

Most evidence for randomness in literature is not as clear as in science, art, or music. The first example is clear, but perhaps some will not want to call it literature at all. In 1965 two senior students at Reed College saw some examples of computer-produced poetry and decided that they could do as well. As their model was symbolist poetry, they did not attempt rhyme or meter, although their program might be extended to cover either or both. The computer program is so organized that the resulting poem is based on a series of random choices. First, the computer chooses randomly a category—possibilities are such themes as "sea" or "rocks." The program then selects (again using a built-in random number generator) a sentence structure from among twenty possibilities. The sentence structure contains a series of parts of speech. The computer randomly puts words into it, keeping within the previously chosen vocabulary stored in the computer memory. Because of the limited memory capacity of the small computer available, only five words occur in a given thematic and grammatical category. There are occasionally some interesting products.

Turning from a student effort to a recently available commercial product, consider the novel *Composition I* by Marc Saporta, which comes in a box containing a large number of separate sheets. Each page concludes with the end of a paragraph. The reader is told to shuffle the pages before beginning to read. Almost no two readers will see the pages in the same order, and the ordering is determined in a random process. For some readers the girl is seduced before she is married, for other readers after she is married. A similar process has been used by William Burroughs in *The Naked Lunch* and elsewhere, except that in this case the shuffling is done by the writer himself. Burroughs writes on many separate pieces of paper and then orders them over and over in different ways until he is satisfied with the arrangement. He has suggested that his work can be read in other orders, and ends *The Naked Lunch* with an "Atrophied Preface."

P. Mayersburg[6] has pointed out elements of chance construction in several other writers' work. He says of Michel Botor: "*Mobile* is constructed around coincidence: coincidence of names, places, signs, and sounds. . . . Coincidence implies the destruction of traditional chronology. It replaces a pattern of cause and effect with one of chance and accident." He sees another chance aspect in these writers: they recognize that they cannot completely control the mind of the reader.

But can we find examples in the work of more important writers? The evidence is less direct. While contemporary artists have openly mentioned their use of randomness, contemporary writers and critics, with a few exceptions, have seldom been willing to admit publicly that randomness plays any role in their writings. But I will argue that randomness is nevertheless often there, although I am aware of the difficulty of establishing it firmly.

The cubist poets, perhaps because of their associations with artists, did experiment consciously with randomness. The story is told of how Apollinaire removed all the punctuation from the proofs of *Alcools* because of typesetting errors, and he continued to use random organization in his "conversation poems" and in other work.

> The "opposite of narration" defines the very quality Apollinaire finally grasped in following cubism into the experimental work of Delaunay, the quality he named simultanism. It represents an effort to retain a moment of experience without sacrificing its logically unrelated variety. In poetry it also means an effort to neutralize the passage of time involved in the act of reading. The fragments of a poem are deliberately kept in a random order to be reassembled in a single instant of consciousness.[7]

It can be argued that James Joyce used random elements in *Ulysses* and *Finnegans Wake*. Several minor stories at least indicate that Joyce was not unfriendly toward the use of random input. For example, when Joyce was dictating to Samuel Beckett, there was a knock at the door. Joyce said, "Come in," and Beckett wrote down, "Come in," thinking that it was part of the book. He immediately

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[6]P. Mayersberg, "The Writer as Spaceman," *The Listener*, October 17, 1963, p. 607.

[7]Roger Shattuck, *The Banquet Years* (Harcourt, Brace, and Co., New York), p. 238.

realized that Joyce had not intended to dictate it; but when he started to erase it, Joyce insisted that it should stay. And it is still there in *Finnegans Wake*, because of a chance occurrence. A related comment is made by Budgin in *James Joyce and the Making of Ulysses*: "... he was a great believer in his luck. What he needed would come to him."

Proceeding from such stories to Joyce's books, I believe that there are random elements in the vocabulary itself. It is well known that much of the vocabulary of *Finnegans Wake* differs from the vocabulary of other English-language books. Some of the words are combinations of other better-known English words, and others are traceable to exotic sources. I do not think that Joyce constructed every new word carefully, but rather that he consciously explored randomly or partially randomly formed words. There is some slight tradition for this procedure in such works as "Jabberwocky."

Another aspect of Joyce's writing, shared with other works of contemporary literature, also has some connection with our theme, although this connection is not generally realized. I refer to the "stream of consciousness" organization. The Victorian novel was ordered in a linear time sequence; there were occasional flashbacks, but mostly the ordering of events in the novel was chronological. The stream of consciousness novel does not follow such an order, but instead the events are ordered as they might be in the mind of an individual. This psychological ordering has distinctly random elements. *Finnegans Wake* has been interpreted as one night in the mental life of an individual. I would not claim that our conscious processes are completely random, but I think it is not impossible to see *some* random elements in them

We mentioned that it has not been customary to admit that randomness is a factor in contemporary literature. Much of the critical literature concerning Joyce exemplifies this. But at least one study sees Joyce as using random components: R. M. Adams' *Surface and Symbol—the Consistency of James Joyce's Ulysses*.[8] Adams relates the story of the "come in" in *Finnegans Wake*, and he tells of Joyce's requesting "any God dam drivel you may remember" of

[8]R. M. Adams, *Surface and Symbol—The Consistency of James Joyce's Ulysses* (Oxford University Press, New York, 1952).

forces on each other and on the walls of the container. To know the positions and velocities of all the particles was impossible because of the multitude of particles; ordinary quantities of gas contained 10^{24}—one followed by twenty-four zeros—particles. This lack of complete information made it necessary to use general properties such as energy conservation in connection with probability considerations. One could not predict where each particle would be, but one could predict average behavior and relate this behavior to observed thermodynamical quantities. Thus statistical thermodynamics introduced statistical modes of thought to the physicist; but the underlying laws were still considered to be deterministic.

A fundamental quantity in thermodynamics, entropy, was found to have a simple statistical interpretation: it was the measure of the degree of randomness in a collection of particles. Entropy could be used as the basis of the most elegant formulation of the second law of thermodynamics: in a closed system the entropy always increases, or the degree of randomness tends to increase.

A special series of technical problems developed over the two kinds of averaging used in statistical considerations: time-averaging, inherently involved in all measurements; and averaging over many different systems, the ensemble averaging of Gibbs used in the calculations. The "ergodic theorems" that were extensively developed to show that these two averages were the same again forced careful and repeated attention on probabilistic considerations.

My second example is the theory of evolution, almost universally acknowledged as the major intellectual event of the last century. Charles Darwin and Alfred Russell Wallace developed the theory independently, using clues from Malthus' essay on population. The basic ideas are well known. Organisms vary, organisms having the fittest variations survive, and these successful variations are passed on to the progeny. The random element of evolution is in the "numerous successive, slight favorable variations"; the offspring differ slightly from the parents. Darwin, lacking an acceptable theory of heredity, had little conception of how these variations come about; he tended to believe, parallel to the developers of statistical thermodynamics, that there *were* exact laws, but that they were unknown.

> I have hitherto sometimes spoken as if the variations . . . had been due to chance. This, of course, is a wholly incorrect expression, but it seems

> to acknowledge plainly our ignorance of the cause of each particular variation.[10]

But others were particularly disturbed by the chance factors apparently at work in variations. This was one of the factors that led Samuel Butler from his initial praise to a later critical view of Darwin. Sir John Herschel was very emphatic:

> We can no more accept the principle of arbitrary and casual variation and natural selection as a sufficient account, *per se*, of the past and present organic world, than we can receive the Laputian method of composing books . . . as a sufficient one of Shakespeare and the *Principia*.[11]

When a usable theory of heredity was developed during the next half century, randomness played a major role, both in the occurrence of mutations in genes and in the genetic inheritence of the offspring. So, almost in spite of Darwin, chance became increasingly important in evolutionary theory. ". . . The law that makes and loses fortunes at Monte Carlo is the same as that of Evolution."[12]

The theory of evolution roused almost every thinking man in the late nineteenth century. Frederick Pollock, writing about the important British mathematician William Kingdon Clifford, says:

> For two or three years the knot of Cambridge friends of whom Clifford was a leading spirit were carried away by a wave of Darwinian enthusiasm: we seemed to ride triumphant on an ocean of new life and boundless possibilities. Natural selection was to be the master-key of the universe; we expected it to solve all riddles and reconcile all contradictions.[13]

This is only one account outside biology, but it illustrates how evolution affected even those *not* directly concerned with it as a scientific theory. It does not seem unreasonable, then, that at the same time evolution contributed to the new attitude toward randomness. I

[10]C. Darwin, *Origin of the Species* (first edition), p. 114.

[11]Sir Herschel, *Physical Geography of the Globe* (Edinburgh, 1861), quoted in John C. Green, *The Death of Adam* (New American Library, New York), p. 296.

[12]M. Hopkins, *Chance and Error—The Theory of Evolution* (Kegan Paul, Trench, Truber & Co., London, 1923).

[13]W. K. Clifford, *Lectures and Essays* (Macmillan, London, 1886), Introduction.

might also mention two other books that are particularly interesting in showing the influence of evolution outside the sciences, offering details we cannot reproduce here. One is Leo J. Henkin's *Darwinism in the English Novel 1860-1910*; the other is Alvar Ellegård's *Darwin and the General Reader.*

There were of course other things happening in the nineteenth century, but these two developments were important and had far-reaching implications outside of their immediate areas. Alfred North Whitehead, in *Science and the Modern World*, claims that in the nineteenth century "four great novel ideas were introduced into theoretical science." Two of these ideas were energy, whose rise in importance was related to thermodynamics, and evolution. It was consistent with established tradition, however, to believe that the use of chance in these areas was not essential. Other non-scientific factors were also important; for example, Lord Kelvin's attitude toward chance was colored by religious considerations. In S. P. Thomson's *Life* we find a speech of his in the *Times* of 1903 arguing that "There is nothing between absolute scientific belief in Creative Power and the acceptance of the theory of a fortuitous concourse of atoms."

According to our splash in the puddle theory, we should be able to point out evidence that two nineteenth-century developments, statistical mechanics and evolution, had very far-reaching effects in areas quite different from their points of origin, effects reflecting interest in randomness. This is a big task, but we will attempt to give some minimal evidence by looking at the writings of two important American intellectuals near the turn of the century, both of whom were consciously influenced by statistical mechanics and Darwinian evolution. The two are Henry Adams and Charles Sanders Peirce.

We have Adams' account of his development in *The Education of Henry Adams.* Even a casual glance shows how much of the language of physics and biology occurs in the book, and how often references are made to those areas. Chapter 15 is entitled "Darwinism," and early in the chapter he says:

> The atomic theory; the correlation and conservation of energy; the mechanical theory of the universe; the kinetic theory of gases; and

> Darwin's law of natural selection were examples of what a young man had to take on trust.

Adams had to accept these because he was not in a position to argue against them. Somewhat later in the book Adams comments, in his usual third person:

> He was led to think that the final synthesis of science and its ultimate triumph was the kinetic theory of gases. . . . so far as he understood it, the theory asserted that any portion of space is occupied by molecules of gas, flying in right lines at velocities varying up to a mile a second, and colliding with each other at intervals varying up to seventeen million seven hundred and fifty thousand times a second. To this analysis—if one understood it right—all matter whatever was reducible and the only difference of opinion in science regarded the doubt whether a still deeper analysis would reduce the atom of gas to pure motion.

And a few pages later, commenting on Karl Pearson's "Grammar of Science":

> The kinetic theory of gases is an assertion of ultimate chaos. In plain, chaos was the law of nature; order was the dream of man.

Later, "Chaos was a primary fact even in Paris," this in reference to Henri Poincare's position that all knowledge involves conventional elements.

Of all Henry Adams' writings, "A Letter to American Teachers of History" is most consistently saturated with thermodynamical ideas. This 1910 paper[14] begins with thermodynamics. It first mentions the mechanical theory of the universe, and then says:

> Toward the middle of the Nineteenth Century—that is, about 1850—a new school of physicists appeared in Europe . . . made famous by the names of William Thomson, Lord Kelvin, in England, and of Clausius and Helmholtz in Germany, who announced a second law of thermodynamics.

He quotes the second law of thermodynamics in both the Thomson and the Clausius forms. It is not always clear how seriously one is to take this thermodynamical model of history.

About fifteen pages into "A Letter," Darwin is presented as

[14]Henry Adams, *The Degradation of the Democratic Dogma* (Macmillan and Co., New York, 1920), pp. 137-366.

Meanwhile, the statistical method had, under that very name, been applied with brilliant success to molecular physics. . . . In the very summer preceding Darwin's publication, Maxwell had read before the British Association the first and most important of his researches on the subject. The consequence was that the idea that fortuitous events may result in physical law and further that this is the way in which these laws which appear to conflict with the principle of conservation of energy are to be explained had taken a strong hold upon the minds of all who are abreast of the leaders of thought. [6.297]

Peirce is not reflecting the historical attitude of the physicists who developed statistical thermodynamics but is reading his own views back into this work.

So it is not surprising that chance plays a fundamental role in Peirce's metaphysics. Peirce generalized these ideas into a general philosophy of three categories, Firstness, Secondness, and Thirdness. These three terms have various meanings in his work, but a frequent meaning of Firstness is chance. He was one of the first to emphasize that chance was not merely for mathematical convenience but was fundamental to the universe. He used the word "Tychism," from the Greek for "chance," the "doctrine that absolute chance is a factor in the universe." [6.2000]

This view of the essential role of chance he opposed to the view that universal necessity determined everything by fixed mechanical laws, in which most philosophers of science in the late nineteenth century still believed. In a long debate between Peirce and Carus concerning this issue, Peirce says:

The first and most fundamental element that we have to assume is a Freedom, or Chance, or Spontaneity, by virtue of which the general vague nothing-in-particular-ness that preceded the chaos took on a thousand definite qualities.

In "The Doctrine of Necessity" Peirce stages a small debate between a believer in his position and a believer in necessity, to show that the usual arguments for absolute law are weak. Everyday experiences make the presence of chance in the universe almost obvious:

The endless variety in the world has not been created by law. It is not of the nature of uniformity to originate variation nor of law to beget circumstance. When we gaze on the multifariousness of nature we are looking straight into the face of a living spontaneity. A day's ramble in the country ought to bring this home to us. [6.553]

> A man in China bought a cow and three days and five minutes later a Greenlander sneezed. Is that abstract circumstance connected with any regularity whatever? And are not such relations infinitely more frequent than those which are regular? [5.342]

The necessity of initial conditions in solving the equations of mechanics is another indication to Peirce of the essential part played by chance. Modern scientists have also stressed the "randomness" of initial conditions: E. P. Wigner writes, "There are . . . aspects of the world concerning which we do not believe in the existence of any accurate regularities. We call these initial conditions."

Peirce tells us we must remember that "Three elements are active in the world: first, chance; second, law; and third, habit taking." [1.409] He imagines what a completely chance world would be like, and comments, "Certainly nothing could be imagined more systematic." For Peirce the universe begins as a state of complete randomness. The interesting problem is to account for the regularity in the universe; law must evolve out of chaos. This evolutionary process is far from complete even now, and presents a continuing process still:

> We are brought, then, to this: Conformity to law exists only within a limited range of events and even there is not perfect, for an element of pure spontaneity or lawless originality mingles, or at least must be supposed to mingle, with law everywhere. [1.407]

Thus Peirce's scheme starts with chaos and out of this by habit orderliness comes, but only as a partial state.

What is of interest to us is the fundamental role of chance or randomness in Peirce's cosmology, and the connection of that role with statistical mechanics and Darwinism, rather than the details of his metaphysics.

The two examples of Henry Adams and C. S. Peirce do not establish the splash in the puddle, but they do serve at least to indicate the influence of the Darwinian and kinetic theory ideas, and they show the rising importance of chance.

Although I have concentrated on the relatively increased attention focused upon randomness in the twentieth century as compared with the nineteenth century, randomness attracted some interest before our century. One can find many earlier examples of the order-randomness dichotomy, and there have been periods when, even before the nineteenth century, random concepts acquired some status. One example containing elements of our present dichotomy is the continuing battle between classicism and romanticism in the arts and in literature. But the twentieth-century interest, as we have indicated, is more intense and of different quality. The chance component has never been totally absent; even the most careful artist in the last century could not be precisely sure of the result of his meticulously controlled brush stroke. The classical painter resisted chance—the goal of his years of training was to gain ever greater control over the brush. By contrast the contemporary painter often welcomes this random element and may even increase it. It is this contrast that I intend to stress. Although I point to this one element, the reader should not falsely conclude that I am not aware of nonrandom elements. Even now randomness is seldom the sole factor. When Pollock painted, the random component was far from the only element in his technique. He chose the colors, he chose his hand motions, and he chose the place on the canvas where he wanted to work. Further, he could, and often did, reject the total product at any time and begin over. Except in the most extreme examples, randomness is not used *alone* anywhere; it is almost always part of a larger situation. This is J. R. Pierce's emphasis on order.

The persistence of chance elements in highly ordered societies suggests a human need for these elements. Perhaps no society ever described was more completely organized than Arthur C. Clarke's fictional city of Diaspar, described in ***The City and the Stars***. Diaspar, with its past, and even to some extent its future, stored in the memory banks of the central computer, has existed with its determined social structure for over a billion years. But the original planners of the city realized that perfect order was too much for man to bear:

> "Stability, however, is not enough. It leads too easily to stagnation, and thence to decadence. The designers of the city took elaborate steps to avoid this. . . . I, Khedron the Jester, am part of that plan. A very small part, perhaps. I like to think otherwise, but I can never be sure. . . . Let us say that I introduce calculated amounts of disorder into the city."[17]

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[17]A. C. Clarke, *The City and the Stars* (Harcourt, Brace and Co., New York, 1953), pp. 47-53.

But our present situation confronts us with something more than a simple dichotomy between order and disorder, as suggested in both of the following passages, one from L. L. Whyte and one from Erwin Schrödinger:

> In his long pursuit of order in nature, the scientist has turned a corner. He is now after *order and disorder* without prejudice, having discovered that complexity usually involves both.[18]

> The judicious elimination of detail, which the statistical system has taught us, has brought about a complete transformation of our knowledge of the heavens. . . . It is manifest on all sides that this statistical method is a dominant feature of our epoch, an important instrument of progress in almost every sphere of public life.[19]

Although the use of random methods in physics and biology at the end of the last century originally assumed that one was dealing with areas that could not be treated exactly, but where exact laws did exist, a subtle change of view has come about, so that now random elements are seen as having a validity of their own. Both Whyte and Schrödinger see the current situation as something more than a choice between two possibilities. Whyte thinks both are essential for something he calls "complexity." But I prefer Schrödinger's suggestion that the two are not necessarily opposed, and that randomness can be a tool for increasing order. Perhaps we have a situation resembling a Hegelian synthesis, combining two themes which had been considered in direct opposition.

Finally I note an important twentieth century reaction to randomness: Joy. The persistence of games of chance through the ages shows that men have always derived some pleasure from randomness; they are important in Clarke's Diaspar, for example:

> In a world of order and stability, which in its broad outlines had not changed for a billion years, it was perhaps not surprising to find an absorbing interest in games of chance. Humanity had always been fascinated by the mystery of the falling dice, the turn of a card, the spin of the pointer . . . however, the purely intellectual fascination of chance remained to seduce the most sophisticated minds. Machines that behaved

[18] L. L. Whyte, "Atomism, Structure, and Form," in *Structure in Art and in Science*, ed. G. Kepes (G. Braziller, New York, 1965) p. 20.

[19] E. Schrödinger, *Science and Human Temperament*, trans. J. Murphy and W. H. Johnston (W. W. Norton, Inc., New York), p. 128.

Everyday observation of waves includes ripples in water or vibrations set up by a slammed door. A wave is a traveling pattern, not the mass movement of matter.

13 Introduction to Waves

Physical Sciences Study Committee

1965

15-1 A Wave: Something Else That Travels

In the last chapter we considered at some length a particle model of light, in which we supposed that light consisted of a stream of particles or corpuscles. We found that this model fails to provide completely satisfactory explanations for some of the behavior of light that we observed. We therefore find ourselves faced with a choice: we can try to construct a better particle model that will succeed where the earlier one failed, or we can look for a new model based on a completely different concept. Let us try the second approach.

The most basic thing to be accounted for in *any* model of light is the fact that light travels through space. In looking for a new theory, we first ask whether there is anything except a particle (or stream of particles) that can move from one point to another. The answer is "yes." Consider, for example, what happens when we drop a pebble into a quiet pond. A circular pattern spreads out from the point of impact. Such a disturbance is called a *wave*, and if you watch the water closely enough, as such a wave moves across the surface, you will find that although the water may be churned and jostled locally, it does not move forward with the wave. This is quite clear if you watch a bit of wood or a small patch of oil that may be floating on the pond. The wood or oil moves up and down as the wave passes; it does not travel along with the wave. In other words, a wave can travel for long distances, but once the disturbance has passed, every drop of water is left where it was before.

at least, a gambler's act of throwing his whole personality—his accidents, his skills, his weaknesses, his luck—against the world."

My final example of randomness is lighter. I am reliably informed that several years ago a group of students at Harvard formed a random number society for propagating interest in random numbers. Among other activities they chose each week a random number of the week, and persuaded a local radio station to announce it!

Although the reader may not accept my thesis, I continue with the assumption that our culture differs from the culture of the previous few centuries partly because of an increased concern with and conscious use of elements which are random in some sense of the word. We have seen this use in seemingly unrelated areas, and in ways previously very uncommon. Now we will enter on an even more difficult problem: assuming that the twentieth century consciously seeks out randomness, can we find any historical reasons for its permeating different fields?

I need hardly remind you of the difficulty of this problem. Theorizing in history has generally seemed unreasonable, except to the theorist himself and to a small group of devoted followers. The present problem is not general history but the even more difficult area of intellectual history. Despite vigorous attempts to understand cultural evolution, or particular aspects of it such as the development of scientific knowledge, I believe it is fair to say that we know far less than we would like to know about how ideas develop. It would, therefore, be unreasonable for me to expect to give a rich theory of how humans modify ideas. Instead I shall grope toward a small piece of such a theory, basing my attempt on the evidence presented on randomness as a twentieth-century theme.

The rough idea I shall bring to your attention might be crudely

If we look around us, we can find all sorts of examples of waves. For instance, we notice an American flag as it ripples in the breeze at the top of a flagpole. The ripples or waves travel out along the cloth. Individual spots on the cloth of the flag, however, hold their positions as the waves pass by. The fourth white star in the bottom line on the field of blue always remains the fourth star in the bottom line and its distances from the four edges of the flag remain unchanged. Just as the water does not travel with the water waves, so the cloth of the flag remains in place when the waves have passed through it.

Some waves are periodic or nearly so; the motion of the material repeats itself over and over. Not all waves, however, have this property. For example, when you slam the door of a room, the air in the doorway is suddenly compressed, and this single short compression passes as a disturbance across the room, where it gives a sudden push to a curtain hanging over the window. Such a wave of short duration is called a *pulse*.

Here is another example of a wave pulse. We place half a dozen pocket-billiard balls (plastic croquet balls will work, too) in a straight line new waves of interest in randomness, which eventually, through the traveling of the wave, affected areas at a distance from the source. Probably one source is not enough; often one needs reinforcement from several disturbances to create a revolution. And the sources themselves must be powerful if the effects are to be felt at great distances in the cultural plane.

I shall note two nineteenth-century events which were powerful sources, and so may have contributed to a new interest in randomness. Both are from science, but this may reflect my own specialization in history of science; I am likely to find examples from the area I know best. My two examples are of unequal weight. The minor one certainly affected profoundly the physicist's attitude toward randomness, but how widespread its effect was is not clear. The second example, however, was the major intellectual event of the century.

The first example is the development of kinetic theory and statistical thermodynamics in the last half of the century, involving Rudolf Clausius, James Clerk Maxwell, Ludwig Boltzmann, Willard Gibbs, and others. Because physicists believed that Newtonian mechanics was the fundamental theory, they thought that all other theories should "reduce" to it, in the same sense that all terms could be defined using only the terms of mechanics, and that the fundamental principles of other areas could be deduced logically from the principles of mechanics. This attitude, applied to thermodynamics, led to kinetic theory and statistical thermodynamics.

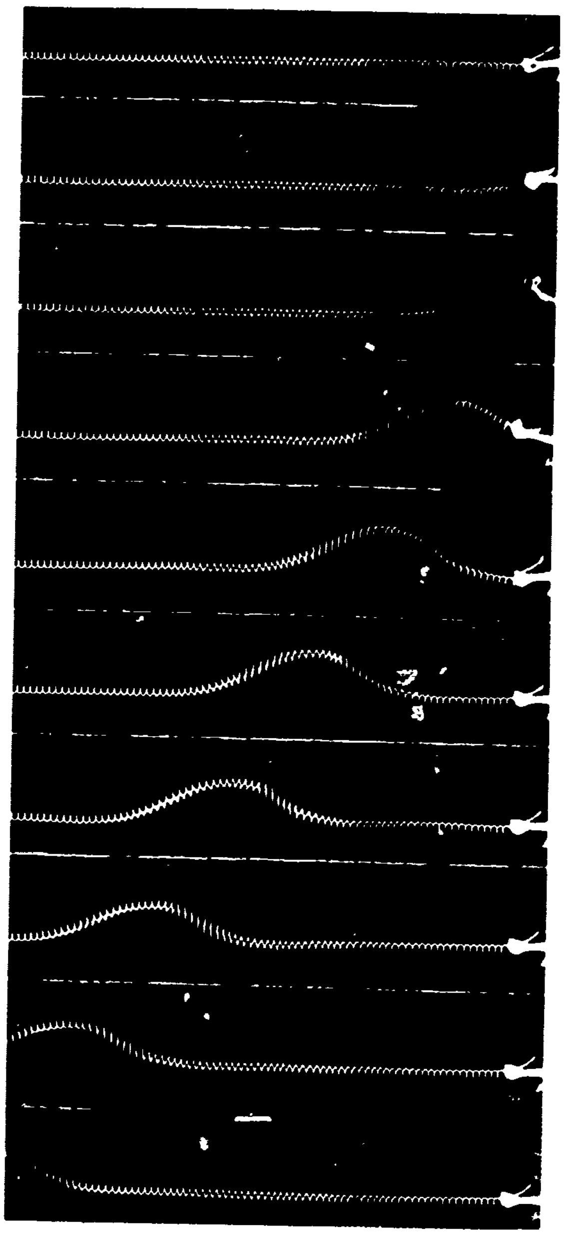
In kinetic theory a gas (a word which may originally have meant "chaos"[9]) was viewed as a very large number of separate particles, each obeying the Newtonian laws of motion, exerting

[9]Pointed out to me by Steven Brush. See J. R. Partington, "Joan Baptist von Helmont," *Annals of Science*, I, 359-384 (1936).

later, the driver of the second car, seeing that the car ahead of him is in motion, starts his car forward. Still later, the driver of the third car gets his car under way, and so on down the line away from the traffic signal. You can see a "pulse" move down the line of cars. It is interesting that this "starting pulse" travels in one direction while the cars travel in the other direction.

Just how fast the starting pulse travels back depends on how fast the various drivers react, and how their cars respond. If we were able to handpick a group of drivers with identical reaction times, and provide them with cars that accelerated in exactly the same way, the starting pulse would travel backward at a uniform speed.

What is alike in all of these examples? In each case the disturbance travels through some medium—through the water, the cloth of a flag, the billiard balls, or the line of cars; but the medium does not go along with the disturbance. Disturbances which travel through media are what we mean by waves. We can now answer the question we asked at the beginning of this section. Is there anything except a particle that can move from one point to another? A wave, a thing which is not itself a particle of matter,



15-1 The generation and motion of a pulse along a spring shown by a series of pictures taken with a movie camera.

15-2 Waves on Coil Springs

Do waves really behave like light? To find out, we must know more about them. When we know how they act, we can compare their behavior with what we know about light and with other things that we can find out about it. The variety of examples we have mentioned also suggests that waves are worth studying for their own sake.

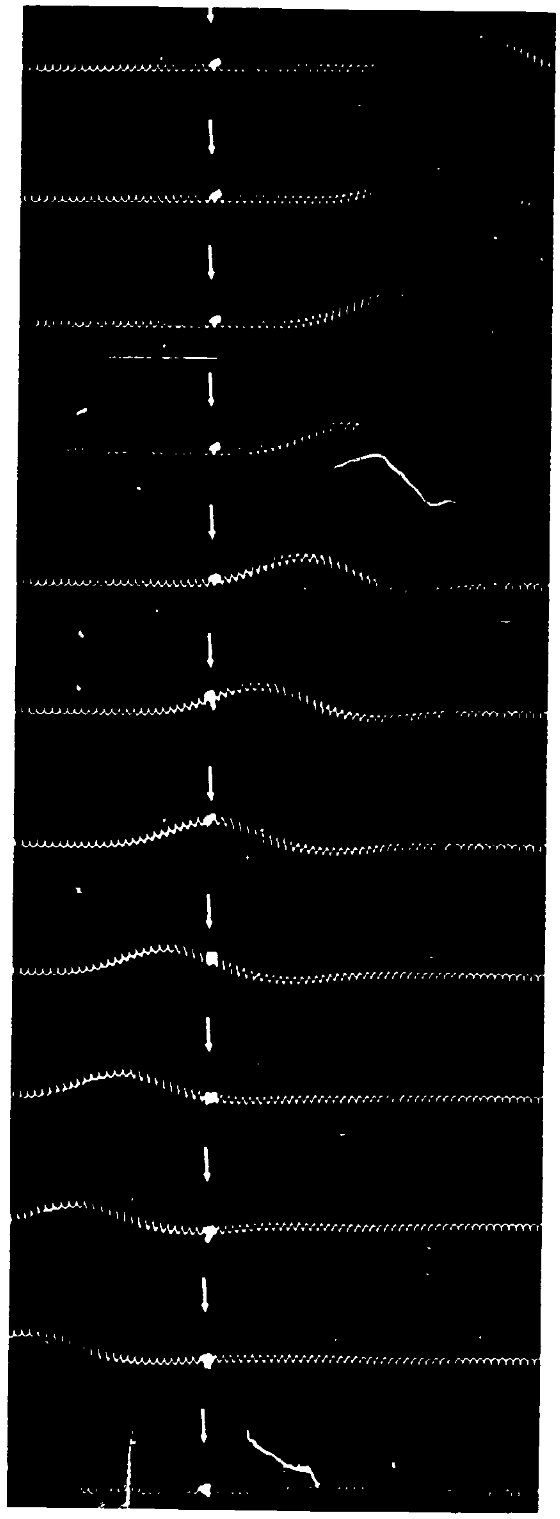
It is convenient to start our study of waves with a coil spring.* Figure 15-1 shows pictures of a pulse traveling along such a spring. These pictures were taken by a movie camera at intervals of $\frac{1}{24}$ of a second.

