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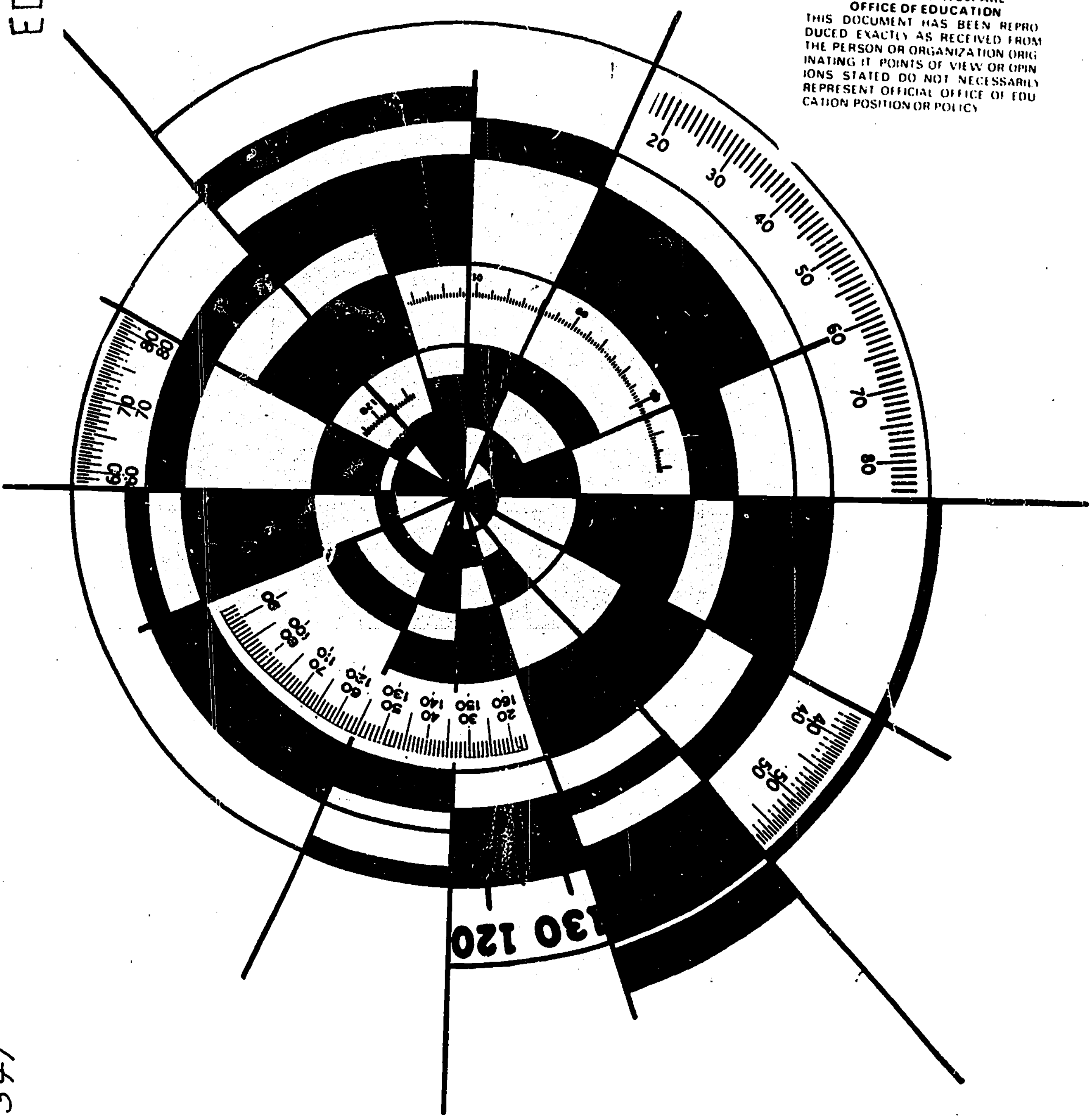
ABSTRACT

This guide to accompany "Angle Measure" contains all of the student information in SE 015 340 plus supplemental teacher materials. A summary of terminal objectives and necessary equipment and teaching aids is given. Discussion topics, teaching suggestions, and answers appear with each section. Related documents are SE 015 334 - SE 015 340 and SE 015 342 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ED 069530

ANGLE MEASURE



TEACHER'S GUIDE

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ANGLE MEASURE

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

BASIC ASSUMPTIONS

It is assumed that the student has not yet achieved the objectives listed for this booklet, although he has been exposed to much of the content. Although he has previously used a protractor, the student's understanding of its construction and use in measuring angle size is probably sketchy and mechanical in nature.

OBJECTIVES

A summary of the terminal objectives for the booklet, indicating lessons where these objectives are developed, follows the **TABLE OF CONTENTS** . In addition, the objectives for each lesson are listed separately lesson by lesson in the **TEACHER'S GUIDE** . Although your teaching and class discussions should not be limited by these objectives, evaluation of the student's progress should be based on the stated objectives for the booklet. Use the objectives as a guide to the intent of each lesson and discuss them with your students.

The terminal objectives are also listed at the beginning of the student booklet. Indicate the location and purpose of this list to your students. As you discuss the objectives of each lesson with your students, refer them to the corresponding objectives in the front of their booklet. The student can use his understanding of the objectives, ✓ **POINT** exercises, and **EXERCISES** to evaluate his own progress.

OVERVIEW

The student is asked to function on two levels throughout the booklet. He is to: (1) exhibit his competence in the mechanics of measuring and estimating angle size, and (2) to make generalizations on the nature of measurement.

The main purpose of the booklet is for the student to review and develop the mechanics of using both a circular and semi-circular protractor, a skill that will be used as a tool in future booklets. However, the more abstract concepts of measurement should not be neglected. Estimation activities and class discussions are given a major role in the lessons.

Allow the student to experiment; to make errors without retribution; to develop his own methods of estimation; and to freely discuss his problems with both you and his classmates.

BOOKLET ORGANIZATION

In achieving these aims, different sections of the booklet serve different functions. The role for some of the major sections are as follows:

EXERCISES denotes those problems that are for supervised study in the classroom, primarily worked on an individual basis. Although it is not anticipated that much homework will be given, portions of these problems could be used as brief homework assignments.

CLASS ACTIVITY denotes those problems to be worked on a cooperative or team basis. These are <u>not</u> appropriate for homework. The role of the teacher is that of a resource person to be used only if needed. Allow the students to gather data and to discover on their own.

DISCUSSION QUESTIONS are an integral part of the lessons. They stress and draw together those concepts that are inherent in all measurement situations. The measurement exercises and activities will be more meaningful to the student if his results are discussed and compared with those of his classmates. These questions are to be used for in-class discussions (not homework). The Teacher's Guide for each lesson gives other discussion topics relevant to the student booklet under the headings **THINGS TO DISCUSS** and **CONTENT AND APPROACH.**

The exercises labeled ✓**POINT** in the student booklet are to be used by the student in evaluating his own progress.

The **APPENDICES** contain material for two purposes: (1) review and remediation, and (2) enrichment activities which can be explored by individuals or groups of students to extend their knowledge of angle measure beyond the stated objectives of this booklet.

In addition to a lesson by lesson list of **EQUIPMENT AND TEACHING AIDS** . a summary list is given on page xiii.

Teaching suggestions and answers to exercises from the student booklet appear. whenever possible, on the teacher's page opposite the corresponding student page. For example, page T5 will contain comments pertaining to page 5 of the student booklet.

USING THE TEACHER'S GUIDE . . .

> A curriculum is not a new set of materials. A curriculum is a set of instructional goals for a particular group of students and a plan for the interaction of teacher, materials, and students for the achievement of these goals. A curriculum is what goes on in your classroom to achieve the goals you have established for your students."
>
> Dr. Jack E. Forbes

The authors concur with the above statements by Dr. Forbes. The total content of the booklet ANGLE MEASURE is not appropriate for all classes or all students. The teaching strategies used and the content covered should vary with the entering behaviors of each class. Use the suggestions for each lesson, teaching techniques, discussion questions, and class activities as a springboard for developing your lesson plans. Adapt the pacing and expected level of mastery of each topic so that the intended outcomes of instruction are compatible with the level of ability and previous background of your students.

TABLE OF CONTENTS

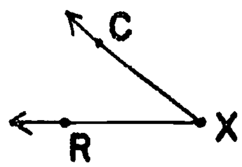
TEACHER COMMENTARY for APPENDICES A, C, E, AND F appears on the pages following page 142.

SUMMARY OF TERMINAL OBJECTIVES

1. Given a labeled representation of an angle, the student will be able to locate and name the

   a. vertex, and

   b. sides of the angle.

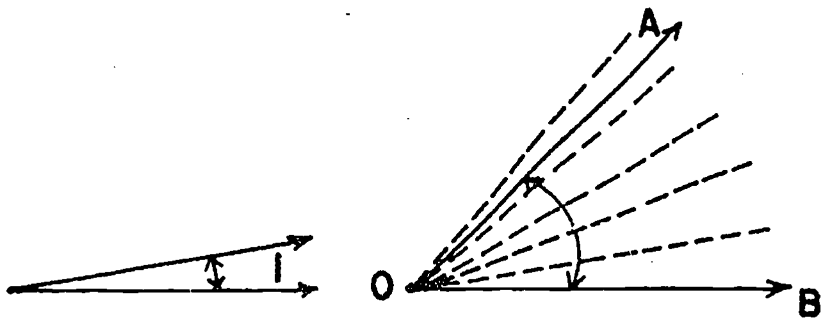


vertex _______

sides _______

2. The student will be able to demonstrate his understanding of <u>unit angle</u> by answering questions such as:

   a.



Given ∠ 1 as the unit angle, complete the following:

(1) _____ units < m(∠ BOA) < _____ units

(2) to the nearest unit, m(∠ BOA) = ____ units

   b. If zips and zaps are units of angle measure and

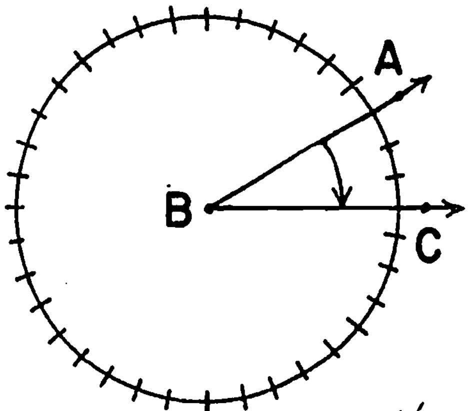
m(∠ BEF) = 6 zips
m(∠ BEF) = 4 zaps

Which is the larger unit of measure, a zip or a zap?

3. The student shall demonstrate his understanding of <u>amount of turn</u> as a measure of an angle.

   a. The student will be able to describe, by drawing the appropriate arrows, clockwise and counterclockwise rotation.

b. Given an angle whose vertex is at the center of a circle with the direction of rotation indicated, the student will be able to measure the angle. The unit of measure is a fractional amount of turn. The circle is divided into congruent arcs as an aid in determining the measure.



$m(\angle ABC) = \frac{3}{36}$ of a turn

c. The student is able to define a degree as $\frac{1}{360}$ of a turn.

d. The student is able to convert an angle measure from a fractional amount of turn to degree measure and vice versa.

e. The student recognizes and can describe a right angle as an angle swept out by a $\frac{1}{4}$ turn and a straight angle as an angle swept out by a $\frac{1}{2}$ turn.

4. The student will be able to estimate the size of an angle.

   a. Given an angle whose measure is between 0° and 360°, the student will be able to give a reasonable estimate (a 10° tolerance) of the measure in degrees.

b. Given thr[illegible] [illegible] least 10°. [illegible] in order of [illegible] without measuring.

c. Given three angles differing in measure by at least 10°, and a given measure, in degrees, the student will be able to select the angle which is closest to the given measure.

5. Given a drawing of an angle whose measure is between 0° and 360°, the student will be able to measure the angle to the nearest degree under the following conditions:

   a. When the angle is determined by

      (1) two intersecting lines

      (2) two rays having a common endpoint.

   b. Using either a

      (1) circular protractor, or

      (2) a semi-circular protractor.

   c. With the protractor in

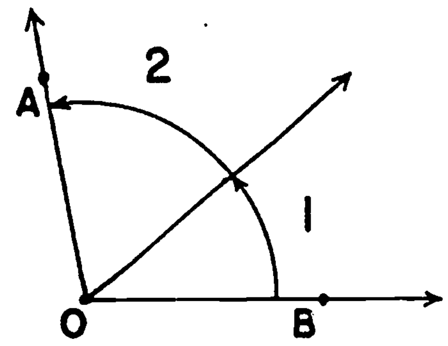
      (1) standard position (one ray through the zero point), or

      (2) non-standard position.

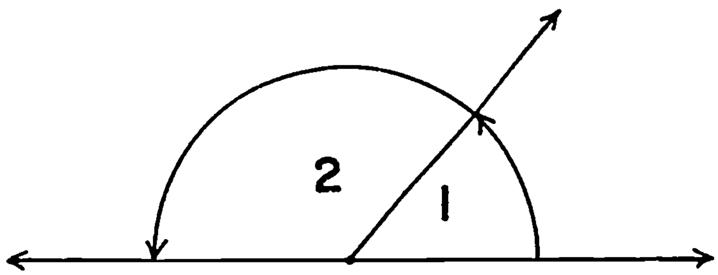
   d. When the "sides" of the angle must be extended to get a reading on the protractor scale.

It is assumed that the protractor used is of sufficient quality to enable an accurate reading to the nearest degree.

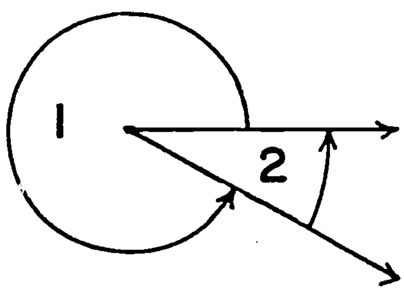
[illegible]. The student shall demonstrate his knowledge of the additive property of measurement and the fact that there are 360° in one complete revolution by answering questions such as the following without using a protractor.



If m(∠1) = 40°, and m(∠2) = 60°, then m(∠BOA) = _____.



If m(∠1) = 50°, then m(∠2) = _____.



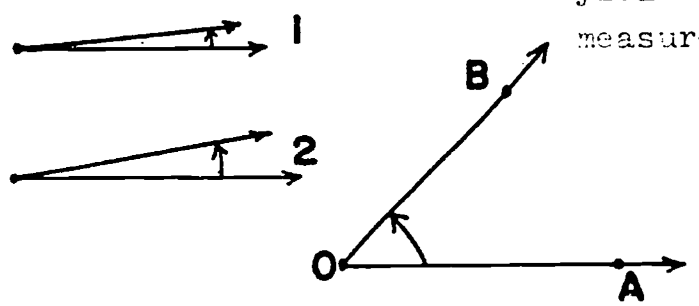
If m(∠1) = 330°, then m(∠2) = _____.

7. Given a measure in degrees between 0° and 180° and a protractor, the student will be able to draw an angle having the given measure.

8. The student is able to state that one degree equals sixty minutes and one minute equals sixty seconds.

9. The student is able to demonstrate his understanding of precision in angle measurement by answering questions such as:

a.



Which unit angle will yield the more precise measure of $\angle AOB$?

b.

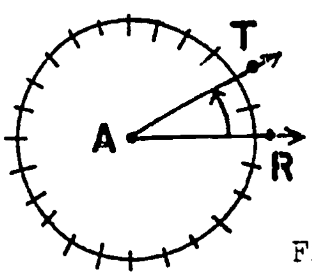


Fig. 1

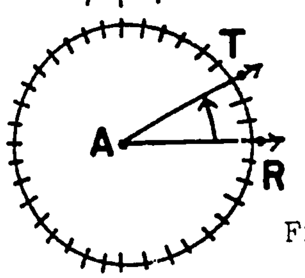


Fig. 2

In Figure 1 circle A is divided into 24 congruent arcs and in Figure 2 circle A is divided into 36 congruent arcs. Which circle helps determine the more precise measure of $\angle RAT$?

c. Which unit gives the more precise measure of an angle: degree, minute, or second?

EQUIPMENT AND TEACHING AIDS

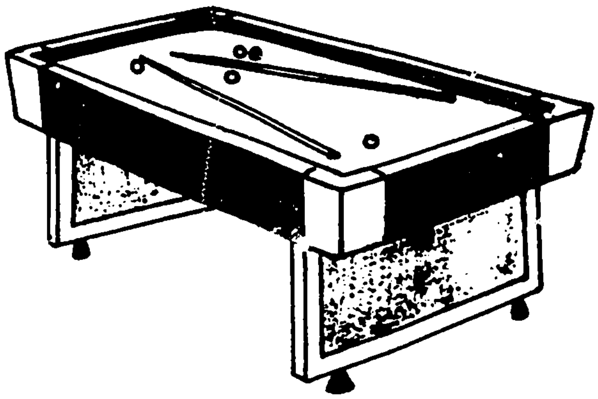
A. STUDENT		LESSON
*1.	packets of plastic unit angle wedges	1,2,8
2.	drawing compass	3-6
3.	straightedge	3,6,11
4.	circular protractor (1 per student)	8-12
5.	semi-circular protractor (1 per student)	9,10,11
*6.	"broken" protractor	10
**7.	ruler (graduated to $\frac{1}{8}$ inch)	12

B. TEACHER		
1.	overhead projector	1-12
2.	projection screen	1-12
**3.	acetate rulers	2
*4.	acetate protractors (arbitrary units)	2
*5.	prepared transparencies (TR 3-1 through TR 3-5)	3-6,8,12
*6.	acetate protractors (in degrees)	
	a. semi-circular	9,10,11
	b. circular	9,10,11
	c. "broken"	10
*7.	acetate overlay (for checking exercise # 1, p. 91)	11

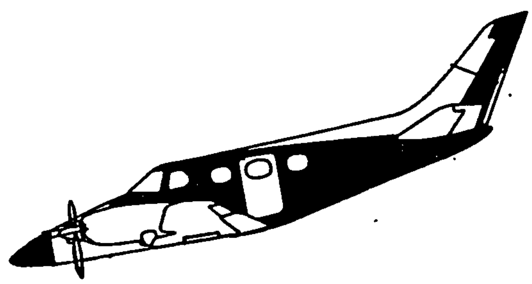
* Indicates those materials provided with the booklets.

** Use same materials provided with the booklet **EXPLORING LINEAR MEASURE** .

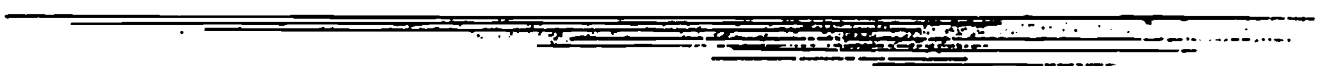
MEASURING THE OPENING

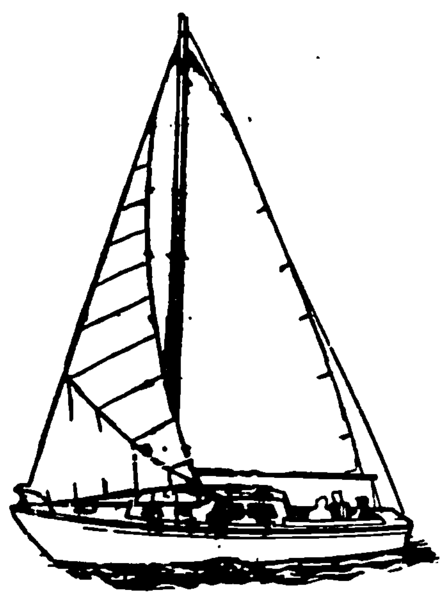


In billiards, a successful "bank shot" depends on the angle of incidence and reflection.



The smoothness of a plane's landing depends on the angle of descent.



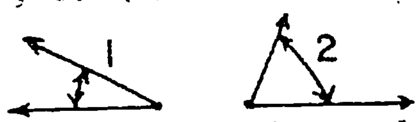


To stay on course, the bearing (in degree measure) of a boat must be measured accurately.

MEASURING THE OPENING

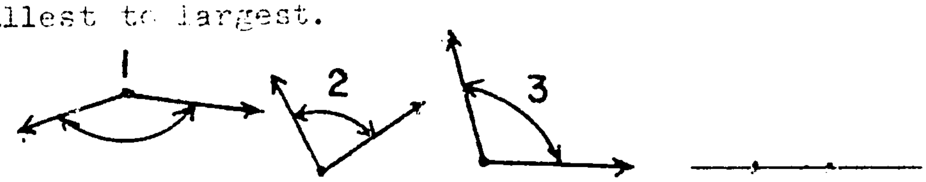
OBJECTIVES

1. The student will demonstrate his ability to judge the relative sizes of angles by working problems such as:
   a. Make the statement true by inserting either $>$, $=$, or $<$ in the blank provided.



$m(\angle 1)$ ____ $m(\angle 2)$

   b. Arrange the following angles in order of size from smallest to largest.



2. Given a labeled representation of an angle, the student will be able to locate and name the
   a. vertex, and
   b. sides of the angle.

3. Given a unit wedge (), the student will be able to...
   a. measure a given angle to the nearest unit.
   b. draw an angle having a given measure.

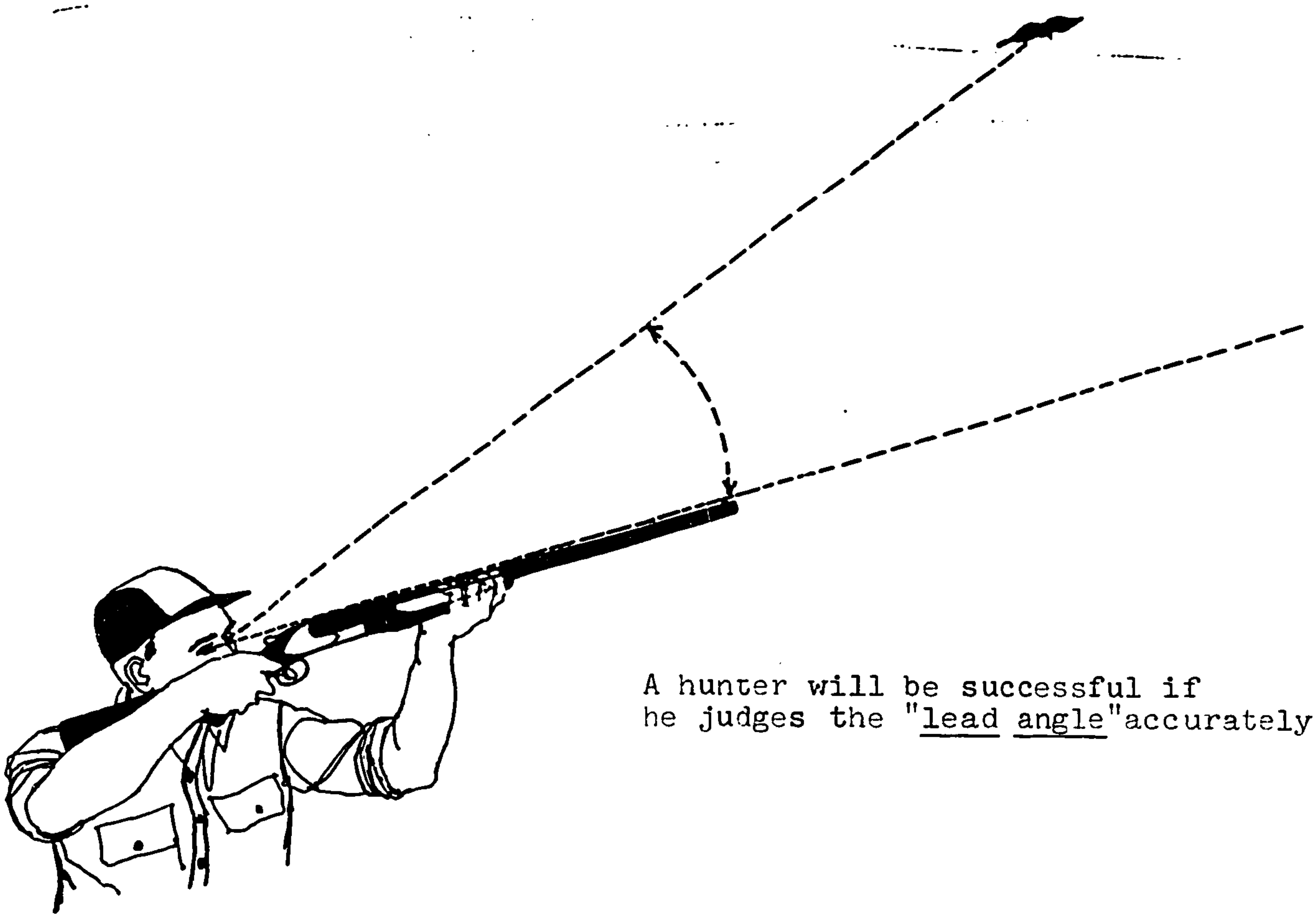
EQUIPMENT AND TEACHING AIDS

A. STUDENT

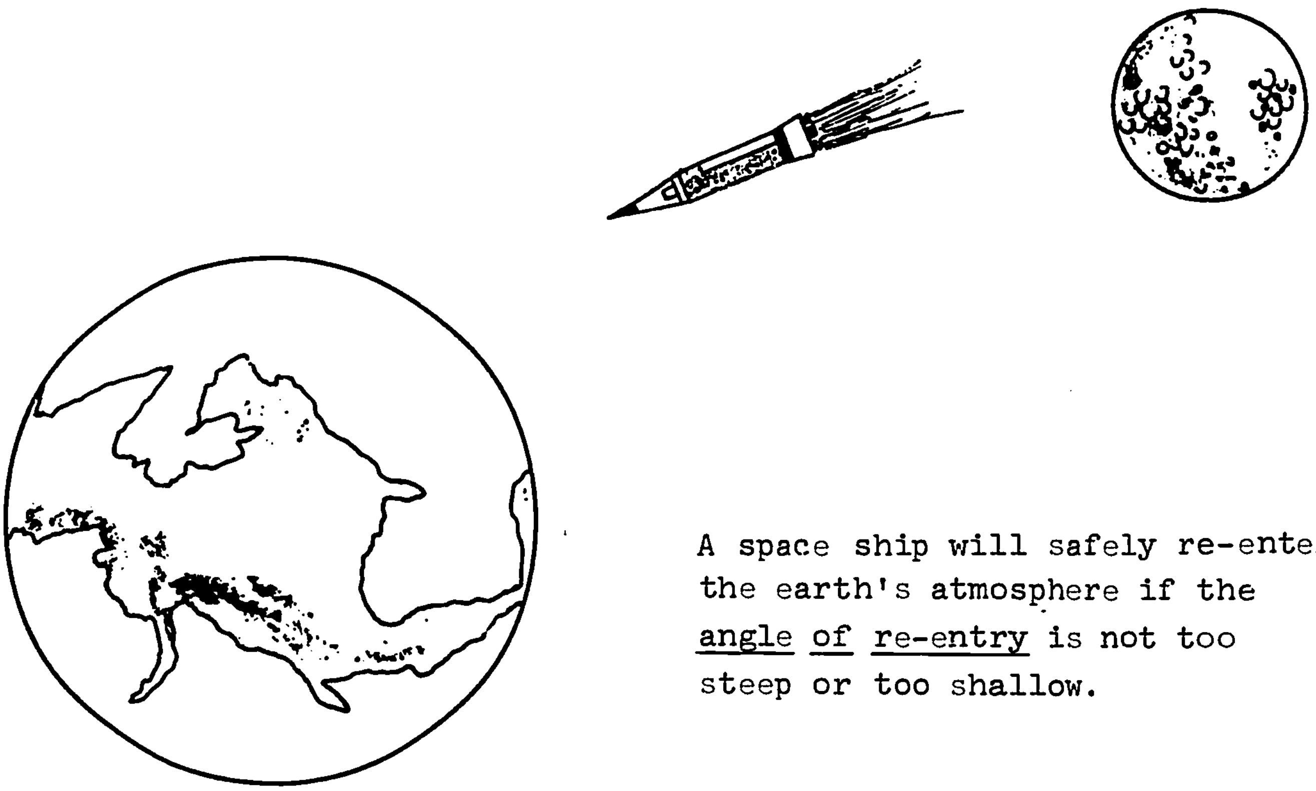
*1. Packets of plastic unit angle wedges (10°, 15°, 30°).

B. TEACHER

1. Overhead projector
2. Projection screen



A hunter will be successful if he judges the "lead angle" accurately.



A space ship will safely re-enter the earth's atmosphere if the angle of re-entry is not too steep or too shallow.

CONTENT AND APPROACH

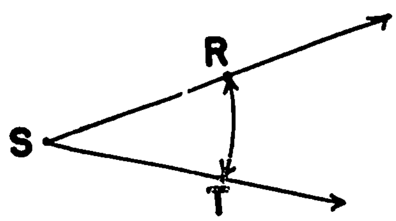
The pictures on pages 1 and 2 illustrate some situations where the determination (by measuring or estimating) of the measure for some angle is useful or necessary. These illustrations may be a partial motivation for the need to study the topic of angle measure. If you desire to have a short discussion on the uses of angle measure illustrated, your students may be able to suggest other uses of angle measure with which they are familiar.

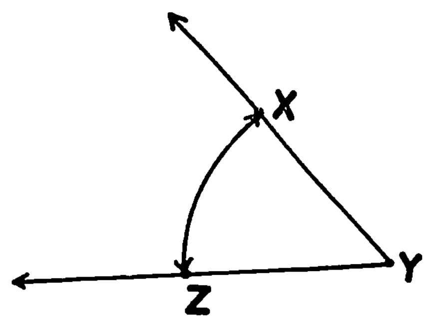
This booklet assumes some understanding of geometric vocabulary and notation such as ray, point, segment, and the distinction between $\overline{AB}$, $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, and $\overrightarrow{BA}$. To aid the student in understanding the material, it may be necessary at times to conduct a quick review of such vocabulary and notation.

THE SIZE OF AN ANGLE IS...

The size of an angle depends on the amount of opening between its sides. The greater the opening, the greater the size of the angle.

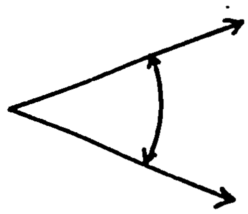
To illustrate, ∠XYZ has a greater size than ∠RST because "the opening between the sides of ∠XYZ is greater than the opening between the sides of ∠RST".

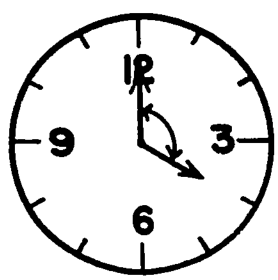


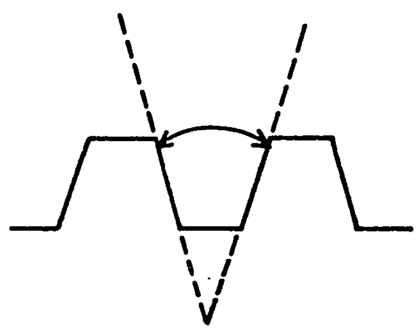


The size of ∠XYZ is greater than the size of ∠RST.

A double-headed arrow will be used to indicate the opening between the sides of an angle.







CONTENT AND APPROACH

In this initial exposure, the size of an angle is defined as the amount of opening between its sides. The measure of an angle is a number determined by (1) the size of the angle and (2) the unit of measure used.

Notice the use of the double-headed arrow to indicate the angle (amount of opening) under consideration. This symbolism will be used until the angle is considered as a rotation. At this time, the double-headed arrow will be dropped and a single-headed arrow will be used to indicate both the direction and amount of turn.

✓ **POINT**

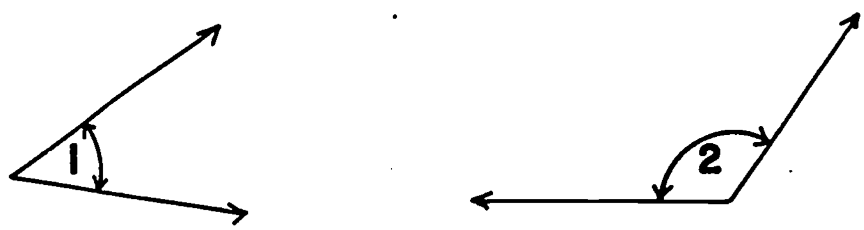
1. What determines the size of an angle?

2. In this booklet, what symbol is used to indicate the amount of opening between the sides of an angle?

WHICH ANGLE IS GREATER?

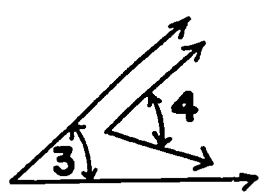
Without measuring, which angle in each pair seems to have the greater opening between the sides? Circle your response.

A.



A. $\angle 1, \angle 2$

B.



B. $\angle 3, \angle 4$

C.

C. $\angle 5, \angle 6$

ANSWERS

✓POINT

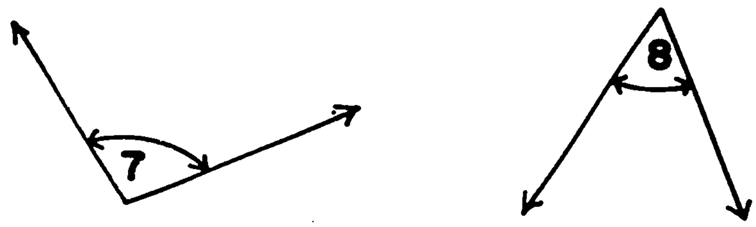
1. The amount of opening between its sides.
2. A double-headed arrow.

CONTENT AND APPROACH

The purpose of the exercises under **WHICH ANGLE IS GREATER?** is to indicate a need for a method to measure angles. Otherwise, it would be impossible to differentiate between angles of different size (See **A METHOD IS NEEDED** , p. 5.). The students should not be expected to answer all four questions correctly-- encourage them to guess.

Notice that "greater angle" is defined as the angle with the greater amount of opening between its sides. One of the major purposes of this booklet is to have the student develop the skill to measure this opening accurately, using either a circular or semi-circular protractor.

D.



∠7, ∠8

A METHOD IS NEEDED . . .

Although the angles in some of the pairs on pages 4 and 5 seem to have the same amount of opening, this is not the case. (The correct responses are ∠2, ∠4, ∠5, and ∠7.)

Before you could accurately determine which angle in each pair has the greater opening, a method is needed for measuring the opening between the sides of an angle.

MEASURING THE OPENING . . .

To measure angles, a method similar to the one for measuring length is usually used.

GIVEN AN ANGLE TO BE MEASURED . . .

(1) Select some angle to use as a unit of measure.

(2) Compare the angle to be measured with the unit angle by counting the number of unit angles it takes to "fill up" the opening of the angle being measured.

This method of measuring angles is illustrated on pages 6 and 7.

CONTENT AND APPROACH

The section **MEASURING THE OPENING** mentions the similarities between the method of angle measure described on pages 5-7 and the way in which length is measured. These similarities will be discussed in some detail in LESSON 2 (See pp. 14-16).

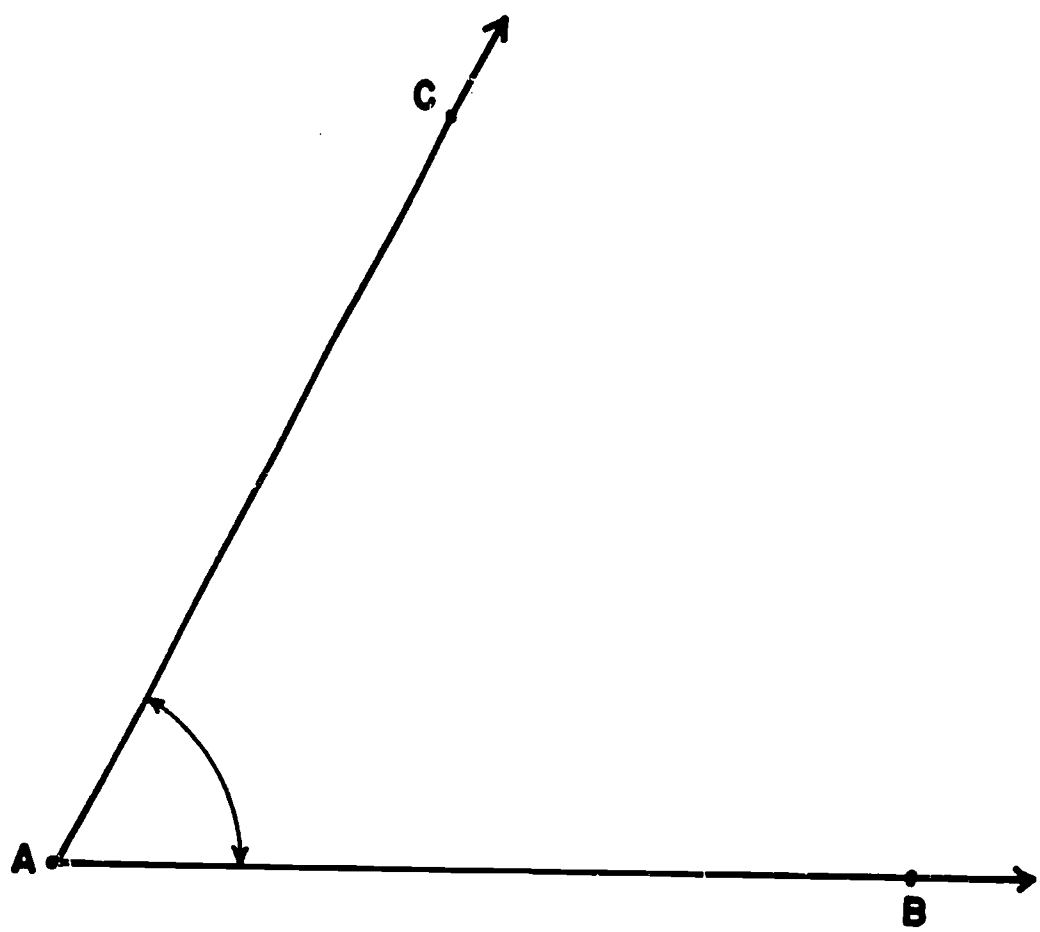
MEASUREMENT IS . . .

THE SCIENCE OF OBTAINING

BETTER APPROXIMATIONS.

EXAMPLE

What is the measure of $\angle$ CAB?



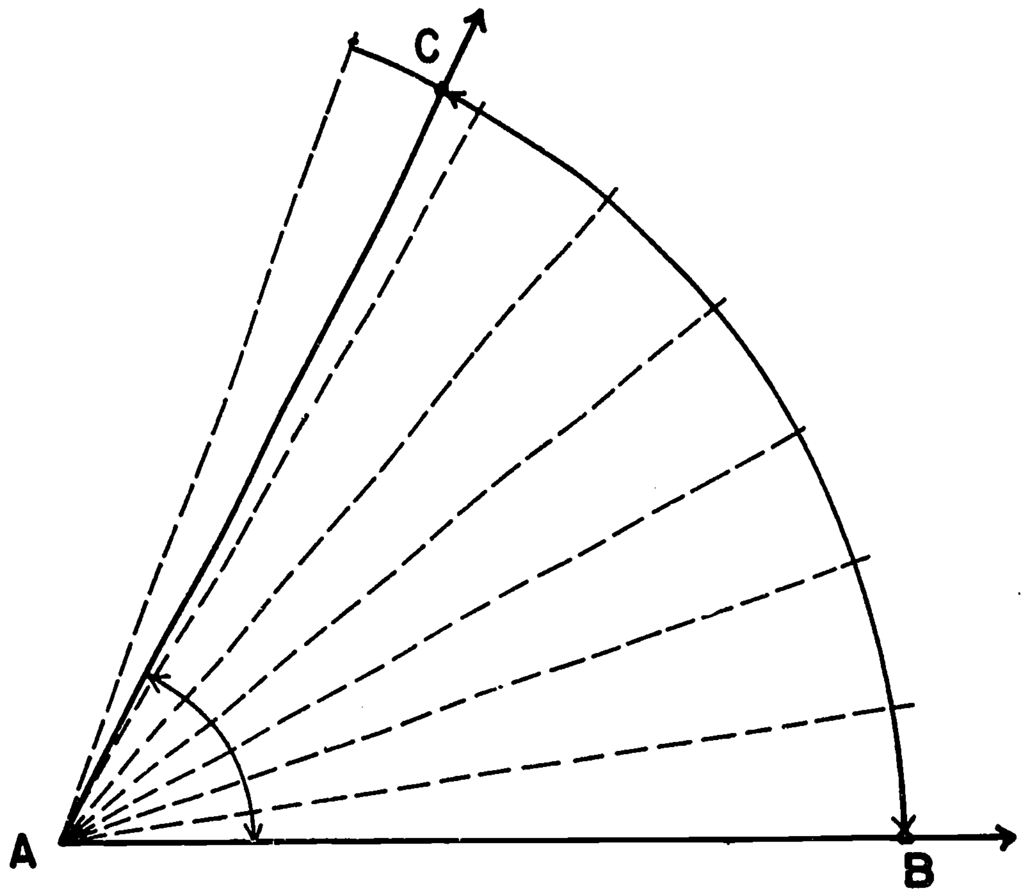
SOLUTION

(1) SUPPOSE THE RED UNIT ANGLE WEDGE FROM THE PACKET OF ANGLE WEDGES IS SELECTED AS THE UNIT OF MEASURE.

CONTENT AND APPROACH

The solution to the example on pages 6 and 7 follows the steps outlined at the bottom of page 5. The unit angle used in the sample solution is the same size as the red angle wedge included in the student packets.
Additional examples can be worked by using a packet of angle wedges and (1) angles drawn on the stage of an overhead projector or (2) angles represented by objects in the classroom vicinity (corner of a desk top,...).

(2) COMPARE THE ANGLE TO BE MEASURED WITH THE UNIT BY COUNTING THE NUMBER OF UNIT ANGLES IT TAKES TO "FILL UP" THE OPENING OF THE ANGLE BEING MEASURED.



6 units < m(∠CAB) < 7 units

It takes more than 6 and less than 7 units to "fill up" the angle being measured. Since the measure (size) of ∠CAB is closer to 6 units than it is to 7 units,...

m(∠CAB) = 6 units when measured to the nearest unit.

[m(∠CAB) is read "the measure of angle CAB".]

CONTENT AND APPROACH

Notice the use of intermediate steps in "thinking out" step number 2. First, the measure of the angle is "trapped" between two consecutive whole units of measure. Secondly, a judgment is made as to which whole number of units the measure is closest. These steps are often done as a matter of habit, especially when a person becomes proficient in the use of the measuring instrument. However, at this stage of development these steps are probably useful and can be used as a model for the kind of questions the student should be asking himself when determining the measure of an angle.

Symbolism such as "6 units $< m(\angle CAB) <$ 7 units" will be used in the first part of this booklet in a similar manner to its use in **EXPLORING LINEAR MEASURE**.

You will be given an envelope containing unit angle wedges of three different sizes. These will be used in the remainder of this lesson.

YET TO BE DONE . . . .

1. Verify the solution on pages 6 and 7 by using the red units from your envelope.
2. Find the measure of $\angle$ CAB to the nearest yellow unit. to the nearest blue unit.

CLASS ACTIVITY

1. You will be given some angles to measure. Measure each angle to the nearest whole red unit, yellow unit, and blue unit.

ANGLE	MEASURE TO THE NEAREST WHOLE...		
	Red Unit	Yellow Unit	Blue Unit

2. Extend the sides of two of the angles listed in the above table. Measure these angles again. Does the length of the sides of an angle have any effect on the measure of the angle?

CHECK AND CORRECT YOUR RESULTS BEFORE GOING TO THE NEXT PAGE.

CONTENT AND APPROACH

The **CLASS ACTIVITY** is designed to be worked by teams of 3-4 students with one packet of unit wedges per team. Space is provided for as many as 8 angles to be measured for question I, each angle being measured with each of the three unit wedges. Fewer than 8 angles will probably be enough as the method of measuring angles will be developed further and practiced throughout the booklet. Use as many angles as you feel is necessary for your class. If possible, provide a few angles drawn on paper and some represented by objects in the classroom vicinity (corner of desk top, braces on a chair,...).

ANSWERS

YET TO BE DONE

2. $m(\angle CAB)$ = 4 yellow units
   $m(\angle CAB)$ = 2 blue units

CLASS ACTIVITY

1. Answers will be determined by the angles provided the student.
2. No. This is an important idea and will be useful when the student must extend the sides of a representation of an angle in order to get a reading on the protractor scale. It is important that the student understand that the measure of an angle does not depend on the length of the sides of its representation.

✓ POINT

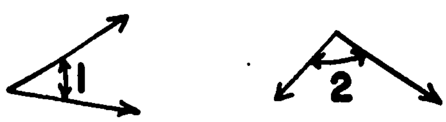
1. Describe the process for measuring angles used in this lesson.
2. Look at your results for the **CLASS ACTIVITY** (p. 8). Does using a different unit of measure change the number used to express the measure of the angle? the amount of opening between the sides of the angle? Give reasons for your answers.

EXERCISES

1. Make each statement true by inserting either > , < or = in the blank provided.

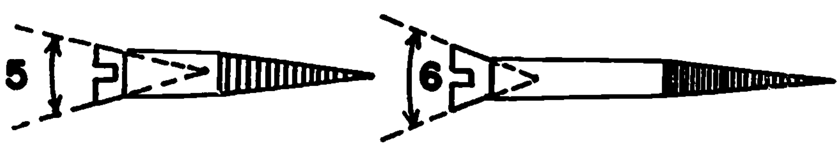
   > means "is greater than"
   < means "is less than"
   = means "is the same as"

   Recall that the measure of an angle depends on the amount of opening between the sides of the angle.



a. $\angle 1$ ___ $\angle 2$

b. $\angle 3$ ___ $\angle 4$



c. $\angle 5$ ___ $\angle 6$

ANSWERS

✓POINT

1. Do not demand a verbatim response from the text (pp. 6-7) but only an indication in the student's words that direct measurement involves two steps: (a) selecting a unit of measure and (b) counting the number of units it takes to "fill up" the object being measured.

2. Yes. No. The amount of opening is constant. Changing the unit of measure will usually change the number of units it takes to "fill up" the opening.

EXERCISES

1. a. $\angle 1 < \angle 2$

   b. $\angle 3 > \angle 4$

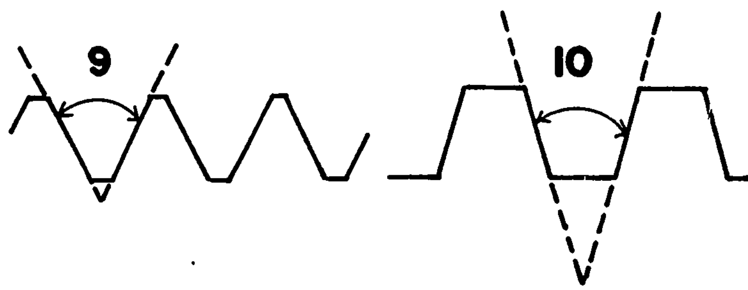
   c. $\angle 5 < \angle 6$

THINGS TO DISCUSS

Notice that Exercise 4 (p. 12) is the reverse of the measurement process demonstrated in the text (pp. 6-7). Students may need help on this one. Prior to the **EXERCISES** and in conjunction with the discussion of the angle measurement process, you may wish to demonstrate on the blackboard and/or overhead stage a procedure for using the angle wedges to draw an angle of a given measure. In Lesson 11, this procedure will be refined as students are required to draw angles of a given degree measure.

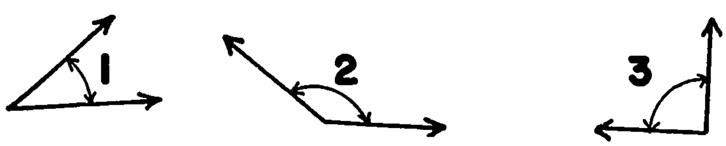
Since we are measuring to the nearest whole unit, would all angles drawn for a given measure (for example, 3 red units) have to be congruent (the same size)? How could congruence be tested? (tracing and superposition)

d. $\angle 7$ ___ $\angle 8$

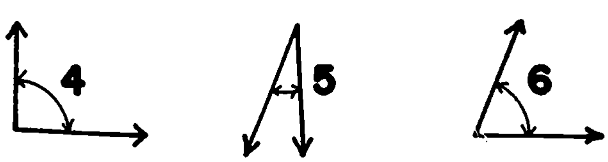


e. $\angle 9$ ___ $\angle 10$

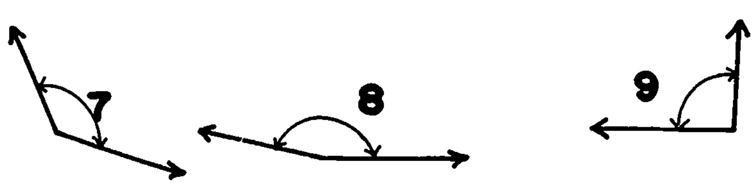
2. For each set of angles, arrange them in order from smallest to largest. Part a is done as an example. Record your answers in the blanks provided.

a. 

a. $\angle 1, \angle 3, \angle 2$

b. 

b. ______________

c. 

c. ______________

d. Of the 9 angles pictured in parts a-c, which angle seems to be the largest? the smallest?

d. ______________

CONTENT AND APPROACH (for page 9)

In relation to question #2 of the ✓**POINT** a distinction will be made between the use of the words <u>size</u> and <u>measure</u>. Size will be used to denote the amount of opening between the sides of an angle and measure will be used to denote the number of units it takes to "fill up" the opening of the angle being measured. Thus, angles having the same measure will not necessarily be the same size. The authors have attempted to be consistent in their usage of these two words.

