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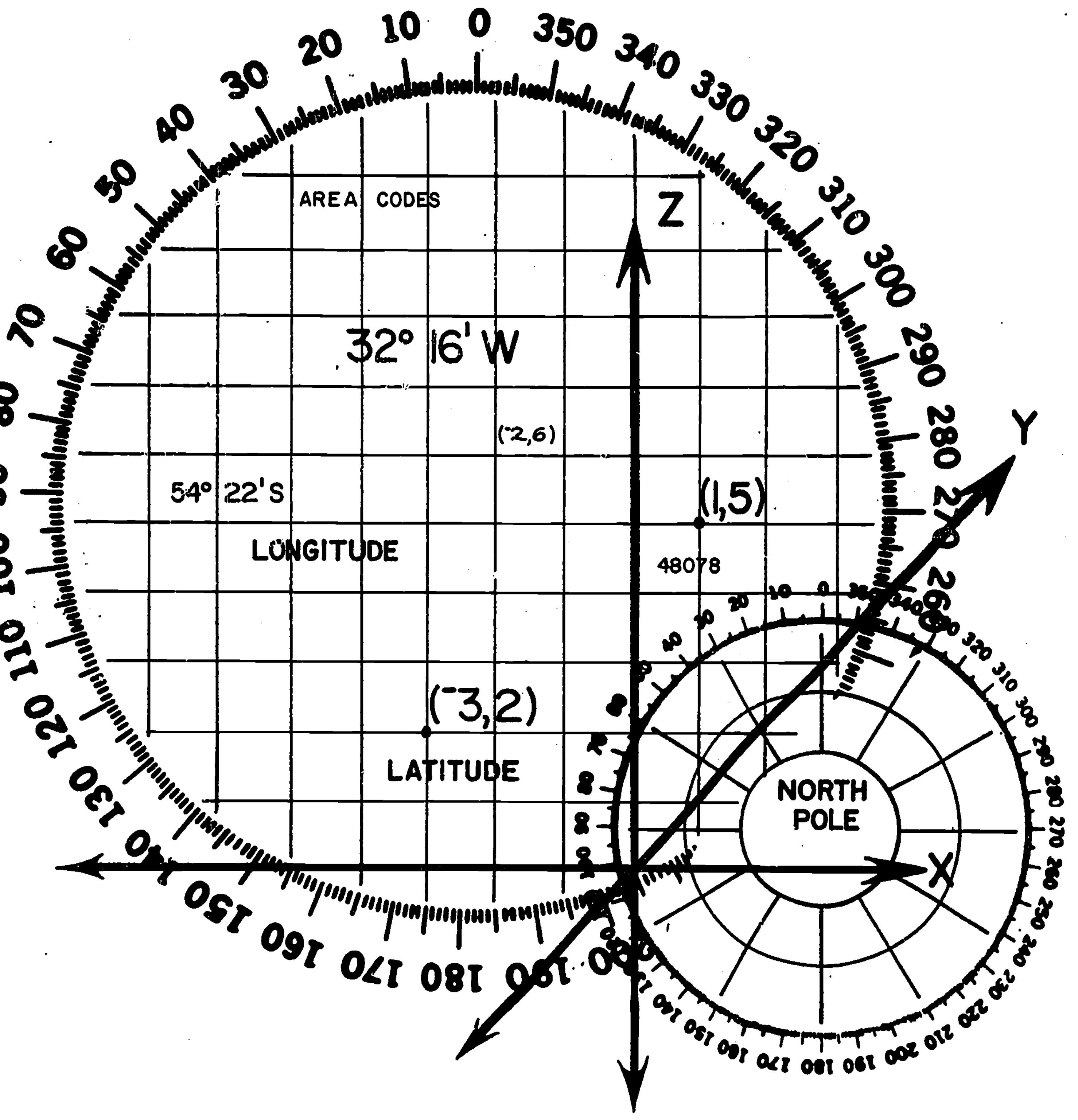
ABSTRACT

This instructional unit presents the coordinate system as a correspondence between a set of numbers and a set of points. A variety of coordinate systems are studied with major emphasis on the rectangular system. Basic problem solving and critical thinking skills are practiced in practical application situations. Related documents are SE 015 334 - SE 015 341 and SE 015 343 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

WHERE IS THE POINT?



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WHERE IS THE POINT?

OAKLAND COUNTY MATHEMATICS PROJECT

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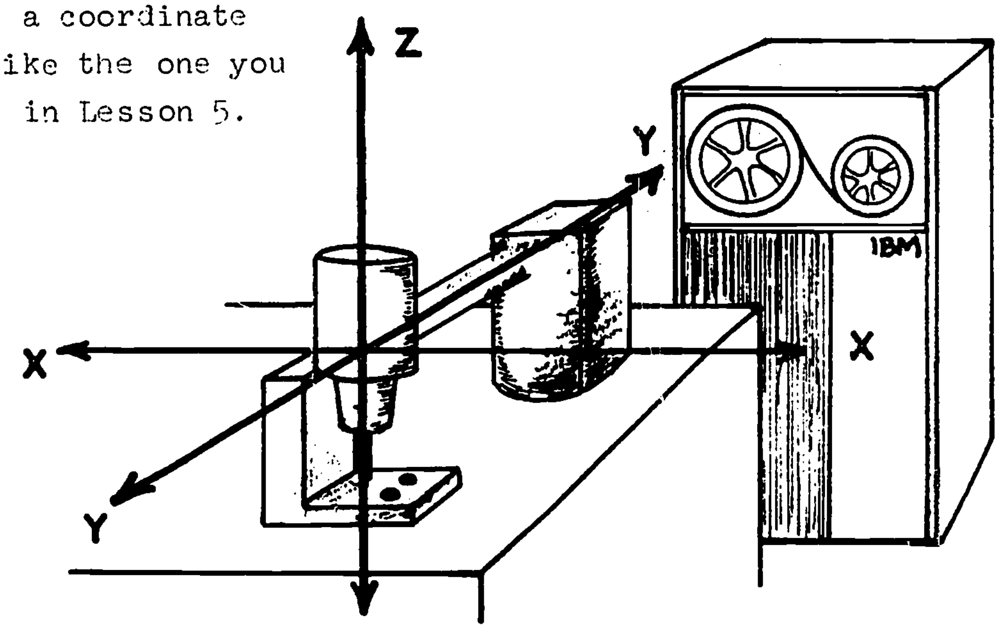
PREFACE

This booklet is about coordinate systems. Essentially, a coordinate system is a way of assigning names to objects so that the name can be used to locate the object. The term object, as it is used here, refers to points, regions on a map, parts on an assembly line, or satellites in space.

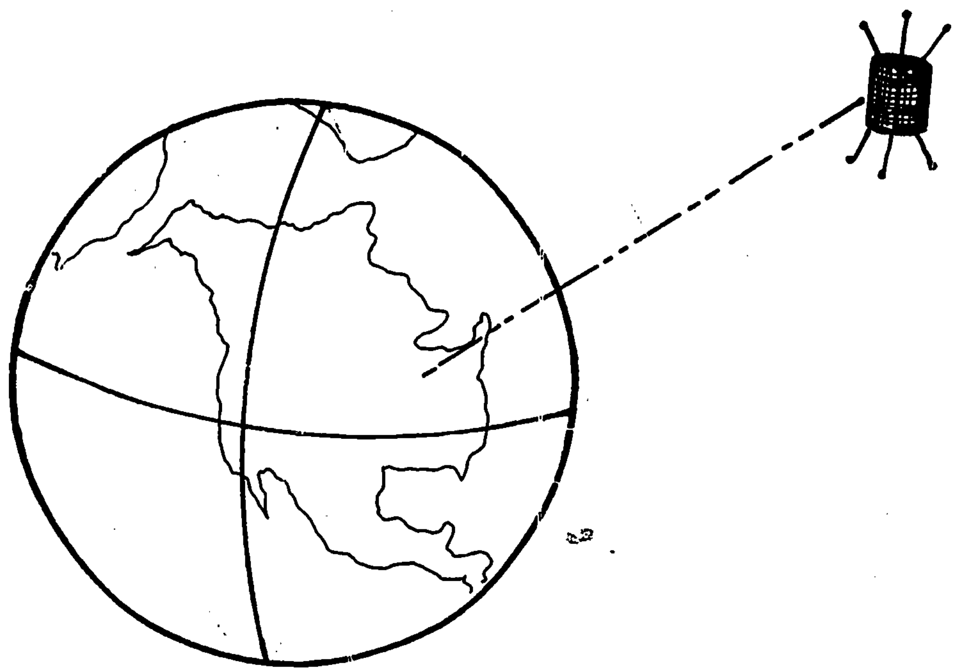
The need for an orderly plan to describe the position of objects is basic to science, technology and many areas of everyday life.

Various coordinate systems help us locate our theater seat; retrieve our car in a parking lot: locate places on maps; even send and receive our mail (zip code).

Machine tools are often programmed by a computer to move from point-to-point as they perform their operations. The basis for measuring on such controlled machine tools is a coordinate system like the one you will see in Lesson 5.



In another lesson you will learn how the position of a satellite could be specified at some instant by giving three numbers: its vertical distance from the Earth's center, its latitude and its longitude.



Each lesson in this booklet is concerned with using some kind of coordinate system. When you finish this booklet you will know quite a bit of useful information about plotting points in different coordinate systems.

You will be able to apply what you learn in this booklet to many areas of mathematics and science - both this year and in the future. We know that you will find the lessons interesting.

TABLE OF CONTENTS

Title	Page
LESSON 1 - MAP COORDINATES	1
LESSON 2 - COORDINATE SYSTEMS	10
LESSON 3 - THE GLOBAL COORDINATE SYSTEM . .	17
LESSON 4 - RULER AND PROTRACTOR COORDINATES. .	29
SECRET CODE TABLE	38
LESSON 5 - SPACE COORDINATES.	40
LESSON 6 - A PUZZLING LINE	47
LESSON 7 - THE LATTICE PLANE.	51
LESSON 8 - PICTURE PLOTTING	57
LESSON 9 - PICTURE DISTORTION	62
LESSON 10 - REGIONS IN THE PLANE	68
LESSON 11 - WHAT IS A COORDINATE SYSTEM? . .	75

MAP COORDINATES

Part I Where is Graham Lake?

The map that you are using in this lesson is the Official Highway Map of Oakland County.

The map shows lakes, cities, highways, airports, parks and many places of interest. Each place has a location on the map. A location is given by using the letters and numbers on the sides of the map.

The location of Graham Lake is Z-24. Find this on your map.

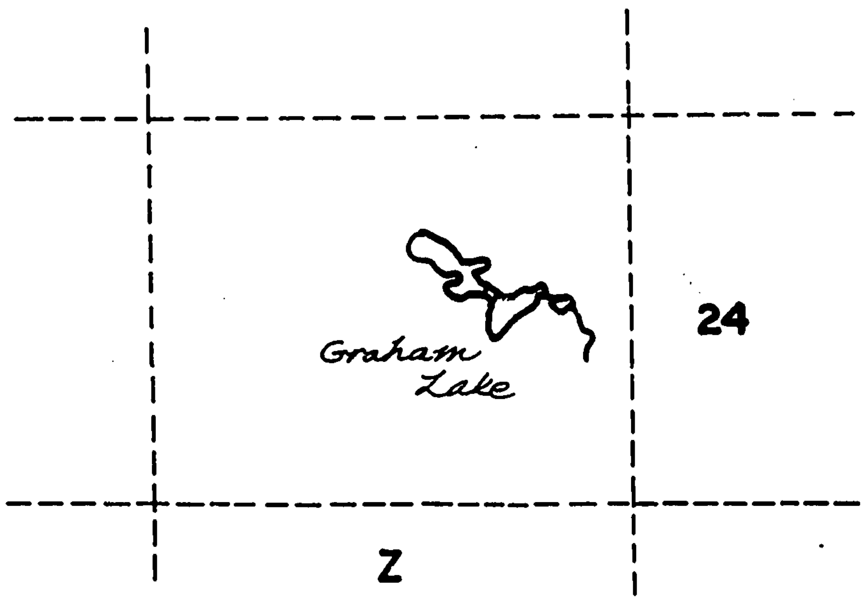


Figure 1

EXERCISES

1-3. A location is given in the right hand column below. Give the name of the lake, city or major intersection you find at that location.

	PLACE	LOCATION
1.	______________	0-9
2.	______________	W-4
3.	______________	BB-29

4-9 Find the location of the places listed in the left hand column.

PLACE	LOCATION
Example 1. Clarkston	N-21
4. Parker Lake (Oxford Township)	______
5. Square Lake	______
6. I-75, M-59 Intersection	______
7. Pontiac Municipal Airport	______
8. Your house	______
9. Your school	______

Figure 2 shows a grid. A grid has vertical lines which form columns and horizontal lines which forms rows. The rows and columns intersect to form <u>cells</u>. If the rows and columns are labeled with letters or numbers, then each cell can be named.

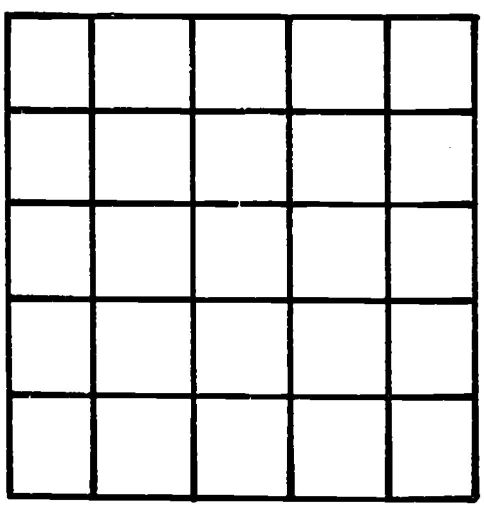
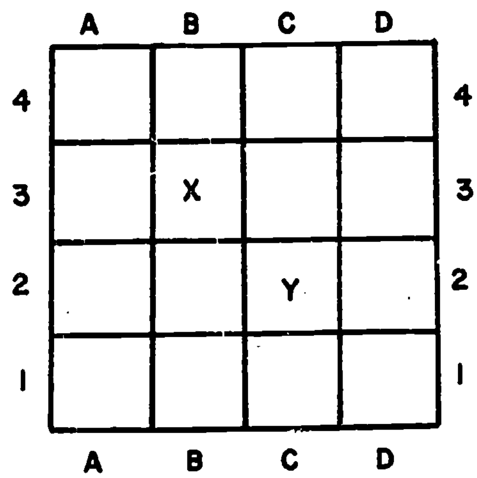


Figure 2

LESSON 1

Figure 3



EXAMPLE : The X, in Figure 3, is in cell B-3.

10. Give the location of the cell containing the letter Y.

11. Print the letter W in cell A-1.

Vertical and horizontal lines are not drawn on the Oakland County map, but letters and numbers are used to locate cells just as in the grid shown in Figure 3.

12. The numbers are printed in a North-South Line. The letters are printed in a (an) ____?____ line.

The letters and numbers used to give locations are called <u>coordinates</u>. W-4 are the <u>coordinates</u> of a certain section in Southfield Township.

The letter(s) is the <u>East-West coordinate</u> and the number is <u>the North-South coordinate</u>.

By using coordinates we can give another person the location of any object on the map and he can find the position of that object.

The first coordinate tells us how far to go horizontally on the map. The second coordinate tells us how far to go vertically on the map.

✓ **POINT**

Which coordinate, the East-West or the North-South, tells how many units to move horizontally?

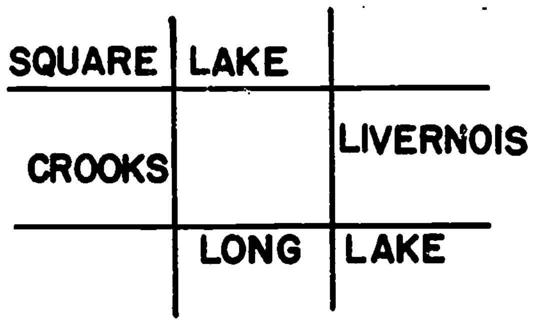
PART 2 TOWNSHIP-SECTION NUMBER

A township is a square parcel of land six miles on a side. Each township is divided into thirty-six sections.

EXERCISES

1. Give the name of a city in Avon Township.
2. What township do you live in?
3. Each township is how long on a side?
4. The area of each section is ____?____ .
5. The mile roads run East-West, in Oakland County, starting with ____?____ road at the southern end of the County.

6-8. Give the location of each section by giving the map coordinates of the section and the name of the township that the section is in.

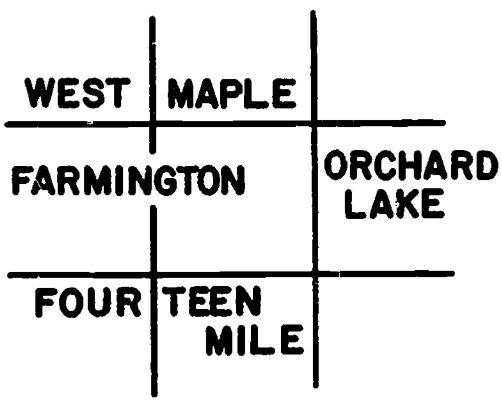


MAP COORDINATES

____

NAME OF TOWNSHIP

____

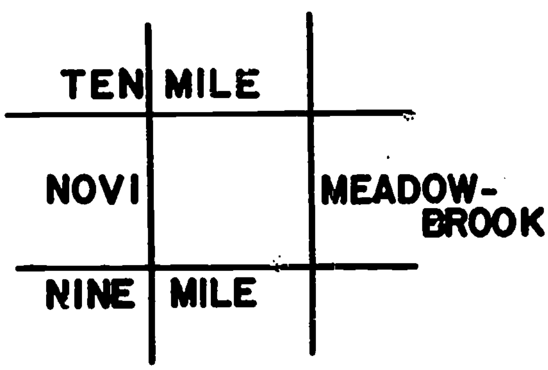


MAP COORDINATES

____

NAME OF TOWNSHIP

____



MAP COORDINATES

____

NAME OF TOWNSHIP

____

What are numbers like those shown in Figure 4 used for? Look at the legend on the map.

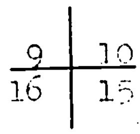


Figure 4

9. Township sections are numbered in a special way. Complete the numbering of each section in Figure 5.

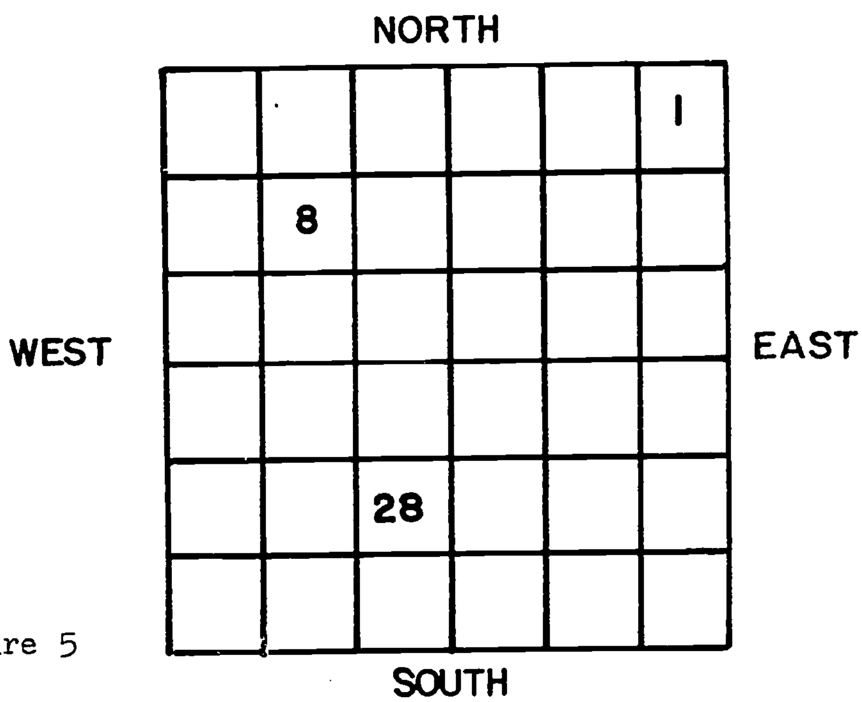
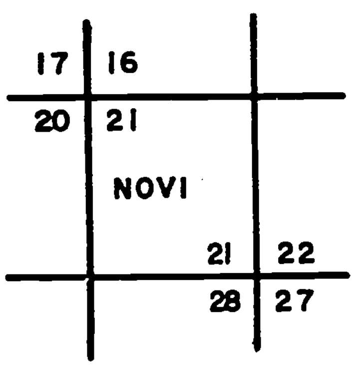


Figure 5

10.



MAP COORDINATES

____

NAME OF TOWNSHIP

____

11. Find section 9 in Troy Township. Give its map coordinates.

12. How many miles is section Y-9 from Y-2?

13-16. Complete the table in Figure 6 so that each township section corresponds to its proper location.

	Township name and Section number	Map Location
13.	Pontiac-9	
14.		V-10
15.	Waterford-21	
16.		F-1

Figure 6

Township names and section numbers can also be used to give map locations. The township name acts as one coordinate and the section number acts as the other coordinate.

17. In giving the location of Childs Lake you now have two choices. You can give its coordinates as

a. ___________ (Letter(s), Number) Or

b. ___________ (Township Name, Section Number)

Do you see any advantages in using township names and section numbers for locations? Any disadvantages?