We see that the shape of the pulse does not change as it moves along. Except for the fact that the pulse moves, its picture at one moment is just like a later picture. Also we see that the pulse moves the same distance in each interval between pictures—it moves along the spring at constant speed.

The spring as a whole is not permanently changed by the passage of the pulse. But what happens to each small piece of spring as the pulse goes by? To help us fix our attention on one piece, we can mark the spot by tying on a bit of white string or ribbon as shown in Fig. 15-2. If we then shake the spring to start a pulse moving along it, we can see how the marked spot is displaced. We find that it moves at right angles to the spring as the pulse passes it.

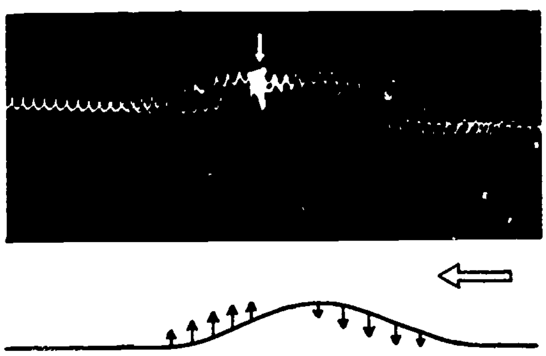
Other pieces of the spring, as well as the marked spot, also move. We can see which pieces are moving and which way they go if we look at two pictures, one of which is taken

* If you find it hard to get a coil spring, a flexible clothesline or a rubber tube will also do pretty well. Tie one end to a doorknob and shake the other. If the clothesline or tube is sufficiently heavy, you will get good pulses that travel slowly enough for easy observation.



15-2 The motion of a pulse from right to left along a spring with a ribbon around one point. The ribbon moves up and down as the pulse goes by, but does not move in the direction of motion of the pulse.

shortly after the other. Here we shall use two successive pictures taken from Fig. 15–2. We have printed these two pictures together in Fig. 15–3 so that we see the pulse in two successive positions just as we would see it in a rapid double exposure. Below the photo in Fig. 15–3 we have traced the pulse in its earlier position, and the gray line shows the later position. As the arrows show, while the pulse moved from



15-3 The relation between the motion of a pulse traveling from right to left and the motion of the coil. The photograph shows the pulse in two successive positions. The arrows in the diagram indicate how the coil moves as the pulse passes. The large, open arrow shows the direction of the motion of the pulse.

right to left, each piece of the coil in the right-hand half of the pulse moved down and each piece of coil in the left-hand half moved up.

If the pulse were moving from left to right, just the reverse would be true, as we show in Fig. 15–4. Here we use a schematic pulse because it is a little easier to work with and we can make the time interval between positions as short as we wish. In this way we can determine the instantaneous motion of the coil. Thus, if we know in which direction the pulse is moving, we can determine how each point of the spring moves at any particular stage in the passage of

1. The Law of Conservation
2. The Law of Dissipation
3. The Law of Evolution

The contrast Adams is making is between Darwin's ideas and Kelvin's ideas.

We find other similar references in Henry Adams, but this should be enough to show his interest in Darwin and kinetic theory. Other aspects of contemporary science also very much influenced him; he often refers to the enormous change produced by the discovery of new kinds of radiation at the turn of the century. He seems to be a particularly rewarding individual to study for an understanding of the intellectual currents at the beginning of the century, as Harold G. Cassidy has pointed out:

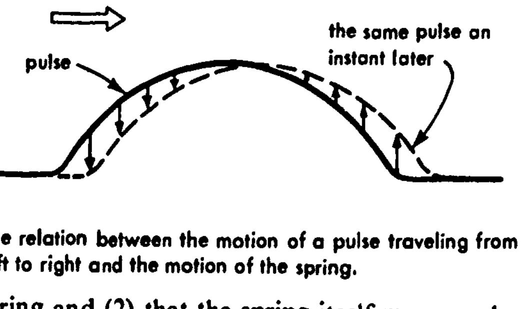
> Henry Adams was an epitome of the non-scientist faced with science that he could not understand, and deeply disturbed by the technological changes of the time. He was a man with leisure, with the wealth to travel. With his enquiring mind he sensed, and with his eyes he saw a great ferment at work in the World. He called it a force, and tried to weigh it along with the other forces that moved mankind. The education he had received left him inadequate from a technical point of view to understand, much less cope with, these new forces. Yet his insights were often remarkable ones, and instructive to us who look at our own period from so close at hand.[15]

As final evidence we consider the work of the seminal American philosopher Charles Sanders Peirce. Peirce, although seldom holding an academic position, played an important role in American philosophy, particularly in the development of pragmatism. He was the leader of the informal "Metaphysical Club" in Cambridge dur-

[15] Harold G. Cassidy, "The Muse and the Axiom," *American Scientist* 51, 315 (1963).

the pulse. On the other hand, if we know how the parts of the spring move, we can determine the direction in which the pulse is traveling.

We now have a good notion of how the pieces of spring move, even though there is no visible motion in any one of our pictures. Really, what we have done is to observe (1) that any pulse moves undistorted at constant speed along the



15-4 The relation between the motion of a pulse traveling from left to right and the motion of the spring.

spring and (2) that the spring itself moves only at right angles to the motion of the pulse. We can combine these two pieces of information to learn how each part of the spring moves at any time. Of course, we have looked only at the simplest waves, and the statement we have just made may not be true of all waves. Even in the awareness of these theories (some of which he did not fully accept) evidenced by many passages in his work, such as these comments in "On the Fixation of Belief":

> Mr. Darwin has purposed to apply the statistical method to biology. The same thing has been done in a widely different branch of science, the theory of gases. We are unable to say what the movements of any particular molecule of gas would be on a certain hypothesis concerning the constitution of this class of bodies. Clausius and Maxwell were yet able, eight years before the publication of Darwin's immortal work, by the application of the doctrine of probabilities, to predict that in the long run such and such a proportion of the molecules would under given circumstances, acquire such and such velocities; that there would take place, every second, such and such a relative number of collisions, etc., and from these propositions were able to deduce certain properties of gases especially in regard to the heat relations. In like manner, Darwin, while unable to say what the operation of variation and natural selection in any individual case will be, demonstrates that, in the long run, they will, or would, adopt animals to their circumstances.[16] [5.362]

A second example in which Peirce links the two theories is in "Evolutionary Lore":

> *The Origin of the Species* was published toward the end of the year 1859. The preceding years since 1846 had been one of the most productive seasons—or if extended so as to cover the book we are considering, *the* most productive period in the history of science from its beginnings until now. The idea that chance begets order, which is one of the cornerstones of modern physics . . . was at that time put into its clearest light. [6.297]

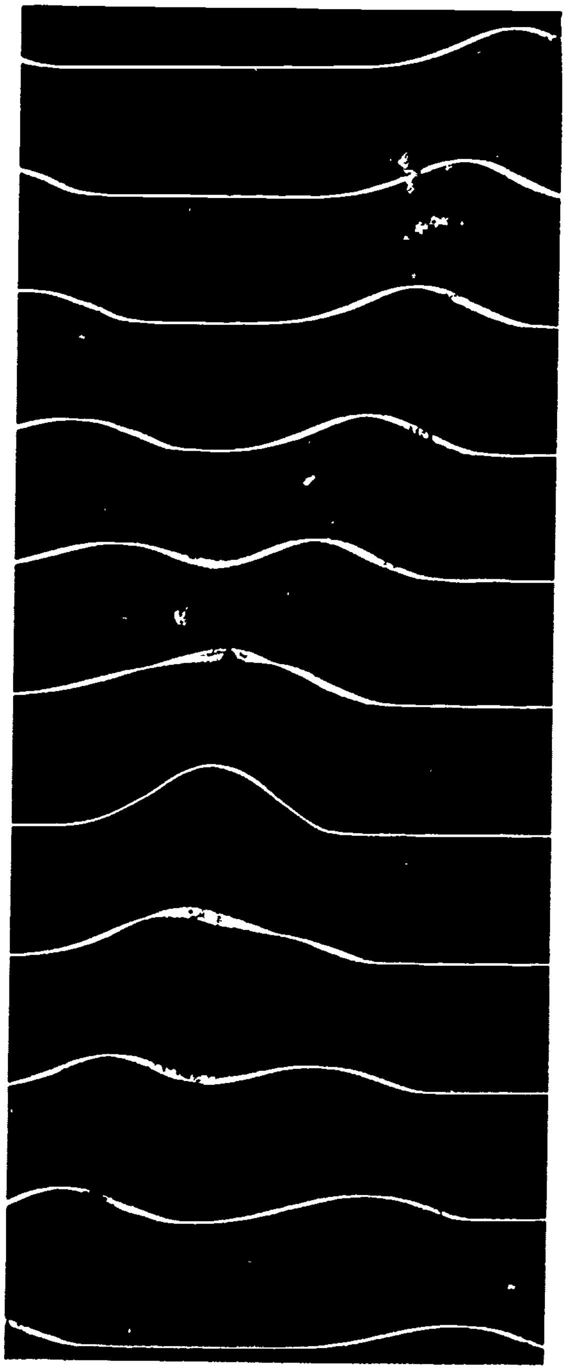
He goes on to mention Quetelet and Buckle, and then begins a discussion of the kinetic theory:

[16]C. S. Peirce, *Collected Papers* ed. C. Hartshorn and P. Weiss (Harvard University Press, Cambridge, Mass.). References are to section numbers.

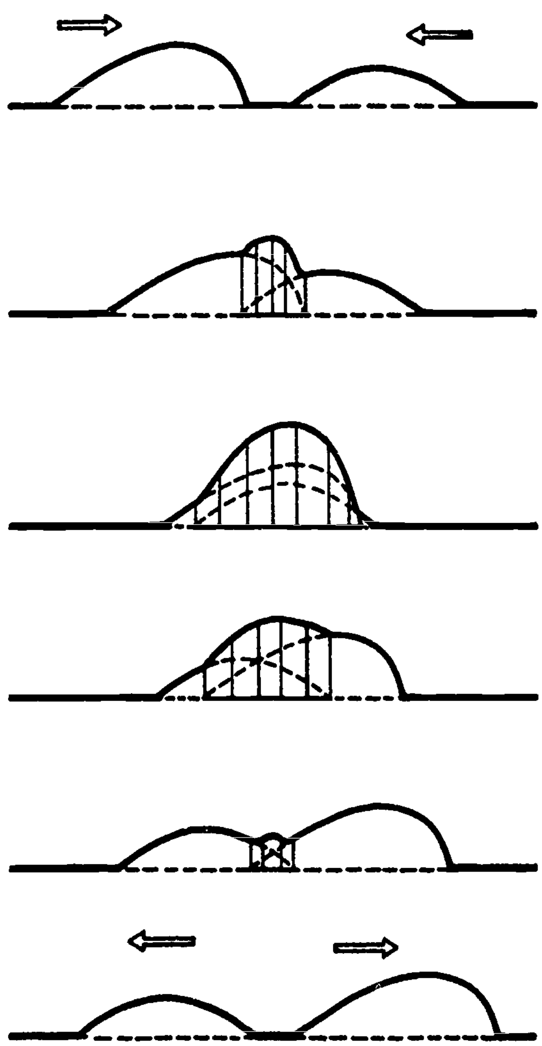
other, or do they somehow knock each other out?

The best way to find out is to try. The photographs in Fig. 15–5 show what happens when two pulses are started at opposite ends of a spring at the same time, one traveling from left to right and one from right to left. The top pictures show the pulses approaching each other as if each had the spring to itself. As they cross each other, the two pulses combine to form complicated shapes. But after having crossed, they again assume their original shapes and travel along the spring as if nothing had happened, as is indicated by the pictures at the bottom. The left-going pulse continues to travel to the left with its original shape. The right-going pulse continues to move on to the right with its earlier form. We can perform this experiment over and over with different pulses. We always get the same general result.

The fact that two pulses pass through each other without either being altered is a fundamental property of waves. If we throw two balls in opposite directions, and they hit each other, their motion is violently changed. The crossing of waves and the crossing of streams of balls made of solid matter are thus two very



15-5 Two pulses crossing each other. Notice thot the two pulses have different shapes. Thus we can see thot the one which was on the left at the beginning is on the right after the crossing, and vice versa.

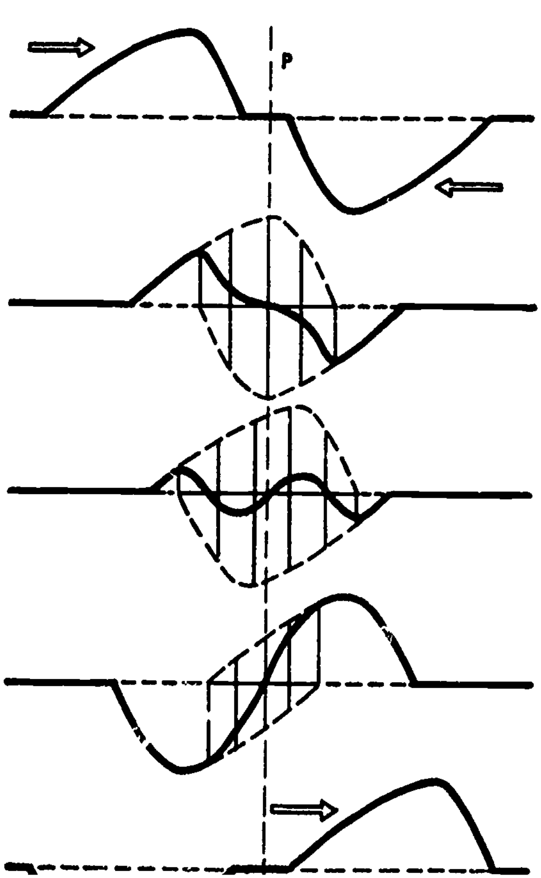


15-6 The superposition of two pulses. The displacement of the combined pulse is the sum of the separate displacements.

matter of fact, it also works for more than two pulses—the displacements due to any number of pulses can be added.

We can summarize the whole situation as follows. To find the form of the total wave disturbance at any time, we add at each point the displacements belonging to each pulse that is proper, so it is not expressed openly. But in other places randomness is clearly acknowledged. We noted that the artist is particularly willing to admit the use of randomness, so it is not surprising to see an artist, Ben Shahn, admitting his pleasure: "I love chaos. It is a mysterious, unknown road with unexpected turnings. It is the way out. It is freedom, man's best hope."[20]

[20]Quoted in *Industrial Design* 13, 16 (1966).



Our second special case is shown in Fig. 15–9. Here we have two similar pulses, one coming from the right and one from the left. In one the displacements are upward and in the other they are downward. These pulses differ from those of Fig. 15–7 in that neither is symmetrical, although the two are alike in shape and size.

Because neither of the pulses is symmetrically shaped, they never completely cancel each other. But there is always one point P on the spring which will stand still. That point is exactly halfway between the two pulses. As the pulses come together, they pass simultaneously through that halfway point in such a way that the highest point of one pulse and the lowest point of the other just cancel each other out. The same argument applies to any other pair of corresponding points on the pulses. They always arrive at the midpoint of the spring together, one on top and one at the bottom. Consequently, the midpoint stands still.

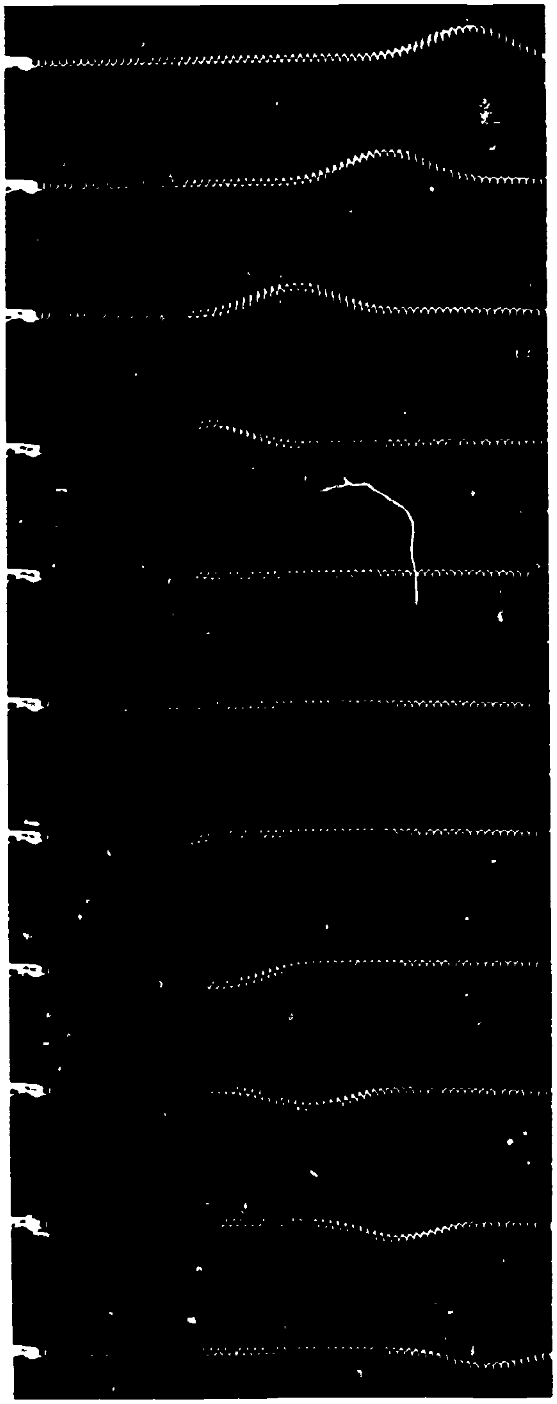
15-4 Reflection and Transmission

When a pulse moving on a spring comes to an end that is held fixed, it bounces back. This reversal of direction is called *reflection*, and the pulse that comes back is called the *reflected pulse*. In Fig. 15–10 the fixed end is on the left. In the original or *incident pulse*, which moves to the left, the displacement is upward. The returning pulse has its displacement downward. The pulse comes back upside down, but with the same shape that it had before it was reflected.

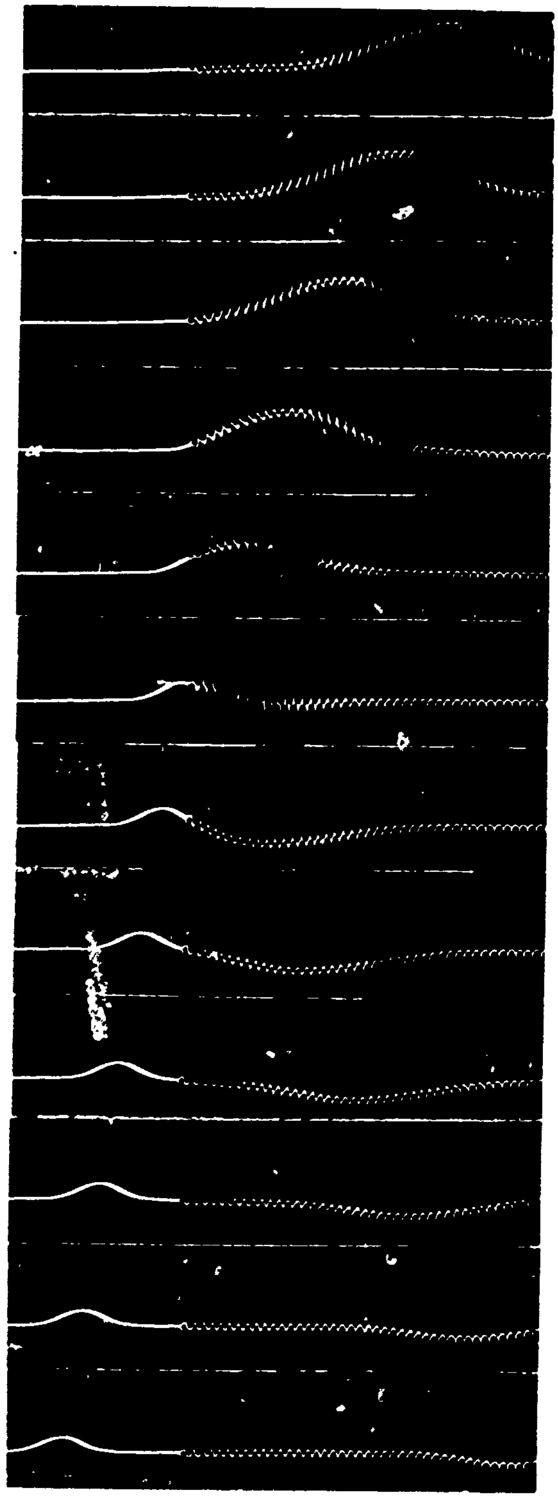
You may wonder why the reflected pulse is upside down. The reason for this behavior is that one point on the spring, in this case the end point held by the hand, does not move. We have already met a situation where a point on the spring remained at rest; this was the point

P in Fig. 15–9. Cover the right-hand half of Fig. 15–9 and you will see an upward pulse moving to the right, "flattening out" as it approaches *P*, and finally being reflected upside down. Now, at the front of an upward pulse, the spring itself moves upward (Fig. 15–3). When the front of the pulse in Fig. 15–9 gets to *P*, the point *P* should move upward. But since *P* remains at rest, the upward motion of the spring must be canceled by a downward motion. The only difference between the situations shown in Figs. 15–9 and 15–10 is that in Fig. 15–9 we supply the necessary downward motion by sending a downward pulse from the right, whereas in Fig. 15–10 we supply the downward motion by simply holding the end point fixed. Forcing the end point to remain at rest is just another way of supplying the downward motion which cancels the motion of the spring due to the original pulse, and then propagates to the right in the form of an upside-down pulse.

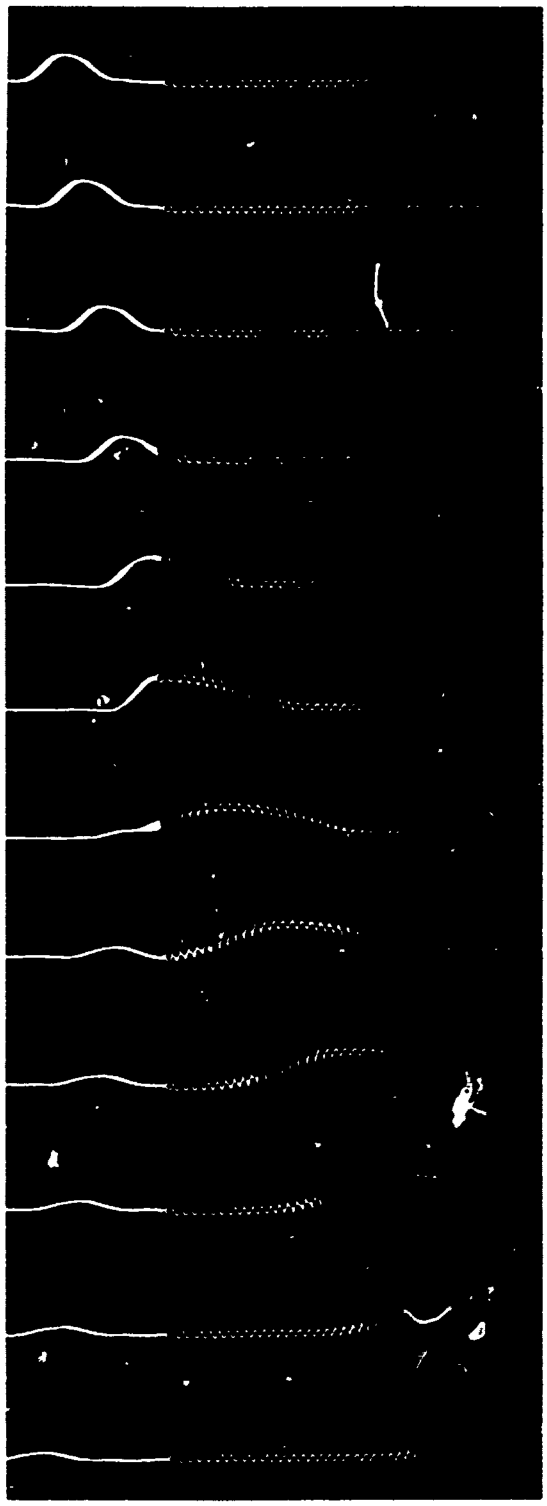
Imagine now that instead of fixing our coil spring at one end, we connect it to another spring which is much heavier and therefore harder to move. Our new arrangement will be somewhere in between the two cases (a) the original spring tied down, and (b) the original spring just lengthened by an additional piece of the same material. In case (a) the whole pulse is reflected upside down; in case (b) the whole pulse goes straight on. We may, therefore, expect that under our new arrangement part of the pulse will be *reflected* upside down, and part of it will go on, or as we say, will be *transmitted.* This effect is shown in Fig. 15–11 where the original pulse comes from the right and the heavier spring is on the left. We see that at the junction or boundary between the two springs —which are the media in which the wave travels—the pulse splits into two parts, a reflected and a transmitted pulse. Like superposition, the splitting into a reflected and a transmitted part is a typical wave property.



15-10 Reflection of a pulse from a fixed end. The reflected pulse is upside down.



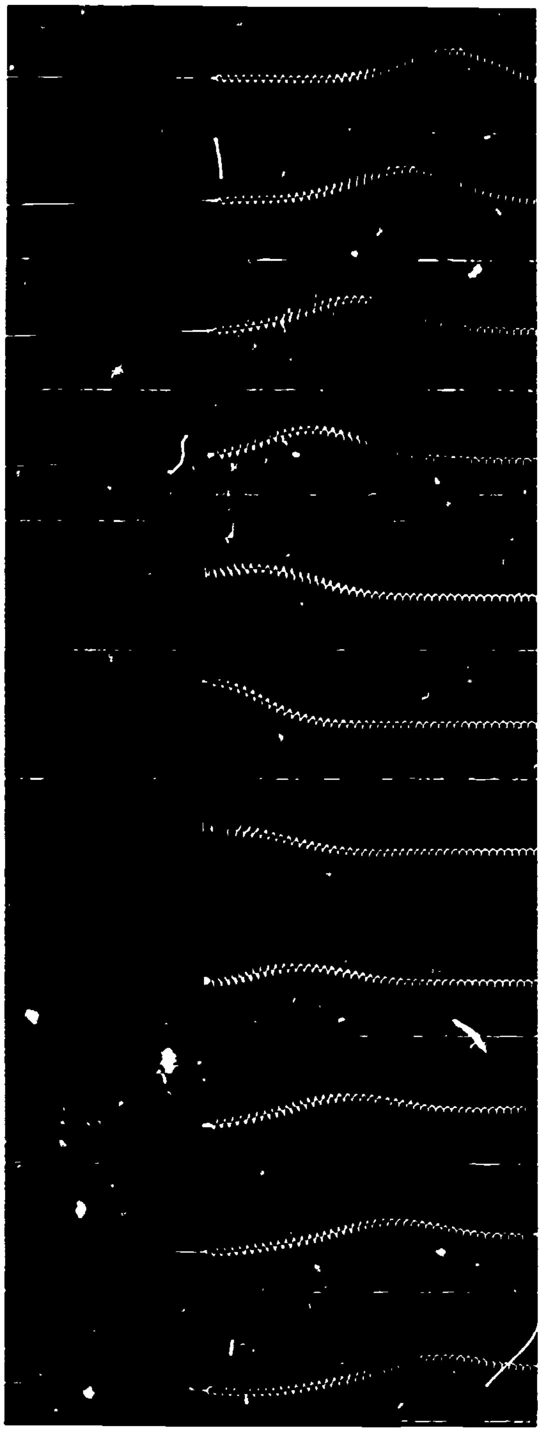
15-11 A pulse passing from a light spring (right) to a heavy spring. At the junction the pulse is partially transmitted and partially reflected. You will note that the reflected pulse is upside down.



15-12 A pulse passing from a heavy spring (left) to a light spring. At the junction the pulse is partially transmitted and partially reflected. The reflected pulse is right side up.

What happens when a pulse goes the other way, traveling along the heavier spring and arriving at the junction between it and the light spring? This is not so easy to foresee. We no longer can bracket the behavior between two situations in which we know the answer. But experiment tells us what takes place. In Fig. 15–12 we see a pulse moving from the left, from a heavy toward a light spring. Here, as in the opposite case, illustrated in Fig. 15–11, part of the pulse is transmitted and part is reflected, but this time the reflected pulse is right-side up.