ANSWERS (for page 10)

1. d. $\angle 7 < \angle 8$

   e. $\angle 9 > \angle 10$

2. a. $\angle 1, \angle 3, \angle 2$

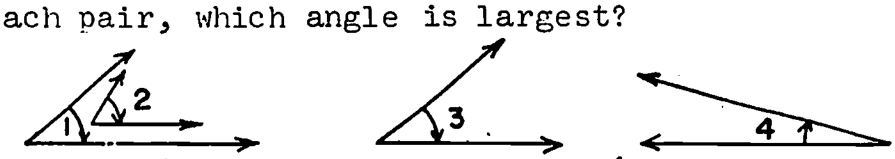
   b. $\angle 5, \angle 6, \angle 4$

   c. $\angle 9, \angle 7, \angle 8$

   d. largest - $\angle 8$
   smallest - $\angle 5$

NOTE: In relation to <u>exercises 1 and 2</u> (pages 9-10), field-testing results for prior versions of this booklet indicate that students have difficulty judging relative sizes of angles in situations like those below.

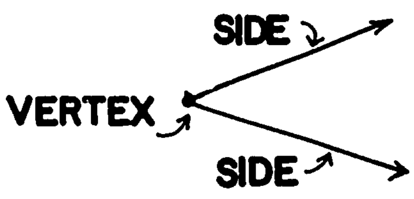
In each pair, which angle is largest?

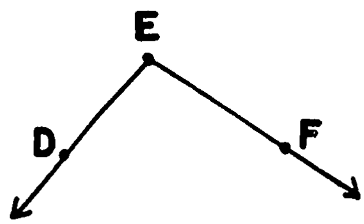


$\angle 1$ or $\angle 2$? $\qquad$ $\angle 3$ or $\angle 4$?

When exercises 1 and 2 are checked, it may be helpful to discuss methods that your class used to determine the angle with the greater amount of opening.

The rays which form an angle are called the <u>sides</u> of the angle. The common endpoint of the sides is called the <u>vertex</u> of the angle.



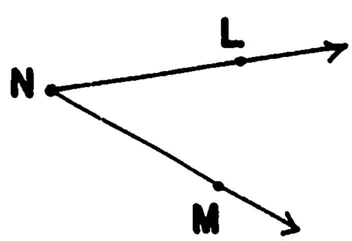


$\overrightarrow{EF}$ is read "ray $\overrightarrow{EF}$".

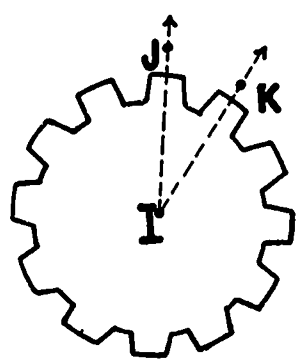
vertex: E
sides: $\overrightarrow{ED}$ and $\overrightarrow{EF}$

3. For each angle, name the (1) <u>vertex</u> and (2) the <u>sides</u> of the angle. Record your answers in the blanks provided.

a.



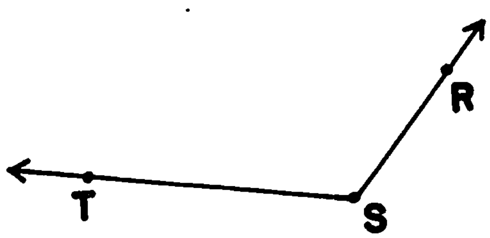
b.



a. vertex ______
sides ______

b. vertex ______
sides ______

c.



c. vertex ______
sides ______

ANSWERS

3. a. vertex N

   sides $\overrightarrow{NL}$, $\overrightarrow{NM}$

   b. vertex I

   sides $\overrightarrow{IJ}$, $\overrightarrow{IK}$

   c. vertex S

   sides $\overrightarrow{ST}$, $\overrightarrow{SR}$

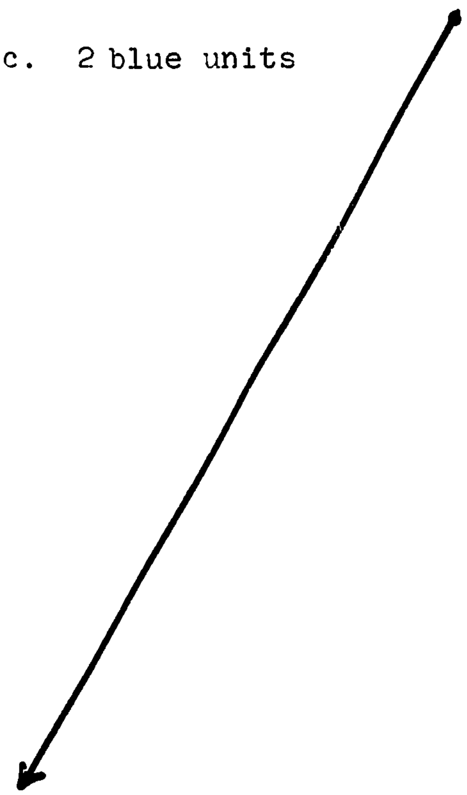
4. Using the unit angles from your envelope as guides, draw angles having the following measures.

   One side (ray) is given for each angle as a starting point. Use the endpoint of the ray as the vertex of the angle.

   a. 4 yellow units

   b. 4 red units

c. 2 blue units



NAMING ANGLES . . .

An angle may be named in several ways. The more common methods of naming angles are listed in **APPENDIX B** at the back of this booklet.

THINGS TO DISCUSS

Use **APPENDIX B** to orient your students to the methods of naming angles which are illustrated in that section. All of these methods will be used in this booklet.

BECOMING MORE PRECISE

Measurement is done by <u>comparison</u>. In the previous lesson, the measure of an angle was found by comparing the opening of the angle being measured with the opening of the unit angle.

This lesson will review and formalize some of the ideas studied in Lesson 1.

ANGLES ARE . . .

Angles are formed by two rays having a common endpoint.

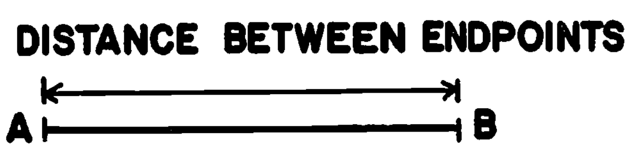
$\angle$ BAC or $\angle$ CAB

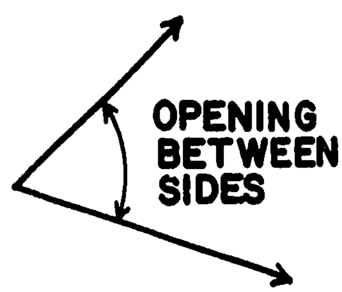
In both cases above, $\angle$ BAC (or $\angle$ CAB) is formed by the rays, $\overrightarrow{AB}$ and $\overrightarrow{AC}$, having a common endpoint, A.

DISTANCES AND OPENINGS . . .

A segment is measured by the distance between its endpoints.

An angle is measured by the amount of opening between its sides.



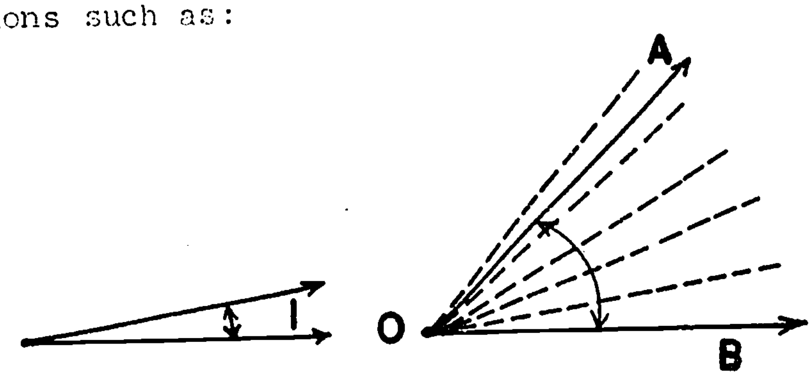


BECOMING MORE PRECISE

OBJECTIVES

1. The student will be able to demonstrate his understanding of unit angle and the measuring process by answering questions such as:

   a.



   Given $\angle 1$ as the unit angle, complete the following:

   (1) ______units < m($\angle$AOB) < ______units

   (2) to the nearest unit, m($\angle$AOB) = _____ units

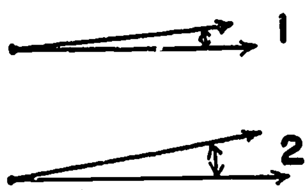
   b. If zips and zaps are units of angle measure and

   m($\angle$BEF) = 6 zips
   m($\angle$BEF) = 4 zaps

   Which is the larger unit of measure, a zip or a zap?

2. The student is able to demonstrate his understanding of precision in angle measurement by answering questions such as:

   a.



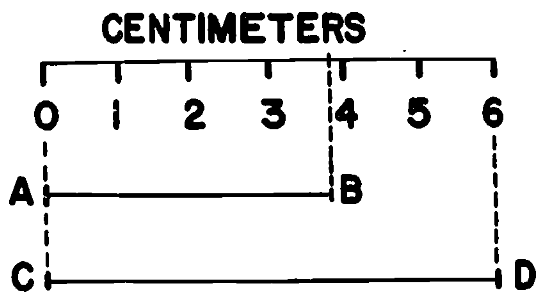
   Which unit angle will yield the more precise measure of $\angle$ AOB?

MEASURING LENGTH . . .

When measuring length,...

(1) a length (foot, pace, centimeter,...) is selected to use as a unit of measure, and...

(2) the length to be measured is compared with the unit of measure by counting the number of unit lengths it takes to "fill up" the length being measured.



Suppose 1 cm. is selected as the unit of measure. The measures of $\overline{AB}$ and $\overline{CD}$ to the nearest whole unit are...

$m(\overline{AB}) = 4$ cm.

$m(\overline{CD}) = 6$ cm.

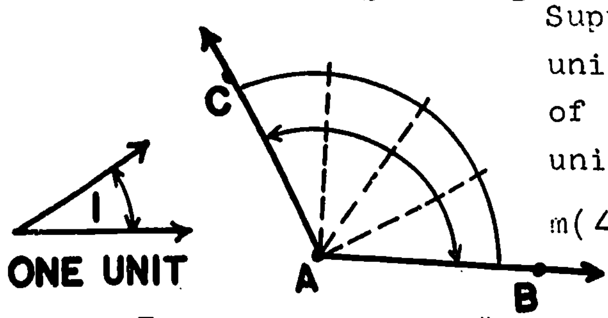
[$m(\overline{AB})$ is read "the measure of segment AB".]

MEASURING ANGLE SIZE . . .

When measuring an angle,...

(1) an angle is selected to use as a unit of measure, and...

(2) the angle to be measured is compared with the unit of measure by counting the number of unit angles it takes to "fill up" the opening of the angle being measured.



Suppose $\angle 1$ is selected as the unit of measure. The measure of $\angle CAB$ to the nearest whole unit is...

$m(\angle CAB) = 4$ units.

[$m(\angle CAB)$ is read " the measure of angle C-A-B".]

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Packets of angle wedges

B. TEACHER

1. Overhead projector
2. Projection screen

*3. Acetate ruler (same packet used in ELM)

4. Three acetate protractors - each using a different unit of measure.

CONTENT AND APPROACH

This lesson formalizes the process of measuring angles described in Lesson 1. Notice that pp. 14-16 indicate the similarities between measurement of length and angle size.

The acetate protractors and rulers can be used to provide additional examples like those on page 15, highlighting the similarities between angle and linear measure.

The examples on page 15 illustrate that <u>the measure of a length or angle</u>...

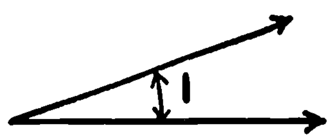
(1) depends on the unit of measure selected, and...

(2) is the number of unit angles or lengths it takes to "fill up" the length or opening of the angle being measured.

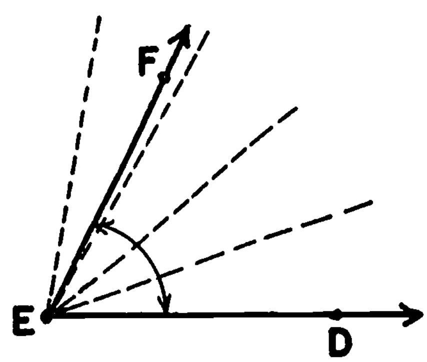
SOME DON'T COME OUT EVEN . . .

When measuring length, most measurements don't "come out even". For example, $m(\overline{AB})$ on page 15 was not exactly 3 centimeters nor 4 centimeters but was somewhere between 3 and 4 centimeters.

The same is true of angle measure. Using $\angle$ 1 below as the unit of measure, $m(\angle DEF)$ is between 3 and 4 units.

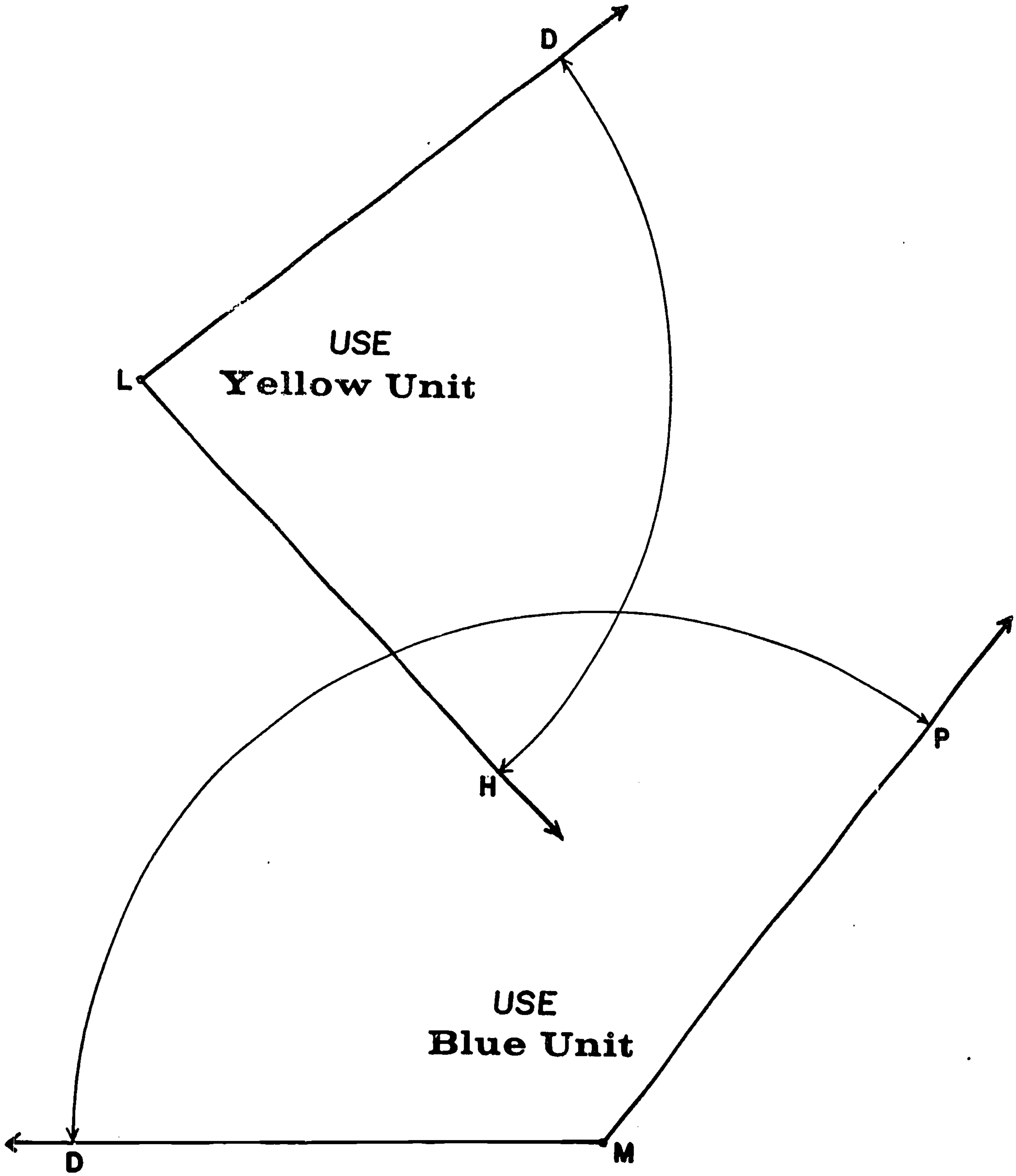


ONE UNIT

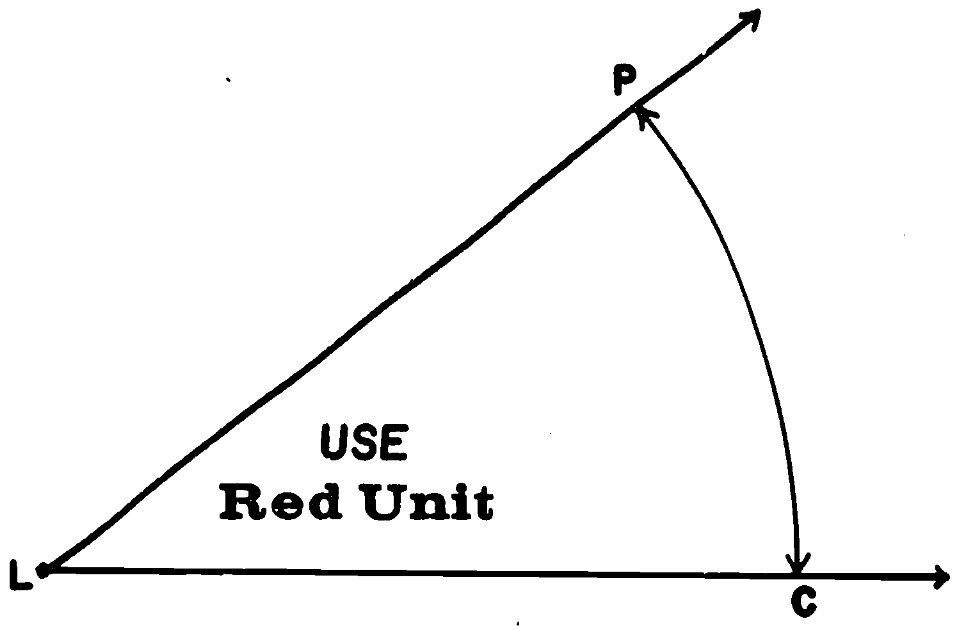


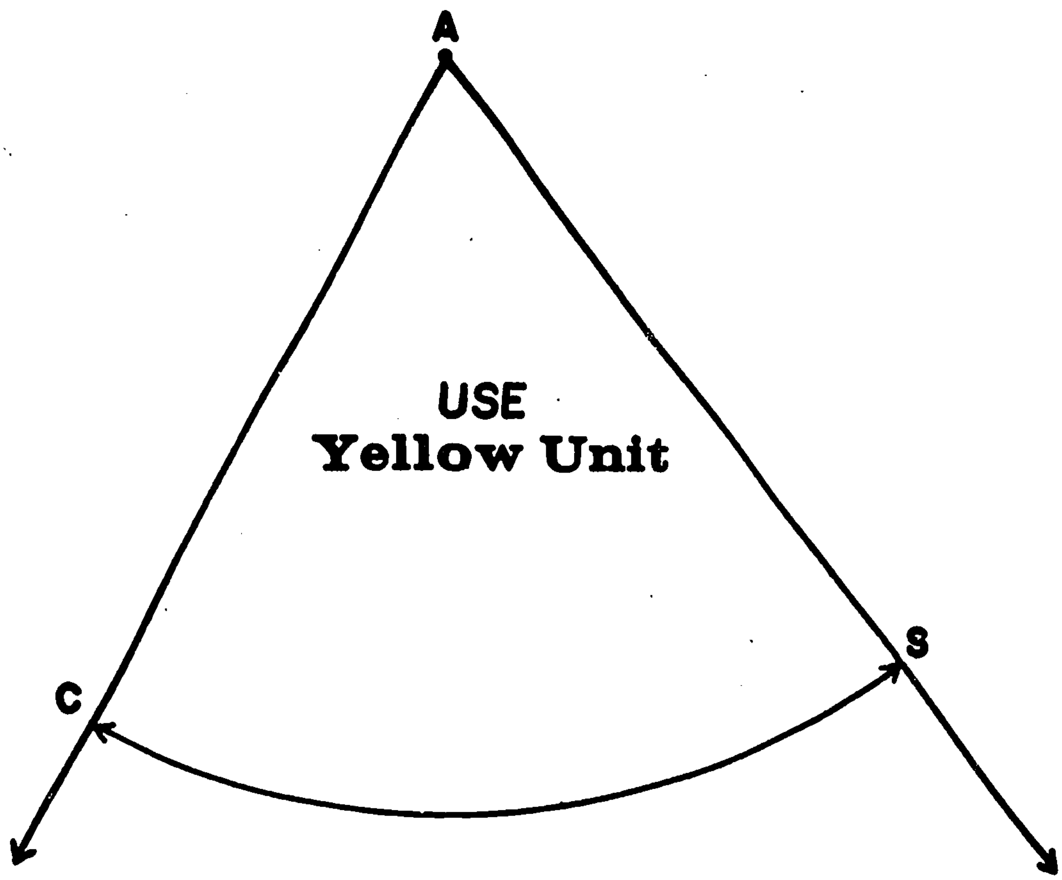
We write this: <u>3 units < m($\angle$ DEF) < 4 units</u>. This statement is read "the measure of $\angle$ DEF is greater than 3 and less than 4."

What is $m(\angle DEF)$ when measured to the nearest whole unit? (Use $\angle$ 1 as the unit of measure.) You have to decide whether $m(\angle DEF)$ is closer to 3 units or 4 units. Determining to which of the two units $m(\angle DEF)$ is closer involves sighting, estimating, and judgment. In order to measure accurately, you must be able to make accurate guesses as to which of two units the measure is closer.



For use with CLASS ACTIVITY , p. 17.





For use with CLASS ACTIVITY , p. 17.

CONTENT AND APPROACH

The statements at the top of page 16 summarize the similarities between linear and angle measure illustrated on page 15.

The section **SOME DON'T COME OUT EVEN** emphasizes the approximate nature of measurement with particular emphasis on angle measure. The format is like that used in the earlier booklet **EXPLORING LINEAR MEASURE**. (See **CONTENT AND APPROACH**, p.T 7 of this booklet.) The comments in the last paragraph emphasize the reliance of accurate measurement upon accurate judgment by the person who is measuring.

CLASS ACTIVITY

For this activity use the unit angle wedges that were used in lesson 1.

INSTRUCTIONS:

Measure the angles pictured on both sides of the insert between pages 16 and 17, using the unit given for each angle.

Record your results in the table below.

ANGLE MEASURED	UNIT OF MEASURE	COMPLETE THE STATEMENTS
1. $\angle$ DLH	Yellow	___ units < m($\angle$ DLH) < ___ units. To the nearest whole unit, m($\angle$ DLH) = ___ units.
2. $\angle$ DMP	Blue	___ units < m($\angle$ DMP) < ___ units. To the nearest whole unit, m($\angle$ DMP) = ___ units.
3. $\angle$ PLC	Red	___ units < m($\angle$ PLC) < ___ units. To the nearest whole unit, m($\angle$ PLC) = ___ units.
4. $\angle$ SAC	Yellow	___ units < m($\angle$ SAC) < ___ units. To the nearest whole unit, m($\angle$ SAC) = ___ units.

BECOMING MORE PRECISE....

All measurements are approximations. A more precise measure of an angle is one in which a better approximation of the angle's size is obtained.

CONTENT AND APPROACH

The angles to be measured in the **CLASS ACTIVITY** are reproduced on both sides of the insert between pages 16 and 17. The units of measure are those included in the packet of unit angle wedges.

If additional practice is needed, additional angles may be given and/or the four angles represented could be measured using unit wedges other than those indicated in the table.

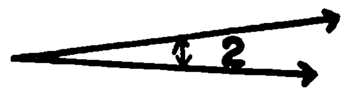
ANSWERS

ANGLE MEASURED	UNIT OF MEASURE	COMPLETE THE STATEMENTS
1. $\angle$ DLH	Yellow	_5_ units < m($\angle$ DLH) < _6_ units. To the nearest whole unit, m($\angle$ DLH) = _6_ units.
2. $\angle$ DMP	Blue	_4_ units < m($\angle$ DMP) < _5_ units. To the nearest whole unit, m($\angle$ DMP) = _4_ units.
3. $\angle$ PLC	Red	_3_ units < m($\angle$ PLC) < _4_ units. To the nearest whole unit, m($\angle$ PLC) = _4_ units.
4. $\angle$ SAC	Yellow	_4_ units < m($\angle$ SAC) < _5_ units. To the nearest whole unit, m($\angle$ SAC) = _4_ units.

BECOMING MORE PRECISE defines precision of angle measure in the same manner precision of linear measure was defined in the booklet **EXPLORING LINEAR MEASURE**. The definition is illustrated by the examples on pages 18 and 19.

EXAMPLE

∠ CAB has been measured to the nearest whole unit, using two different units of measure. Which unit gives the more precise measure (the better approximation of the size of ∠CAB)?



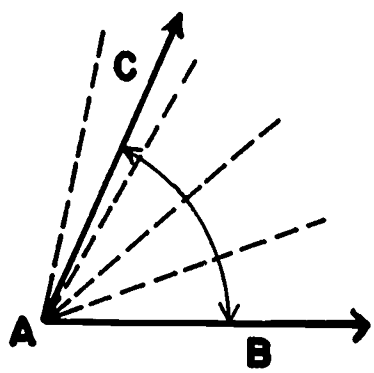


Figure 1

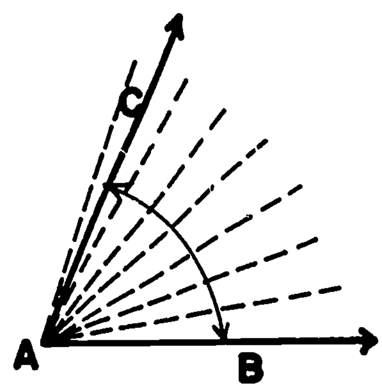


Figure 2

Using ∠ 1 as the unit of measure,...

3 units < m(∠CAB) < 4 units

Using ∠ 2 as the unit of measure,...

6 units < m(∠CAB) < 7 units

In Figure 1:

m(∠CAB) lies somewhere in the interval between 3 units and 4 units. The measure is located in an interval equal to the size of ∠ 1, the unit of measure.

In Figure 2:

m(∠CAB) lies somewhere in the interval between 6 units and 7 units. The measure is located in an interval equal to the size of ∠ 2, the unit of measure.

CONTENT AND APPROACH

Use the example (pp. 18-19) to illustrate the definition of precision given. The key point is that a more precise measure will give a better approximation of the size of the object (angle) being measured.

Which of the two measurements are more precise? The more precise measurement will give the better approximation of the angle size.

Since the interval is smaller in Figure 2, the measure obtained for $\angle$ CAB, in Figure 2, is a better approximation of its actual size.

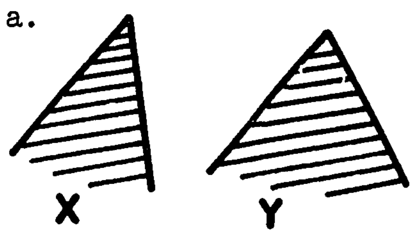
Therefore $\angle$ 2 gives a more precise measure than $\angle$ 1 as it locates m($\angle$ CAB) in a smaller interval.

PRECISION	— depends on the unit of measure. The smaller unit of measure, the more precise the measurement.

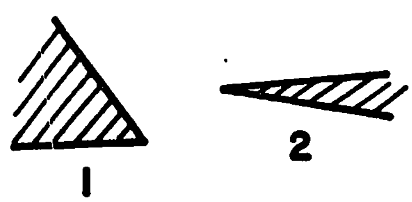
✓ **POINT**

1. Which unit in each pair will give the more precise measurement?

a.



b.



2. Suppose blobs and gobs are units for measuring angles. When $\angle$ 1 is measured to the nearest unit,...

   2 blobs < m($\angle$1) < 3 blobs
   4 gobs < m($\angle$1) < 5 gobs

   a. Which is the larger unit of measure, a blob or a gob?

   b. Which unit will give the more precise measurement?

CONTENT AND APPROACH

The student should use the ✓POINT to check his understanding of precision. The definition in the box summarizes the discussion of pp. 17-19. The exercises in the ✓POINT are similar to those in questions 4 and 5 of the EXERCISES.

ANSWERS

✓POINT

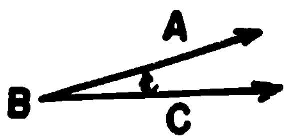
1. a. x b. 2

2. a. blob

   b. gob

EXERCISES

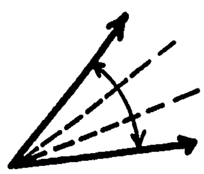
1. Suppose an angle with an opening as large as that of $\angle$ ABC has a measure of 1 unit.



ONE UNIT

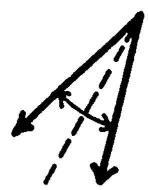
If the measure of $\angle$ ABC is one unit, what is the measure of the following angles?

a.



_____**UNITS**

d.

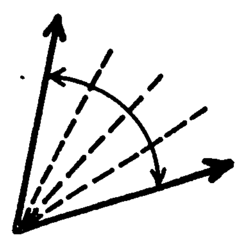


_____ **UNITS**

b.

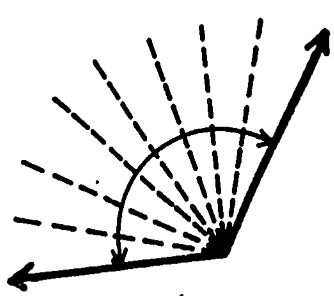
_____**UNITS**

e.



_____**UNITS**

c.



_____**UNITS**

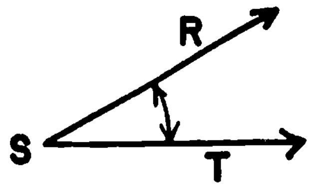
2. The measure of the angle in 1(c) is how many times the measure of the angle in 1(d)? _____ How many times the measure of the angle in 1 (e)? _____

ANSWERS

1. a. <u>3</u> units
   b. <u>5</u> units
   c. <u>8</u> units
   d. <u>2</u> units
   e. <u>4</u> units

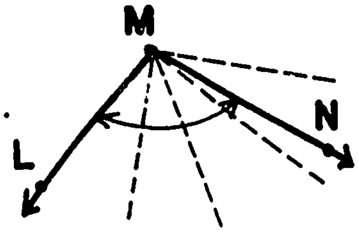
2. 4 times ; 2 times

3. Use $\angle$RST as the unit angle. For each angle pictured, complete the blanks to make true statements.



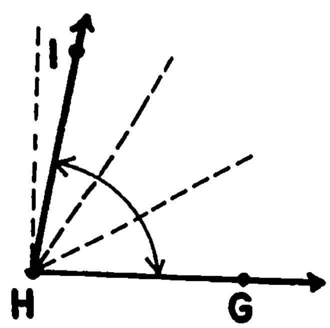
m($\angle$RST) = 1 UNIT

a.



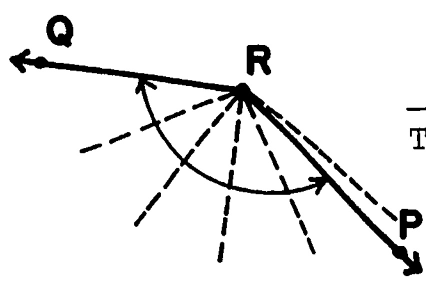
___ units < m($\angle$ LMN) < ___ units.
To the nearest whole unit,
m($\angle$ LMN) = ___ units.

b.



___ units < m($\angle$ GHI) < ___ units.
To the nearest whole unit,
m($\angle$ GHI) = ___ units.

c.



___ units < m($\angle$ QRP) < ___ units.
To the nearest whole unit,
m($\angle$ QRP) = ___ units.

4. Suppose zips and zaps are units for measuring angles. When $\angle$ TGC is measured,...

m($\angle$ TGC) = 10 zips
m($\angle$ TGC) = 6 zaps

a. Which is the smaller unit of measure?
b. Which unit will give a more precise measurement?

ANSWERS

3. a. $\underline{3}$ units < $m(\angle LMN) <$ $\underline{4}$ units.

   To the nearest whole unit,
   $m(\angle LMN) = \underline{3}$ units.

   b. $\underline{2}$ units < $m(\angle GHI) <$ $\underline{3}$ units.

   To the nearest whole unit,
   $m(\angle GHI) = \underline{3}$ units.

   c. $\underline{4}$ units < $m(\angle QRP) <$ $\underline{5}$ units.

   To the nearest whole unit,
   $m(\angle QRP) = \underline{5}$ units.

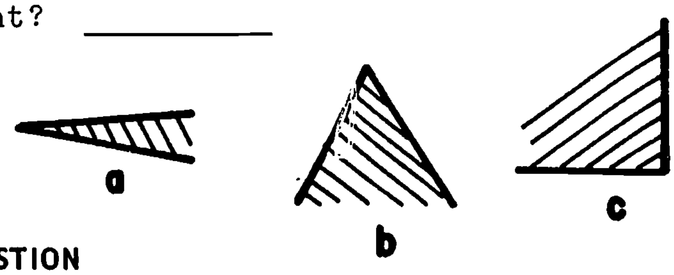
4. a. A <u>zip</u> is smaller

   b. <u>zip</u>

THINGS TO DISCUSS

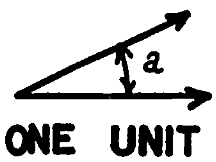
Notice that the angles in 3(a) and 3(b) have the same measure when measured to the nearest whole unit but are not the same size. The students could be asked if angles of the same measure are always the same size, reinforcing the distinction between the measure and size of an angle.

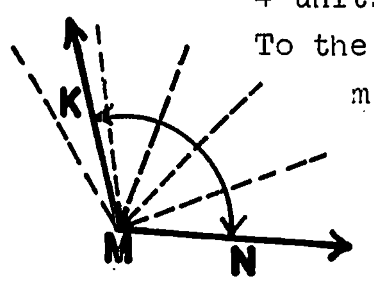
5. Which of the following units will give the most precise measurement? __________



DISCUSSION QUESTION

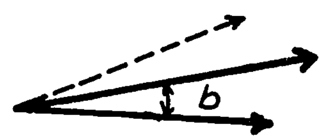
1. Using $\angle a$ as the unit of measure,





4 units $<$ m($\angle$ NMK) $<$ 5 units.
To the nearest whole unit,
m($\angle$ NMK) = 4 units.

Suppose a new unit of measure ($\angle$ b) is selected which is $\frac{1}{2}$ the size of $\angle$ a.



Would the following statements be true for $\angle$ NMK? Why or why not?

8 units $<$ m($\angle$ NMK) $<$ 10 units.
To the nearest whole unit,
m($\angle$ NMK) = 8 units.

USING OTHER METHODS . . .

This booklet describes the most common method of measuring angles, using a unit angle as the unit of measure.

Another possibility for measuring angle size is discussed in the section entitled **CAN A RULER BE USED TO MEASURE ANGLES?** in **APPENDIX C** at the back of this booklet.

ANSWERS

EXERCISES

5. unit a

DISCUSSION QUESTION

The use of this question is <u>optional</u>. If used, however, it is preferable to explore and discuss it with the entire class. The first statement (8 units $<$ m($\angle$ NMK) $<$ 10 units) is true. The second statement is not necessarily true.

The two statements concerning m($\angle$ NMK) when $\angle$ a is used as the unit of measure imply that 4 units $<$ m($\angle$ NMK) $<$ $4\frac{1}{2}$ units. The following possibilities exist concerning m($\angle$ NMK) to the nearest whole unit with $\angle$ b as the unit of measure.

UNIT OF MEASURE

$\angle$ a	$\angle$ b
If 4 units $<$ m($\angle$ NMK) $<$ $4\frac{1}{4}$ units,	then 8 units $<$ m($\angle$ NMK) $<$ $8\frac{1}{2}$ units. To the nearest whole unit, m($\angle$ NMK) = 8 units.
If $4\frac{1}{4}$ $\leq$ m($\angle$ NMK) $<$ $4\frac{1}{2}$ units,	then $8\frac{1}{2}$ units $\leq$ m($\angle$ NMK) $<$ 9 units. To the nearest whole unit, m($\angle$ NMK) = 9 units.

THINGS TO DISCUSS

The section **USING OTHER METHODS** refers to **APPENDIX C**. The activity in the Appendix is an <u>enrichment</u> activity designed to challenge some of your better students.

ANGLES FORMED BY ROTATIONS

In Lessons 1 and 2, angles were thought of as the union of two rays having a common endpoint. Another way to think of angles is to consider the amount of rotation that is made when an object turns.

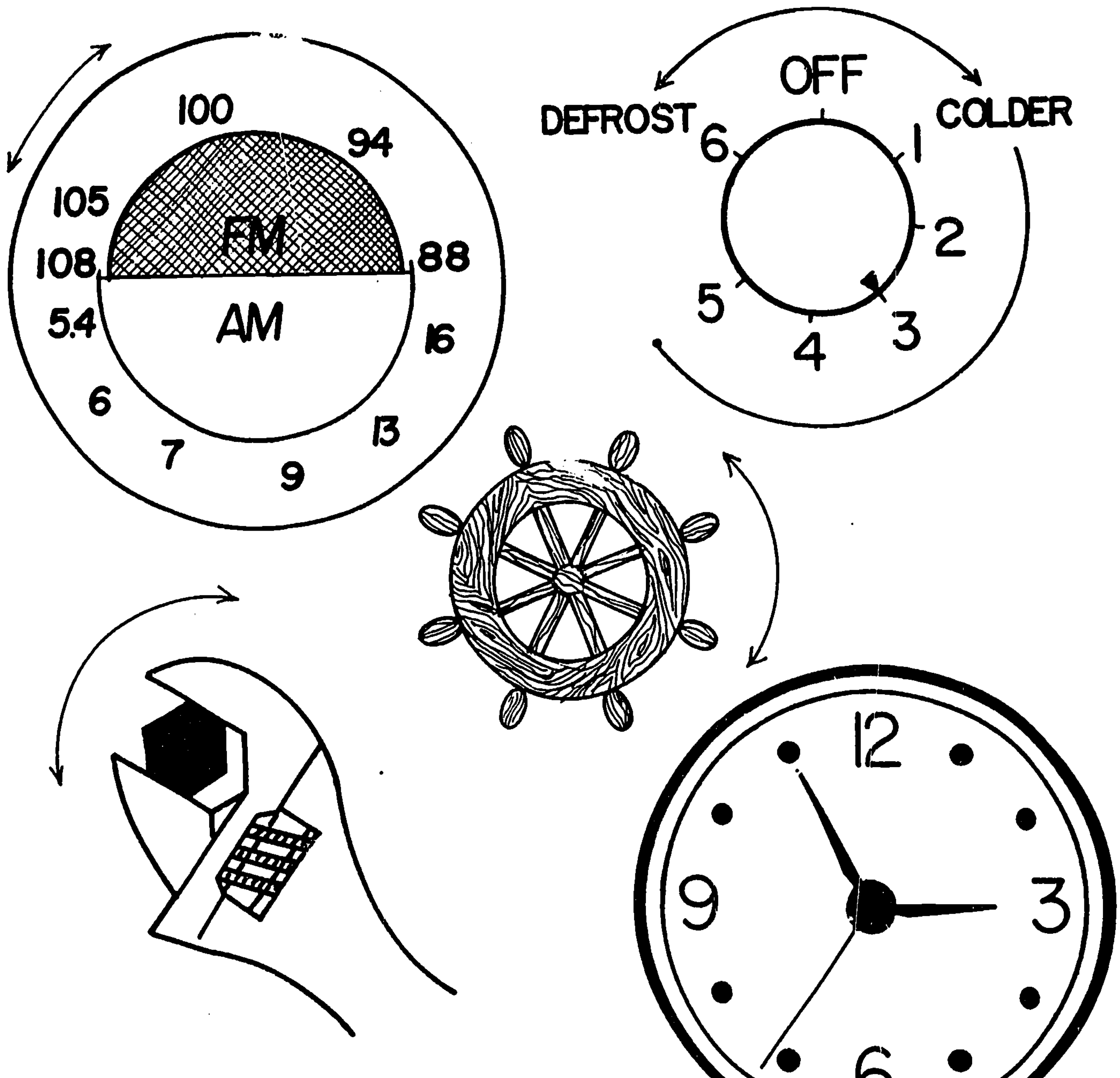


Figure 1

ANGLES FORMED BY ROTATIONS

OBJECTIVES

1. To introduce the student to the concept of an angle formed by a rotation.

2. The student shall demonstrate his understanding of the measure of an angle swept out by a rotation. A turn is defined as one complete rotation. The student will be able to measure an angle swept out in an amount less than, equal to or greater than one turn. The unit of measure is a fractional amount of turn. Congruent arcs are laid off on the circle as an aid in determining the fractional amount of turn. The direction of rotation is indicated.

EQUIPMENT AND TEACHING AIDS

A drawing compass is recommended for each student to aid in visualizing an angle formed by an amount of turn.

TRANSPARENCY

3-1. An acetate circle fastened to a base so it can pivot at its center. Two dark radius lines determine an angle of rotation.

3-2. Six-inch circle;eight congruent arcs.

3-3. Six-inch circle;twenty-four congruent arcs.

3-4. Six-inch circle;thirty-six congruent arcs.

3-5. Six-inch circle;three hundred and sixty congruent arcs.

Purpose:

To illustrate the measure of an angle formed by a rotation.

Lay a pencil down on your desk. Hold the eraser end firm and rotate the free end. Notice the amount of turn.

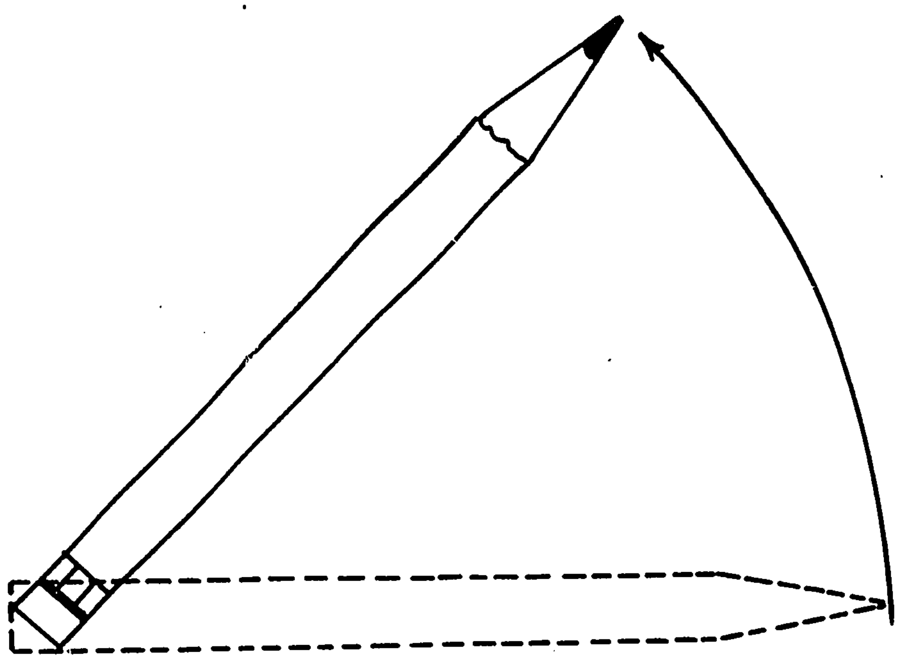


Figure 2

The diagram in Figure 2 indicates that the pencil has made about one-eighth of a turn.

A turn is one complete revolution.

CONTENT AND APPROACH

The notion of an angle formed by two rays with a common endpoint is rather static. This lesson presents an alternate notion: the rotation of a ray in the plane. The measure of an angle formed by a rotation is the amount of turn.

Start the lesson off by discussing angles determined by a rotation. Many such angles can be seen in everyday life. Refer to the drawings in Figure 1. The drawings are: (1) a radio station selection dial; (2) a refrigerator control dial; (3) a steering wheel; (4) a wrench turning a bolt; and (5) a clock face.

The main idea to this discussion is to make the notion of an amount of opening determined by a rotation more meaningful.

Demonstrate the rotation of a pencil on the stage of the overhead projector. Emphasize the definition: A turn is one complete revolution. Demonstrate such angles as those formed by $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{[illegible]}$ of a turn. Make a mark to record the starting position. The students will find it difficult to estimate the amount of turn. This is good since it will make the use of scaled circles (See page 27) more meaningful.

See Appendix A, Number 1

The second hand on a clock makes <u>one turn</u> every 60 seconds. What part of a turn does it make in 15 seconds? In 30 seconds?

The measure of an angle swept out by a rotation is the amount of turn.

Thus the measure of the angle swept out by the second hand of a clock in 20 seconds is $\frac{1}{3}$ of a turn.

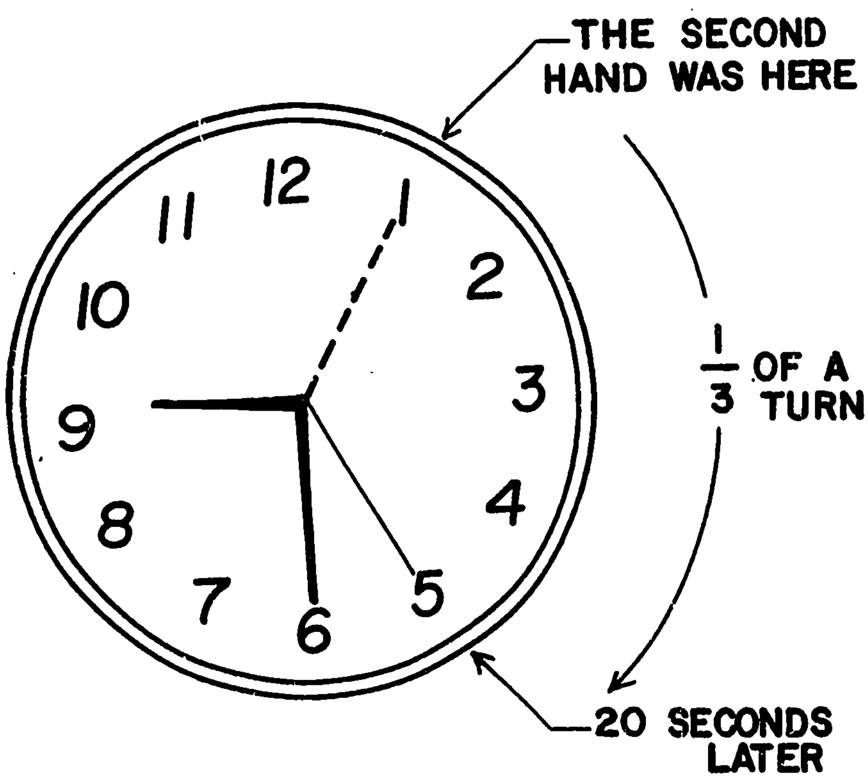


Figure 3

To determine the amount of turn you must keep track of the starting position.

What is the measure of the angle swept out by the minute hand in 45 minutes?

The clock face provides a familiar object to discuss the measure of an angle swept out by a rotation. The second hand sweeps out $\frac{1}{4}$ of a turn in 15 seconds and $\frac{1}{2}$ of a turn in 30 seconds.

The measure of an angle swept out by a rotation is the amount of turn. The question in the student's mind is how one determines the amount of turn. The clock face is divided into 12 congruent arcs. In 20 seconds the second hand sweeps by 4 of these 12 arcs. Hence, 4 out of 12 is $\frac{1}{3}$ of a turn.

It is hoped that this type of thinking will lead to more understanding, in later lessons, of the protractor as a means of measuring angles.

NOTE: Lessons 1-5 are written so that it is unimportant whether or not the student knows what a degree, in angle measure, is.

The circle provides a useful way of studying the angle swept out by an amount of turn.

A scale is provided in cases where the amount of turn must be measured carefully.