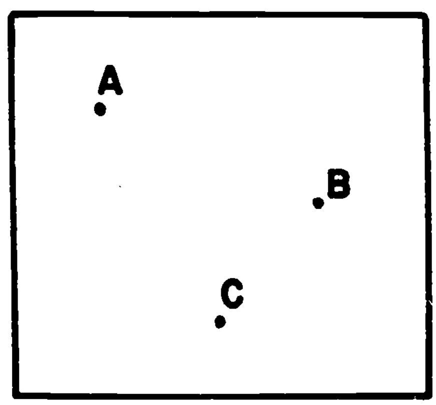
Part 3 (Optional) Grid Size

Many places on the map (especially cities, large lakes and parks) do not lie entirely within the boundaries of any one section. This is not a real disadvantage if you are just interested in general location.

EXAMPLE: Your friend asks, "Where is South Lyon?" Would you say it's at A-2, B-2, A-3, or B-3? Your friend should be able to find South Lyon by using any one of these four locations.

Each location like BB-17 names a section on the map. Each section measures one mile on a side. Thus, each symbol like BB-17 or F-7, etc., locates one square mile. Think of a way to locate areas on the map which are smaller than one square mile.

Figure 7



EXERCISES

1. Let points A, B, and C, in Figure 7, represent places in one of the square mile township sections. Draw a grid over Figure 7 so that each point is in a separate cell.

2. Think of a way to label the rows and columns on the grid shown in Figure 8. Use your labels to give the location of the corner of Seventh and Ludlow.

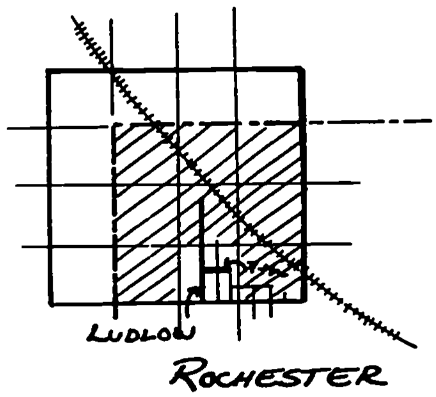


Figure 8

3. What is the area, in square miles, of each cell shown in Figure 8?

COORDINATE SYSTEMS

Part I Knowing the label can you locate the object?

In Lesson 1, an object was located on the map by using its map coordinates.

Different types of coordinate systems are used in many places every day.

A coordinate system assigns a label to an object. Knowing the label you can locate the object that is named by the label.

A ticket to a football game gives the coordinates of a seat in the stadium.

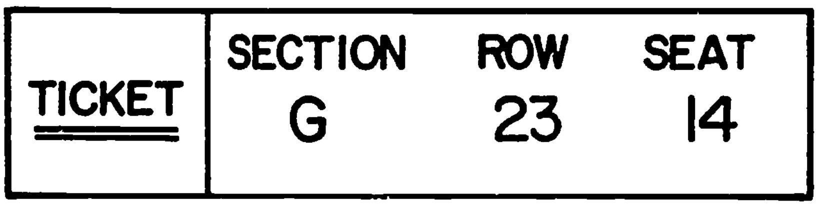


Figure 1

Knowing the coordinates, G-23-14, you can locate the seat.

A shopping center parking lot uses a coordinate system as an aid in locating your car.

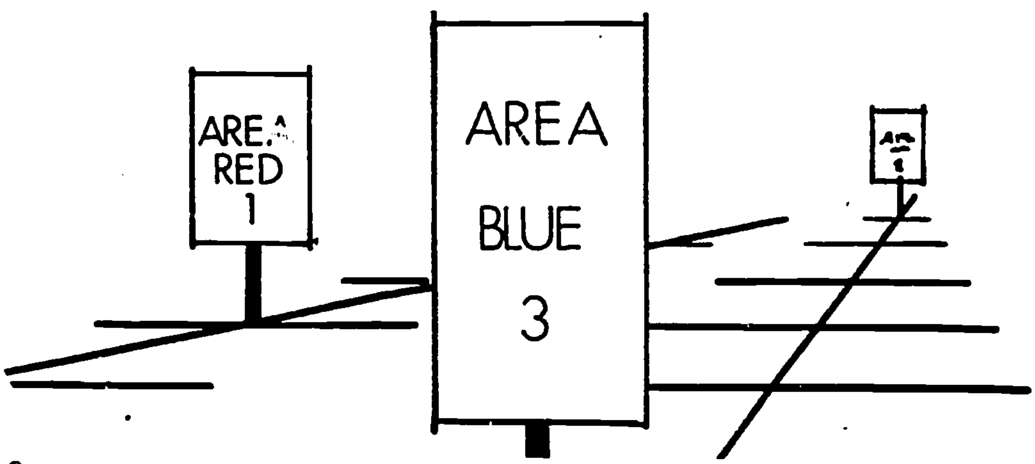


Figure 2

The parking lot shown in Figure 2 is divided into sections and each section is divided into rows.

The coordinates Red-3 pinpoint a certain row in a certain section. How would you improve this coordinate system so that the coordinates would locate an individual parking space in a row?

Zip code is a five-digit coordinate system that locates areas within the United States and its possessions for purposes of simplifying the distribution of mail.

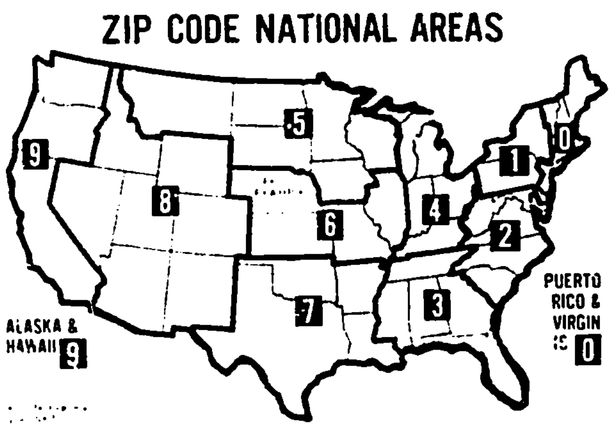


Figure 3

In the Zip code, the United States and its possessions are divided up into 10 large geographic areas. Each area consists of three or more states or possessions and is given a number between 0 and 9, inclusive. What number area is Michigan in?

Each of these large Zip code areas is divided into two or more sub-areas.

Together, the first three digits of any Zip code number stand for either a particular Sectional Center or a metropolitan city. (See Figure 4.)

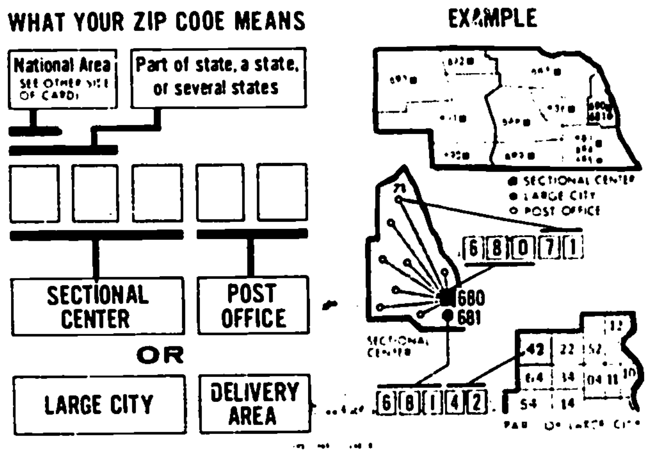


Figure 4

The last two digits stand for a post office or a delivery area. What is your Zip code? Do you think a letter could be delivered that has just a name and the Zip code for an address? Why?

DISCUSSION QUESTIONS

1. Explain how each of the following are used as a coordinate system. What is located? How is it located?
   a. House numbers in a city
   b. Room numbers in a hotel
   c. Dewey decimal system (in the library)
   d. Ticket to a play
   e. Latitude and longitude on a world map
2. Can you suggest other types of coordinate systems?

EXERCISES

Scott Wright has been chosen to be chairman of the coat-room committee for the Annual Junior Senior Dance. The chief part of his job will be to organize a way to check coats in and out.

Scott wants to make it easy to locate and return a coat when the check stub is turned in.

The coatroom will be in Mr. Rolfe's classroom.

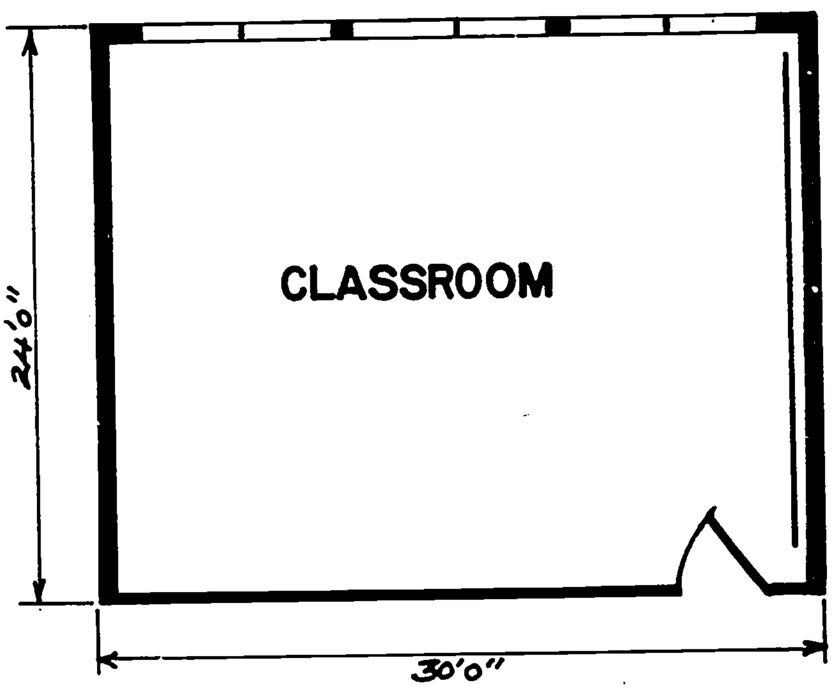


Figure 5

Nine coat-racks are to be used.

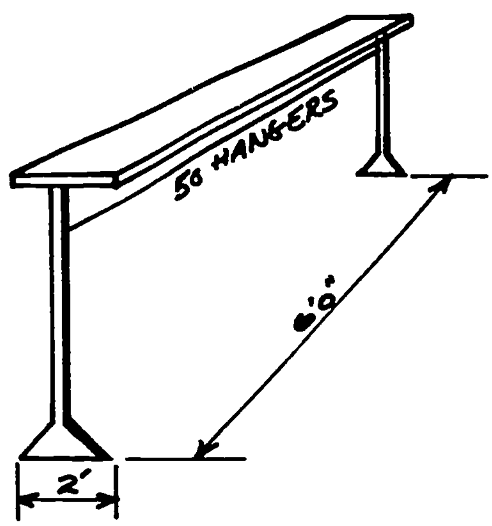
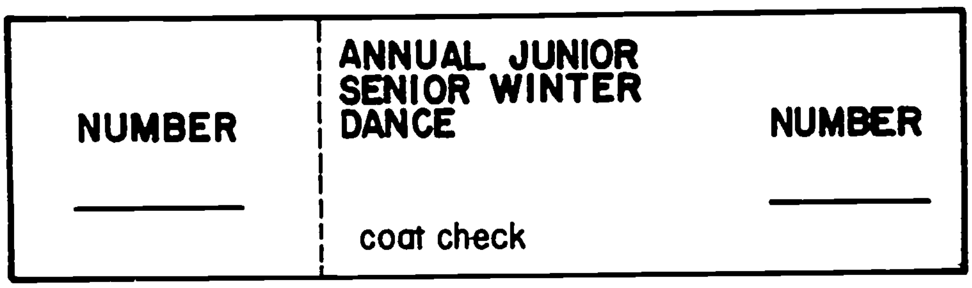


Figure 6

Help Scott think of a smooth way to check coats in and out of the coatroom.

1. Place the coatracks in Mr. Rolfe's room. Use your own design. Show your answer by drawing the coatracks, on the picture in Figure 5.
2. The numbering used on the coatchecks should be part of your coordinate system for locating the coats. Show how you would label the coatchecks. Answer by labeling a sample ticket below.



3. Mark an "x" on your design in Figure 5 to show where the coat is located that goes with your answer to Exercise 2.
4. Compare your coordinate system with others in your class. Can there be more than one answer?

Part 2 Seating Chart Numbers

(Your teacher will handle this part of the lesson using an overhead transparency of the seating chart for your class.)

EXERCISES

Sandra	David	Al	Judy M.	Fred	Harry
John		June	Phil	Mark	Lynn
Mike B.	Donna	Steve	Diane		Lonnie
Evelyn	Bill		Dan	Judy R.	Roy
Martha	Carol	Debbie	Jean	Terry	Bob
Peter	Mary	Stu	Phyllis	Ginny	Mike W.

The following exercises use the seating chart shown above.

1. Jean's seating chart coordinates are (3,1). Her row coordinate is 3. One is her _?_ coordinate.
2. Peter is sitting at the <u>origin</u> of the seating chart coordinate system. What are his coordinates?
3. Everyone sitting on the <u>seat axis</u> has what number for their row coordinate?
4. Everyone sitting on the <u>row axis</u> has what number for their seat coordinate?
5. Give Steve's coordinates.
6. Who is sitting at (3,2)?
7. Do the coordinates (3,2) locate the same position as the coordinates (2,3)?

8. We use the symbol (R,S) for the coordinates of a place on the seating chart. R is a number along the <u>row axis</u> and S is a number along the <u>seat axis</u>.

a. Give the names of those students sitting where R=5.

(5,0) ____________ (5,3) ____________

(5,1) ____________ (5,4) ____________

(5,2) ____________ (5,5) ____________

b. Give the names of those students sitting where R=S.

____________ ____________

____________ ____________

____________ ____________

c. Give the names of those students sitting where $R + S = 4$.

____________ ____________ ____________

____________ ____________

d. Give the names of those students sitting where $R + S = 7$.

____________ ____________ ____________

____________ ____________

THE GLOBAL COORDINATE SYSTEM

A ship in distress out in the Atlantic Ocean radios its position to potential rescuers.

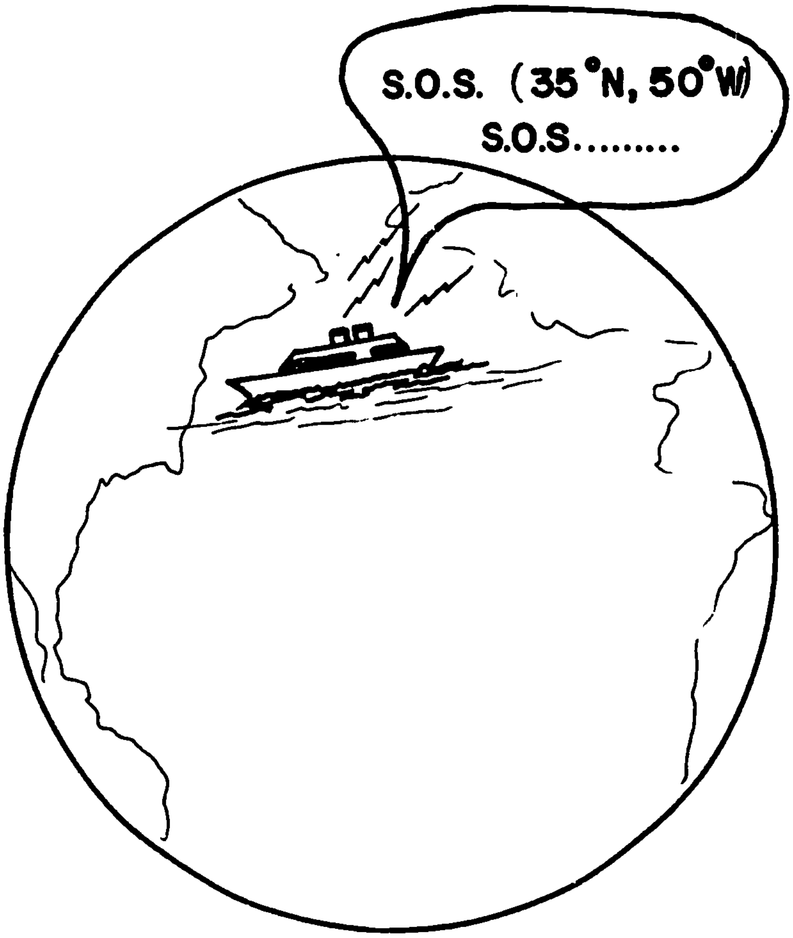


Figure 1

How will the rescuing ships or planes use the location information?

Lines on the globe form a sort of grid. They help us locate the position of an object anywhere on the surface of the earth.

The lines running from the North Pole to the South Pole are called lines of longitude. Lines running around the globe, parallel to the Equator, are called lines of latitude.

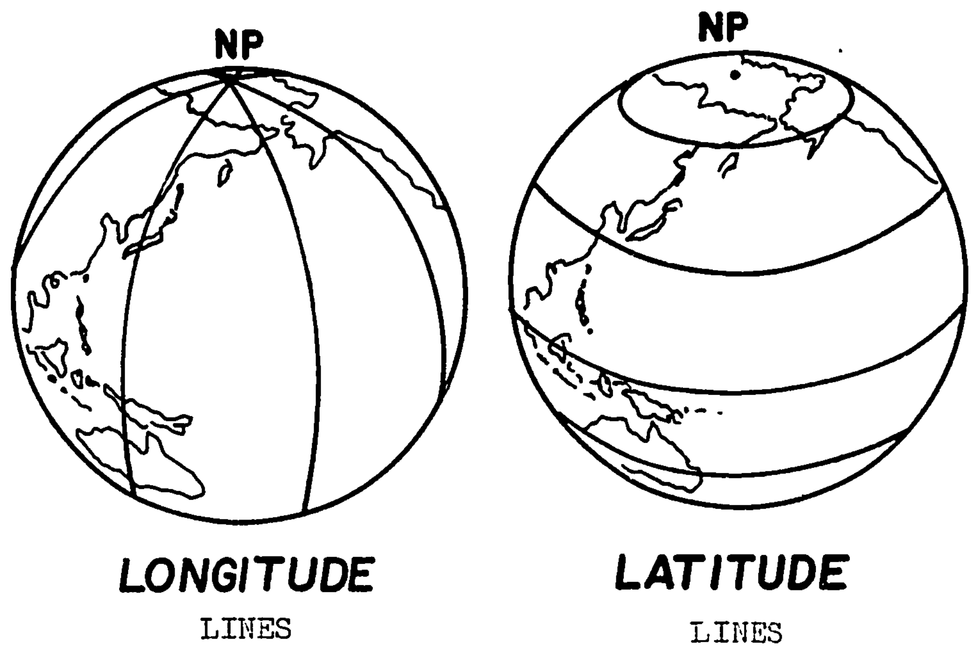


Figure 2

Latitude is measured, in degrees, north and south from the Equator. The Equator is 0° latitude. The Equator divides the globe into two halves, north latitude and south latitude.

To see an angle of latitude you have to imagine the globe with a slice taken out.

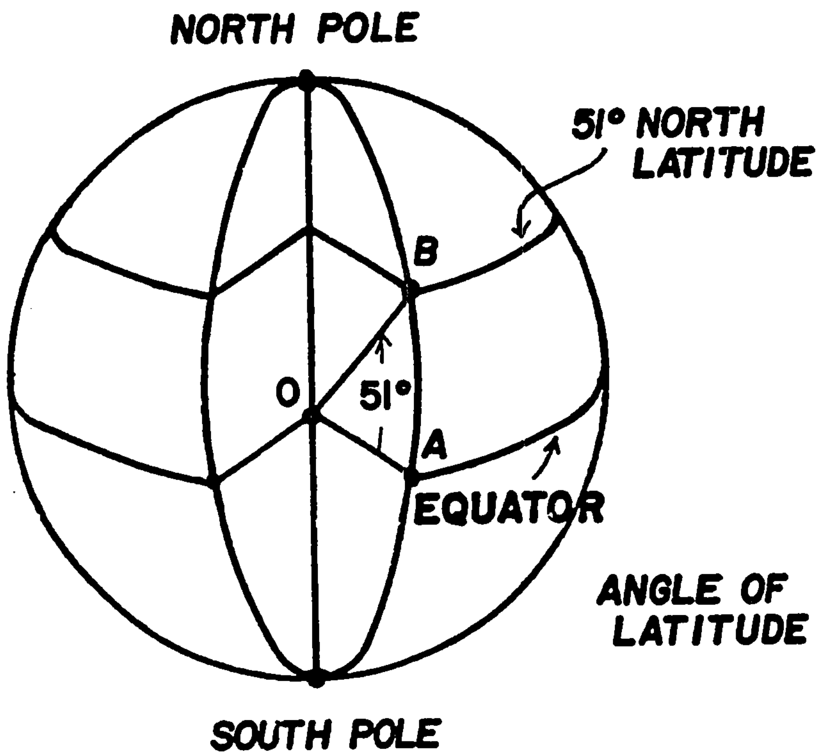


Figure 3

Point A, in Figure 3, is on the Equator. Point O is the center of the Earth. The measure of angle AOB is 51°. Every point on the 51° north latitude circle is 51° above the Equator.

The North Pole is at 90° north latitude. Where is the South Pole located?

The <u>longitude lines</u> are called <u>great circles</u> because the center of each circle of longitude is also the center of the Earth.

Great circles are important because the shortest distance between two points on the globe is along an arc of a great circle.

If you slice the globe along any great circle, you will cut the globe in half.