In summary, then, when a pulse is sent along a spring toward a junction with a second spring, we observe that the whole pulse is reflected upside down whenever the second spring is very much heavier than the first. As the second spring is replaced by lighter and lighter springs, the reflected pulse becomes small and a larger and larger transmitted pulse is observed to go on beyond the junction. When the second spring is only as massive as the first, no reflected pulse is left and the original pulse is completely transmitted. Then if the second spring is made still lighter, reflection sets in again, this time with the reflected pulse right-side up. The lighter the second spring, the larger is the reflected pulse. When the second spring is negligible the reflected pulse is nearly the same size as the pulse sent in. This can be demonstrated with a heavy spring tied to a thin nylon thread (Fig. 15–13).



15-13 A pulse on a spring reflected from a junction with a very light thread. The whole pulse returns right side up. The blurring of the thread in the middle frames of the sequence of pictures indicates that the particles of the thread are moving at higl ,peed as the pulse passes. Can you determine the direction of this motion in each of the frames?

Two masters of physics introduce the wave concept in this section from a well-known popular book.

14 What is a Wave?

Albert Einstein and Leopold Infeld

1961

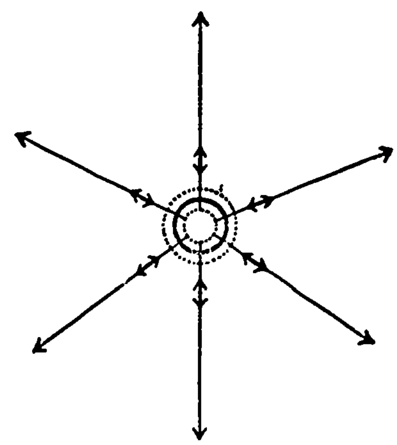
A bit of gossip starting in Washington reaches New York very quickly, even though not a single individual who takes part in spreading it travels between these two cities. There are two quite different motions involved, that of the rumor, Washington to New York, and that of the persons who spread the rumor. The wind, passing over a field of grain, sets up a wave which spreads out across the whole field. Here again we must distinguish between the motion of the wave and the motion of the separate plants, which undergo only small oscillations. We have all seen the waves that spread in wider and wider circles when a stone is thrown into a pool of water. The motion of the wave is very different from that of the particles of water. The particles merely go up and down. The observed motion of the wave is that of a state of matter and not of matter itself. A cork floating on the wave shows this clearly, for it moves up and down in imitation of the actual motion of the water, instead of being carried along by the wave.

In order to understand better the mechanism of the wave let us again consider an idealized experiment. Suppose that a large space is filled quite uniformly with water, or air, or some other "medium." Somewhere in the center there is a sphere. At the beginning of the experiment there is no motion at all. Suddenly the sphere begins to "breathe" rhythmically, expanding and contracting in volume, although retaining its spherical shape. What will happen in the medium? Let us begin our examination at the moment the sphere begins to expand. The particles of the medium in the immediate vicinity of the sphere are pushed out, so that the density of a spherical shell of water, or air, as the case may be, is increased above its normal value. Similarly, when the sphere contracts, the density of that part of the medium immediately surrounding it will be decreased. These changes of density are propagated throughout the entire medium. The particles constituting the medium perform only small vibrations, but the whole motion is that of a progressive wave. The essentially new thing here is that for the first time we consider the motion of something which is not matter, but energy propagated through matter.

Using the example of the pulsating sphere, we may introduce two general physical concepts, important for the characterization of waves. The first is the velocity with which the wave spreads. This will depend on the medium, being different for water and air, for example. The second concept is that of *wave-length*. In the case of waves on a sea or river it is the distance from the trough of one wave to that of the next, or from the crest of one wave to that of the next. Thus sea waves have greater wave-length than river waves. In the case of our waves set up by a pulsating sphere the wave-length is the distance, at some definite time, between two neighboring spherical shells showing maxima or minima of density. It is evident that this distance will not depend on the medium alone. The rate of pulsation of the sphere will certainly have a great effect, making the wave-length shorter if the pulsation becomes more rapid, longer if the pulsation becomes slower.

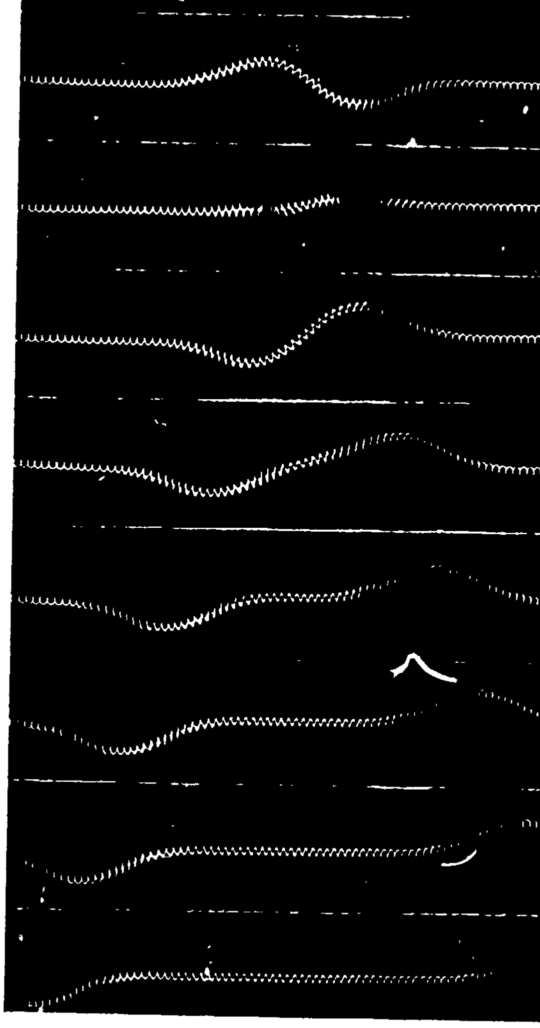
This concept of a wave proved very successful in physics. It is definitely a mechanical concept. The phenomenon is reduced to the motion of particles which, according to the kinetic theory, are constituents of matter. Thus every theory which uses the concept of wave can, in general, be regarded as a mechanical theory. For example, the explanation of acoustical phenomena is based essentially on this concept. Vibrating bodies, such as vocal cords and violin strings, are sources of sound waves which are propagated through the air in the manner explained for the pulsating sphere. It is thus possible to reduce all acoustical phenomena to mechanics by means of the wave concept.

It has been emphasized that we must distinguish between the motion of the particles and that of the wave itself, which is a state of the medium. The two are very different but it is apparent that in our example of the pulsating sphere both motions take place in the

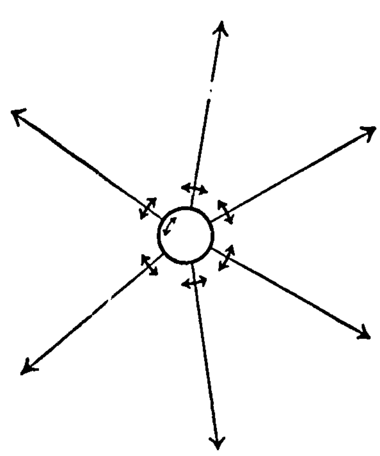


same straight line. The particles of the medium oscillate along short line segments, and the density increases and decreases periodically in accordance with this motion. The direction in which the wave spreads and the line on which the oscillations lie are the same. This type of wave is called *longitudinal.* But is this the only kind of wave? It is important for our further considerations to realize the possibility of a different kind of wave, called *transverse.*

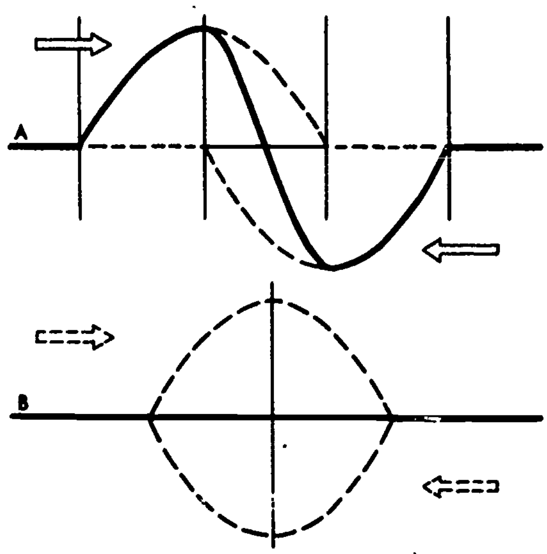
Let us change our previous example. We still have the sphere, but it is immersed in a medium of a different kind, a sort of jelly instead of air or water. Furthermore, the sphere no longer pulsates but rotates in one direction through a small angle and then back again,



15-7 The superposition of two equal and opposite pulses on a coil spring. In the fifth picture they almost cancel each other.

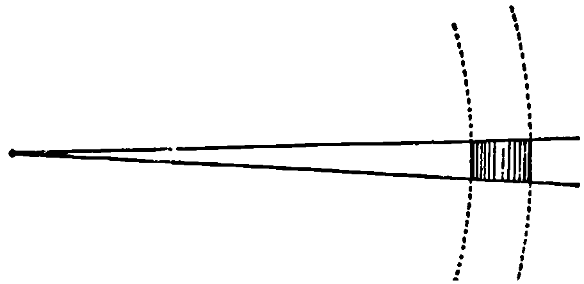


always in the same rhythmical way and about a definite axis. The jelly adheres to the sphere and thus the adhering portions are forced to imitate the motion. These portions force those situated a little further away to imitate the same motion, and so on, so that a wave is set up in the medium. If we keep in mind the distinction between the motion of the medium and the motravels along the spring from the right-hand end with one that displaces the spring upward and travels from the left. Suppose that the two pulses have exactly the same shape and size and that each is symmetrical. Notice that in one picture the addition of equal displacements upward (plus) and downward (minus) leaves us with a net displacement of zero. There is clearly a moment, as the pulses pass each other, when the whole spring appears undisplaced. (See also the drawing of Fig. 15-8.) Why does the picture not look exactly like a spring at rest? Let us consider the difference between an undisplaced spring carrying two equal and oppo-



15-8 The superposition of two equal and opposite pulses. (A) Before complete cancellation. (B) At complete cancellation.

One more remark: the wave produced by a pulsating or oscillating sphere in a homogeneous medium is a *spherical* wave. It is called so because at any given moment all points on any sphere surrounding the source behave in the same way. Let us consider a portion of such a sphere at a great distance from the source. The farther away the portion is, and the smaller we take it, the more it resembles a plane. We can say, without trying to be too rigorous, that there is no essential difference between a part of a plane and



The basic equation which summarizes the properties of waves is developed as an example of the application of mathematics to physics. Lindsay's detailed discussion will be rewarding for the student who has some knowledge of calculus coupled with persistence to work on the more advanced passages.

15 Wave Motion and Acoustics

Robert Bruce Lindsay

1940

1. Concept of a Wave. In the discussion of radiation as a type of heat transfer (Sec. 4, Chapter XVII) mention was made of the explanation of this in terms of *wave motion.* It now becomes necessary to elucidate this important concept, which is basic for acoustics, optics, and a large part of modern physics.

No one who has observed the phenomenon taking place when a stone is dropped on a water surface can have failed to be impressed by the way in which the disturbance spreads out in all directions from the point where it is first produced. In an almost uncanny fashion the motion involved in the original disturbance of the water surface is transferred to distant parts of the surface without any motion of the water itself from the original point to the distant ones. In other words, we here have to deal primarily with the motion not of material but of a change in the *configuration* of material. This type of motion is known as *wave* motion, and a wave may be briefly defined as any propagated disturbance in a continuous medium.

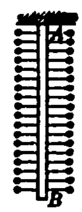


Fig. 26.1

A few well-known illustrations will serve to focus attention on the meaning of the wave concept. (1) A kink produced in a long string or rubber hose by shaking at one point appears to move along the string. (2) A long metallic ribbon (*AB* in Fig. 26.1) with perpendicular side bars attached may be twisted at the bottom and the twist will be observed to travel up the ribbon and return again after reaching the top. We call this a *torsional* wave. (3) A solid metal rod *AB* is rigidly clamped (Fig. 26.2). A vertically suspended ivory ball *C* rests lightly against the end *B*. If one taps the rod lightly at *A*, the ball after an extremely short interval flies away from *B*. We say that a compressional elastic wave has traveled along the rod; the elastic "squeeze" produced by the impact at *A* has been propagated to *B*. (4) A person *speaks* and another person at some distance *hears* him; the elastic disturbance produced in the air in front of the mouth of the speaker travels through the air to the ear of the hearer; if the air is removed the propagation fails. (5) The electromagnetic disturbance in the antenna of a

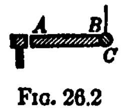


Fig. 26.2

radio station travels through space to be picked up by a radio receiver: this more complicated phenomenon we call propagation of electromagnetic waves. Light waves are a special case.

We intend to study all kinds of waves in this part of the book. First, however, let us note some general properties. The propagation of the disturbance always takes place with a definite *velocity* which depends on the medium and on the nature of the disturbance. This is called the velocity of the wave and is evidently a very important wave characteristic. Wave velocities may range from a few meters per second as in waves in a string to 344 meters/sec for sound in air at room temperature and to 3×10^8 meters/sec for the velocity of light in vacuo.

When the propagated disturbance is a displacement of the medium from its equilibrium condition in the direction of propagation the wave is said to be *longitudinal.* As examples, we note the compressional wave in a solid rod and sound waves in air. When the disturbance is a displacement perpendicular to the direction of propagation the wave is called *transverse.* As examples, we can mention the torsional waves in the ribbon previously cited and light or electromagnetic waves in general. Waves on the surface of a liquid like water are a very common illustration of a combination of transverse and longitudinal waves.

2. Mathematical Representation of Wave Motion. If we are to make an effective study of waves we must have a way of representing them mathematically. This boils down to the need for a mathematical function to represent a disturbance moving with definite velocity V through a medium. Let us simplify our picture by supposing that the disturbance is a displacement denoted by ξ and is at any instant a function of x alone. But since it moves it must also be a function of t, the time. Hence our task is to find the function $\xi = f(x, t)$ which depicts a wave traveling along the positive x axis with velocity V. Consider the function

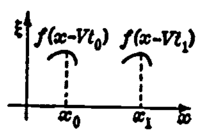


FIG. 26.3

$$\xi = f(x - Vt), \tag{1}$$

where the argument is the combination of x and t in the form $x - Vt$. To understand the physical meaning of such a function, take an arbitrary time $t = t_0$; then $\xi = f(x - Vt_0)$ is a function of x alone. In Fig. 26.3 we have indicated the plot of this function in the neighborhood of $x = x_0$. Now consider a *later* time $t = t_1$, and plot the function $f(x - Vt_1)$, which is again a function of x alone. In Fig. 26.3 we have plotted a portion of this in the vicinity of $x = x_1$. The value of ξ for $t = t_0$ at the point $x = x_0$ is clearly $f(x_0 - Vt_0)$. The value of ξ

for $t = t_1$ at the point $x = x_1$ is just as clearly $f(x_1 - Vt_1)$. These two values will be *equal* if we *choose*

$$x_0 - Vt_0 = x_1 - Vt_1,$$

or

$$x_1 - x_0 = V(t_1 - t_0). \tag{2}$$

In other words, the value of the function at time $t = t_0$ at point $x = x_0$ reappears at the point $x = x_1$ at the later time $t = t_1$ where these quantities are related by eq. 2. Any two points indeed are related in this way, which means that the whole function of x changes with time in such a fashion that it appears to be moving along the positive x axis with velocity V. But this is what we mean by wave motion. Hence we see that $\xi = f(x - Vt)$ represents a wave traveling in the positive x direction with velocity V. Note that nothing is said about the shape of the function: this is so far quite arbitrary, e.g., possible forms of $f(x - Vt)$ are $C_1(x - Vt)$, $C_2(x - Vt)^2$, $e^{C_3(x-Vt)}$, etc., where C_1, C_2, C_3 are constants put in to secure the correct dimensionality but numerically arbitrary until more conditions are laid on the wave.

It is left for the reader to show by precisely the same reasoning as above that $\xi = f(x + Vt)$ is the mathematical representation of a wave progressing in the negative x direction with velocity V.

Let us again emphasize that the function $f(x - Vt)$ is a function of both space and time, i.e., at any instant of time it varies from place to place along the x axis, while at any particular place it varies as time passes. Its ability to represent wave motion is inherent in the way in which the space and time dependence are tied together, so to speak, in the argument of the function.

3. Wave Velocity. Since the velocity of a wave is such an important characteristic we ought to devote some attention to its evaluation. Going back to (1), let us differentiate both sides partially with respect to x. Applying the ordinary calculus rule about the differentiation of a function of a function, we get

$$\frac{\partial \xi}{\partial x} = \frac{\partial f(x - Vt)}{\partial(x - Vt)} \cdot \frac{\partial(x - Vt)}{\partial x} = \frac{\partial f(x - Vt)}{\partial(x - Vt)},$$

and a second differentiation yields likewise

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{\partial^2 f(x - Vt)}{\partial(x - Vt)^2}. \tag{3}$$

Similarly if we differentiate first once and then twice partially with respect to t, the results are

$$\frac{\partial\xi}{\partial t} = \frac{\partial f(x - Vt)}{\partial(x - Vt)} \frac{\partial(x - Vt)}{\partial t} = -V \frac{\partial f(x - Vt)}{\partial(x - Vt)},$$

$$\frac{\partial^2\xi}{\partial t^2} = V^2 \frac{\partial^2 f(x - Vt)}{\partial(x - Vt)^2}. \tag{4}$$

Comparison of (3) and (4) yields

$$V^2 = \frac{\partial^2\xi}{\partial t^2} \bigg/ \frac{\partial^2\xi}{\partial x^2},$$

or

$$V = \sqrt{\frac{\partial^2\xi}{\partial t^2} \bigg/ \frac{\partial^2\xi}{\partial x^2}}, \tag{5}$$

where, of course, we take the positive sign of the radical. We may consider (5) the fundamental equation of wave motion. From our present standpoint it is significant because it gives us a means of finding out something about V. The reader might indeed query why we went to the trouble of conducting two differentiations, since we might have at once

$$V = -\frac{\partial\xi}{\partial t} \bigg/ \frac{\partial\xi}{\partial x}.$$

The trouble with this is its lack of generality. It holds only for a wave in the positive x direction. The reader can readily show that, for a wave in the negative x direction, one gets $V = \frac{\partial\xi}{\partial t} \Big/ \frac{\partial\xi}{\partial x}$. However, the form (5) holds precisely for this case as it does for the positive x direction. Hence we take (5) to be more fundamental.

To find V in any specific case we must evaluate the ratio of the derivatives in (5). As an illustration, let us do this for the longitudinal waves in a solid rod already mentioned in Sec. 1. Consider the rod a cylinder of cross-sectional area S placed with its axis along the x axis. (Fig. 26.4.) Suppose that at some point of the rod a longitudinal tensile *stress* F is applied. The result is a displacement of every point of the rod from its equilibrium position, i.e., a dilatational *strain* (cf. Sec. 1, Chapter XI). The measure of this strain is the increase in length per unit length of any element of

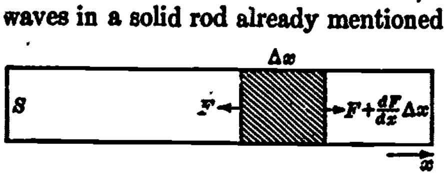


FIG. 26.4

4. Harmonic Waves. As our previous discussion should have amply indicated, there is nothing essentially periodic about wave motion. Periodicity implies the repetition at regular intervals of time of the same phenomenon at a particular place. But the passage of a solitary hump of displacement through a medium with velocity V does not fulfill this requirement though it *is* a wave. Nevertheless it turns out that the most important kinds of waves for physical problems *are* periodic waves of which the simplest type is the simple harmonic or sinusoidal variety. In such a wave the disturbance, e.g., a displacement of the medium, is a sine or cosine function of both position and time, with these quantities entering in the characteristic combination $x - Vt$. Since we do not associate meaning with the sine or cosine of a *distance*, we must write for the displacement in a sinusoidal wave

$$\xi = A \sin k(x - Vt), \tag{18}$$

where k is a constant having the dimensions of reciprocal length, so as to make $k(x - Vt)$ non-dimensional. If we plot ξ as a function of x for a particular instant $t = t_0$, we get the usual sine curve indicated in Fig. 26.6. The maximum displacement A is called the *amplitude* of the harmonic wave. The question of the physical significance of k arises.

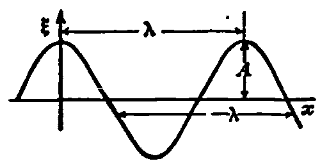


FIG. 26.6

Let us call the interval along the x axis after which the displacement repeats itself in the same way λ (cf. the figure). Then clearly

$$A \sin (kx - Vkt_0) = A \sin [k(x + \lambda) - Vkt_0].$$

But this means that

$$kx - kVt_0 + 2\pi = k(x + \lambda) - kVt_0$$

whence

$$k = \frac{2\pi}{\lambda}. \tag{19}$$

The quantity λ is known as the *wave length* of the harmonic wave. Technically speaking, it is the distance between *successive* points at which the wave differs in phase by 2π, e.g., the distance between successive maximum displacements or *crests*, or the equal distance between successive minimum displacements or *troughs*.

We can also express k in terms of the number of waves which pass any point in unit time, i.e., the frequency, ν. For at any particular point ξ oscillates in time with a period P, let us say. Take the point x_0. Then

$$\xi = A \sin k(x_0 - Vt).$$

After the expiration of time P, the displacement must return to the same value. Thus

$$A \sin k(x_0 - Vt) = A \sin k[x_0 - V(t + P)],$$

whence

$$k(x_0 - Vt) = k[x_0 - V(t + P)] + 2\pi$$

or

$$kVP = 2\pi,$$

so that

$$k = \frac{2\pi}{VP} = \frac{2\pi\nu}{V}, \tag{20}$$

since the frequency $\nu = 1/P$. The combination of (19) and (20) gives the very important wave formula

$$\lambda = VP = \frac{V}{\nu}. \tag{21}$$

This could have been seen indeed at once from the definition of wave length, frequency, and period. Consulting Fig. 26.6, we see that a crest travels distance λ in time P, progressing with constant velocity V. Hence (21) follows at once. It holds for *all* types of harmonic waves and indicates that waves of high frequency have short wave length, and vice versa.

It is customary to define the *phase* of a harmonic wave as the argument of the sine in the expression (18). Thus

$$\text{Phase of harmonic wave} = k(x - Vt). \tag{22}$$

This may in turn be written in various ways, depending on the manner in which k is expressed. The reader will have no difficulty verifying the equivalence of

$$\begin{aligned} k(x - Vt) &= 2\pi\left(\frac{x}{\lambda} - \nu t\right) = (kx - 2\pi\nu t) \\ &= 2\pi\left(\frac{x}{\lambda} - \frac{t}{P}\right) = 2\pi\left(\frac{x}{\lambda} - \frac{V}{\lambda}t\right). \end{aligned} \tag{23}$$

Convenience dictates the choice to be made for any particular purpose.

5. Wave Front. Huygens' Principle. So far we have spoken only of a wave progressing along the positive or negative x directions. But we recall that this is indeed a special case: water waves spread over a surface and sound and light waves spread in general through three-dimensional space. Evidently we need another concept to take account of this spatial distribution. This is the *wave front*, defined as the surface at all points of which the phase is the same at the same instant. The simplest type of wave front is a *plane*, and we can see its meaning best by considering this case. In Fig. 26.7 we have indicated a plane PP' perpendicular to the x axis. Let us suppose that a disturbance is being propagated along the x axis in such a way that the phase is the same at some instant at all points of the plane indicated. This means that the *displacement* ξ is the same at all points of the plane and, moreover, that the displacement velocity $\partial\xi/\partial t$ is also the same. Of course the plane will not in practice extend to infinity, but will have finite dimensions.

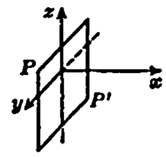


FIG. 26.7

If a wave spreads out from a single point in a medium having the same properties in every part, the wave front will be a *sphere*. This is approximately true, for example, of sound emitted from a small source. A portion of a spherical wave front far away from the source will be approximately plane.

The use of wave fronts is an important means of studying the propagation of waves. The question arises: If one knows what the wave front of a particular progressive wave is at one instant, how can one ascertain what it is at some subsequent instant? The answer to this question is provided by a fundamental principle first enunciated by Huygens. In Fig. 26.8, let us represent the wave front at the instant t by AB. This is strictly, of course, its trace on the plane of the paper. Draw from every point of AB a hemispherical wavelet of radius equal to Vdt, where V is the wave velocity in the medium and dt is a small interval of time. Now draw the surface which touches all these wavelets, i.e., the mathematical envelope of the set. This will form the wave surface $A'B'$, which according to Huygens' principle constitutes the new wave surface at the later time $t + dt$.

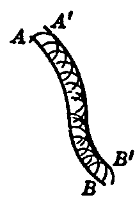


FIG. 26.8

6. Reflection and Refraction of Plane Waves. One of the uses of Huygens principle is the determination of the laws of reflection and refraction of a wave meeting the surface separating two media. The phenomenon of reflection is a common observation with all kinds of waves, e.g., water waves from a pier, sound waves from a high wall (echo), and light waves from a mirror. The law governing the geometrical characteristics of reflection has been known for a long time. We want now to examine it carefully in the light of the fundamental principle of the propagation of wave fronts.

We shall confine our attention to the reflection of a plane wave at a plane surface. In Fig. 26.9, let SS' denote the trace on the plane of the paper of the plane surface separating the media I and II. Imagine that a plane wave front AB (strictly, of course, AB is the trace on the plane of the paper of the wave front which itself is assumed normal to the plane of the paper, but for simplicity we shall use the notation just indicated) is incident on the surface at the angle $i = \angle BAC$. This is also the angle which the normal to the wave front OA makes with the normal to the surface NN'. We shall call it the angle of incidence. At the instant when we contemplate the wave front its end A is in contact with the surface. We wish to construct the wave front after the expiration of time t, where this is the time taken for the disturbance to travel from the end B to the surface. Draw BC normal to the wave front. Then if the velocity in the medium I is V_1 we have $BC = V_1t$. Strictly speaking, we ought to trace the successive positions of the wave front as the disturbance proceeds from B to C. Actually, since the wave front is plane it suffices to draw, using A as center, a semicircle with radius V_1t. Then we know that, by the time the disturbance from B has reached C, the reflected disturbance from A has reached some point on the hemisphere of which this semicircle is the trace. Hence the trace of the reflected wave front will be the line CD through C tangent to this semicircle. This can be verified by constructing the intermediate wavelets between C and D by means of Huygens' principle. The tangent just drawn will be seen to be tangent to them all. Since AD is normal to CD and $BC = AD$ is normal to AB, it follows that the angle which CD makes with SS' is equal to the angle which AB makes with SS'. We may call the former angle the angle of reflection. But then we have shown that in the reflection of a plane wave front from a plane surface the angle of incidence is equal to the angle of reflection. This is the law of reflection for plane waves. Note that it can be giv n a very simple expression in terms of the normals to the wave fronts. Thus OA, the normal to the incident wave front, will be called the *incident ray.* Similarly ADR, the normal to the reflected wave front CD, will be called the *reflected ray.* From the construction it is clear that these make the same angle with the normal to the surface. We shall often find it convenient in treating wave motion to replace wave fronts by rays. Another essential part of the law of plane wave reflection is the result, easily evident from Fig. 26.9, that the reflected ray lies in the same plane as the incident ray, namely the plane of the diagram.

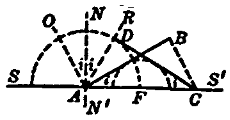


Fig. 26.9

Let us now go on to a discussion of the refr . . plar.e wave at a plane surface. Consider again (referrin) . . 26 10) the bounding surface SS' between media I and II in which the wave velocities are V_1 and V_2, respectively. The plane wave front AB is incident on SS' at angle i. The problem is to construct the wave front in medium II at the end of time t where $t = BC/V_1$. We shall assume that $V_2 < V_1$. In time t the disturbance from A will be somewhere on the surface of the hemispherical wave front whose trace is the circle of radius V_2t. Draw the tangent from C to this circle, i.e., CF. This is the wave front desired, as may be readily verified by constructing other wavelets in accordance with Huygens' principle. The *refracted ray* is AF, which makes with the normal NN' the same angle r (the so-called angle of refraction) which CF makes with the surface SS'. From Fig. 26.10 we see that

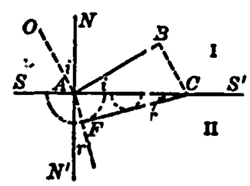


FIG. 26.10

$$\sin i = \frac{BC}{AC} = \frac{V_1t}{AC},$$

$$\sin r = \frac{AF}{AC} = \frac{V_2t}{AC},$$

whence

$$\frac{\sin i}{\sin r} = \frac{V_1}{V_2}. \tag{24}$$

This is the law of refraction for plane waves of every variety and is usually called *Snell's law*, because Snell discovered it for light. The ratio V_1/V_2 may be called the *index of refraction* of medium II with respect to medium I. Obviously its value depends on the kind of wave being considered.