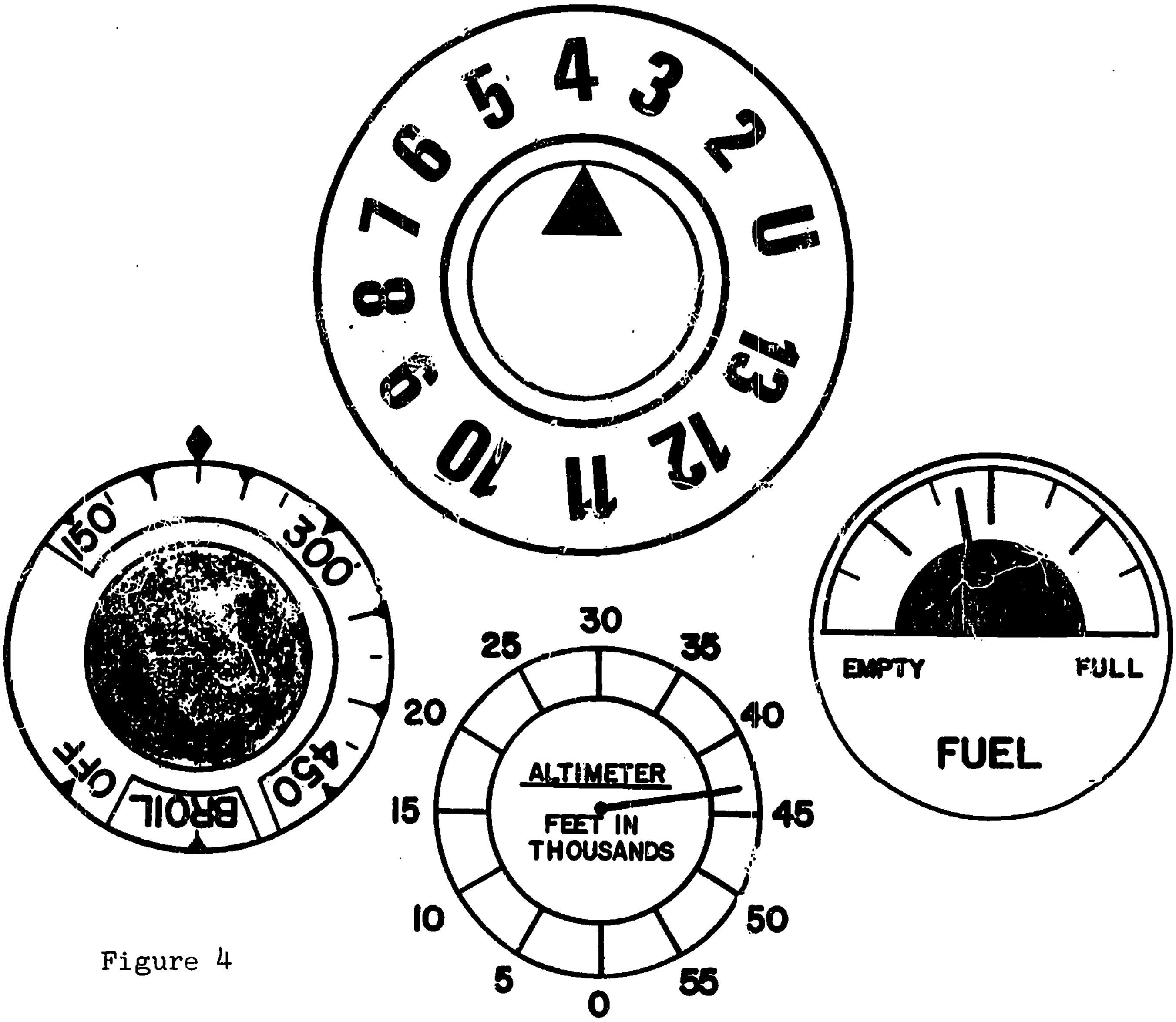


Figure 4

What part of a turn is necessary to change the T.V. from channel 2 to 7? Use both directions. What do you notice about the sum of the two fractions?

The illustrations in Figure [illegible] show a heat control on an electric frying pan; a T.V. channel selector; a fuel gauge and an altimeter. The students should be encouraged to suggest other objects from everyday life that utilize an amount of turn on some sort of circular scale.

It takes $\frac{5}{13}$ of a turn to go from channel 2 to channel 7, turning the selector dial to the right. It takes $\frac{8}{13}$ of a turn to go from 2 to 7 when turning the dial to the left.

The students should notice that: $\frac{8}{13} + \frac{5}{13} = 1$.

They should be able to generalize, although it is not necessary to do so at this time, that the two fractions in this type of a situation add up to one. The booklet deals with the question of indicated direction of rotation in Lesson 4.

Here are some computation problems that you may use at this point: (See also page T 31.)

Solve for N.

(1) $\frac{1}{8} + \frac{7}{8} = N$, $N = 1$

(2) $\frac{3}{24} + \frac{21}{24} = N$, $N = 1$

(3) $1 - \frac{3}{8} = N$, $N = \frac{5}{8}$

(4) $\frac{1}{16} + N = 1$, $N = \frac{15}{16}$

(5) $N - \frac{3}{12} = \frac{9}{12}$, $N = 1$

(6) $1 - N = \frac{3}{20}$, $N = \frac{17}{20}$

The circle in Figure 5 has been separated into 8 congruent arcs.

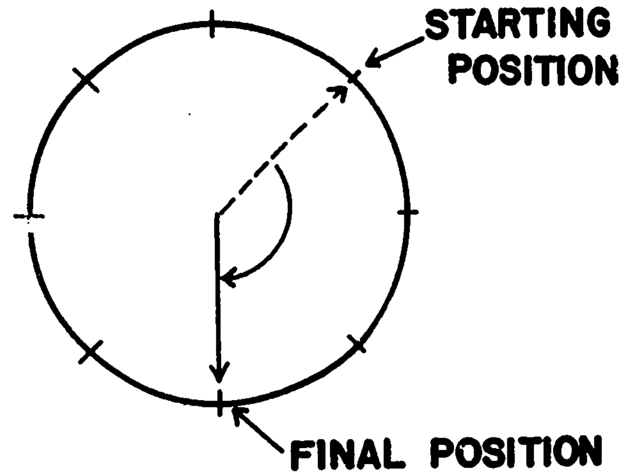


Figure 5

Each arc is what part of the circle?

The measure of the angle swept out is $\frac{3}{8}$ of a turn. Why?

The circle in Figure 6 has been separated into 12 congruent arcs.

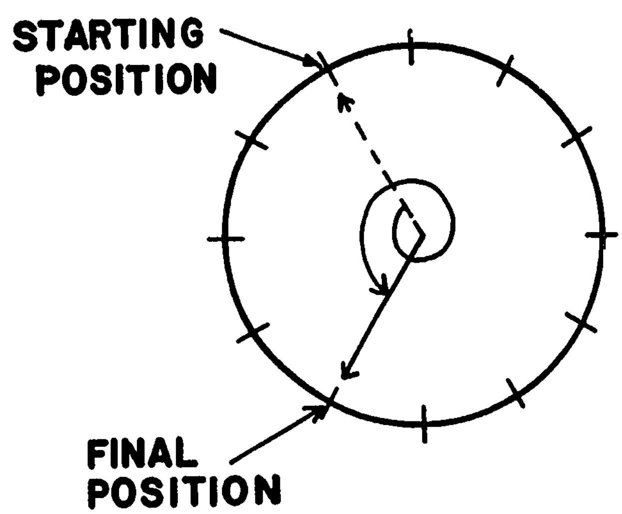


Figure 6

The measure of the angle swept out is $\frac{16}{12}$ turns. Why?

Explain congruence in terms of same size and same shape. Point out that if two circles, having different radii, are both divided into 8 congruent arcs, the arcs of one circle will not be congruent to the arcs of the other circle.

In Figure 5, each arc is $\frac{1}{8}$ of the circle. The measure of the indicated angle is $\frac{3}{8}$ of a turn because the ray has swept through 3 of the 8 arcs.

Use Transparencies 3-1 through 3-5 to present a variety of angles to the class. Check to see if they have the idea: The measure of an angle is the ratio of the number of arcs swept through to the total number of arcs in the circle. Give them situations where the angle swept out is larger than one turn.

THINGS TO DISCUSS

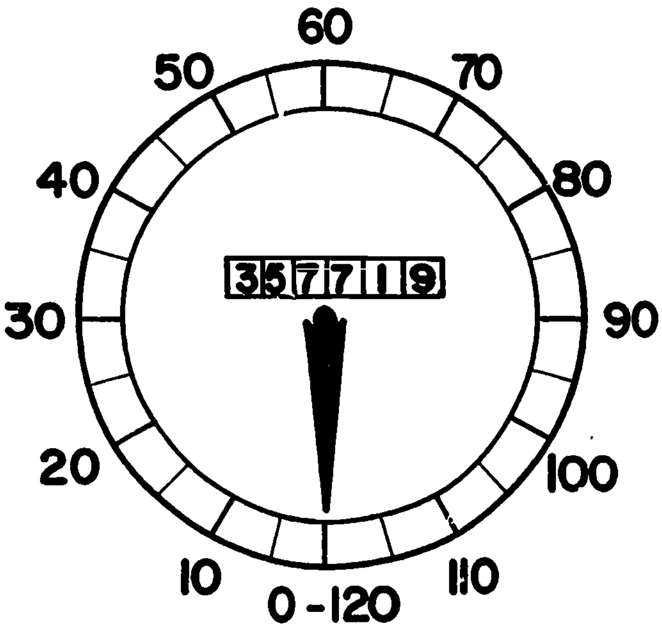
1. Everyday situations where angles are formed by a rotation. (e.g. wheel, door, fan, etc.)
2. Give situations where the measure of one angle is given and the other is found by subtracting from 1.
3. Discuss how the circle, divided into congruent arcs, helps determine the amount of turn. Compare amount of turn with amount of opening.
4. Give an angle whose measure is an improper fraction and ask how many complete rotations were made. (e.g. $\frac{25}{12}$ turns indicates 2 complete rotations plus $\frac{1}{12}$ of a turn more.)

✓ POINT

1. What is a turn?
2. What is the measure of an angle swept out by a rotation?
3. What was the measure of an angle as described in Lessons 1 and 2?
4. Each arc of the circle in Figure 6, page 27, is what part of the circle?

EXERCISES

1. The hours marked on a circular clock face divide the circle into 12 congruent arcs. Each arc is what part of the circle?
2. What is the measure of the angle you turn your T.V. channel selector to go from channel 2 to channel 4? Use the shorter direction. (See Figure 4, page 26.)
3. What is the measure of the angle swept out by the hour hand in 7 hours?
4. Suppose an automobile has a speedometer with a circular dial.



The dial is scaled from 0 mph to 120. Each mark represents 5 mph.

a) Each small arc is what part of the circle?

b) What is the measure of the angle swept out when accelerating from 0 to 40 mph?

ANSWERS TO ✓ POINT

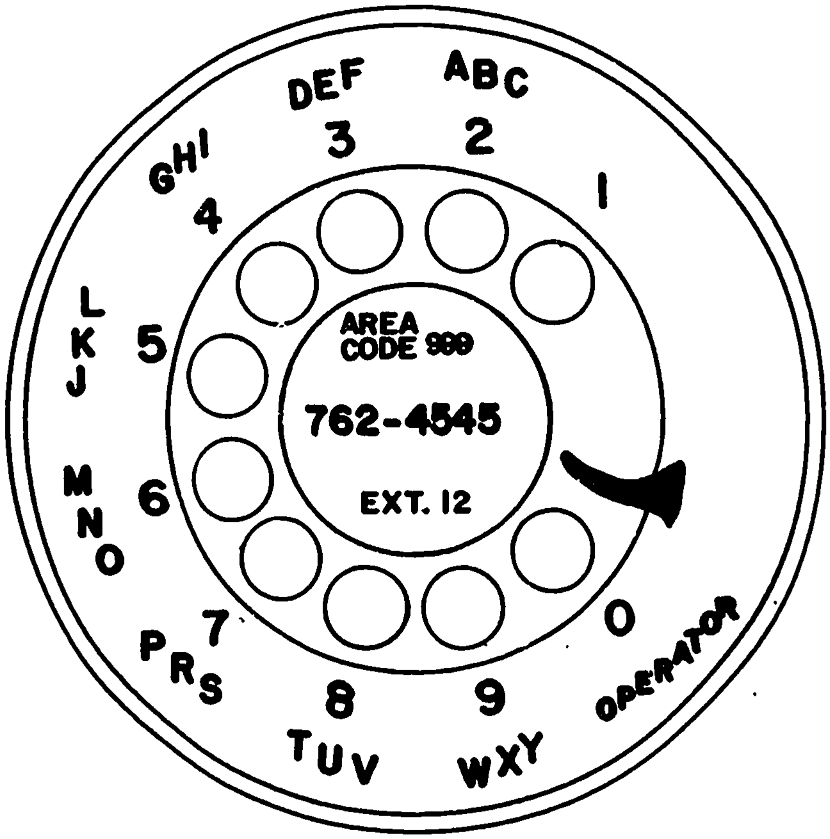
1. A turn is one complete revolution.
2. The measure of an angle swept out by a rotation is the amount of turn.
3. The angle to be measured was compared to a unit angle. The measure was the number of unit angles necessary to fill up the opening of the angle to be measured.
4. $\frac{1}{12}$

ANSWERS TO EXERCISES

1. $\frac{1}{12}$
2. $\frac{2}{13}$ of a turn.
3. $\frac{7}{12}$ of a turn.
4. a.) $\frac{1}{24}$

   b.) $\frac{8}{24}$ of a turn or $\frac{40}{120}$ of a turn. (Note: Equivalent fractions will be discussed in Lesson 4. Do not demand reduced fractions.)

5. Through approximately what size angle do you turn your phone dial when dialing the number 2? The number 5?



6. Through what size angle does the minute hand of a clock turn in:

a) 1 hour	b) 30 min.	c) 20 min.
d) 3 hours	e) 15 min.	f) 30 seconds

7. One complete turn of the pedals (with no coasting) makes John's bike travel nearly 14 feet. What angle do the pedals turn through if the bike travels:

a) 42 feet	b) 10 feet	c) 21 feet

ANSWERS CONT'D

5. (The centers of the holes in the dial telephone are approximately 30° apart. There is room for 12 evenly spaced circles.)

   a.) Notice that when you dial the number 2, each circle is displaced 3 numbers, e.g., 9 moves to 6. Thus the answer is $\frac{3}{12}$ of a turn.

   b.) $\frac{6}{12}$ of a turn.

6. a.) 1 turn. b.) $\frac{30}{60}$ of a turn. c.) $\frac{20}{60}$ of a turn.

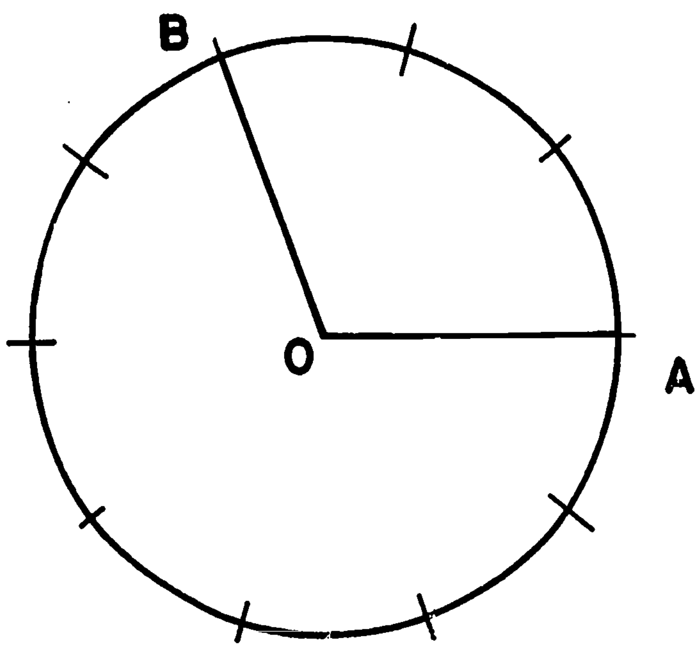
   d.) 3 turns. e.) $\frac{15}{60}$ of a turn. f.) $\frac{1}{120}$ of a turn.

7. a.) 3 turns. b.) $\frac{5}{7}$ of a turn. c.) $1\frac{1}{2}$ turns.

   The student may solve these Exercises by setting up a proportion. Example: 7. a.) $\frac{1}{14} = \frac{x}{42}$; $x = 3$.

CENTRAL ANGLES

An angle determined by two radii (pronounced ray-dee-i) of a circle is called a central angle.



Radii OA and OB of circle O form central angle AOB.

Figure 1

The measure of the angle determined by the two radii is the amount of turn necessary to make the two radii coincide.

What is the measure of $\angle AOB$? This question cannot be answered until there is an indication of the direction and amount of rotation that is intended.

CENTRAL ANGLES

OBJECTIVES

1. The student will be able to describe, by drawing the appropriate arrows, clockwise and counterclockwise rotation.

2. The student will be able to define a central angle as an angle determined by two radii of a circle.

3. Given a central angle with the direction of rotation indicated, the student will be able to measure the angle. The unit of measure is a fractional amount of turn. The circle is divided into congruent arcs as an aid in determining the measure.

4. Given an angle measure as a fractional amount of turn, an indicated direction of rotation, and a circle divided into congruent arcs (a multiple of the denominator of the given measure), the student will be able to draw a central angle having the given measure.

EQUIPMENT AND TEACHING AIDS

A drawing compass, straightedge and Transparencies 3-1 through 3-5.

CONTENT AND APPROACH

This lesson gives additional work on the ability to measure an angle using the amount of turn as the measure. The concept of a central angle helps relate the measure of the angle to the number of congruent arcs swept out in a rotation.

For Example: (See Figure 1, page 30.)

Case 1. Hold OB fixed and turn OA in a counterclockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{3}{10} \text{ of a turn.}$$

Case 2. Hold OB fixed and turn OA in a clockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{7}{10} \text{ of a turn}$$

Case 3. Hold OA fixed and turn OB in a clockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{3}{10} \text{ of a turn.}$$

Case 4. Hold OA fixed and turn OB in a counterclockwise direction to meet OA. Then

$$m(\angle AOB) = \frac{7}{10} \text{ of a turn.}$$

What do you notice about the sum of the fractions in Case 1 and Case 2?

In Lesson 8, the use of a circular protractor will be related to the central angle.

The measure of an angle formed by a rotation is related to the direction of the rotation.

The student should notice that the sum of the two fractions in cases 1 and 2 and cases 3 and 4 is 1.

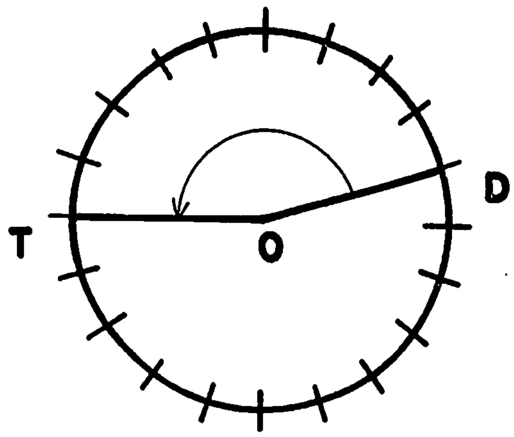
There is a small technical problem with the use of central angles since the sides of these angles are radii and not rays.

Notice that we say that the angle is determined by the radii. In Lesson 5, the students will see that these segments can be extended and not change the measure of the angle.

Test analysis from the first edition of this booklet indicates that students were weak in answering the following type of question: "A circle used to measure an angle is divided into 20 congruent arcs. What unit is being used to measure the angle?" Answer: $\frac{1}{20}$ of a turn. Give special attention to the unit of measure.

To avoid the confusion caused by the various ways in which a rotation can be made to make the radii coincide, we use an arrow.

The arrow in Figure 2 indicates how the rotation is made.



The arrow indicates how OD is rotated to meet OT. The direction is counterclockwise and the amount of turn is $\frac{9}{20}$.

Figure 2

$$m(\angle DOT) = \frac{9}{20} \text{ of a turn}$$

✓ **POINT**

1. What are the sides of a central angle?
2. Where is the vertex of a central angle?
3. How is the measure of a central angle determined?

EXERCISES

1. Imagine that you are looking down at the Earth from the North Pole. Is the Earth turning in a clockwise or a counterclockwise direction?

THINGS TO DISCUSS

1. <u>The name for an angle in this lesson follows the direction of rotation.</u> For example in ∠DCT, page 32, D is on the starting side, C is the center of the circle and T is on the terminal side.

2. Discuss the statement: The measure of the angle determined by the two radii is the amount of turn necessary to make the two radii coincide. The students should visualize the rotation of one radius as determined by the direction and amount of turn.

ANSWERS TO ✓ POINT

1. Radii.

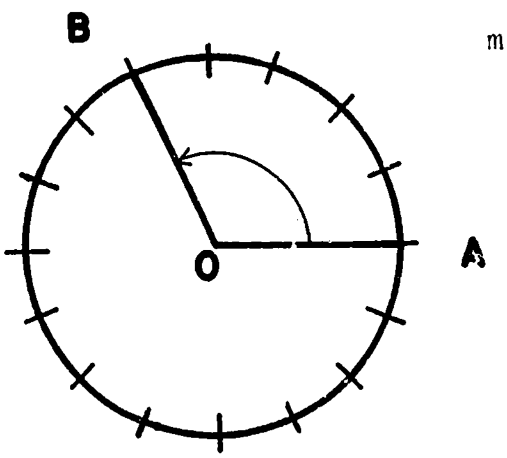
2. At the center of the circle.

3. The amount of turn necessary to make the two radii coincide.

ANSWERS TO EXERCISES

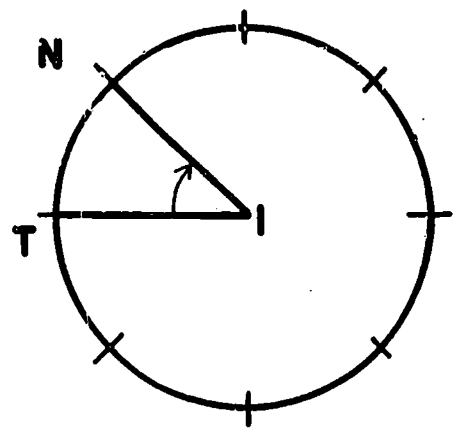
1. Counterclockwise direction.

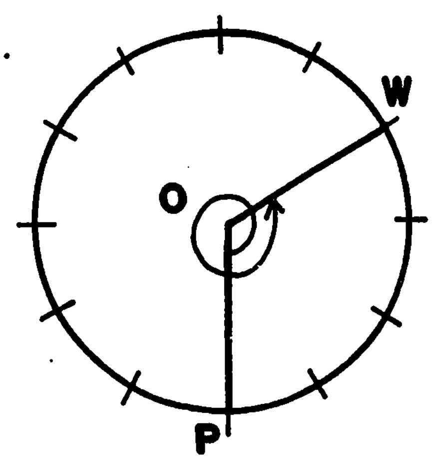
2-6. Determine the measure of each of the following central angles.

2. 

m(∠AOB) = ___ of a turn.

3. m(∠TIN) = ____



4. 

m(∠POW) = ___

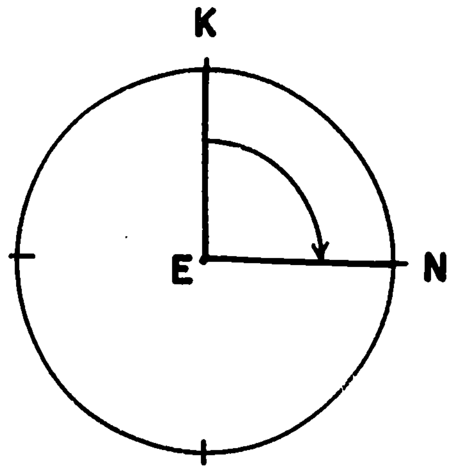
THINGS TO DISCUSS CONT'D

3. Ask what the measure of the angle is if more than one complete rotation is made prior to the final meeting of the two radii. (Answer: A measure which is an improper fraction results - the sum of the whole turns plus the fractional part remaining.) There are times when the number of whole rotations is important. For example, $\angle$ DCT may be formed by a rotating machine part. If the measure of $\angle$ DCT is $5\frac{9}{20}$, the angle appears the same as when its measure is $\frac{9}{20}$; but the fact that there were 5 complete turns in addition to $\frac{9}{20}$ of a turn could be of importance to the man running the machine.

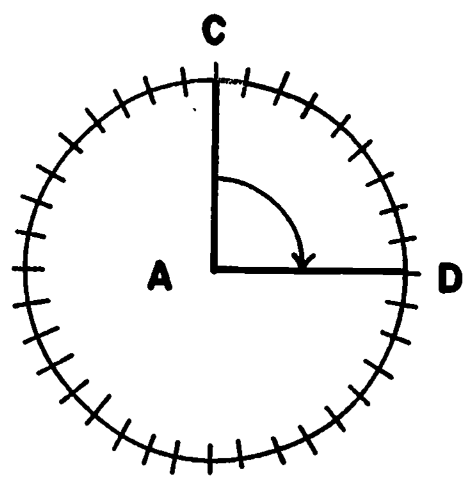
ANSWERS CONT'D

2. $\frac{5}{16}$
3. $\frac{1}{8}$ of a turn
4. $1 + \frac{4}{12} = \frac{12}{12} + \frac{4}{12} = \frac{16}{12}$ or $1\frac{4}{12}$ turns. If necessary, take this opportunity to discuss changing a mixed number to an improper fraction and vice versa.

5. m(∠ KEN) = ____



6.

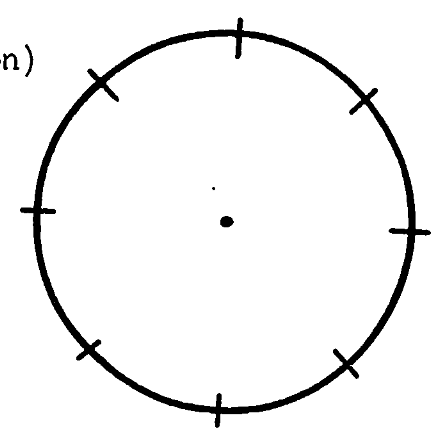


m(∠CAD) = ____

7. Which of the Exercises 2-6, show a central angle formed by a clockwise rotation?

8-10. Draw a central angle having the given measure and direction.

8. $\frac{3}{8}$ of a turn. (clockwise direction)



ANSWERS CONT'D

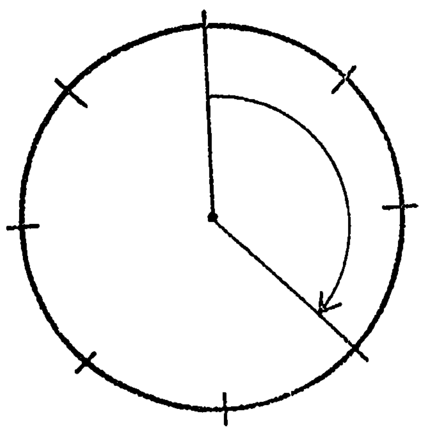
5. $\frac{1}{8}$ of a turn

6. $\frac{9}{36}$ of a turn. Note: The students should see that ∠KEN and ∠CAD "look the same". Indeed, when you reduce $\frac{9}{36}$ to lowest terms it is $\frac{1}{4}$.

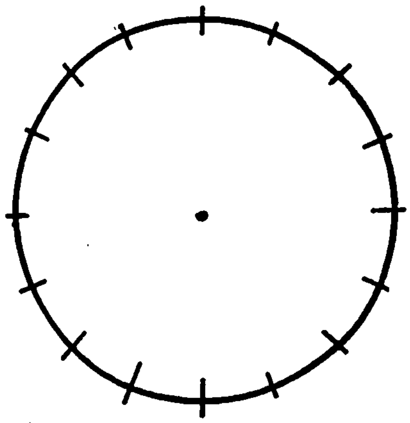
7. 3, 5 and 6.

Answers to Exercises 8-10 will vary as to the exact position of the angle as there are several positions in which to draw the first radius.

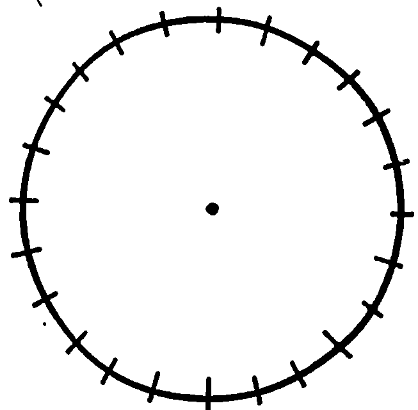
8.



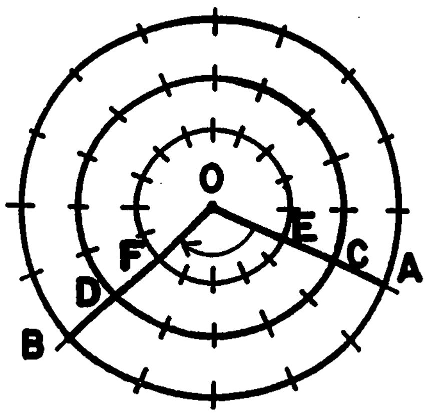
9. $1\frac{5}{16}$ turns. (counterclockwise direction)



10. $\frac{3}{12}$ of a turn. (counterclockwise direction)



11. Three concentric circles are shown below.



a) How many congruent arcs are marked off on each circle?

b) How do the lengths of the arcs on the three circles compare?

c) Give the measure of each of the three central angles.

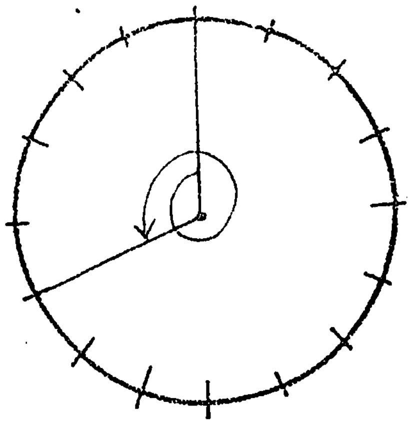
m(∠AOB) = ____

m(∠COD) = ____

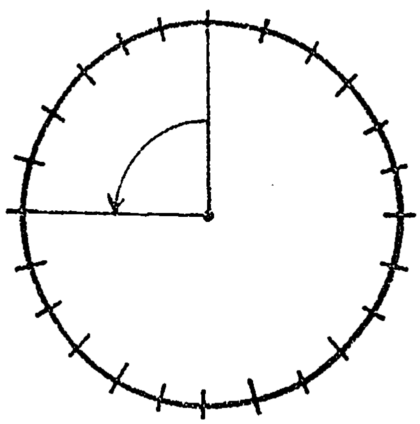
m(∠EOF) = ____

ANSWERS CONT'D

9.



10.



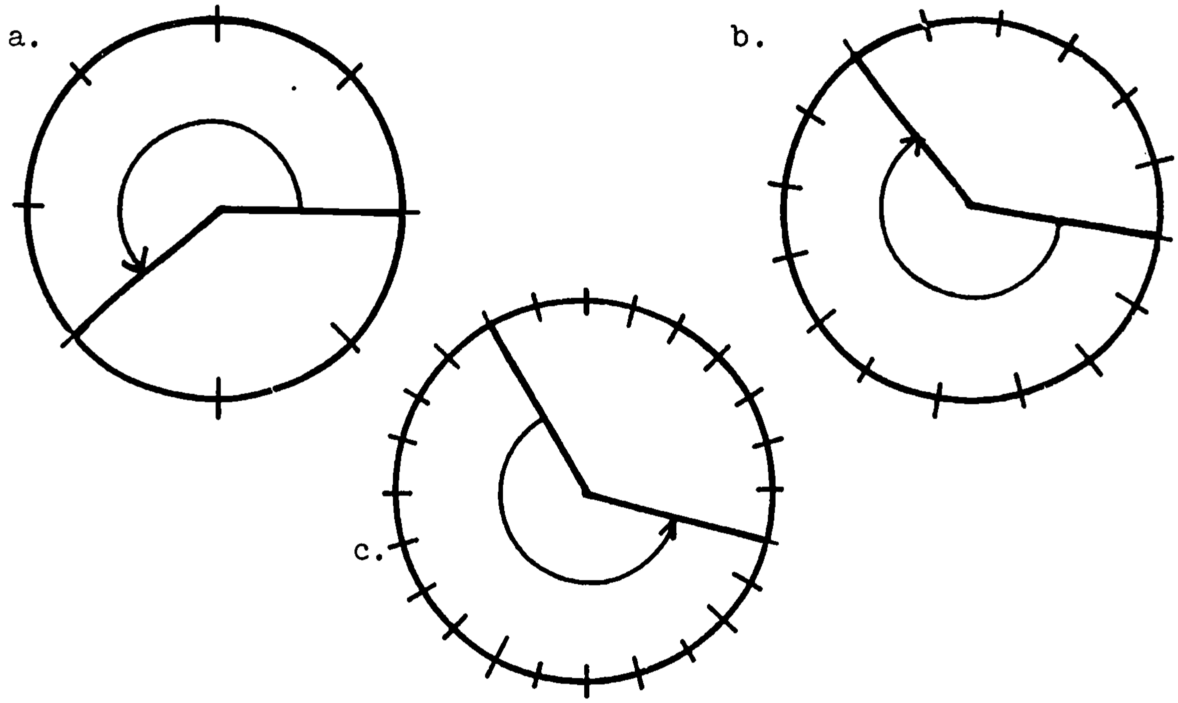
Some students will get this wrong because they will count 3 arcs and draw the angle. There are 24 arcs, so to make $\frac{3}{12}$ of a turn they will have to count 6 arcs.

11. a. 16

b. The arcs increase in length as the circles increase in radius.

c. Each measure is $\frac{5}{16}$ of a turn. This Exercise will be followed up in Lesson 5.

12. Three central angles are shown below. Give the measure of each angle. What do you conclude about the measures of these three angles?



13. A set of fractions is given in each exercise below. In each set, circle the fractions which are equivalent.

a) $\frac{1}{2}, \frac{6}{10}, \frac{16}{32}, \frac{40}{20}$.

b) $\frac{2}{7}, \frac{2}{6}, \frac{4}{12}, \frac{5}{15}$.

c) $\frac{15}{20}, \frac{1}{3}, \frac{9}{12}, \frac{1}{8}$.

d) $\frac{1}{2}, \frac{6}{48}, \frac{2}{3}, \frac{5}{40}$.

e) $\frac{10}{15}, \frac{2}{6}, \frac{2}{3}, \frac{6}{9}$.

ANSWERS CONT'D

12. a. $\frac{5}{8}$ of a turn

    b. $\frac{10}{16}$ of a turn

    c. $\frac{15}{24}$ of a turn

    The measures are equivalent. Note, however, that the direction of rotation is different for b.

13. a. $\frac{1}{2}$ and $\frac{16}{32}$

    b. $\frac{2}{6}$, $\frac{4}{12}$, and $\frac{5}{15}$

    c. $\frac{15}{20}$ and $\frac{9}{12}$

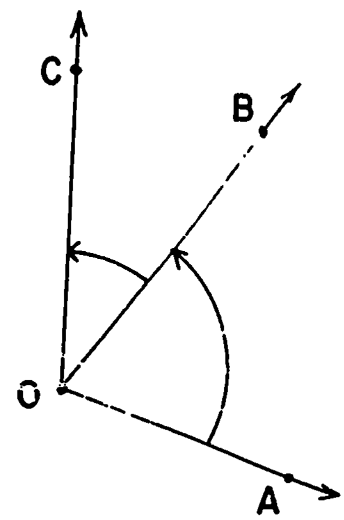
    d. $\frac{6}{48}$ and $\frac{5}{40}$

    e. $\frac{10}{15}$, $\frac{2}{3}$ and $\frac{6}{9}$

    Take this opportunity to work in equivalent fractions. Take the circle in 12 c and double the number of congruent arcs. The students should see that when the denominator for the measure of the angle is doubled, the numerator is also doubled. $\frac{15 \times 2}{24 \times 2} = \frac{30}{48}$.

See Appendix A for Worksheets Number 2, Number 3, and Number 4.

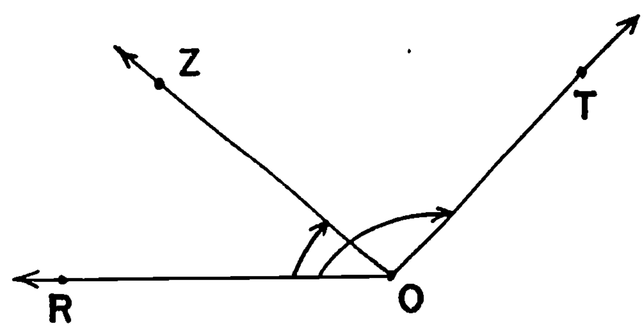
Adding and Subtracting Angles



1. The measure of angle AOB is $\frac{1}{[illegible]}$ of a turn.

   The measure of angle BOC is $\frac{1}{[illegible]}$ of a turn.

   The measure of angle AOC is _____ of a turn.

2. 

   m($\angle$ ROT) = $\frac{3}{8}$ of a turn

   m($\angle$ ROZ) = $\frac{1}{9}$ of a turn

   m($\angle$ ZOT) = __ of a turn

3. Ray MN is rotated $\frac{11}{24}$ of a turn in a counterclockwise direction. Then MN is rotated $\frac{1}{6}$ of a turn in a clockwise direction. How much farther will MN have to be rotated in a clockwise direction before it returns to its original position?

4-10 Solve for N

4. $\frac{2}{5} + \frac{2}{3} = N$

5. $1\frac{1}{6} + \frac{1}{2} = N$

6. $\frac{1}{3} + \frac{4}{5} = N$

7. $\frac{3}{4} - \frac{1}{8} = N$

8. $1 + N = \frac{13}{9}$

9. $2 + N = \frac{17}{6}$

10. $1\frac{3}{8} + 1\frac{1}{3} = N$

Answers. Adding and Subtracting Angles

1. $\frac{3}{10}$ of a turn

2. $\frac{19}{72}$ of a turn

3. $\frac{7}{24}$ of a turn

4. $\frac{16}{15}$ or $1\frac{1}{15}$

5. $1\frac{4}{6}$ or $1\frac{2}{3}$

6. $\frac{17}{15}$ or $1\frac{2}{15}$

7. $\frac{5}{8}$

8. $\frac{4}{9}$

9. $\frac{5}{6}$

10. $2\frac{17}{24}$

THE SIZE OF AN ANGLE

In Lesson 1 it was made clear that the size of an angle does not depend on the length of its sides. The five angles pictured in Figure 1 are all the same size.

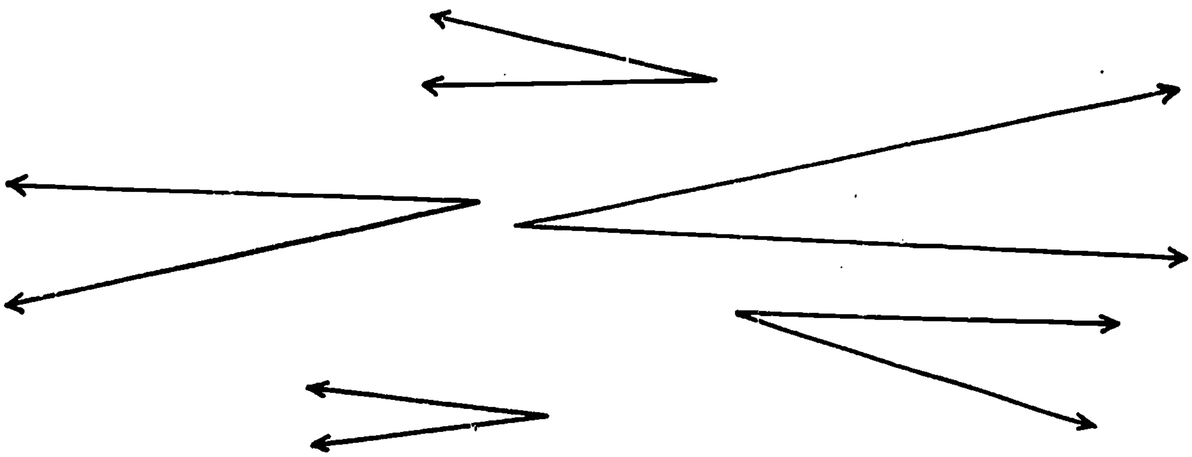


Figure 1

Remember that the sides of an angle can be thought of as <u>rays</u>. A ray can be extended indefinitely.

In Lessons 3 and 4 circles divided into congruent arcs were used to help measure the size of angles. Does the measure of an angle depend on the size of the circle used to measure it?

Each of the three circles in Figure 2 have been divided into 36 congruent arcs. Notice that the larger the circle--the longer the arc.

THE SIZE OF AN ANGLE

OBJECTIVES

1. The student is able to demonstrate his understanding that the size of an angle does not depend on the size of the circle used to make the measurement. He will be able to measure an angle using two or more different size circles.

2. Given an angle whose vertex is at the center of a circle divided into congruent arcs, where the sides of the angle do not lie on any of the marks, the student will be able to approximate the measure of the given angle. The unit is a fractional amount of turn.

3. The student is able to demonstrate his understanding of precision in angle measurement by answering a question like this:

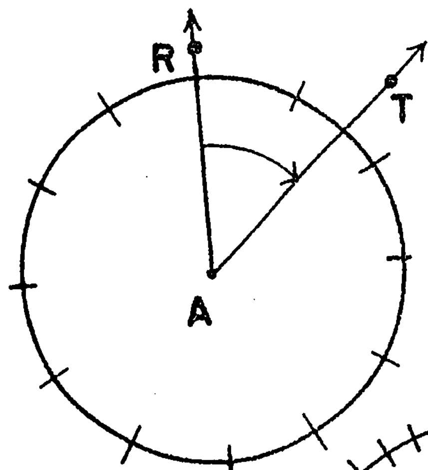


Fig. 1

In Figure 1 circle A is divided into 12 congruent arcs and in Figure 2 circle A is divided into 36 congruent arcs. Which circle helps determine the more precise measure of $\angle$ RAT?

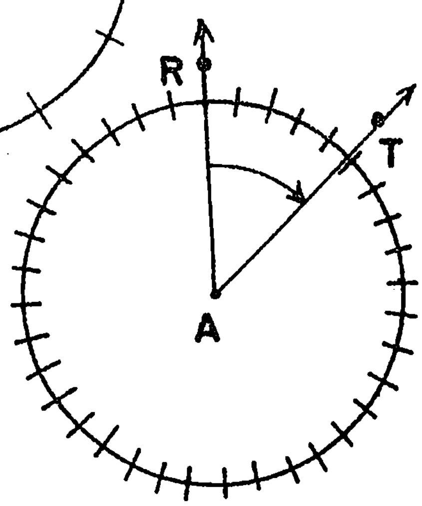
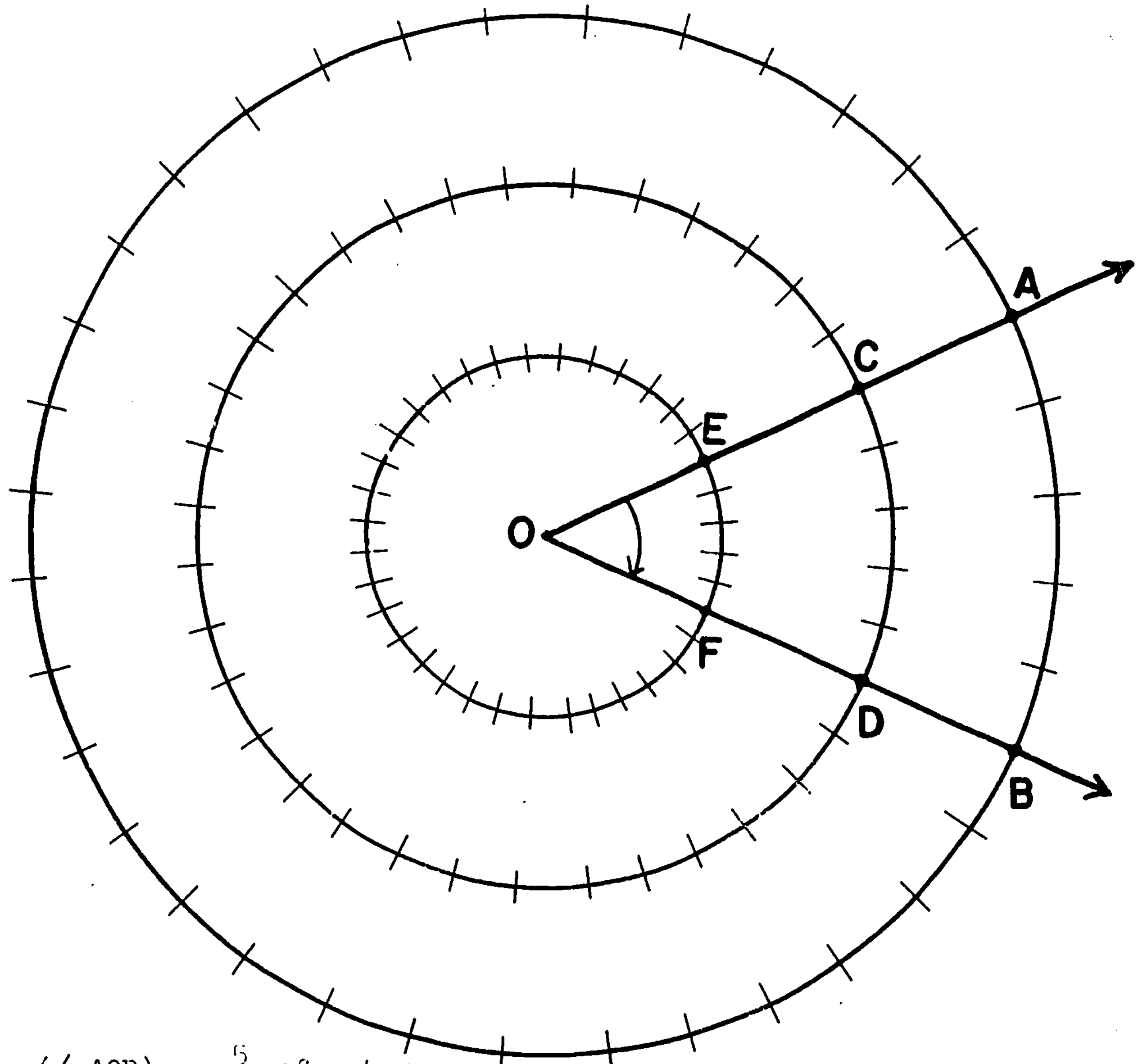


Fig. 2



$m(\angle AOB) = \frac{5}{36}$ of a turn

Figure 2

$m(\angle COD) = \frac{5}{36}$ of a turn

$m(\angle EOF) = \frac{5}{36}$ of a turn

Notice that, in each circle, the angle cuts off the same number of arcs.

THE SIZE OF AN ANGLE IS DETERMINED BY THE AMOUNT OF TURN

The size of an angle does not depend on the size of the circle used to measure it.

EQUIPMENT AND TEACHING AIDS

A drawing compass and Transparencies 3-1 through 3-5.

CONTENT AND APPROACH

This lesson zeroes in on two main ideas. The first is that the size of an angle does not depend on the length of its sides nor the size of the circle used to measure it.

The second relates to the ideas of approximation and precision.

Figure 2 on page 39 is useful for illustrating that the size of an angle does not depend on the length of its sides nor the size of the circle. Emphasize that the measure of an angle is a measure of the amount of opening. The measure of an angle will depend upon the number of congruent arcs in the circle used to measure the angle. The size of the angle, however, will stay constant regardless of the size of the circle or the number of congruent arcs used.

Note that we are not using central angles in this lesson. We have retained the circle with the vertex of the angle at the center, but now the sides of the angle are rays.

Using the notion of a ray we can always extend the sides of an angle so that they will cut the circle used to measure the angle.

The circle in Figure 3 has been separated into 20 congruent arcs. The sides of the angle, in the figure, do not lie on any of the marks. To measure this angle we must make an approximation.

Example 1. Measure $\angle$ TOM to the nearest whole unit.

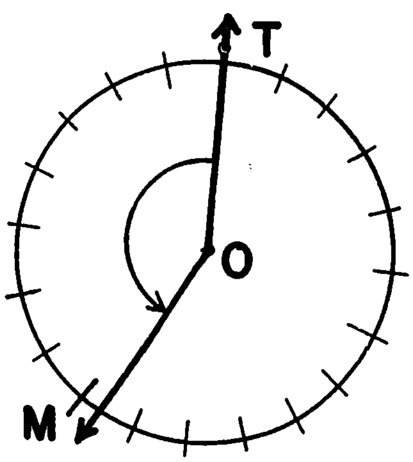


Figure 3

Since the circle is divided into 20 congruent arcs, the unit is $\frac{1}{20}$ of a turn.

$\angle$ TOM cuts off 8 units plus a little more.

$m(\angle \text{TOM}) \approx 8(\frac{1}{20} \text{ of a turn})$

$\approx \frac{8}{20}$ of a turn.

But to the nearest unit $m(\angle \text{TOM}) = \frac{8}{20}$ of a turn.

Example 2. Measure $\angle$ RAT.

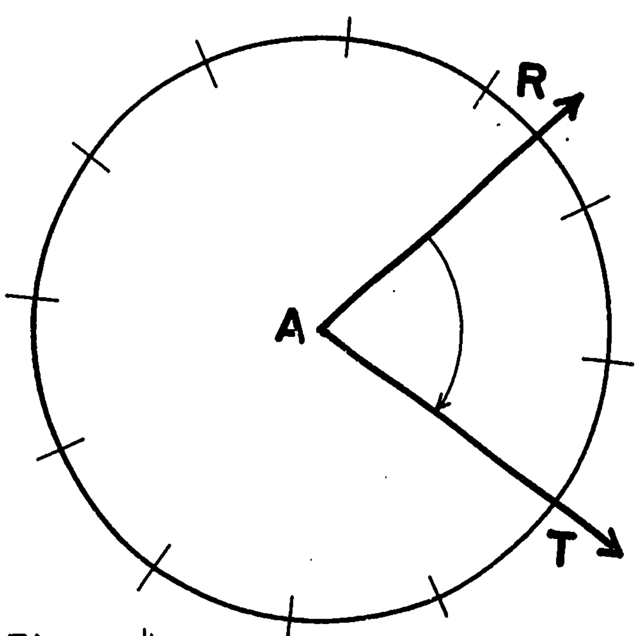


Figure 4

$\angle$RAT cuts off less than 3 of the arcs. We write

$m(\angle \text{RAT}) < \frac{3}{12}$ of a turn.

$\angle$RAT cuts off more than 2 of the arcs. We write

$m(\angle \text{RAT}) > \frac{2}{12}$ of a turn.

All measurements are approximations and the matter of precision plays a central role in approximation. This lesson attempts to relate precision to the size of the unit and the size of the interval in which the measure lies.

These two ideas are equivalent: the smaller the unit of measure the more precise the measurement; the smaller the interval in which the measure lies, the more precise the measure.