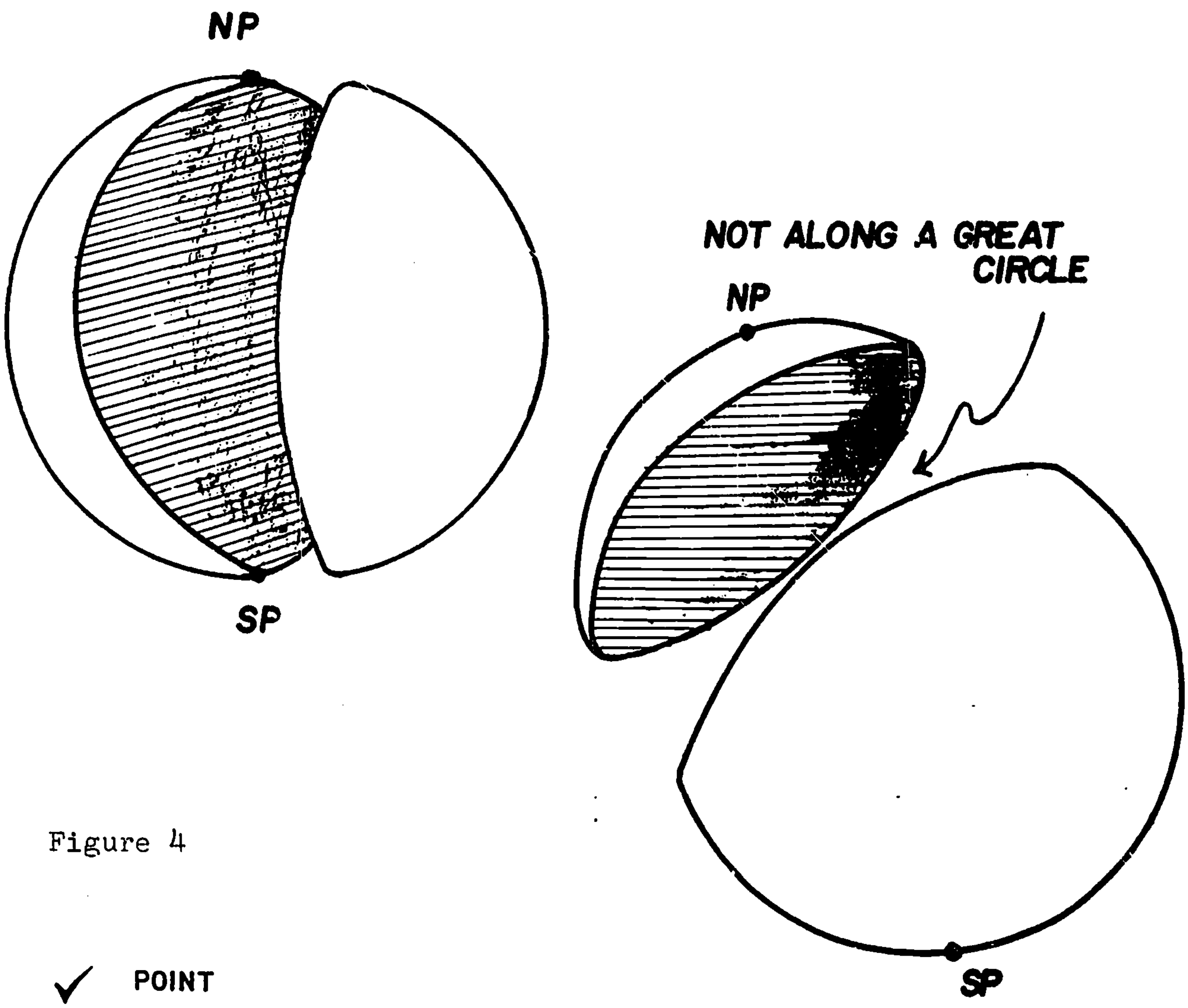


Figure 4

✓ **POINT**

1. There is only one line of latitude which is a great circle. What is its name?
2. Which lines are parallel to the Equator?
3. The latitude of the Equator is ___?___ degrees.

An important line of longitude runs through Greenwich, England. Greenwich is just outside London.

Actually, this important circle of longitude has two parts. The half circle that runs through Greenwich is often called the Prime Meridian. The other half of this circle is called the International Dateline.

This circle made up of the Prime Meridian and the International Dateline divides the globe into two halves, east longitude and west longitude.

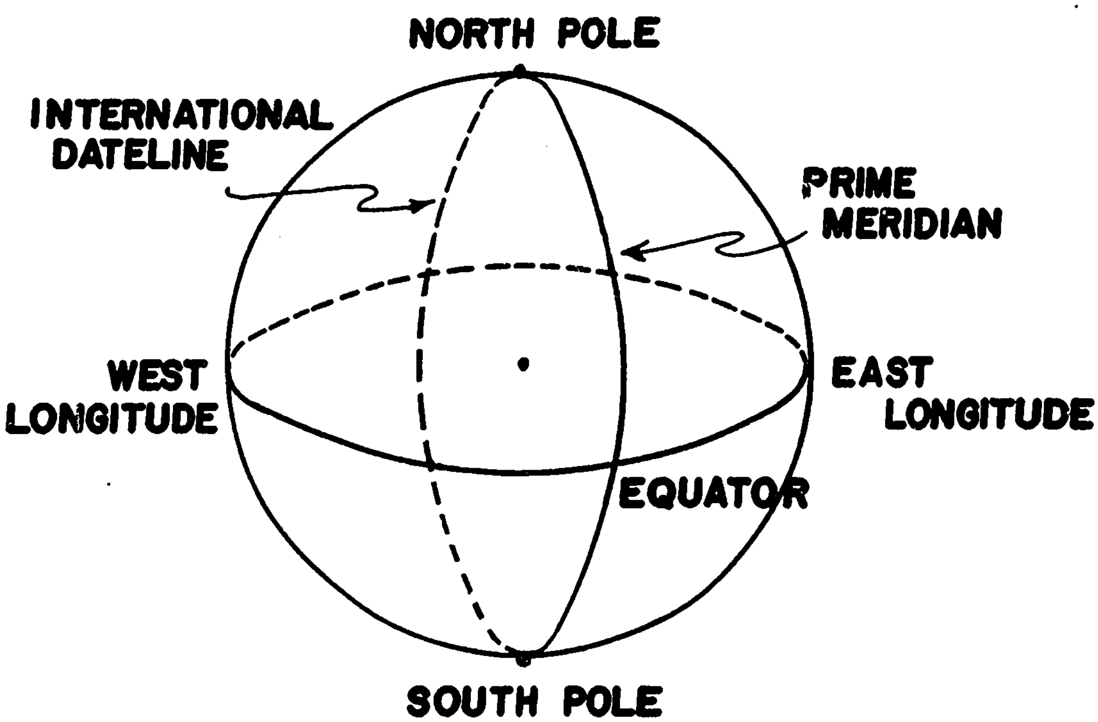


Figure 5

Each line of longitude "cuts" the Equator. Beginning with the Prime Meridian, lines of longitude are numbered from 0° to 180° in both the east and west directions.

The Prime Meridian is 0° longitude. The International Dateline is 180° longitude.

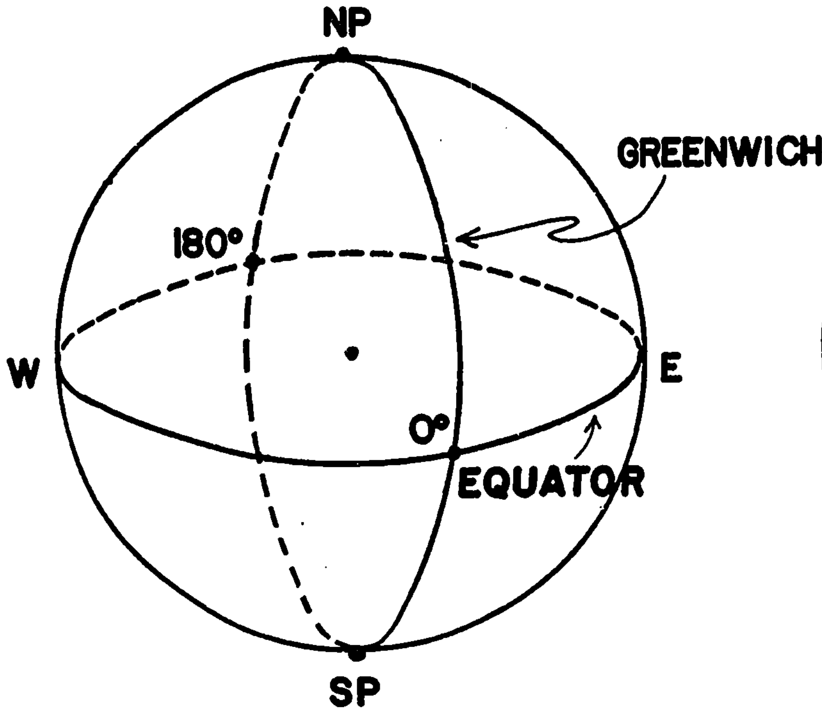


Figure 6

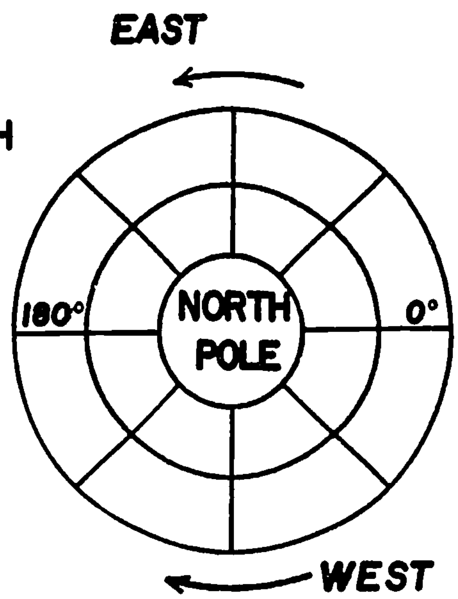


Figure 7

To see an angle of longitude you must again imagine the globe with a slice taken out.

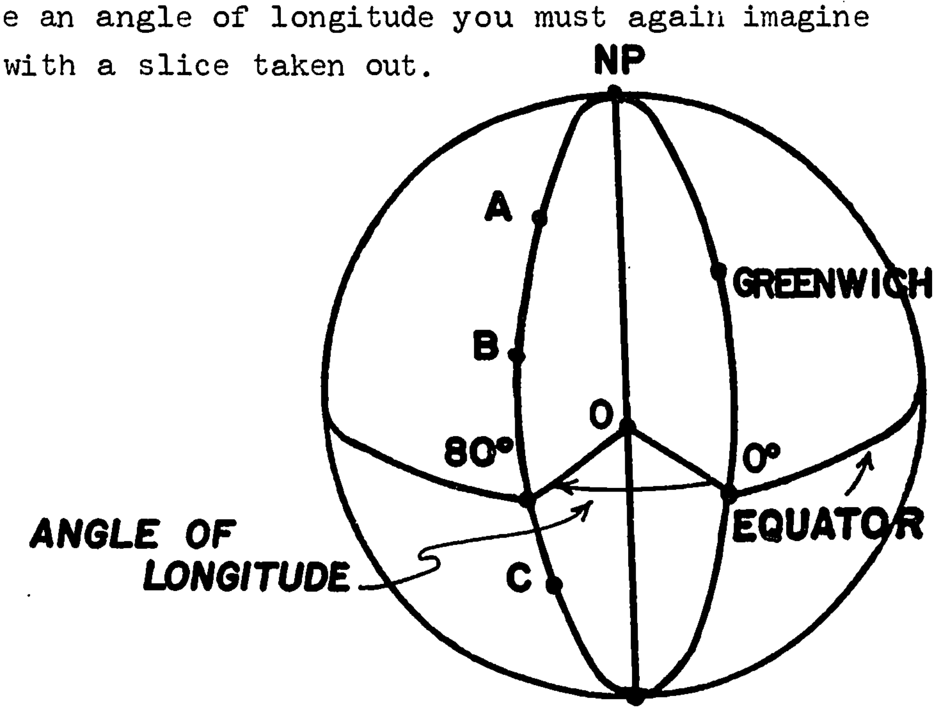


Figure 8

Points A, B, and C in Figure 8 all lie on the 80° west longitude line.

Suppose you were to travel around the Earth on a circle which passes through both Poles.

Using Figure 8, to be specific, suppose you start your trip at the South Pole and travel north along the 80° west longitude line.

As you travel along this arc, passing through points C, B and A you are always 80° west of the Prime Meridian.

However, when you cross over the North Pole heading south you are on a different longitude line. How many degrees east of the Prime Meridian are you now?

✓ **POINT**

1. The circle formed by the Prime Meridian and the International Dateline divides the globe into two halves, ? longitude and ? longitude.
2. Longitude is measured in degrees east and west from the ________ ________.
3. All lines of longitude pass through which points on the globe?

Latitude and longitude lines form a set of geographic coordinates. Any position on Earth can be located by giving two angle measures: The number of degrees latitude (stating whether north or south) and the number of degrees longitude (stating whether east or west). By agreement, latitude is always given first.

Two great circles act as the reference lines or axes of the geographic coordinate system.

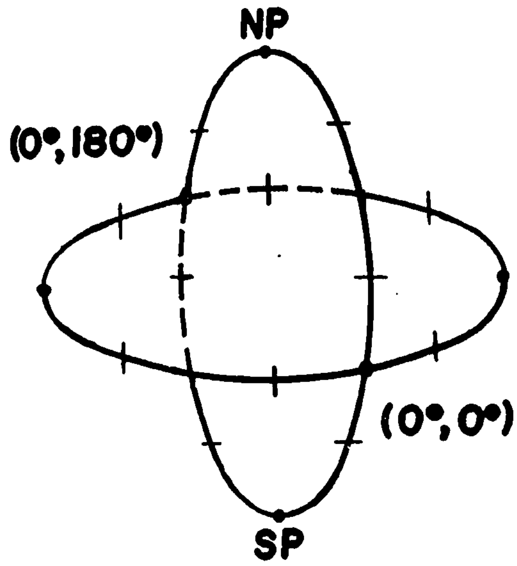


Figure 9

The point where the Prime Meridian and the Equator intersect is called the origin of the global coordinate system.

Degrees of latitude are marked off on the Greenwich Line. starting at the origin and proceeding to the North Pole or to the South Pole.

Degrees of longitude are marked off on the Equator starting at the origin and proceeding east or west.

EXAMPLE:

Mexico City is located approximately at (19° N, 99° W). Find this on your globe.

Geographic coordinates for the two Poles need a special agreement. Since all lines of longitude pass through both the North and South Poles we have to agree on what to use for their second coordinate.

We will agree to use 0° for the longitude coordinate of the poles.

	Geographic Coordinates
Scuth Pole	$(90^{\circ}$ S, $0^{\circ})$
North Pole	$(90^{\circ}$ N, $0^{\circ})$

✓ **POINT**

1. What are the coordinates of the origin? Are the abbreviations N, S, E or W necessary?
2. What are the coordinates of the point where the International Dateline and the Equator intersect?
3. Explain how the Equator and the circle formed by the Prime Meridian and the International Dateline are like the axes used in Lessons 1 and 2.

EXERCISES

In which half do the following countries lie?

COUNTRY	EAST OR WEST LONGITUDE
1. Brazil	____________
2. Japan	____________
3. United States	____________
4. Australia	____________
5. France	____________

COUNTRY	NORTH OR SOUTH LATITUDE
6. Canada	____________
7. India	____________
8. Australia	____________
9. Argentina	____________
10. Sweden	____________

11. Which line of latitude is a great circle?

12. Navigators talk about traveling the "great circle route". What does that mean?

13. The 0° longitude line is called the ______________ line.

14. Can the circumference of the earth be measured along any circle of latitude?

15-18. Give the coordinates (to the nearest degree) of each city given below.

CITY	LOCATION
15. Paris, France	__________
16. New York City	__________
17. Pontiac, Michigan	__________
18. Greenwich, England	__________

19. The center of which country is located approximately at (25° S, 135° E)?

20. Find the origin on the globe. Describe its geographic location.

21. Imagine a line running from a point on the globe, through the center of the globe and exiting on the "opposite" side of the globe.

    Give the coordinates of the point located on the "opposite" side of the globe from the given point.

    a. (0°, 50° W) ... __________

    b. (30° N, 0°) ... __________

    c. (45° N, 180°)... __________

    d. (30° N, 80°W) .. __________

22. Find the locations of Boulder, Colorado and Philadelphia, Pennsylvania. Subtract their longitude coordinates to obtain the number of degrees longitude between them.

POWER QUESTIONS

1. Each rectangular section on the coordinate system used on the map (See Lesson 1) was the same size. Explain why this is not so on the globe.

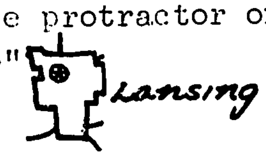
2. A hunter set out from his camp one day. He walked one mile south and then two miles east. He then shot a bear and towed it one mile north and arrived at his camp. Where was the hunter's camp?

FOR YOU TO DO

Look up the coordinates of the geographic center of the United States.

RULER AND PROTRACTOR COORDINATES

Part I The Origin at Lansing

Fix the full circle protractor on the map so that the center is on the "star" 

and the 0-180 degree line runs north-south. See the picture below.

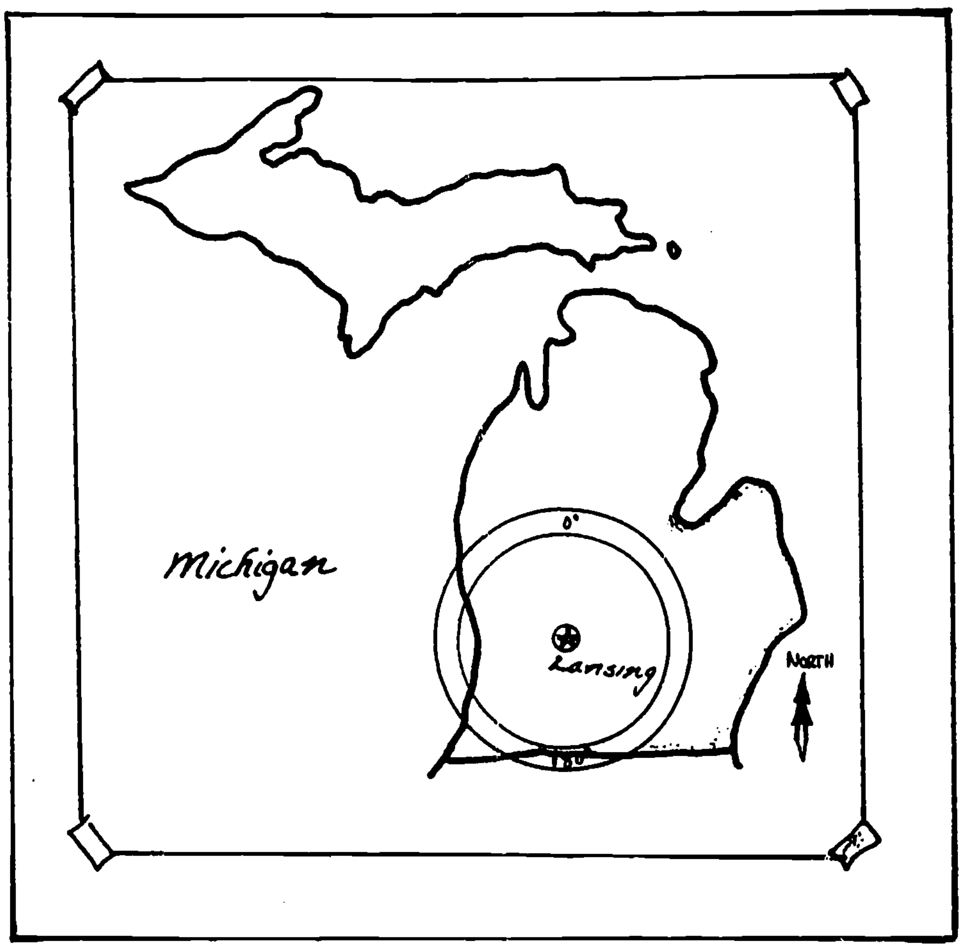


Figure 1

U.S. Highway 27 runs north-south out of Lansing, just below St. Johns. You can use it to line up your protractor.

The direction of an object on the map is obtained by measuring the angle between the north line and the line joining the center of the protractor to the object.

The direction angle is measured in degrees from north in a clockwise direction. This angle is called the bearing of the object from the center of the protractor. Thus a bearing of 90° would be due East.

A bearing and a distance can be used to locate any place on the map. When we give the location of a place on the map we give its coordinates.

(distance, bearing)

By agreement, the distance is listed first.

EXAMPLE: The coordinates of Cadillac are $(5\frac{7}{8}'', 338^{\circ})$.

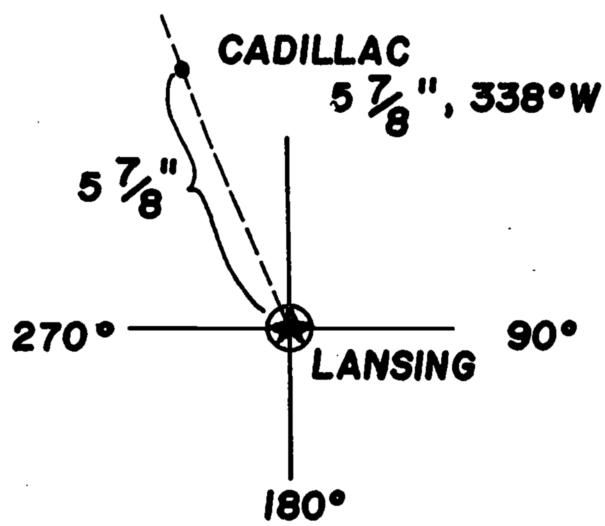


Figure 2

The distance coordinate for Lansing is zero. What is the bearing for Lansing?