It ought to be emphasized that, although we have derived the laws of reflection and refraction for a plane wave at a plane surface, they can be readily generalized to wave fronts and surfaces of arbitrary form. In general it is easier to work with the normals to the wave front or the *rays*. We shall see good examples of this when we come to light. We shall also have occasion to note where this procedure does *not* work.

We have neglected one important phenomenon associated with the reflection of waves at a boundary. This can be understood in terms of the experiment on torsional waves in a metal ribbon referred to in Sec. 1 of this chapter. If the ribbon is rigidly fastened at the ceiling, when the " twist " wave reaches this point, the ribbon is observed to stop twisting in the original direction and to begin to twist in the *opposite* direction as the wave is reflected downwards. This corresponds to what we shall term a *change in phase* on reflection. Closer analysis of the problem shows that, if we define the phase of a harmonic wave as in eq. 22, the total change in phase when a wave is reflected at a rigid boundary is equal to π, corresponding to a half-wave-length change in x. On the other hand, when a wave is reflected at a boundary which is perfectly free to move, the change of phase is *zero*. This can be tested experimentally in the case of the torsional wave in the ribbon by giving the ribbon perfect freedom of rotation where it is attached to the ceiling. The behavior of the phase of an elastic wave on reflection can be stated in even more general fashion, viz.: when an elastic wave is reflected at a boundary in going from an elastically less rigid to an elastically more rigid medium (where here the effective elasticity is measured by the product of the density and the velocity of the elastic wave) the phase changes by π; while when the reflection takes place in going from an elastically more rigid to a less rigid medium, the change of phase is zero. Even in light waves there is reason to believe that such phase changes take place on reflection (cf. Sec. 2, Chapter XXVIII).

7. Stationary Waves. When a progressive wave in a medium is reflected by a surface or barrier of some kind, reflection gives rise to a wave in the opposite direction. Thus if one end of a string or rubber hose is tied to a rigid support while the other end is shaken, in addition to the wave traveling down the string from the hand, a wave traveling back to the hand from the support is also observed. In the general case of harmonic waves proceeding in opposite directions in a medium a very interesting phenomenon can arise. In the first place we must note that the resulting disturbance is the algebraic sum of the disturbances in the two waves. If the disturbances are harmonic displacements with the same frequency and amplitude we have

$$\begin{aligned} \xi_+ &= A \cos 2\pi\left(\nu t - \frac{\nu x}{V}\right), \\ \xi_- &= A \cos 2\pi\left(\nu t + \frac{\nu x}{V}\right), \end{aligned} \tag{25}$$

the plus and minus signs referring to the waves in the positive and negative directions respectively. The resultant displacement is

$$\xi = \xi_+ + \xi_- = 2A \cos 2\pi\nu t \cos \frac{2\pi\nu x}{V}, \tag{26}$$

if the indicated trigonometric operations are carried out. If we fix our attention on some value of x, the displacement in general varies in time with frequency ν, but there are certain points at which *no* motion ever takes place. These are the points for which

$$\cos \frac{2\pi\nu x}{V} = 0, \tag{27}$$

and where x therefore has the values

$$x = \frac{(2n+1)V}{4\nu}, \tag{28}$$

n being any integer. The points corresponding to these values of x are called *nodal points* or *nodes*. In Fig. 26.11 we have plotted the displacement ξ given in eq. 26 as a function of x for four successive instants, indicated by the numbers 1, 2, 3, 4 at the left side of the figure. Though the four curves differ they all agree in passing through the points N_1, N_2, N_4, etc., and the displacement never differs from zero at these points: they are the nodes. From (28) it follows that the distance between successive nodes is $V/2\nu$, which, however, from eq. 21 is equal to $\lambda/2$ or a half wave length. Evidently the motion fluctuates between the extreme positions marked 1 and 4, the largest possible displacement being $2A$. This is attained periodically at intervals equal to $P = 1/\nu$ at points intermediate between the nodes, i.e., at L_1, L_2, etc. These points are known as *loops*. Here, of course, $\cos \frac{2\pi\nu x}{V} = 1$. The distance between successive loops is also equal to $\lambda/2$.

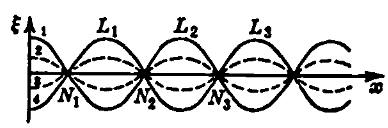


Fig. 26.11

The whole phenomenon we have been discussing is known as a *stationary* or *standing* wave. The production of such a wave affords a very satisfactory method of estimating the velocity of the wave motion in question, for, if the frequency is known and the distance between successive nodes in the standing wave pattern is measured, the velocity can at once be computed from eq. 21. For example, this can be done very nicely by *Melde's experiment* (Fig. 26.12), in which a string of length l has one end fastened to a prong of an electrically driven tuning fork while the other end passes over a frictionless pulley and terminates in a weight W. The frequency of the fork remaining constant, one gets different standing wave patterns, i.e., different

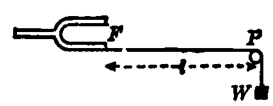


Fig. 26.12

numbers of nodes, by altering the weight W (so changing the velocity in accordance with eq. 16). The effect is very striking.

Let us consider a horizontal string of length l fastened rigidly at both ends. When struck or plucked it becomes the seat of transverse stationary waves. Since the ends must be nodes the simplest possible type of standing-wave pattern is that shown in (a) of Fig. 26.13, where there is a loop in the center of the string and the motion fluctuates between the two extreme positions ACB and ADB. It is clear that we have here $\lambda/2 = l$ or

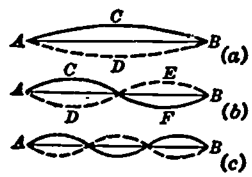


FIG. 26.13

$$\lambda = 2l \tag{29}$$

corresponding to frequency

$$\nu = \frac{V}{2l}. \tag{30}$$

This is called the *fundamental* mode of oscillation of the stretched string, and (29) and (30) give the fundamental wave length and frequency respectively. The next possible mode is shown in (b) of Fig. 26.13 with

$$\lambda = l, \qquad \nu = \frac{V}{l}. \tag{31}$$

This is the first *harmonic* of the stretched string. The second harmonic corresponds to the situation depicted in (c) with

$$\lambda = \frac{2l}{3}, \qquad \nu = \frac{3V}{2l}. \tag{32}$$

The set of frequencies $\nu_1 = V/2l$, $\nu_2 = 2V/2l$, $\nu_3 = 3V/2l \cdots$, $\nu_n = nV/2l, \cdots$, are the characteristic frequencies of the vibrating string. Note that they increase proportionately to the natural numbers: the harmonics are respectively 2, 3, 4, $\cdots n \cdots$ times the fundamental frequency.

We shall meet precisely the same type of stationary-wave phenomena in connection with sound and light waves. They are clearly independent of whether the waves are transverse or longitudinal.

8. Interference of Waves. The production of standing or stationary waves described in the previous section is but one illustration of the combination of progressive waves. There is no reason why we cannot envisage the passage of many harmonic progressive waves of different frequency in various directions in a medium. To find the resultant disturbance at any point we merely add algebraically the individual disturbances. This resultant will vary periodically with the time (unless indeed it happens to occur at a *node*). The combination of waves to form a resultant wave is often termed *superposition*, but we can also describe it by the term *interference*, the simplest type of which is called *constructive* and is illustrated by the superposition of harmonic waves of the same frequency, traveling with the same velocity in the same direction and having the *same phase*. In Fig. 26.14 we indicate two such waves 1 and 2 traveling in the same direction and having slightly different amplitudes. Their resultant is wave 3 of the same frequency

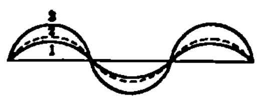


FIG. 26.14

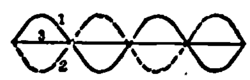


FIG. 26.15

and velocity but of larger amplitude than either of the component waves. A quite different situation is shown in Fig. 26.15, where the two waves 1 and 2 (here of the same amplitude) are precisely *out* of *phase* with each other. This means that crest of one falls on trough of the other, and vice versa, so that the resultant displacement is zero at all times everywhere. This is called *destructive interference*.

A very interesting illustration of interference is to be found in the wave pattern produced when a plane harmonic water wave encounters a rigid obstacle having two orifices through which the disturbance can go. The situation corresponds to Fig. 26.16, where we represent the obstacle by OO'. The approaching plane wave traveling upward toward the obstacle can be represented by the traces of its successive wave fronts on the plane of the diagram. Dotted lines such as TT' will represent *troughs* (i.e., at every point of TT' the displacement is a minimum at the same time) while full lines like CC' indicate *crests*. The distance between the lines CC' and TT' is, of course, $\lambda/2$. The orifices are assumed to have dimensions small compared with λ. Then the disturbance will spread out from each opening in the form of semicircular wave fronts as indicated in the picture, the full semicircles denoting crests and the dotted ones troughs. The wave fronts from the two orifices will overlap and interfere. Where crest falls on crest or trough on trough, the interference is constructive and the displacement will be large in magnitude. Where crest of one wave system falls on trough of the other, the interference is destructive and the displacement is zero. Hence there results the interesting crisscross pattern which can be

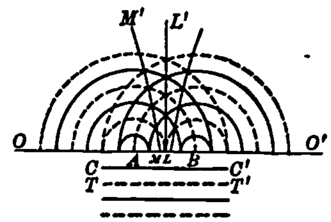


FIG. 26.16

readily observed on a water surface where circular or semicircular wave fronts overlap. In the figure the line LL' is a line connecting points of constructive interference, while MM' is a line connecting points of destructive interference. We shall find this diagram very useful when we discuss the interference of light waves.

So far our description of interference has assumed that the frequency of the interfering waves is the same. This need not be true. Consider Fig. 26.17 which shows two harmonic progressive waves 1 and 2, where 2 has frequency double that of 1. Suppose at the point A the waves are in phase. Then in the neighborhood of B and D they are out of phase. Hence the resultant wave produced by the superposition of 1 and 2 is 3, which corresponds to large displacement in the neighborhood of A, C, and E but very small displacement around B and D, i.e., large amplitude succeeds small amplitude in periodic succession. We note that the frequency of the resultant disturbance is just the difference between the frequencies of the components. It is called the *beat* frequency of the components.

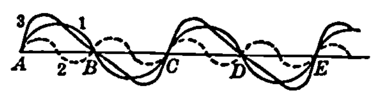


FIG. 26.17

9. Diffraction. Among the very pretty experiments which can be performed with water waves in a ripple tank is that in which a plane wave meets an obstacle OO' parallel to the wave front (Fig. 26.18). One might suppose that the obstacle would allow the plane wave to pass above the line OA but completely cut it off in the region below OA. That is, we might expect the obstacle to produce a water wave *shadow* so that no disturbance gets down into the region AOO' and all of it moves forward as a plane wave beyond the obstacle. As a matter of fact this does *not* happen. Experiment indicates that in addition to the plane waves in the region above OA there are some circular wave fronts in the region below OA, i.e., some wave propagation in the direction OB: the advancing wave acts as if it were able to *bend around* the obstacle. This ability of a wave to bend around an obstacle is known as *diffraction* and is an extremely significant property of waves of all kinds. It is not, of course, restricted to harmonic waves. Experiment indicates, and we shall later show indeed for light waves, that long-wave-length harmonic waves can be more readily diffracted than short waves.

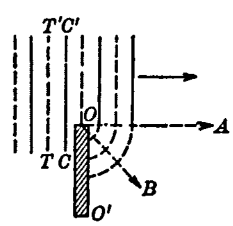


FIG. 26.18

10. Polarization. An instructive experiment which can be performed with transverse waves in a string or rubber hose consists in

$\xi + \frac{\partial \xi}{\partial x} dx$. Hence the change in length of the original piece dx is $d\xi = \frac{\partial \xi}{\partial x} dx$, or the change in length per unit length or strain is $\frac{d\xi}{dx} = \frac{\partial \xi}{\partial x}$, as given in (6). If the rod may be treated as an elastic solid, or better if the stress and strain are within the elastic limit, we may use Hooke's law (Sec. 3, Chapter XI) and write

$$\frac{F}{\dfrac{\partial \xi}{\partial x}} = Y, \tag{7}$$

where Y is Young's modulus. Now let us consider the actual *motion* of any small element of length of the rod, say Δx, as indicated in the figure. It is under tension, and the tension force at the left-hand end is simply SF directed toward the left while that at the right-hand end is $SF + S\frac{\partial F}{\partial x}\Delta x$ directed toward the right, since we have to suppose that the tension changes with x and $\partial F/\partial x$ represents its rate of change. Hence the *net* or *resultant* force on the element of length Δx appears as the *difference* of the two or

$$S\frac{\partial F}{\partial x}\Delta x. \tag{8}$$

We must now apply the fundamental equation of dynamics: force = mass times acceleration. The mass of the element is $\rho S \Delta x$, where ρ is the average density. The acceleration is $\partial^2\xi/\partial t^2$. Hence the fundamental equation of motion takes the form

$$S\Delta x\frac{\partial F}{\partial x} = \rho S \Delta x \frac{\partial^2 \xi}{\partial t^2},$$

or simplified

$$\frac{\partial F}{\partial x} = \rho\frac{\partial^2 \xi}{\partial t^2}. \tag{9}$$

making the string pass through a narrow slot cut in a piece of cardboard (cf. Fig. 26.19). If the lengthwise direction of the slot is parallel to the direction of displacement in the string, the wave motion readily passes through the slot (as, for example, through S_1S_2 in the figure). On the other hand, if the slot is perpendicular to the direction of displacement (S_3S_4 in the figure), the wave motion fails to get through the slot. In general, if the slot is placed at an angle with the direction of displacement, only the component of the displacement parallel to the slot gets through. If then a wave with arbitrary or changing displacement direction encounters such a slot, when it emerges the displacement will be parallel to the slot. We shall say that the wave becomes *polarized*, and indeed in this case *plane polarized*, since the displacement then lies continually in a plane containing the direction of propagation. Evidently a slot like the one described can act as a plane polarizer for transverse waves. At the same time if a plane-polarized transverse wave encounters such a slot, the slot can act as an *analyzer* for the wave, since it will allow the wave through with its original amplitude only for a certain orientation.

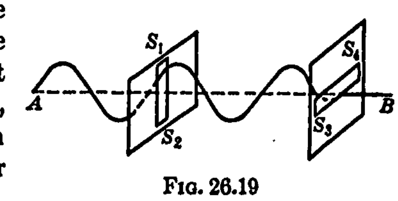


Fig. 26.19

We have emphasized throughout the discussion in the preceding paragraph that the wave is *transverse*. It should be clear to the reader that the effects described will be absent for *longitudinal* waves. This provides an interesting test of the nature of particular types of wave motion: if one can find no evidence for the polarization of a wave, it is clear that it must be longitudinal. On the other hand, phenomena

$$V = \sqrt{\frac{Y}{\rho}}. \qquad (11)$$

For an illustration we recall that for steel $\rho = 7.8$ grams/cm^3, approximately, and $Y = 2.0 \times 10^{12}$ dynes/cm^2. Eq. 11 gives $V = 5{,}000$ meters/sec, approximately.

The same sort of analysis can be carried out for elastic waves of any sort in any material medium. Interestingly enough, it is found that V is always expressible as the square root of the ratio of two factors, the numerator representing an elasticity factor, e.g., an elastic constant or combination of such, and the denominator an inertia factor, e.g., the density as above in (11). We can therefore say qualitatively that the velocity of elastic waves in very dense media which are not very elastic is smaller than that in more elastic, lighter media. We shall now give a few illustrations, quoting results without working them out in all cases.

For a *torsional* wave (cf. Sec. 1) in a bar or rod, $V = \sqrt{\mu/\rho}$, where μ = shear modulus or rigidity (cf. Sec. 3, Chapter XI).

For a compressional or "squeeze" wave in a fluid medium $V = \sqrt{p_e/\rho_e}$, where p_e is the *excess* pressure brought about in the fluid by the compressional disturbance and ρ_e is the associated excess density. Thus we have to visualize the passage of such a wave through a fluid as the motion of a "squeeze," i.e., a state of compression, through the fluid. As it progresses, the pressure at any particular point rises momentarily above its equilibrium value and then falls below it. As the pressure changes so does the density and indeed in the same direction. The relation between the changes in pressure and density can be obtained from a knowledge of the elastic properties of the fluid. Let the fluid be a liquid with bulk modulus k. Then a change of pressure Δp is associated with a change of volume Δv through the expression (cf. Sec. 3, Chapter XI).

$$\frac{\Delta p}{\Delta v/v} = -k. \qquad (12)$$

Note that we are here using v for volume to avoid confusion with V,

Many aspects of the music produced by instruments, such as tone, consonance, dissonance, and scales, are closely related to physical laws.

16 Musical Instruments and Scales

Harvey E. White

1940

MUSICAL instruments are often classified under one of the following heads: *strings*, *winds*, *rods*, *plates*, and *bells*. One who is more or less familiar with instruments will realize that most of these terms apply to the material part of each instrument set into vibration when the instrument is played. It is the purpose of the first half of this chapter to consider these vibrating sources and the various factors governing the frequencies of their musical notes, and in the second part to take up in some detail the science of the musical scale.

16.1. Stringed Instruments. Under the classification of *strings* we find such instruments as the *violin, cello, viola, double bass, harp, guitar*, and *piano*. There are two principal reasons why these instruand therefore (12) becomes

$$\frac{\Delta p}{\Delta\rho/\rho} = k\,.$$

But $\Delta p = p_e$ and $\Delta\rho = \rho_e$ in the meaning we have just given to these quantities. Hence

$$\frac{p_e}{\rho_e} = \frac{k}{\rho}.$$

and the velocity of a compressional wave in a liquid becomes

$$V = \sqrt{\frac{k}{\rho}}, \tag{13}$$

Thus for water $\rho = 1$ gram/cm^3, $k = 2.14 \times 10^{10}$ dynes/cm^2, whence $V = 1{,}460$ meters/sec.

For a compressional wave in a solid *not* in the form of a long thin rod, it can be shown that the velocity has the form

$$V = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}. \tag{14}$$

Here both shear and bulk moduli enter into the velocity of a compressional wave.

A compressional wave in a gas has already been treated in Sec. 4, Chapter XVI, and we shall merely recall for reference that $V = \sqrt{p_e/\rho_e}$ reduces in this case to

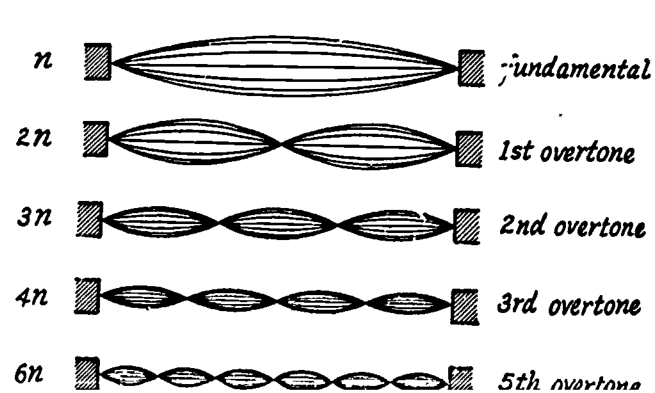
$$V = \sqrt{\frac{\gamma p}{\rho}}, \tag{15}$$

where p and ρ are the equilibrium or average pressure and density, respectively, and γ is the ratio of the specific heat at constant pressure to that at constant volume ($\gamma = 1.41$ for air). Note here how the quantity γp plays the rôle of elasticity factor for a gas.

Accurate measurements with vibrating strings, as well as theory, show that the frequency n is given by the following formula:

$$n = \frac{1}{2L}\sqrt{F/m}, \tag{16a}$$

where L is the distance in centimeters between two consecutive nodes, F is the tension on the string in dynes, and m the mass in grams of one centimeter length of string. The equation gives the exact pitch of a string or the change in pitch due to a change in *length, mass*, or *tension.* If the length L is doubled the frequency is halved, i.e., the pitch is lowered one octave. If m is increased n decreases, and if the tension F is increased n increases. The formula shows that to double the frequency by tightening a string the tension must be increased fourfold.



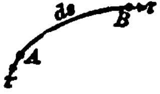


Fig. 26.5

The deduction of eq. 16 is worth going through, though it follows the same general scheme as that used in deriving eq. 11. In Fig. 26.5 a small portion of the string ds, with mass $\rho\, ds$, is shown in its displaced condition (displacement ξ) with the tension τ acting *along* it at both ends. To write the equation of motion we must equate the net vertical force acting on the element to the mass times the acceleration. Since we are assuming *small* transverse displacements from equilibrium we can replace ds by its projection dx on the equilibrium position of the string without serious error. The vertical component of the tension at the end A is then approximately $\tau \frac{\partial \xi}{\partial x}$, where the derivative is taken at this point. (We assume $\partial\xi/\partial x \doteq \partial\xi/\partial s$, approximately.) At B we have $\tau \frac{\partial \xi}{\partial x} + \tau \frac{\partial^2 \xi}{\partial x^2} dx$. The net upward force is $\tau \frac{\partial^2 \xi}{\partial x^2} dx$. The equation of motion then becomes

$$\tau \frac{\partial^2 \xi}{\partial x^2} dx = \rho \frac{\partial^2 \xi}{\partial t^2} dx,$$

or

$$V = \sqrt{\frac{\partial^2 \xi}{\partial t^2} \Big/ \frac{\partial^2 \xi}{\partial x^2}} = \sqrt{\frac{\tau}{\rho}}.$$

Waves on the surface of water or other liquids are rather complicated affairs. If they are merely ripples, they are mainly due to surface tension. On the other hand, if they are primarily due to gravity and the water depth is not too great the velocity comes out to be

$$V = \sqrt{gh}, \tag{17}$$

where g is the acceleration of gravity and h is the depth of the water. For other types of water waves the reader must consult a treatise on theoretical physics.

frequency and several overtones simultaneously. This is accomplished by plucking or bowing the string vigorously. To illustrate this, a diagram of a string vibrating with its fundamental and first overtone is shown in Fig. 16C. As the string vibrates with a node at the center and a frequency $2n$, it also moves up and down as a whole with the fundamental frequency n and a node at each end.

It should be pointed out that a string set into vibration with nodes and loops is but an example of *standing waves*, see Figs. 14K and 14L. Vibrations produced at one end of a string send a continuous train of waves along the string to be reflected back from the other end. This is true not only for transverse waves but for longitudinal or torsional waves as well. Standing waves of the latter two types can be demonstrated by stroking or twisting one end of the string of a sonometer or violin with a rosined cloth.

Fig. 16C—String vibrating with its fundamental and first overtone simultaneously.

16.3. Wind Instruments. Musical instruments often classified as "wind instruments" are usually divided into two subclasses, "wood-winds" and "brasses." Under the heading of wood-winds we find such instruments as the *flute*, *piccolo*, *clarinet*, *bass clarinet*, *saxophone*, *bassoon*, and *contra bassoon*, and under the brasses such instruments as the *French horn*, *cornet*, *trumpet*, *tenor trombone*, *bass trombone*, and *tuba* (or *bombardon*).

In practically all wind instruments the source of sound is a vibrating proper length, standing waves will be set up and the air column will resonate to the frequency of the tuning fork. In this experiment the proper length of the tube for the closed pipes is obtained by slowly pouring water into the cylinder and listening for the loudest response. Experimentally, this occurs at several points as indicated by the first three diagrams; the first resonance occurs at a distance of one and one-quarter wave-lengths, the second at three-quarters of a wave-length, and the third at one-quarter of a wave-length. The reason for these

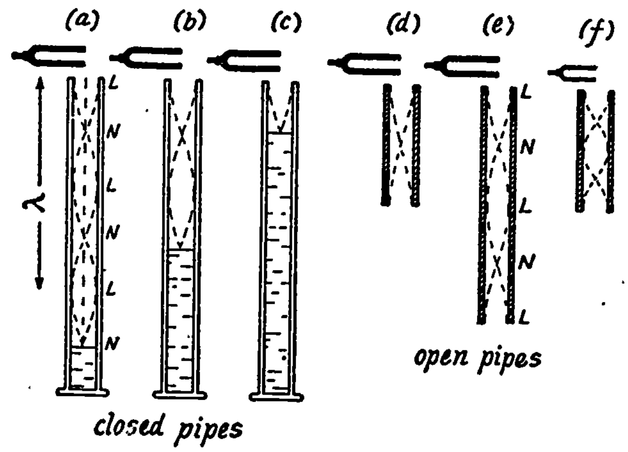


FIG. 16D—The column of air in a pipe will resonate to sound of a given pitch if the length of the pipe is properly adjusted.

odd fractions is that only a node can form at the closed end of a pipe and a loop at an open end. This is true of all wind instruments.

For open pipes a loop forms at both ends with one or more nodes in between. The first five pipes in Fig. 16D are shown responding to a tuning fork of the same frequency. The sixth pipe, diagram (f), is the same length as (d) but is responding to a fork of twice the frequency of the others. This note is one octave higher in pitch. In other words, a pipe of given length can be made to resonate to various frequencies. Closed pipe (a), for example, will respond to other forks whose waves are of the right length to form a node at the bottom, a loop at the top and any number of nodes in between.

The existence of standing waves in a resonating air column may be demonstrated by a long hollow tube filled with illuminating gas as shown in Fig. 16E. Entering through an adjustable plunger at the left the gas escapes through tiny holes spaced at regular intervals in a row along the top. Sound waves from an organ pipe enter the gas column by setting into vibration a thin rubber sheet stretched over the right-hand end. When resonance is attained by sliding the plunger to the correct position, the small gas flames will appear as shown. Where the nodes occur in the vibrating gas column the air molecules are not moving, see Fig. 14L (b); at these points the pressure is high and the flames are tallest. Half way between are the loops; regions where the molecules vibrate back and forth with large amplitudes, and the flames are low. Bernoulli's principle is chiefly responsible for the pressure

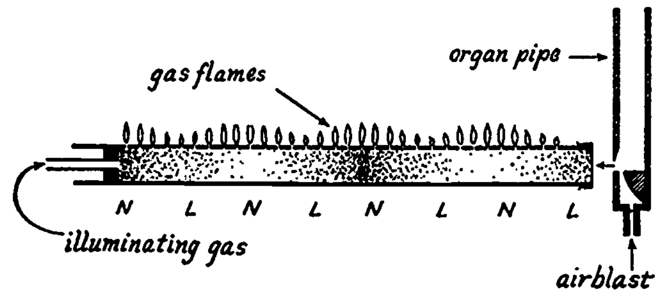


FIG. 16E—Standing waves in a long tube containing illuminating gas.

differences, see Sec. 10.8, for where the velocity of the molecules is high the pressure is low, and where the velocity is low the pressure is high.

The various notes produced by most wind instruments are brought about by varying the length of the vibrating air column. This is illustrated by the organ pipes in Fig. 16F. The longer the air column the lower the frequency or pitch of the note. In a regular concert organ the pipes vary in length from about six inches for the highest note to almost sixteen feet for the lowest. For the middle octave of the musical scale the open-ended pipes vary from two feet for *middle* C to one foot for C^1 one octave higher. In the wood-winds like the flute the length of the column is varied by openings in the side of the instrument and in many of the brasses like the trumpet, by means of valves. A valve is a piston which on being pressed down throws in an additional length of tube.

The frequency of a vibrating air column is given by the following formula,

$$n = \frac{1}{2L}\sqrt{K\frac{p}{d}},$$

where L is the length of the air column, K is a number representing the compressibility of the gas, p is the pressure of the gas, and d is its density. The function of each factor in this equation has been verified by numerous experiments. The effect of the length L is illustrated in Fig. 16F. To lower the frequency to half-value the length must be doubled. The effect of the density of a gas on the pitch of a note may be demonstrated by a very interesting experiment with the human

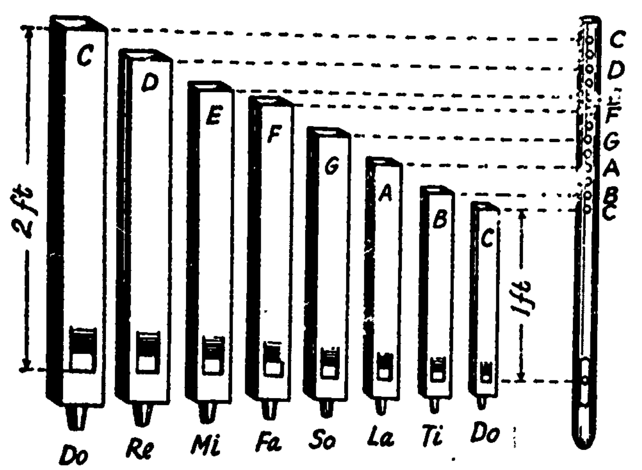


FIG. 16F—Organ pipes arranged in a musical scale. The longer the pipe the lower is its fundamental frequency and pitch. The vibrating air column of the flute is terminated at various points by openings along the tube.

voice. Voice sounds originate in the vibrations of the vocal cords in the larynx. The pitch of this source of vibration is controlled by muscular tension on the cords, while the quality is determined by the size and shape of the throat and mouth cavities. If a gas lighter than air is breathed into the lungs and vocal cavities, the above equation shows that the voice should have a higher pitch. The demonstration can be best and most safely performed by breathing helium gas, whose effect is to raise the voice about two and one-half octaves. The experiment must be performed to be fully appreciated.