We favor the latter statement as one to start with because it seems to give the student a clearer understanding of what is meant by a better approximation (i.e., a more precise measurement).

The student should view the unit shown in Figure 3 as if a ruler has been wrapped around the circle. $\frac{1}{10}$ of a turn is a unit just like $\frac{1}{4}$ or $\frac{1}{8}$ of an inch are units.

When we talk about the measure of $\angle RAT$, in Figure 4, we establish that it lies in an interval between $\frac{2}{12}$ and $\frac{3}{12}$ of a turn.

The measure of $\angle$ RAT lies somewhere in an interval between $\frac{2}{12}$ of a turn and $\frac{3}{12}$ of a turn.

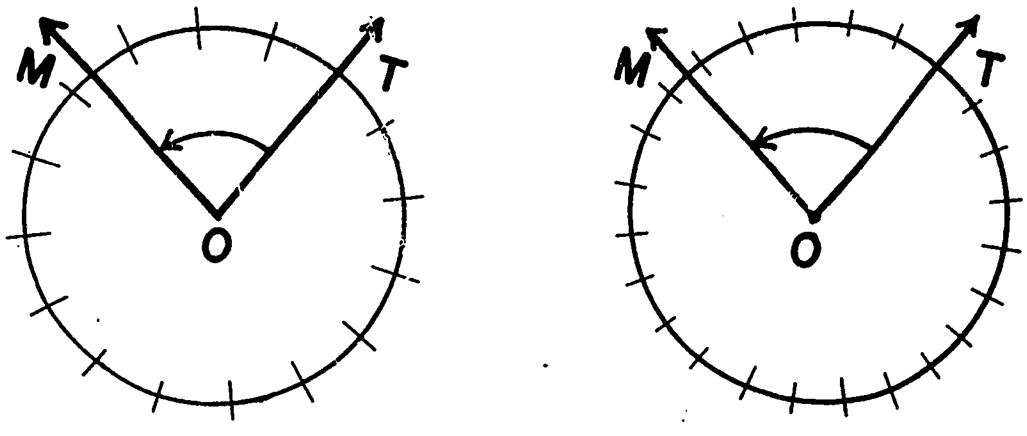
$$\frac{2}{12} \text{ of a turn} < m(\angle \text{RAT}) < \frac{3}{12} \text{ of a turn.}$$

Notice that the length of this interval is the unit used to measure $\angle$ RAT.

It appears (looking at Figure 4) that the measure of $\angle$ RAT is nearer $\frac{3}{12}$ of a turn. The measure of $\angle$ RAT to the nearest whole unit is $\frac{3}{12}$ of a turn.

All measurements are approximations. A more precise measure of an angle is one in which a better approximation is obtained.

To have a more precise measure of an angle means the measure lies within a smaller interval.



5a

5b

Figure 5

The measure of $\angle$ RAT is something that we shall <u>never</u> know exactly. We can only hope to obtain a better approximation of this measure by making the interval smaller. We make the interval smaller by using a smaller unit of measure.

Make the analogy to linear measurement. If the length of a steel rod is known to be in an interval between 5.01 and 5.02 centimeters, we have a better approximation than if we have an interval of say, 5.1 to 5.2 centimeters.

The students should gradually see that the length of the interval depends on the size of the unit used.

In Figure 5a:

$$\frac{3}{16} \text{ of a turn} < m(\angle TOM) < \frac{4}{16} \text{ of a turn.}$$

The interval is $\frac{1}{16}$ of circle A.

In Figure 5b:

$$\frac{5}{24} \text{ of a turn} < m(\angle TOM) < \frac{6}{24} \text{ of a turn.}$$

The interval is $\frac{1}{24}$ of circle A.

The smaller the interval that the measure lies in, the better the approximation.

Thus, $\frac{5}{24}$ of a turn is a better approximation of the measure of $\angle$TOM than $\frac{4}{16}$ of a turn.

The measure of an angle can always be placed in an interval whose length is equal to the size of the unit of measure.

Dividing a circle into more and more congruent arcs makes the size of the unit of measure smaller.

The smaller the unit, the more precise the measure.

What can be done to the circle in Figure 4, page 40, to get a more precise measure of $\angle$RAT?

The students should be able to explain the statement in bold type on page 42. Have them explain what can be done to the circle in Figure 3 to obtain a more precise measure of $\angle$ RAT.

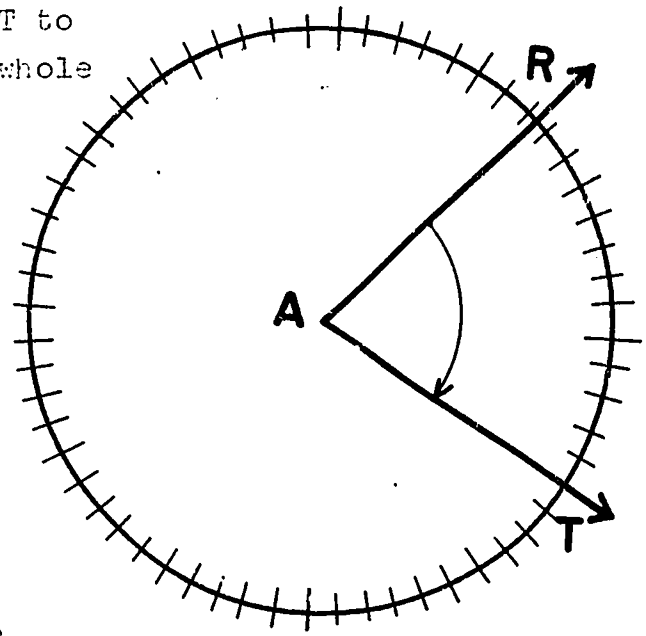
THINGS TO DISCUSS

1. Discuss what an amount of opening means and how it is related to: (a) the length of the sides of an angle; (b) the size of the circle used to measure the angle; (c) the number of congruent arcs that the circle is divided into; and (d) the direction of rotation.

2. Discuss what "to the nearest whole unit" means. Emphasize that even after the measure of an angle is "captured" in an interval, the student must make the decision as to which end of the interval he believes the measure to be nearest. That is, he must round off to the nearest whole unit.

See Appendix A, Number 5 and Number 6

Example 3. Measure $\angle$ RAT to the nearest whole unit.



Solution:

$m(\angle RAT) = \frac{13}{60}$ of a turn.

Figure 6

The unit used in Example 2, page 40, is $\frac{1}{12}$ of circle A. The unit used in Example 3 above is $\frac{1}{60}$ of circle A.

$$\frac{1}{60} \text{ of circle A} < \frac{1}{12} \text{ of circle A.}$$

thus, $\frac{13}{60}$ of a turn is a better approximation of $\angle$RAT than $\frac{3}{12}$ of a turn.

Dividing the same circle into more congruent arcs gives a more precise measure of the angle.

✓ **POINT**

1. What can be done to circle O in Figure 3, page 40, in order to obtain a more precise measure of $\angle$TOM?

THINGS TO DISCUSS CONT'D

3. Discuss what can be done to a circle in order to obtain a more precise measure. Use two circles on the overhead to measure the same angle. Make one circle have more congruent arcs than the other. Try this with two circles having the same radus and two circles having different radii. It is true that the circle with the most congruent arcs will give the more precise measurement - even if the radii vary.

ANSWERS TO ✓ POINT

1. Divide the circle into more than 20 congruent arcs.

2. The "size" of an angle depends on which of the following?

   a. The length of its sides.

   b. The amount of opening between its sides.

   c. The "size" of the circle used to measure it.

   d. The "size" of the unit angle used to measure it.

3. A circle used to measure an angle is divided into 40 congruent arcs. What is the unit being used to measure the angle?

4. Explain how dividing a circle into more congruent arcs gives a more precise measure of the angle.

EXERCISES

1. Circle R is divided into 18 congruent arcs. In using Circle R to measure an angle to the nearest unit, what is the unit? (Use amount of turn.)

2. Circle O is divided into 360 congruent arcs.

   a) In using circle O to measure an angle to the nearest unit, what is the unit?

   b) Besides a part of a turn, what other name is given to this unit?

ANSWERS TO ✓ POINT CONT'D

2. b. The size of an angle is constant. The measure of an angle varies with the size of the unit used.

3. $\frac{1}{40}$ of a turn

4. It makes the unit of measure smaller and this causes the measure to lie in a smaller known interval.

ANSWERS TO EXERCISES

1. $\frac{1}{18}$ of a turn

2. a. $\frac{1}{360}$ of a turn

   b. Some students may know this to be a degree. Do not mark them incorrect if they do not know. This question opens up a discussion leading to the next lesson.

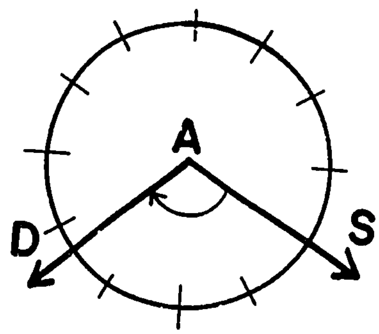
3. Which will give the more precise measure of an angle:

   a) A circle divided into 50 congruent arcs or

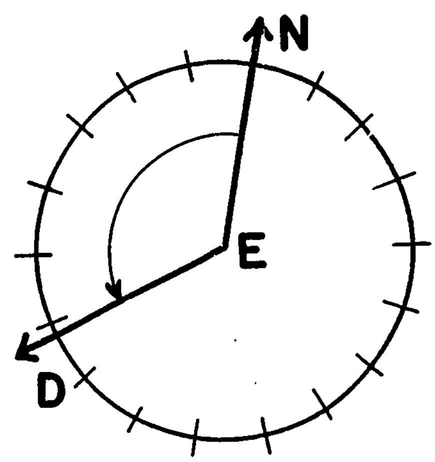
   b) the same circle divided into 100 congruent arcs?

4-6. Measure each of the following angles to the nearest whole unit. Use the indicated unit.

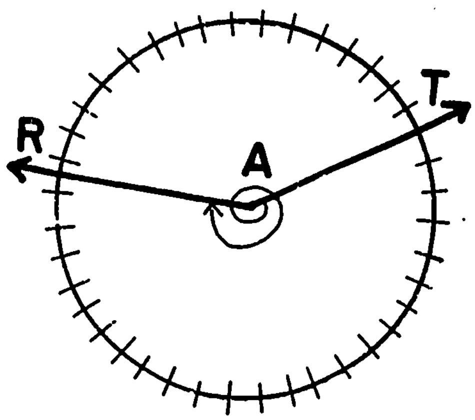
4. m(∠SAD) = ____ of a turn



5. m(∠NED) = ____



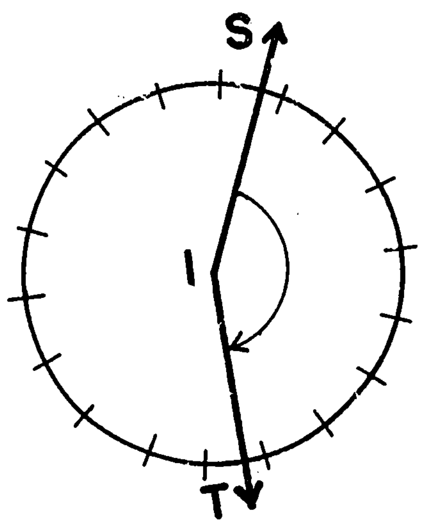
6. m(∠TAR) = ____



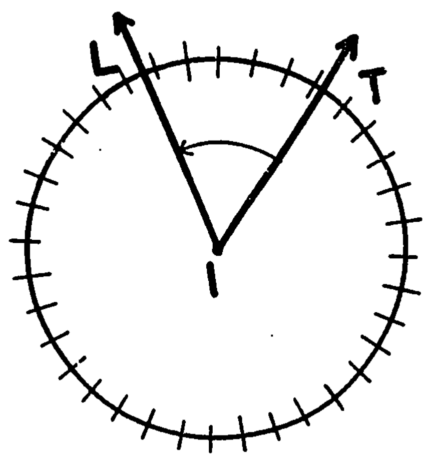
ANSWERS CONT'D

3. b.
4. $\frac{4}{12}$ of a turn or $\frac{1}{3}$ of a turn
5. $\frac{6}{18}$ of a turn or $\frac{1}{3}$ of a turn
6. $\frac{64}{40}$ or $1\ \frac{24}{40}$ turns or $1\ \frac{3}{5}$ turns

7-9. Estimate the measure of each of the following angles to the nearest whole unit.

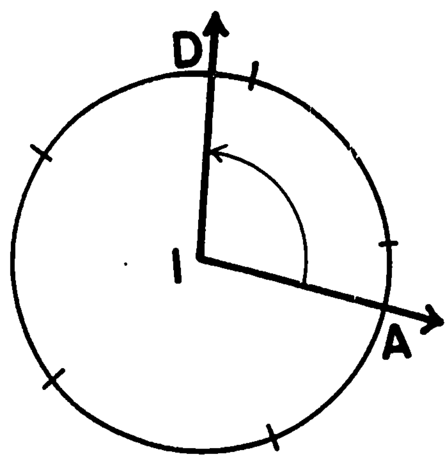


7. m(∠SIT)= ____ of a turn.

8. 

m(∠TIL)= ____

9. m(∠AID) = ____



ANSWERS CONT'D

Exercises 7-9 are more difficult than Exercises 4-6 in that they show angles where neither side lies on a scale mark.

7. $\frac{8}{18}$ of a turn or $\frac{4}{9}$ of a turn
8. $\frac{6}{36}$ of a turn or $\frac{1}{6}$ of a turn
9. $\frac{1}{5}$ of a turn

WHAT IS A DEGREE?

When an angle is formed by the rotation of a ray, the measure of the angle is the amount of turn.

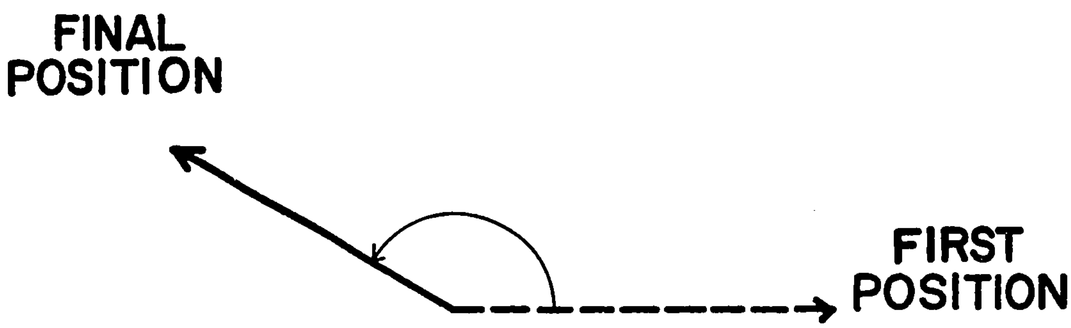


Figure 1

In Lessons 3, 4 and 5, the circle was an aid in determining the amount of turn.

A REVIEW (Complete the following statements.)

(1) The vertex of the angle was located at ____________,

(2) The circle was divided into ____________________.

(3) An arrow indicated ____________________________.

(4) A ratio of the number of arcs "swept out" by one side of the angle to ____________________________ is a measure of the angle.

WHAT IS A DEGREE?

OBJECTIVES

1. To introduce the degree as a unit of measure for angles.

2. The student is able to define a degree as $\frac{1}{360}$ of a turn.

3. The student is able to convert an angle measure from a fractional amount of turn to degree measure and vice versa.

4. The student will be able to determine the measure of an angle in degrees when (1) the vertex of the angle is at the center of a circle, (2) an arrow indicates the direction and amount of rotation, (3) the circle is divided into congruent arcs, and (4) the sides of the angle cut the circle.

5. The student recognizes and can describe a right angle as an angle swept out by a $\frac{1}{4}$ turn and a straight angle as an angle swept out by a $\frac{1}{2}$ turn.

6. Given a measure in degrees and a circle divided into congruent arcs, the student will be able to draw an angle having that measure.

EQUIPMENT AND TEACHING AIDS

A drawing compass, straightedge and Transparencies 3-1 through 3-5.

ANSWERS TO REVIEW QUESTIONS

(1) the center of a circle.
(2) a number of congruent arcs.
(3) the direction of rotation.
(4) the total number of arcs in the circle

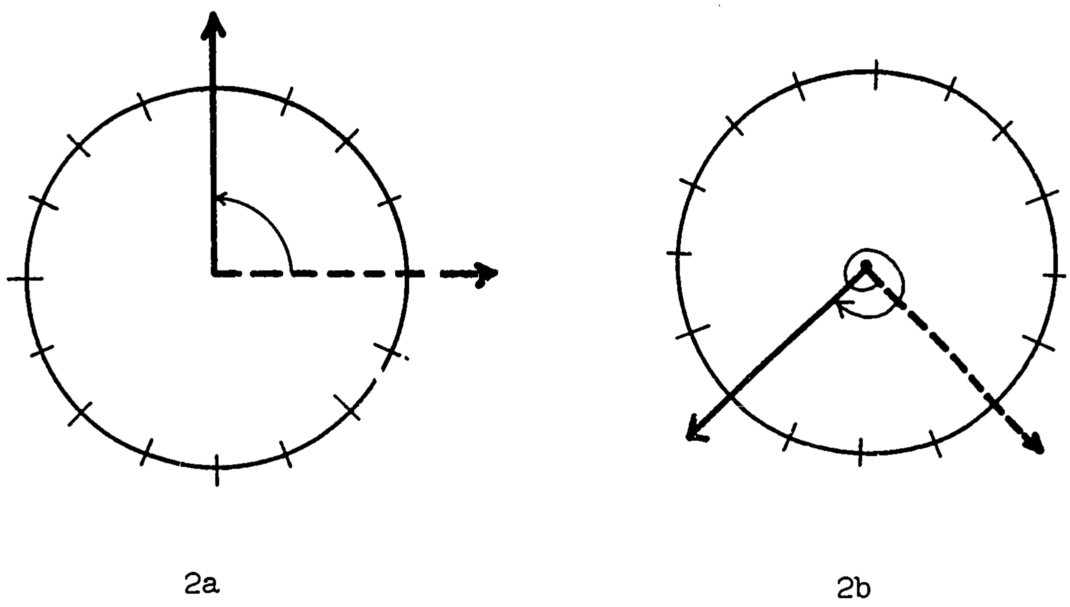


Figure 2

Example 1. Figure 2a shows an angle formed by a rotation in a counterclockwise direction. The amount of rotation is less than a turn. Since the circle is divided into 16 congruent arcs, the unit for measuring the amount of rotation is $\frac{1}{16}$ of a turn. The measure of the angle is $\frac{4}{16}$ of a turn.

Example 2. Figure 2b shows an angle formed by a rotation in a clockwise direction. The amount of rotation is greater than a turn. The measure of the angle is $\frac{20}{16}$ or $1\frac{4}{16}$ turns.

We call a quarter turn a <u>right angle.</u> How many right angles are there in a complete turn?

The angle swept out in Figure 2a is a right angle. How many right angles were swept out by the rotation shown in Figure 2b?

CONTENT AND APPROACH

It is assumed that most students know what a right angle is (they can identify or draw one) and that they will remember that the measure of a right angle is 90°. Review with them that two perpendicular lines form four right angles.

We now relate the degree to arcs of a circle. Lessons 3, 4 and 5 have given us sufficient background to allow the student to understand the degree as a unit of angle measure.

Transparency 3-5 is a circle divided into 360 congruent arcs. Each arc determines an angle (whose vertex is at the center of the circle) of one degree.

See Appendix A, Number 7 and Number 8

The angle swept out by a half turn also has a special name. It is called a straight angle. How many quarter turns are there in a straight angle?

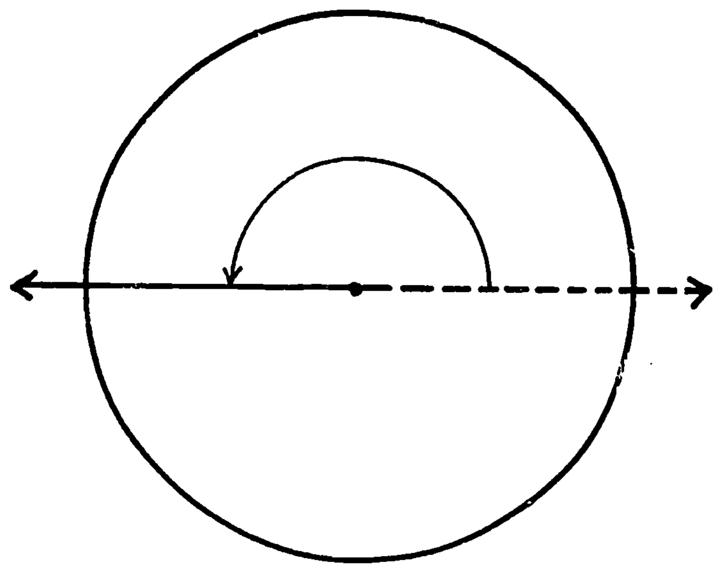


Figure 3

From previous work in mathematics you may remember that the measure of a right angle is 90 degrees.

A degree is the most commonly used unit for measuring angles.

What is a degree and where did it come from?

The degree is related to arcs of a circle. We inherited the degree from the ancient Sumerians who lived near the Tigris and Euphrates Rivers several thousand years ago. The Sumerians used a calendar of 12 months, each month having 30 days.

OTHER UNITS OF ANGLE MEASURE (Supplementary Information)

Radian. A central angle subtended in a circle by an arc whose length is equal to the radius of the circle. Thus the radian measure of an angle is the ratio of the arc it subtends to the radius of the circle in which it is a central angle. Radian measure is frequently used in scientific work.

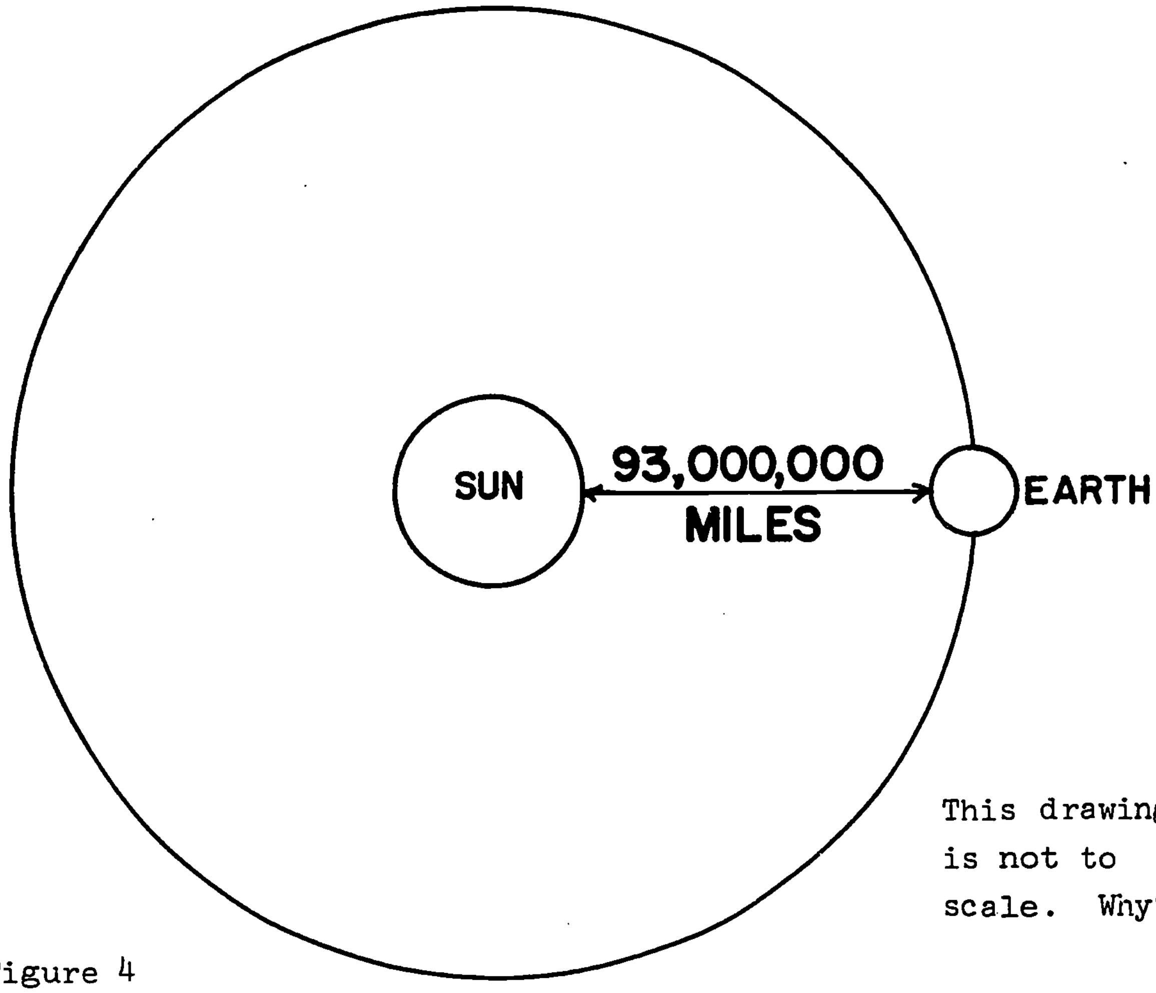
Arc length can be determined by using the formula $S = \theta R$ where S is the arc length; θ is the measure of the subtended central angle in radians and R is the radius of the circle.

$$2\pi \text{ radians} = 360^\circ$$
$$1 \text{ radian} = 57^\circ\ 17'\ 44''$$

Mil. This unit lends itself to accurate approximation in measuring at great distances. It is sometimes used in certain military activities. A mil is approximately an angle that subtends a length of one unit at a distance of one thousand units. In a complete revolution there would be $2\pi \cdot 1000$ or $6283.18 \cdots$ such units. For convenience the mil is defined as $\frac{1}{6400}$ of a complete rotation.

$$1 \text{ mil} = .05625^\circ = .000982 \text{ radians}$$
$$1^\circ = 17.7777 \cdots \text{ mils}$$

The Earth travels around the sun once each year. The path of its travel is almost circular.



This drawing is not to scale. Why?

Figure 4

If the year were exactly 360 days (every so often the Sumerians had to add extra days to their year), the Earth in its annual passage around the sun would pass through one degree per day.

The degree comes from the sexagesimal measure of an angle. In this system, one complete revolution is divided into 360 parts. Each part is called a degree. One degree is divided into 60 parts, with each part called a minute and one minute is divided into 60 parts, with each part called a second.

THINGS TO DISCUSS

1. Discuss the review questions on page 47 as they will provide a basis for introducing the full circle protractor in Lesson 8.

2. Relate right angles to perpendicular lines. A straight angle is related to a line by pointing out (See Figure 3) that when a ray is rotated $\frac{1}{2}$ of a turn, its original position and its final position determine a line.

3. There are many common units of linear measure, but the typical student only hears of one unit of angle measure, i.e., the degree. Point out the arbitrariness of this unit. Let them make up a special unit, say $\frac{1}{20}$ of a turn, and give it a name.

Suggested Activity. Have Students Convert Their Locker Combination to Degree Measure.

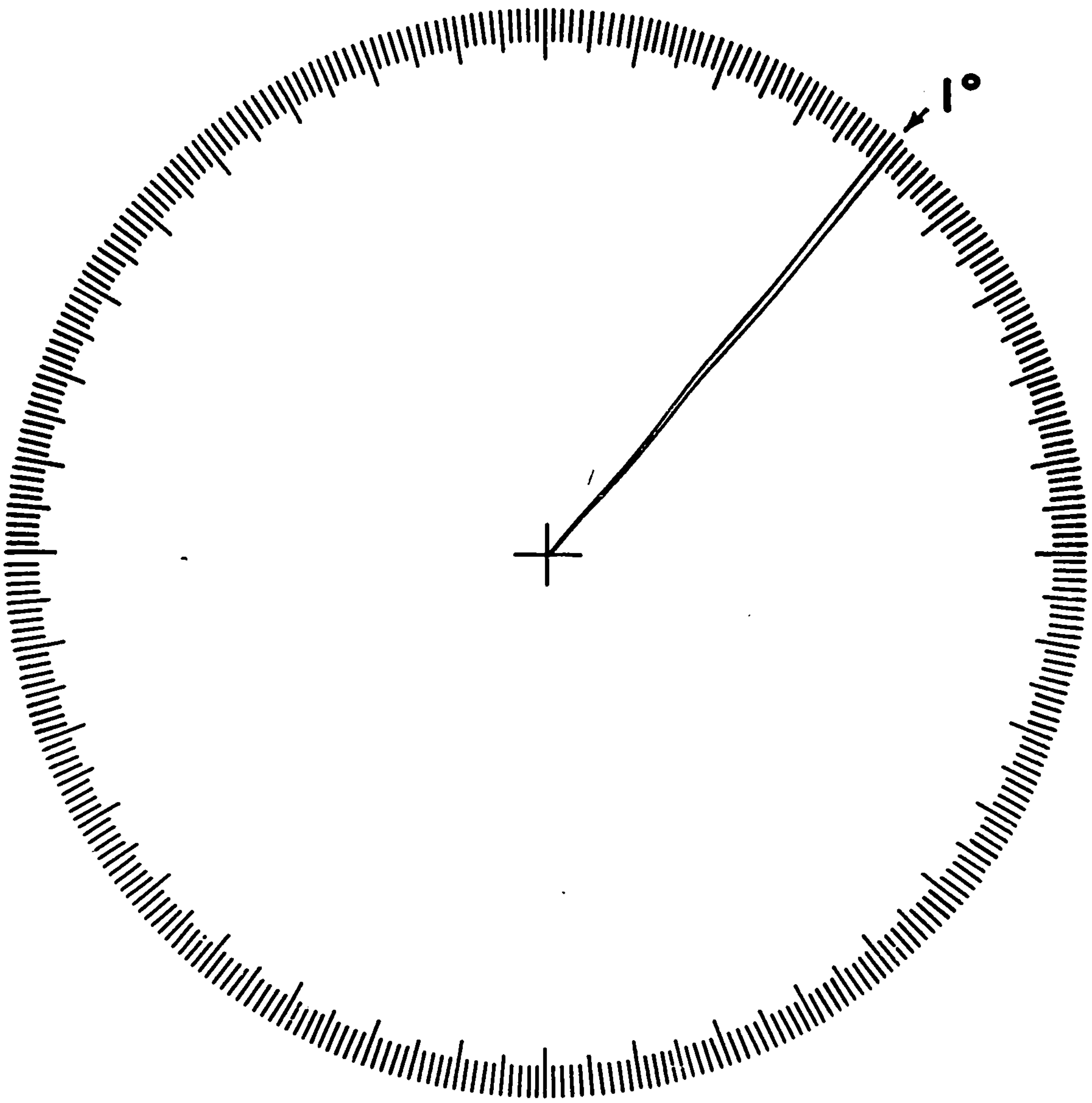


Figure 5

The circle in Figure 5 is divided into 360 congruent arcs. Each arc determines a central angle of one degree.

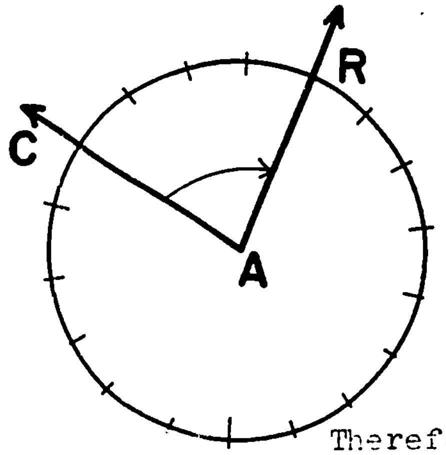
$\frac{1}{360}$ **of a turn = 1 degree**

THINGS TO DISCUSS CONT'D

They can use this special unit to measure angles. The degree has survived down through the ages because it is a convenient unit. A small unit is necessary because one wants to assign a different measure (when rounding to the nearest whole unit) for angles which vary slightly in their amount of opening. For example, if $\frac{1}{20}$ of a turn were used as a standard unit, then two angles, say $m(\angle A) = 20^\circ$ and $m(\angle B) = 28^\circ$, would both receive the same measure, namely 1 unit, when rounded off to the nearest whole unit. For most cases, there is too much of a difference between $\angle A$ and $\angle B$ to round off the measure to one unit and thus it would be necessary to give a fractional part to the measure of each of these angles.

Point out to the students that a smaller amount of turn than $\frac{1}{360}$ would cause the scale markings to be nearly illegible even on a circle as large as the one in Figure 5.

Example 3. Give the measure of $\angle CAR$ as a part of a turn and in degrees.



$m(\angle CAR) = \frac{4}{18}$ of a turn.

$$\frac{4}{18} = \frac{x}{360}$$

$$\frac{4 \cdot 20}{18 \cdot 20} = \frac{80}{360}.$$ Thus,

$x = 80$.

Therefore, $m(\angle CAR) = \frac{80}{360}$ of a turn and

$\frac{80}{360}$ of a turn $= 80^\circ$.

The symbol "$^\circ$" is read degrees.

Example 4. How many degrees is $\frac{1}{3}$ of a turn?

There are 360° in one turn.

$\frac{1}{3}(360^\circ) = 120^\circ$

Example 5. Convert 135° to the corresponding fractional amount of turn.

$$\frac{135^\circ}{360^\circ} = \frac{3 \cdot \cancel{45^\circ}}{8 \cdot \cancel{45^\circ}} = \frac{3}{8}$$

Answer: $135^\circ = \frac{3}{8}$ of a turn.

THINGS TO DISCUSS CONT'D

4. Discuss the Examples on page 52. Some time should be spent here in helping the students convert from amounts of turn to degree measure and vice versa. Each example shows a different approach. In Example 3, a proportion is set up. This provides a good time to work with proportions.

   In Example 4, one turn is 360° so $\frac{1}{3}$ of a turn is $\frac{1}{3}$ times 360°. Here the students can obtain some practice in multiplying fractions by whole numbers.

   In Example 5, the student is reducing a fraction.

5. Discuss questions on page 57.

Test analysis from the first edition of this booklet indicates that students were weak in converting degree measure to an amount of turn and vice versa. Special emphasis should be put on Examples 3 - 5, page 52.

✓POINT

1. What special name is given to angles formed by
   (a) one-quarter of a turn?
   (b) one-half turn?

2. What makes a central angle different from angles in general?

3. A degree is what part of a turn?

4. Explain how a degree is determined.

EXERCISES

1. How many degrees in:

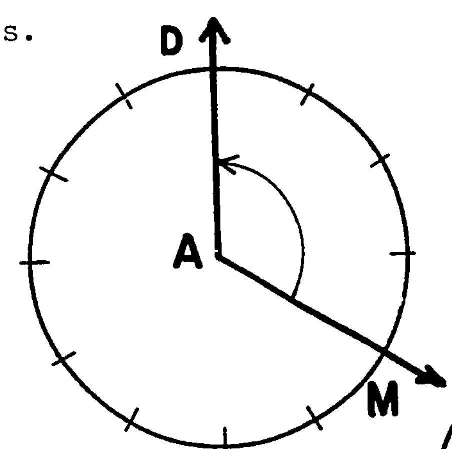
   a) $\frac{1}{4}$ of a turn   b) $\frac{1}{2}$ of a turn   c) 1 turn

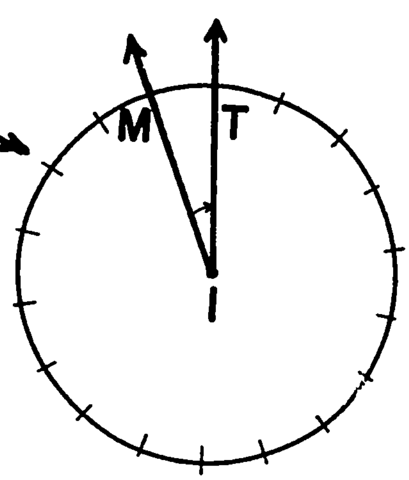
   d) $\frac{1}{3}$ of a turn   e) $1\frac{3}{8}$ turns   f) $3\frac{1}{4}$ turns

2. How many degrees in
   a) a right angle?
   b) a straight angle?

3. Give the measure of each of the following angles in degrees.

   a) m(∠ MAD) = ___°





   b) m(∠MIT) = ____

ANSWERS TO ✓ POINT

1. (a) a right angle
   (b) a straight angle

2. A central angle is determined by two radii of a circle. Thus, its vertex is at the center of a circle and its sides are segments. It is an angle in the sense that the segments can be extended to form rays.

3. $\frac{1}{360}$ of a turn.

4. When a circle is divided into 360 congruent arcs, each arc determines a central angle whose measure is 1°.

ANSWERS TO EXERCISES

1. a.) 90° b.) 180° c.) 360°
   d.) 120° e.) 495° f.) 1170°

2. a.) 90° b.) 180°

3. a.) 120° b.) 20°

6. A circle is divided into 8 congruent arcs.

   a) How many degrees will be measured by each arc?

   b) A central angle of this circle cuts off three of the eight arcs. What is the measure of this angle in degrees?

7. A circle is divided into 24 congruent arcs.

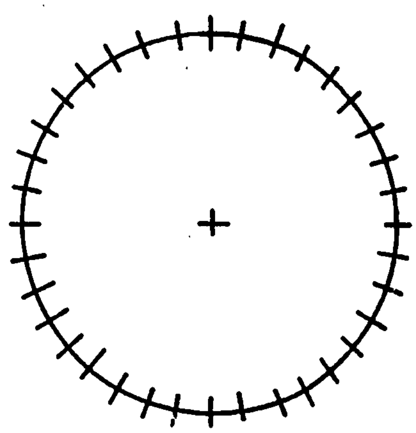
   a) How many degrees will be measured by each arc?

   b) A central angle of this circle cuts off 18 of the 24 arcs. What is the measure of this angle in degrees?

8. A circle is divided into 36 congruent arcs.

   a) How many degrees will be measured by each arc?

   b) Central angle STU of this circle has a measure of $2\frac{3}{36}$ turns in a counterclockwise direction. Draw $\angle$STU.



   c) What is the measure of $\angle$STU in degrees?

ANSWERS CONT'D

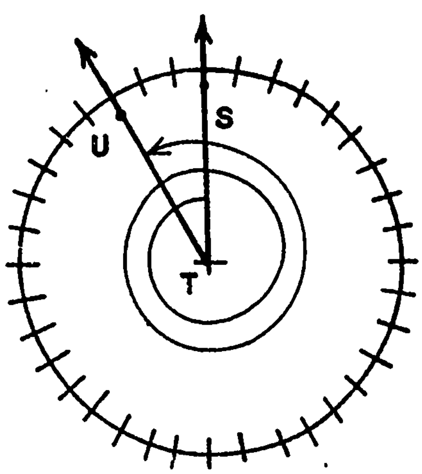
6. a) 45° b) 135°

7. a) 15° b) 270°

8. a) 10°

c) 750°

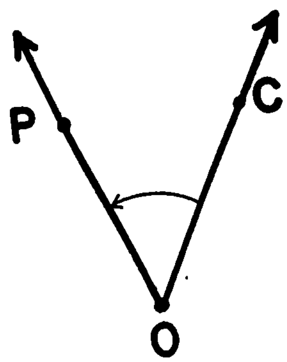
b)



9-12. Estimate the measure of each of the following angles in degrees by:

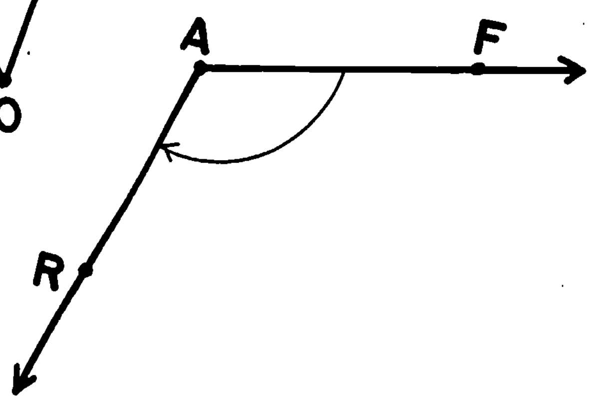
(1) Use a drawing compass to draw a circle using the vertex of the angle as the center. You decide what radius to use.

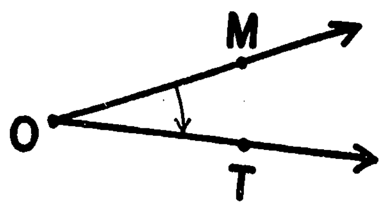
(2) Divide the circle into a number of congruent arcs. Use your judgment as to how many arcs to use. Mark off the arcs by estimating their positions.

9. 

$m(\angle COP) \approx$ _____ °

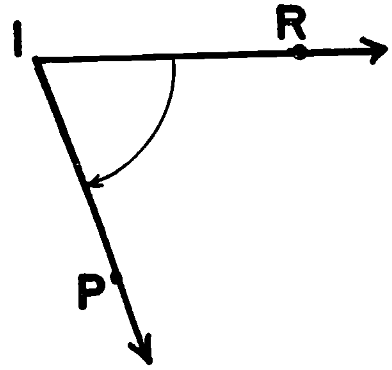
10. $m(\angle FAR) \approx$ ____



11. 

$m(\angle MOT) \approx$ ____

12. $m(\angle RIP) \approx$ ____



SUGGESTION: This page is too crowded to draw circles for each angle. Have the students trace each angle onto separate pieces of paper. One way to obtain convenient arc marks is to fold the paper, through the center of the circle, into fourths, eighths, etc., and then estimate the position of any needed intermediate marks.

ANSWERS CONT'D

9. 50°
10. 120° (Answers will vary by about 5° either way.)
11. 25°
12. 70°

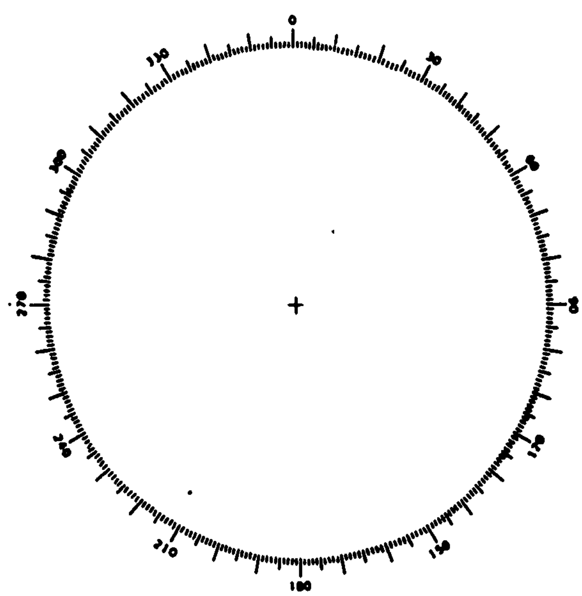
DISCUSSION QUESTIONS

1. What is a degree?

2. How are a degree and an inch alike?

3. Discuss applications where an angle whose measure is greater than 360° might be found.

4. The circle in Figure 5, page 51, is divided into 360 congruent arcs. The circle drawn below is also divided into 360 congruent arcs. Discuss the following statement: Each arc determines a central angle of one degree <u>regardless of the size of the circle.</u>



ANSWERS TO DISCUSSION QUESTIONS

1. A unit of angle measure- $\frac{1}{360}$ of a turn. Ask what makes the degree special. Does anyone know of any other standard units for angle measure? See pages T 49-T 50.

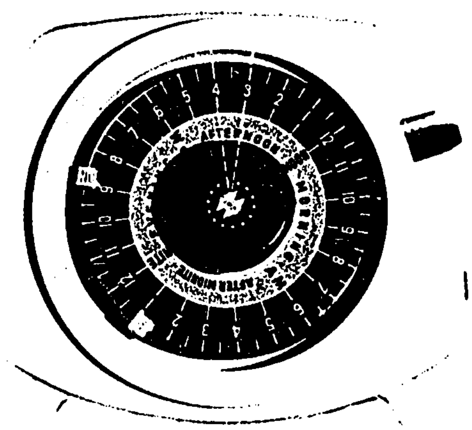
2. They are both units of measure. The inch measures an amount of opening between two points on a line. The degree measures an amount of opening between two rays having a common endpoint.

3. In situations where an angle has been swept out and the total number of complete rotations is important. Some Exercises in Lesson 3 presented situations where the measure of the angle was greater than 360°.

4. Although the arcs on the circle on page 57 are much smaller than the arcs on page 51, one degree is still determined by one arc. Refer back to Exercise 11, page 35. The students should see that for different size circles the arcs change size, but the size of one degree is constant.

DIALS AND GAUGES

Many dials and gauges are either (1) circular or (2) have a scale which is based on a circular design. Some examples are included in the following exercises.



1. An electronic timer such as the one pictured to the left will turn appliances and lights on (and off) automatically.

   Using the settings shown in the picture, the lights will go "on" at 9 p.m. and "off" at 1 a.m.

When the markings are extended, the smaller angle formed by each numbered marking with the numbered markings on either side has a measure of 15°. (The 15° angle formed by the 3 p.m. and 4 p.m. markings is indicated by dotted rays.)

On the setting shown above, the smaller angle formed by the 9 p.m. and 1 a.m. markings has a measure of 60° ($4 \times 15^\circ$).

(a) What will be the angle formed by the markings for the following settings? (Move from the 1st to the 2nd setting in a counter-clockwise direction.)

First Setting	Second Setting	Angle Measure
9 p.m.	5 a.m.	120° ($8 \times 15^\circ$)
7 p.m.	12 midnight	____
6 p.m.	2 a.m.	____
8 p.m.	6 a.m.	____

DIALS AND GAUGES

OBJECTIVES

1. Given a circular dial or gauge, the student will be able to determine the measures of angles determined by given markings on the scale of the circular dial or gauge.

CONTENT AND APPROACH

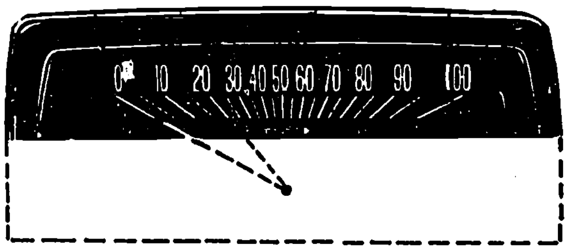
As indicated, the use of this lesson is optional. The examples in the **EXERCISES** illustrate dials and gauges whose scales are based on a circular design (in some cases, only part of a circle is utilized). It is not necessary that the student understand angle measure prior to being able to read the gauges shown. However, it would be impossible to calibrate or design these gauges without an understanding of angle measure.

ANSWERS

	First Setting	Second Setting	Angle Measure
1. (a)	9 p.m.	5 a.m.	120°
	7 p.m.	12 midnight	75°
	6 p.m.	2 a.m.	120°
	8 p.m.	6 a.m.	150°

(b) For each angle measure, give a pair of settings whose markings, when extended, will form an angle of that size. (There are several for each part.)

Angle Measure	Settings
135°	on at _____, off at _____
90°	on at _____, off at _____
210°	on at _____, off at _____
15°	on at _____, off at _____



2. On the speedometer to the left, part of the needle which indicates speed is hidden from view (indicated by dotted line).

The entire needle pivots on a point and the end of the needle follows a circular path as it moves. In the above speedometer as the needle moves from 0 to 20 m.p.h., the degree measure of the angle formed is approximately 20°.

(a) As the car accelerates or decelerates from the first speed to the second speed, give the measure of the angle formed by the needle as it moves from the first speed to the second speed.

First Speed	Second Speed	Angle Measure
0 m.p.h.	40 m.p.h.	40°
20 m.p.h.	80 m.p.h.	_____
30 m.p.h.	50 m.p.h.	_____

(b) Using the speedometer needle, describe three different ways for determining an angle of 40°.

First Speed ______ ______ ______

Second Speed ______ ______ ______

ANSWERS

1.(b) There are several possible answers for each part. Conditions for correct answers are given below.

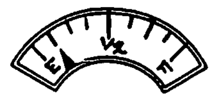
Angle Measure	Settings
135°	9 hour difference
90°	6 hour difference
210°	14 hour difference
15°	1 hour difference

2.(a)

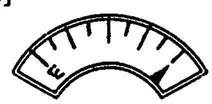
First Speed	Second Speed	Angle Measure
0 m.p.h.	40 m.p.h.	40°
20 m.p.h.	80 m.p.h.	60°
30 m.p.h.	50 m.p.h.	20°

(b) There are several possibilities. To determine an angle of 40°, the difference between the first speed and second speed should be 40 m.p.h.

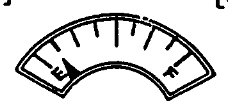
3. The gasoline gauge on a car looked like this when there were 2 gallons of gasoline in the tank. Estimate the number of gallons of gasoline in the tank for each gauge-reading below.



[A]



[B]



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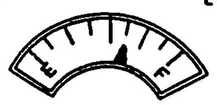
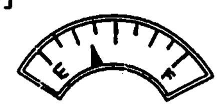


[D]



(a) ________ (b) ________ (c) ________ (d) ________

4. The timer pictured at the right will time intervals up to 60 minutes long.

The timer shown is set for an interval of 40 minutes. (Assume the vertical pointer is on "0".)

(a) The circular dial on the timer has been divided into how many congruent arcs? ____________

(b) Each arc is what fraction of the circle? ____________

(c) What is the degree measure of each arc? ____________

ANSWERS

3. (a) 16 gal. (b) 2 gal. (c) 10 gal. (d) 5 gal.