Since there is no angle between Lansing and itself we will agree to call the bearing of Lansing 0°.

✓ POINT

Explain

1. Where the origin of this coordinate system is.
2. What are the coordinates of the origin?

EXERCISES

1. Give the name of the city located by the given coordinates. (The measurements have been rounded off to the nearest $\frac{1}{8}$ inch and nearest 1 degree.)

COORDINATES	CITY
a. $(5\frac{5}{8}'', 80^\circ)$	a. ________
b. $(4'', 225^\circ)$	b. ________
c. $(10\frac{3}{8}'', 2^\circ)$	c. ________
d. $(1\frac{6}{8}'', 167^\circ)$	d. ________
e. $(0'', 0^\circ)$	e. ________
f. $(15\frac{4}{8}'', 332^\circ)$	f. ________

2. Determine the coordinates of the following cities. Measure to the center of the circle used to indicate cities on the map.

CITY	COORDINATES
a. Muskegon	a. ________
b. Midland	b. ________
c. Kalamazoo	c. ________

d. Traverse City d. ____________

e. Pontiac e. ____________

f. Lansing f. ____________

3. What is the distance coordinate of Wyandotte?

4. What is the angle coordinate for Detroit?

5. Could the order of the coordinates be changed? That is, could the angle measure be the first coordinate? Why?

Use the map of Michigan on the following page to do these exercises.

6. Place the center of your protractor on the dot labeled O. Point 0° north. Plot the following points.

A. $(1\frac{7}{8}'', 208^{\circ})$ C. $(1\frac{3}{4}'', 333^{\circ})$

B. $(1\frac{3}{8}'', 130^{\circ})$ D. $(3'', 348^{\circ})$

7. Give the names of the cities that you think are located at points A, B, C, D and O.

8. What are the coordinates of point O?

33/34

Part 2 The Tornado Plotting Map

A distance-angle coordinate system is used by the weather bureau to keep track of tornados or other severe weather.

The circles shown on the map all have a common center (they are concentric circles) located at the Detroit radar equipment site. This is the origin of this coordinate system.

The circles are called range marks because they help determine the distance from the origin to some point on the map.

The 100 mile (nautical miles are used) range mark is divided up like a protractor. This helps determine the bearing of a point on the map.

Notice that lines of latitude and longitude are also shown on the Tornado Plotting Map.

When a tornado is reported, its range (distance from the origin) and bearing are used to give its coordinates.

EXAMPLE: A tornado is spotted near London, Ontario (top right section on the map). Its range is approximately 109 miles and its bearing is approximately 63°.

Thus, its coordinates are (103 mi., 63°).

EXERCISES (All questions refer to the Tornado Plotting Map)

1. Determine the range (approximately) of each of the following cities:

   a. Port Huron
   b. Pontiac
   c. Lansing
   d. Cleveland, Ohio
   e. Fort Wayne, Indiana
   f. Canton, Ohio

2. Determine the bearing (approximately) of each of the cities, a through f in Exercise 1.

3. A tornado is spotted at (100 mi., 130°) heading north. What large city is in danger?

4. A thunderstorm is reported at Lansing. What are its coordinates (approximately)?

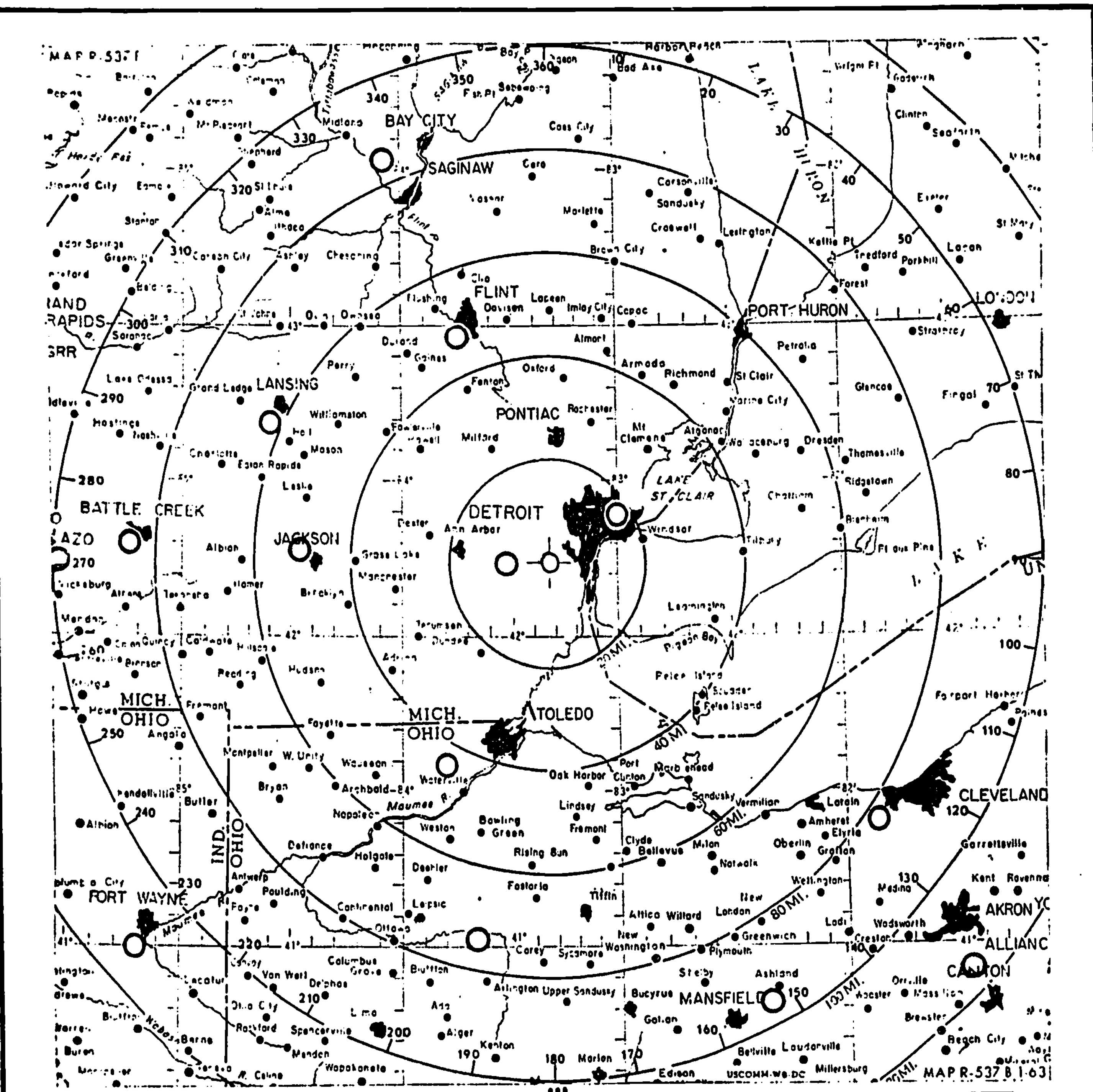
5. What are the coordinates of Pontiac?

6. What city is located at (100 mi., 175°)? In what state?

7. The geographic coordinates of St. Johns, Mich. are (43° N, 84° 33' W). Find this point on your map and give its range and bearing.

8. What city is located at (41° 5' N, 85° 8' W)?

9. Perry, Mich. is at (54 mi., 313°) and Edmore, Mich. is at (104 mi., 313°). How far, in nautical miles, is Edmore from Perry?



TORNADO PLOTTING MAP — An actual work sheet used by the weather bureau radar meteorologist to track severe weather, this map has as its center the location of the Detroit radar equipment. The map provides those listening to the radio or television a quick reference when severe weather occurs, giving both distance in nautical miles and direction in degrees from Detroit. The prominent range marks are at 20-mile intervals.

SECRET CODE TABLE

Each ordered pair of numbers will lead you to a letter of the alphabet. Use the Secret Code Table.

Secret Code Table

4	J	S	E	P	H
3	U	C	O	B	Y
2	I	L	F	X	N
1	G	R	V	D	W
0	A	M	Q	T	K
	0	1	2	3	4

1. Decode the following secret message:

   (4,3) (2,3) (0,3) – (0,0) (1,1) (2,4) – (1,3) (1,2) (2,4) (2,1) (2,4) (1,1).

2. Which letter of the alphabet is not in the table? Think of a pair of coordinates that could be used to code this letter.

3. Which letter is at the <u>origin</u> in the Secret Code Table?

4. A message can be coded by altering the coordinates.

WORD	COORDINATES	CODE	CODED WORD
C	(1,3)	(1,2)	L
O	(2,3)	(2,2)	F
P	(3,4)	(3,3)	B
Y	(4,3)	(4,2)	N

What was done to change the coordinates in Column 2 to the coordinates in Column 3?

5. Another way to code a word is to exchange the first and second coordinates.

EXAMPLE: (3,1) becomes (1,3). Give the coded word for the word **MAN**.

6. Something has been done to make this next message harder to decode. You will have to think a little before you can spell out this secret message:

$(3,2)\ (1,2)\ (^-1,2) - (0,3)\ (3,3)\ (1,2)\ (^-1,2)\ (0,1)$
$(2,0) - (^-1,0)\ (1,3)\ (2,^-1) - (^-1,^-1)\ (3,1) - (^-1,^-1) -$
$(2,^-1)\ (1,2)\ (2,0)\ (^-1,^-1)\ (3,2)$.

7. Use the Secret Code Table to code your first name. Do something to the coordinates to make your code hard (but not impossible) to figure out. Give your coded name to a friend and see if he can break the code.

SPACE COORDINATES

Julie Austin was watching a fly buzz around the classroom. She wondered if there was a way to keep track of the fly's position using mathematics. As the fly flew near a corner of the room, Julie thought of using space coordinates.

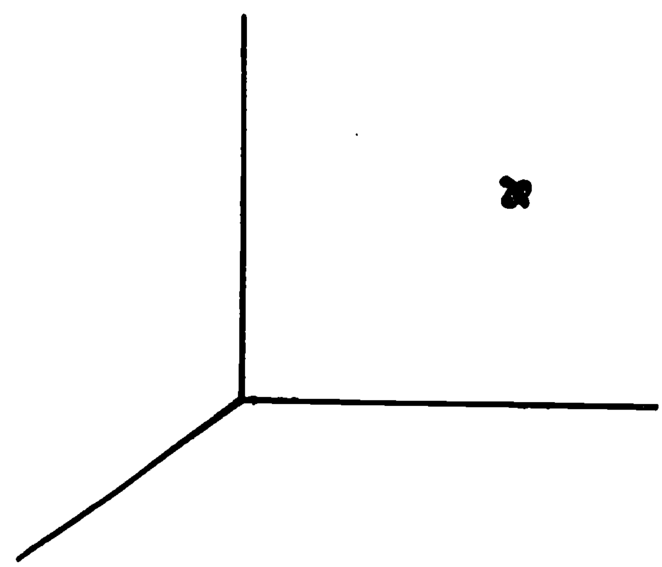


Figure 1

The lines where the walls and floor meet are perpendicular to each other. These lines could be thought of as axes. The walls and the floor can be thought of as planes. Each pair of axes lie in a plane.

With a way of counting off distance on each axis the position of the fly at any one instant in time could be determined.

In Figure 2, a scale has been placed on each axis. The units on each scale are the same. The axes are labeled x, y and z so that we can keep track of them. The point where the axes intersect is called the <u>origin</u>.

It is very difficult to draw the axes for space coordinates on paper. That y-axis is really supposed to be sticking out of the paper right at you.

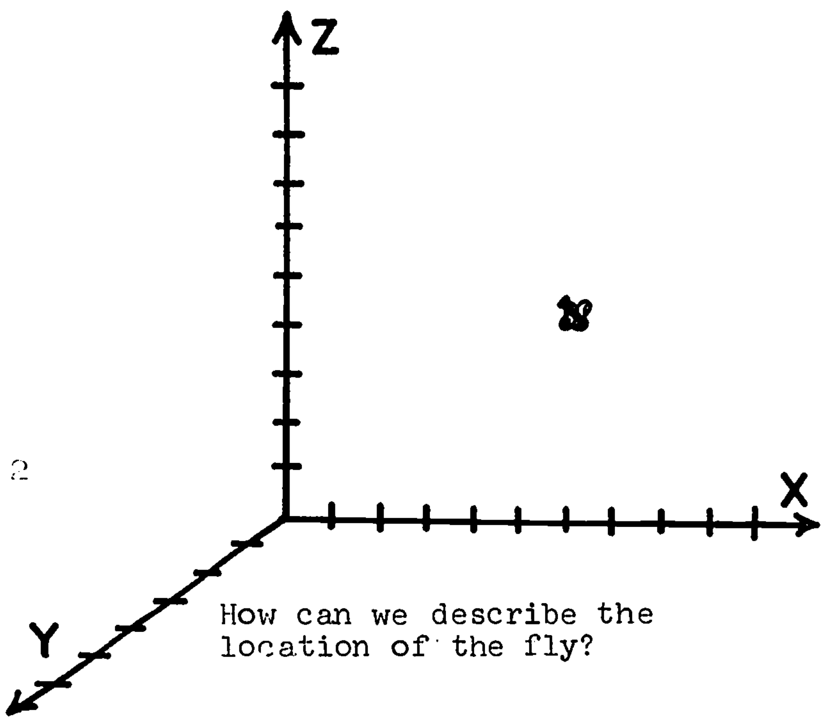


Figure 2

The fly is located at a certain distance from each plane. To describe his coordinates we must determine these distances.

Recall that two intersecting lines determine a plane. In describing the fly's coordinates we shall refer to the xy plane, the yz plane, and xz plane. Look at a corner in your classroom to visualize these three planes.

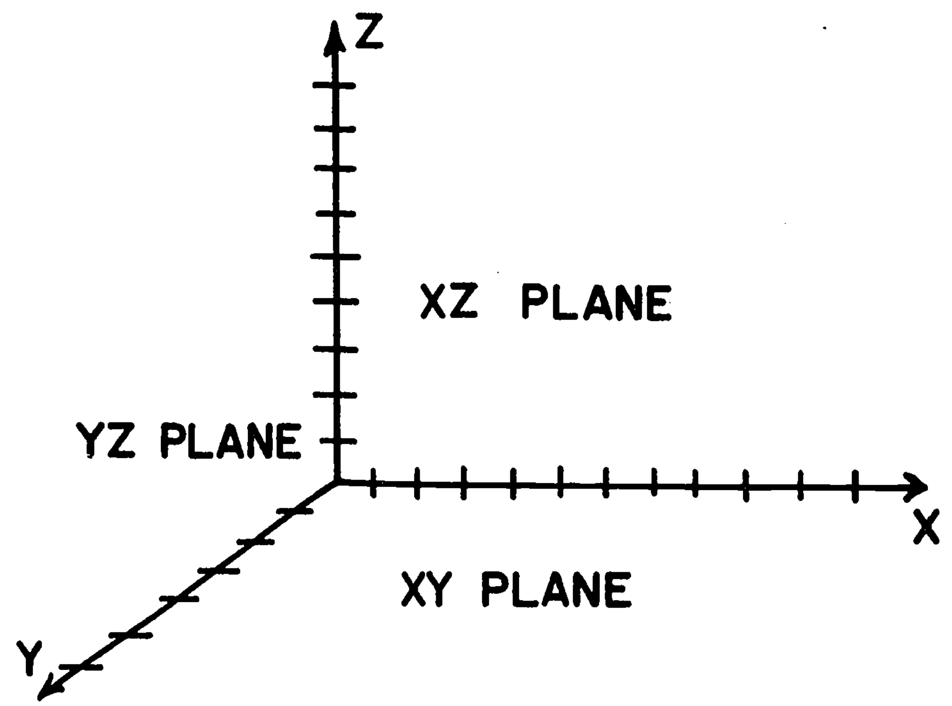


Figure 3

In Figure 4, the fly is 7 units from the xy plane. This distance is counted along the z-axis.

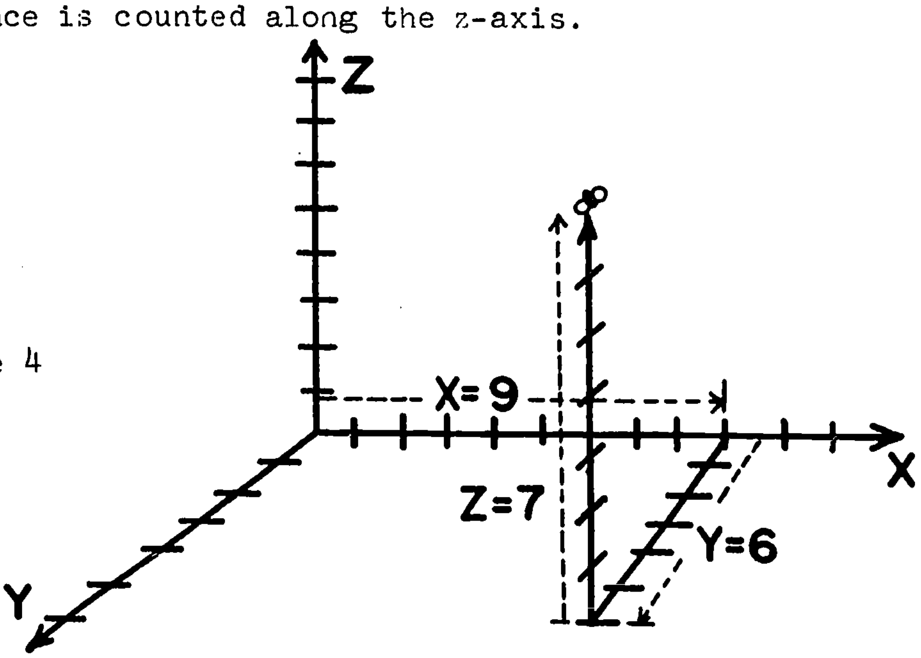


Figure 4

The coordinates of the fly's position at the instant shown in Figure 4 are determined by three numbers. The symbol (x, y, z) is used to give the space coordinates of a point.

✓ POINT

1. The symbol (x,y,z) is called an ordered triple. Explain why.

2. Explain what the number used for y tells us in the ordered triple (x,y,z). Explain what z tells us.

3. Which two axes lie in the xz plane?

EXERCISES

1. Give the coordinates of the fly's position as shown in Figure 4.

2. Which plane represents the floor, the xy plane, the yz plane, or the xz plane?

3. In the ordered triple (x,y,z), the number for y gives the distance of the point from which plane?

4. All points in the xy plane have a value of _?_ for the z coordinate.

5. The point (5,18,3) is located how many units from the xy plane?

6. What are the coordinates of the origin?

7. The point (2,4,0) lies in which plane?

8. The point (5,3,1) is located how many units from the xz plane?

9. The point (a,b,c,) lies in the xy plane. What is the value of c?

10. The point (r,s,t) is located 5 units from the xy plane, 7 units from the xz plane, and 4 units from the yz plane. Give the values for r, s, and t.

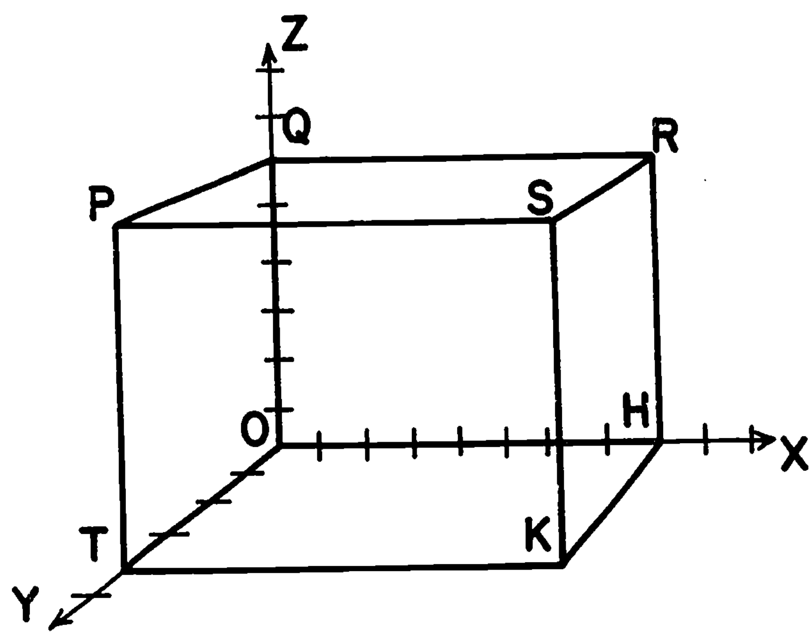


Figure 5

11. Give the coordinates of the following points.

T ________	O ________
P ________	H ________
Q ________	K ________

12. There is one coordinate which is the same for all of the points T, P, Q and O. Which coordinate has the same value for all of these points?

13. The __?__ coordinate of the points T, K, and H, has the same value for each point.

14. The x-coordinate of the points R, S, K and H will have the same value for each point. What will x equal for these points?

15. The z coordinate will have a value of 6 for which vertices in Figure 5, page 44?

16. The side of the box, in Figure 5, labeled OTKH is in the xy plane. Which coordinate, x, y or z is the same for all points in this plane?