16.4. Edge Tones. When wind or a blast of air encounters a small obstacle, little whirlwinds are formed in the air stream behind the obstacle. This is illustrated by the cross-section of a flue organ pipe in Fig. 16G. Whether the obstacle is long, or a small round object, the whirlwinds are formed alternately on the two sides as shown. The air stream at B waves back and forth, sending a pulse of air first

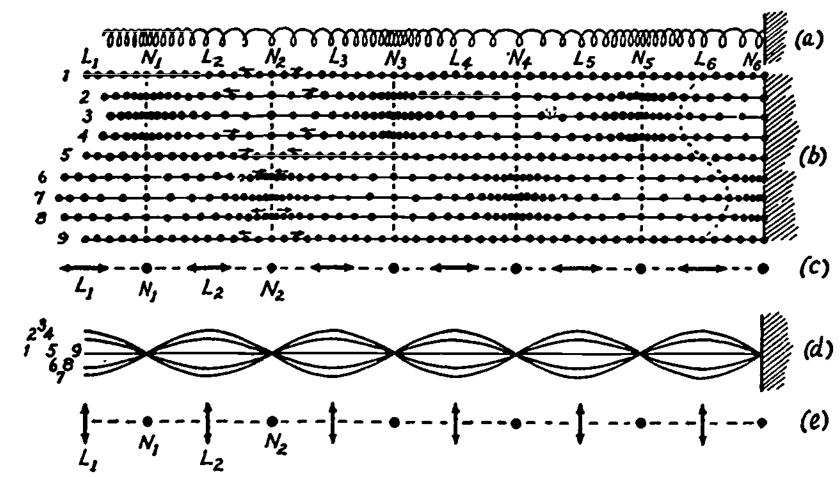


FIG. 14L—Illustrating standing waves as they are produced with (*a*) the longitudinal waves of a spring, (*b*) the longitudinal waves of sound in the air, and (*d*) the transverse waves of a rope. (*c*) and (*e*) indicate the direction of vibration at the loops.

up one side and then the other. Although the wind blows through the opening A as a continuous stream, the separate whirlwinds going up each side of the obstacle become periodic shocks to the surrounding air. Coming at perfectly regular intervals these pulses give rise to a musical note often described as the whistling of the wind. These notes are called "edge tones."

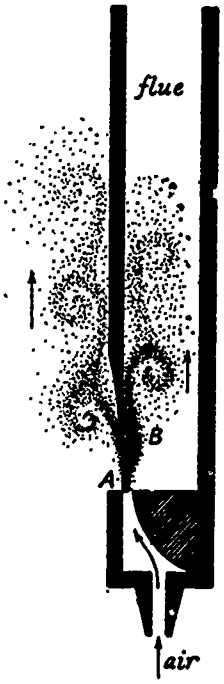


FIG. 16G—A steady stream of air blown across the lip of an organ pipe sets up whirlwinds along both sides of the partition.

The number of whirlwinds formed per second, and therefore the pitch of the edge tone, increases with the wind velocity. When the wind howls through the trees the pitch of the note rises and falls, its frequency at any time denoting the velocity of the wind. For a given wind velocity smaller objects give rise to higher pitched notes than large objects. A fine stretched wire or rubber band when placed in an open window or in the wind will be set into vibration and give out a musical note. Each whirlwind shock to the air reacts on the obstacle (the wire or rubber band), pushing it first to one side and then the other. These are the pushes that cause the reed of a musical instrument to vibrate and the rope of a flagpole to flap periodically in the breeze, while the waving of the flag at the top of a pole shows the whirlwinds that follow each other along each side.

These motions are all "forced vibrations" in that they are forced by the wind. A stretched string or the air column in an organ pipe has its own natural frequency of vibration which may or may not coincide with the frequency of the edge tone. If they do coincide, resonance will occur, the string or air column will vibrate with a large amplitude, and a loud sound will result. If the edge tone has a different frequency than the fundamental of the string, or air column, vibrations will be set up but not as intensely as before. If the frequency of the edge tone of an organ pipe, for example, becomes double that of the fundamental, and this can be obtained by a stronger blast of air, the pipe will resonate to double its fundamental frequency and give out a note one octave higher.

16.5. Vibrating Rods. If a number of small sticks are dropped upon the floor the sound that is heard is described as a *noise.* If one stick alone is dropped one would also describe the sound as a noise, unless, of course, a set of sticks of varying lengths are arranged in order of length and each one dropped in its order. If this is done, one notices that each stick gives rise to a rather definite musical note and the set of sticks to a musical scale. The use of vibrating rods in the design of a musical instrument is to be found in the *xylophone*, the *marimba*, and the *triangle*. Standing waves in a rod, like those in a stretched string, may be any one of three different kinds, transverse, longitudinal, and torsional. Only the first two of these modes of vibration will be treated here.

Transverse waves in a rod are usually set up by supporting the rod at points near each end and striking it a blow at or near the center. As

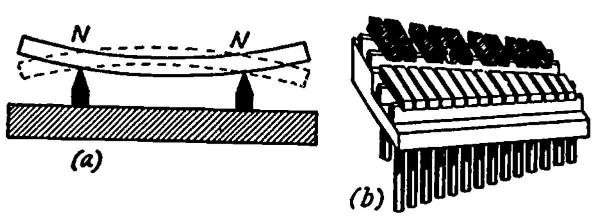


FIG. 16H—The bars of the marimba or xylophone vibrate transversely with nodes near each end.

illustrated in Fig. 16H(a) the center and ends of the rod move up and down, forming nodes at the two supports. Like a stretched string of a musical instrument, the shorter the rod the higher is its pitch, and the longer and heavier the rod the lower is its frequency of vibration and pitch.

The *xylophone* is a musical instrument based upon the transverse vibrations of wooden rods of different lengths. Mounted as shown in Fig. 16H(b) the longer rods produce the low notes and the shorter ones the higher notes. The *marimba* is essentially a xylophone with a long, straight hollow tube suspended vertically under each rod. Each tube is cut to such a length that the enclosed air column will resonate to the sound waves sent out by the rod directly above. Each resonator tube, being open at both ends, forms a node at its center.

Longitudinal vibrations in a rod may be set up by clamping a rod at one end or near the center and stroking it with a rosined cloth. Clamped in the middle as shown in Fig. 16I the free ends of the rod move back and forth while the middle is held motionless, maintaining a node at that point. Since the vibrations are too small to be seen with the eye a small ivory ball is suspended near the end as shown. The bouncing of this ball is indicative of the strong longitudinal vibrations. This type of vibration in a rod is not used in musical instruments.

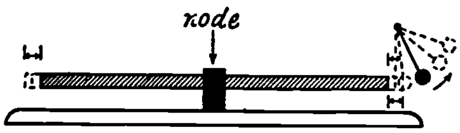


FIG. 16I—Diagram of a rod vibrating longitudinally with a node at the center.

16.6. Vibrating Plates. Although the *drum* or the *cymbals* should hardly be called musical instruments they are classified as such and made use of in nearly all large orchestras and bands. The noise given out by a vibrating drumhead or cymbal plate is in general due to the high intensity of certain characteristic overtones. These overtones in turn are due to the very complicated modes of vibration of the source.

Cymbals consist of two thin metal disks with handles at the centers. Upon being struck together their edges are set into vibration with a clang. A drumhead, on the other hand, is a stretched membrane of

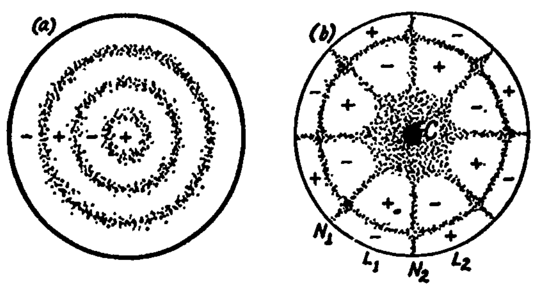


FIG. 16J—Chladni's sand figures showing the nodes and loops of (*a*) a vibrating drumhead (clamped at the edge) and (*b*) a vibrating cymbal plate (clamped at the center).

leather held tight at the periphery and is set into vibration by being struck a blow at or near the center.

To illustrate the complexity of the vibrations of a circular plate, two typical *sand patterns* are shown in Fig. 16J. The sand pattern method of studying the motions of plates was invented in the 18th century by Chladni, a German physicist. A thin circular metal plate is clamped at the center C and sand sprinkled over the top surface. Then while touching the rim of the plate at two points N_1 and N_2 a violin bow is drawn down over the edge at a point L. Nodes are formed at the stationary points N_1 and N_2 and loops in the regions of

L_1 and L_2. The grains of sand bounce away from the loops and into the nodes, the regions of no motion. At one instant the regions marked with a + sign all move up, while the regions marked with a — sign all move down. Half a vibration later the + regions are moving down and the — regions up. Such diagrams are called *Chladni's sand figures.*

With cymbal plates held tightly at the center by means of handles a node is always formed there, and loops are always formed at the periphery. With a drumhead, on the other hand, the periphery is always a node and the center is sometimes but not always a loop.

16.7. Bells. In some respects a bell is like a cymbal plate, for when it is struck a blow by the clapper, the rim in particular is set

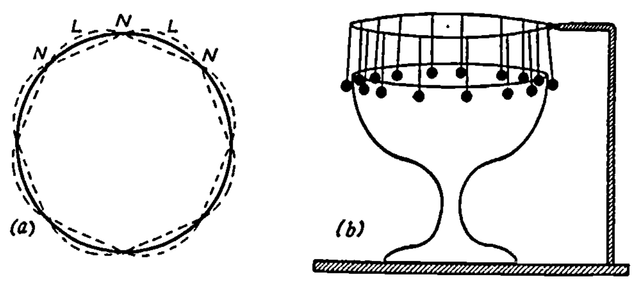


FIG. 16K—Experiment illustrating that the rim of a bell or glass vibrates with nodes and loops.

vibrating with nodes and loops distributed in a symmetrical pattern over the whole surface. The vibration of the rim is illustrated by a diagram in Fig. 16K(a) and by an experiment in diagram (b). Small cork balls are suspended by threads around and just touching the outside rim of a large glass bowl. A violin bow drawn across the edge of the bowl will set the rim into vibration with nodes at some points and loops at others. The nodes are always even in number just as they are in cymbal plates and drumheads, and alternate loops move *in* while the others move *out.*

Strictly speaking, a bell is not a very musical instrument. This is due to the very complex vibrations of the bell surface giving rise to so many loud overtones. Some of these overtones harmonize with the fundamental while others are discordant.

16.8. The Musical Scale. The musical scale is based upon the relative frequencies of different sound waves. The frequencies are so chosen that they produce the greatest amount of *harmony.* Two notes are said to be harmonious if they are pleasant to hear. If they are not pleasant to hear they are discordant.

The general form of the musical scale is illustrated by the symbols, letters, terms, and simple fractions given in Fig. 16L.

The numbers indicate that whatever the frequency of the *tonic C*, the frequency of the octave C^1 will be twice as great, that G will be three halves as great, F four thirds as great, etc. These fractions below

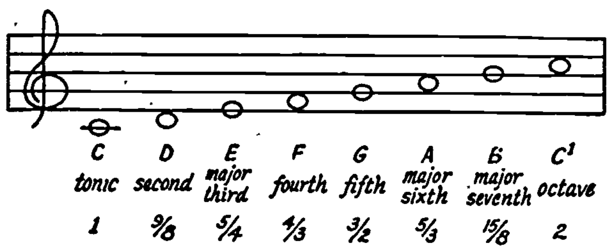


FIG. 16L—Diagram giving the names, and fractional ratios of the frequencies, of the different tone intervals on the diatonic musical scale.

each note are proportional to their frequencies in whatever octave of the musical scale the notes are located.

The musical pitch of an orchestral scale is usually determined by specifying the frequency of the A string of the *first violin*, although sometimes it is given by *middle C* on the piano. In the history of modern music the standard of pitch has varied so widely and changed so frequently that no set pitch can universally be called standard.* For many scientific purposes the A string of the violin is tuned to a frequency of 440 vib/sec, while in a few cases the slightly different scale of 256 vib/sec is used for the *tonic*, sometimes called *middle C.*

16.9. The Diatonic Scale. The middle octave of the diatonic musical scale is given in Fig. 16M assuming as a standard of pitch $A = 440$. The *vocal notes* usually sung in practicing music are given in the second row. The *ratio numbers* are the smallest whole numbers proportional to the *scale ratios* and to the actual *frequencies.*

The *tone ratios* given at the bottom of the scale indicate the ratio between the frequencies of two consecutive notes. *Major tones* have a ratio of 8 : 9, *minor tones* a ratio of 9 : 10, and diatonic semitones a

* For a brief historical discussion of normal standards of pitch the student is referred to the book "The Science of Musical Sounds" by D. C. Miller. For other treatments of the science of music see "Sound" by Capstick, "Science and Music" by James Jeans, and "Sound and Music" by J. A. Zahn.

ratio 15 : 16. (The major and minor tones on a piano are called *whole tones* and the semitones are called *half tones.*)

Other tone intervals of interest to the musician are the following:

Interval	*Frequency Ratio*	*Examples*
Octave..................	1 : 2	CC^1, DD^1, EE^1
Fifth....................	2 : 3	CG, EB, GD^1
Fourth..................	3 : 4	CF, EA, GC^1
Major third.............	4 : 5	CE, FA, GB
Minor third.............	5 : 6	EG, AC^1
Major sixth.............	3 : 5	CA, DB, GE^1
Minor sixth.............	5 : 8	EC^1, AF^1

A scientific study of musical notes and *tone intervals* shows that harmony is based upon the frequency ratios between notes. The smaller the whole numbers giving the ratio between the frequencies of

major tone   *minor tone*   *semitone*   *major tone*   *minor tone*   *major tone*   *semitone*   *majortone*

scale notes	C	D	E	F	G	A	B	C^1	D^1
vocal notes	Do	Re	Mi	Fa	So	La	Ti	Do	Re
ratio numbers	24	27	30	32	36	40	45	48	54
frequencies	264	297	330	352	396	440	495	528	594
scale ratios	1	9/8	5/4	4/3	3/2	5/3	15/8	2	9/4

tone ratios   8:9   9:10   15:16   8:9   9:10   8:9   15:16   8:9

FIG. 16M—The diatonic musical scale illustrated by the middle octave with C as the tonic and $A = 440$ as the standard pitch.

two notes the more *harmonious*, or *consonant*, is the resultant. Under this definition of harmony the octave, with a frequency ratio of 1 : 2, is the most harmonious. Next in line comes the fifth with a ratio 2 : 3, followed by the fourth with 3 : 4, etc. The larger the whole numbers the more *discordant*, or *dissonant*, is the interval.

Helmholtz was the first to give a physical explanation of the various degrees of consonance and harmony of these different intervals. It is based in part upon the beat notes produced by two notes of the interval.

As shown by Eq. (15*a*) the *beat frequency* between two notes is equal to their frequency difference. Consider, for example, the two notes *C* and *G* of the middle octave in Fig. 16M. Having frequencies of 264 and 396, the beat frequency is the difference, or 132. This is a frequency fast enough to be heard by the ear as a separate note, and in pitch is one octave below *middle C*. Thus in sounding the *fifth*, *C* and *G*, three harmonious notes are heard, 132, 264, 396. They are harmonious because they have ratios given by the smallest whole numbers 1 : 2 : 3.

Harmonious *triads* or *chords* are formed by three separate notes each of which forms a harmonious interval with the other two, while the highest and lowest notes are less than an octave apart. Since there are but six such triads they are shown below.

Harmonic Triads or Chords	*Frequency Ratio*	*Example*
Major third followed by minor third.....	4 : 5 : 6	C E G
" " " fourth..........	3 : 4 : 5	C F A
Minor third " " major third.....	5 : 6, 4 : 5	E G B
Minor third " " fourth..........	5 : 6, 3 : 4	E G C^1
Fourth " " major third.....	4 : 5, 3 : 4	C E A
Fourth " " minor third.....	3 : 4, 5 : 6	E A C^1

Consider the *beat notes* or *difference tones* between the various pairs of notes in the second triad above. The notes themselves have frequencies $C = 264$, $F = 352$, and $A = 440$. The difference tones $F\text{-}C = 88$, $A\text{-}F = 88$, and $A\text{-}C = 176$. Being exactly one and two octaves below *C*, one of the notes of the triad, they are in harmony with each other. Grouping the first two beat frequencies as a single note, all the frequencies heard by the ear have the frequencies 88, 176, 264, 352, and 440. The frequency ratios of these notes are 1 : 2 : 3 : 4 : 5, the first five positive whole numbers.

16.10. The Chromatic Scale. Contrary to the belief of many people the *sharp* of one note and the *flat* of the next higher major or minor tone are not of the same pitch. The reason for this false impression is that on the piano the black keys represent a compromise. The piano is not tuned to the *diatonic scale* but to an *equal tempered scale.* Experiments with eminent musicians, and particularly violinists, have shown that they play in what is called *pure intonation*, that is, to a *chromatic scale* and not according to equal temperament as will be described in the next section.

On the *chromatic scale* of the musician the ratio between the frequency of one note and the frequency of its sharp or flat is 25 : 24.

This ratio is just the difference between a *diatonic semitone* and a *minor tone*, i.e., $^{15}/_{16} \div {}^{9}/_{10} = {}^{25}/_{24}$. The actual frequencies of the various sharps and flats for the middle octave of the chromatic scale, based upon $A = 440$, are shown above in Fig. 16N. $C^{\#}$ for example has

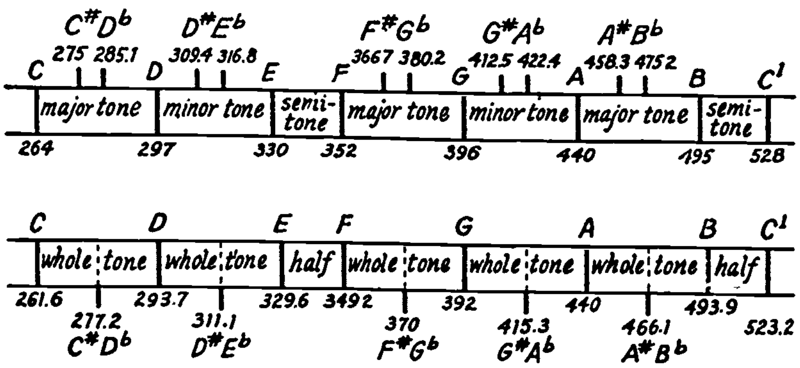


FIG. 16N—Scale diagrams showing the diatonic and chromatic scale above and the equal tempered scale below.

a frequency of 275 whereas D^{b} is 285.1. This is a difference of 10 vib/sec, an interval easily recognized at this pitch by most everyone. (The sharps and flats of the semitone intervals are not shown.)

16.11. The Equal Tempered Scale. The white keys of the piano are not tuned to the exact frequency ratios of the diatonic scale; they are tuned to an equal tempered scale. Each octave is divided into twelve equal ratio intervals as illustrated below in Fig. 16N. The *whole tone* and *half tone* intervals shown represent the white keys of the piano, as indicated in Fig. 16O, and the sharps and flats represent the black keys. Including the black keys, all twelve tone intervals in every octave are exactly the same. The frequency of any note in the equal tempered scale turns out to be 6 percent higher than the one preceding it. More accurately, the frequency of any one note multiplied by the decimal

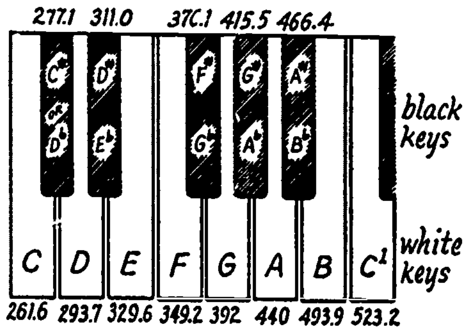


FIG. 16O—The equal tempered scale of the piano illustrating the frequencies of the middle octave based upon $A = 440$ as the standard pitch.

1.05946 gives the frequency of the note one-half tone higher. For example, $A = 440$ multiplied by 1.05946 gives $A^{\sharp}$ or $B^{\flat}$ as 466.1 vib/sec. Similarly, 466.1×1.05946 gives 493.9.

The reason for tuning the piano to an equal tempered scale is to enable the pianist to play in any key and yet stay within a given pitch range. In so doing, any given composition can be played within the range of a given person's voice. In other words, any single note can be taken as the *tonic* of the musical scale.

Although the notes of the piano are not quite as harmonious as if they were tuned to a diatonic scale, they are not far out of tune. This can be seen by a comparison of the actual frequencies of the notes of the two scales in Fig. 16N. The maximum differences amount to about 1 percent, which for many people is not noticeable, particularly in a modern dance orchestra. To the average musician, however, the difference is too great to be tolerated, and this is the reason most symphony orchestras do not include a piano. The orchestral instruments are usually tuned to the *A* string of the *first violin* and played according to the chromatic and diatonic scale.

16.12. Quality of Musical Notes. Although two musical notes have the same pitch and intensity they may differ widely in tone quality. Tone quality is determined by the number and intensity of the overtones present. This is illustrated by an examination either of the vibrating source or of the sound waves emerging from the source. There are numerous experimental methods by which this is accomplished.

A relatively convenient and simple demonstration is given in Fig. 16P, where the vibrating source of sound is a stretched piano string. Light from an arc lamp is passed over the central section of the string which, except for a small vertical slot, is masked by a screen. As the string vibrates up and down the only visible image of the string is a very short section as shown at the right, and this appears blurred. By reflecting the light in a rotating mirror the section of wire draws out a wave *W* on a distant screen.

If a string is made to vibrate with its fundamental alone, its own motion or that of the emitted sound waves have the form shown in diagram (a) of Fig. 16Q. If it vibrates in two segments or six segments (see Fig. 16B) the wave forms will be like those in diagrams (b) and (c) respectively. Should the string be set vibrating with its fundamental and first overtone simultaneously, the wave form will appear something like diagram (d). This curve is the sum of (a) and (b) and is obtained graphically by adding the displacement of corresponding points. If in addition to the fundamental a string vibrates with

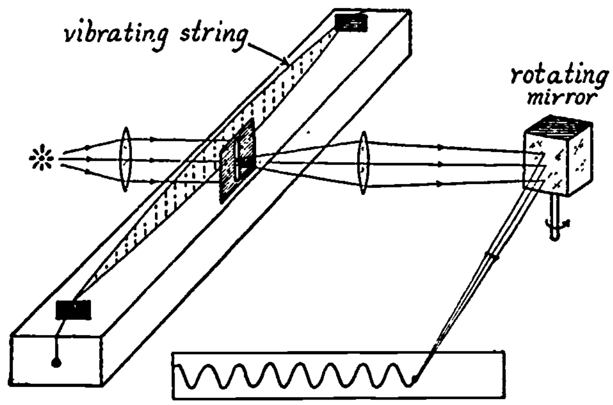


FIG. 16P—Diagram of an experiment demonstrating the vibratory motion of a stretched string.

the first and fifth overtones the wave will look like diagram (e). This is like diagram (d) with the fifth overtone added to it.

It is difficult to make a string vibrate with its fundamental alone. As a rule there are many overtones present. Some of these overtones

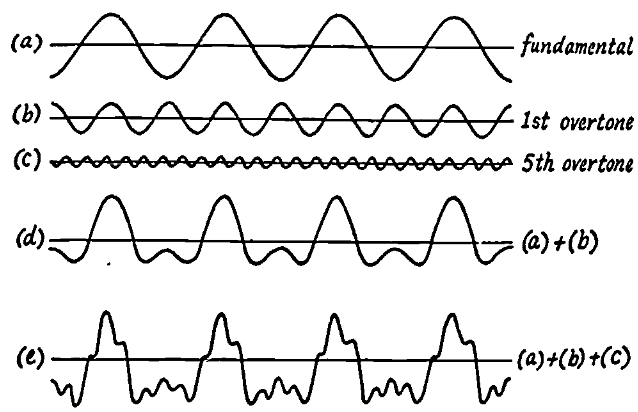


FIG. 16Q—Illustrating the form of the sound waves resulting from the addition of overtones to the fundamental.

harmonize with the fundamental and some do not. Those which harmonize are called *harmonic overtones*, and those which do not are called *anharmonic overtones.* If middle $C = 264$ is sounded with its first eight overtones, they will have 2, 3, 4, 5, 6, 7, and 8 times 264 vib/sec. These on the diatonic scale will correspond to notes C^1, G^1, C^2, E^2, G^2, X, and C^3. All of these except X, the sixth overtone, belongs to some harmonic triad. This sixth overtone is anharmonic and should be suppressed. In a piano this is accomplished by striking the string one-seventh of its length from one end, thus preventing a node at that point.

16.13. The Ranges of Musical Instruments. The various octaves above the middle of the musical scale are often labeled with numerical superscripts as already illustrated, while the octaves below the middle are labeled with numerical subscripts.

The top curve in Fig. 16Q is typical of the sound wave from a tuning fork, whereas the lower one is more like that from a violin. The strings of a violin are tuned to intervals of the *fifth*, $G_1 = 198$, $D = 297$, $A = 440$, and $E^1 = 660$. The various notes of the musical scale are obtained by touching a string at various points, thus shortening the section which vibrates. The lowest note reached is with the untouched G_1 string and the highest notes by the E^1 string fingered about two-thirds of the way up toward the *bridge*. This gives the violin a playing range, or compass, of 3½ octaves, from $G_1 = 198$ to $C^3 = 2112$.

The *viola* is slightly larger in size than the violin but has the same shape and is played with slightly lower pitch and more sombre tone quality. Reaching from C_1 to C^2, it has a range of three octaves.

The *cello* is a light bass violin which rests on the floor, is played with a bow, has four strings pitched one octave lower than the viola, C_2, G_2, D_1, and A_1, and has a heavy rich tone quality. The *double bass* is the largest of the violin family, rests on the floor and is played with a bow. The strings are tuned to two octaves below the viola and one octave below the cello. In modern dance orchestras the bow is often discarded and the strings are plucked with the fingers.

Of the wood-wind instruments the *flute* is nearest to the human voice. It consists essentially (see Fig. 16R) of a straight narrow tube about 2 feet long and is played by blowing air from between the lips across a small hole near the closed end. The openings along the tube are for the purpose of terminating the vibrating air column at various points. See Fig. 16F. With all holes closed a loop forms at both ends with a node in the middle. See Fig. 16D(d). As each hole is opened one after the other, starting from the open end, the vibrating air column with a loop at the opening grows shorter and shorter, giving out higher and higher notes. To play the scale one octave higher, one blows harder to increase the frequency of the *edge tones* and set the air column vibrating, as in Fig. 16D(e), with three loops and two nodes. Starting at *middle C* the flute can be extended in pitch for two octaves, up to C^2. The *piccolo* is a small flute, 1 foot long, and sings one octave higher. The tone is shrill and piercing and the compass iis C^1 to A^2.

The *oboe* is a melodic double-reed keyed instrument, straight and about 2 feet long. It has a reedy yet beautiful quality, and starting at

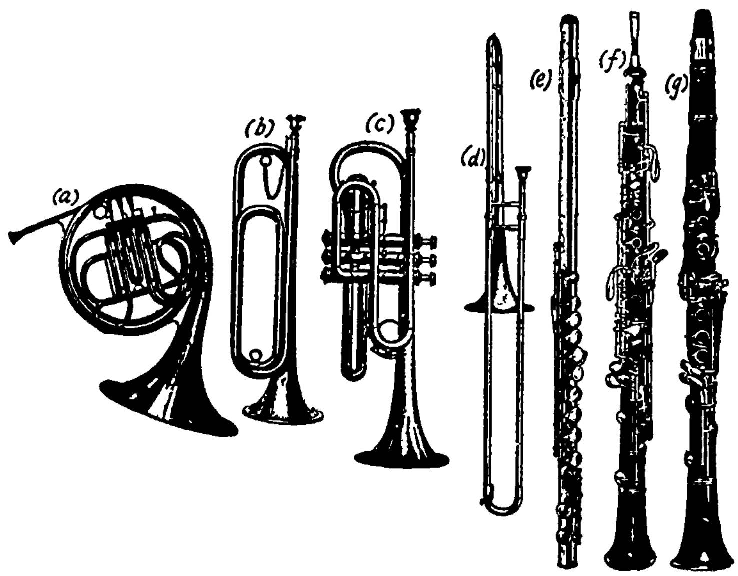


FIG. 16R—Musical instruments. Brasses: (*a*) horn, (*b*) bugle, (*c*) cornet, (*d*) trombone. Wood-winds: (*e*) flute, (*f*) oboe, and (*g*) clarinet.

B_1 has a range of about two octaves. The *clarinet*, sometimes called the violin of the military band (see Fig. 16R), is a single-reed instrument about 3 feet long. It has a range of over three octaves starting at E_1. The *bass clarinet* is larger than the clarinet, but has the same shape and plays one octave lower in pitch.

The *bassoon* is a bass double-reed keyed instrument about 4 feet long. The tone is nasal and the range is about two octaves starting at $B^b{}_3$.

The *horn* is a coiled brass tube about 12 feet in length (see Fig. 16R) but interchangeable according to the number of crooks used. It has a soft mellow tone and starting at C_2 has a range of three octaves. The *cornet*, not usually used in symphony orchestras (see Fig. 16R), is a coiled conical tube about 4½ feet long with three valves. It has a mellow tone starting at *middle C* and extends for two octaves. The trumpet is a brass instrument having a similar shape as, and slightly larger than, the cornet. Having three valves, it extends to two octaves above *middle C*. The purpose of the valves is to vary the length of the vibrating air column.