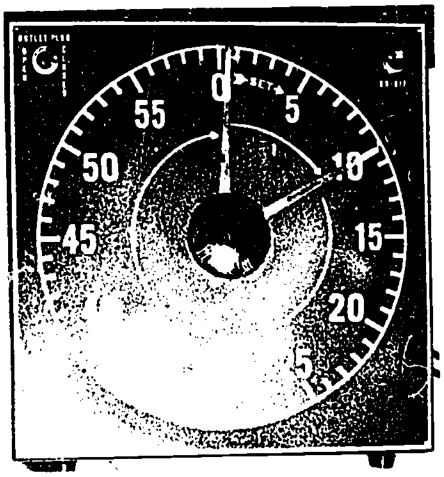
4. (a) 60

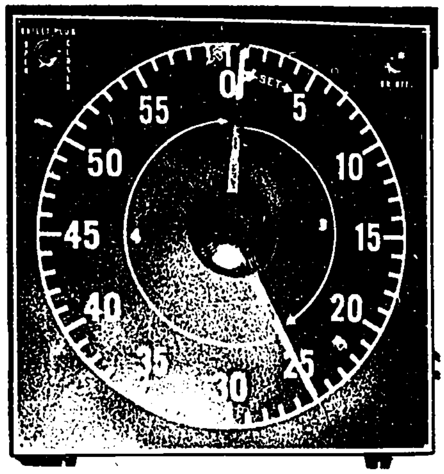
   (b) $\frac{1}{60}$

   (c) 6°

NOTE: The answers for exercise 4 are based on counting all markings on the dial. If only the numbered markings were counted, the answers would be 12, $\frac{1}{12}$, and 30°.

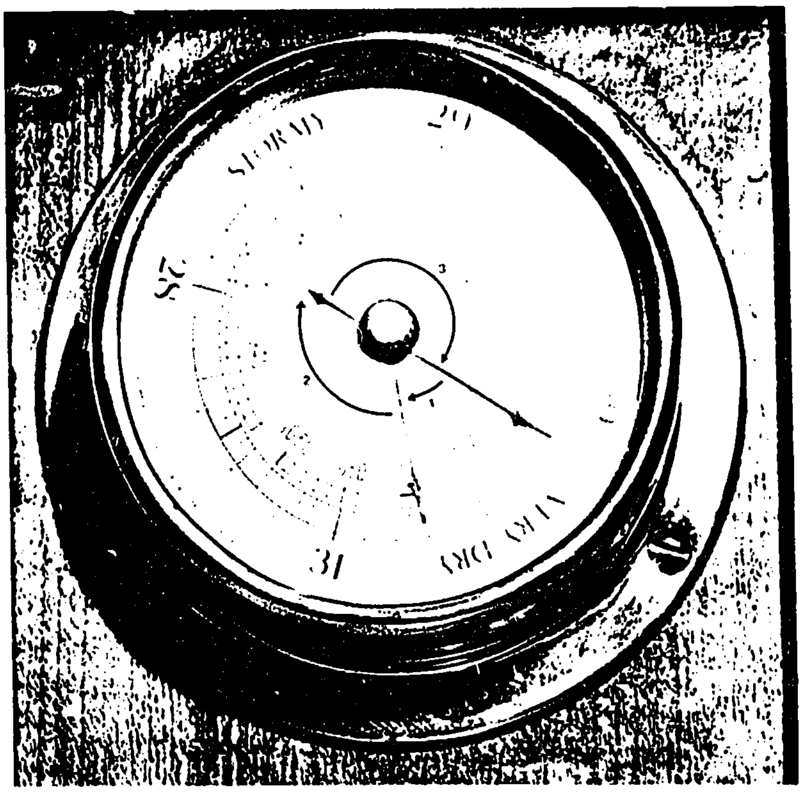
(d) Give the degree measure of each angle indicated.





m (∠1) = ________ m (∠3) = ________

m (∠2) = ________ m (∠4) = ________



5. The outer dial of the barometer pictured to the left is divided into 40 congruent arcs. (Using numbered markings only.)

(a) What is the degree measure of the smaller angle (∠1) pictured?

m(∠1) = ________

(b) Using the settings pictured above, . . .

m (∠2) = ________ m (∠3) = ________

ANSWERS

4.(d) $m(\angle 1) =$ 60° $m(\angle 3) =$ 150°

$m(\angle 2) =$ 300° $m(\angle 4) =$ 210°

5. (a) $m(\angle 1) =$ 45°

(b) $m(\angle 2) =$ 135° $m(\angle 3) =$ 180°

NOTE: The answers above assume the following:

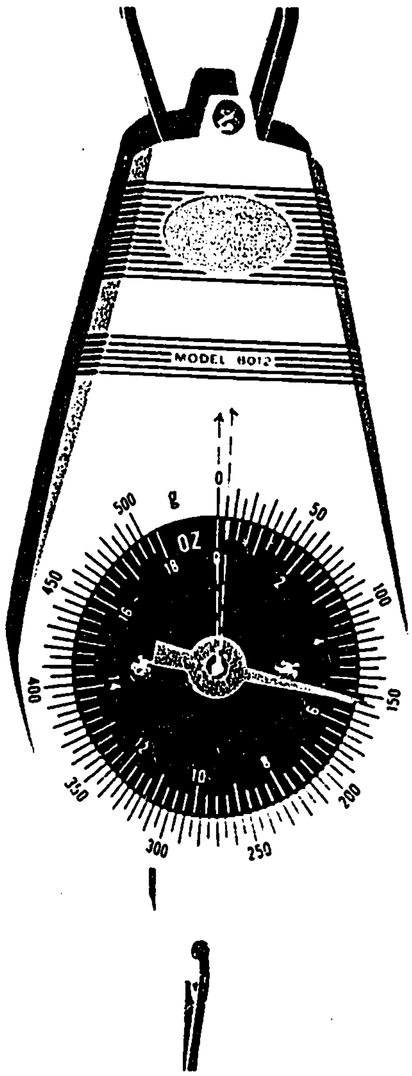
4(d): The vertical pointer is set on "0".

5(a-b): The pointers are set on the nearest numbered marking.

6. The outer dial (0 to 500g.) of the spring scale pictured to the right is divided into 100 congruent arcs.

   As the pointer moves from 0 to 500g., it moves through an angle of 330°. Therefore, the small angle indicated by the dotted rays has a measure of 3.3°.

   (a) What is the measure of the angle formed as the pointer moves from 0g. to each of the following settings?



Setting	Degree Measure
155 g.	102.3° (31 x 3.3°)
100 g.	______
250 g.	______
400 g.	______

The pointer above indicates a weight of 155 g.

(b) Using the dials of the scale, give the approximate weight in grams equivalent to each of the following weights in ounces. (The ounce scale is on the inner part of the circle.)

ANSWERS

6.(a)

Setting	Degree Measure
155 g	102.3°
100 g	66°
250 g	165°
400 g	264°

Weight in ounces	Weight in grams
6 oz.	170 g.
9 oz.	______
16 oz.	______
4 oz.	______
$9\frac{1}{2}$ oz.	______

(c) A pound is equivalent to approximately how many grams? ______

FOLLOW-UP . . .

The dials and gauges are only a sample of the dials and gauges that could have been shown. Bring in dials and gauges (or pictures of them) whose scales are based on a circular design. Describe the angles formed as the indicator or needle moves from one marking to another.

ANSWERS

6. (b)

Weight in ounces	Weight in grams
6 oz.	170 g.
9 oz.	255 g.
16 oz.	455 g. (or 454 g.)
4 oz.	115 g.
$9\frac{1}{2}$ oz.	270 g.

(c) 455 g. (or 454 g.)
Answer depends on the preciseness of the approximation.

CONTENT AND APPROACH

The section entitled **FOLLOW-UP** indicates a class or individual project that could be used in conjunction with this lesson. Pictures of gauges could be used to make overhead transparencies to aid class discussion of the angle measures involved with these other gauges.

Some of the pictures in this section were taken from an angle and may cause some scale distortion. The answers given assume that no scale distortion has occurred.

THE CIRCULAR PROTRACTOR

A circular protractor is an instrument for measuring angles. It is a copy of a circle that has been divided into congruent arcs. (Usually 360 arcs are used so that each arc corresponds to one degree.)

A circular protractor measures an angle by treating the angle as if it were a central angle.

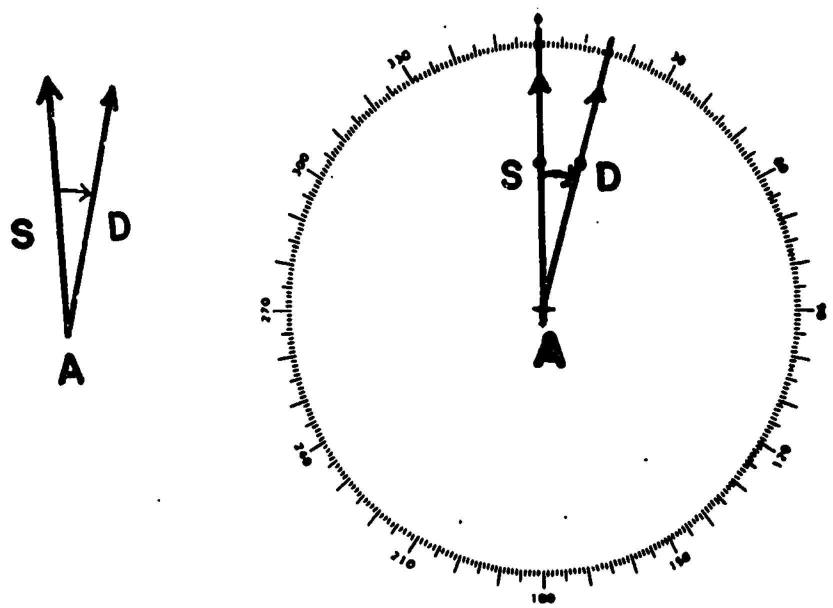


Figure 1

The center of the circular protractor is placed on the vertex of the angle.

The sides of the angle are extended, if necessary, so that they cut the circle.

What is the measure of ∠SAD to the nearest degree?

THE CIRCULAR PROTRACTOR

OBJECTIVES

1. The student is able to demonstrate his understanding of the statement: A circular protractor measures an angle by treating the angle as if it were a central angle. He will be able to make a statement like: "The center of the circular protractor is placed on the vertex of the angle and the "sides" of the angle are extended, if necessary, so that they cut the circle."

2. The student is able to compare the "unit angle" concept with the "unit arc" concept in using a circular protractor. He will be able to make a statement like: "The unit angle, whose vertex is at the center of the protractor, will determine a unit arc on the protractor."

3. The student is able to state that one degree equals sixty minutes and one minute equals sixty seconds.

EQUIPMENT AND TEACHING AIDS

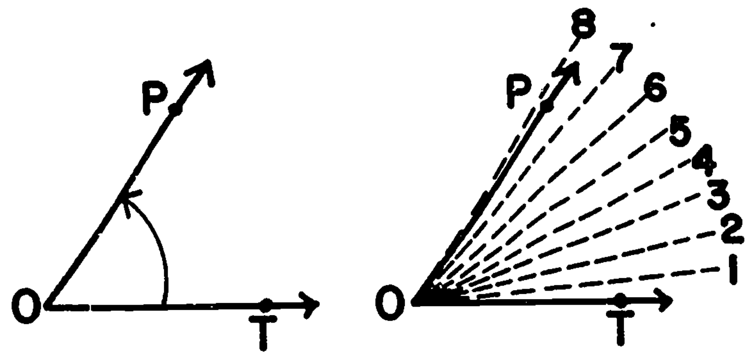
Plastic unit angle wedges to serve as unit angles. Full circle protractors (one for each student). Transparencies 3-2 through 3-5.

CONTENT AND APPROACH

The student should see how placing a circular protractor over an angle, so that the vertex coincides with the center, is related to his previous experience in Lessons 3,4,5 and 6.

A COMPARISON

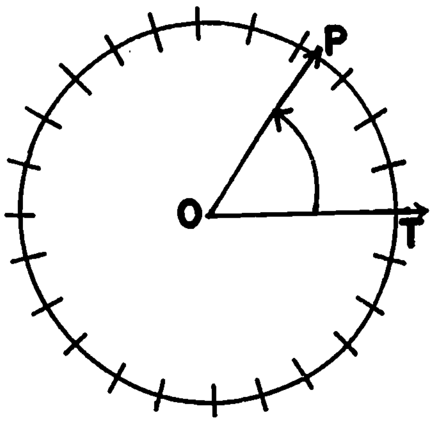
In Lessons 1 and 2 an angle was measured by first selecting a unit angle, then determining how many copies of this unit angle would "fill up" the opening of the other angle.



Unit Angle The Angle to be Measured

Figure 2 m(∠TOP) = 8 unit angles (To the nearest whole unit.)

In Lessons 3-7 we have been looking at angles formed by a rotation. The measure of such an angle is the amount of turn. A circle divided into congruent arcs makes it easier to measure an amount of turn.



m(∠TOP) = $\frac{4}{24}$ of a turn.

(To the nearest whole unit.)

Figure 3

The unit used in Figure 3 is $\frac{1}{24}$ of a circle. What is the unit used in Figure 1? Which unit is more precise? Why?

Draw an angle like $\angle$POT, page 65, on the overhead projector. Use the smallest plastic angle wedge as a unit angle and measure $\angle$ POT. This will review the use of a unit angle. Then place transparency 3-4 over $\angle$ POT. Now compare the use of the 36 congruent arcs in measuring $\angle$ POT to the use of the 10° angle wedge.

<u>The students should come to the determination that these two concepts are tightly related and that in both instances it is the amount of opening that is being determined.</u>

Thus when the student is using the protractor he should see that he is using a unit angle (1°) in the form of counting unit arcs.

DISCUSSION QUESTIONS

1. How does the circular protractor use both the "unit angle" (Figure 2) and the "unit arc" (Figure 3) ideas for measuring angles?

2. How is the unit angle of 1° related to the circle divided into 21,600 congruent arcs?

3. In what way does the circular protractor treat every angle like it was a central angle?

4. How can a circular protractor be used to measure an angle whose measure is greater than 360°?

ANSWERS TO DISCUSSION QUESTIONS

1. An angle is determined by an arc and an arc determines an angle. When we use a protractor we count arcs, but we can think we are counting unit angles and it amounts to the same thing.

2. A central angle of 1° will subtend 60 of these congruent arcs. The unit angle of 1°, in this case, would not yield as precise a measure as the unit arc, i.e. $\frac{1}{21,600}$ of a turn.

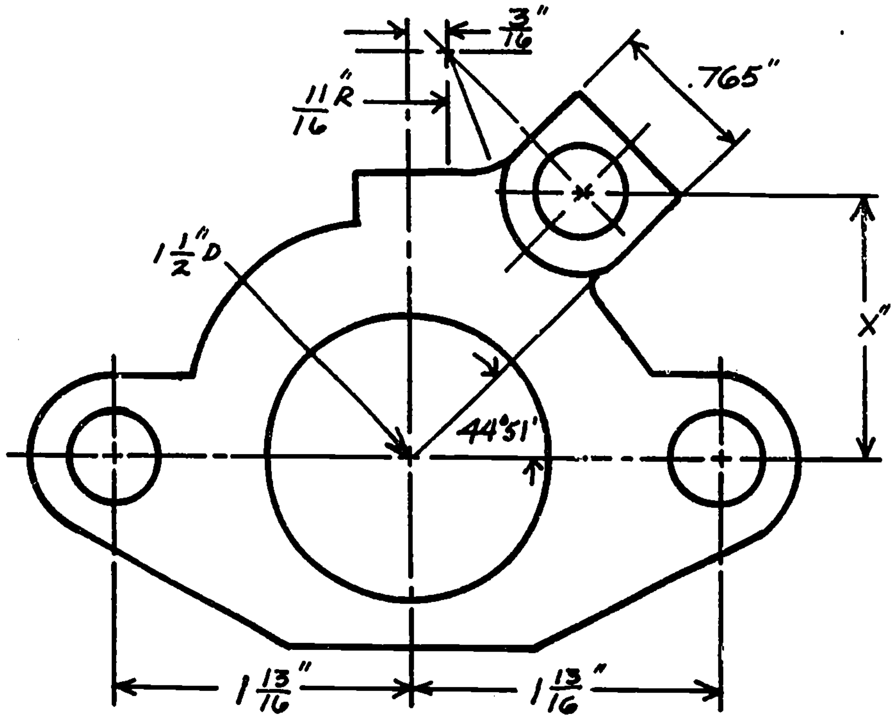
3. The center of the protractor is placed on the vertex of the angle and the two radii are determined by where the sides of the angle meet the circle.

4. By counting 360° for each full rotation; measuring the angle whose measure is less than 360°; and adding these two measurements.

MINUTES AND SECONDS

Modern mass production methods require that parts be machined to size limitations which allow parts to be easily interchanged.

Figure 4



In many industrial applications angles must be measured precisely and the degree is not a small enough unit. In these cases the degree is divided up into smaller units.

One degree is divided into 60 parts.
Each part is called a <u>minute</u>.

$1^{\circ} = 60'$ (The symbol ' is read minutes.)

One minute is divided into 60 parts.
Each part is called a <u>second</u>.

$1' = 60''$ (The symbol " is read seconds.)
$1^{\circ} =$ __?__ "

A measurement of 38 degrees, 42 minutes, 16 seconds is written as $38^{\circ}\ 42'\ 16''$ and means: $38^{\circ} + 42' + 16''$.

Part two of this lesson delves into the conversion of decimal degrees and minutes and seconds. The student is not expected to master the computation, but to know that these smaller units exist and why.

The names degree, minute and second have other uses. Heat is often measured in degrees, but degrees of temperature and degrees of angle are not related. Minutes and seconds of angle and minutes and seconds of time are related in the sense that 60 seconds make 1 minute in either situation.

The symbols for minutes and seconds of angle measure can be confused with the symbols for feet and inches respectively. In Figure 4, the angle measure shown is 44 degrees, 51 minutes. The linear dimensions are given in inches.

The symbol for degrees can also be confused with the symbol for a zero exponent.

Remind the students that of the three units: minute, second and degree, the second is the smallest and therefore the more precise.

Degree measures are often written in decimal form. Thus, it is sometimes necessary to convert these measures to the correct number of degrees, minutes and seconds.

Example 1. Convert 62.82° to the correct number of degrees, minutes and seconds.

Solution: $62.82^\circ = 62^\circ + .82^\circ$. We must convert $.82^\circ$ to minutes and seconds.

Since $1^\circ = 3{,}600''$ we can set up a proportion:

$$\frac{3600''}{1^\circ} = \frac{N}{.82^\circ} . \qquad N = \frac{.82^\circ \times 3600''}{1^\circ} .$$

Thus $N = 2{,}952''$. Now $1' = 60''$. Think: What number times $60''$ equals $2{,}952''$.

$60 \overline{)2952.}$ By division we see that

$$\begin{aligned} 2{,}952'' &= 49 \times 60'' + 12''. \\ &= 49 \times 1' + 12'' \end{aligned}$$

Thus, $.82^\circ = 2{,}952'' = 49' + 12''$.

So $62.82^\circ = 62^\circ\ 49'\ 12''$.

Discuss Examples 1 and 2 with the class, outlining the reasons for each step. Do not require that the students master this computation.

Example 1. It is the $.82^\circ$ that must be converted. A proportion can be set up or the students can be led to see that if $1^\circ = 3{,}600''$ than $.82 \times 1^\circ = .82 \times 3{,}600''$. Dividing $2{,}952''$ by $60''$ gives the number of minutes in 2,952 seconds.

In expanded form: $62^\circ\ 45'\ 12'' = 62^\circ + 45' + 12''$

$$= (62 + \frac{49}{60} + \frac{12}{3600})^\circ.$$

Example 2. $14'\ 24''$ means $14' + 24''$. Converting $14'$ to seconds allows us to find the total number of seconds. Than we must find what part this is of a degree.

An alternate way (rather than use a proportion) would be to see $18^\circ\ 14'\ 24''$ as

$$(18 + \frac{14}{60} + \frac{24}{3600})^\circ.$$

$$\frac{14}{60} + \frac{24}{3600} = \frac{14 \times 60 + 24}{3600}$$

$$= \frac{864}{3600} = .24\ .$$

Example 2. Convert $18^\circ\ 14'\ 24''$ to degrees in decimal form.

Solution: We must convert $14'\ 24''$ to degrees.

$1' = 60''$ so $14' = 14 \times 60'' = 840''$.

Thus $14'\ 24'' = 840'' + 24'' = 864''$.

$$1^\circ = 3600''. \quad \frac{1^\circ}{3600''} = \frac{N}{864''}. \quad N = \frac{1^\circ \times 864''}{3600''}.$$

Thus $N = .24^\circ$ and $18^\circ\ 14'\ 24'' = 18.24^\circ$.

✓ POINT

1. How many minutes are there in one degree?
2. Is an angle measure of $2{,}785''$ greater than or less than 1°?

EXERCISES

1. How many seconds are there in one degree?
2. A circle would have to be divided into how many congruent arcs if each arc is to measure 1 second of angle?
3. Complete the following conversions:

   (a) $180' = \_\_\_\_^\circ$ (c) $\frac{1}{3}^\circ = \_\_\_\_'$

   (b) $900' = \_\_\_\_^\circ$ (d) $18{,}000'' = \_\_\_\_^\circ$
4. Convert the following degree measures to the correct amount of degrees, minutes and seconds.

   (a) 5.5° (b) 47.25°

   (c) 13.19° (Hint: Multiply .19 times 3600".)
5. Convert the following angle measures to degrees in decimal form.

   (a) $5^\circ\ 15'$ (b) $137^\circ\ 42'$ (c) $58^\circ\ 28'\ 12''$

ANSWERS TO ✓ POINT

1. $1^\circ = 60'$
2. Less than 1° since $1^\circ = 3{,}600''$.

Answers to Exercises

1. $3{,}600'' = 1^\circ$
2. 1,296,000 congruent arcs
3. a) 3° c) $20'$

   b) 15° d) 5°
4. a) $5^\circ\ 30'$ b) $47^\circ 15'$

   c) $13^\circ\ 11'\ 24''$
5. a) 5.25° b) 137.7° c) 58.47°

HOW MANY DEGREES?

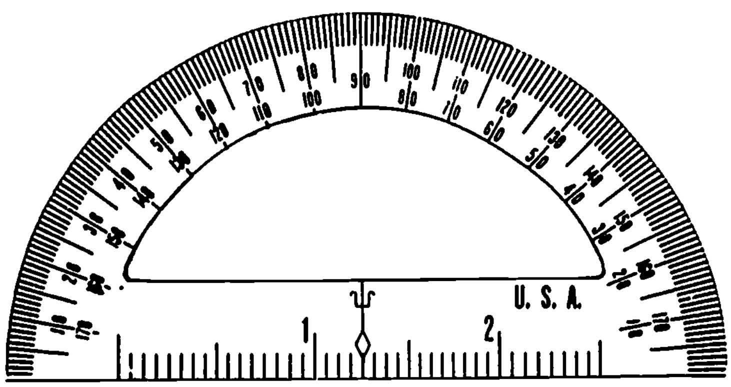
A VARIETY OF SHAPES AND SIZES

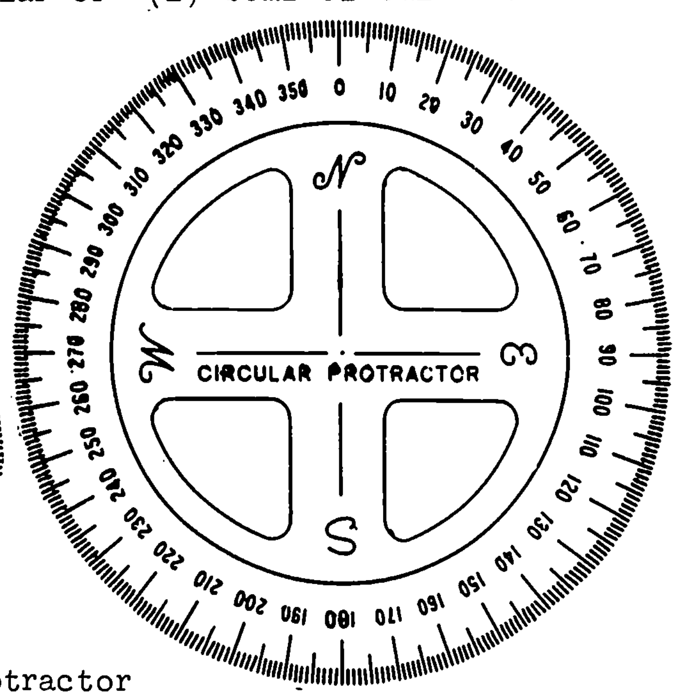
Protractors come in a variety of shapes and sizes. Some of these are pictured in the back of this booklet in **APPENDIX D** .

Regardless of shape or size, all protractors are read in a similar manner. If the protractor you are using is not like the ones pictured in this lesson and you can not determine how it is read, consult **APPENDIX D** or your teacher for help.

THE SHAPE OF A PROTRACTOR IS . . .

The protractors you will use in this booklet are for the most part either (1) circular or (2) semi-circular in shape.





TO BE EXPLORED . . .

Is it necessary for a protractor to be circular or semi-circular?

To explore the question above, refer to **APPENDIX E** at the back of this booklet.

HOW MANY DEGREES?

OBJECTIVES

1. Given a drawing of an angle whose measure is between 0° and 360°, the student will be able to measure the angle to the nearest degree under the following conditions:

   a. When the angle is determined by...

      (1) two intersecting lines, or
      (2) two rays having a common endpoint.

   b. Using either a...

      (1) circular protractor, or...
      (2) a semi-circular protractor.

   c. With the protractor in standard position (one ray through the zero point).

   d. When the "sides" of the angle must be extended to get a reading on the protractor scale.

   It is assumed that the protractor used is of sufficient quality to enable an accurate reading to the nearest degree.

2. Given an angle whose measure is between 0° and 360°, the student will be able to give a reasonable estimate (a 10° tolerance) of the measure in degrees.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

1. Semi-circular protractor (1 per student)
2. Circular protractor (1 per student)

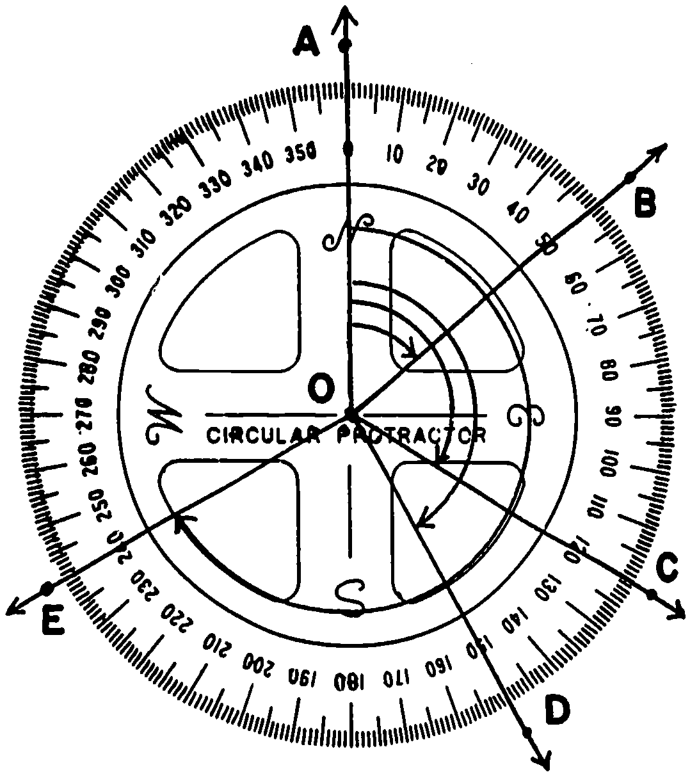
B. TEACHER

1. Overhead projector
2. Projection screen

*3. Acetate protractors (circular and semi-circular)

USING A CIRCULAR PROTRACTOR . . .

In some situations, a circular protractor may be more suitable for measuring angles.



Using the circular protractor to the left,...

$m(\angle AOB) = 50^\circ$

$m(\angle AOC) = 120^\circ$

$m(\angle AOD) = 150^\circ$

$m(\angle AOE) = 240^\circ$

When reading the angle measures listed above, . . .

(1) the center point of the protractor is on the vertex of the angle.

(2) one side (ray) of the angle passes through the zero point on the protractor scale.

(3) the number on the protractor scale corresponding to the other ray gives the measure of the angle.

CONTENT AND APPROACH (for page 70)

Protractors differ in their markings, especially in the manner the center point of the circular scale is indicated. The protractors pictured in the booklet represent what seem to be more commonly available in department stores, drug stores, etc.

APPENDIX D is to be used as a reference for the student and illustrates the use of several common protractors in measuring angle size. This section may be of help to the student who has a protractor unlike the one pictured in the booklet and exposes the student to the variety of protractors available on the commercial market.

APPENDIX E is designed as an enrichment lesson and project for those students who are not sufficiently challenged by the text material.

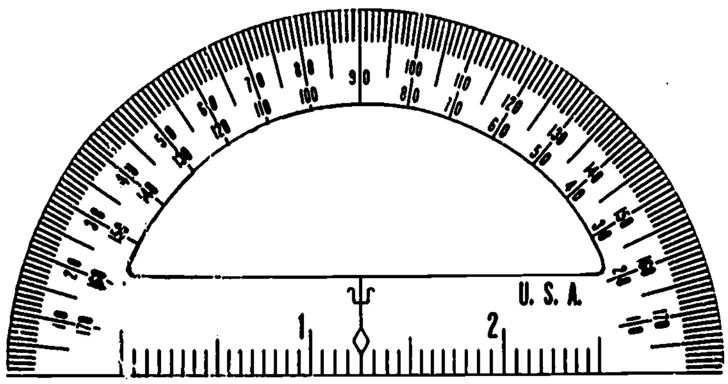
CONTENT AND APPROACH (for page 71)

The use of a circular protractor is demonstrated when it is placed in standard position (one ray through the zero point on the protractor scale). Use the acetate protractor and the overhead projector to further illustrate the three points listed at the bottom of the page. The student was initially exposed to a circular protractor in Lesson 8.

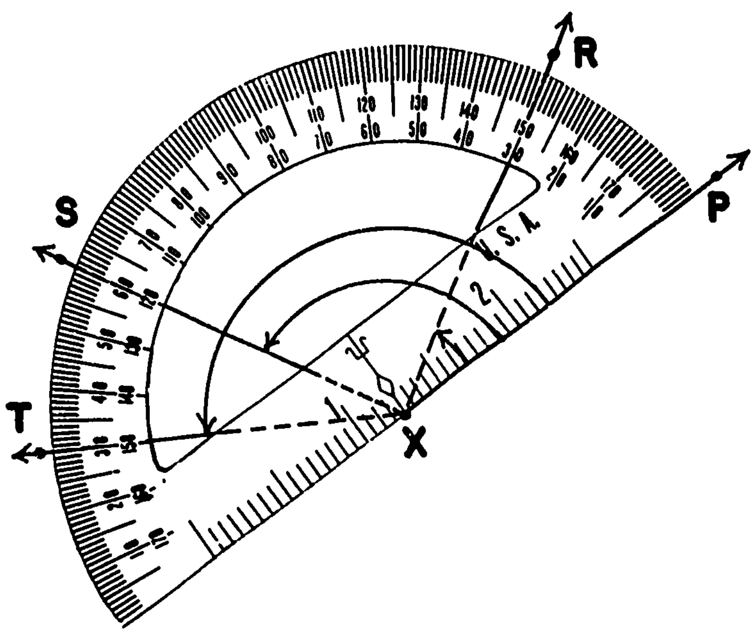
Conditions under which a circular protractor is more convenient to use than a semi-circular protractor is a topic for discussion at a later date after the student has had experience in using both. When demonstrating both the circular protractor and the semi-circular protractor (p. 72), include situations where the sides must be extended to get a reading on the protractor scale (see p. 8).

USING A SEMI-CIRCULAR PROTRACTOR . . .

Most of the protractors which you see for sale in department stores, drug stores, etc.... are semi-circular. Both the circular and semi-circular protractors have advantages in different situations.



SEMI-CIRCULAR PROTRACTOR



Using the semi- circular protractor at the left,...

$m(\angle PXR) = 30^\circ$

$m(\angle PXS) = 120^\circ$

$m(\angle PXT) = 150^\circ$

Notice that the semi-circular protractor is used in the same manner as the circular protractor.

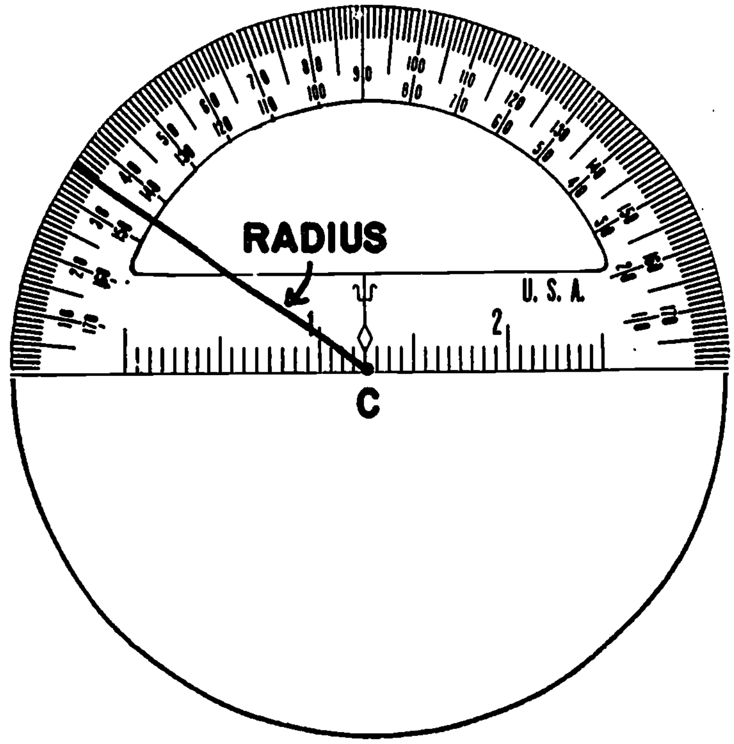
CONTENT AND APPROACH

The use of a semi-circular protractor is demonstrated when it is placed in standard position. The sentence at the bottom of the page refers to the three points listed at the bottom of page 71. Use the acetate protractor and overhead projector to further illustrate the use of a semi-circular protractor.

SOME SIMILARITIES . . .

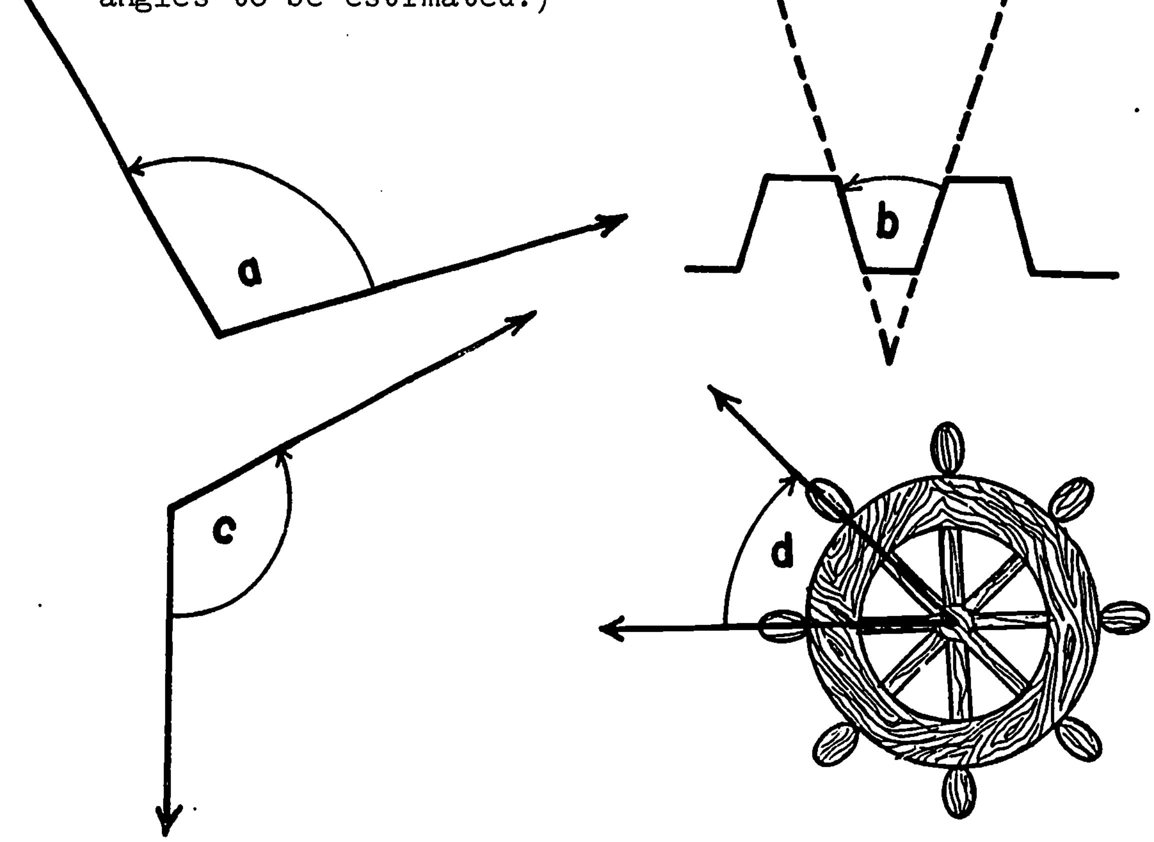
The point of a semi-circular protractor which is placed on the vertex of the angle being measured is also the <u>center</u> of a circle. (See picture to the right.)

Thus, any semi- circular protractor could be considered as one-half of a circular protractor.



EXERCISES I

1. Estimate in degrees the measure of each angle. Record your estimates in **TABLE 9-1** . (The arrows indicate the angles to be estimated.)



CONTENT AND APPROACH

The section at the top of the page indicates similarities between the circular and semi-circular protractor. By placing two acetate semi-circular protractors together alongside an acetate circular protractor on the stage of an overhead projector, similarities mentioned in the text can be vividly demonstrated.

Estimating angle size is not a skill which is developed in one set of exercises. (See **KEEP IN PRACTICE**, p.79.) Do not treat any estimates as incorrect. (Some may be close.) Encourage the students to guess. The second portion of LESSON 9 (pp. 76 - 79) gives some tips on refining estimates of angle size. You may wish to use **EXERCISES 1** as a contest to determine the best estimator in your class.

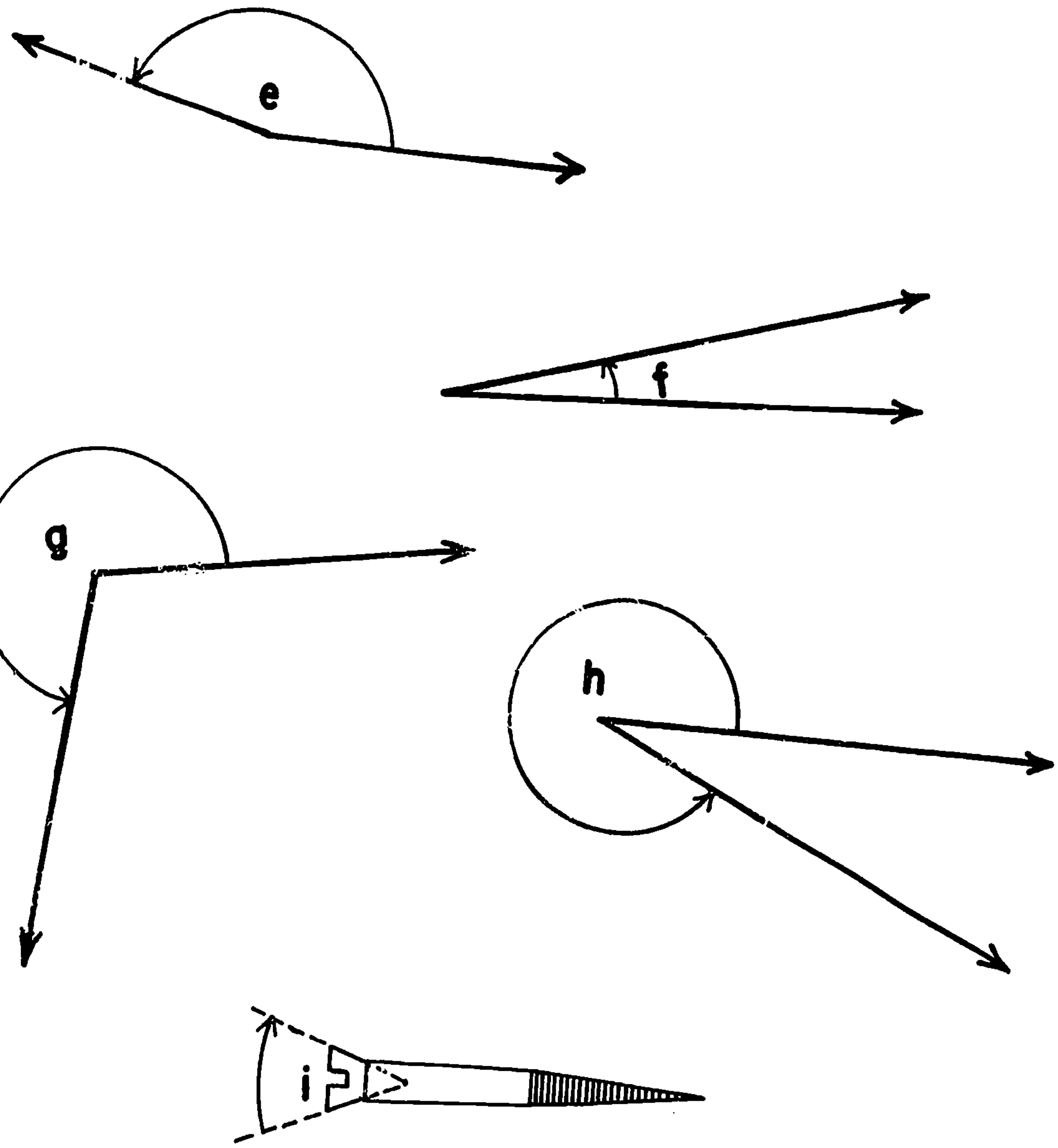

e
f
g
h
i

TABLE 9-1

ANGLE	ESTIMATE IN DEGREES...	MEASURE TO NEAREST DEGREE...
a		
b		
c		
d		
e		
f		
g		
h		
i		

2. For each angle in exercise 1, measure to the nearest degree using either ...

   a. a circular protractor, or

   b. a semi-circular protractor.

   Record your results in TABLE 9-1.

   If necessary, extend the sides of the angles so the measure can be read.

How close were your estimates? If your estimates were within 10° of the measured size, your estimates were very good.

ANSWERS

TABLE 9-1

ANGLE	ESTIMATE IN DEGREES... *	MEASURE TO NEAREST DEGREE... * *
a		103°
b		35°
c		119°
d		45°
e		165°
f		14°
g		255°
h		335°
i		44°

* Estimates will vary.

* * See page T79 for comments concerning the accuracy of these measurements.

CONTENT AND APPROACH

In exercise 2, provide each student with experience using both protractors. As an aid in discussing the questions at the top of page 76, it would be helpful to have each angle measured by both protractors. For example, if the class were divided into 2 groups, each student in one group could use a semi-circular protractor for angles a-d and a circular protractor, for angles e-i, with the students in the other group following the reverse of these directions.

Notice the comment about "extending" the sides of an angle. As indicated in **LESSON 1** (p.8), the student should be aware that the length of the side represented does not affect the size of the angle.

It is not expected that the student's estimates will come within 10° of the measured size at this stage. This is, however, a goal for the student to work toward with continued practice.

DISCUSSION QUESTIONS

1. In Exercise 2, would you get the same measure regardless of which protractor was used? Why or why not?

2. Explain how to use a semi-circular protractor to measure an angle whose measure is greater than 180° and less than 360°.

3. When measuring angles, would the ability to make a reasonable estimate for the measure of an angle help you to check your work? Why or why not?

REFINING YOUR ESTIMATES...

To estimate accurately, it is necessary to develop a "feeling" or mental picture of the units of measure involved. For example, to estimate the length of a room in feet, it is helpful to be able to visualize the length of one foot or one yard (3 feet).

As a guide, an angle of 1° is probably too small to be of use when estimating angle size. (See Figure 1.)

Figure 1

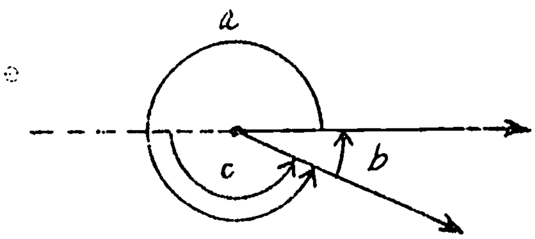
SOME USEFUL GUIDES...

Some useful estimation guides are those angles whose measures are multiples of 30° (30°, 60°, 90°, 120°.,...) and 45° (45°, 90°, 135°.,...).

DISCUSSION QUESTIONS

1. Yes. Refer to the similarities between the protractors—they use the same scale.

2. There are some alternatives. Refer to the diagram below. Suppose you were to measure $\angle$ a.



   (i) Extend one ray in the opposite direction. Measure $\angle$ c. Add m($\angle$ c) to 180° to get m($\angle$ a).

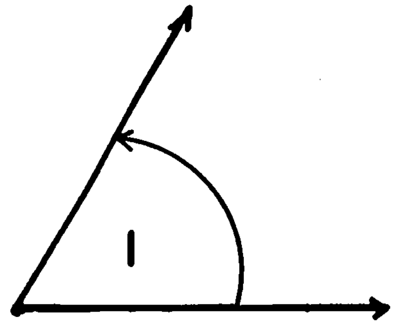
   (ii) Measure $\angle$ b. To get m($\angle$ a) subtract m($\angle$ b) from 360°.

3. Yes. It will not check the correctness of the measurement but will enable you to determine whether or not your answer is reasonable, thus eliminating many errors due to extreme carelessness or reading of the wrong scale.

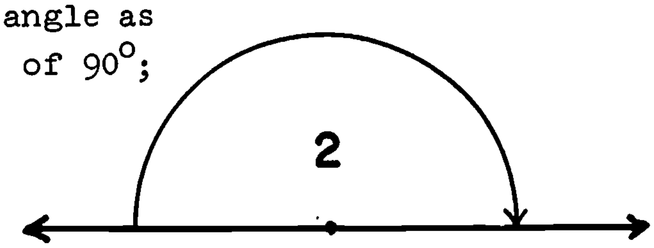
CONTENT AND APPROACH

Use the examples in this section (pp.76-79) as one suggested method only. Individuals vary in the angles which are easy for them to visualize. One quality of a good estimation guide is the ability of the person to visualize it. This is one reason an angle of 1° is useless as a guide. Capitalize on the student's previous experience in estimating length to indicate the need for a good estimation guide. This section is important for work in future sections of the booklet (see **NOTE**, p. T 86).

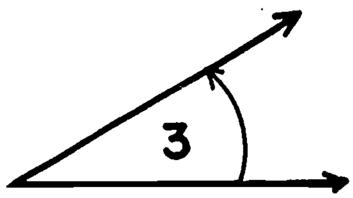
$m(\angle 1) = 60^\circ$. Using this angle as a guide, determine an angle of 30°.



$m(\angle 2) = 180^\circ$. Using this angle as a guide, determine an angle of 90°; of 45°.



$m(\angle 3) = 30^\circ$. Using this angle as a guide, determine an angle of 120°. ($120^\circ = 4 \times 30^\circ = 90^\circ + 30^\circ$)



Several angle measures can be interpreted as combinations of these angles. For example, $150^\circ = 90^\circ + 60^\circ$ or $5 \times 30^\circ$. Complete the following.

$135^\circ = 90^\circ +$ ________ $= 3 \times$ _________.

$225^\circ = 180^\circ +$ ________.

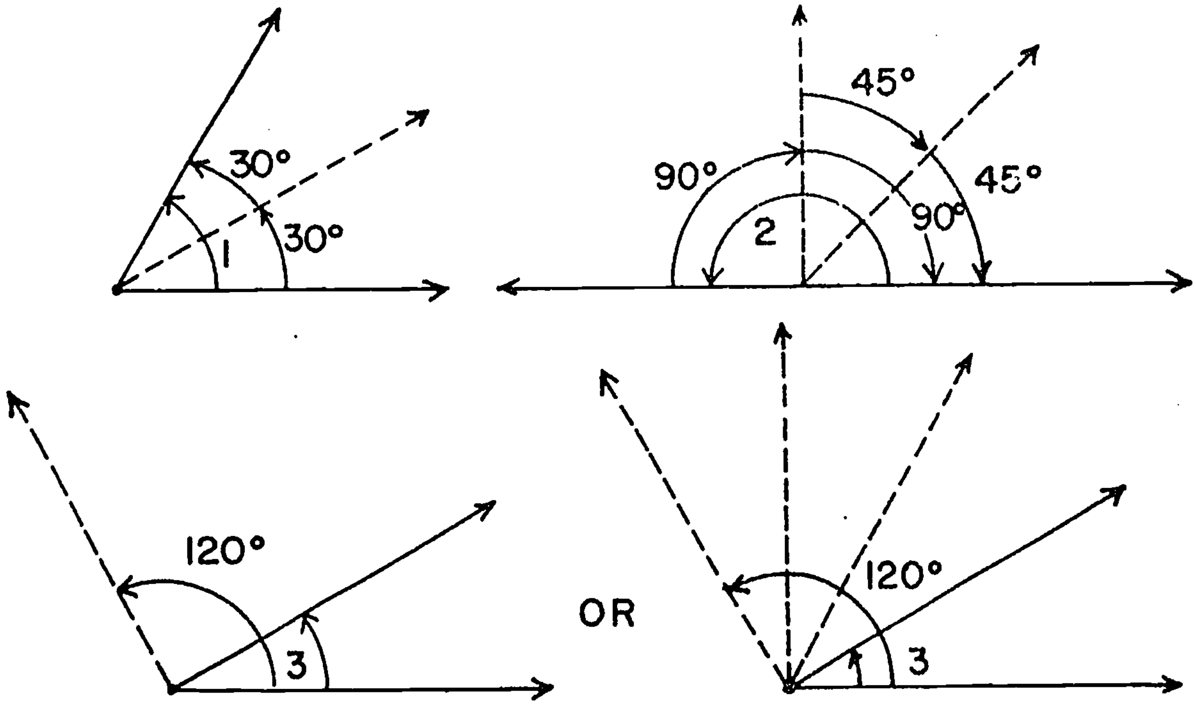
$270^\circ = 180^\circ +$ _______ $= 3 \times$ _________.