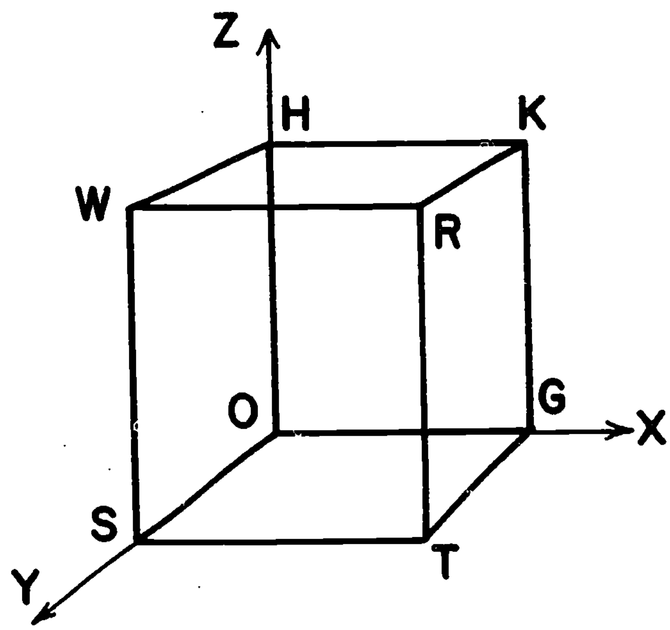


Figure 6

17. The box shown in Figure 6 is a cube.

a. Give the coordinates of each vertex. Use the length of each side of the cube as the unit distance on the scale.

O	________	T	________
H	________	G	________
W	________	K	________
S	________	R	________

b. There is a rectangular region determined by points O, H, R and T. What is true about the x and y coordinates of any point on this region?

POWER QUESTIONS

1. Stuart coordinatized the vertices of a cube using stick-on labels. Unfortunately he dropped the cube and all but three of the labels fell off. (HINT: Get a cube and label the vertices as shown in Figure 7.) Which vertex did Stuart use as the origin?

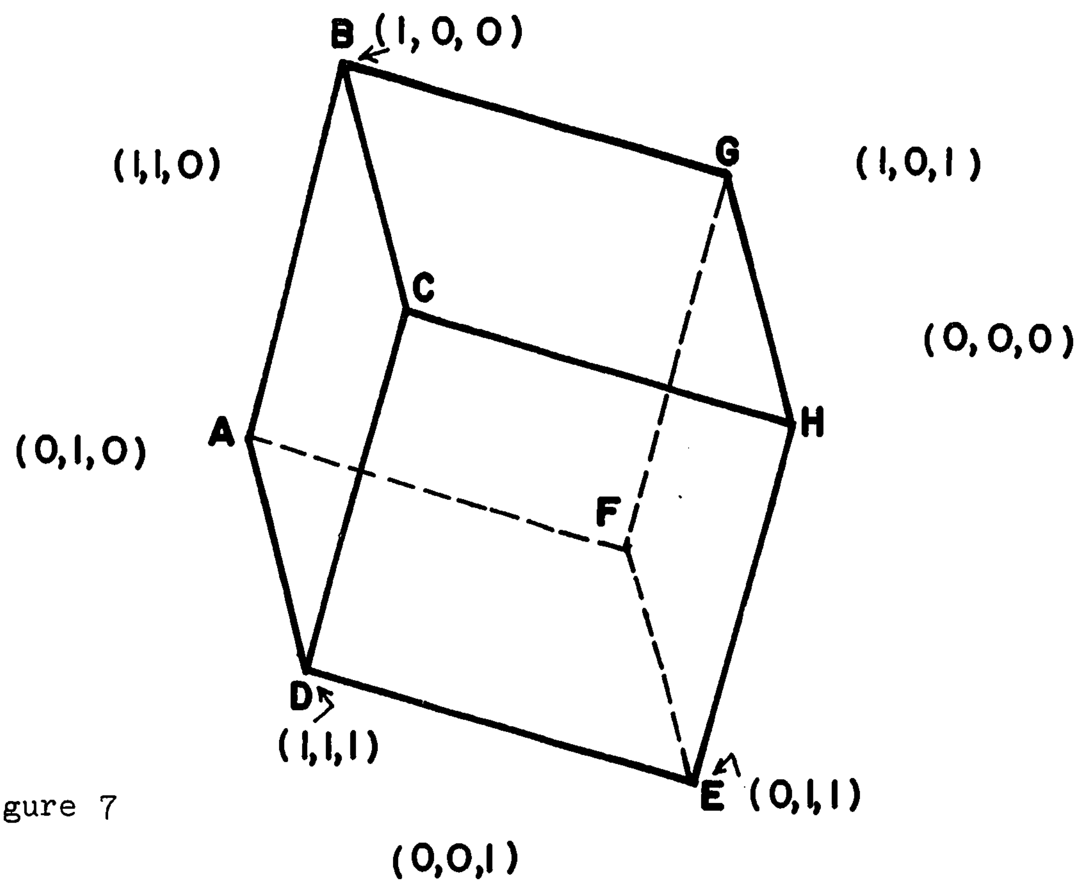


Figure 7

2. Explain where the point $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is in Stuart's cube.

3. Which segment (edge of the cube) lies along the x-axis?

4. What are the coordinates of vertex F?

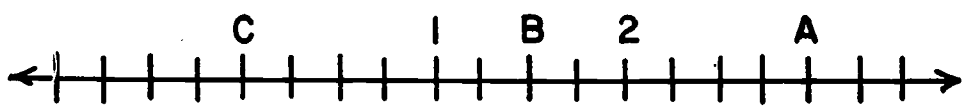
A PUZZLING LINE

In an investigation, a detective only needs to find one or two clues to solve a mystery.

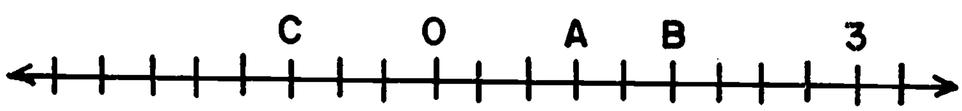
Use the clues given in the following questions to solve the puzzles on the number line.

EXERCISES

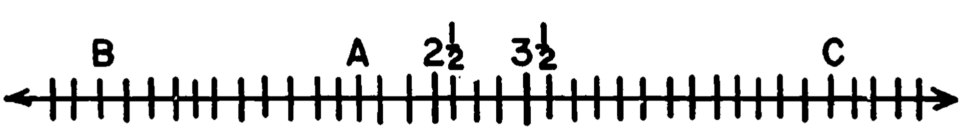
1. Fill in the missing numbers.

a) 

A = ______ B = ______ C = ______

b) 

A = ______ B = ______ C = ______

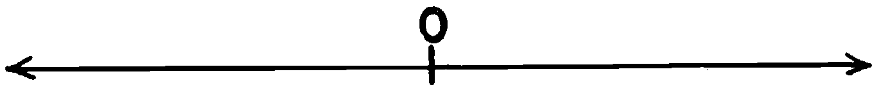
c) 

A = ______ B = ______ C = ______

2. Positive numbers are greater than zero. Thus if, b is a positive number, this can be written as

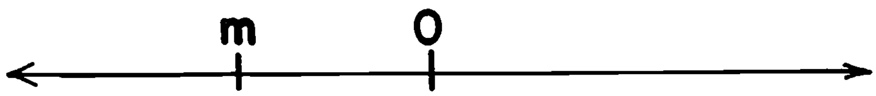
$$b > 0.$$

On which side of zero, on the number line, is b located if $b > 0$?



(answer by placing b on the correct side of zero)

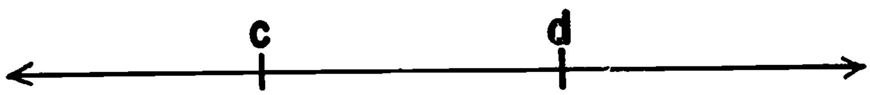
3. The number m is shown on the number line.



Is m a positive or negative number?

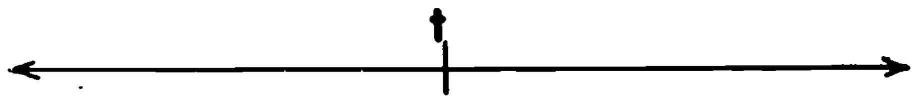
4. In question 3, what about the numbers between m and zero? Are they all positive; all negative; or some of each?

5. The numbers c and d are shown on the number line.



c is a negative number, d is a positive number.
Make a mark on the number line where you think the number zero might be located.

6. The number t is shown on the number line.



s is a number which is greater than t.

$$s > t$$

Place s on the number line so that it is on the correct side of t.

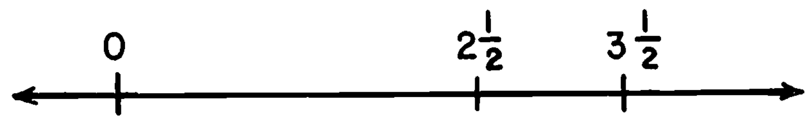
7. Do the statements about the numbers agree with their position on the number line?

Answer True, False, or Cannot tell.

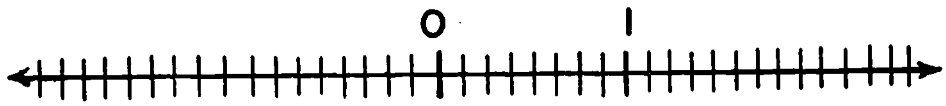
<u>Circle your answer</u>.

a)	$x > y$	T	F	?
b)	$x > 0$	T	F	?
c)	$z < x$	T	F	?
d)	x is twice as large as y.	T	F	?
e)	$y < 0$	T	F	?

8. Locate, by some means other than guessing, the number one on this number line.



9. Place each given number on the number line.

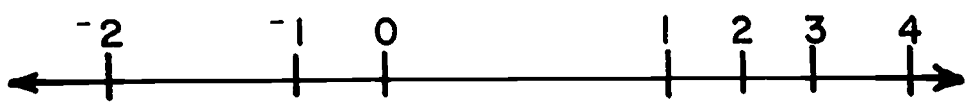


a) $1\frac{7}{8}$	c) $^{-}2$	e) .75	g) $^{-}2.25$
b) $2\frac{1}{2}$	d) $^{-}1\frac{1}{3}$	f) 1.6	h) $\frac{1}{8}$

10. I am thinking of a number. It is not negative and it is not positive. What number am I thinking about?

POWER QUESTIONS

1. Tell what is wrong with this number line.



2. Construct your own number line. Make room enough to label the line segment with the integers from $^{-}5$ to 5. This line segment will be your <u>unit distance</u>. You can

use a ruler or a compass to make the number line.

THE LATTICE PLANE

Choose a point near the center of your lattice plane. Call this point O for origin.

Draw a vertical and a horizontal line through point O.

These two lines are called <u>axes</u>. Label the vertical axis y and label the horizontal axis x. Most mathematics textbooks call these axes the x-axis and the y-axis.

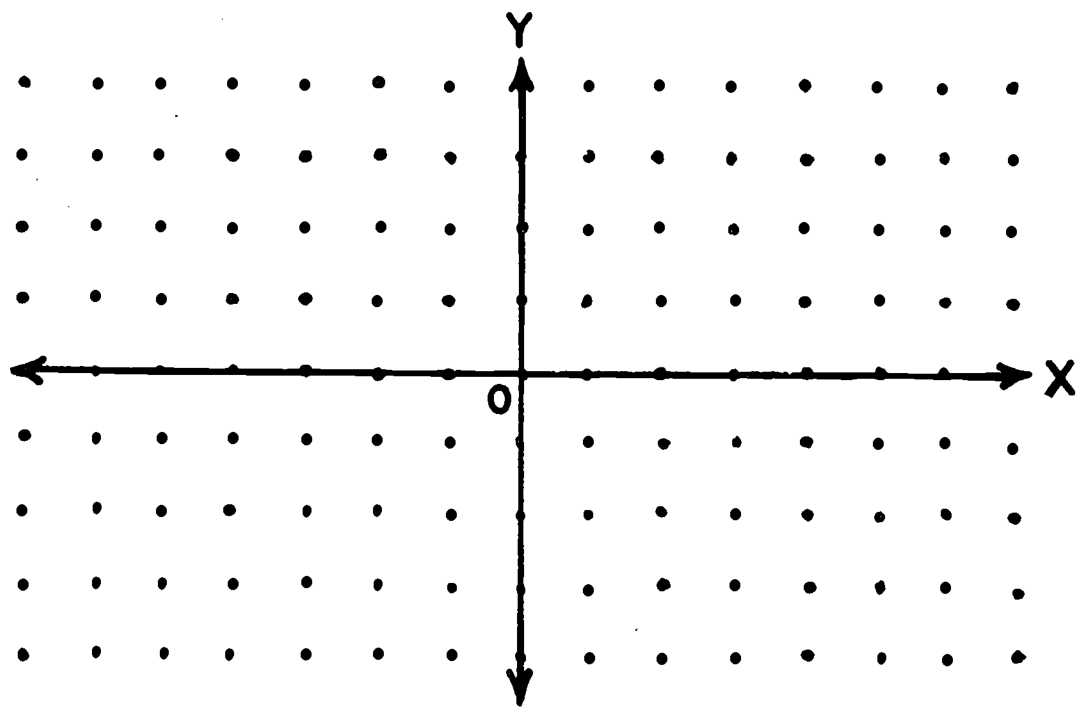


Figure 1

Label the lattice point, on the x-axis (on your paper) just to the right of the origin with a 1.

EXERCISES

1. Label the lattice point just to the left of the origin on the x-axis.

2. Label all of the lattice points on the x-axis with integers.

3. Label all of the lattice points on the y-axis with integers.

4. The coordinates of point A, figure 2, are (5,3). What are the coordinates of point B?

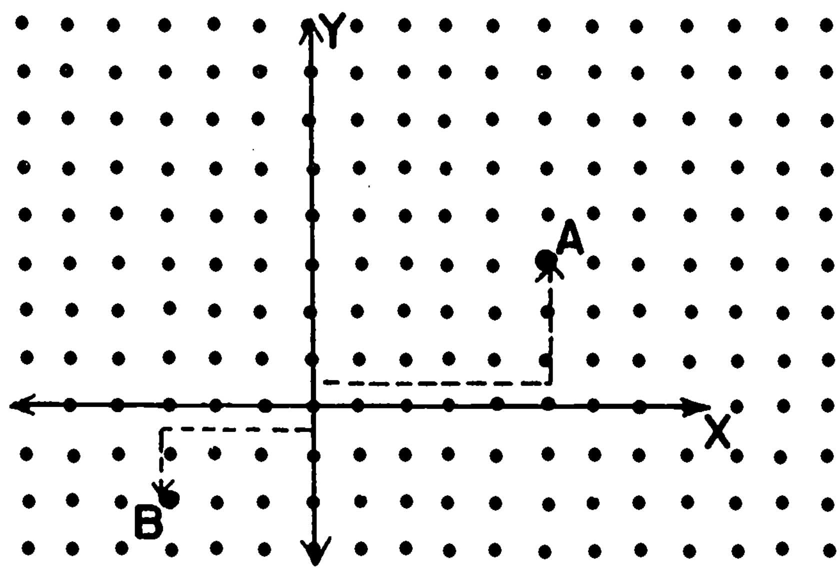


Figure 2

5. Start at the origin (on your paper) and follow these directions:

   (1) Count four lattice points to the right (the positive direction) on the x-axis. Mark this point. (Circle it lightly.)

   (2) Draw a vertical line through this point. The line should be parallel to the y-axis.

   (3) Starting at the origin again, count three lattice points up (the positive direction) on the y-axis. Mark this point also.

   (4) Draw a horizontal line through this point. The line should be parallel to the x-axis.

6. The two lines you drew should intersect at a lattice point. Give the coordinates of this lattice point.

Figure 3 shows some lattice points labeled A, B, C, D, E, F, G, H, I.

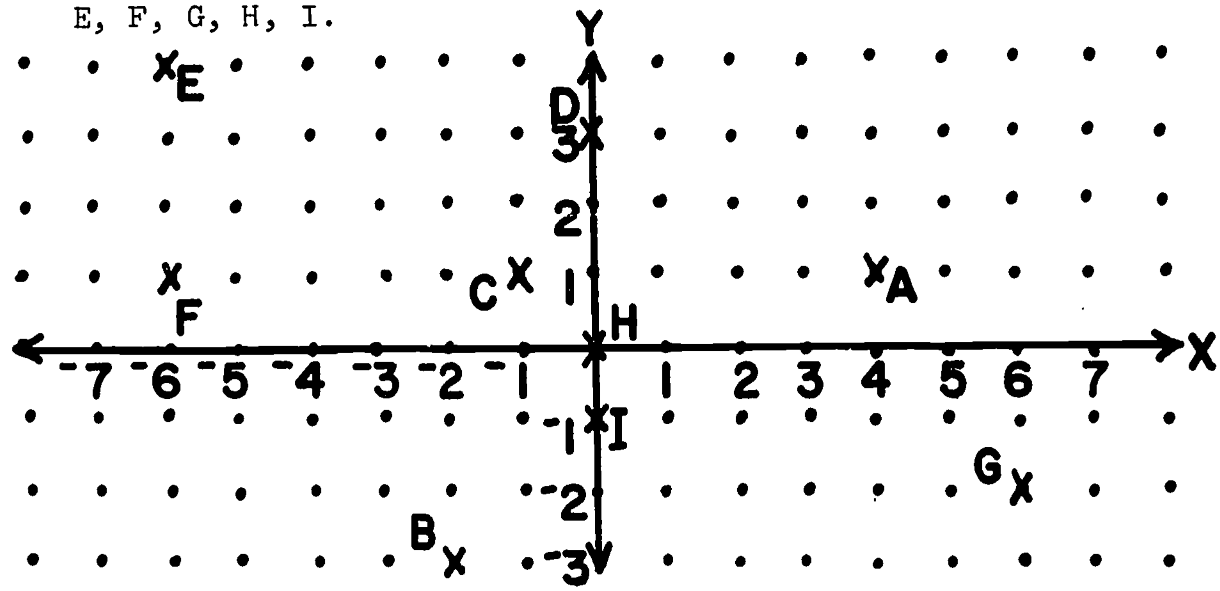


Figure 3

7. What is the x-coordinate of point A?

8. What is the y-coordinate of point B?

9. What are the coordinates of point E?

10. What are the coordinates of point G?

11. Some of the lettered lattice points have the same y-coordinate. Which ones are these?

12. List the lettered lattice points which have zero as their first coordinate.

13. Lattice points which have zero as their first coordinate are all on which axis?

14. The points whose coordinates are (1,0), (2,0), (3,0) (6,0), (⁻4,0), and (0,0) are all on which axis?

15. Lattice points which have zero as their second coordinate are all on which axis?

16. The points whose coordinates are (0,8), (0,$^-$2), (0,$^-$3), (0,2) and (0,0) are on which axis?

17. Give the coordinates of the point which lies on both axes. What is this point called?

18. Draw a line through those lattice points whose x-coordinate is 6. (Draw the line on your paper.)

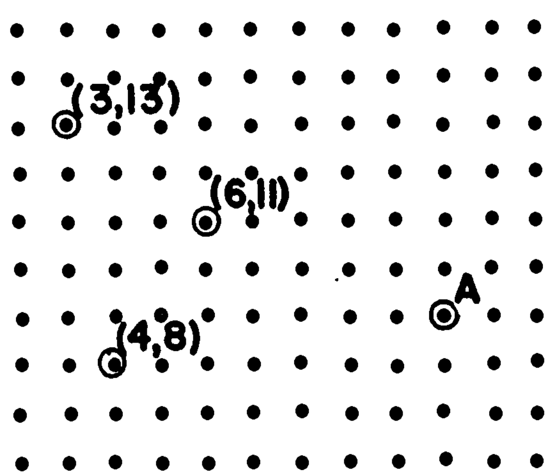
19. Draw a line through those lattice points whose y-coordinate is $^-$2.

20. The point of intersection of the lines is ____________. (Give the coordinates of this point.)

21. All of the lattice points which lie on the x-axis have __________ for their second coordinate.

22. Some lattice points have x and y-coordinates that are the same. Draw a line through these points.

23. Part of the lattice plane is shown below. Use the given information to determine the coordinates of point A.



POWER QUESTIONS

1. Several incomplete coordinates are given on the lattice plane below. Find the point which is the origin of the indicated rectangular coordinate system. Circle your answer.