The *trombone* is a brass instrument played with a slide, is a conical tube about 9 feet long when straightened (see Fig. 16R), and has a tone range from F_2 to C^1. Since the length of the vibrating air column can be varied at will it is easily played to the chromatic scale. The *tuba* is the largest of the saxhorns and has a range from F_3 to F_1.

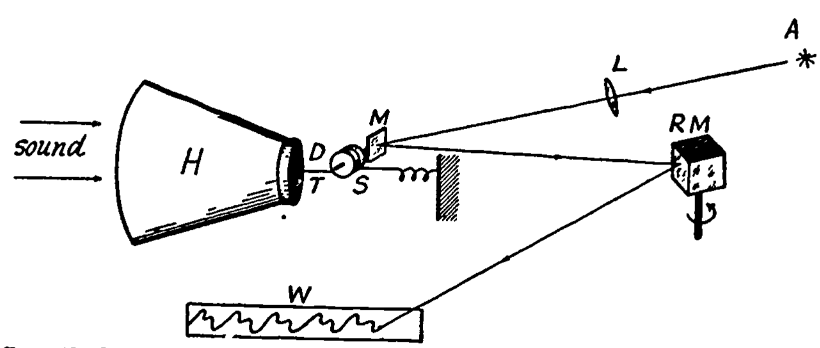


FIG. 16S—Diagram of a phonodeik. An instrument for observing the form of sound waves.

The *bugle* (see Fig. 16R) is not capable of playing to the musical scale but sounds only certain notes. These notes are the harmonic overtones of a fundamental frequency of about 66 vibrations per second. With a loop at the mouthpiece, a node in the center, and a loop at the flared end, this requires a tube 8 feet long. The second, third, fourth, and fifth overtones have the frequencies $66 \times 3 = 198$, $66 \times 4 = 264$, $66 \times 5 = 330$, and $66 \times 6 = 396$ corresponding to G_1, C, E, and G, the notes of the bugle. By making the lips vibrate to near these frequencies the air column is set resonating with 3, 4, 5, or 6 nodes between the two open ends.

16.14. The Phonodeik. The phonodeik is an instrument designed by D. C. Miller for photographing the minute details and wave forms of all audible sounds. The instrument consists of a sensitive diaphragm D (see Fig. 16S), against which the sound waves to be studied are allowed to fall. As the diaphragm vibrates back and forth under the impulses of the sound waves the thread T winds and unwinds on the spindle S, turning the tiny mirror M up and down. A beam of light from an arc lamp A and lens L is reflected from this mirror onto a rotating mirror RM. As RM spins around the light sweeps across a distant screen, tracing out the sound wave. The trace may be either photographed or observed directly on the screen. Persistence of vision enables the whole curve to be seen for a fraction of a second.

Several sound curves photographed by Miller are redrawn in Fig. 16T. In every graph except the one of the piano, the sound is

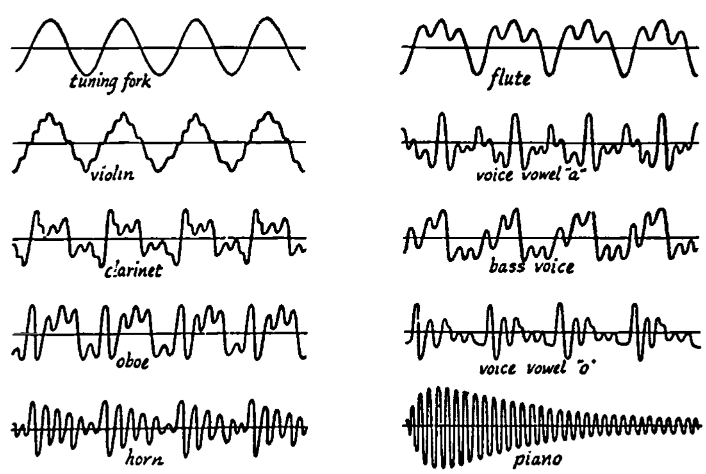


FIG. 16T—Various types of sound waves in music as observed with a phonodeik or cathode ray oscillograph.

maintained at the same frequency so that the form of each wave, no matter how complex, is repeated the same number of times. The tuning fork is the one instrument which is readily set vibrating with its fundamental alone and none of its harmonics. Although each different instrument may sound out with the same note, that is, the same fundamental, the various overtones present and their relative loudness determines the quality of the note identified with that instrument.

QUESTIONS

1. Name the five classes of musical instruments and give at least one example of each.
2. What three factors determine the fundamental frequency of a vibrating string? How does the frequency change as each of these factors is changed?

3. What four factors determine the fundamental frequency of a vibrating air column? How does the frequency change as each of these factors is changed?
4. Define or explain each of the following: (a) fundamental frequency, (b) overtones, (c) harmonics, (d) edge tones, (e) anharmonic overtones.
5. Diagram one octave of the diatonic musical scale showing (a) scale notes, (b) vocal notes, (c) ratio numbers, (d) frequencies, (e) scale ratios, and (f) tone ratios.
6. What is the essential difference between the diatonic and the even tempered musical scales?
7. Illustrate each of the following musical intervals by giving the scale notes and frequency ratios: (a) octave, (b) fifth, (c) fourth, (d) major third, (e) minor third, (f) major sixth, (g) minor sixth, (h) major seventh, (i) major tone, (j) minor tone, and (k) semitone.
8. Briefly explain the chromatic scale.

PROBLEMS

1. A piano string 100 cm long, with a mass of 0.1 gm per cm length, is under a tension of forty million dynes. Find the fundamental frequency.
2. Find the tension in the A string of a violin assuming the length to be 50 cm and the mass per cm length of string to be 0.04 gm.
3. If both ends of an organ pipe, 4 ft long, are open find its fundamental frequency.
4. Calculate the frequencies of the notes on the diatonic musical scale of Fig. 16M, for (a) the first octave above, and (b) the first octave below the one shown.
5. Give the frequencies of the first eight harmonic overtones and the first four anharmonic overtones to middle C having a frequency of 264 vib/sec.

The four members of the violin family have changed very little in hundreds of years. Recently, a group of musicians and scientists have constructed a "new" string family.

17 Founding a Family of Fiddles

Carleen M. Hutchins

1967

New measurement techniques combined with recent acoustics research enable us to make violin-type instruments in all frequency ranges with the properties built into the violin itself by the masters of three centuries ago. Thus for the first time we have a whole family of instruments made according to a consistent acoustical theory. Beyond a doubt they are musically successful.

by Carleen Maley Hutchins

FOR THREE OR FOUR centuries string quartets as well as orchestras both large and small, have used violins, violas, cellos and contrabasses of classical design. These wooden instruments were brought to near perfection by violin makers of the 17th and 18th centuries. Only recently, though, has testing equipment been good enough to find out just how they work, and only recently have scientific methods of manufacture been good enough to produce consistently instruments with the qualities one wants to design into them. Now, for the first time, we have eight instruments of the violin family constructed on principles of proper resonance for desired tone quality. They represent the first successful application of a consistent acoustical theory to a whole family of musical instruments.

The idea for such a gamut of violins is not new. It can be found in Michael Praetorius's *Syntagma Musicum* published in 1619. But incomplete understanding and technological obstacles have stood in the way of practical accomplishment. That we can now routinely make fine violins in a variety of frequency ranges is the result of a fortuitous combination: violin acoustics research—showing a resurgence after a lapse of 100 years—and the new testing equipment capable of responding to the sensitivities of wooden instruments.

As is shown in figure 1, our new instruments are tuned in alternate intervals of a musical fourth and fifth over the range of the piano keyboard. Moreover each one has its two main resonances within a semitone of the tuning of its middle strings. The result seems beyond a doubt successful musically. Over and over again we hear the comment, "One must hear the new instruments to believe such sounds are possible from strings."

Catgut Acoustical Society

Groundwork in the scientific investigation of the violin was laid by such men as Marin Mersenne (1636), Ernst Chladni (1802), Felix Savart (1819) and Hermann L. F. Helmholtz (1860). Savart, who can rightly be considered the grandfather of violin research, used many ingenious devices to explore the vibrational characteristics of the violin. But he was unable to gain sufficient knowledge of its complicated resonances to apply his ideas successfully to development and construction of new instruments. Recent research that has led to our new fiddle family is largely the work of Hermann Backhaus, Herman Meinel, Gioacchino Pasqualini, Ernst Rohloff, Werner Lottermoser and Frieder Eggers in Europe and of the late Frederick A. Saunders, John C. Schelleng, William Harvey Fletcher and myself in the United States.

Saunders, widely known for his work on Russell-Saunders coupling, pioneered violin research on this side of the Atlantic. He was a former chairman of the physics department of Harvard University, a fellow of the National Academy of Sciences and president of the Acoustical Society of America. In his work on violin acoustics, Saunders gradually became associated with colleagues who were highly competent in various scientific and musical disciplines. These associates greatly furthered the development of his work and contributed valuable technical knowledge, but they had little time for experimentation. Some were skillful musicians living under the pressure of heavy teaching and concert schedules. Nevertheless some were able to find time for the testing, designing and craftsmanship needed in the development of experimental instruments. In 1963 about 30 persons associated with Saunders in this project labeled themselves the "Catgut Acoustical Society." This informal society now has more than 100 members (see box on page 26), publishes a semiannual newsletter and holds one or two meetings each year. Among its members are acousticians, physicists, chemists, engineers, instrument makers, composers, performing musicians, musicologists, patrons and others who believe that insufficient attention has been paid to the inherent potentialities of bowed string instruments. They are making a coördinated effort to discover and develop these potentialities and are encouraged that many members of the violin fraternity share their aims.

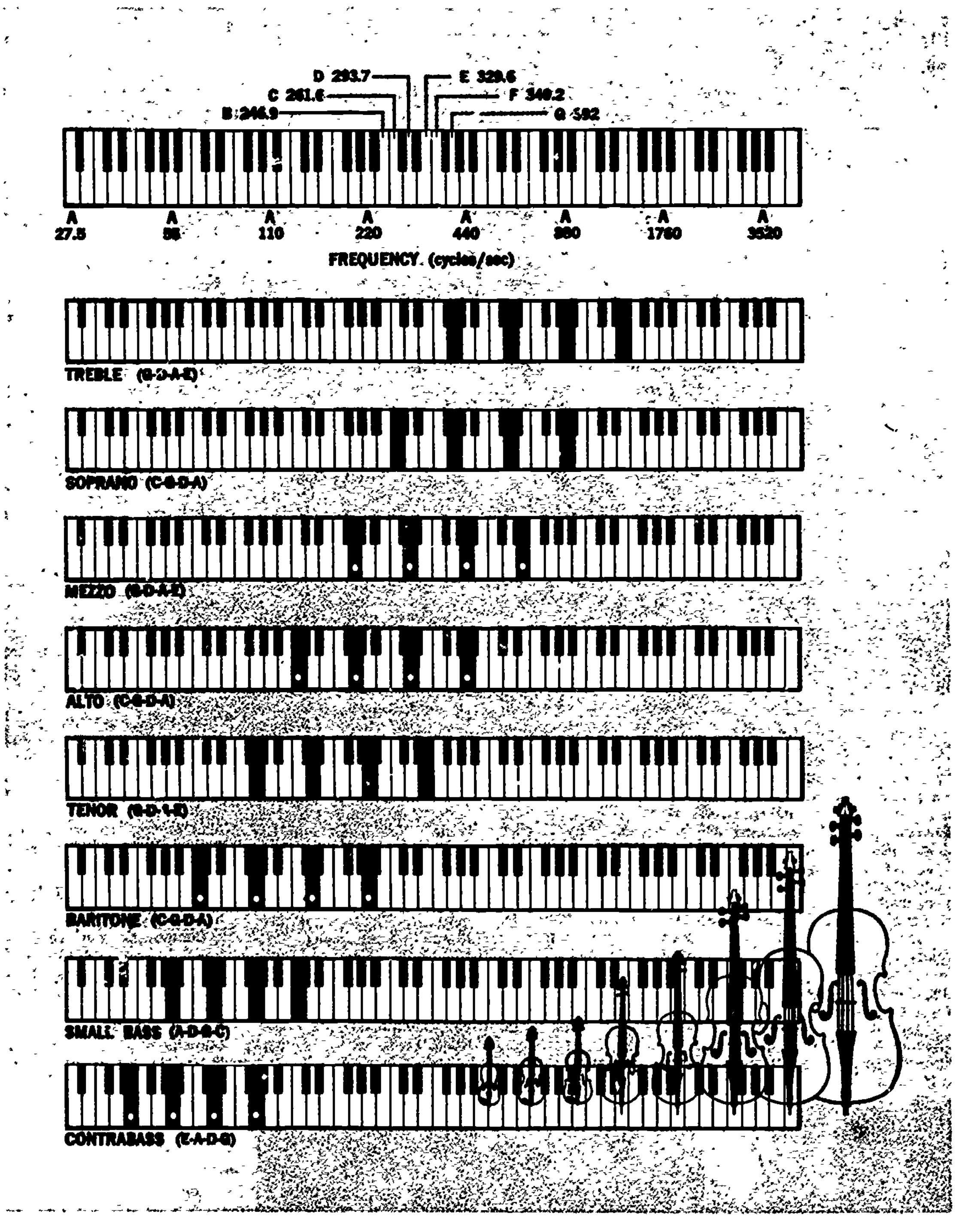
Among other accomplishments of our Catgut Acoustical Society is a concert played t Harvard last summer during the meeting of the Acoustical Society of America. It was dedicated to Saunders and the instruments were our eight new fiddles, which are the outgrowth of research he began. I write about the concert and about the instruments as a member of the society and as one who worked with Saunders from 1948 until his death in 1963. My activities include reconciliation of the wisdom of experienced musicians and violin makers, coördination of much technical information from widely separated sources, and design, construction and testing of experimental instruments. In 1937 Saunders reported[1] in the *Journal of the Acoustical Society of America* what later proved to be basic to the development of the new violin family, namely the position of the main body resonance as well as the main cavity resonance in a series of excellent violins. (The main body resonance is the lowest fundamental resonance of the wood structure; the cavity resonance is that of the air in the instrument cavity.) But the necessary knowledge of how to place these resonances with any degree of predictability in instruments of good tone quality was not evolved and reported until 1960.[2] The tonal effect of this placement of the two main resonances for each instrument and the necessary scaling theory was not reported until 1962.[3]

Between 1950 and 1958 Saunders and I undertook a long series of experiments to test various features of violin construction one at a time. We determined effect of variations in length, shape and placement of the f holes, position of the bass bar and sound post, significance of the inlay of purfling around the edges of top and back plates and frequency of the cavity resonance as a function of rib height and f hole areas (see figure 2). Because many of these experiments needed definitive testing equipment not then available, most of the results are still unpublished in Saunders's notebooks.

One sobering conclusion we reached was that with many alterations in such features as size and shape of f holes, position of the bass bar and sound post, the best tonal qualities resulted when conventional violin-making rules were followed. In other words, the early violin makers, working empirically by slow trial and error, had evolved a system that produced practically optimal relationships in violin construction.

In 1958, during a long series of experiments to test the effect of moving violin and viola resonances up and down scale, the composer in residence at Bennington College, Henry Brant, and the cellist, Sterling Hunkins, proposed development of eight violin-type instruments in a series of tunings

In addition to nurturing her fiddle family, the author shows interest in children. After graduating from Cornell she taught for 15 years in New York schools, acquiring an MA from New York University meanwhile. She also acquired a chemist husband and two children, all of whom live in Montclair, N. J.



NEW INSTRUMENT TUNING spans the piano range with eight fiddles that range in size from 210-cm contrabass to a 27-cm treble. The conventional violin is the mezzo of the new series. Colored keys show tuning of new instruments and white dots that of conventional instruments. —FIG. 1

and sizes to cover substantially the whole pitch range used in written music; these instruments would start with an oversize contrabass and go to a tiny instrument tuned an octave above the violin. Their request was so closely related to our experimental work that after half an hour's discussion Saunders and I agreed that a serious attempt would be made to develop the set. The main problem would be to produce an instrument in each of the eight frequency ranges having the dynamics, the expressive qualities and overall power that are characteristic of the violin itself, in contrast to the conventional viola, cello and string bass.

Research and new fiddles

The problem of applying basic research results to actual design and construction of new instruments now faced us. From the previous ten

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Who's Who in Catgut Acoustics

Without cross fertilization of ideas from experts in many related disciplines our new fiddle family could not have evolved in the short period of nine or ten years. No listing of names and activities can do justice to each one whose thinking and skills have been challenged and who has given time, energy and money. Their only reward is sharing in the project.

The spirit of the group has been likened to the informal coöperation that flourished among scientists in the 18th century. In addition many of the active experimenters are themselves enthusiastic string players so that a technical session is likely to end with chamber-music playing.

In the following list I try to include all those who have helped along the way, listing those who have been most active first even though they are not all members of CAS. Some of the numerous musicians are not actually familiar with the new instruments, but their comments on earlier experimental models of conventional violins, violas and cellos have provided musical insights and information necessary to the new instruments.

Physicists. Basic research and scaling for the new instruments: Frederick A. Saunders, John C. Schelleng and myself. Theory of vibrations, elasticity, shear and damping in the instruments and their parts: Arthur H. Benade, Frieder Eggers, Roger Kerlin, Max V. Mathews, Bernard W. Robinson, Robert H. Scanlan, John C. Schelleng, Eugen J. Skudrzyk, Thomas W. W. Stewart, Sam Zaslavski.

Chemists. Effects of varnish and humidity on the instruments; varnish research: Robert E. Fryxell, Morton A. Hutchins, Louis M. Condax.

Architect. Basic design and development of patterns for the new violin family, and maker of bows for them: Maxwell Kimball.

Electronic engineers. Norman Dooley, Francis L. Fielding, Sterling W. Gorrill, A. Stuart Hegeman, Alvin S. Hopping.

Translators. Mildred Allen, Edith L. R. Corliss, Donald Fletcher.

Editors. Harriet M. Bartlett, Dennis Flanagan, Robert E. Fryxell, Mary L. Harbold, Martha Taylor, Alice Torrey, Howard Van Sickle.

Photographers. Louis M. Condax, Russell B. Kingman, Douglas Ogawa, Peter N. Pruyn, J. Kellum Smith.

Artist. Irving Geis.

Lawyers. Harvey W. Mortimer, J. Kellum Smith, Robert M. Vorsanger.

General consultants. Alice T. Baker, Donald Engle, Cushman Haagensen, Mary W. Hinckley, Ellis Kellert, Henry Allen Moe, Ethel and William R. Scott.

Secretaries. Lorraine Elliott, Belle Magram.

Violin experts and makers. Karl A. Berger, René Morel, Simone F. Sacconi, Rembert Wurlitzer, myself—and Virginia Apgar, Armand Bartos, William W. Bishop, Donald L. Blatter, William Carboni, Louis M. Condax, Fred Dautrich, Jean L. Dautrich, Louis Dunham, Jay C. Freeman, Louis Grand, Jerry Juzek, Otto Kaplan, Gordon McDonald, William E. Slaby.

Violinists. Charles F. Aue, Broadus Erle, William Kroll, Sonya Monosoff, Helen Rice, Louis E. Zerbe—and Samuel Applebaum, Catherine Drinker Bowen, Marjorie Bram, Ernestine Briemeister, Alan Branigan, Nicos Cambourakis, Roy B. Chamberlin Jr., Frank Clough, Louis M. Condax, Yoko Matsuda Erle, Sterling Gorrill, Walter Grueninger, Ann Haworth, H. T. E. Hertzberg, Carol Lieberman, Max Mandel, Max V. Mathews, David Montagu, Max Pollikoff, Bernard W. Robinson, Booker Rowe, Frances Rowell, Robert Rudie, Florence DuVal Smith, Jay C. Rosenfeld.

Violists. Robert Courte, Lilla Kalman, Maxwell Kimball, David Mankovitz, Louise Rood, Frederick A. Saunders—and John A. Abbott, Alice Schradieck Aue, Virginia Apgar, Emil Bloch, Harold Coletta, Helene Dautrich, John D'Janni, Lillian Fuchs, Raphael Hillyer, Henry James, Boris Kroyt, Eugene Lehner, Rustin McIntosh, John Montgomery, Elizabeth Payne, Werner Rose, David Schwartz, Emanuel Vardi, Eunice Wheeler, Bernard Zaslav, Sam Zaslavski, myself.

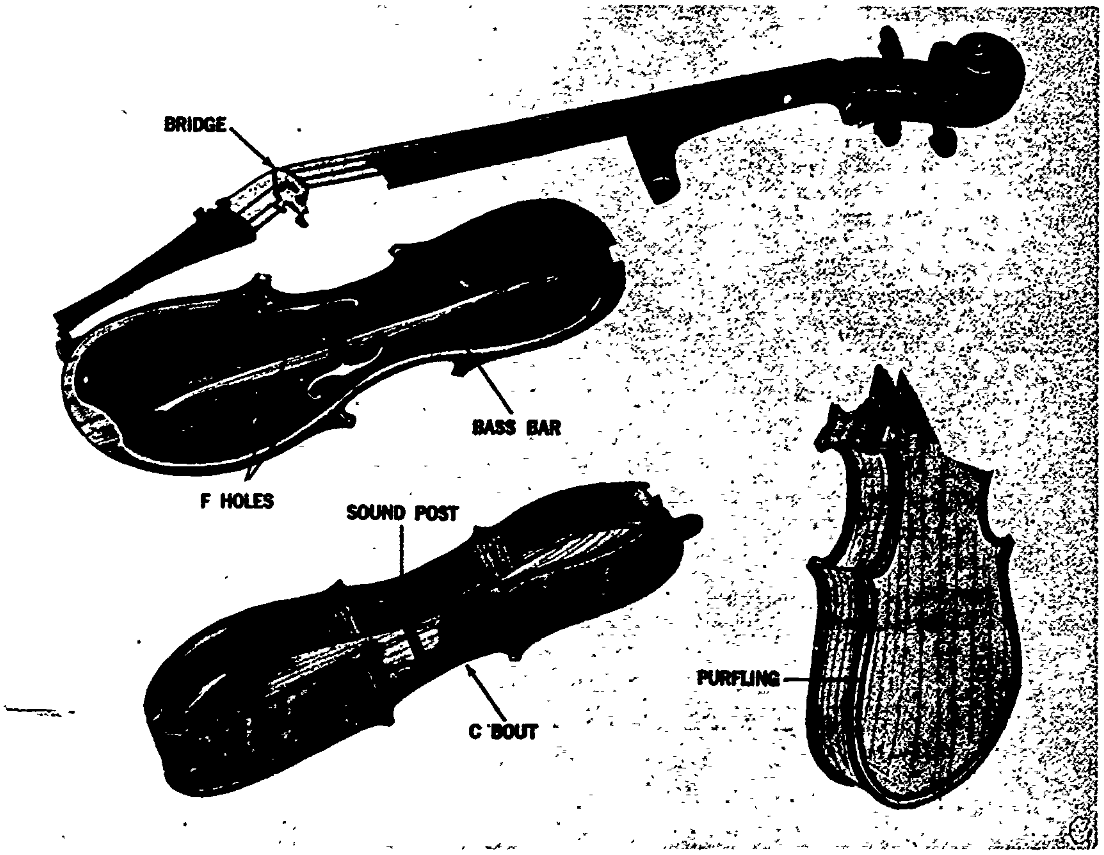
Cellists. Robert Fryxell, John C. Schelleng, India Zerbe—and Charles F. Aue, Joan Brockway, Roy B. Chamberlin, Frank Church, Elwood Culbreath, Oliver Edel, Maurice Eisenberg, George Finckel, Marie Goldman, Barbara Hendrian, Arnold Kvam, Russell B. Kingman, Charles McCracken, Stephen McGee, George Ricci, Peter Rosenfeld, Mary Lou Rylands, True Sackrison, Mischa Schneider, Sanford Schwartz, Joseph Stein, Mischa Slatkin, Joseph Tekula.

Bassists. Julius Levine, Alan Moore, Ronald Naspo, David Walter—and Alvin Brehm, John Castronovo, Garry Karr, Stuart Sankey, Charel Traeger, Howard Van Sickle, Ellery Lewis Wilson.

Composers and conductors. Henry Brant—and Marjorie Bram, Justin Connolly, Herbert Haslam, Frank Lewin, Marc Mostovoy, Harold Oliver, Quincy Porter, Cornelia P. Rogers, Leopold Stokowski, Arnold M. Walter.

PHOTO BY J. KELLUM SMITH

REHEARSAL for a concert with Henry Brant conducting an octet of fiddles.



INSTRUMENT PARTS, except for scaling, have remained the same since master makers brought the violin to near perfection about three centuries ago. —FIG. 2

years' experimentation, the following four working guides were at hand:

1. location of the main body and main cavity resonances of several hundred conventional violins, violas and cellos tested by Saunders and others,[1, 4-9]

2. the desirable relation between main resonances of free top and back plates of a given instrument, developed from 400 tests on 35 violins and violas during their construction, [2,10,11]

3. knowledge of how to change frequencies of main body and cavity resonances within certain limits (learned not only from many experiments of altering plate thicknesses, relative plate tunings and enclosed air volume but also from construction of experimental instruments with varying body lengths, plate archings and rib heights) and of resultant resonance placements and effects on tone quality in the finished instruments,[2,4,11]

4. observation that the main body resonance of a completed violin or viola is approximately seven semitones (quarter notes) above the average of the main free-plate resonances, usually one in the top and one in the back plate of a given instrument.[2] This observation came from electronic plate testing of free top and back plates of 45 violins and violas under construction. It should not be inferred that the relation implies a *shift* of free-plate resonances to those of the finished instrument. The change from two free plates to a pair of plates coupled at their edges through intricately constructed ribs and through an off-center soundpost, the whole under varying stresses and loading from fittings and string tension, is far too complicated to test directly or calculate.[12]

What is good?

In developing the new instruments our main problem was finding a measurable physical characteristic of the violin itself that would set it apart from its cousins, the viola, cello and contrabass. The search for this controlling characteristic, unique to the violin, led us through several hundred response and loudness curves of violins, violas and cellos. The picture was at first confusing because many variations were found in the placement of the two main resonances. However, Saunders's tests on Jasha Heifetz's Guarnerius violin[13] showed the main-body resonance was near the frequency of the unstopped A 440-cycles-per-second string and the main cavity resonance at the unstopped D 294 string. Thus the two main resonances of this instrument were near the frequencies of its *two unstopped middle strings*.

Ten violins, selected on the basis that their two main resonances were within a whole tone of their two open middle strings, were found to be some of the most musically desirable instruments—Amatis, Stradivaris, Guarneris and several modern ones. In marked contrast to these were all violas and cellos tested, which characteristically had their main body and cavity resonances three to four semitones *above* the frequencies of their two open middle strings although they still had the same separation, approximately a musical fifth, between these two main resonances.

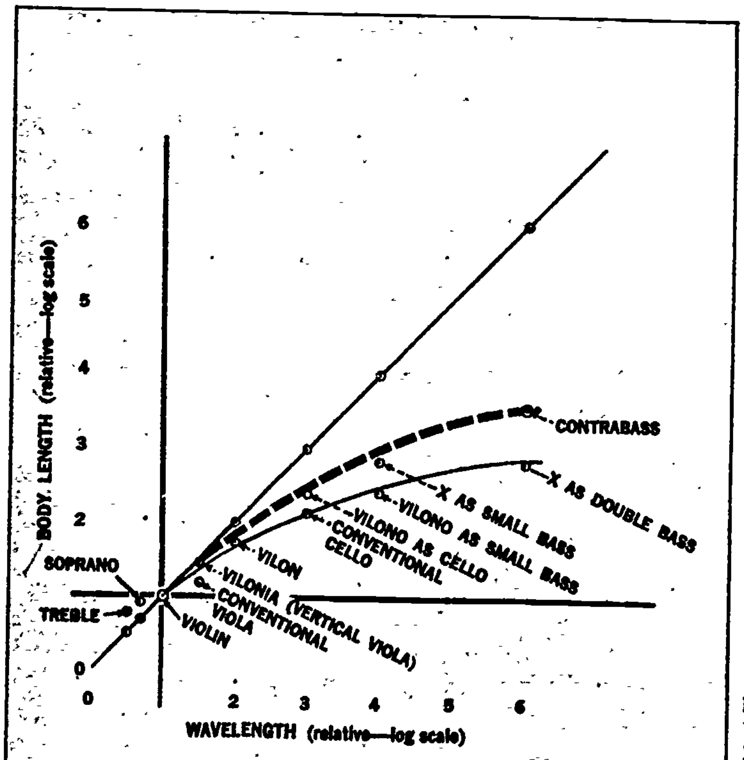
We reasoned that the clue to our problem might be this placement of the two main resonances relative to the tuning of the two open middle strings. A search through many small violins and cellos, as well as large and small violas, showed enormous variation in the placement of these two resonances. We hoped to find some instrument in which even one of these resonances would approximate what we wanted for the new instruments.

In one quarter-size cello the body resonance was right for viola tuning, D 294, but the cavity resonance was too low at D 147. We bought this chubby little cello and reduced the rib height nearly 4 in. (10 cm), thereby raising the frequency of the cavity resonance to the desired G 196. When it was put back together, it looked very thin and strange with ribs only 1.5 in. (3.8 cm) high and a body length of over 20 in. (51 cm), but strung as a viola it had tone quality satisfactory beyond expectations!