$315^\circ = 270^\circ +$ ________.

The examples on the next page will illustrate how these angles may be used as guides in estimating.

CONTENT AND APPROACH

Use the three examples at the top of the page as models of one way to determine other angle sizes using the angles whose measures are multiples of 30° and 45°. The dotted lines indicate the intent of each example.



The intent of the completion blanks is to illustrate that several angles are either multiples and/or combinations of a very few common angles such as 30°, 45°, 60°, 90°, and 180°. The examples on page 78 illustrate how these angles can be used to estimate angle size if a "feeling" for the sizes of these basic angles is developed.

ANSWERS

$135^\circ = 90^\circ + \underline{45^\circ} = 3 \times \underline{45^\circ}$

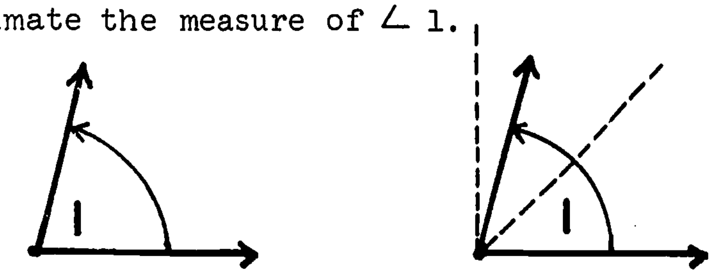
$225^\circ = 180^\circ + \underline{45^\circ}$

$270^\circ = 180^\circ + \underline{90^\circ} = 3 \times \underline{90^\circ}$

$315^\circ = 270^\circ + \underline{45^\circ}$

EXAMPLES:

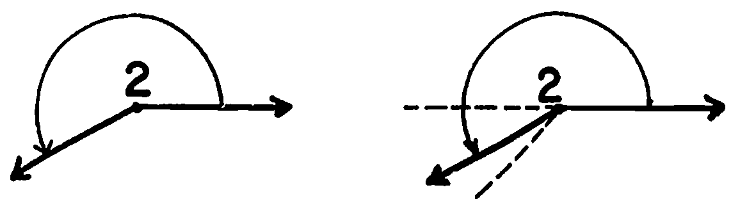
1. Estimate the measure of $\angle 1$.



SOLUTION: $m(\angle 1)$ is less than 90° and more than 45°.
$m(\angle 1)$ is a little more than half-way between 45° and 90°.
$67\frac{1}{2}^\circ$ is half-way between 45° and 90°.
<u>$m(\angle 1) \approx 70^\circ$.</u>

($\approx$ is read "approximately the same as".)

2. Estimate the measure of $\angle 2$.



SOLUTION: $m(\angle 2)$ is more than 180° and less than 225°.
$m(\angle 2)$ is closer to 225° and more than half-way between 180° and 225°.
<u>$m(\angle 2) \approx 215^\circ$.</u>

CHECK: Measure $\angle 1$ and $\angle 2$ with a protractor.
To the nearest degree,...
$m(\angle 1) = 76^\circ$
$m(\angle 2) = 210^\circ$

NOTE: The above angles could have been estimated in a variety of ways. The solutions are meant only as an example of <u>one</u> way these angle sizes could be estimated.

CONTENT AND APPROACH

As indicated at the bottom of the page, these examples show only one way these angle sizes could be estimated. For example, $\angle 1$ could have been estimated using angles of 90° and 60° as estimation guides.

EXERCISES II

1. For each angle pictured,...
   (1) estimate the measure.
   (2) measure to the nearest degree using a protractor.
   (3) check the closeness of each estimate by computing the difference between your estimate and your measurement.
   (4) record your results in **TABLE 9-2** .

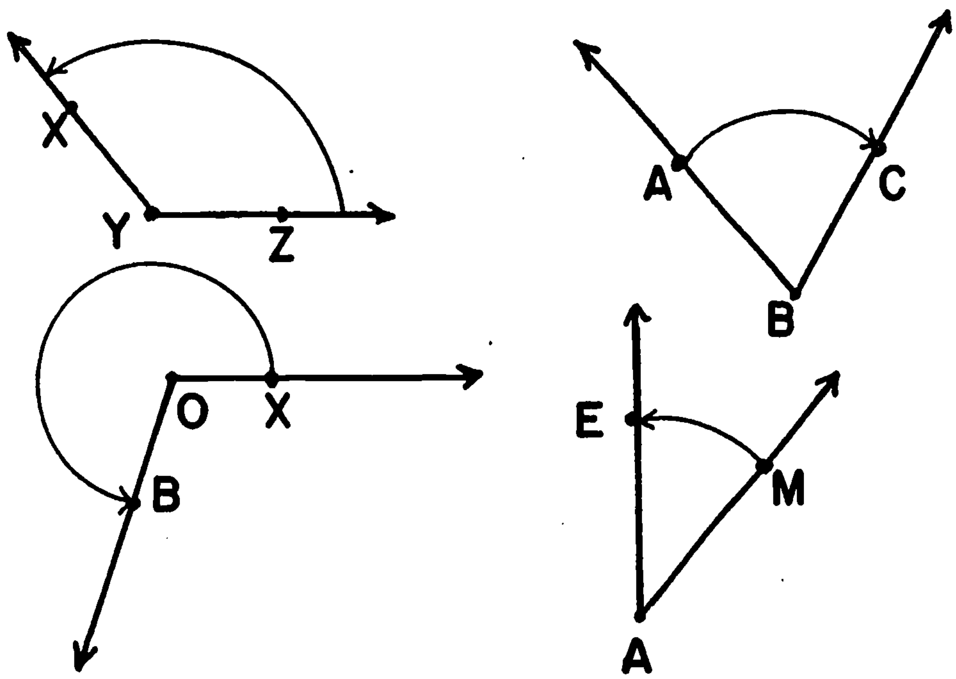


TABLE 9-2

ANGLE	ESTIMATE	MEASUREMENT	DIFFERENCE
∠ ZYX			
∠ ABC			
∠ XOB			
∠ MAE			

KEEP IN PRACTICE . . .

The ability to make reasonable estimates is not a skill which can be developed in one lesson. Keep in practice by estimating whenever possible. If you can make reasonable estimates, you will be able to check the reasonableness of any measuring you do.

CONTENT AND APPROACH

The choice of protractors is optional. Use whatever you feel is most appropriate or allow the students to choose which one they prefer to use. As in **EXERCISES I**, no estimate is incorrect. Have the student evaluate his own progress by comparing the closeness of his estimates in this set of exercises with the closeness of his estimates in **EXERCISES I**.

The measurements that the students obtain for **TABLE 9-2** may vary slightly (1 or 2 degrees) from the results given due to the thickness of the rays represented, an inability to determine the exact location of the vertex, and the fact that the protractors are not precision instruments. The measurements given were obtained by lining up the protractor scale markings with the center of the rays pictured. Use these answers given as guides. Discuss with your students reasons for small variations in measurements and how these variations could be corrected if variations occur.

The above comments also apply to the results of **TABLE 9-1** (page 75) and any other measurements in this booklet which are made to the nearest degree.

ANSWERS

TABLE 9-2

ANGLE	ESTIMATE	MEASUREMENT	DIFFERENCE
∠ ZYX	*	129°	**
∠ ABC		70°	
∠ XOB		251°	
∠ MAB		40°	

* Estimates will vary.

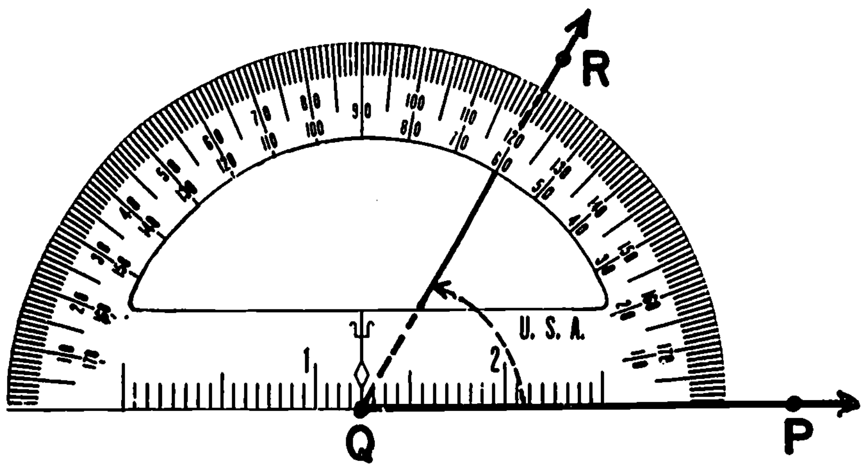
** The difference is a function of the estimate and measurement.

CONTENT AND APPROACH

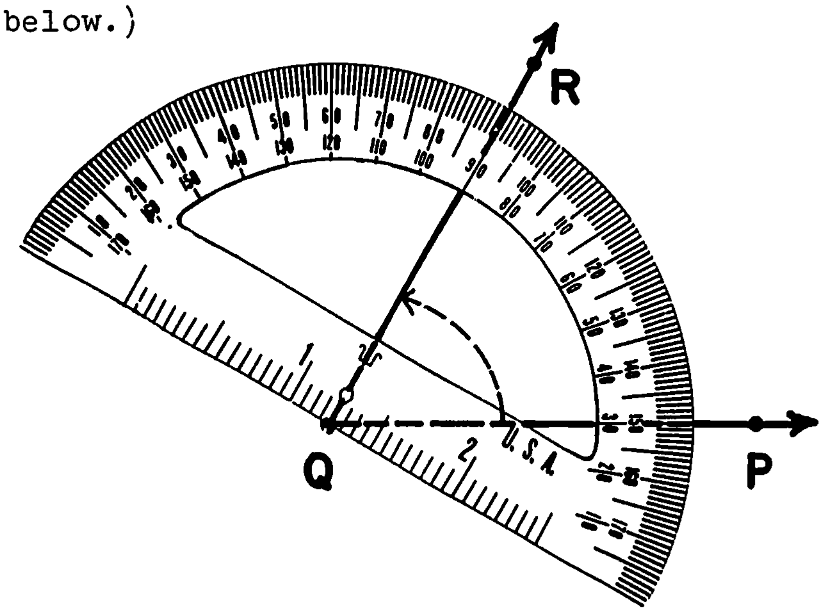
The statements under **KEEP IN PRACTICE** are very important. Ask your students to estimate whenever appropriate. The ability to estimate well can only accomplished through continued practice.

BROKEN PROTRACTORS

ANY WAY YOU WANT TO . . .



$\angle$ PQR pictured above has a measure of 60°. Suppose a protractor was placed on $\angle$ PQR in a different manner. (See picture below.)



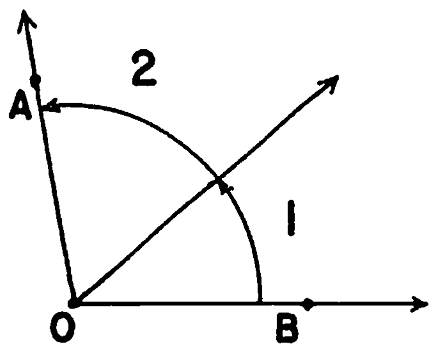
The measure of $\angle$ PQR as measured above is still 60° ($90^{\circ} - 30^{\circ} = 60^{\circ}$). In both cases the center point of the protractor is placed on the vertex of the angle (Q).

BROKEN PROTRACTORS

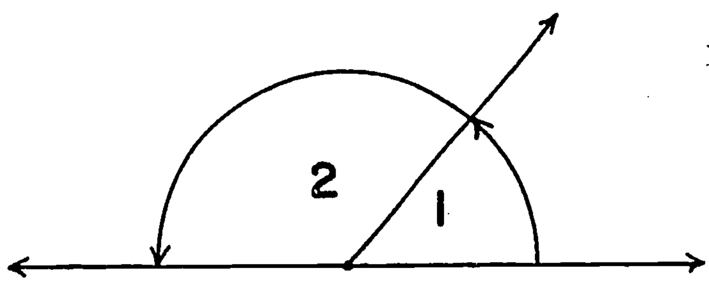
OBJECTIVES

1. Given a drawing of an angle whose measure is between 0° and 360°, the student will be able to measure the angle to the nearest degree with the protractor in non-standard position.

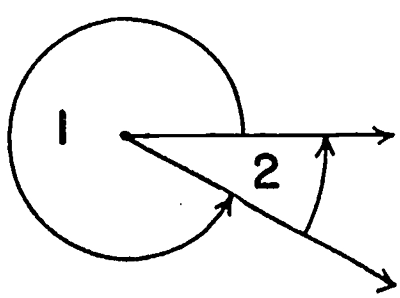
2. The student shall demonstrate his knowledge of the additive property of measurement and the fact that there are 360° in one complete revolution by answering questions such as the following without using a protractor.



If $m(\angle 1) = 40^\circ$, and
$m(\angle 2) = 60^\circ$, then
$m(\angle BOA) =$ _____.



If $m(\angle 1) = 50^\circ$, then
$m(\angle 2) =$ _____.



If $m(\angle 1) = 330^\circ$, then
$m(\angle 2) =$ ______.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

1. Circular protractor
2. Semi-circular protractor

*3. "Broken" protractor (1 per student)

B. TEACHER

1. Overhead projector
2. Projection screen
3. Acetate protractors (circular, semi-circular, broken)

CONTENT AND APPROACH

One of the major purposes of Lesson 10 is to indicate that any point of the protractor scale may be used as the "zero point". The pictures on page 80 illustrate this point. Other examples can be given using the acetate protractors and overhead projector.

Discussion of the material on page 80 and development of **OBJECTIVE #2** are incorporated in the **DISCUSSION EXERCISES** (pp. 81 - 82). Give the students an opportunity to work the **DISCUSSION EXERCISES** and discover the results on their own prior to discussing the results with the entire class. (See page T 82).

DISCUSSION EXERCISES

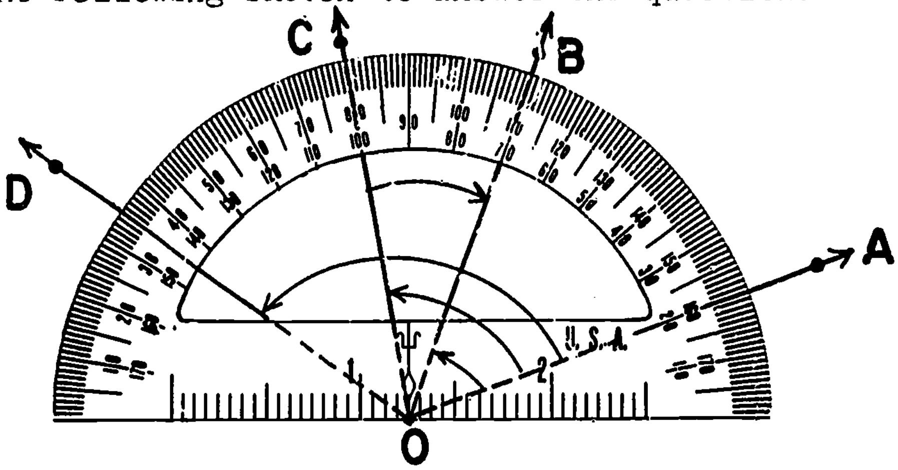
1. Name some other ways to measure $\angle PQR$.

2. When measuring an angle with a protractor, is it necessary for one of the rays of the angle to pass through the zero point on the protractor scale? Why or why not?

3. Are some points of the protractor scale more convenient to use as the "zero point" than others?

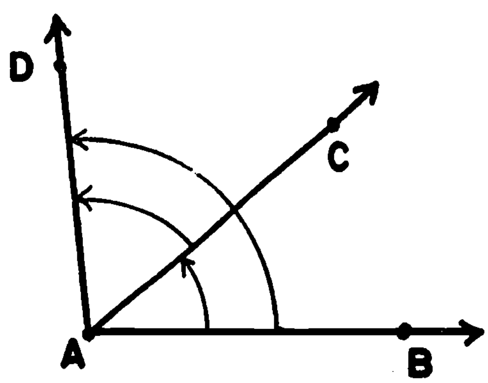
4. If $\angle$ PQR was measured with a circular protractor graduated in degrees, would $m(\angle PQR) = 60^\circ$?

5. Use the following sketch to answer the questions below.



$m(\angle AOB)$ = ______
$m(\angle AOC)$ = ______
$m(\angle AOD)$ = ______
$m(\angle COB)$ = ______

6.



If $m(\angle BAC) = 40^\circ$ and $m(\angle CAD) = 55^\circ$,

$m(\angle BAD)$ = ______

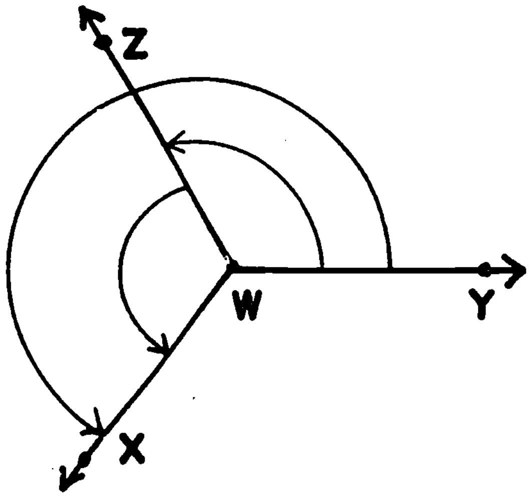
CONTENT AND APPROACH

Exercises 1-4 discuss the implications of the material on page 80. (**OBJECTIVE #1**) Exercises 5-7 develop the topics incorporated in **OBJECTIVE #2** . This second objective is developed through examples. After discussing Exercises 1-4, have the students attempt Exercises 5-7 on their own. Discuss the results, providing additional examples if necessary and relating it to previous work in this booklet.

ANSWERS

1. Other "zero points" can be used.
2. No. - See illustration on page 80.
3. Yes. - Using points such as 5°, 10°, 15°,... usually results in easier computation necessary for obtaining the result.
4. Yes.
5. $m(\angle AOB) = 50^{\circ}$
   $m(\angle AOC) = 80^{\circ}$
   $m(\angle AOD) = 125^{\circ}$
   $m(\angle COB) = 30^{\circ}$
6. $m(\angle BAD) = 95^{\circ}$

7.



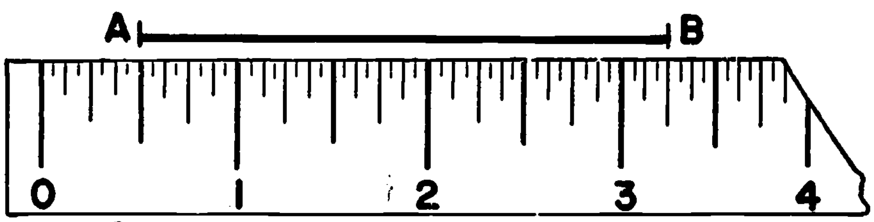
If m(∠ YWZ) = 120°
and m(∠ YWX) = 230°,

m(∠ ZWX) = ______

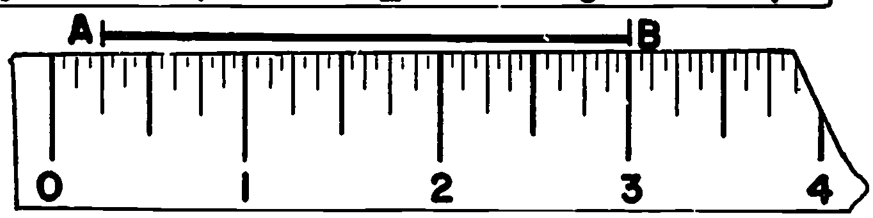
START WHERE YOU WANT TO . . .

The previous section illustrates one similarity between protractor and ruler measurement.

(1) When using a ruler, any point of the ruler scale can be used as the "zero point".

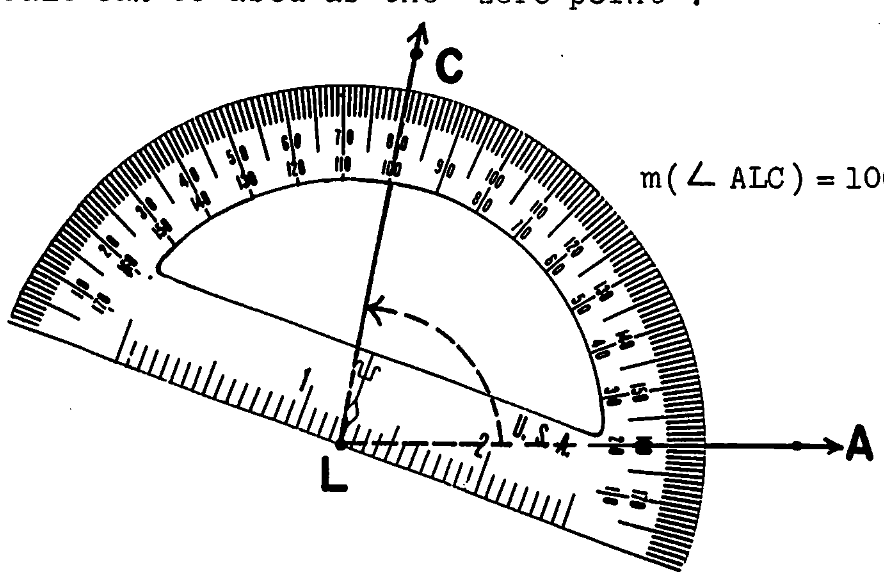


m($\overline{AB}$) = $2\frac{3}{4}$ in.



m($\overline{AB}$) = $2\frac{3}{4}$ in.

(2) When using a protractor, any point of the protractor scale can be used as the "zero point".



m(∠ ALC) = $100^\circ - 20^\circ = 80^\circ$

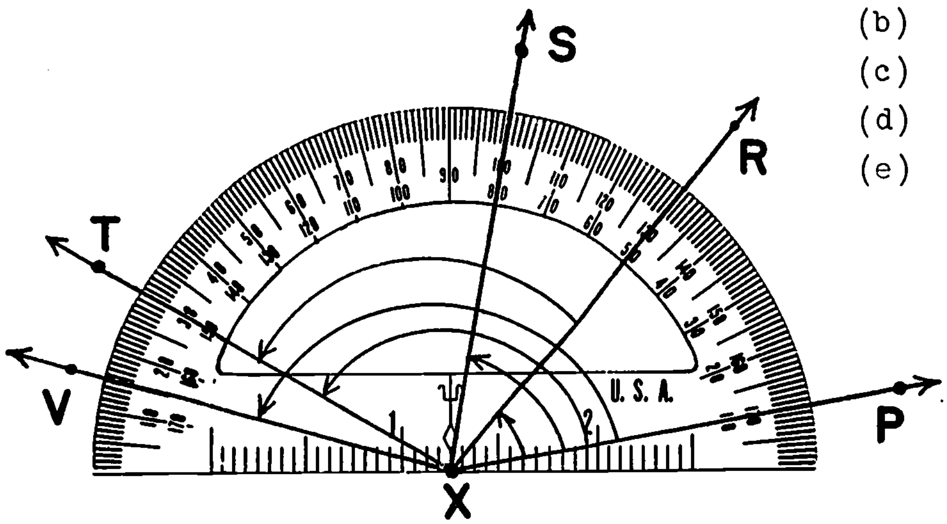
ANSWERS

7. $m(\angle ZWX) = 110^\circ$

CONTENT AND APPROACH

START WHERE YOU WANT TO summarizes the development of **OBJECTIVE #1**, indicating the similarities between the arbitrary selection of a "zero point" in linear and angle measure.

EXERCISES



1. Complete the following:

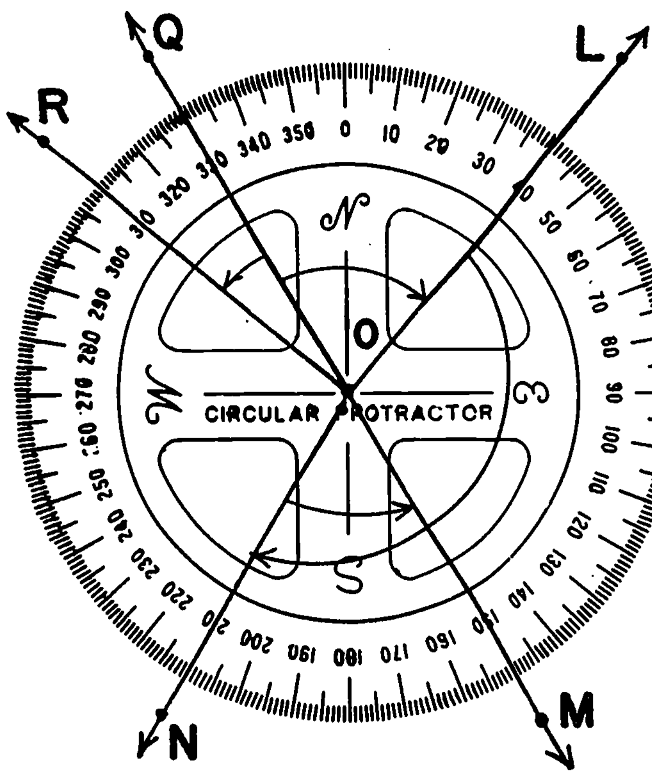
   (a) m(∠ PXR) = _____.

   (b) m(∠ PXS) = _____.

   (c) m(∠ PXT) = _____.

   (d) m(∠ PXV) = _____.

   (e) m(∠ RXT) = _____.



2. Complete the following:

   (a) m(∠ QOL) = _____.

   (b) m(∠ LON) = _____.

   (c) m(∠ QOR) = _____.

   (d) m(∠ NOM) = _____.

ANSWERS

1. a. m(∠PXR) = <u>40°</u>.
   b. m(∠PXS) = <u>70°</u>.
   c. m(∠PXT) = <u>140°</u>.
   d. m(∠PXV) = <u>155°</u>.
   e. m(∠RXT) = <u>100°</u>.

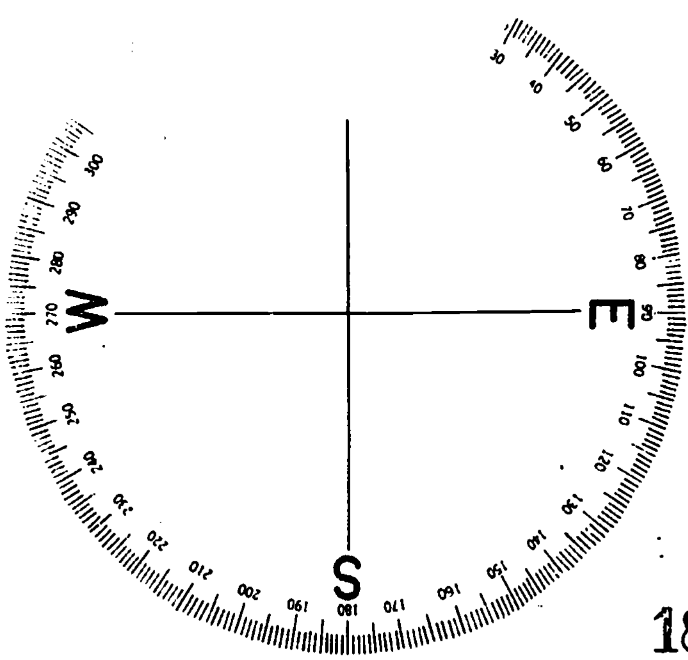
2. a. m(∠QOL) = <u>70°</u>.
   b. m(∠LON) = <u>170°</u>.
   c. m(∠QOR) = <u>20°</u>.
   d. m(∠NOM) = <u>60°</u>.

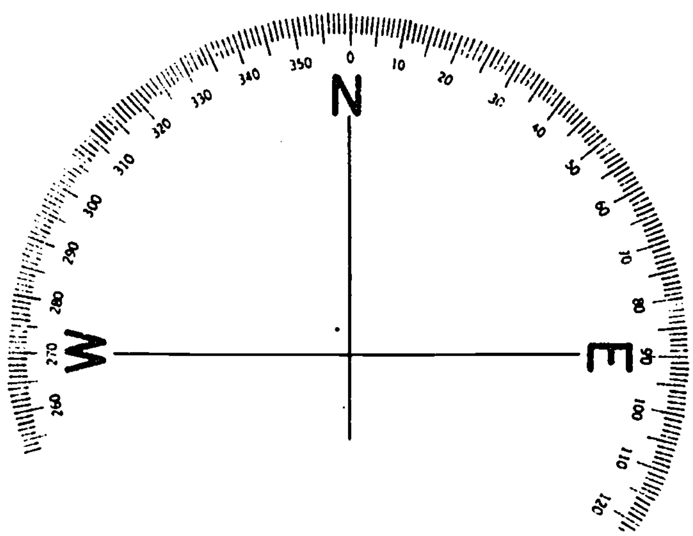
CONTENT AND APPROACH (Exercises 3-4, pp. 84-85)

Three possible strategies for using Exercises 3-4, pp. 84-85, are outlined below and on pages T84 and T85.

STRATEGY I

To follow the format on pages 84-85, have each student cut out a protractor from the insert between pages 84 and 85 so that a portion (no more than one-half) of the scale is missing and the center point of the protractor is still indicated. To gain maximum benefit from discussing the results have students cut out different portions of the scale on each protractor (see examples below). Sample discussion questions for exercises 3-4 are given on page T85 under the heading **THINGS TO DISCUSS**.





3. For this exercise, you will use a "broken" protractor. Use this "broken" protractor to measure each of the angles below. Record your results in **TABLE 10-1** (in column labeled **BROKEN PROTRACTOR**).

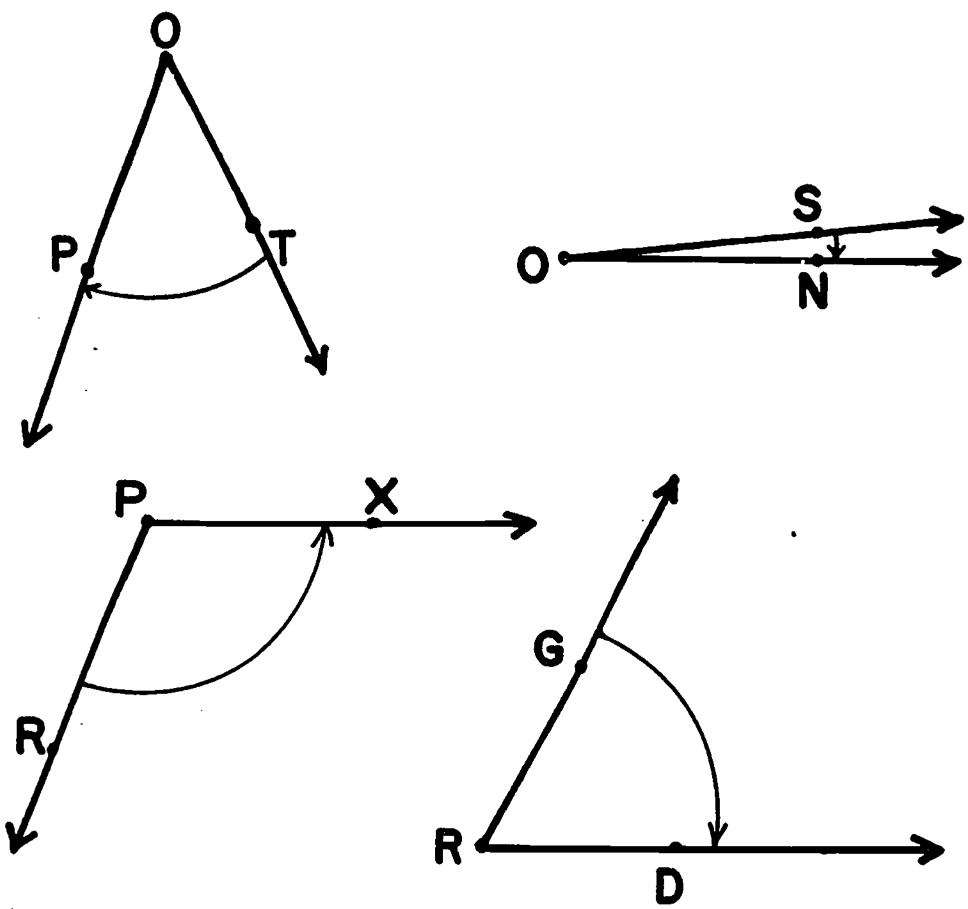
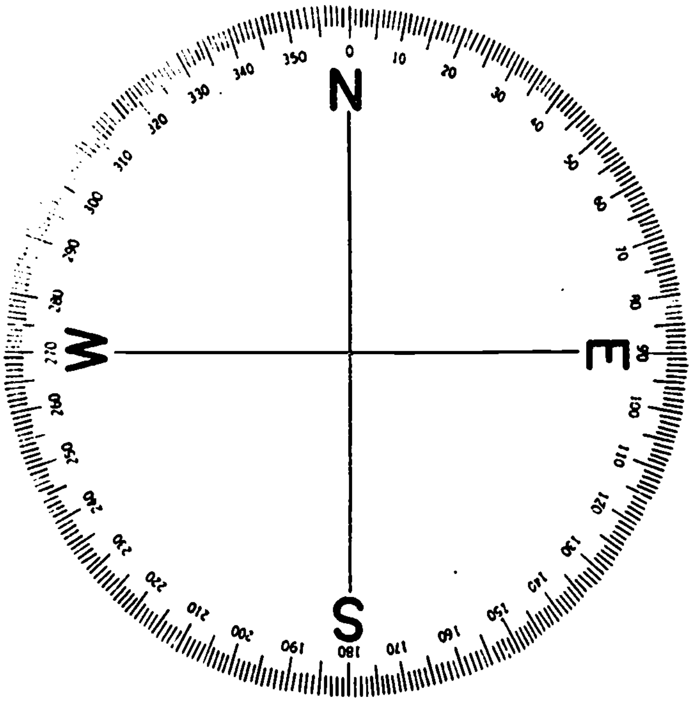
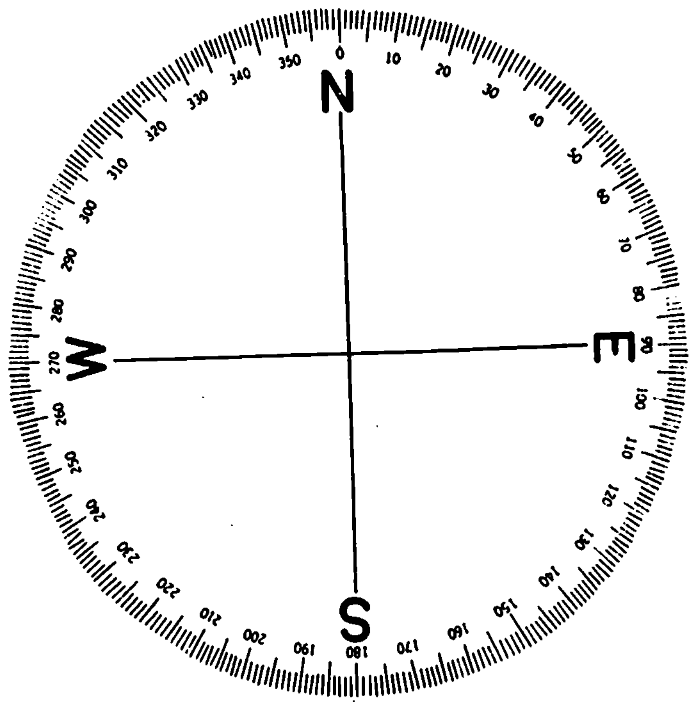
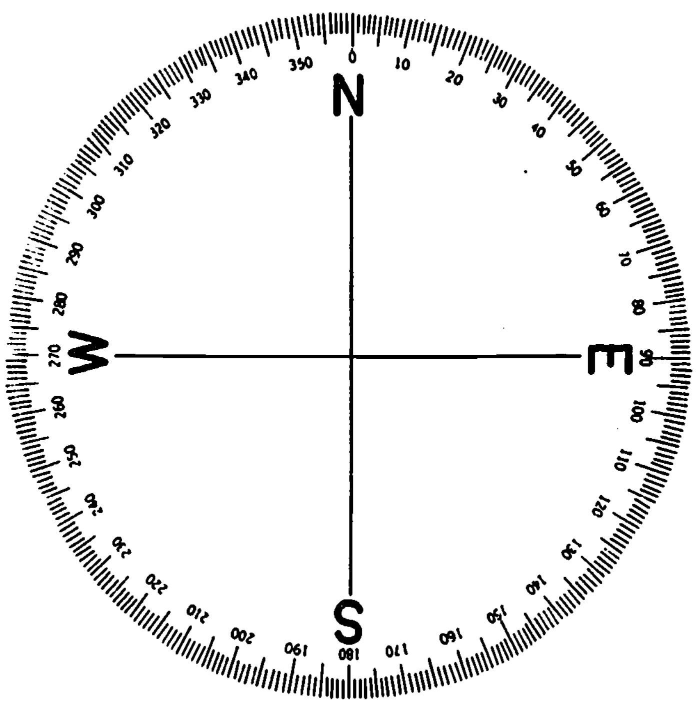


TABLE 10-1

ANGLE	MEASURE TO NEAREST DEGREE USING...	
	BROKEN PROTRACTOR	UNBROKEN PROTRACTOR
∠ TOP		
∠ SON		
∠ RPX		
∠ GRD		







FOR USE WITH EXERCISES 3-4, pp. 84-85.

STRATEGY 2

Using the sheet of protractors and page 84 as masters, make one acetate sheet of each to use with an overhead projector.

Cut out a different portion of the scale for each protractor. Project the four angles from page 84 on the screen. As each broken protractor is placed on an angle, have each student record his results in a table like the one below.

	MEASURING TO NEAREST DEGREE USING ...			
ANGLE	B.P.1.	B.P.2	B.P.3	U.P.
∠ TOP				
∠ SON				
∠ RPX				
∠ GRD				

Continue this process until all angles have been measured with each of the protractors (12 measurements in all). Have the students then measure each of the angle with an unbroken protractor, using the same angles pictured in the booklet. The results can then be compared and discussed, using the sample discussion questions as a guide (p. T85).

ANSWERS

3-4.

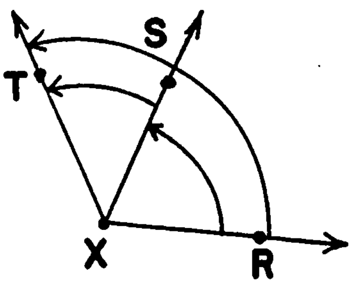
TABLE 10-1

ANGLE	MEASURE TO NEAREST DEGREE USING...	
	BROKEN PROTRACTOR	UNBROKEN PROTRACTOR
∠ TOP	47°	47°
∠ SON	6°	6°
∠ RPX	112°	112°
∠ GRD	62°	62°

4. Measure each of the angles in exercise 3 using an unbroken protractor. Record your results in **TABLE 10-1** (column labeled **UNBROKEN PROTRACTOR**).

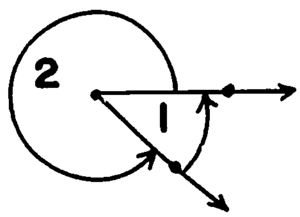
5. Complete the following:

a.



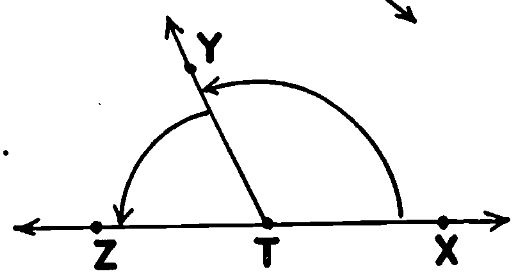
m(∠ RXS) = 70°
m(∠ SXT) = 50°
m(∠ RXT) = _____

b.



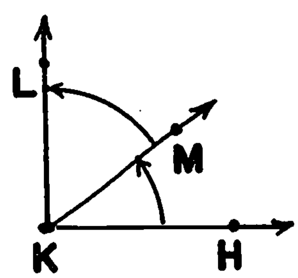
m(∠ 1) = 40°
m(∠ 2) = _____

c.



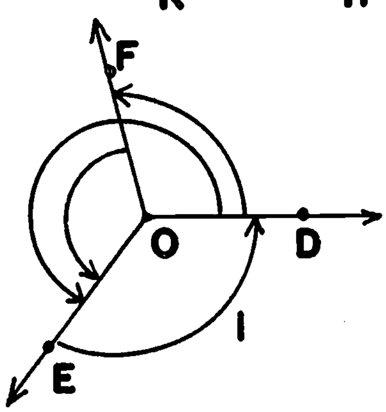
m(∠ YTZ) = 65°
m(∠ XTY) = _____

d.



∠ HKL is a right angle.
m(∠ MKL) = 55°
m(∠ HKM) = _____

e.



m(∠ DOE) = 230°
m(∠ DOF) = 105°
m(∠ FOE) = _____
m(∠ 1) = _____

ANSWERS

4. See TABLE 10-1 on page T [illegible].

[illegible]. a. m(∠ [illegible]) = [illegible]°.

   b. m(∠ [illegible]) = [illegible]°.

   c. m(∠ XTY) = 115°.

   d. m(∠ HKM) = [illegible]°.

   e. m(∠ FGE) = [illegible]°.

   m(∠ [illegible]) = 1[illegible]0°.

STRATEGY 3

Use the plastic circular protractors and a format like that outlined in either Strategy I or Strategy 2. To construct a "broken protractor", place masking tape over a portion of the outer scale.

THINGS TO DISCUSS

Questions similar to those below could be used to discuss the results of Exercises 3 and 4 (TABLE 10-1).

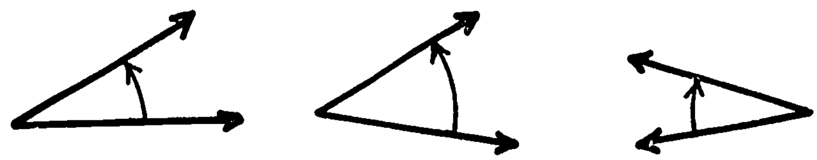
1. Were the measures in exercises 3 and 4 using either the broken protractor or the unbroken protractor the same? Why or why not?

2. Did every one in the class use the same point on the protractor as their "zero point"?

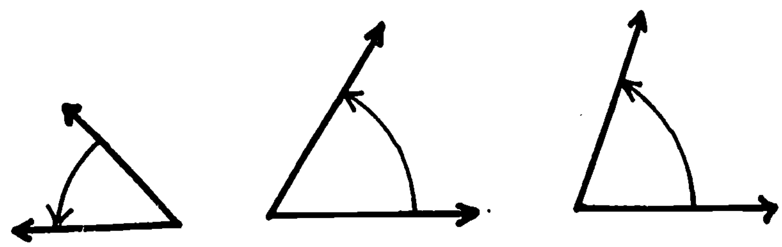
3. Could you use any point on the protractor scale as the "zero point"? Would some points be more convenient to use than others?

6. Each part has three angles pictured. In each part, circle the angle which you believe to be closest to the measure given.

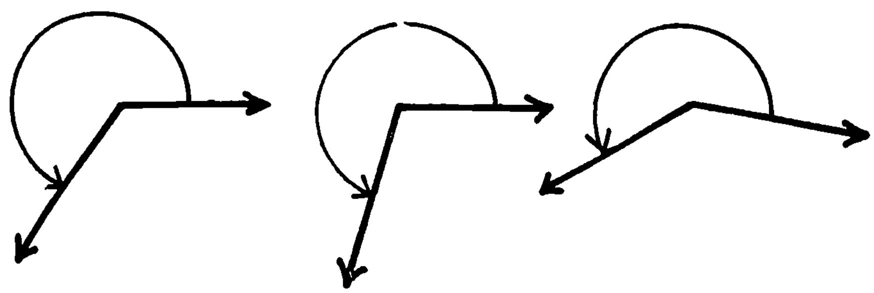
a. 25°



b. 60°



c. 240°



7. Check the accuracy of your guesses in exercise #6 by measuring each angle.

*8. (a) For each triangle pictured on page 87, measure...
(i) each side to the nearest $\frac{1}{16}$ inch, and
(ii) each angle to the nearest degree.
Record your results in the blanks provided.

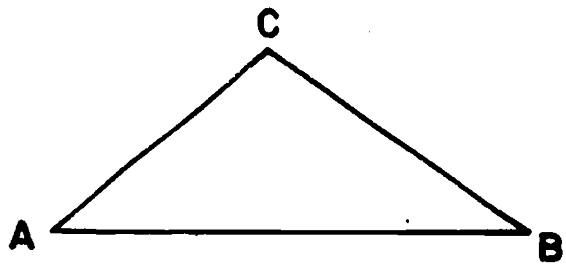
ANSWERS

6. a. Third angle
   b. Second angle
   c. First angle

These angles should be circled.

8. This exercise is optional. The exercise allows the student to practice skills in both linear and angle measurement, and in making conjectures concerning these measurements.

NOTE: Field-testing indicates that students experience some difficulty with problems like those in exercise 6. Pages 76-78 discuss the use of commonly used angles (30°, 45°, 60°, 90°,...) as guides in estimating angle measures. This section should be discussed thoroughly. Having students practice on exercises like exercise 2 on page 91 using measures of 30°, 45°, 60°,... would probably be of help in developing the ability to work problems like those in exercise 6.

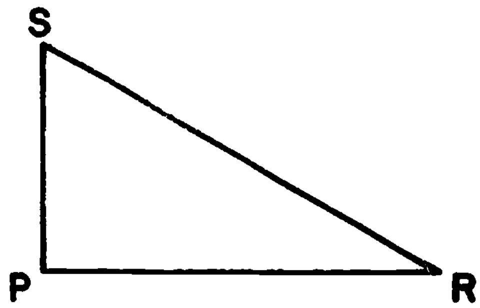


$m(\overline{AB})$ = ____ $m(\overline{BC})$ = ____

$m(\angle C)$ = ____ $m(\angle A)$ = ____

$m(\overline{CA})$ = ____

$m(\angle B)$ = ____

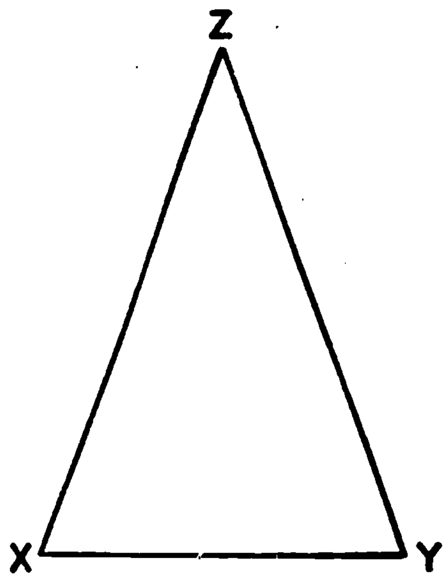


$m(\overline{SR})$ = ____ $m(\overline{RP})$ = ____

$m(\angle P)$ = ____ $m(\angle S)$ = ____

$m(\overline{PS})$ = ____

$m(\angle R)$ = ____



$m(\overline{XZ})$ = ____ $m(\overline{ZY})$ = ____

$m(\angle Y)$ = ____ $m(\angle X)$ = ____

$m(\overline{YX})$ = ____

$m(\angle Z)$ = ____

(b) Write a conjecture about the relationship between the size of each angle and the length of the sides opposite the angles.

(c) Write any other conjectures which seem to be true on the basis of your measurements.

ANSWERS

8. $m(\overline{AB}) = \underline{2\frac{1}{2}\text{ in.}}$ $m(\overline{BC}) = \underline{1\frac{11}{16}\text{ in.}}$

$m(\angle C) = \underline{106^\circ}$ $m(\angle A) = \underline{39^\circ}$

$m(\overline{CA}) = \underline{1\frac{1}{2}\text{ in.}}$

$m(\angle B) = \underline{35^\circ}$

$m(\overline{SR}) = \underline{2\frac{3}{8}\text{ in.}}$ $m(\overline{RP}) = \underline{2\frac{1}{16}\text{ in.}}$

$m(\angle P) = \underline{90^\circ}$ $m(\angle S) = \underline{60^\circ}$

$m(\overline{PS}) = \underline{1\frac{3}{16}\text{ in.}}$

$m(\angle R) = \underline{30^\circ}$

$m(\overline{XZ}) = \underline{2\frac{13}{16}\text{ in.}}$ $m(\overline{ZY}) = \underline{2\frac{13}{16}\text{ in.}}$

$m(\angle Y) = \underline{70^\circ}$ $m(\angle X) = \underline{70^\circ}$

$m(\overline{YX}) = \underline{1\frac{15}{16}\text{ in.}}$

$m(\angle Z) = \underline{40^\circ}$

(b) The conjecture intended (your students may find others) is that the angle opposite the longest side of a triangle has a greater measure than either of the other two angles.