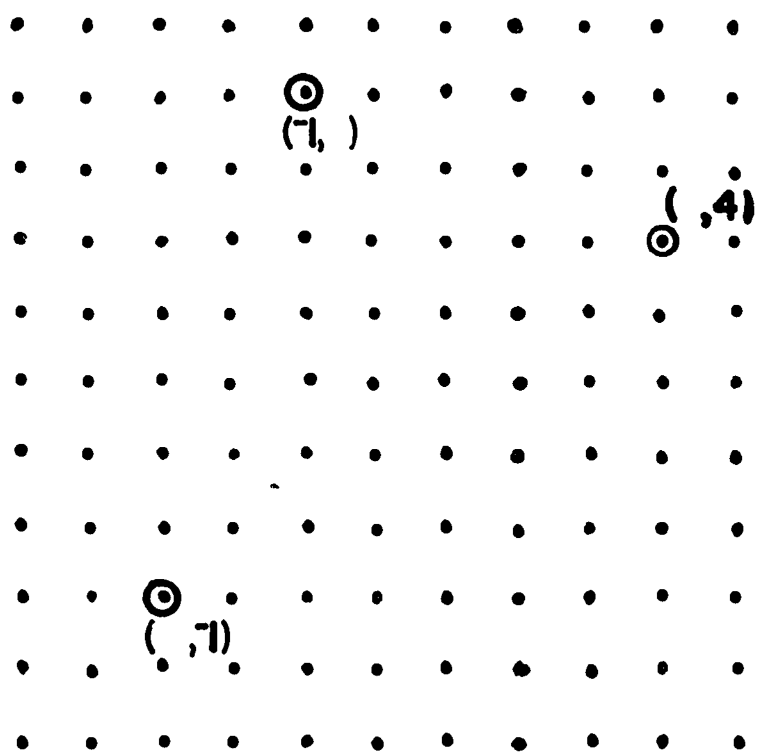


Figure 4

2. Draw an x-axis and a y-axis on a new lattice plane. Make the x-axis run the length of the paper. For each exercise plot the lattice points and connect them with a line in order.

   a) $(^{-}1, 5)$, $(^{-}1, 2)$, $(1, 2)$.

   b) $(^{-}9, 7)$, $(^{-}8, 4)$, $(^{-}7, 5)$, $(^{-}6, 4)$, $(^{-}5, 7)$.

   c) $(^{-}3, ^{-}3)$, $(^{-}4, ^{-}2)$, $(^{-}3, ^{-}1)$, $(^{-}2, ^{-}2)$, $(^{-}3, ^{-}3)$.

   d) $(6, ^{-}1)$, $(4, ^{-}1)$, $(4, ^{-}2)$, $(5, ^{-}2)$, $(4, ^{-}2)$, $(4, ^{-}3)$, $(6, ^{-}3)$

   e) $(^{-}2, 4)$, $(^{-}4, 4)$, $(^{-}4, 5)$, $(^{-}3, 5)$, $(^{-}4, 5)$, $(^{-}4, 6)$, $(^{-}2, 6)$

   f) $(^{-}6, ^{-}2)$, $(^{-}7, ^{-}1)$, $(^{-}8, ^{-}1)$, $(^{-}8, ^{-}3)$, $(^{-}7, ^{-}3)$, $(^{-}6, ^{-}2)$

   g) $(2, 3)$, $(2, 0)$, $(4, 0)$.

   h) There is a missing letter. Do you know what it should be? Describe the missing letter by giving the coordinates which could be used to draw this letter.

PICTURE PLOTTING

EXERCISES

1. In the first table of ordered pairs given in Exercise 2 below, the largest value of all of the first coordinates is 24. The smallest value for all of the first coordinates is $^{-}22$.

   a. The largest value for all of the second coordinates is ___?___.

   b. The smallest value for all of the second coordinates is ___?___.

   In numbering the x-axis you will have to go from $^{-}22$ to 24. This means using the unit distance to mark off 22 units to the left and 24 units to the right on the x-axis.

   c. How many total units are necessary on the x-axis?

   d. How many total units are necessary on the y-axis?

   e. How can you use the above information to make the points fit on your graph paper?

2. Plot the following points and connect them with segments in the order that you plot them.

a.	$(^{-}22,9)$	j.	$(^{-}4,14)$
b.	$(^{-}21,8)$	k.	$(0,10)$
c.	$(^{-}17,^{-}6)$	l.	$(^{-}3,10)$
d.	$(15,^{-}6)$	m.	$(^{-}4,14)$
e.	$(24,6)$	n.	$(^{-}9,14)$
f.	$(11,6)$	o.	$(^{-}8,10)$
g.	$(9,10)$	p.	$(^{-}12,10)$
h.	$(5,10)$	q.	$(^{-}14,8)$
i.	$(4,14)$	r.	$(^{-}21,8)$

3. The flag is missing from the pole in Number 2 (see $(^-22,9)$). Design your own flag and give the coordinates of its points.

4. For the table of ordered pairs below determine the total number of units needed on the x-axis and the total number needed on the y-axis.

a.	$(^-3\frac{1}{2}, ^-10)$	l.	$(1\frac{1}{2}, 12\frac{1}{2})$
b.	$(^-3\frac{1}{2}, ^-15)$	m.	$(2, 3\frac{1}{2})$
c.	$(^-5\frac{2}{3}, ^-17)$	n.	$(2\frac{1}{2}, 2\frac{1}{2})$
d.	$(^-5\frac{2}{3}, ^-12)$	o.	$(2\frac{3}{4}, ^-5)$
e.	$(^-3\frac{1}{2}, ^-10)$	p.	$(3\frac{1}{2}, ^-6)$
f.	$(^-3\frac{1}{2}, ^-6)$	q.	$(3\frac{1}{2}, ^-15)$
g.	$(^-2\frac{3}{4}, ^-5)$	r.	$(5\frac{2}{3}, ^-17)$
h.	$(^-2\frac{1}{2}, 2\frac{1}{2})$	s.	$(5\frac{2}{3}, ^-12)$
i.	$(^-2, 3\frac{1}{2})$	t.	$(3\frac{1}{2}, ^-10)$
j.	$(^-1\frac{1}{2}, 12\frac{1}{2})$	u.	$(3\frac{1}{2}, ^-15)$
k.	$(0, 14\frac{1}{3})$	v.	$(^-3\frac{1}{2}, ^-15)$

Plot the points and connect them with segments in the order that you plot them.

5. a. $(15,^{-}20)$
b. $(^{-}2,^{-}25)$
c. $(^{-}20,^{-}42)$
d. $(0,^{-}32)$
e. $(^{-}5,^{-}30)$
f. $(15,^{-}25)$
g. $(20,20)$
h. $(15,30)$
i. $(^{-}5,30)$
j. $(^{-}14,20)$
k. $(^{-}6,21)$
l. $(^{-}4,19)$
m. $(^{-}6,20)$
n. $(^{-}14,20)$
o. $(^{-}15,14)$
p. $(^{-}19,14)$
q. $(^{-}14,12)$
r. $(^{-}15,11)$
s. $(^{-}15,13)$
t. $(^{-}27,^{-}1)$
u. $(^{-}25,^{-}2)$
v. $(^{-}13,^{-}1)$
w. $(^{-}11,^{-}7)$
x. $(^{-}2,^{-}5)$
y. $(^{-}8,^{-}13)$
z. $(0,^{-}15)$
aa. $(^{-}5,^{-}30)$

6. a. $(^{-}14,14)$
b. $(^{-}15,11)$
c. $(^{-}17,10)$
d. $(^{-}11,8)$
e. $(2,13)$
f. $(17,8)$
g. $(26,8)$
h. $(37,15)$
i. $(30,6)$
j. $(12,4)$
k. $(1,^{-}4)$
l. $(0,^{-}10)$
m. $(^{-}11,^{-}17)$
n. $(^{-}11,^{-}15)$
o. $(^{-}2,^{-}9)$
p. $(^{-}1,0)$
q. $(^{-}5,0)$
r. $(^{-}5,^{-}4)$
s. $(^{-}4,^{-}7)$
t. $(^{-}6,^{-}5)$
u. $(^{-}7,0)$
v. $(^{-}10,0)$
w. $(^{-}18,6)$
x. $(^{-}21,4)$
y. $(^{-}22,5)$
z. $(^{-}20,9)$
aa. $(^{-}14,14)$

7. a. $(^-5, ^-12)$
b. $(^-4, ^-13)$
c. $(^-3, ^-12)$
d. $(^-3, 4)$
e. $(1, 4)$
f. $(1, 2)$
g. $(^-1, 2)$
h. $(^-3, 4)$
STOP (Do not draw a line from h to i.)
i. $(1, 2)$
j. $(1, ^-12)$
STOP
k. $(^-1, 2)$
l. $(^-1, ^-12)$
m. $(0, ^-13)$
n. $(1, ^-12)$
o. $(0, ^-11)$
p. $(^-1, ^-12)$
STOP
q. $(3, 7)$
r. $(5, 5)$
s. $(5, ^-12)$
t. $(4, ^-13)$
u. $(3, ^-12)$
v. $(4, ^-11)$
w. $(5, ^-12)$
STOP
x. $(^-3, ^-12)$
y. $(^-4, ^-11)$
z. $(^-5, ^-12)$
aa. $(^-5, 7)$
bb. $(3, 7)$
cc. $(3, ^-12)$

8. a. $(8, 2)$
b. $(6, 0)$
c. $(^-8, 3)$
d. $(^-8, ^-2)$
e. $(10, ^-5)$
f. $(8, ^-7)$
g. $(^-10, ^-4)$
h. $(^-10, 5)$
i. $(8, 2)$
j. $(8, ^-3)$
k. $(^-6, 0)$
l. $(^-6, 2\frac{1}{2})$
STOP
m. $(^-6, 0)$
n. $(^-8, ^-2)$
STOP
o. $(6, 0)$
p. $(6, ^-2\frac{1}{2})$
STOP
q. $(^-10, 5)$
r. $(^-8, 7)$
s. $(10, 4)$
t. $(10, ^-5)$
STOP

9.

a. $(1, 11\frac{1}{3})$
b. $(3\frac{1}{2}, 11)$
c. $(4, 10)$
d. $(3\frac{2}{3}, 9\frac{2}{3})$
e. $(2\frac{1}{2}, 10)$
f. $(^{-}\frac{1}{2}, 10)$
g. $(^{-}\frac{3}{4}, 12)$
h. $(^{-}1, 12\frac{1}{2})$
i. $(^{-}1\frac{1}{2}, 14\frac{1}{2})$
j. $(^{-}2\frac{1}{2}, 15)$
k. $(^{-}2, 15\frac{1}{2})$
l. $(^{-}3, 15\frac{1}{3})$
m. $(^{-}2\frac{2}{3}, 15)$
n. $(^{-}3, 15)$
o. $(^{-}4, 14)$
p. $(^{-}3\frac{1}{2}, 12\frac{1}{2})$
q. $(^{-}3, 12)$
(STOP
r. $(9, 10)$
s. $(9\frac{1}{2}, 10)$
t. $(11\frac{1}{2}, ^{-}\frac{1}{2})$
u. $(8\frac{1}{2}, ^{-}1)$
v. $(8\frac{1}{2}, ^{-}7)$
w. $(^{-}6\frac{1}{2}, ^{-}7)$
x. $(^{-}6\frac{1}{2}, ^{-}1)$
y. $(^{-}9\frac{1}{2}, ^{-}\frac{1}{2})$
z. $(^{-}7\frac{1}{2}, 10)$

aa. $(^{-}3\frac{1}{2}, 10)$
bb. $(^{-}4\frac{1}{3}, 11)$
cc. $(^{-}3\frac{1}{2}, 12\frac{1}{2})$
STOP
dd. $(4, 10)$
ee. $(8, 10)$
ff. $(7\frac{3}{4}, 12\frac{3}{4})$
gg. $(6\frac{3}{4}, 13\frac{1}{2})$
hh. $(6, 12\frac{2}{3})$
ii. $(6\frac{3}{4}, 11)$
jj. $(5\frac{2}{3}, 11)$
kk. $(4\frac{2}{3}, 11\frac{3}{4})$
ll. $(3\frac{2}{3}, 13)$
mm. $(2\frac{1}{3}, 13\frac{2}{3})$
nn. $(1\frac{1}{2}, 13\frac{1}{2})$
oo. $(0, 12)$
pp. $(^{-}\frac{3}{4}, 12)$
qq. $(^{-}\frac{1}{2}, 10)$
rr. $(^{-}1\frac{3}{4}, 10)$
ss. $(^{-}2\frac{1}{3}, 7\frac{1}{2})$
tt. $(^{-}2\frac{1}{2}, 7)$
uu. $(^{-}3, 7\frac{1}{2})$
vv. $(^{-}3, 10)$
ww. $(^{-}3\frac{1}{2}, 10)$

PICTURE DISTORTION

In most cases a rectangular coordinate system is laid out so that the unit distance is the same on each axis.

Figure 1 shows a graph of a five-pointed star. The coordinates are giver in the table.

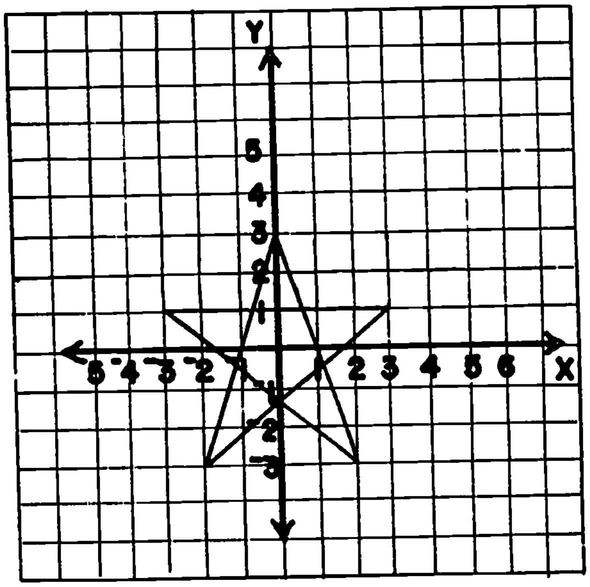


Figure 1

Notice that the scale is marked off the same on each axis.

Table of coordinates for the five-pointed star

X	Y
$^-3$	1
3	1
$^-2$	$^-3$
0	3
2	$^-3$

The five-pointed star in Figure 2 has the same coordinates as the star shown in Figure 1. However, the two stars do not look the same.

Marking off the scale differently on each axis caused <u>distortion.</u>

The scale on the x-axis in Figure 2 does not have the same unit distance as the scale on the y-axis.

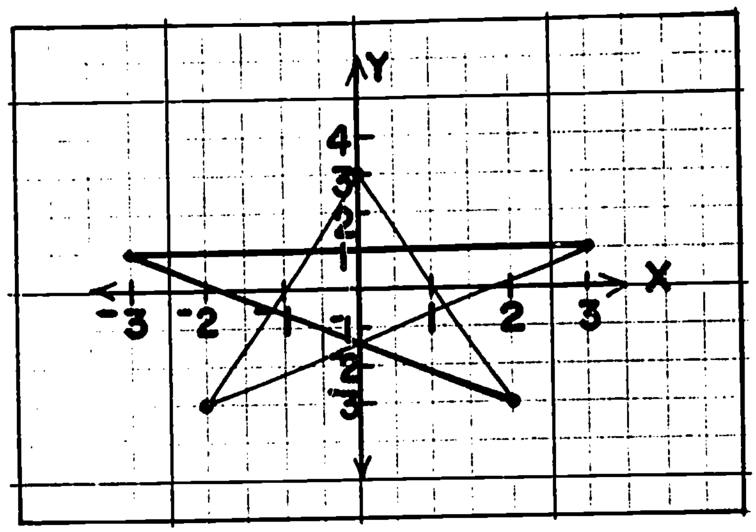


Figure 2

On the graph paper shown in Figures 2 and 3 there are 5 grid lines per inch. The grid lines are the vertical and horizontal lines that make up the graph paper.

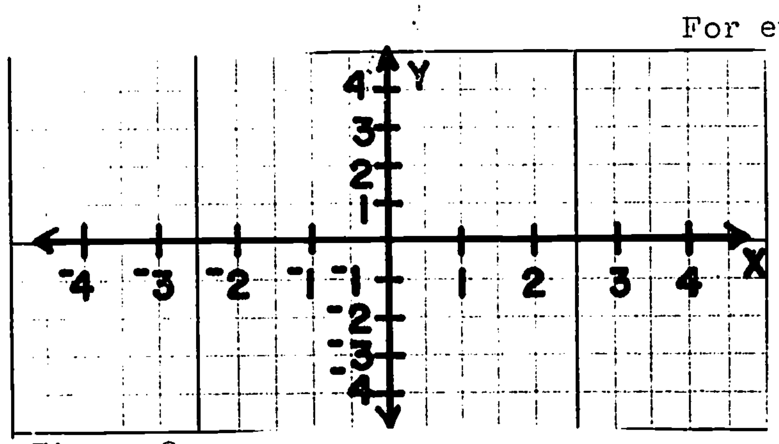


Figure 3

For every 2 grid lines cutting the x-axis there is one grid line cutting the y-axis.

$$\frac{\text{x-axis unit distance}}{\text{y-axis unit distance}} = \frac{\text{2 grid lines}}{\text{1 grid line}}$$

EXERCISES

1. Number the y-axis 1, 2, 3, and so on, using every third grid line on your graph paper. Number the x-axis using each grid line in turn. Number the negative part of each axis in the same manner.

   a. Thus, for every ______ grid line(s) on the y-axis there is one grid line on the x-axis.

   b. The ratio of the x-axis unit distance to the y-axis unit distance is ____?____ .

   c. Plot the coordinates of the five-pointed star. (Use the number pairs given in Figure 1.) Draw the star and compare its shape to those in Figures 1 and 2.

2. When the axes of a rectangular coordinate system are scaled so that the unit distances are the same; the coordinates

   (0,0), (2,0), (2,2), and (0,2)

   are vertices of a square. (See Figure 4.)

   a. Number the axis on your graph paper so that the unit distance on the x-axis is 5 grid marks long and the unit distance on the y-axis is 2 grid marks long.

   b. What is the ratio of the x-axis unit distance to the y-axis unit distance?

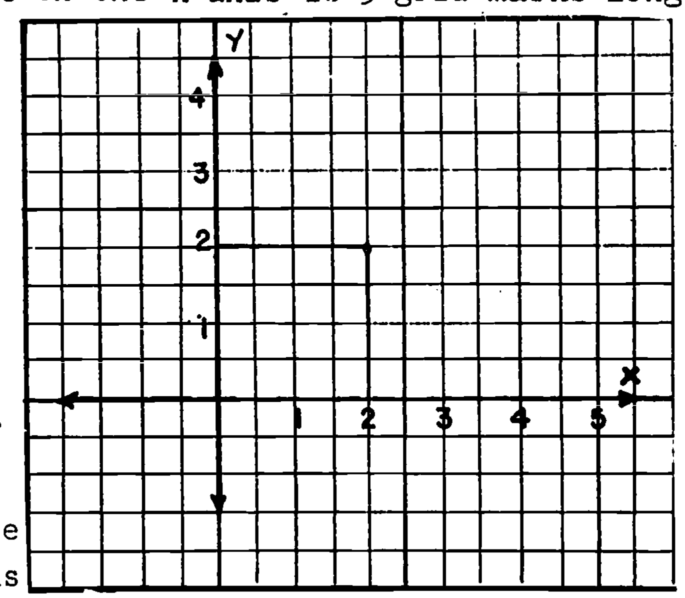


Figure 4

c. Plot the coordinates (0,0), (2,0), (2,2), and (0,2). Connect the points in the order given. Is this figure a square? Each side is two units long. What makes the difference?

3. How are axes scaled in Figure 5?

   a. Number the axes on your graph paper so that for every 4 grid marks on the y-axis there is one grid mark on the x-axis.

   b. What is the ratio of the x-axis unit distance to the y-axis unit distance.

   c. Plot the points (1,5), ($5, ^-2$), and ($^-3, ^-2$) and draw in the triangle. Does it appear to have the same shape as the triangle shown above?

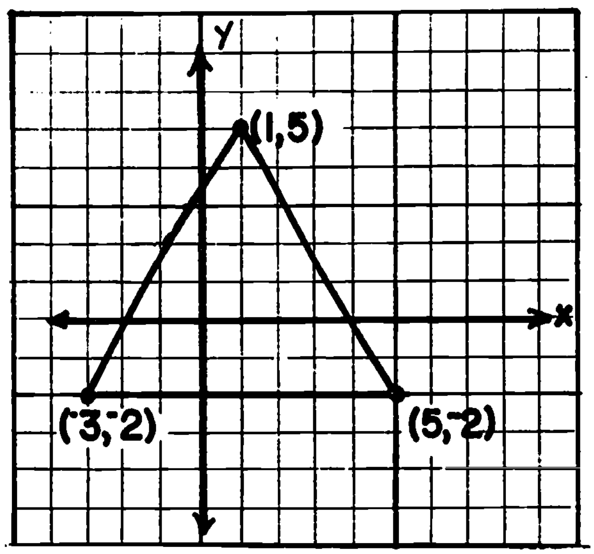
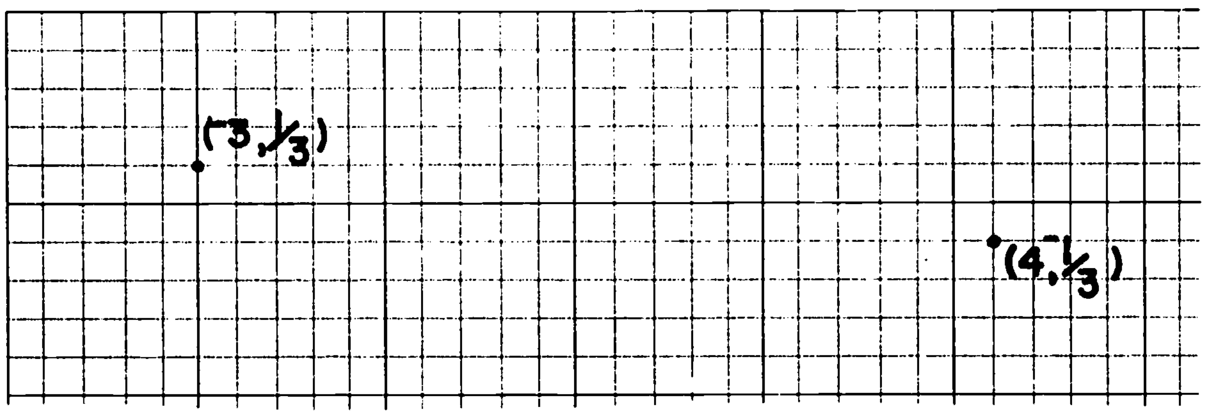


Figure 5

4. The coordinates of the points shown below are correct. Draw in the proper x-axis and y-axis unit distances.



A SECOND WAY TO DISTORT A GRAPH IS TO CHANGE THE ANGLE BETWEEN THE AXES.