An experimental small viola that I had made for Saunders proved to have its two main resonances just a semitone below the desired frequency for violin tone range. When strung as a violin, this shallow, heavy-wooded instrument had amazing power and clarity of tone throughout its range. It sounded like a violin although the quality on the two lower strings was somewhat deeper and more viola-like that the normal violin.

The next good fortune was discovery and acquisition of a set of three instruments made by the late Fred L. Dautrich of Torrington, Conn., during the 1920's and '30's. He had described them in a booklet called *Bridging the Gaps in the Violin Family.*[17] His *vilonia*, with a body length of 20 in. (51 cm) was tuned as a viola and played cello-fashion on a peg. The *vilon*, or tenor, which looked like a half-size cello, was tuned an octave below the violin, G-D-A-E. His *vilono*, or small bass, with strings tuned two octaves below the violin, filled the gap between the cello and the contrabass. These represented three of the tone ranges we had projected for the new violin family. Tests showed that their resonances lay within working range of our theory. A year of work, adjusting top and back plate wood thicknesses for desired resonance frequencies and rib heights for proper cavity resonances in each of the three instruments gave excellent results. The vilono proved to have exactly the resonance frequencies projected for the enlarged cello, or baritone. So it was moved up a notch in the series and tuned as a cello with extra long strings.

Dautrich's pioneering work had saved years of cut and try. We now had four of the new instruments in playing condition; mezzo, alto (verti-



BODY LENGTHS for new instruments were determined by plotting lengths of known instruments against wavelength, then extending data in a smooth curve to include treble at one end and contrabass at the other. Identified points show where old and new instruments fall. —FIG. 3

cal viola), tenor and baritone. I was able to add a fifth by making a soprano, using information gained from many tests on three-quarter- and half-size violins.

With five of the new instruments developed experimentally and in playing condition, we decided to explore their musical possibilities and evaluate the overall results of our hypothesis of resonance placement. In October 1961 the working group gathered at the home of Helen Rice in Stockbridge, Mass., where Saunders and his associates had, for some years, met frequently to discuss violin acoustics and play chamber music. Short pieces of music were composed for the five instruments, and the musicians gave the new family of fiddles its first workout. The consensus was that our hypothesis was working even better than we had dared to hope! Apparently the violin-type placement of the two main resonances on the two open middle strings of each instrument was enabling us to project the desirable qualities of the violin into higher and lower tone ranges.

The next step was to explore the resonances of various size basses to help in developing the small bass and the contrabass. A small three-quarter-size bass with arched top and back proved to have just about proper resonances for the small bass. With removal of its low E 41 string and the addition of a high C 131 string to bring the tuning to A-D-G-C (basses are tuned in musical fourths for ease of fingering) it fitted quite well into the series as the small bass. But as yet no prototype for the contrabass could be located. This final addition to the series was to come later.

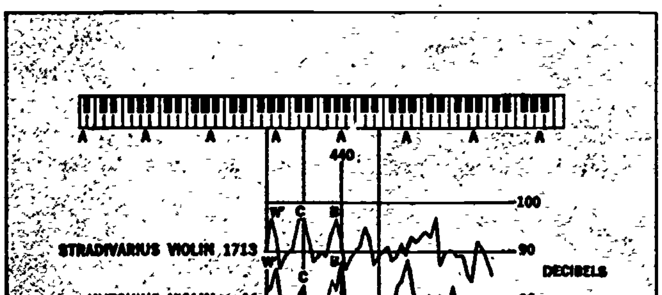
First musical success

By January 1962 we were ready for a real test in which experts could hear our six new instruments and compare them with good conventional violins, violas and cellos. Composers arranged special music, and professional players had a chance to practice on the new instruments.

Ensemble results exceeded all our expectations. We had violin-like quality and brilliance through the entire range of tones. Our soprano produced a high clear quality that carried well over the other instruments although the high positions on its two lower strings were weak. The mezzo tone was powerful and clear although somewhat viola-like on the two lower strings. The alto (vertical viola) was judged a fine instrument even with inadequate strings. The unique tone of the tenor excited all who heard it. The baritone produced such powerful and clear tones throughout its range that the cellist playing it in a Brahms sonata commented, "This is the first time I have been able to hold my own with the piano!" The small bass was adequate but needed more work. General comments told us that the new instruments were ready to stand on their own, musically, although much more work was to be done on adjustments, strings and proper bows.

End-of-scale problems

With the helpful criticisms and suggestions that came from the first musical test we were encouraged to



LOUDNESS CURVES are useful evaluations of instrument characteristics. Each is made by bowing an instrument to maximal loudness at 14 semitones on each string and plotting the resulting loudness ceiling against frequency of sound. —FIG. 4

tackle the problems of the largest and smallest instruments. No existing instruments could be adapted experimentally. We had to design and build them.

The largest bass available for testing was a huge Abraham Prescott, with a 48-in. (122-cm) body length, made in Concord, N.H., in the early 1800's but even that was not big enough! A tiny pochette, or pocket fiddle, from the Wurlitzer collection, with a body length of 7 in. (18 cm) had the right cavity resonance, but its body resonance was much too low.

The body length of each of the new instruments has been one of the controlling factors in all of our experiments. Thus it was decided that the best way to arrive at the dimensions for the largest and smallest would be to plot a curve of body lengths of known instruments, to check against their resonance placement and string tuning. This working chart is shown in figure 3 in which linear body length is plotted against the logarithm of wavelength. The curve for the new instruments was extended in a smooth arc to include the contrabass frequency at the low end and the treble frequency at the upper end, an octave above the normal violin. This procedure gave a projected body length of 51 in. (130 cm) for the contrabass and 10.5 in. (26.5 cm) for the treble. Of course rib height and enclosed air volume were separately determined by other considerations.

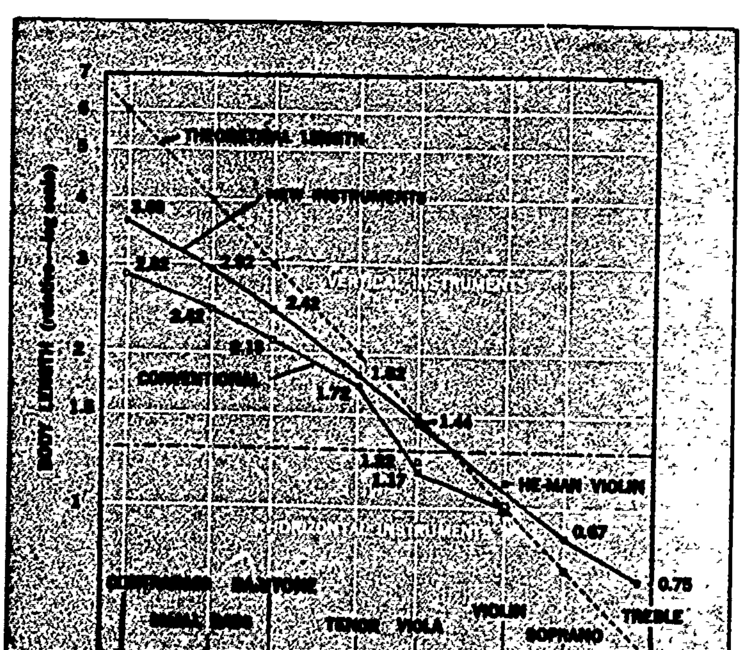
Current design practice

From all of this experience we have developed what we might call a "design philosophy." It depends mainly on resonance placement and loudness curves.

Our resonance principle, according to which each member of the new violin family has been made, can be stated as follows: The main body resonance of each of the instruments tuned in fifths is placed at the frequency of the open third string, and the main cavity resonance at the frequency of the open second string. Another way of stating the principle, and one that includes the instruments tuned in fourths as well as those tuned in fifths, is this: Wood prime is placed two semitones above the lowest tone, and the cavity resonance is a fourth above that. (Wood prime is the strengthened frequency one octave below the main body—"wood"—resonance.) These conditions are exemplified in Heifetz's Guarnerius violin and many other good ones, but they are not found in all good violins.

The loudness curve developed by Saunders is one of our most useful measures for evaluating overall instrument characteristics. We make such a curve by bowing an instrument as loudly as possible at 14 semitones on each string and plotting maximal loudness against frequency. Despite unavoidable variations in any test that requires a musician to bow an instrument, the loudness curve is significant because there is a fairly definite limit to the momentary volume an experienced player can produce with a short rapid bow stroke.

As you will see in figure 4, the loudness ceiling varies for each semitone on a given instrument. The curves of this figure were made by bowing each instrument without vibrato at a constant distance from a sound meter. From them you can see the placement of main body and cavity resonances in eight conventional instruments—two violins, two violas, two cellos and two basses. You can see that in the violins the wood prime adds power to the low range of the G string. In the violas, cellos and basses the two main resonances, which are higher in frequency relative to string tuning, create



SCALING FACTORS for old and new instruments are a useful reference guide for designers. —FIG. 5

a condition of somewhat weaker response on the lowest four or five semitones.

Fitting fiddles to players

After you decide what kind of acoustics you want, you still have another problem: You have to make fiddles that people can play. For years we worked toward design of an acoustically good instrument with genuine viola tone. Meanwhile we had to keep in mind such conflicting requirements as large radiating areas in the plates and adequate bow clearance in the C bouts (figure 2). Relation of string length to other dimensions that define tone spacing on the fingerboard —the violin maker's "mensure"—is another consideration important to the player. With our acoustic pattern as a model we undertook enlarging, scaling and redesigning all our new instruments, always keeping violin placement of resonances in each tone range.

From our set of experimentally adapted instruments, which represent a variety of styles and designs in violin making, we had learned many things. The vertical viola was about right in body dimensions, but its strings were too long for viola fingering and too short for cello fingering. The tenor was too small, and the cellists were asking for it to have strings as long as possible. The baritone was right for body size, but it had much too long strings. The bass players were asking for a long neck on the small bass and a short one on the large bass with string lengths as close as possible to conventional.

From such comments we realized that there were two basic designs for ease of playing in relation to string lengths and overall mensure of each instrument. Controlling factor in the instrument mensure is placement of the notches of the f holes because a line drawn between these two points dictates the position of the bridge and the highest part of the arch of the top plate. Mensure for the tenor and small bass would need to be as great as possible and for the vertical viola and baritone it would need to be as small as possible. Since the relative areas of the upper and lower bouts are critical factors in plate tuning, adjustment of these mensures posed quite a set of problems.

We developed a series of scaling factors[3] based on relative body length, relative resonance placement and relative string tuning that could be used as a reference guide in actual instrument construction. Figure 5 shows the set which has proved most useful in making the eight new instruments as well as those of conventional instruments.

We had a problem in measuring responses of plates of many sizes—all the way from the 10.5-in. (26-cm) one of the treble violin to the 51-in. (130-cm) one of the contrabass. We solved it by redesigning our transducer from a magnet-armature to a moving-coil type. Then the wooden fiddle plate, suspended at its corners by elastic bands, was made to vibrate as the cone of a loudspeaker (figure 6).

Using the know-how developed in making and testing several hundred violin, viola and cello plates, I could tune the plates of new instruments so that not only did each pair of top and back plates have the desired frequency relation,[2] but it also had its wood thicknesses adjusted to give a reasonable approach to what would be an optimal response.[15]

As a starting guide in adjusting plate frequencies I used the finding that a seven-semitone interval should separate the main body resonance of the finished violin from the average of the two frequencies of the free plates. It was soon obvious, however, that this relationship was not going to hold as the instruments increased in size. As the instrument gets larger the interval becomes smaller, but we do not have enough data yet to make a precise statement about it.

We used scaling theory and the three basic acoustical tools of scientific violin making: (a) frequency relationship between free top and back plates, (b) optimal response in each plate and (c) interval between body resonance and average of free-plate frequencies. We are able not only to create new instruments of the violin family but also to improve the present members. But we have to combine the acoustical tools with the highest art of violin making.

Traits of family members

Any family has its resemblances and its differences. So it is with our violins. They make a family (figure 7) with basic traits in common. But they also have their own personalities.

Treble (G-D-A-E). The main problem with our treble has been to get the frequencies of body and cavity resonances high enough and still keep the mensure long enough for a player to finger consecutive semitones without having to slide his fingers around. We projected a theoretical body length of 10.5 in. (26.7 cm) and a string length of 10 in. (25.4 cm), but to have the proper cavity resonance in this size body, the ribs would be only 3 mm high—a potentially dangerous structural condition! Besides we knew of no string material that could be tuned to E 1320 at a length of 25.4 cm without breaking. At one point we thought we might have to resort to a three-stringed instrument in this range as was indicated by Michael Praetorius in 1619.[16]

The cavity-resonance problem was solved by making six appropriately sized holes in the ribs to raise its frequency to the desired D 587. A string material of requisite tensile strength to reach the high E 1320 was finally found in carbon rocket wire, made by National Standard Company. This proved suitable not only for the high E string but for a number of others on the new instruments. As a temporary measure the ribs were made of soft aluminum to prevent the holes from unduly weakening the structure. Redesign should eliminate the nasal quality found on the lower strings and improve the upper ones. Despite this nasal quality many musicians are pleased with the degree in which the upper strings surpass the normal violin in the same high range.

Plans are to redesign this instrument in several different ways in an effort to discover the best method of achieving desired tone quality throughout its entire range.

Soprano (C-G-D-A). The soprano was designed to have as large a plate area as possible, with resulting shallow ribs and fairly large f holes to raise the cavity resonance to the desired G 392. The overall tone has been judged good and is most satisfactory on the three upper strings. The instrument needs redesign, however, for a better quality on the lower strings. The mensure is as long as possible for playing convenience. J. S. Bach wrote for an in-

PHOTOS BY PETER PRUYN

TESTING FIDDLES. New techniques enable today's makers to achieve results their predecessors could not produce. Redesigned transducer measures response of plate that is made to vibrate like a loudspeaker cone in operation. —FIG. 6

strument in this tuning, which Sir George Grove describes in Grove's dictionary:[17] "The violino piccolo is a small violin, with strings of a length suitable to be tuned a fourth above the ordinary violin. It existed in its own right for playing notes in a high compass....It survives as the 'three-quarter violin' for children. Tuned like a violin, it sounds wretched, but in its proper pitch it has a pure tone color of its own, for which the high positions on the ordinary violin gave no substitute."

Mezzo (G-D-A-E). The present mezzo with a body length of 16 in. (40.5 cm) was added to the new violin family when musicians found that even an excellent concert violin did not have the power of the other members of the group. According to scaling theory[18] this instrument, which is 1.14 times as long as the violin, has somewhat more power than necessary to match that of the others. So a second instrument has been developed that is 1.07 times as long as the violin. It has violin placement of resonances yet is adjusted to have conventional violin mensure for the player.[19] It has more power than the normal violin and seems most satisfactory. In fact several musicians have indicated that it may be the violin of the future.

Alto (vertical viola) (C-G-D-A). The greatest difficulty with the alto is that it puts the trained viola player at a distinct disadvantage by taking the viola from under his chin and setting it on a peg, cello fashion on the floor. Even with an unusual body length of 20 in., its mensure has been adjusted to that of a normal 17.5-in. (44.5-cm) viola, and some violists with large enough physique have been able to play it under the chin. Cello teachers have been impressed by its usefulness in starting young children on an instrument that they can handle readily as well as one they can continue to follow for a career. The greatest advantage is the increase in power and overall tone quality.[20] Leopold Stokowski said when he heard this instrument in concert, "That is the sound I have always wanted from the violas in my orchestra. No viola has ever sounded like that before. It fills the whole hall."

Tenor (G-D-A-E). The body length of the tenor was redeveloped from the Dautrich vilon which had a length ratio of 1.72 to the violin. The pres-

PHOTOS BY J. KELLUM SMITH

THE WHOLE FAMILY poses for pictures with performers trying them out. —FIG. 7

MAX POLLIKOFF treble — ERNESTINE BREIMEISTER soprano — LILLA KALMAN mezzo

ent tenor has a ratio of 1.82 with other factors adjusted accordingly, and the strings as long as possible for convenience in cello fingering. Many musicians have been impressed with its potential in ensemble as well as solo work. They are amazed to find that it is not a small cello, musically, but a large octave violin.

The main problem for this instrument is that there is little or no music for it as yet. Early polyphonic music, where the tenor's counterpart in the viol family had a voice, has been rearranged for either cello or viola. It has no part in classical string or orchestral literature, and only a few contemporary compositions include it. Grove[17] has this to say: "The gradual suppression of the tenor instrument in the 18th century was a disaster; neither the lower register of the viola nor the upper register of the violoncello can give its effect. It is as though all vocal part music were sung without any tenors, whose parts were distributed between the basses and contraltos! It is essential for 17th century concerted music for violins and also for some works by Handel and Bach and even later part-writing. In Purcell's *Fantasy on One Note* the true tenor holds the sustained C. . . The need for a real tenor voice in the 19th century is evidenced by the many abortive attempts to create a substitute."

Baritone (C-G-D-A). The body resonance of our baritone is nearly three semitones lower than projected, and this departure probably accounts for the somewhat bass-like quality on the low C 65.4 string. Its strings are 0.75 in. (1.8 cm) longer than those of the average cello. One concert cellist said after playing it for half an hour, "You have solved all the problems of the cello at once. But I would like a conventional cello string length." Thus a redesign of this instrument is desirable by shortening the body length a little. This redesign would raise the frequency of the body resonance and at the same time make possible a shorter string.

Small bass (A-D-G-C). Our first newly constructed instrument in the bass range is shaped like a bass viol with sloping shoulders, but has both top and back plates arched and other features comparable to violin construction. This form was adopted partly to discover the effect of the sloping shoulders of the viol and partly because a set of half-finished bass plates was available. The next small bass is being made on violin shape with other features as nearly like the first one as possible. Bass players have found the present instrument has a most desirable singing quality and extreme playing ease. They particularly like the bass-viol shape. It has proved most satisfactory in both concert and recording sessions.

Contrabass (E-A-D-G). Our contrabass[21] is 7 ft (210 cm) high overall; yet it has been possible to get the string length well within conventional bass mensure at 43 in. (110 cm) so that a player of moderate height has no trouble playing it except when he reaches into the higher positions near the bridge. For sheer size and weight it is hard to hold through a 10-hr recording session as one bassist did. When it was first strung up, the player felt that only part of its potential was being realized. The one constructional feature that had not gone according to plan was rib thickness. Ribs were 3 mm thick, whereas violin making indicated they needed to be only 2 mm thick. So the big fiddle was opened; the lining stripes cut out, and the ribs planed down on the inside to an even 2 mm all over—a job that took 10 days. But when the contrabass was put together and strung up, its ease of playing and depth of tone delighted all who played or heard it. Henry Brant commented, "I have waited all my life to hear such sounds from a bass."

How good are they really?

All who have worked on the new instruments are aware of the present lack of objective tests on them—aside from musician and audience comments. In the near future we plan to compare comments with adequate tonal analyses and response curves of these present instruments as well as new ones when they are made. The

STERLING HUNKINS alto — PETER ROSENFELD tenor — JOSEPH TEKULA baritone — DAVID WALTER small bass — STUART SANKEY contrabass

only objective evaluation so far comes from A. H. Benade at Case Institute: "I used my 100-W amplifier to run a tape recorder alternately at 60 and 90 cps while recording a good violin with the machine's gearshift set at the three nominal 1-, 3.5- and 7.5-in/sec speeds. This was done in such a way as to make a tape which, when played back at 3.5 in/sec, would give forth sounds at the pitches of the six smaller instruments in the new violin family (small bass and contrabass excluded). There were some interesting problems about the subjective speed of low- compared with high-pitch playing, but the musician was up to it and we managed to guess reasonably well. The playing was done without vibrato. It is a tribute to everyone involved in the design of those fiddles that they really do sound like their scientifically transposed cousin violin."

But as yet we know only part of *why* this theory of resonance placement is working so well. Probing deeper into this "why" is one of the challenges that lie ahead. Still unsolved are the problems of the intricate vibrational patterns within each free plate as compared to those in the assembled instrument; the reasons for the effect of moisture and various finishes on the tone of a violin and the possibility of some day being able to write adequate specifications for a fabricated material that will equal the tone qualities of wood!

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This work has received support from the John Simon Guggenheim Memorial Foundation, the Martha Baird Rockefeller Fund for Music, the Alice M. Ditson Fund of Columbia University, the Catgut Acoustical Society and private contributions.

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18 The Seven Images of Science

Gerald Holton

1960

Pure Thought and Practical Power

Each person's image of the role of science may differ in detail from that of the next, but all public images are in the main based on one or more of seven positions. The first of these goes back to Plato and portrays science as an activity with double benefits: Science as pure thought helps the mind find truth, and science as power provides tools for effective action. In book 7 of the *Republic*, Socrates tells Glaucon why the young rulers in the Ideal State should study mathematics: "This, then, is knowledge of the kind we are seeking, having a double use, military and philosophical; for the man of war must learn the art of number, or he will not know how to array his troops; and the philosopher also, because he has to rise out of the sea of change and lay hold of true being. . . . This will be the easiest way for the soul to pass from becoming to truth and being."

The main flaw in this image is that it omits a third vital aspect. Science has always had also a mythopoeic function—that is, it generates an important part of our symbolic vocabulary and provides some of the metaphysical bases and philosophical orientations of our ideology. As a consequence the methods of argument of science, its conceptions and its models, have permeated first the intellectual life of the time, then the tenets and usages of everyday life. All philosophies share with science the need to work with concepts such as space, time, quantity, matter, order, law, causality, verification, reality. Our language of ideas, for example, owes a great debt to statics, hydraulics, and the model of the solar system. These have furnished powerful analogies in many fields of study. Guiding ideas—such as conditions of equilibrium, centrifugal and centripetal forces, conservation laws, feedback, invariance, complementarity—enrich the general arsenal of imaginative tools of thought.

A sound image of science must embrace each of the three functions. However, usually only one of the three is recognized. For example, folklore often depicts the life of the scientist either as isolated from life and from beneficent action or, at the other extreme, as dedicated to technological improvements.

Iconoclasm

A second image of long standing is that of the scientist as iconoclast. Indeed, almost every major scientific advance has been interpreted—either triumphantly or with apprehension—as a blow against religion. To some extent science was pushed into this position by the ancient tendency to prove the existence of God by pointing to problems which science could not solve at the time. Newton thought that the regularities and stability of the solar system proved it "could only proceed from the counsel and dominion of an intelligent and powerful Being," and the same attitude governed thought concerning the earth's formation before the theory of geological evolution, concerning the descent of man before the theory of biological evolution, and concerning the origin of our galaxy before modern cosmology. The advance of knowledge therefore made inevitable an apparent conflict between science and religion. It is now clear how large a price had to be paid for a misunderstanding of both science and religion: to base religious beliefs on an estimate of what science cannot do is as foolhardy as it is blasphemous.

The iconoclastic image of science has, however, other components not ascribable to a misconception of its functions. For example, Arnold Toynbee charges science and technology with usurping the place of Christianity as the main source of our new symbols. Neo-orthodox theologians call science the "self-estrangement" of man because it carries him with idolatrous zeal along a dimension where no ultimate—that is, religious—concerns prevail. It is evident that these views fail to recognize the multitude of divergent influences that shape a culture, or a person. And on the other hand there is, of course, a group of scientists, though not a large one, which really does regard science as largely an iconoclastic activity. Ideologically they are, of course, descendants of Lucretius, who wrote on the first pages of *De rerum natura*, "The terror and darkness of mind must be dispelled not by the rays of the sun and glittering shafts of day, but by the aspect and the law of nature; whose first principle we shall begin by thus stating, nothing is ever gotten out of nothing by divine power." In our day this ancient trend has assumed political significance owing to the fact that in Soviet literature scientific teaching and atheistic propaganda are sometimes equated.

Ethical Perversion

The third image of science is that of a force which can invade, possess, pervert, and destroy man. The current stereotype of the soulless, evil scientist is the psychopathic investigator of science fiction or the nuclear destroyer —immoral if he develops the weapons he is asked to produce, traitorous if he refuses. According to this view, scientific morality is inherently negative. It causes the arts to languish, it blights culture, and when applied to human affairs, it leads to regimentation and to the impoverishment of life. Science is the serpent seducing us into eating the fruits of the tree of knowledge—thereby dooming us.

The fear behind this attitude is genuine but not confined to science: it is directed against all thinkers and innovators Society has always found it hard to deal with creativity, innovation, and new knowledge. And since science assures a particularly rapid, and therefore particularly disturbing, turnover of ideas, it remains a prime target of suspicion.

Factors peculiar to our time intensify this suspicion. The discoveries of "pure" science often lend themselves readily to widespread exploitation through technology. The products of technology—whether they are better vaccines or better weapons—have the characteristics of frequently being very effective, easily made in large quantities, easily distributed, and very appealing. Thus we are in an inescapable dilemma—irresistibly tempted to reach for the fruits of science, yet, deep inside, aware that our metabolism may not be able to cope with this ever-increasing appetite.

Probably the dilemma can no longer be resolved, and this increases the anxiety and confusion concerning science. A current symptom is the popular identification of science with the technology of superweapons. The bomb is taking the place of the microscope, Wernher von Braun, the place of Einstein, as symbols for modern science and scientists. The efforts to convince people that science itself can give man only knowledge about himself and his environment, and occasionally a choice of action, have been largely unavailing. The scientist *as scientist* can take little credit or responsibility either for facts he discovers—for he did not create them—or for the uses others make of his discoveries, for he generally is neither permitted nor specially fitted to make these decisions. They are controlled by considerations of ethics, economics, or politics and therefore are shaped by the values and historical circumstances of the whole society.

There are other evidences of the widespread notion that science itself cannot contribute positively to culture. Toynbee, for example, gives a list of "creative individuals," from Xenophon to Hindenburg and from Dante to Lenin, but does not include a single scientist. I cannot forego the remark that there is a significant equivalent on the level of casual conversation. For when the man in the street—or many an intellectual—hears that you are a physicist or mathematician, he will usually remark with a frank smile, "Oh, I never could understand that subject"; while intending this as a curious compliment, he betrays his intellectual dissociation from scientific fields. It is not fashionable to confess to a lack of acquaintance with the latest ephemeron in literature or the arts, but one may even exhibit a touch of pride in professing ignorance of the structure of the universe or one's own body, of the behavior of matter or one's own mind.

The Sorcerer's Apprentice

The last two views held that man is inherently good and science evil. The next image is based on the opposite assumption—that man cannot be trusted with scientific and technical knowledge. He has survived only because he lacked sufficiently destructive weapons; now he can immolate his world. Science, indirectly responsible for this new power, is here considered ethically neutral. But man, like the sorcerer's apprentice, can neither understand this tool nor control it. Unavoidably he will bring upon himself catastrophe, partly through his natural sinfulness, and partly through his lust for power, of which the pursuit of knowledge is a manifestation. It was in this mood that Pliny deplored the development of projectiles of iron for purposes of war: "This last I regard as the most criminal artifice that has been devised by the human mind; for, as if to bring death upon man with still greater rapidity, we have given wings to iron and taught it to fly. Let us, therefore, acquit Nature of a charge that belongs to man himself."

When science is viewed in this plane —as a temptation for the mischievous savage—it becomes easy to suggest a moratorium on science, a period of abstinence during which humanity somehow will develop adequate spiritual or social resources for coping with the possibilities of inhuman uses of modern technical results. Here I need point out only the two main misunderstandings implied in this recurrent call for a moratorium.

First, science of course is not an occupation, such as working in a store or on an assembly line, that one may pursue or abandon at will. For a creative scientist, it is not a matter of free choice what he shall do. Indeed it is erroneous to think of him as advancing toward knowledge; it is, rather, knowledge which advances towards him, grasps him, and overwhelms him. Even the most superficial glance at the life and work of a Kepler, a Dalton, or a Pasteur would clarify this point. It would be well if in his education each person were shown by example that the driving power of creativity is as strong and as sacred for the scientist as for the artist.

The second point can be put equally briefly. In order to survive and to progress, mankind surely cannot ever know too much. Salvation can hardly be thought of as the reward for ignorance. Man has been given his mind in order that he may find out where he is, what he is, who he is, and how he may assume the responsibility for himself which is the only obligation incurred in gaining knowledge.

Indeed, it may well turn out that the technological advances in warfare have brought us to the point where society is at last compelled to curb the aggressions that in the past were condoned and even glorified. Organized warfare and genocide have been practiced throughout recorded history, but never until now have even the war lords openly expressed fear of war. In the search for the causes and prevention of aggression among nations, we shall, I am convinced, find scientific investigations to be a main source of understanding.

Ecological Disaster

A change in the average temperature of a pond or in the salinity of an ocean may shift the ecological balance and cause the death of a large number of plants and animals. The fifth prevalent image of science similarly holds that while neither science nor man may be inherently evil, the rise of science happened, as if by accident, to initiate an ecological change that now corrodes the only conceivable basis for a stable society. In the words of Jacques Maritain, the "deadly disease" science set off in society is "the denial of eternal truth and absolute values."

The main events leading to this state are usually presented as follows. The abandonment of geocentric astronomy implied the abandonment of the conception of the earth as the center of creation and of man as its ultimate purpose. Then purposive creation gave way to blind evolution. Space, time, and certainty were shown to have no absolute meaning. All a priori axioms were discovered to be merely arbitrary conveniences. Modern psychology and anthropology led to cultural relativism. Truth itself has been dissolved into probabilistic and indeterministic statements. Drawing upon analogy with the sciences, liberal philosophers have become increasingly relativistic, denying either the necessity or the possibility of postulating immutable verities, and so have undermined the old foundations of moral and social authority on which a stable society must be built.