(c) Accept any valid conjecture. One purpose of the exercise is to give the student practice in making conjectures based on measurements obtained in an experimental situation.

DRAWING ANGLES

Given an angle, you have used protractors (both circular and semi-circular) to find its measure to the nearest degree.

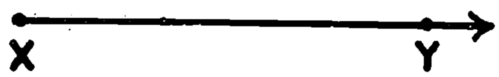
Consider the opposite of this situation. Suppose you were given an angle measure, say 67°, and were to draw an angle that size.

One such example is worked out below.

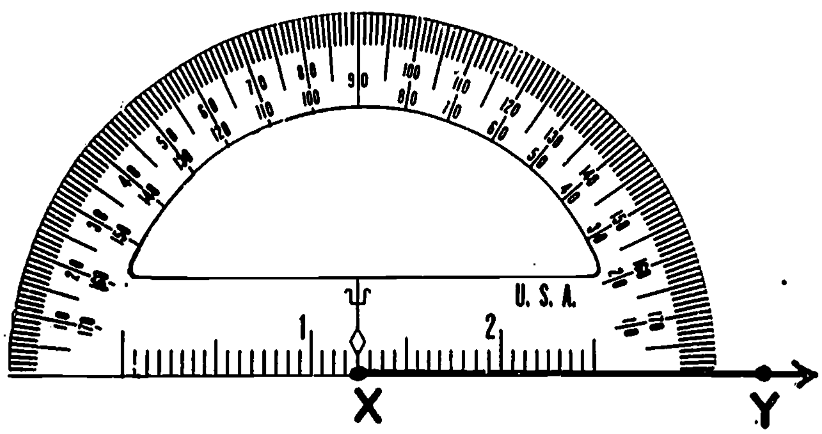
PROBLEM: Using a protractor, draw an angle whose measure is 67°.

SOLUTION:

1. Draw a ray. ($\overrightarrow{XY}$)



2. Place the center point of the protractor on the endpoint (X) of the ray and "line up" the ray with the zero point of the protractor scale.



DRAWING ANGLES

OBJECTIVES

1. Given any measure in degrees between 0° and 360° the student will be able to use a protractor (either circular or semi-circular) to draw an angle having the given measure.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

1. Circular protractor
2. Semi-circular protractor
3. Straightedge

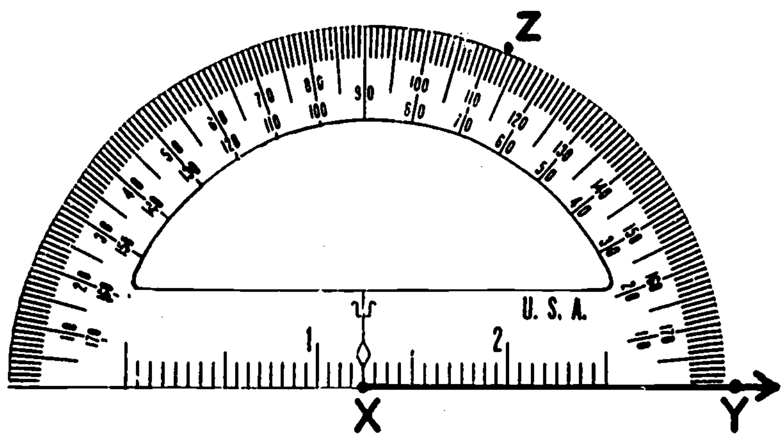
B. TEACHER

1. Acetate protractors (circular and semi-circular)
2. Overhead projector
3. Projection screen
4. Acetate overlay (for checking exercise # 1, p.91)

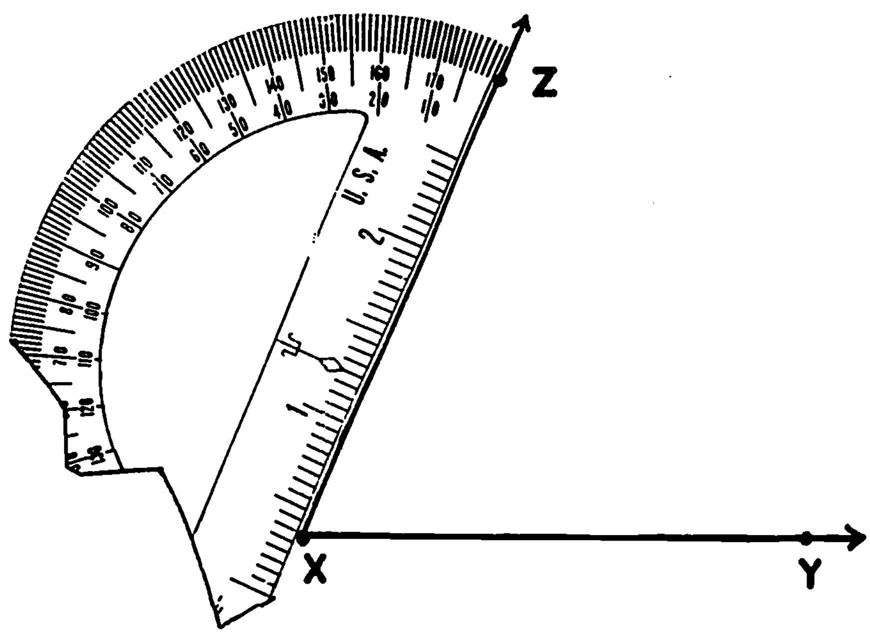
CONTENT AND APPROACH

The diagrams on pages 88 and 89 illustrate how a semi-circular protractor may be used to draw an angle whose measure is between 0° and 180°. In Lesson 1 (pp. 12-13), the students drew angles having a given measure using the unit angle wedges. The acetate semi-circular protractor can be used with an overhead projector to give further illustrations of the procedure outlined on pages 88 and 89.

3. Locate 67° on the protractor scale and make a dot (call it Z) at that point.



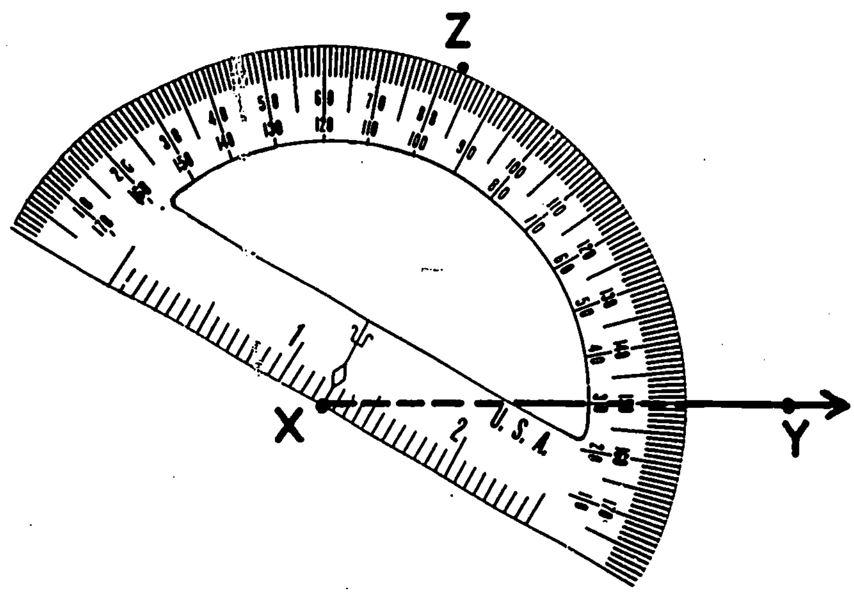
4. Remove the protractor and draw $\overrightarrow{XZ}$. $m(\angle XYZ) = 67^\circ$.

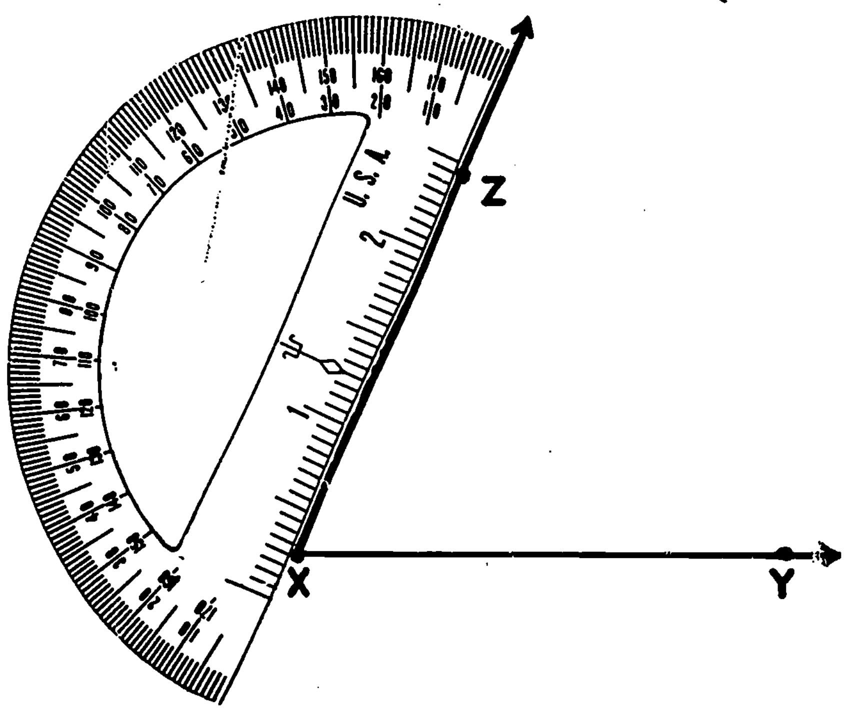


NOTE: There are other methods of drawing an angle of a given size. The method illustrated above is one of those more commonly used.

DISCUSSION QUESTIONS

1. After step 1, Stu "lined up" the ray with the 30° mark on the protractor scale and drew an angle as shown below.





Stu claimed that $m(\angle YXZ) = 67^{\circ}$. Was he correct? Why or why not?

CONTENT AND APPROACH

The example on pages 88 and 89 illustrates only the drawing of an angle between 0° and 180° with a semi-circular protractor. After this procedure is discussed the DISCUSSION QUESTIONS are to be used with the entire class to illustrate the use of a circular protractor in drawing angles, using an arbitrary "zero point", and the drawing of angles between 180° and 360° using either a circular or semi-circular protractor.

DISCUSSION QUESTIONS

1. Yes. Other points on the protractor scale can be used as the "zero point". The only condition that must be satisfied is that the appropriate number of units exists between the two points where the sides of the angle intersect the protractor scale.

In this booklet (Lesson 10) and the booklet **EXPLORING LINEAR MEASURE** (Lesson 9) students used arbitrary "zero points" in measuring angle size and linear distances.

2. Are there other ways of drawing an angle of 67°? If so, describe them.

3. Explain how a semi-circular protractor may be used to draw an angle whose measure is 240°.

4. Explain how a circular protractor may be used to draw an angle whose measure is (a) 121° (b) 220°.

EXERCISES

1. Using a protractor, draw angles whose measure is:

   (a) 78° (c) 127°

   (b) 323° (d) 254°

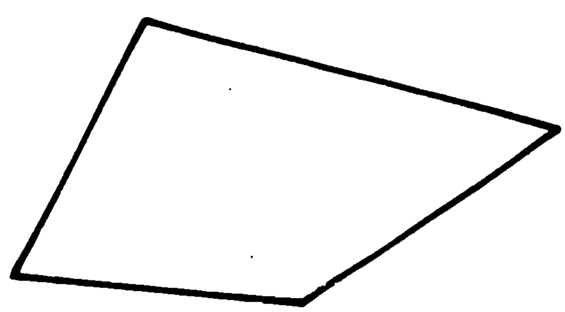
2. Using only a straightedge, draw an angle which you believe to have a measure of:

   (a) 49° (c) 153°

   (b) 300°

3. Check the closeness of your guesses in #2 by measuring each angle drawn and computing the difference between the measured size and the size you attempted to draw. (If you came within 10°, you did a very good job.)

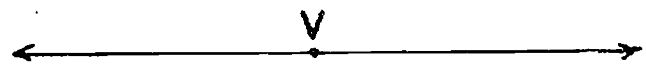
4. Using a ruler and protractor, make a drawing which is the same size and shape as the one below.



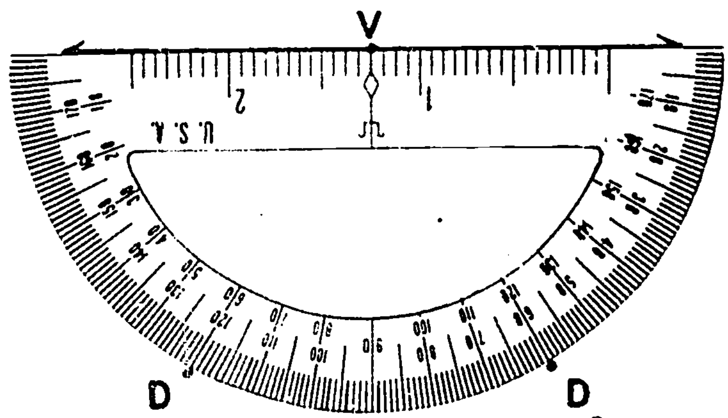
DISCUSSION QUESTIONS

2. Yes. Other "zero points" can be used.

3. The first step would be to draw a line and indicate a point on that line to be used as the vertex (v) of the angle.

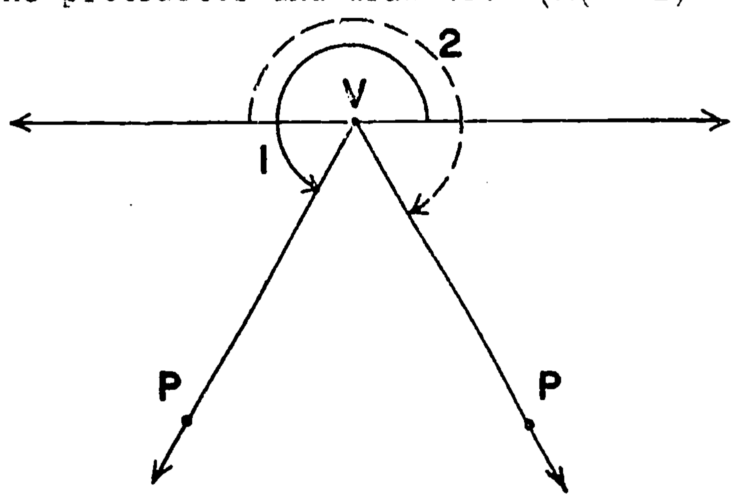


Place the center point of the protractor on the vertex (v) and line up the line with the 0° and 180° points of the protractor scale.



Since $240 = 180 + 60 = 360 - 120$, locate either 60° or 120° on the protractor scale and make a dot (call it D) at that point (there are two possibilities).

Remove the protractor and draw VP. $(m(\angle 1) = m(\angle 2) = 240^{\circ})$



DISCUSSION QUESTIONS

4. Procedure is like those when using a semi-circular protractor. Any point of the protractor scale may be used as the "zero point".

EXERCISES

1. Students may use either a circular or semi-circular protractor for this exercise. If you wish, require the use of either one or both protractors for the parts of this exercise. The accuracy of the drawings may be checked by using the acetate overlay provided.

2-3. Remember that no estimate should be considered as incorrect. Students at this stage will probably not come within 10° but this is a goal toward which they can work. Measurement skills are reviewed when they check the closeness of their estimates.

4. The student could be asked how to test whether or not their drawing is the same size and shape as the original (tracing and/or superposition).


FORTY-TWO STEPS DUE NORTH...
TWENTY-FIVE DEGREE TURN EAST...
5 STEPS AND JUMP!
155 DEGREE RIGHT TURN... TEN STEPS DUE SOUTH
STOP!
MY TREE!
YIPPEE!
AS YOU CAN SEE IT WAS WELL WORTH IT!!

The cartoon on page 92 shows the use of compass bearings to establish or trace a route. Lesson 12 takes up the study of compass bearings as an application of angle measure.

LESSON 12

COMPASS BEARINGS

The positions of ships, planes, forest fires and tornadoes, are often given in terms of their distance from a known point, and their direction.

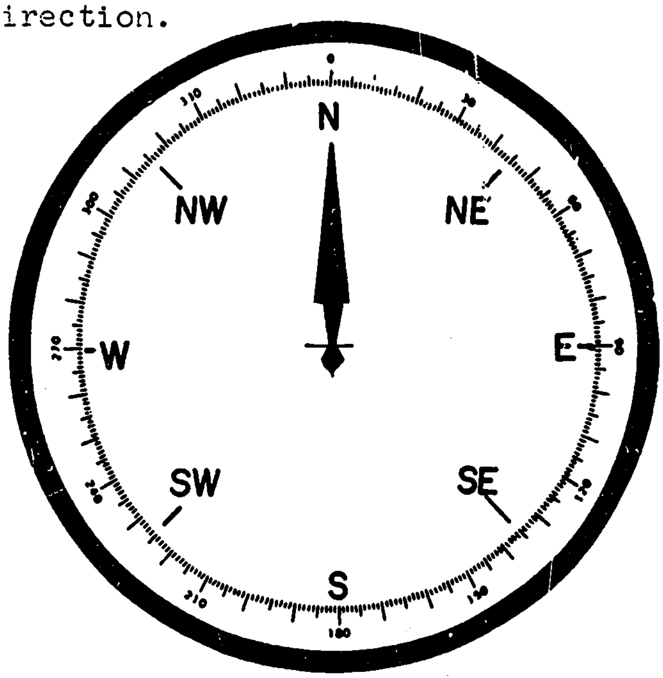


Figure 1

The magnetic compass is used to establish the north line. The direction of an object is then obtained by measuring the angle between the north line and the line joining the fixed point to the object.

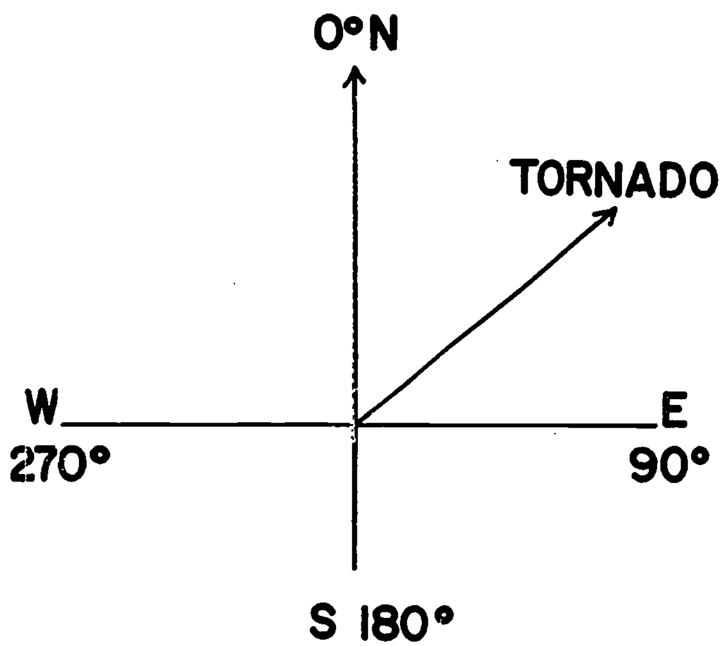


Figure 2

COMPASS BEARINGS

OBJECTIVES

1. To introduce the student to some applications of angle measure.

2. Given a north line and a starting point, the student will be able to plot a ray corresponding to a given bearing.

3. Given a diagram with north and east indicated the student will be able to determine the bearing of a given point.

EQUIPMENT AND TEACHING AIDS

Full circle protractors (one for each student). Rulers for drawing lines and measuring lengths to the nearest $\frac{1}{8}$ inch. Transparency 3-5 can be labeled with a marking pen to represent a compass. If possible, draw several concentric circles on transparency 3-5 to represent range marks on a radar screen.

CONTENT AND APPROACH

The purpose of this lesson is to give several interesting applications of angle measure. Relate this lesson to Lessons 3,4,5,6 and 8 where an angle is swept out by a rotation.

The direction angle is measured in degrees from north in a clockwise direction. This angle is called the bearing of the object from that particular point. Thus a bearing of 90° would be due East.

Estimate, in degrees, the bearing of the tornado in Figure 2.

A large forest has several lookout stations to help guard against forest fires. When a lookout spots a fire he immediately takes its bearing from his station.

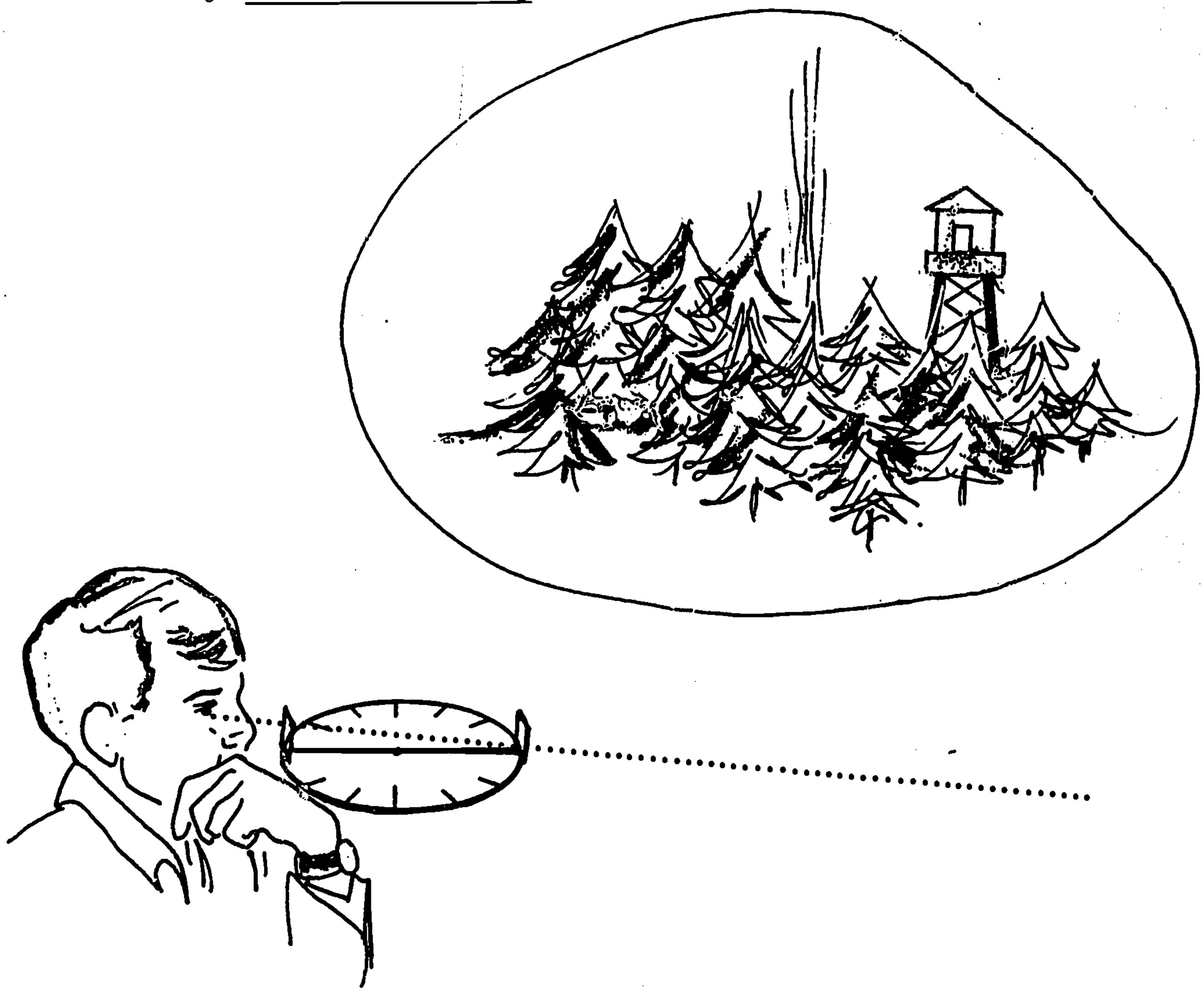


Figure 3

The direction angle, measured from the north-line in a clockwise direction,is called the bearing of the object from the fixed point.

The first application involves fire spotting. The bearing of a fire is the clockwise angle from north to the line-of-sight of the fire. The fixed point is the rangers' lookout tower.

He then calls a second station. The second station takes the fire's bearing from his position. These two readings are plotted on a map and the fire is pinpointed.

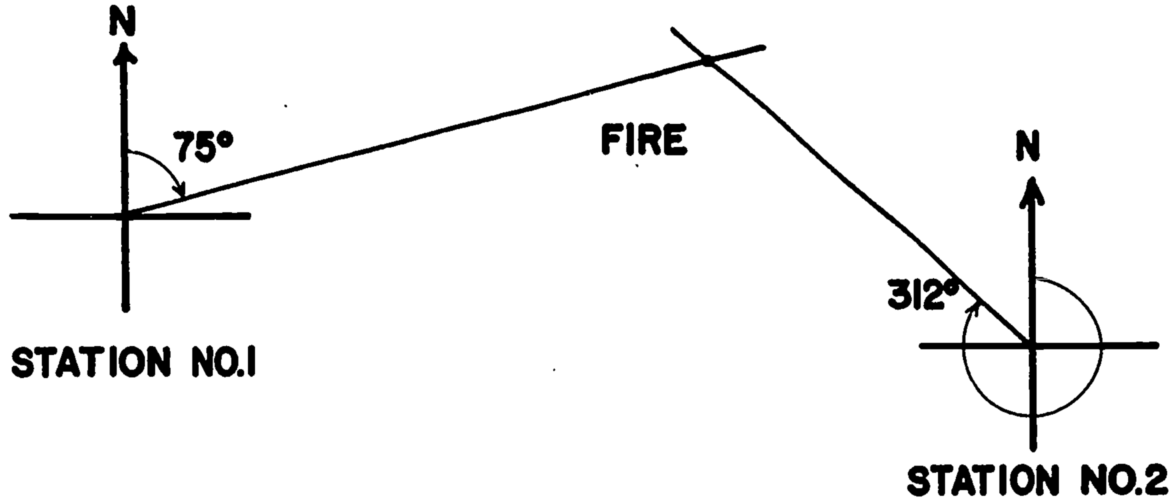
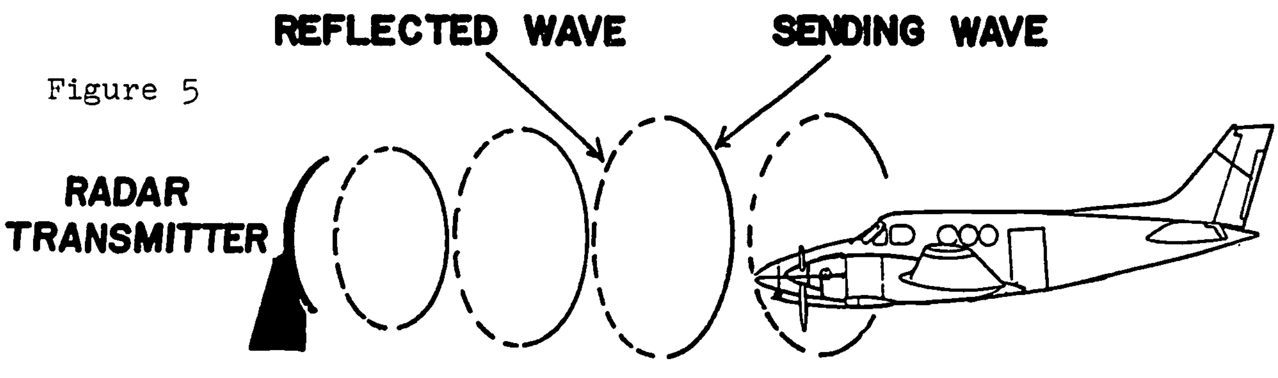


Figure 4

A radar set is used to obtain distance and direction of objects like ships, planes or tornadoes. The radar set sends out radio waves which bounce off objects they meet and return to the set.

Figure 5



The bearing is found from the direction of the radar antenna when the waves are returned. The distance is

The second application involves the radar screen. An "arm" rotates on the radar screen and any nearby object (which reflects the radio waves) appears as a "blip" in a relative position on the screen as this "arm" rotates.

The distance of the object from the radar antenna is determined by the time that it takes for a radio wave to be reflected back from the object.

Radar was first used extensively during W.W.II. Radar is short for radio detecting and ranging.

See Appendix A, Number 9. Lesson 12 has little to do with adding fractions but maintenance of this skill is necessary.

computed from the time taken by the wave to reach the object and return to the set. (Radio waves travel 186,272 miles per second.)

An object picked up by radar will appear as a bright spot on the radar screen in a position corresponding to its actual position.

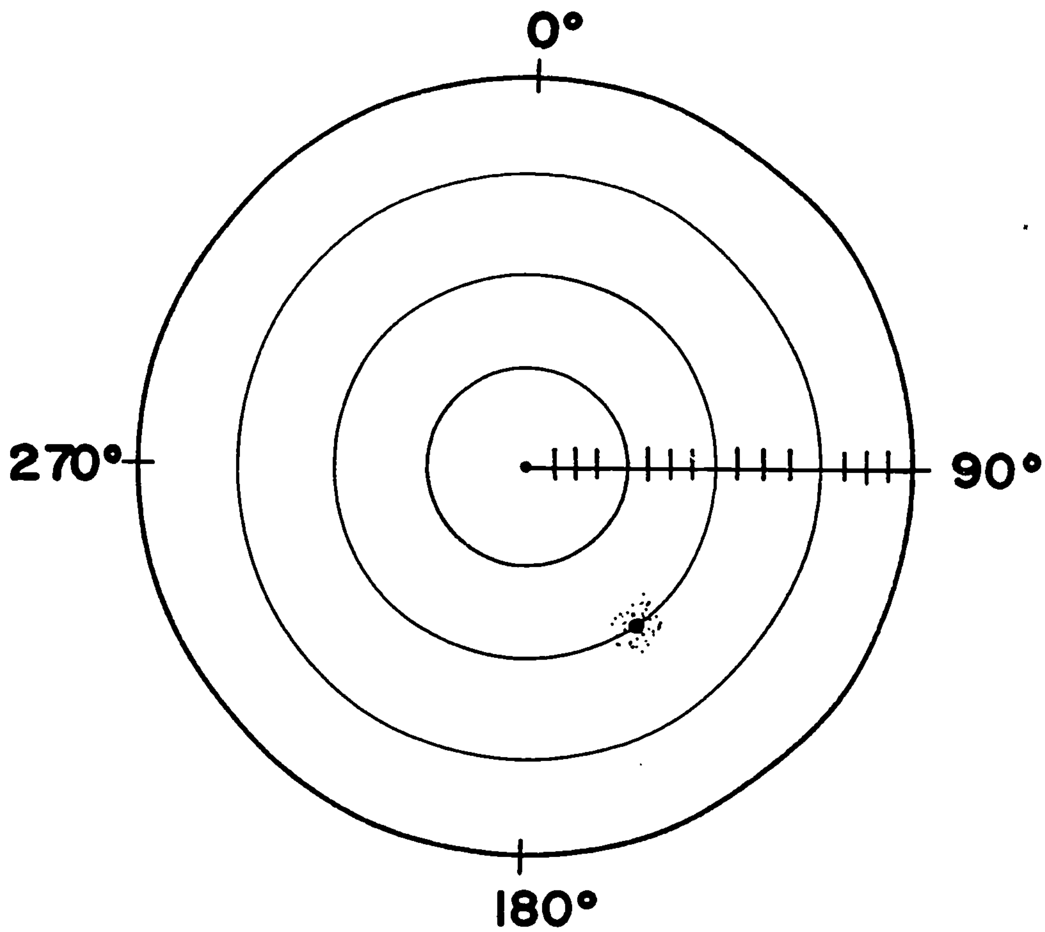


Figure 6

✓ POINT

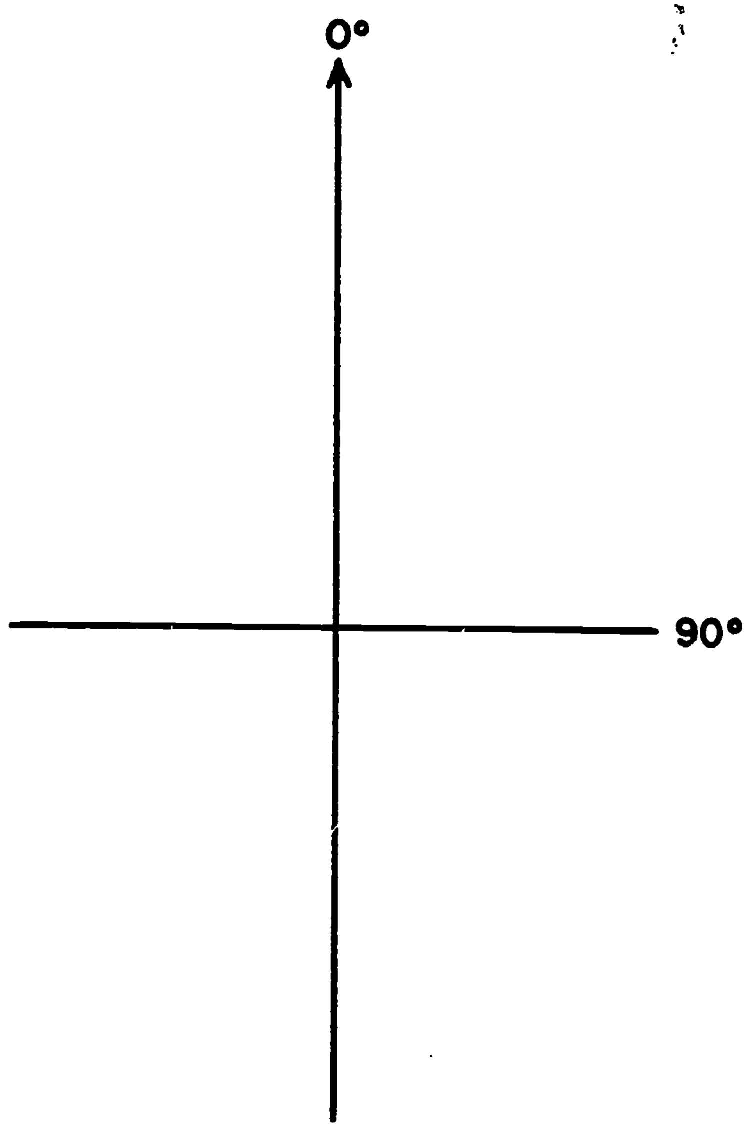
1. What is a bearing?

2. What is the measure of the angle corresponding to the south-east direction?

EXERCISES

1. Draw two perpendicular lines to represent the directions N., S., E., and W. as in Figure 2, page 93. On this diagram draw arrows to indicate the following bearings.

   (a) 135° (b) 68° (c) 317° (d) 275°



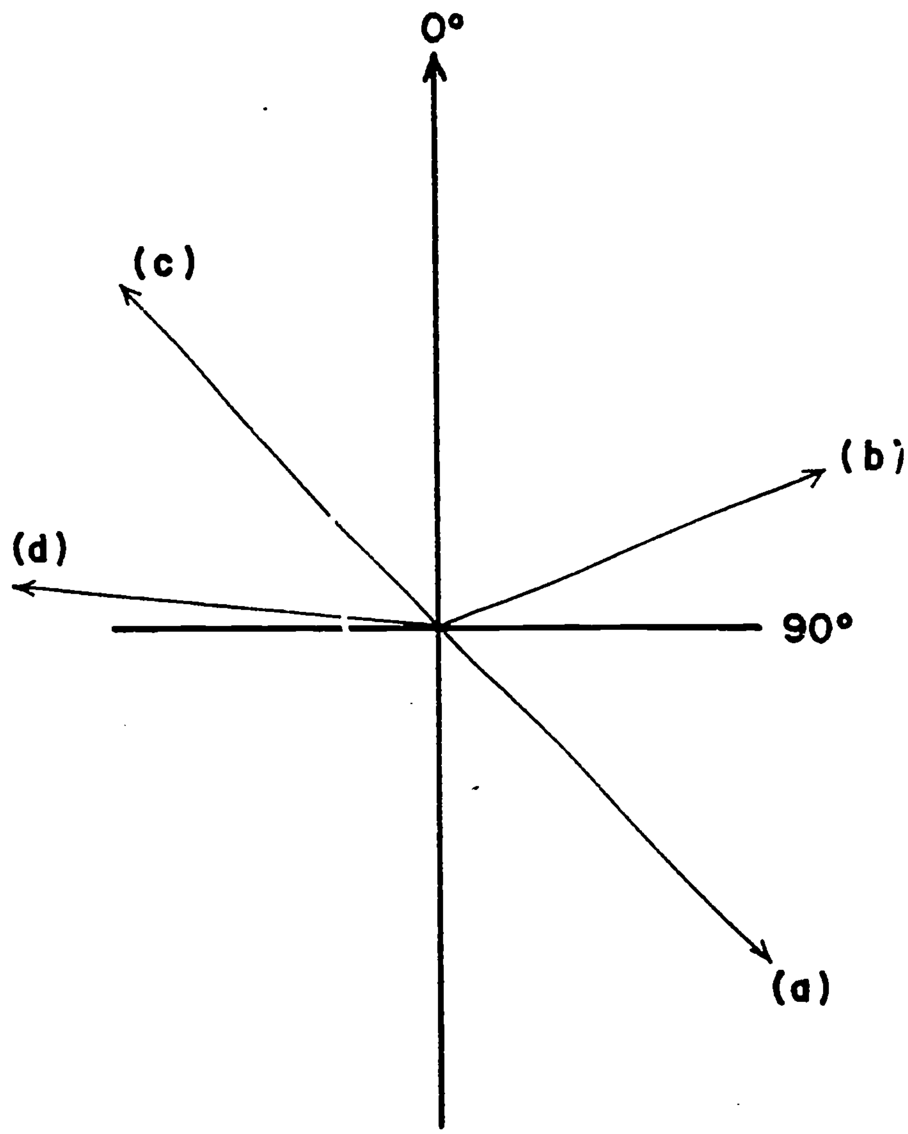
ANSWERS TO ✓ POINT

1. The direction angle, measured in degrees from north in a clockwise direction.

2. 135°

ANSWERS TO EXERCISES

1.



2. What is the final bearing after:

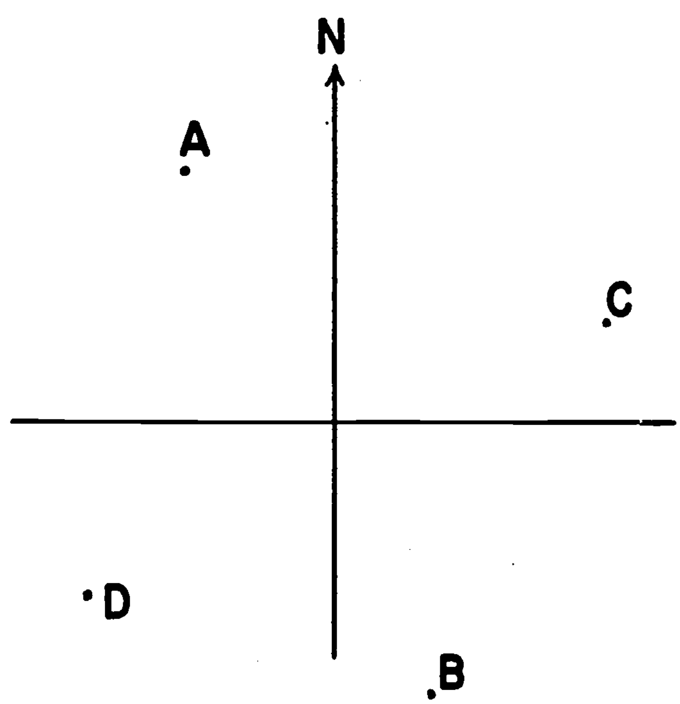
   (a) a clockwise turn of 100° from the direction S.;

   (b) a clockwise turn of 300° from the direction E;

   (c) a counterclockwise turn of 43° from the direction S.W.;

   (d) a counterclockwise turn of 80° from the direction N.E.?

3. Use your protractor to obtain the bearings of points A, B, C and D on the diagram below.



ANSWERS CONT'D

2. a) 280°

   b) 30°

   c) 182°

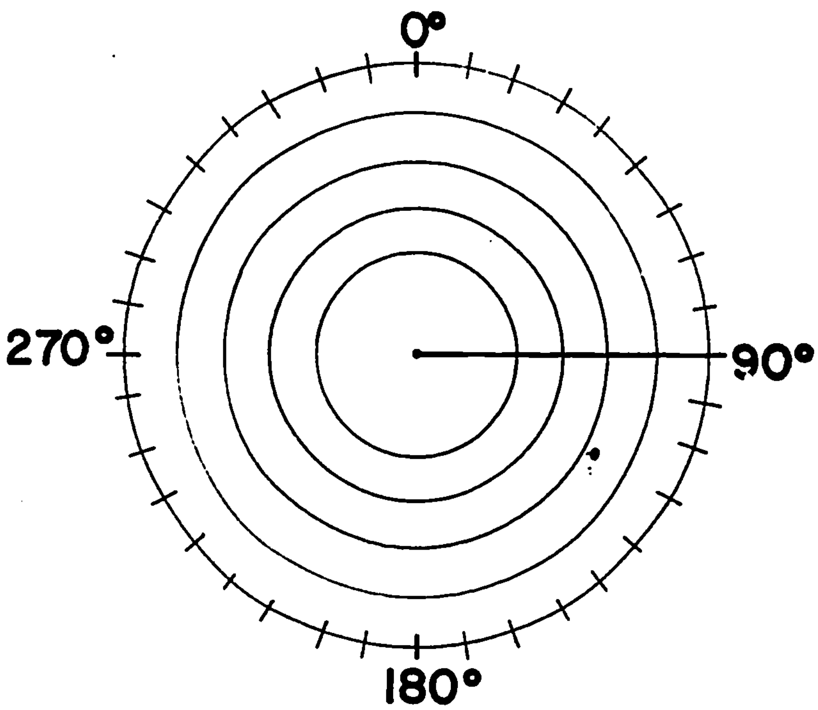
   d) 325°

3. A. $328^\circ - 330^\circ$ B. $160^\circ - 162^\circ$

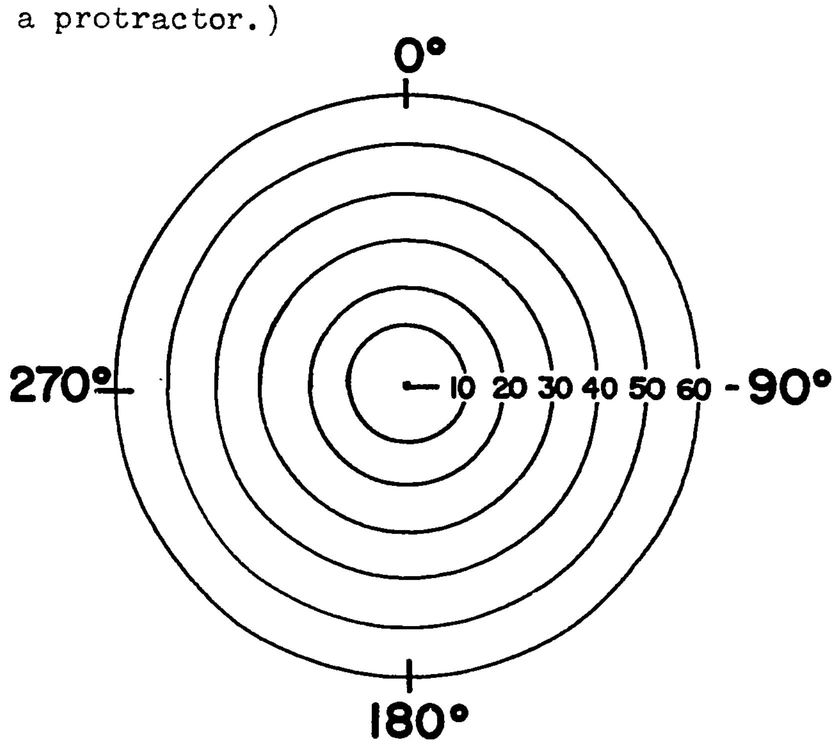
   C. $69^\circ - 71^\circ$ D. $234^\circ - 236^\circ$

A tolerance interval of 2° is given for these measurements since the black dots are about 2° wide.

4. A tornado has been picked up on the radar screen shown below. What is its bearing?



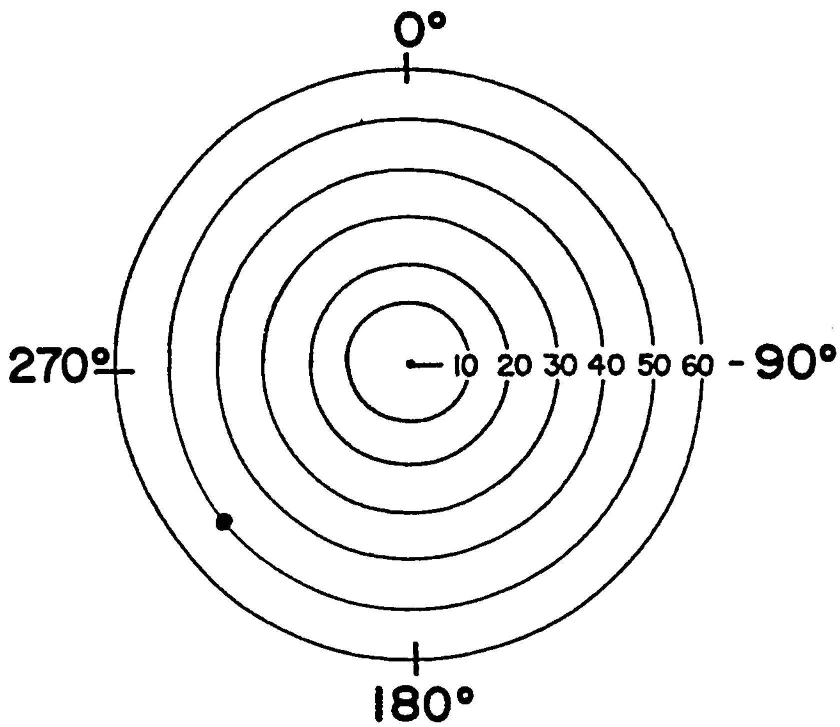
5. The radar set has picked up a ship. The ship's distance is 50 miles, bearing 230°. Draw a dot on the radar screen below to show the ship's position. (Use a protractor.)



ANSWERS CONT'D

4. 120°

5.



6. Two lookout towers have phoned in information on a forest fire. Station No. 1 reports the fire's bearing as 110°. Station No. 2 reports the fire's bearing as 263°. Use a protractor and straightedge to plot the position of the fire on the diagram below.

7. Old Miner Tom buried his gold and drew a map of its location. He put the distances on the map, but left off the bearings. Use your protractor to find the bearing of each point along the path.

(a) From the Miner's cabin to the large Oak tree.

Bearing ________

(b) From the large Oak tree to the Boulder.

Bearing ________

(c) From the Boulder to the Creek.

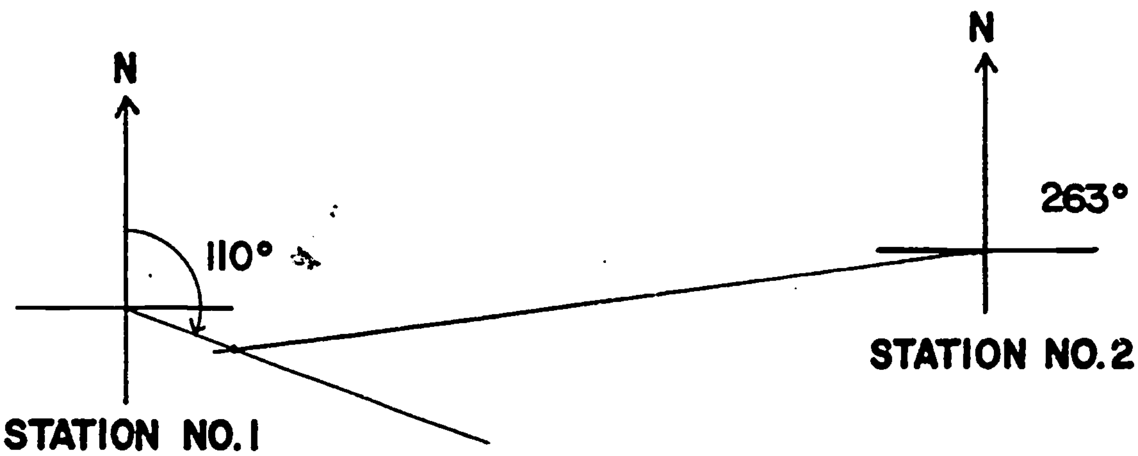
Bearing ________

(d) From the Creek to the Gold.