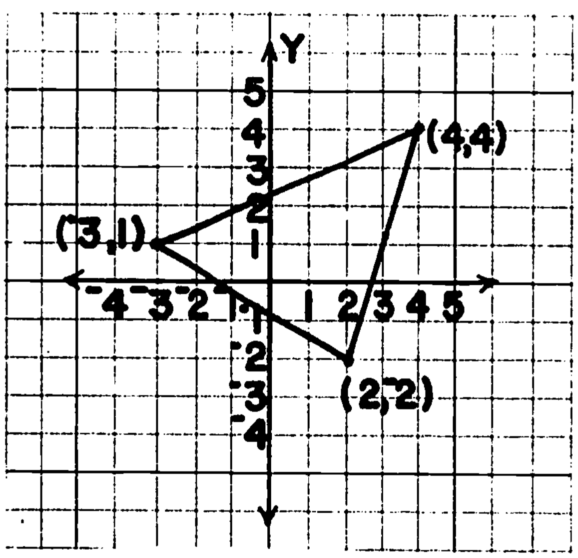


Figure 7a

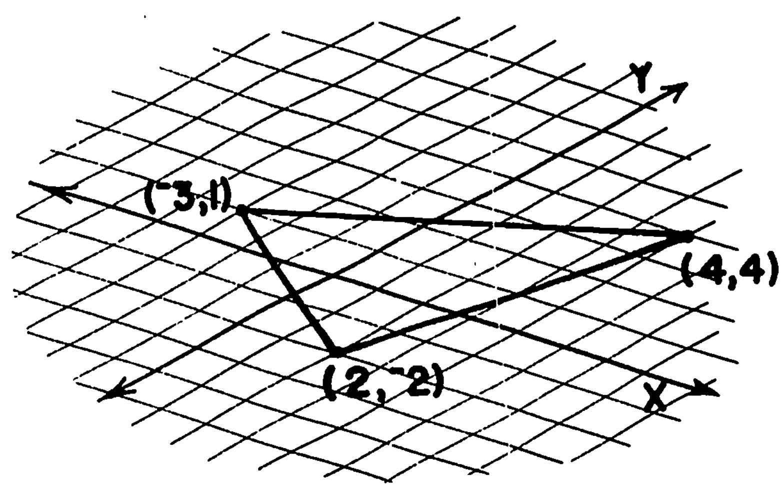


Figure 7b

Figure 7a shows a triangle graphed on the familiar rectangular coordinate system. Figure 7b shows the "same" triangle (the same coordinates that is) plotted on a coordinate system where the axes are not perpendicular.

6. The coordinates of the points shown below are correct. Draw in the proper x- and y-axis.

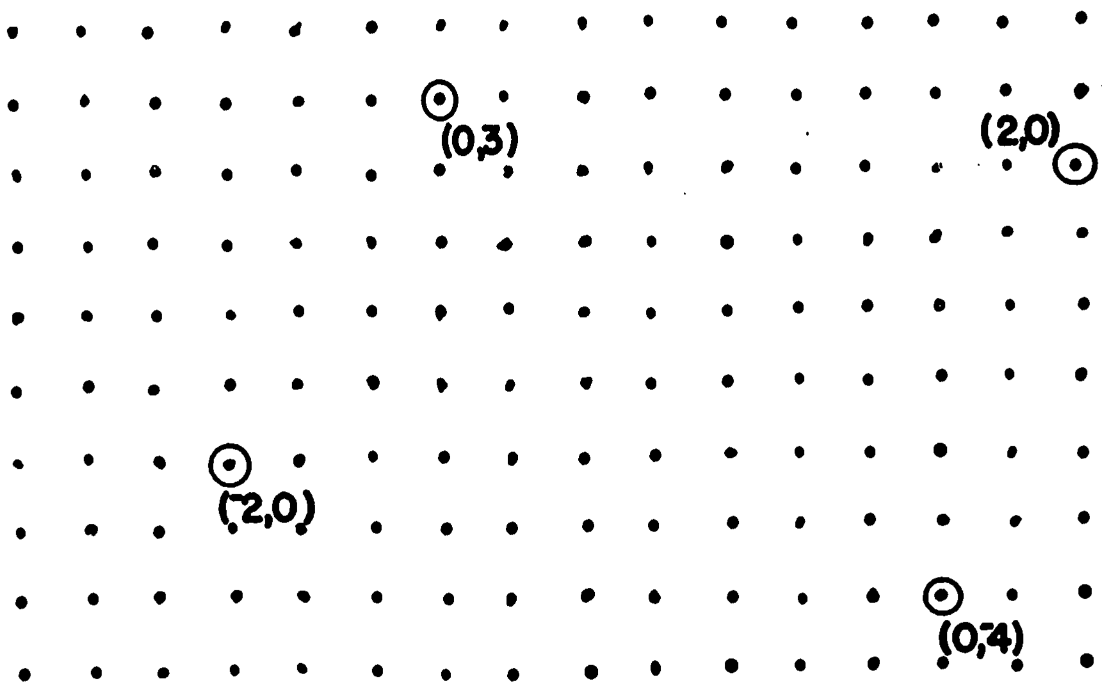


Figure 8

7. When plotting points in a coordinate system where the axes are not perpendicular, it is important that you move parallel to an axis in counting off the units. What are the coordinates of point A below?

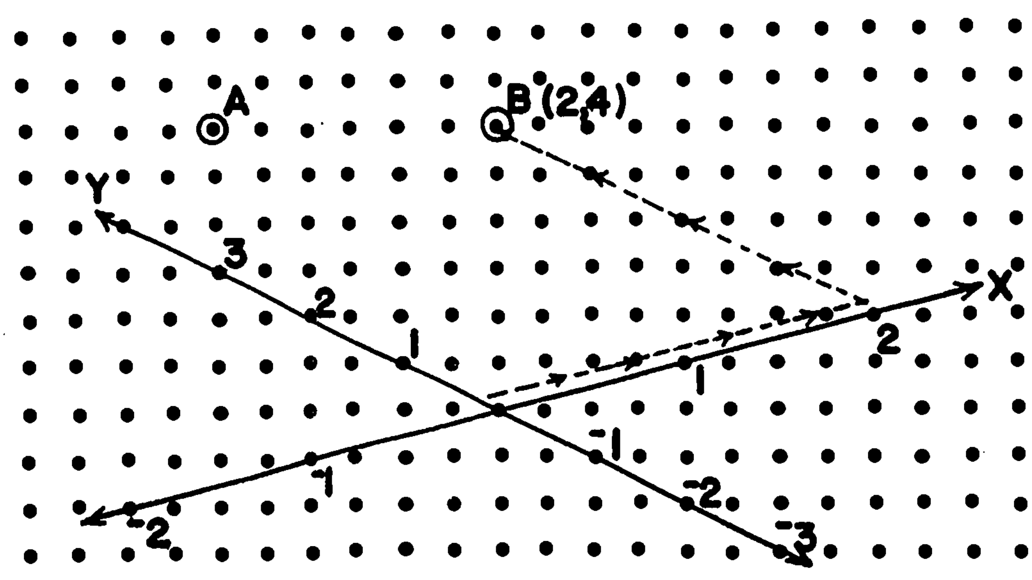


Figure 9

POWER QUESTIONS

1. Use a piece of unlined paper to construct the following graph. (Use a straightedge and a compass.)
    a. Place three noncollinear points (points that are not all on the same line) in a cluster near the center of your paper.
    b. Select one of the three points to be the origin. Draw two lines; each line is determined by the origin and one of the other two points.
    c. Each line is an axis. Let the segment formed by the origin and the other point on the axis determine the unit distance for that axis.
    d. Use a compass to mark off each axis with its unit distance. Number the axes.

2. Plot the coordinates given in Figure 1 and draw the five-pointed star on your coordinate system.

REGIONS IN THE PLANE

Part I The Quadrants

The perpendicular axes of the familiar rectangular coordinate system are shown in Figure 1.

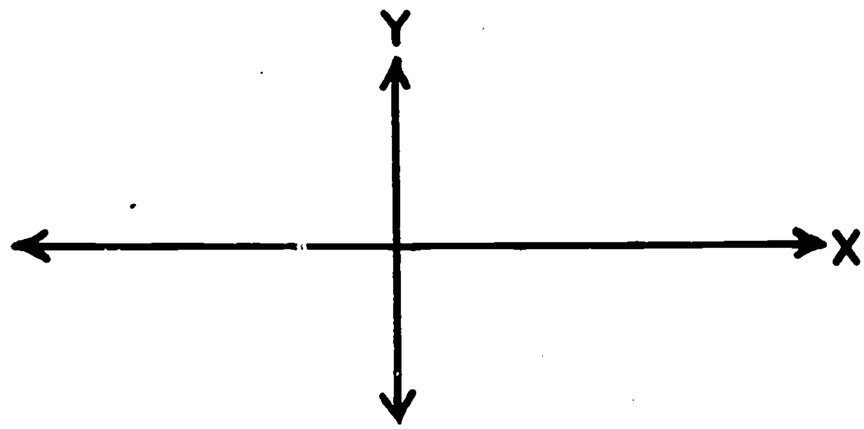


Figure 1

On which side of the y-axis are the points whose x-coordinate is positive?

Imagine the two axes in Figure 1 extended in the plane indefinitely in the four directions. The two axes divide the plane into four regions.

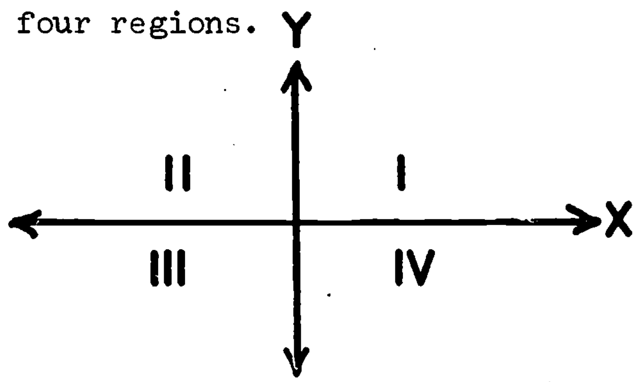


Figure 2

The four regions shown in Figure 2 are called Quadrants. The Quadrants are always numbered I, II, III, and IV in a counterclockwise direction. This way, mathematicians always agree on where, say, the third Quadrant is located.

The axes are not included as part of any of the four Quadrants.

The coordinates of the points in each Quadrant share a common property. For example, each point in Quadrant I has a positive x-coordinate and a positive y-coordinate.

The y-axis divides the plane into three sets of points:

All points whose x-coordinates are positive make up one set of points; all points whose x-coordinates are negative make up a second set; and all points whose x-coordinate is zero make up a third set.

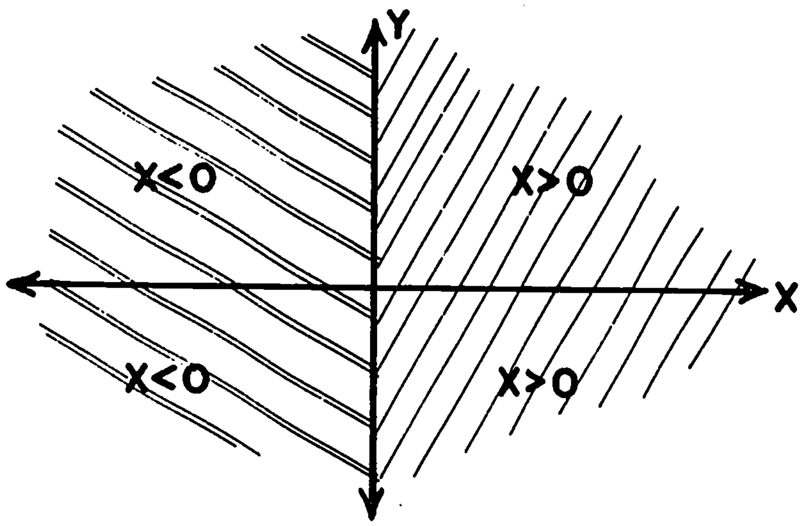


Figure 3

The x-axis also divides the plane into three sets of points:

All points whose y-coordinates are positive; all points whose y-coordinates are negative; and all points whose y-coordinates are zero.

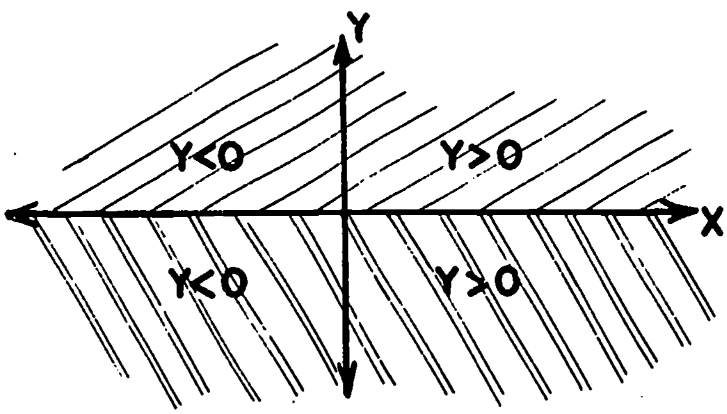


Figure 4

EXERCISES

1. In which Quadrants are the points whose x-coordinate is negative?

2. In which Quadrants does the y-axis lie?

3. Draw an x- and y-axis on your graph paper. Label the four Quadrants.

4. In which Quadrant does the origin lie?

5. Complete the following table:

QUADRANT	The common property of all of the points, (x, y), in the Quadrant
I	$x > 0$ and ________
II	________ and $y > 0$
III	________ and ________
IV	________ ___ ________

6. All of the points on the ___ axis have the property that the x-coordinate is zero.

7. Draw a vertical line through the point (3, 0). Every point on this line has the property that its x-coordinate has what value?

8. Every point on a horizontal line through the point (0 , 5) has what property?

9. A vertical line is drawn through the point $(^-3, 0)$. Consider the points on this line which are also in Quadrant II. Describe the x- and y-coordinates of each of these points.

   Every x-coordinate ______ Every y-coordinate ______

10. Which coordinate remains unchanged for every point on a horizontal line?

11. Which coordinate tells you how far a point is from the x-axis?

12. Draw a line through the points $(2, 2)$, $(^-3, ^-3)$ and $(0, 0)$. Describe the coordinates of all the points on this line.

POWER QUESTIONS

1. The coordinates of point W are (a, b) and the coordinates of point H are (c, d). Here are some facts about these coordinates:

   (1) $a < 0$ and $b > 0$.

   (2) $c > 0$ and $d < 0$.

   Describe the positions of W and H as completely as you can.

Part 2 Graphing Inequalities

Figure 5 shows a vertical line drawn through the point (4, 0). This line is the graph of the sentence $x = 4$. Every point on this line has the property that the x-coordinate has a value of 4.

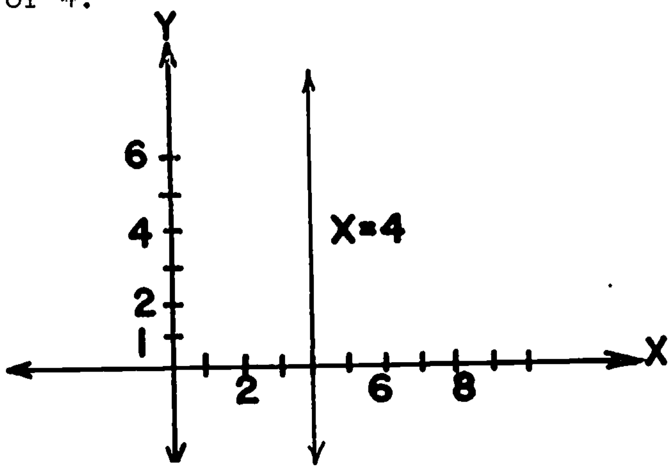
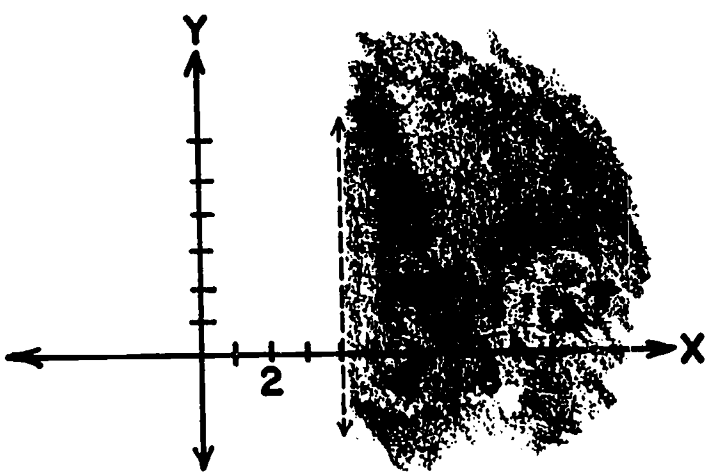


Figure 5

What about the points in the region to the right of the line $x = 4$?

The region to the right of the line $x = 4$ can be described by using an inequality sentence.

All points (x, y) where $x > 4$ are in the shaded region shown in Figure 6.



A "dash line" is used when the line is <u>not</u> to be included as part of of the shaded region.

Figure 6

Notice that y can be any number. It is only the x-coordinate that is restricted.

If the line, $x = 4$, is to be included in the shaded region, then we say

$x \geq 4$ which is read
"x is greater than or equal to 4"

The symbol $>$ means strictly "greater than" and does not include "equal to".

EXERCISES

For each exercise below draw an x-axis and a y-axis on your graph paper. Do not use a whole sheet of graph paper for each exercise--a quarter section will do. Shade in the region described.

1. The x-coordinate is less than 2.

2. The y-coordinate is greater than $^-3$.

3. All points (x, y) where $x < 0$ and $y > 0$. Which Quadrant is this?

4. All points (x, y) where $x \geq 1$ and $y \geq 2$.

5. All points (x, y) where $x > 3$ and $x < 5$. y can be any number.

6. All points (x, y) where $y \geq 1$ and $y < 5$ and $x > 0$.

7. All points (x, y) where $y < 3$ and $y > {}^-1$ and $x > {}^-2$ and $x < 5$.

POWER QUESTION

1. Watch out for this one. All points (x, y) where $x < 5$ and $y \geq 3$ and $x \geq y$.

WHAT IS A COORDINATE SYSTEM?

There are many different coordinate systems. Which one you use would depend on several things. Do you wish to coordinatize a plane, the surface region of a geometric solid, all of space or perhaps a line? You may wish to use angle measures for some of the coordinates. If you use axes, then you may want them to be oblique instead of perpendicular.

The main purpose of this lesson is to have you draw together and examine some of the things that you have learned about coordinate systems.

Part I Coordinate Systems Without Angles

In Lesson 5, only points with non-negative coordinates were considered. Figure 1, below, shows the three axes extended in their negative directions.

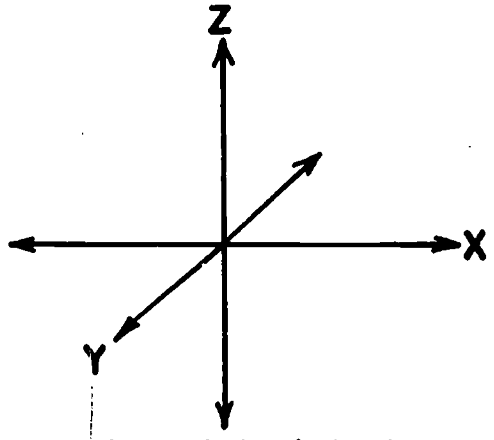


Figure 1

The negative y-axis points into the paper and the negative z-axis points down.

The way in which axes are labeled varies from book to book. There is no "always do it this way" rule. It is wise however, to select one way and stick to it to avoid confusion.

The three axes form three planes. In space these three planes can be used to locate a point by giving its directed distance from each of the planes.

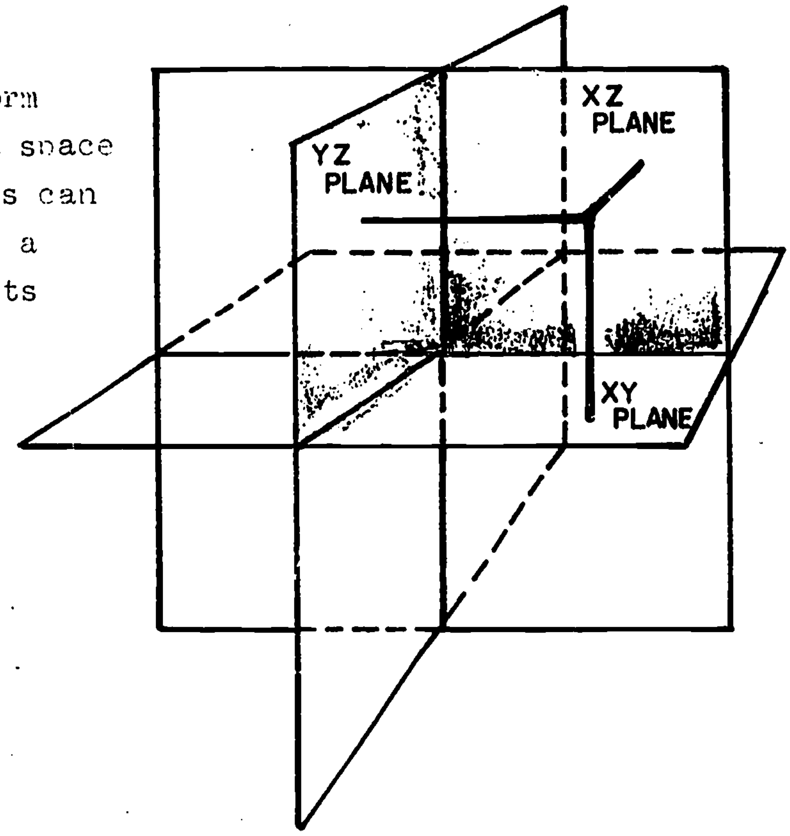


Figure 2

Each directed distance is a coordinate of the point's position. The coordinates of a point in space are given in the form of an ordered triple of numbers.