It should be noted in passing that many applications of recent scientific concepts outside science merely reveal ignorance about science. For example, relativism in nonscientific fields is generally based on farfetched analogies. Relativity theory, of course, does not find that truth depends on the point of view of the observer but, on the contrary, reformulates the laws of physics so that they hold good for every observer, no matter how he moves or where he stands. Its central meaning is that the most valued truths in science are wholly independent of the point of view. Ignorance of science is also the only excuse for adopting rapid changes within science as models for antitraditional attitudes outside science. In reality, no field of thought is more conservative than science. Each change necessarily encompasses previous knowledge. Science grows like a tree, ring by ring. Einstein did not prove the work of Newton wrong; he provided a larger setting within which some contradictions and asymmetries in the earlier physics disappeared.

But the image of science as an ecological disaster can be subjected to a more severe critique. Regardless of science's part in the corrosion of absolute values, have those values really given us always a safe anchor? A priori absolutes abound all over the globe in completely contradictory varieties. Most of the horrors of history have been carried out under the banner of some absolutistic philosophy, from the Aztec mass sacrifices to the auto-da-fé of the Spanish Inquisition, from the massacre of the Huguenots to the Nazi gas chambers. It is far from clear that any society of the past did provide a meaningful and dignified life for more than a small fraction of its members. If, therefore, some of the new philosophies, inspired rightly or wrongly by science, point out that absolutes have a habit of changing in time and of contradicting one another, if they invite a re-examination of the bases of social authority and reject them when those bases prove false (as did the Colonists in this country), then one must not blame a relativistic philosophy for bringing out these faults. They were there all the time.

In the search for a new and sounder basis on which to build a stable world, science will be indispensable. We can hope to match the resources and structure of society to the needs and potentialities of people only if we know more about man. Already science has much to say that is valuable and important about human relationships and problems. From psychiatry to dietetics, from immunology to meteorology, from city planning to agricultural research, by far the largest part of our total scientific and technical effort today is concerned, indirectly or directly, with man—his needs, relationships, health, and comforts. Insofar as absolutes are to help guide mankind safely on the long and dangerous journey ahead, they surely should be at least strong enough to stand scrutiny against the background of developing factual knowledge.

Scientism

While the last four images implied a revulsion from science, scientism may be described as an addiction to science. Among the signs of scientism are the habit of dividing all thought into two categories, up-to-date scientific knowledge and nonsense; the view that the mathematical sciences and the large nuclear laboratory offer the only permissible models for successfully employing the mind or organizing effort; and the identification of science with technology, to which reference was made above.

One main source for this attitude is evidently the persuasive success of recent technical work. Another resides in the fact that we are passing through a period of revolutionary change in the nature of scientific activity—a change triggered by the perfecting and disseminating of the methods of basic research by teams of specialists with widely different training and interests. Twenty years ago the typical scientist worked alone or with a few students and colleagues. Today he usually belongs to a sizable group working under a contract with a substantial annual budget. In the research institute of one university more than 1500 scientists and technicians are grouped around a set of multimillion-dollar machines; the funds come from government agencies whose ultimate aim is national defense.

Everywhere the overlapping interests of basic research, industry, and the military establishment have been merged in a way that satisfies all three. Science has thereby become a large-scale operation with a potential for immediate and world-wide effects. The results are a splendid increase in knowledge, and also side effects that are analogous to those of sudden and rapid urbanization—a strain on communication facilities, the rise of an administrative bureaucracy, the depersonalization of some human relationships.

To a large degree, all this is unavoidable. The new scientific revolution will justify itself by the flow of new knowledge and of material benefits that will no doubt follow. The danger—and this is the point where scientism enters—is that the fascination with the *mechanism* of this successful enterprise may change the scientist himself and society around him. For example, the unorthodox, often withdrawn individual, on whom most great scientific advances have depended in the past, does not fit well into the new system. And society will be increasingly faced with the seductive urging of scientism to adopt generally what is regarded—often erroneously—as the pattern of organization of the new science. The crash program, the breakthrough pursuit, the megaton effect are becoming ruling ideas in complex fields such as education, where they may not be applicable.

Magic

Few nonscientists would suspect a hoax if it were suddenly announced that a stable chemical element lighter than hydrogen had been synthesized, or that a manned observation platform had been established at the surface of the sun. To most people it appears that science knows no inherent limitations. Thus, the seventh image depicts science as magic, and the scientist as wizard, *deus ex machina*, or oracle. The attitude toward the scientist on this plane ranges from terror to sentimental subservience, depending on what motives one ascribes to him.

Science's greatest men met with opposition, isolation, and even condemnation for their novel or "heretic" ideas. But we should distinguish between the heretical innovator and the naive crank.

19 Scientific Cranks

Martin Gardner

1957

Cranks vary widely in both knowledge and intelligence. Some are stupid, ignorant, almost illiterate men who confine their activities to sending "crank letters" to prominent scientists. Some produce crudely written pamphlets, usually published by the author himself, with long titles, and pictures of the author on the cover. Still others are brilliant and well-educated, often with an excellent understanding of the branch of science in which they are speculating. Their books can be highly deceptive imitations of the genuine article—well-written and impressively learned. In spite of these wide variations, however, most pseudoscientists have a number of characteristics in common.

First and most important of these traits is that cranks work in almost total isolation from their colleagues. Not isolation in the geographical sense, but in the sense of having no fruitful contacts with fellow researchers. In the Renaissance, this isolation was not necessarily a sign of the crank. Science was poorly organized. There were no journals or societies. Communication among workers in a field was often very difficult. Moreover, there frequently were enormous social pressures operating against such communication. In the classic case of Galileo, the Inquisition forced him into isolation because the Church felt his views were undermining religious faith. Even as late as Darwin's time, the pressure of religious conservatism was so great that Darwin and a handful of admirers stood almost alone against the opinions of more respectable biologists.

Today, these social conditions no longer obtain. The battle of science to free itself from religious control has been almost completely won. Church groups still oppose certain doctrines in biology and psychology, but even this opposition no longer dominates scientific bodies or journals. Efficient networks of communication within each science have been established. A vast cooperative process of testing new theories is constantly going on—a process amazingly free (except, of course, in totalitarian nations) from control by a higher "orthodoxy." In this modern framework, in which scientific progress has become dependent on the constant give and take of data, it is impossible for a working scientist to be isolated.

The modern crank insists that his isolation is not desired on his part. It is due, he claims, to the prejudice of established scientific groups against new ideas. Nothing could be further from the truth. Scientific journals today are filled with bizarre theories. Often the quickest road to fame is to overturn a firmly-held belief. Einstein's work on relativity is the outstanding example. Although it met with considerable opposition at first, it was on the whole an intelligent opposition. With few exceptions, none of Einstein's reputable opponents dismissed him as a crackpot. They could not so dismiss him because for years he contributed brilliant articles to the journals and had won wide recognition as a theoretical physicist. In a surprisingly short time, his relativity theories won almost universal acceptance, and one of the greatest revolutions in the history of science quietly took place.

It would be foolish, of course, to deny that history contains many sad examples of novel scientific views which did not receive an unbiased hearing, and which later proved to be true. The pseudoscientist never tires reminding his readers of these cases. The opposition of traditional psychology to the study of hypnotic phenomena (accentuated by the fact that Mesmer was both a crank and a charlatan) is an outstanding instance. In the field of medicine, the germ theory of Pasteur, the use of anesthetics, and Dr. Semmelweiss' insistence that doctors sterilize their hands before attending childbirth are other well known examples of theories which met with strong professional prejudice.

Probably the most notorious instance of scientific stubbornness was the refusal of eighteenth century astronomers to believe that stones actually fell from the sky. Reaction against medieval superstitions and old wives' tales was still so strong that whenever a meteor fell, astronomers insisted it had either been picked up somewhere and carried by the wind, or that the persons who claimed to see it fall were lying. Even the great French *Académie des Sciences* ridiculed this folk belief, in spite of a number of early studies of meteoric phenomena. Not until April 26, 1803, when several thousand small meteors fell on the town of L'Aigle, France, did the astronomers decide to take falling rocks seriously.

Many other examples of scientific traditionalism might be cited, as well as cases of important contributions made by persons of a crank variety. The discovery of the law of conservation of energy by Robert Mayer, a psychotic German physician, is a classic instance. Occasionally a layman, completely outside of science, will make an astonishingly prophetic guess—like Swift's prediction about the moons of Mars (to be discussed later), or Samuel Johnson's belief (expressed in a letter, in 1781, more than eighty years before the discovery of germs) that microbes were the cause of dysentery.

One must be extremely cautious, however, before comparing the work of some contemporary eccentric with any of these earlier examples, so frequently cited in crank writings. In medicine, we must remember, it is only in the last fifty years or so that the art of healing has become anything resembling a rigorous scientific discipline. One can go back to periods in which medicine was in its infancy, hopelessly mixed with superstition, and find endless cases of scientists with unpopular views that later proved correct. The same holds true of other sciences. But the picture today is vastly different. The prevailing spirit among scientists, outside of totalitarian countries, is one of eagerness for fresh ideas. In the great search for a cancer cure now going on, not the slightest stone, however curious its shape, is being left unturned. If anything, scientific journals err on the side of permitting *questionable* theses to be published, so they may be discussed and checked in the hope of finding something of value. A few years ago a student at the Institute for Advanced Studies in Princeton was asked how his seminar had been that day. He was quoted in a news magazine as exclaiming, "Wonderful! Everything we knew about physics last week isn't true!"

Here and there, of course—especially among older scientists who, like everyone else, have a natural tendency to become set in their opinions—one may occasionally meet with irrational prejudice against a new point of view. You cannot blame a scientist for unconsciously resisting a theory which may, in some cases, render his entire life's work obsolete. Even the great Galileo refused to accept Kepler's theory, long after the evidence was quite strong, that planets move in ellipses. Fortunately there are always, in the words of Alfred Noyes, "The young, swift-footed, waiting for the fire," who can form the vanguard of scientific revolutions.

It must also be admitted that in certain areas of science, where empirical data are still hazy, a point of view may acquire a kind of cult following and harden into rigid dogma. Modifications of Einstein's theory, for example, sometimes meet a resistance similar to that which met the original theory. And no doubt the reader will have at least one acquaintance for whom a particular brand of psychoanalysis has become virtually a religion, and who waxes highly indignant if its postulates are questioned by adherents of a rival brand.

Actually, a certain degree of dogma—of pig-headed orthodoxy—is both necessary and desirable for the health of science. It forces the scientist with a novel view to mass considerable evidence before his theory can be seriously entertained. If this situation did not exist, science would be reduced to shambles by having to examine every new-fangled notion that came along. Clearly, working scientists have more important tasks. If someone announces that the moon is made of green cheese, the professional astronomer cannot be expected to climb down from his telescope and write a detailed refutation. "A fairly complete textbook of physics would be only part of the answer to Velikovsky," writes Prof. Laurence J. Lafleur, in his excellent article on "Cranks and Scientists" (*Scientific Monthly,* Nov., 1951), "and it is therefore not surprising that the scientist does not find the undertaking worth while."

The modern pseudo-scientist—to return to the point from which we have digressed—stands entirely outside the closely integrated channels through which new ideas are introduced and evaluated. He works in isolation. He does not send his findings to the recognized journals, or if he does, they are rejected for reasons which in the vast majority of cases are excellent. In most cases the crank is not well enough informed to write a paper with even a surface resemblance to a significant study. As a consequence, he finds himself excluded from the journals and societies, and almost universally ignored by the competent workers in his field. In fact, the reputable scientist does not even know of the crank's existence unless his work is given widespread publicity through non-academic channels, or unless the scientist makes a hobby of collecting crank literature. The eccentric is forced, therefore, to tread a lonely way. He speaks before organizations he himself has founded, contributes to journals he himself may edit, and—until recently—publishes books only when he or his followers can raise sufficient funds to have them printed privately.

A second characteristic of the pseudo-scientist, which greatly strengthens his isolation, is a tendency toward paranoia. This is a mental condition (to quote a recent textbook) "marked by chronic, systematized, gradually developing delusions, without hallucinations, and with little tendency toward deterioration, remission, or recovery." There is wide disagreement among psychiatrists about the causes of paranoia. Even if this were not so, it obviously is not within the scope of this book to discuss the possible origins of paranoid traits in individual cases. It is easy to understand, however, that a strong sense of personal greatness must be involved whenever a crank stands in solitary, bitter opposition to every recognized authority in his field.

If the self-styled scientist is rationalizing strong religious convictions, as often is the case, his paranoid drives may be reduced to a minimum. The desire to bolster religious beliefs with science can be a powerful motive. For example, in our examination of George McCready Price, the greatest of modern opponents of evolution, we shall see that his devout faith in Seventh Day Adventism is a sufficient explanation for his curious geological views. But even in such cases, an element of paranoia is nearly always present. Otherwise the pseudo-scientist would lack the stamina to fight a vigorous, single-handed battle against such overwhelming odds. If the crank is insincere—interested only in making money, playing a hoax, or both—then obviously paranoia need not enter his make-up. However, very few cases of this sort will be considered.

There are five ways in which the sincere pseudo-scientist's paranoid tendencies are likely to be exhibited.

(1) He considers himself a genius.

(2) He regards his colleagues, without exception, as ignorant blockheads. Everyone is out of step except himself. Frequently he insults his opponents by accusing them of stupidity, dishonesty, or other base motives. If they ignore him, he takes this to mean his arguments are unanswerable. If they retaliate in kind, this strengthens his delusion that he is battling scoundrels.

Consider the following quotation: "To me truth is precious.... I should rather be right and stand alone than to run with the multitude and be wrong.... The holding of the views herein set forth has already won for me the scorn and contempt and ridicule of some of my fellowmen. I am looked upon as being odd, strange, peculiar.... But truth is truth and though all the world reject it and turn against me, I will cling to truth still."

These sentences are from the preface of a booklet, published in

1931, by Charles Silvester de Ford, of Fairfield, Washington, in which he proves the earth is flat. Sooner or later, almost every pseudo-scientist expresses similar sentiments.

(3) He believes himself unjustly persecuted and discriminated against. The recognized societies refuse to let him lecture. The journals reject his papers and either ignore his books or assign them to "enemies" for review. It is all part of a dastardly plot. It never occurs to the crank that this opposition may be due to error in his work. It springs solely, he is convinced, from blind prejudice on the part of the established hierarchy—the high priests of science who fear to have their orthodoxy overthrown.

Vicious slanders and unprovoked attacks, he usually insists, are constantly being made against him. He likens himself to Bruno, Galileo, Copernicus, Pasteur, and other great men who were unjustly persecuted for their heresies. If he has had no formal training in the field in which he works, he will attribute this persecution to a scientific masonry, unwilling to admit into its inner sanctums anyone who has not gone through the proper initiation rituals. He repeatedly calls your attention to important scientific discoveries made by laymen.

(4) He has strong compulsions to focus his attacks on the greatest scientists and the best-established theories. When Newton was the outstanding name in physics, eccentric works in that science were violently anti-Newton. Today, with Einstein the father-symbol of authority, a crank theory of physics is likely to attack Einstein in the name of Newton. This same defiance can be seen in a tendency to assert the diametrical opposite of well-established beliefs. Mathematicians prove the angle cannot be trisected. So the crank trisects it. A perpetual motion machine cannot be built. He builds one. There are many eccentric theories in which the "pull" of gravity is replaced by a "push." Germs do not cause disease, some modern cranks insist. Disease produces the germs. Glasses do not help the eyes, said Dr. Bates. They make them worse. In our next chapter we shall learn how Cyrus Teed literally turned the entire cosmos inside-out, compressing it within the confines of a hollow earth, inhabited only on the inside.

(5) He often has a tendency to write in a complex jargon, in many cases making use of terms and phrases he himself has coined. Schizophrenics sometimes talk in what psychiatrists call "neologisms" —words which have meaning to the patient, but sound like Jabber-

wocky to everyone else. Many of the classics of crackpot science exhibit a neologistic tendency.

When the crank's I.Q. is low, as in the case of the late Wilbur Glenn Voliva who thought the earth shaped like a pancake, he rarely achieves much of a following. But if he is a brilliant thinker, he is capable of developing incredibly complex theories. He will be able to defend them in books of vast erudition, with profound observations, and often liberal portions of sound science. His rhetoric may be enormously persuasive. All the parts of his world usually fit together beautifully, like a jig-saw puzzle. It is impossible to get the best of him in any type of argument. He has anticipated all your objections. He counters them with unexpected answers of great ingenuity. Even on the subject of the shape of the earth, a layman may find himself powerless in a debate with a flat-earther. George Bernard Shaw, in *Everybody's Political What's What?*, gives an hilarious description of a meeting at which a flat-earth speaker completely silenced all opponents who raised objections from the floor. "Opposition such as no atheist could have provoked assailed him"; writes Shaw, "and he, having heard their arguments hundreds of times, played skittles with them, lashing the meeting into a spluttering fury as he answered easily what it considered unanswerable."

In the chapters to follow, we shall take a close look at the leading pseudo-scientists of recent years, with special attention to native specimens. Some British books will be discussed, and a few Continental eccentric theories, but the bulk of crank literature in foreign tongues will not be touched upon. Very little of it has been translated into English, and it is extremely difficult to get access to the original works. In addition, it is usually so unrelated to the American scene that it loses interest in comparison with the work of cranks closer home.

ALFRED M. BORK

Alfred M. Bork was born in 1926, received his Ph.D. from Brown University, and is now Professor of Physics at Reed College. He is a consultant to Harvard Project Physics and one of the organizers of the Irvine Conference on the use of computers in the teaching of physics. He was a scholar at the Dublin Institute for Advanced Studies and has served on the faculty of the University of Alaska. His areas of interest include the history of late nineteenth and early twentieth century physics, the teaching of science to nonscience majors, and the production of films with computers. Dr. Bork is the editor of Science and Language and coeditor of Science and Ideas.

JACOB BRONOWSKI

Jacob Bronowski, who received his Ph.D. from Cambridge University in 1933, is now a Fellow of the Salk Institute of Biological Studies in California. He has served as Director of General Process Development for the National Coal Board of England, as the Science Deputy to the British Chiefs of Staff, and as head of the Projects Division of UNESCO. In 1953 he was Carnegie Visiting Professor at the Massachusetts Institute of Technology.

ALEXANDER CALANDRA

Alexander Calandra, Associate Professor of Physics at Washington University, St. Louis, since 1950, was born in New York in 1911. He received his B.S. from Brooklyn College and his Ph.D. in statistics from New York University. He has been a consultant to the American Council for Education and for the St. Louis Public Schools, has taught on television, and has been the regional counselor of the American Institute of Physics for Missouri.

ARTHUR C. CLARKE

Arthur C. Clarke, British scientist and writer, is a Fellow of the Royal Astronomical Society. During World War II he served as technical officer in charge of the first aircraft ground-controlled approach project. He has won the Kalinga Prize, given by UNESCO for the popularization of science. The feasibility of many of the current space developments was perceived and outlined by Clarke in the 1930's. His science fiction novels include Childhoods End and The City and the Stars.

ROBERT MYRON COATES

Robert Myron Coates, author of many books and articles, was born in New Haven, Connecticut in 1897 and attended Yale University. He is a member of the National Institute of Arts and Letters and has been an art critic for The New Yorker magazine. His books include The Eater of Darkness, The Outlaw Years, The Bitter Season, and The View From Here.

E. J. DIJKSTERHUIS

E. J. Dijksterhuis was born at Tilburg, Holland, in 1892, and later became a professor at the University of Leyden. Although he majored in mathematics and physics, his school examinations forced him to take Latin and Greek, which awakened his interest in the early classics of science. He published important studies on the history of mechanics, on Euclid, on Simon Steven and on Archimedes. Dijksterhuis died in 1965.

ALBERT EINSTEIN

Albert Einstein, considered to be the most creative physical scientist since Newton, was nevertheless a humble and sometimes rather shy man. He was born in Ulm, Germany, in 1879. He seemed to learn so slowly that his parents feared that he might be retarded. After graduating from the Polytechnic Institute in Zurich, he became a junior official at the Patent Office at Berne. At the age of twenty-six, and quite unknown, he published three revolutionary papers in theoretical physics in 1905. The first paper extended Max Planck's ideas of quantization of energy, and established the quantum theory of radiation. For this work he received the Nobel Prize for 1929. The second paper gave a mathematical theory of Brownian motion, yielding a calculation of the size of a molecule. His third paper founded the special theory of relativity. Einstein's later work centered on the general theory of relativity. His work has a profound influence not only on physics, but also on philosophy. An eloquent and widely beloved man, Einstein took an active part in liberal and anti-war movements. Fleeing from Nazi Germany, he settled in the United States in 1933 at the Institute for Advanced Study in Princeton. He died in 1955.

R. J. FORBES

R. J. Forbes, professor at the University of Amsterdam, was born in Breda, Holland, in 1900. After studying chemical engineering, he worked for the Royal Dutch Shell Group in their laboratories and in refineries in the East Indies. Interested in archaeology and museum collections, he has published works on the history of such fields as metallurgy, alchemy, petroleum, roadbuilding, technology, and distillation.

GEORGE GAMOW

George Gamow, a theoretical physicist from Russia, received his Ph.D. in physics at the University of Leningrad. At Leningrad he became professor after being a Carlsberg fellow and a university fellow at the University of Copenhagen and a Rockefeller fellow at Cambridge University. He came to the United States in 1933 to teach at the George Washington University and later at the University of Colorado. His popularizations of physics are much admired.

MARTIN GARDNER

Martin Gardner, well-known editor of the "Mathematical Games" department of the Scientific American, was born in Tulsa, Oklahoma, in 1914. He received a B.A. in philosophy from the University of Chicago in 1939, worked as a publicity writer for the University, and then wrote for the Tulsa Tribune. During World War II he served in the Navy. Martin Gardner has written humorous short stories as well as serious articles for such journals as Scripta Mathematica and Philosophy of Science, and is the best-selling author of The Annotated Alice, Relativity for the Million, Math, Magic, and Mystery, as well as two volumes of the Scientific American Book of Mathematical Puzzles and Diversions.

CARLEEN MALEY HUTCHINS

Carleen Hutchins was born in Springfield, Massachusetts, in 1911. She received her A.B. from Cornell University and her M.A. from New York University. She has been designing and constructing stringed instruments for years. Her first step was in 1942 when "I bought an inexpensive weak-toned viola because my musical friends complained that the trumpet I had played was too loud in chamber music, as well as out of tune with the strings - and besides they needed a viola." In 1947, while on a leave of absence from the Brearley School in New York, she started making her first viola - it took two years. She has made over fifty, selling some to finance more research. In 1949 she retired from teaching and then collaborated with Frederick A. Saunders at Harvard in the study of the acoustics of the instruments of the violin family. She has had two Guggenheim fellowships to pursue this study.

GERALD HOLTON

Gerald Holton received his early education in Vienna, at Oxford, and at Wesleyan University, Connecticut. He has been at Harvard University since receiving his Ph.D. degree in physics there in 1948; he is Professor of Physics, teaching courses in physics as well as in the history of science. He was the founding editor of the quarterly Daedalus. Professor Holton's experimental research is on the properties of matter under high pressure. He is a co-director of Harvard Project Physics.

LEOPOLD INFELD

Leopold Infeld, a co-worker with Albert Einstein in general relativity theory, was born in 1898 in Poland. After studying at the Cracow and Berlin Universities, he became a Rockefeller Fellow at Cambridge where he worked with Max Born in electromagnetic theory, and then a member of the Institute for Advanced Study at Princeton. For eleven years he was Professor of Applied Mathematics at the University of Toronto. He then returned to Poland and became Professor of Physics at the University of Warsaw and until his death on 16 January 1968 he was director of the Theoretical Physics Institute at the university. A member of the presidium of the Polish Academy of Science, Infeld conducted research in theoretical physics, especially relativity and quantum theories. Infeld was the author of The New Field Theory, The World in Modern Science, Quest, Albert Einstein, and with Einstein The Evolution of Physics.

ROBERT BRUCE LINDSAY

Robert Bruce Lindsay, born in New Bedford, Massachusetts, in 1900, is the Hazard Professor of Physics at Brown University. Between his studies at Brown and Massachusetts Institute of Technology, he went to Copenhagen as an American-Scandinavian Foundation fellow. He was chairman of the physics department and dean of the graduate school at Brown. Currently he is on the governing board of the American Institute of Physics and editor-in-chief of the Journal of the Acoustical Society of America. In addition to acoustics and quantum mechanics, Professor Lindsay is interested in history and philosophy of science.

JAMES CLERK MAXWELL

See J. R. Newman's articles in Readers 3 and 4.

JAMES ROY NEWMAN

James R. Newman, lawyer and mathematician, was born in New York City in 1907. He received his A.B. from the College of the City of New York and LL.B. from Columbia. Admitted to the New York bar in 1929, he practiced there for twelve years. During World War II he served as chief intelligence officer, U. S. Embassy, London, and in 1945 as special assistant to the Senate Committee on Atomic Energy. From 1956-57 he was senior editor of The New Republic, and since 1948 had been a member of the board of editors for Scientific American where he was responsible for the book review section. At the same time he was a visiting lecturer at the Yale Law School. J. R. Newman is the author of What is Science?, Science and Sensibility, and editor of Common Sense of the Exact Sciences, The World of Mathematics, and the Harper Encyclopedia of Science. He died in 1966.

ERIC MALCOLM ROGERS

Eric Malcolm Rogers, Professor of Physics at Princeton University, was born in Bickley, England, in 1902. He received his education at Cambridge and later was a demonstrator at the Cavendish Laboratory. Since 1963 he has been the organizer in physics for the Nuffield Foundation Science Teaching Project. He is the author of the textbook, Physics for the Inquiring Mind.

PETER GUTHRIE TAIT

Peter Guthrie Tait, collaborator of William Thomson (Lord Kelvin) in thermodynamics, was born at Dalkeith, Scotland, in 1831. He was educated at the Academy at Edinburgh (where James Clerk Maxwell was also a student), and at Peterhouse, Cambridge. He remained at Cambridge as a lecturer before becoming Professor of Mathematics at Queen's College, Belfast. There he did research on the density of ozone and the action of the electric discharge of oxygen and other gases. From 1860 until his death in 1901 he served as Professor of Natural Philosophy at Edinburgh. In 1864 he published his first important paper on thermodynamics and thermoelectricity and thermal conductivity. With Lord Kelvin he published the textbook Elements of Natural Philosophy in 1867.

BARON KELVIN, WILLIAM THOMSON

Baron Kelvin, William Thomson, British scientist and inventor, was born in Belfast, Ireland, in 1824. At the age of eleven he entered the University of Glasgow where his father was professor of mathematics. In 1841 he went to Peterhouse, at Cambridge University. In 1848 Thomson proposed a temperature scale independent of the properties of any particular substance, and in 1851 he presented to the Royal Society of Edinburgh a paper reconciling the work on heat of Sadi Carnot with the conclusions of Count von Rumford, Sir Humphrey Davy, J. R. von Mayer and J. P. Joule. In it he stated the Second Law of Thermodynamics. Lord Kelvin worked on such practical applications as the theory of submarine cable telegraphy and invented the mirror galvanometer. In 1866 he was knighted, 1892 raised to peerage, and in 1890 elected president of the Royal Society. He died in 1907.

LEONARDO DA VINCI

Leonardo da Vinci, the exemplar of "l'uomo universale," the Renaissance ideal, was born in 1452 near Vinci in Tuscany, Italy. Without a humanistic education, he was apprenticed at an early age to the painter-sculptor Andrea del Verrocchio. The first 10 years of Leonardo's career were devoted largely to painting, culminating in the "Adoration of the Magi." Defensive to criticisms on his being "unlettered," Leonardo emphasized his ability as inventor and engineer, becoming a fortification expert for the militarist Cesare Borgia. By 1503 he was working as an artist in almost every field. "Mona Lisa" and "The Last Supper" are among the world's most famous paintings. Besides his engineering feats such as portable bridges, machine guns, tanks, and steam cannons, Leonardo contrived highly imaginative blueprints such as the protohelicopter and a flying machine. His prolific life terminated in the Castle of Claux near Amboise on May 2, 1519.

HARVEY ELLIOTT WHITE

Harvey Elliott White, Professor of Physics at the University of California, Berkeley, was born in Parkersburg, West Virginia in 1902. He attended Occidental College and Cornell University where he received his Ph.D. in 1929. In 1929-30 he was an Institute Research Fellow at the Physics and Technology Institute in Germany. His special interests are atomic spectra and ultraviolet and infrared optics.

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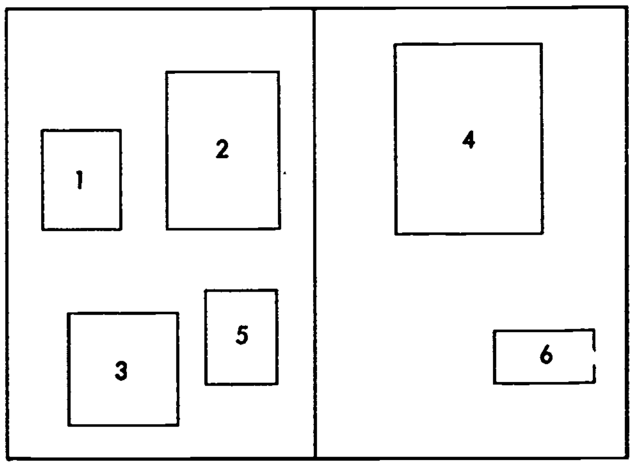

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