If the three axes are mutually perpendicular, then the coordinates are called rectangular coordinates. The word rectangular comes from right-angled.

The three coordinate planes (the xy-plane, the xz-plane and the yz-plane) divide space up into compartments. Each compartment is called an Octant. The axes are not included in any Octant.

DISCUSSION QUESTIONS

1. Why is the term Octant used to name the compartments created by the intersecting coordinate planes?

2. Explain why three coordinates are necessary to describe a point's location in space.

EXERCISES

1. The Octant which is usually number one is the one where each coordinate is positive. This is the Octant we used in Lesson 5. The other Octants can be numbered as follows:

   The Octants II, III and IV are numbered counterclockwise around the positive z-axis
   Octant number V is directly below Octant I.
   Then Octants VI, VII and VIII are numbered counterclockwise around the negative z-axis.

   Complete the following table:

	OCTANT	The common property of all The points (x,y,z) in the Octant
	I	$x > 0$, $y > 0$, $z > 0$
a	II	
b	III	
c	IV	
d	V	
e	VI	
f	VII	
g	VIII	

2. Give the Octant number for each of the following points:

   a. $(3, {}^{-}2, 1)$ c. $(0, 0, 0)$

   b. $({}^{-}4, {}^{-}2, 3)$ d. $(3, {}^{-}2, {}^{-}4)$

3. The segment connecting points $(3,5,2)$ and $({}^{-}4,3,8)$ will pass through which coordinate plane?

In this day of modern technology, computers often control certain machine operations. Such a setup is referred to as "numerical control".

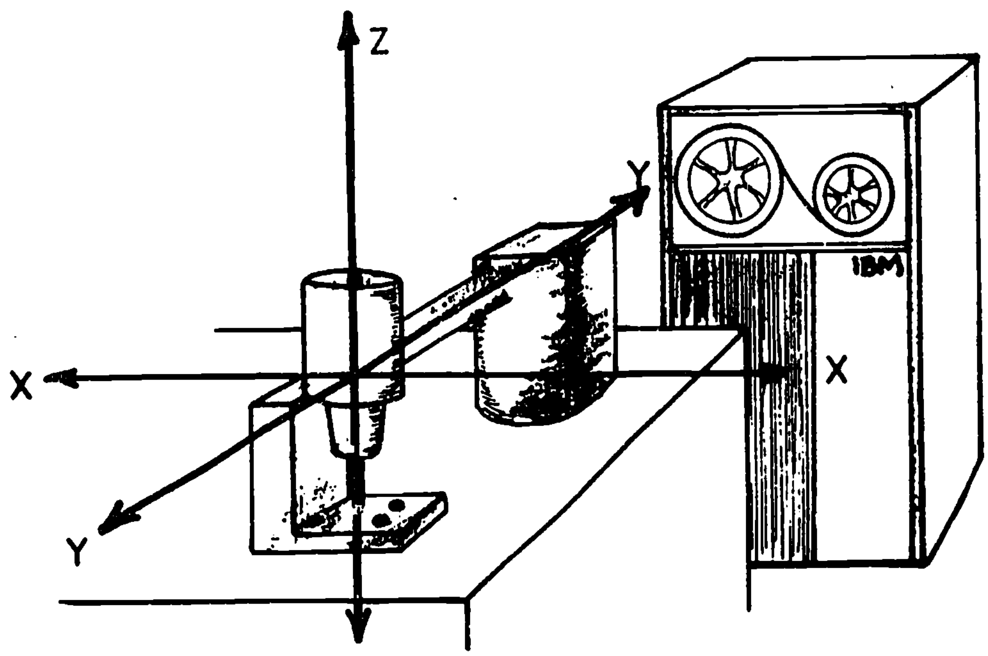


Figure 3

4. The drill shown in Figure 3 began drilling at the point (3 in., 5.2 in., 6.4 in.) and drilled to the point (3 in., 5.2 in., 2.1 in.). How far did the drill go?

5. The drill returned to the point (3 in., 5.2 in., 6.4 in.) and then moved $^{+}$.75 in. along the y-axis. What are the coordinates of the drill now?

6. (Optional)

The Acme Warehouse stores electronic parts. There are thousands of different types of parts (tubes, switches, relays, wire, transistors, etc,) and it is quite a job deeping track of them.

The parts are stored on shelves. Each section of shelves contains 40 storage boxes (called bins).

One section of 40 bins.

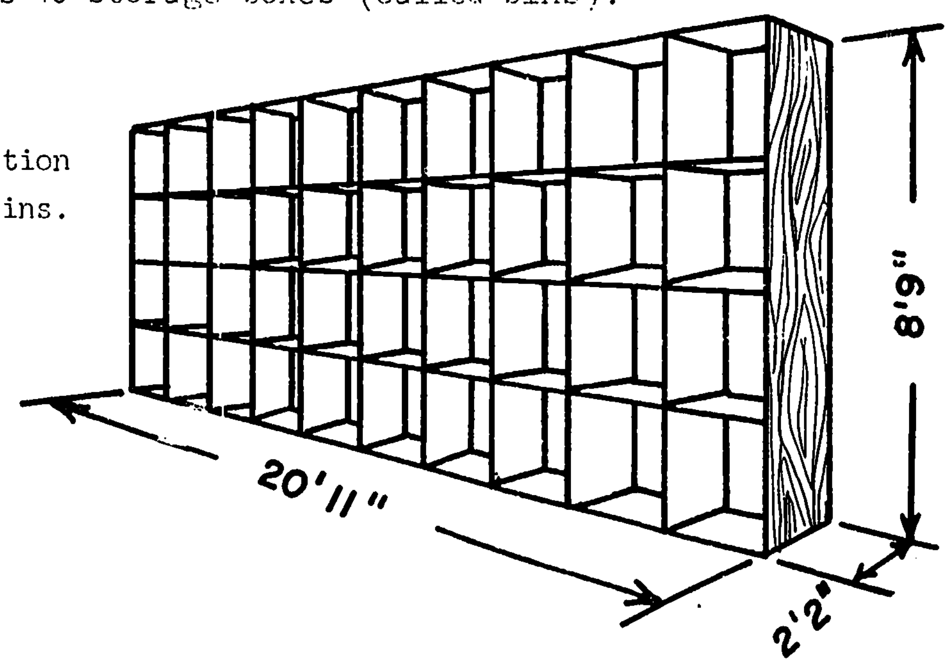


Figure 4

Each bin contains a different type of part.

The warehouse has 6 floors and each floor contains 48 sections of bins.

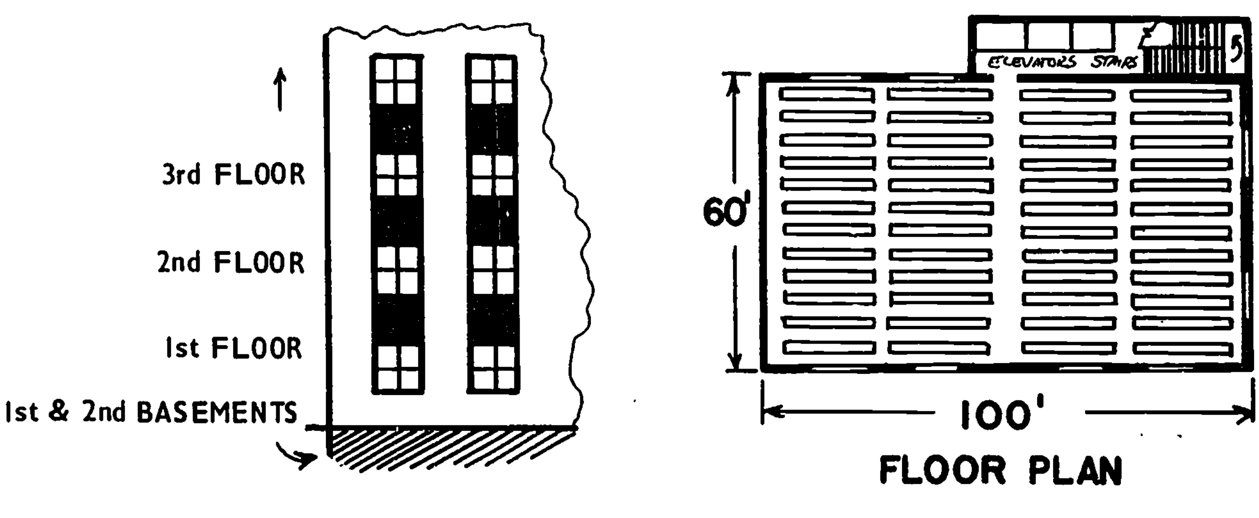
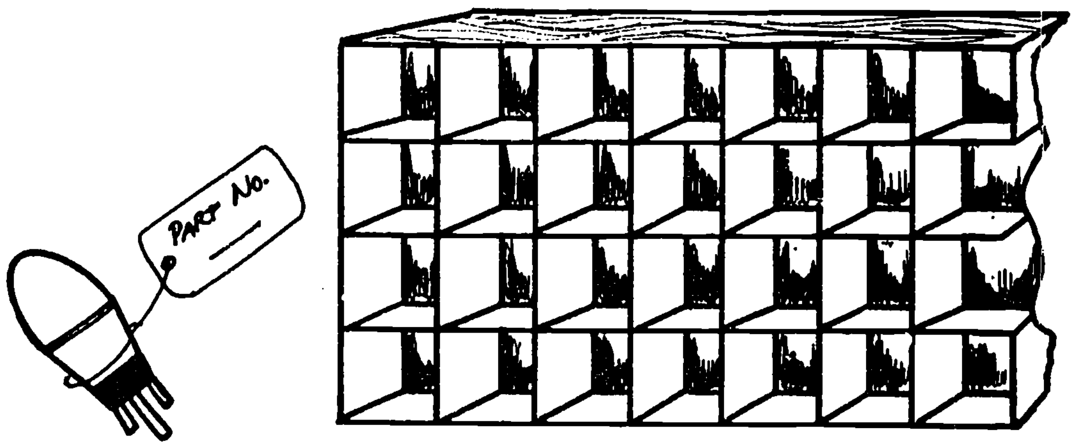


Figure 5

The manager of the warehouse would like to invent a coordinate system to help keep track of the parts. He would like to assign a label to each part so that the label can be used to locate where the part is stored in the warehouse.



THE PART NO. SHOULD TELL WHERE THE PART IS LOCATED.

Figure 6

Help the manager invent a coordinate system so that each part has a different set of coordinates and each set of coordinates locates a different part in the warehouse.

LINE COORDINATES

When we move from space to the plane we lose a coordinate. A plane has only two dimensions. Space has three dimensions. In Lessons 7, 8, 9 and 10 you worked with coordinates of points in the plane so we won't spend extra time on them here.

We didn't, however, discuss coordinates of a point on a line. How do you coordinatize a line?

Figure 7

The line shown in Figure 7 resembles our familiar number line. However, it has not been coordinatized yet.

First, a point must be selected for the origin. The coordinate of this point will be zero.

Second, we select a point whose coordinate will be one. These selections are arbitrary.

The distance between the point whose coordinate is zero and the point whose coordinate is one is the unit distance.

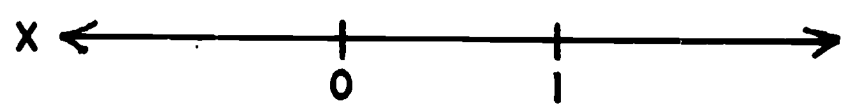


Figure 8

Once the unit distance is known, the coordinate of any point on the line can be determined. Can you explain why the point whose coordinate is 1 could have been selected to the left of the origin?

EXERCISES CONTINUED

7. What is the coordinate of the point located five units to the right of zero on the line?

8. Give the coordinate of the point located two and one-half units to the left of zero.

9. Complete the following table:

Region to be Coordinatized	Number of Axes	Number of Coordinates	Coordinates of the origin
a) Space			
b) Plane			
c) Line			

10. The letter x is used to label the horizontal axis in rectangular coordinate systems for space, the plane, and the line. The graph of $x = 3$, on the line, is a dot located three units to the right of zero. Describe the graph of $x = 3$ (a) in the plane and (b) in space.

Part 2 **Space Coordinates With Angles**
(optional)

In Lesson 5, an ordered triple of numbers was used to describe the coordinates of a point in space.

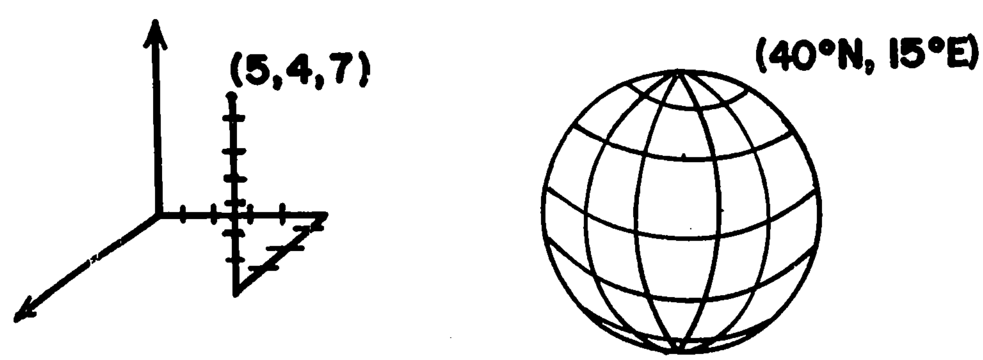


Figure 9

In Lesson 3, points on the globe were determined by an ordered pair of angle measures. Figure 10, on page 83, suggests a way in which these ideas could be combined to obtain space coordinates.

Let P be a point somewhere out in space. A segment, OP, connects the center of the Earth, O, with point P.

The first space coordinate of point P will be the distance OP. The other two coordinates will be the latitude and longitude of the point where OP cuts the Earth's surface.

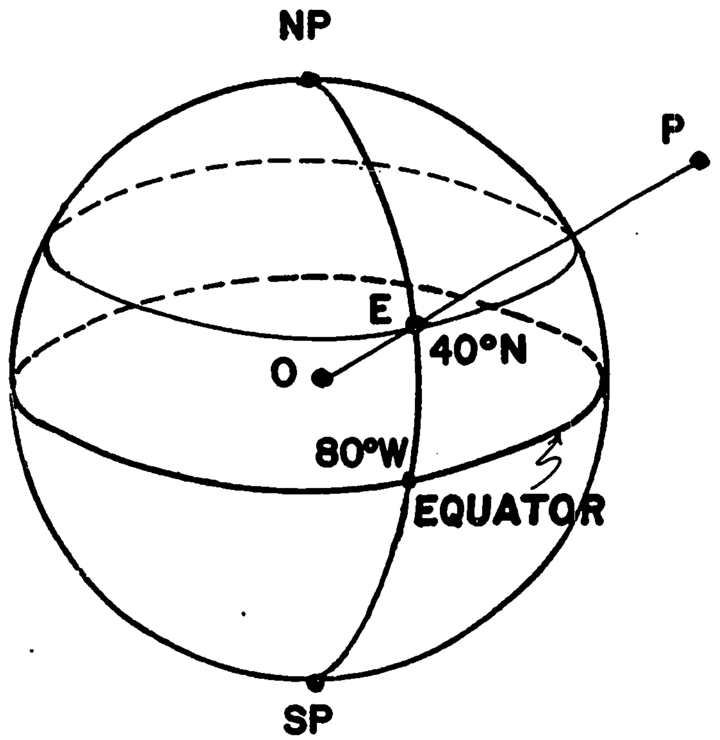


Figure 10

EXAMPLE: Point P, in Figure 10, is 423 miles directly over point E on the Earth's surface. Point E is 3,959 miles from the center of the Earth. The geographic coordinates of point E are (40° N, 80° W), so the space coordinates of point P are

$$(4{,}382 \text{ mi.}, 40^\circ \text{ N}, 80^\circ \text{ W})$$

As in Lesson 3, the two poles lie on all circles of longitude so we have to make a special agreement. We will use 0° for their third coordinate.

	SPACE COORDINATES
North Pole	(3,959 mi., 90° N, 0°)
South Pole	(3,959 mi., 90° S, 0°)

We are assuming that the Earth is a perfect sphere and that all points on the Earth's surface are 3,959 miles from the center.

The origin of this space coordinate system is the center of the Earth. Since any angle of latitude or longitude would be correct for the Earth's center, we will agree to use 0° for both the second and third coordinates of the origin.

EXERCISES

1. A stationary communications satellite is 138 miles directly overhead from a point on Earth whose coordinates are (57° N, 36° W). Give the space coordinates of the satellite.

2. What are the coordinates of the origin of this space coordinate system?

3. Take a point on a segment half-way between the center of the Earth and the South Pole. What are the coordinates of this point?

4. Describe where the point (3,959 mi., 0^{o}, 40^{o} E) is.

5. A plane is flying over Columbus, Ohio (geographic coordinates (40^{o} N, 83^{o} W)) at an altitude of 35,000 feet. Give the coordinates of the plane. (Round off to the nearest tenth of a mile.)

6. At one instant in its flight, Apollo 11's space coordinates were (4,078 mi., 26^{o} N, 80^{o} W). What city was it flying over and how high, from the Earth's surface was it?

7. A satellite is in a circular orbit about the Earth. Its coordinates at this instant are (4,307 mi., 70^{o} N, 110^{o} W). In less than an hour the satellite will be directly opposite the Earth from where it is now. What will be its coordinates then?

Part 3 Invent Your Own Coordinate System

This unit in coordinate geometry has been concerned with one central idea: the location of points or regions using various methods of coordinatization.

You will be given several geometric solids - cylinders, cones, spheres, cubes, etc.

Take one of the geometric solids and examine its surface region. Your assignment is this:

1. Think of a way to give coordinates to each point on the surface of your solid. Apply one of the methods you have studied or invent your own.

2. Display your coordinates scheme so that another student can see how yours works. Use the materials available to make your method of locating points understandable.

Your second assignment is similar to the first. A spacecraft is on its way to the moon.

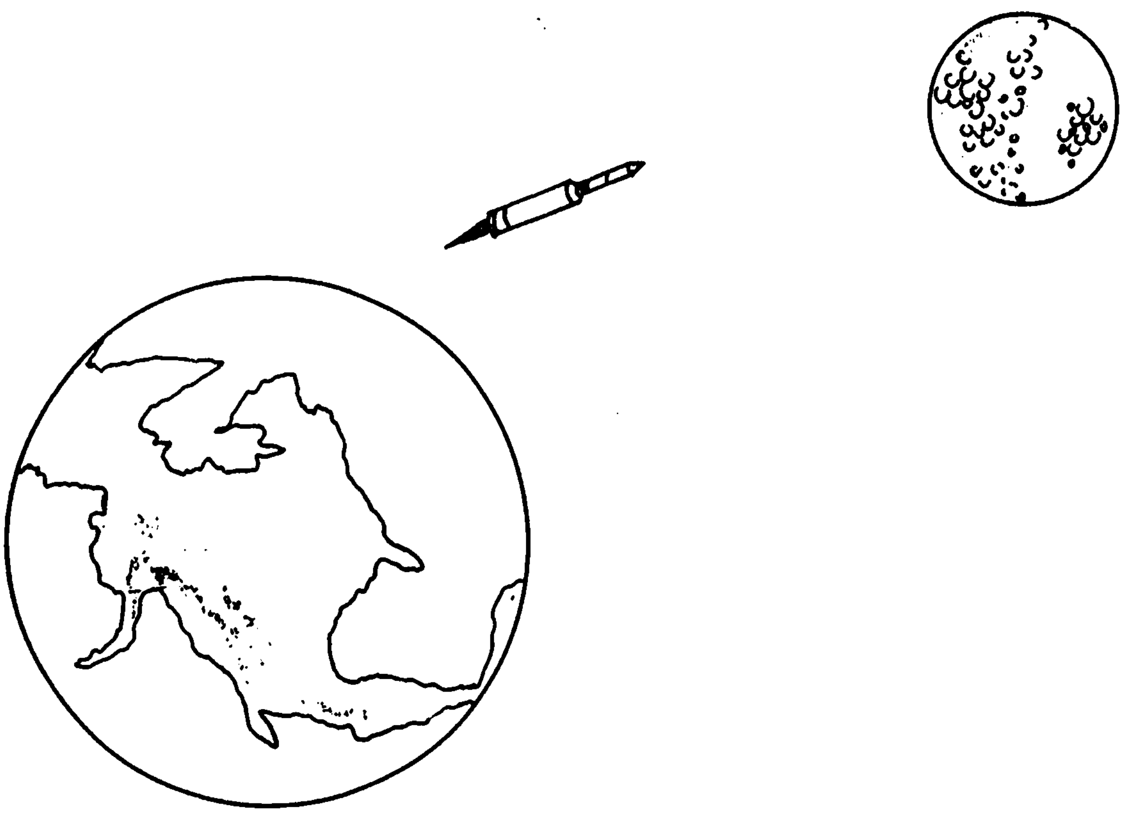


Figure 11

3. Assume that the spacecraft is at some fixed point in space. Think of a way to describe its location using coordinates. You may apply one of the methods you have studied or invent one of your own.

4. Explain your method to the other students in your group.