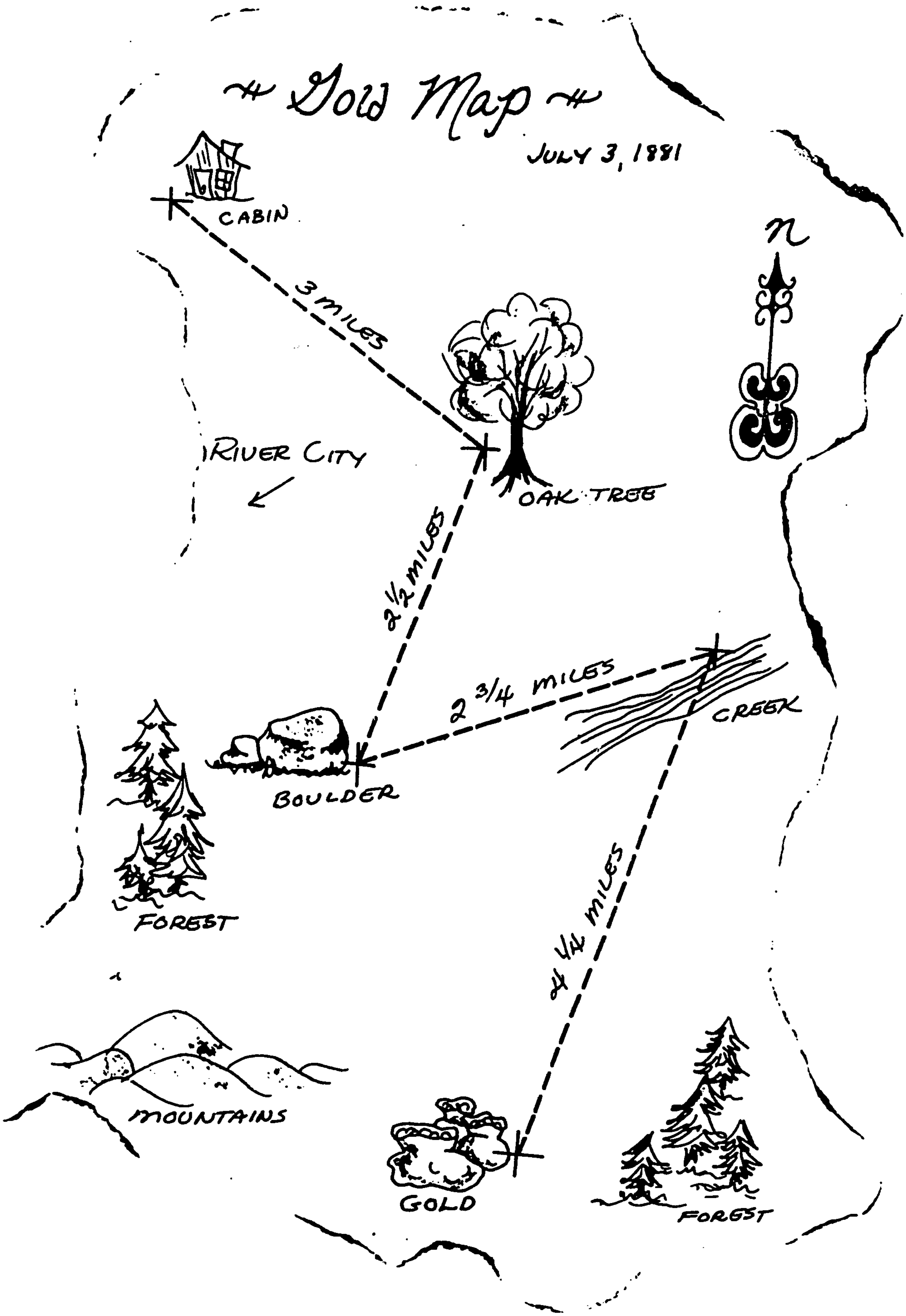
Bearing ________

ANSWERS CONT'D

6.



7. a) About 125°
   b) About 200°
   c) About 69°
   d) About 196°

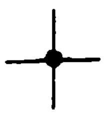

Gold Map
JULY 3, 1881
CABIN
3 MILES
OAK TREE
n
RIVER CITY
2 1/2 MILES
2 3/4 MILES
CREEK
BOULDER
FOREST
4 1/4 MILES
MOUNTAINS
GOLD
FOREST

8. Plot the following course starting with point A below.

   (1) From point A move 3" at a bearing of 45°. Label this point B.

   (2) From point B move $1\frac{1}{2}$" at a bearing of 350°. Label this point C.

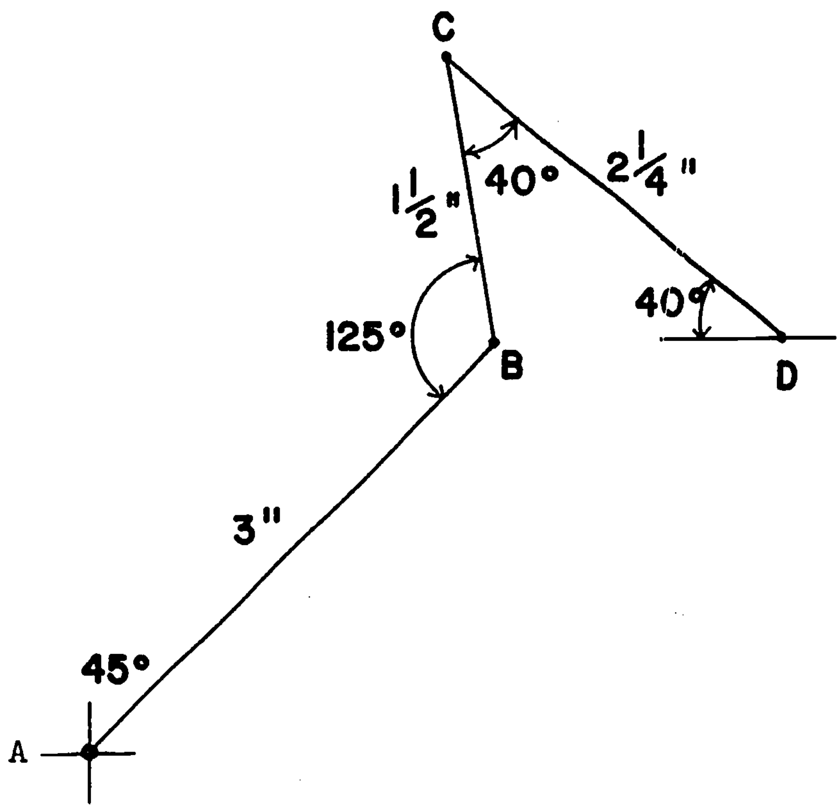
   (3) From point C move $2\frac{1}{4}$" at a bearing of 130°. Label this point D.

A 

9. In Exercise 8, above, what is the bearing of point D from point A?

ANSWERS CONT'D

8.



9. 58° (approximately)

APPENDICES

A. SUPPLEMENTARY EXERCISES

B. NAMING ANGLES

C. CAN A RULER BE USED TO MEASURE ANGLES?

D. USING OTHER PROTRACTORS

E. DO PROTRACTORS HAVE TO BE CIRCULAR?

F. GLOSSARY

SUPPLEMENTARY EXERCISES

The Supplementary Exercises enclosed in this appendix are designed to help measure the student's strengths and weaknesses on some objectives relevant to work with fractional numbers.

The student is encouraged to keep track of his own progress on the enclosed chart. The teacher is encouraged to oversee the individual progress of the students.

Suggestions for when to use the Supplementary Exercises are given in the Teacher's Guide Section of Lessons 3-6, 12. Students should do these exercises on an individual basis.

These exercises are not meant for drill work. They will serve the teacher and the student best by highlighting the ideas, skills and concepts with which the student needs additional help.

SUPPLEMENTARY WORKSHEETS

PERFORMANCE CHART

Worksheet Number	Objective	Rating: Achieved —— Needs Work
1	The student will be able to demonstrate his understanding of the following meaning of fractions: $\frac{a}{b}$ means the unit interval is divided into "b" congruent parts and "a" tells the number of parts taken.	
2	The students use "other names" for one to change a fraction to higher terms.	
3	The student can apply the principle: multiplying or dividing the terms of a fraction by the same number leaves the value of the fraction unchanged.	
4	The student can apply the cross product rule to determine whether two fractions are equivalent or not.	
5	The student can apply a rule to change a whole number into a fraction with a given denominator.	

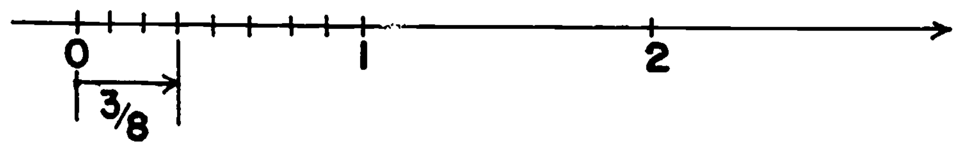
Worksheet Number	Objective	Rating: Achieved —— Needs Work
6	The student is able to apply a rule to determine which of two fractions has the larger value.	
7	The student is able to apply a rule to change an improper fraction to a mixed number.	
8	The student is able to apply a rule to change a mixed number to an improper fraction.	
9	The student is able to apply a rule to add or subtract fractions.	

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Have you ever wondered what is meant by a fraction? How do you picture the fraction $\frac{3}{8}$?

One way to look at $\frac{3}{8}$ is to take a unit length on a number line and subdivide it into 8 equal parts.

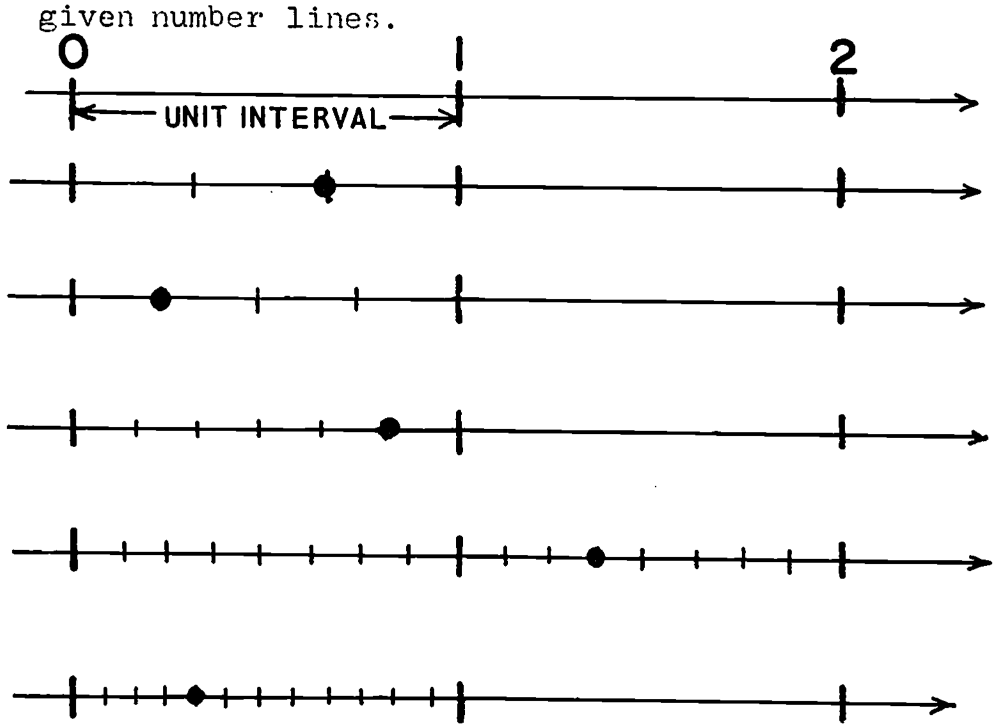
Mark off the first three parts from 0.



$\frac{3}{8}$ **Numerator** tells the number of equal parts from 0. **Denominator** tells the number of equal parts in the unit length.

EXERCISES

1-5. Name the fraction indicated by the mark on each of the given number lines.



1.

2.

3.

4.

5.

ANSWERS

1.

2.

3.

4.

5.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. Suppose you have two fractions that have different numerators and different denominators. Can these two fractions have the same value?

2. $\frac{\square}{9}$ is another name for one. What number does $\square$ represent?

3. $\triangle$ represents a "magic" number. If you take any number and multiply it by $\triangle$ you get the number you started with for your product. What number does $\triangle$ represent?

4. How many different names can you write for the number one?

You can change the numerator and denominator of a fraction and not change its value by multiplying the fraction by another name for one.

Example: $\frac{2}{3} \cdot 1 = \frac{2}{3}$ and $1 = \frac{4}{4}$

so $\frac{2}{3} \cdot \frac{4}{4} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$.

Thus $\frac{2}{3} = \frac{8}{12}$.

EXERCISES

5-7. Fill in the blanks.

5. $\frac{1}{4} \cdot \frac{?}{?} = \frac{3}{12}$

6. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{?}{15} = \frac{12}{?}$

7. $\frac{?}{5} = \frac{?}{20} = \frac{9}{?} = \frac{18}{30} = \frac{?}{10}$

8. What name for one can be used to change $\frac{3}{8}$ to 48ths?

ANSWERS

1.

2.

3.

4.

5. ___ ___

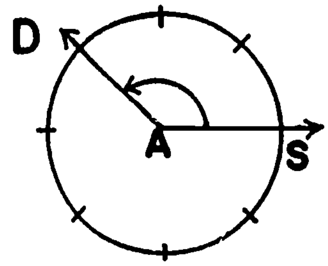
6. ___ ___

7. ___ ___
___ ___

8.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. Suppose that the circle shown below were divided into 24 congruent arcs. What would then be the measure of $\angle SAD$?



Principle for changing fractions:

Multiplying or dividing the terms of a fraction by the same number leaves the value of the fraction unchanged.

Examples:

(a) Change $\frac{63}{72}$ to lower terms.

$$\frac{63}{72} = \frac{63 \div 9}{72 \div 9} = \frac{7}{8}.$$

(b) Change $\frac{2}{3}$ to 24ths.

$$\frac{2}{3} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{16}{24}.$$

EXERCISES

2-10. Find the missing term so that the value of the fraction is unchanged.

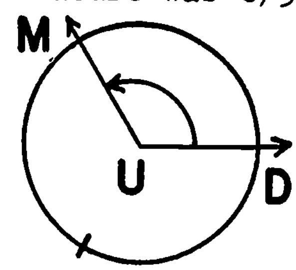
2. $\frac{6}{30} = \frac{\quad}{5}$
3. $\frac{16}{24} = \frac{\quad}{3}$
4. $\frac{45}{108} = \frac{\quad}{12}$
5. $\frac{2}{5} = \frac{\quad}{35}$
6. $\frac{1}{7} = \frac{8}{\quad}$
7. $\frac{3}{4} = \frac{\quad}{36}$
8. $\frac{4}{6} = \frac{\quad}{42}$
9. $\frac{36}{64} = \frac{\quad}{8}$
10. $\frac{4}{7} = \frac{52}{\quad}$

ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. John said that the measure of $\angle$DUM was 20/30 of a turn. Sally said that its measure was 6/9 of a turn. Who was right?



The cross product rule enables you to determine whether two fractions are equivalent or not.

Examples:

(a) $\frac{12}{8}$ equals $\frac{6}{4}$ because $12 \times 4 = 8 \times 6$

(b) $\frac{3}{4}$ does not equal $\frac{7}{12}$ because 3×12 is not equal to 4×7.

EXERCISES

2-10. Determine which pairs of fractions are equivalent. Answer with = or $\neq$.

2. $\frac{4}{8} \bigcirc \frac{6}{9}$ 3. $\frac{8}{12} \bigcirc \frac{4}{6}$ 4. $\frac{2}{3} \bigcirc \frac{5}{6}$

5. $\frac{4}{9} \bigcirc \frac{32}{72}$ 6. $\frac{7}{12} \bigcirc \frac{35}{60}$ 7. $\frac{3}{4} \bigcirc \frac{7}{12}$

8. $\frac{5}{8} \bigcirc \frac{3}{4}$ 9. $\frac{10}{15} \bigcirc \frac{6}{9}$ 10. $\frac{7}{10} \bigcirc \frac{9}{12}$

ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Can you score 100% on this test? Try and see.

TEST. Write the following whole numbers as fractions using the given denominator.

1. $6 = \frac{\quad}{3}$ 2. $7 = \frac{\quad}{5}$ 3. $3 = \frac{\quad}{9}$

4. $13 = \frac{\quad}{11}$ 5. $9 = \frac{\quad}{9}$

If you didn't score 100%, you probably need some practice.

Try this: $6 = \frac{?}{3}$. Ask what divided by 3 gives 6 and set up this proportion:

$$\frac{6}{1} = \frac{N}{3}.$$

Use the cross-product to obtain the answer.

EXERCISES

6-10. Write the whole number as a fraction using the given denominator.

6. $7 = \frac{\quad}{6}$ 7. $11 = \frac{\quad}{9}$ 8. $23 = \frac{\quad}{12}$

9. $15 = \frac{\quad}{8}$ 10. $13 = \frac{\quad}{5}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

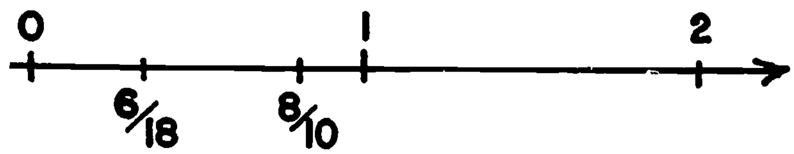
8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Can you always tell which of two fractions has the larger value? How do you do it?



Remember that the larger number is always to the right of the smaller number on the number line?

EXERCISES

1-7. Answer with <, =, or >.

1. $\frac{3}{8} \bigcirc \frac{5}{8}$ 2. $\frac{9}{12} \bigcirc \frac{7}{12}$ 3. $\frac{8}{9} \bigcirc \frac{8}{5}$

4. $\frac{5}{3} \bigcirc \frac{5}{4}$ 5. $\frac{6}{8} \bigcirc \frac{18}{24}$ 6. $\frac{25}{36} \bigcirc \frac{3}{4}$

7. $\frac{5}{8} \bigcirc \frac{9}{12}$

8. Suppose two fractions have the same denominator and different numerators. Which fraction has the larger value?

9. Suppose two fractions have the same numerator and different denominators. Which fraction has the larger value?

10. If $a \cdot b > r \cdot t$, then which fraction is larger; a/r or t/b?

ANSWERS

1.

2.

3.

4.

5.

6.

7.

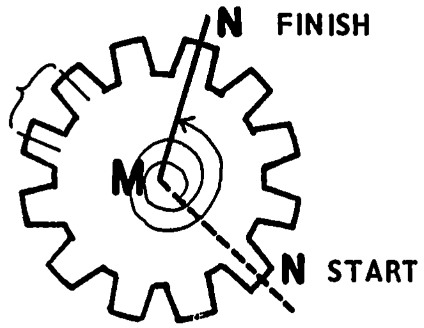
8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. What angle has been swept out by segment $\overline{MN}$?



Can you change $\frac{28}{12}$ to a mixed number?

(a) $\frac{28}{12}$ means $28 \div 12$.

$$12\overline{\smash{)}28}^{\,2} \quad \begin{array}{r} 24 \\ \hline 4 \end{array} \quad . \quad \text{Thus } \frac{28}{12} = 2\frac{4}{12} \quad . \quad \text{OR} \ldots$$

(b) $\frac{28}{12} = \frac{24 + 4}{12} = \frac{24}{12} + \frac{4}{12} = 2 + \frac{4}{12}$

$= 2\frac{4}{12}$.

EXERCISES

2-10. Change each of the following improper fractions to mixed numbers.

2. $\frac{4}{3}$	3. $\frac{49}{16}$	4. $\frac{53}{8}$
5. $\frac{5}{2}$	6. $\frac{20}{3}$	7. $\frac{34}{5}$
8. $\frac{47}{4}$	9. $\frac{61}{7}$	10. $\frac{100}{6}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

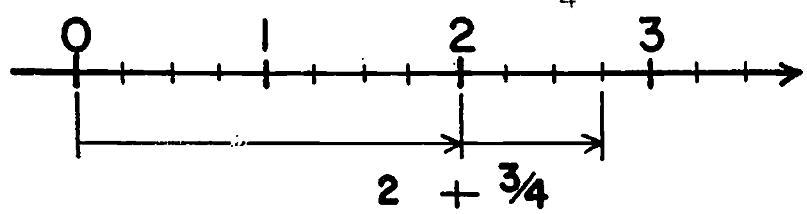
8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. How many quarter inches is $2\frac{3}{4}$ inches?



Can you change $2\frac{3}{4}$ to an improper fraction?

Examples

(a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$.

Or...

(b) $2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{8+3}{4} = \frac{11}{4}$.

EXERCISES

2-10. Change each of the following mixed numbers to improper fractions.

2. $6\frac{2}{3}$	3. $5\frac{3}{4}$	4. $13\frac{2}{5}$
5. $5\frac{1}{8}$	6. $7\frac{1}{2}$	7. $21\frac{1}{3}$
8. $4\frac{7}{12}$	9. $3\frac{9}{16}$	10. $15\frac{3}{5}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

8.

9.

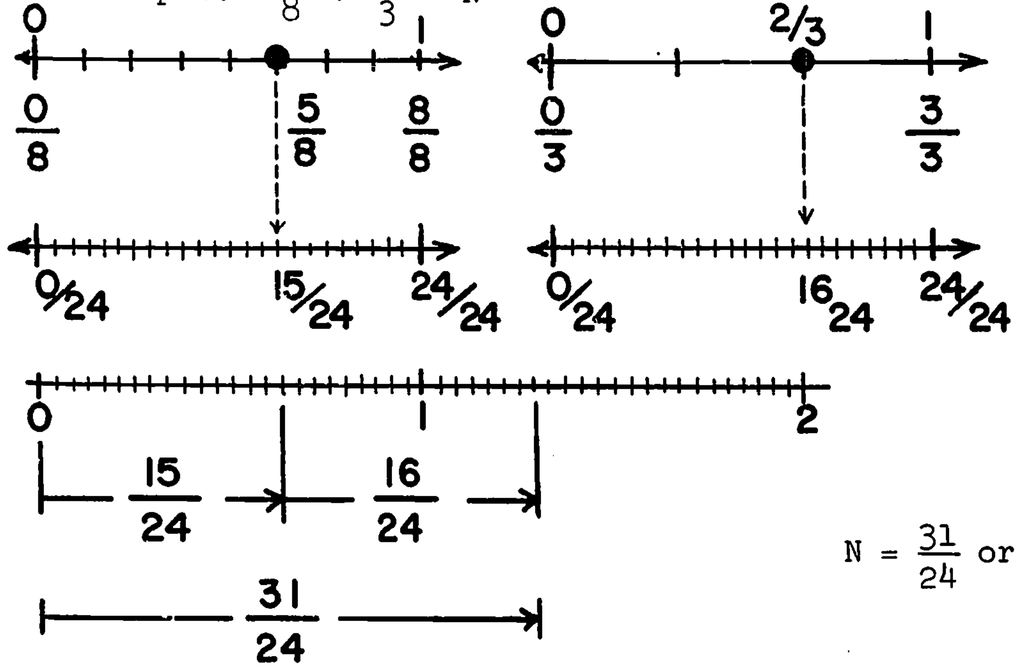
10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Do you ever have trouble adding or subtracting fractions?

Fractions cannot be added or subtracted unless they have the same denominator.

Example: $\frac{5}{8} + \frac{2}{3} = N$



$N = \frac{31}{24}$ or $1\frac{7}{24}$

EXERCISES

1-10. Find the value of N.

1. $\frac{1}{2} + \frac{1}{3} = N$
2. $\frac{5}{8} - \frac{1}{4} = N$
3. $\frac{3}{4} + \frac{2}{5} = N$
4. $\frac{6}{8} - \frac{2}{3} = N$
5. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = N$
6. $\frac{15}{16} - \frac{1}{2} = N$
7. $\frac{5}{4} + \frac{3}{8} = N$
8. $\frac{7}{16} - \frac{3}{12} = N$
9. $\frac{9}{12} + \frac{1}{4} = N$
10. $\frac{8}{15} + \frac{1}{5} = N$

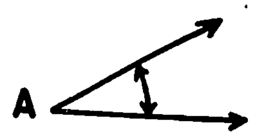
ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

NAMING ANGLES

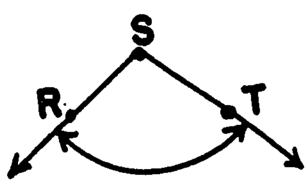
An angle may be named by...

1. ... a capital letter at the vertex, read "∠ A".

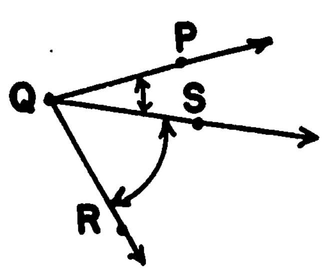


2. ... a small letter or number written inside the angle, read "∠b or ∠ 1".

3. ... three capital letters, read "∠RST or ∠TSR". (Note that the letter at the vertex is always read in the middle.)



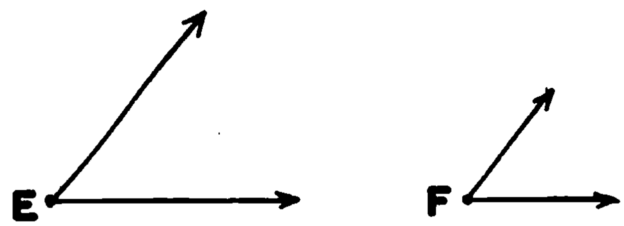
The following picture shows why three letters are sometimes needed to name an angle. If you say "∠Q", it is not clear which angle is meant. If you say "∠PQS" or "∠RQS", it is clear.



CAN A RULER BE USED TO MEASURE ANGLES?

A mathematics class was discussing the following question.

Which angle in the pair below has the greater measure?

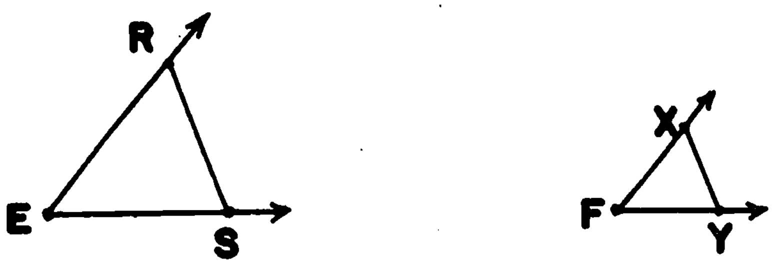


$\angle$ E or $\angle$ F

There was a disagreement when they compared $\angle$E and $\angle$F. Al said that m($\angle$ E) $>$ m($\angle$ F) because it "opened up" more. He drew the following picture to back up his argument.

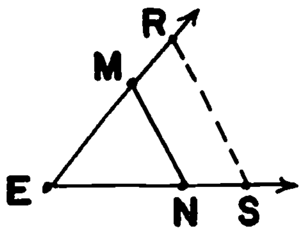
<u>Al's Argument</u>

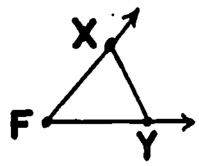
$\overline{RS}$ is longer than $\overline{XY}$. Therefore, $\angle$ E "opens up" more than $\angle$ F and m($\angle$ E) $>$ m($\angle$ F).



Dan disagreed with Al. Dan said that Al's drawing was unfair because he did not measure the "opening" between the sides at the "same place" on the sides.

Dan changed Al's drawing by erasing $\overline{RS}$ and drawing $\overline{MN}$ such that $m(\overline{EN}) = m(\overline{FY})$ and $m(\overline{EM}) = m(\overline{FX})$.





Dan's Argument

I measured the opening between the sides of the angles at the "same place". $\overline{MN}$ is the same length as $\overline{XY}$. Therefore, $\angle E$ "opens up" the same amount as $\angle F$ and $m(\angle E) = m(\angle F)$.

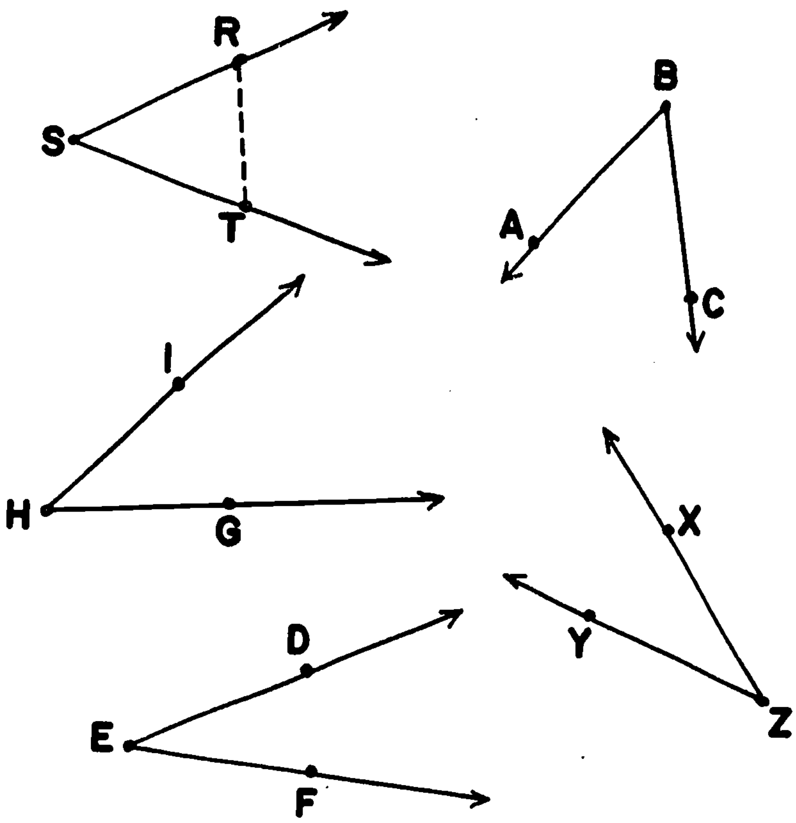
EXERCISES

1. Measure both $\angle E$ and $\angle F$ with a protractor. Make the following statement true by inserting either >, <, or = .

   $m(\angle E)$ ____ $m(\angle F)$

   Whose argument do you think is correct - Al's, Dan's, or neither? Why?

2. (a) Using the labeled points on the sides of the angles as endpoints, measure the "opening" of each angle on page 130 by measuring the linear distance between the sides to the nearest $\frac{1}{16}$ in. Record your results in **TABLE C-1** . (page 130).



(The linear distance to be measured for $\angle$RST is indicated by the dotted segment $\overline{RT}$.)

TABLE C-1

ANGLE	OPENING TO NEAREST $\frac{1}{16}$ in.
$\angle$RST	
$\angle$ABC	
$\angle$DEF	
$\angle$GHI	
$\angle$XZY	

(b) Arrange these five angles in order from smallest to largest.

_____ , _____ , _____ , _____ , _____

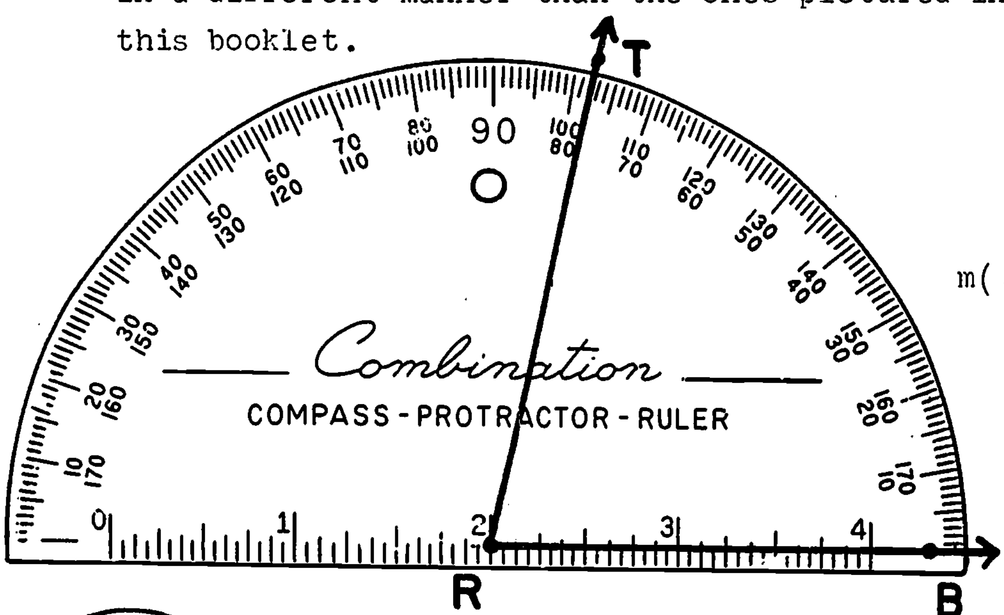
(c) Use a protractor to measure each of the angles to the nearest degree. Will these measurements give you the same order as in 2 (b) ?

3. Is it possible to measure angle size with a ruler? If your answer is yes, the measurements must be done under what conditions?

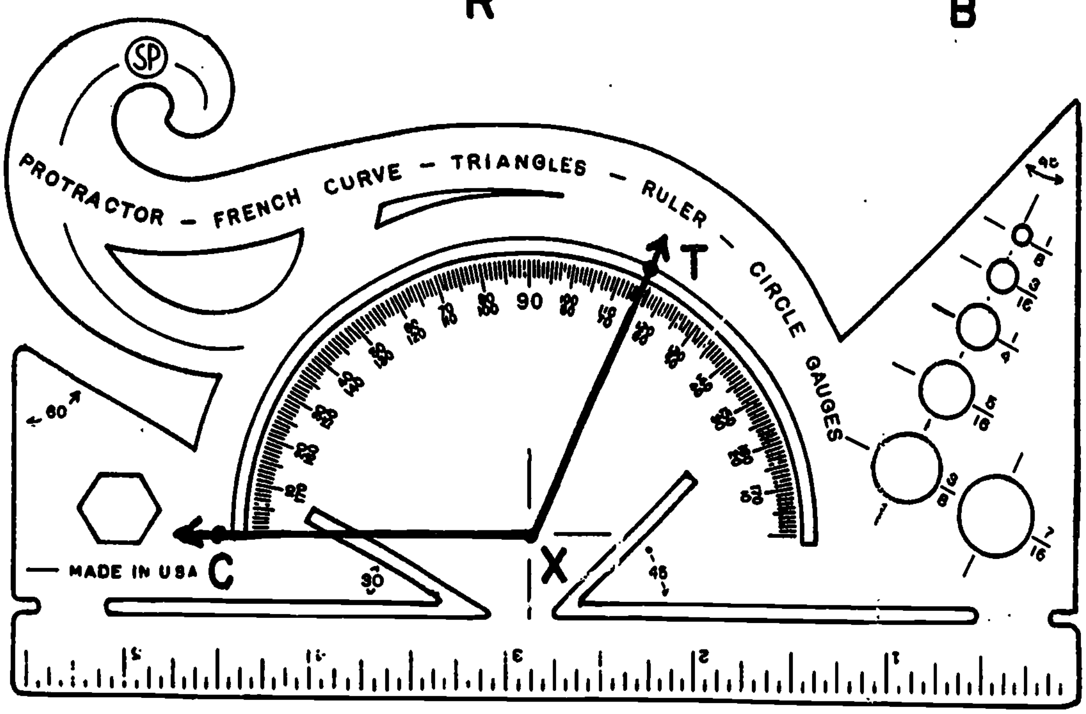
USING OTHER PROTRACTORS

Although the scales are usually marked the same way, protractors often differ in the manner the center point of the protractor (to be placed on the vertex of the angle) is indicated.

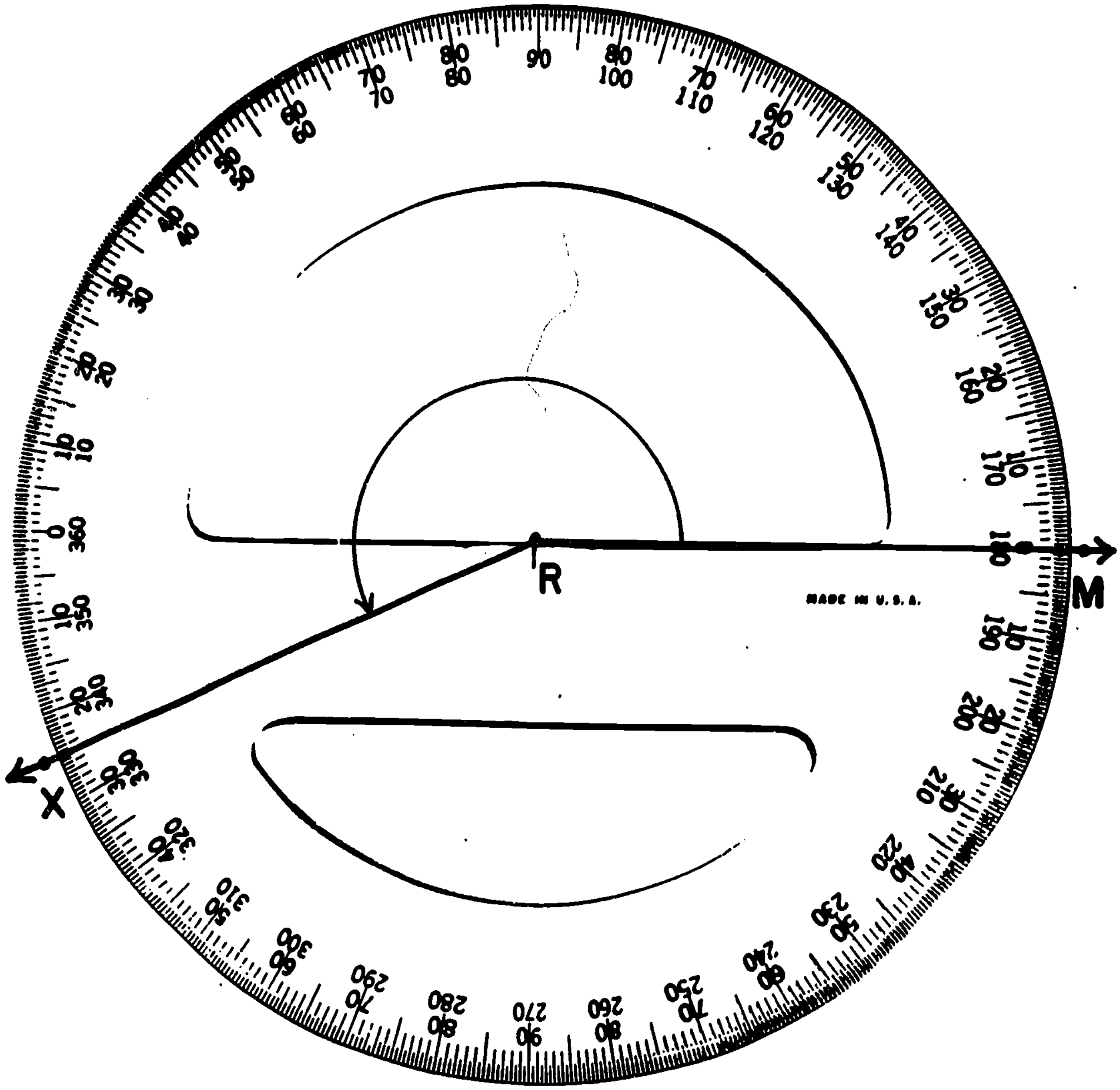
The diagrams below and on page 132 illustrate the use of some commonly used protractors whose center point is indicated in a different manner than the ones pictured in Lessons 9-11 of this booklet.



m(∠ BRT) = 78°



m(∠ CXT) = 115°



m($\angle$ MRX) = 205°

DO PROTRACTORS HAVE TO BE CIRCULAR?

The protractors you have used in this booklet have been either (1) circular or (2) semi-circular in shape.

TO BE EXPLORED . . .

Is it necessary for a protractor to be circular or semi-circular?

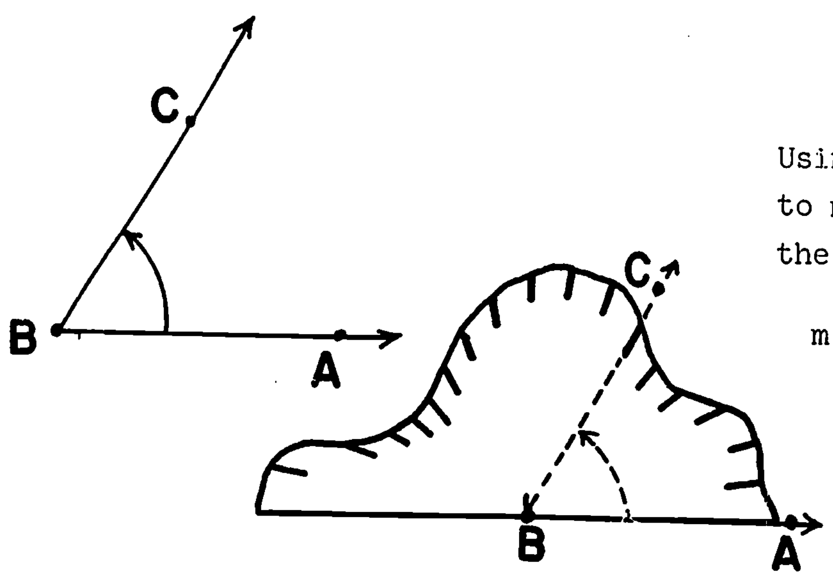
INSTRUCTIONS . . . . .

The protractors (A through G) which are to be cut out and used in Parts I and II of this activity are printed on the two inserts following page 138.

Angles 1 through 4 and **TABLE D-I** are on page 137. Angles 5 through 7 and **TABLE D-2** are on page 138.

PART I

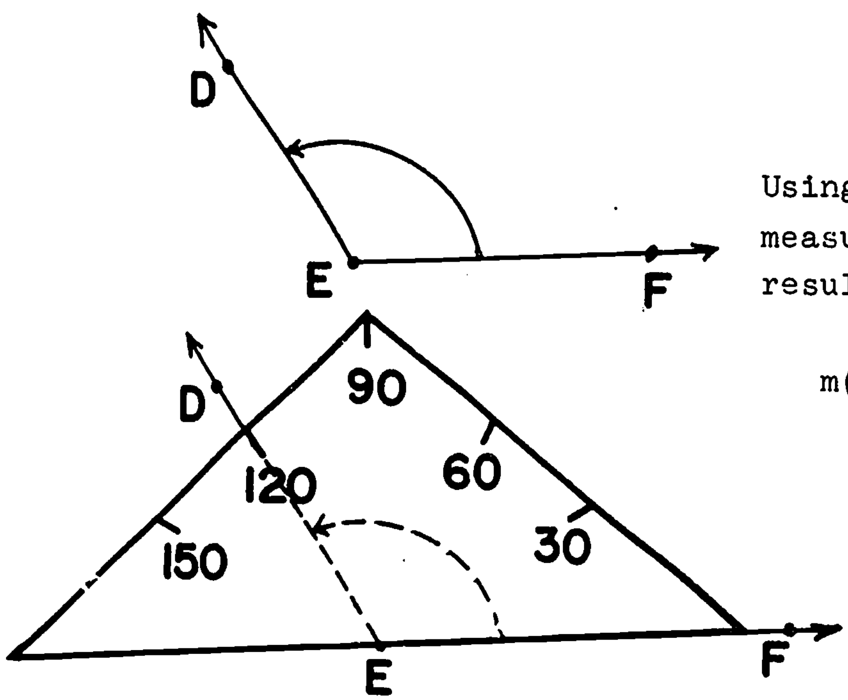
1. Look at $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Estimate the measure of each angle in degrees. Record your estimates in **TABLE D-I** . (Under the heading **ESTIMATED MEASURE** .)
2. Cut out protractor A. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-I** .



Using protractor A to measure $\angle ABC$, the result is:

$m(\angle ABC) = 60°$

3. Cut out protractor B. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1** .



Using protractor B to measure $\angle DEF$, give the results:

$m(\angle DEF) = 120°$

4. Cut out protractor C. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1** .

5. Cut out protractor D. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1** .

Was your estimate close to the measure of each angle? Assuming your measurements were correctly done, your estimates were close if you were within 10 degrees of the measured size.

POWER QUESTIONS I

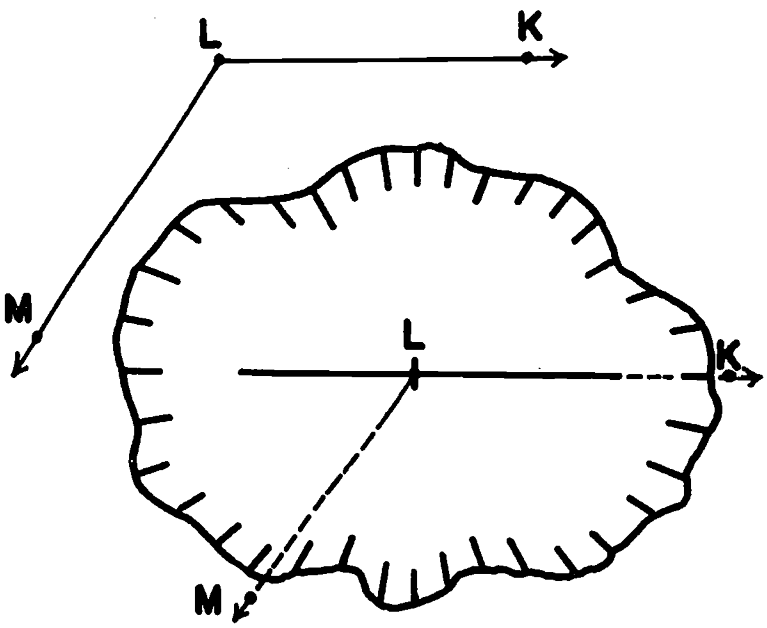
1. Were your measurements for each angle the same, regardless of the protractor used? Why or why not?

2. Explain how protractor E (with scale from 0° to 90°) could be used to measure $\angle 2$ and $\angle 4$.

PART II

1. Look at ∠5, ∠6 and ∠7. Estimate the measure of each angle in degrees. Record your estimates in **TABLE D-2**. (Under the heading **ESTIMATED MEASURE**.)

2. Cut out protractor F. Use this protractor to measure ∠5, ∠6 and ∠7. Record your measurements in **TABLE D-2**.



Using protractor F to measure ∠ KLM, the result is:

$m(\angle KLM) = 235°$

3. Cut out protractor G. Use this protractor to measure ∠5, ∠6 and ∠7. Record your measurements in **TABLE D-2**.

Was your estimate close to the measure of each angle? Assuming your measurements were accurate, your estimates were close if you were within 10 degrees of the measured size.

POWER QUESTIONS II

1. Were your measurements for each angle the same, regardless of the protractor used? Why or why not?

2. Explain how protractors A-D could be used to measure any angle from 0° to 360°.

PART III

ON YOUR OWN . . . .

Design a protractor of arbitrary shape which can be used to measure an angle whose measure is from 0° to 360°.

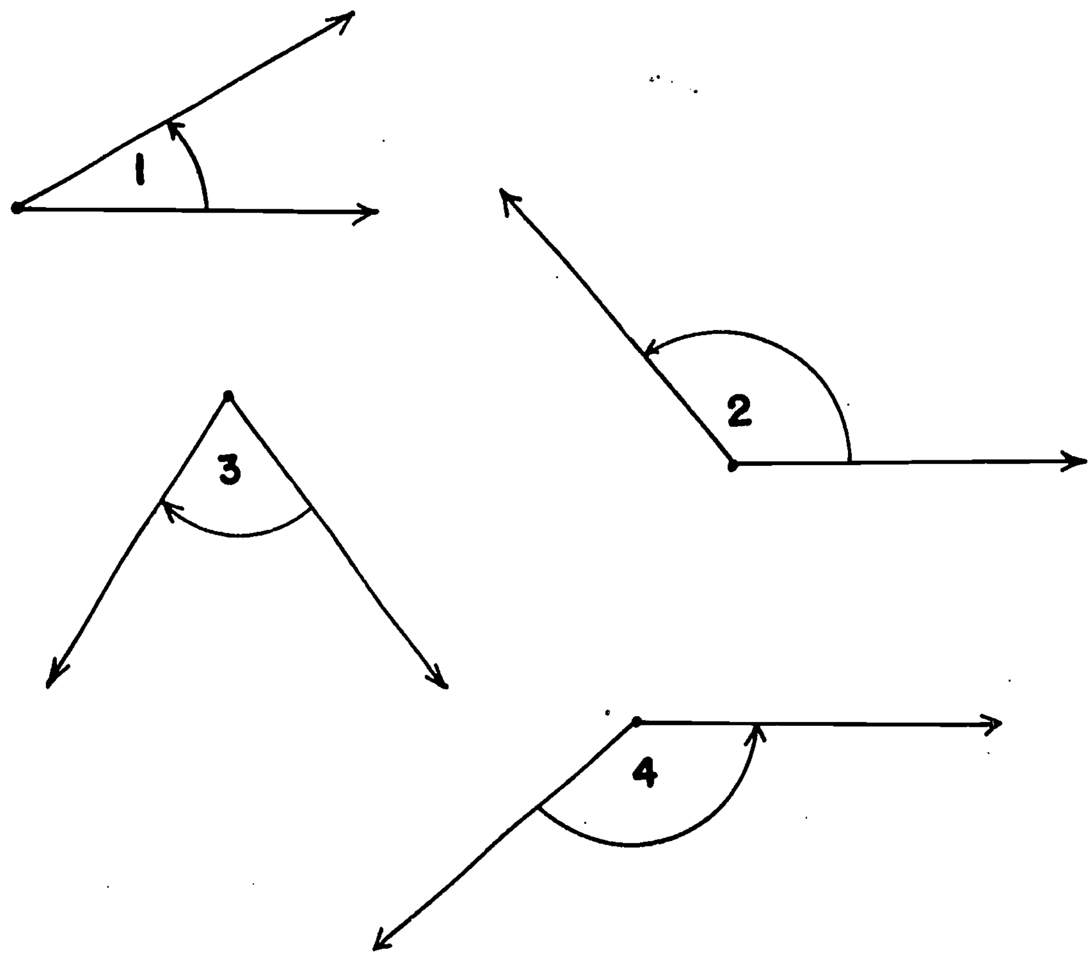


TABLE D-1

ANGLE	MEASURE USING PROTRACTOR: A	B	C	D	ESTIMATED MEASURE
1					
2					
3					
4					

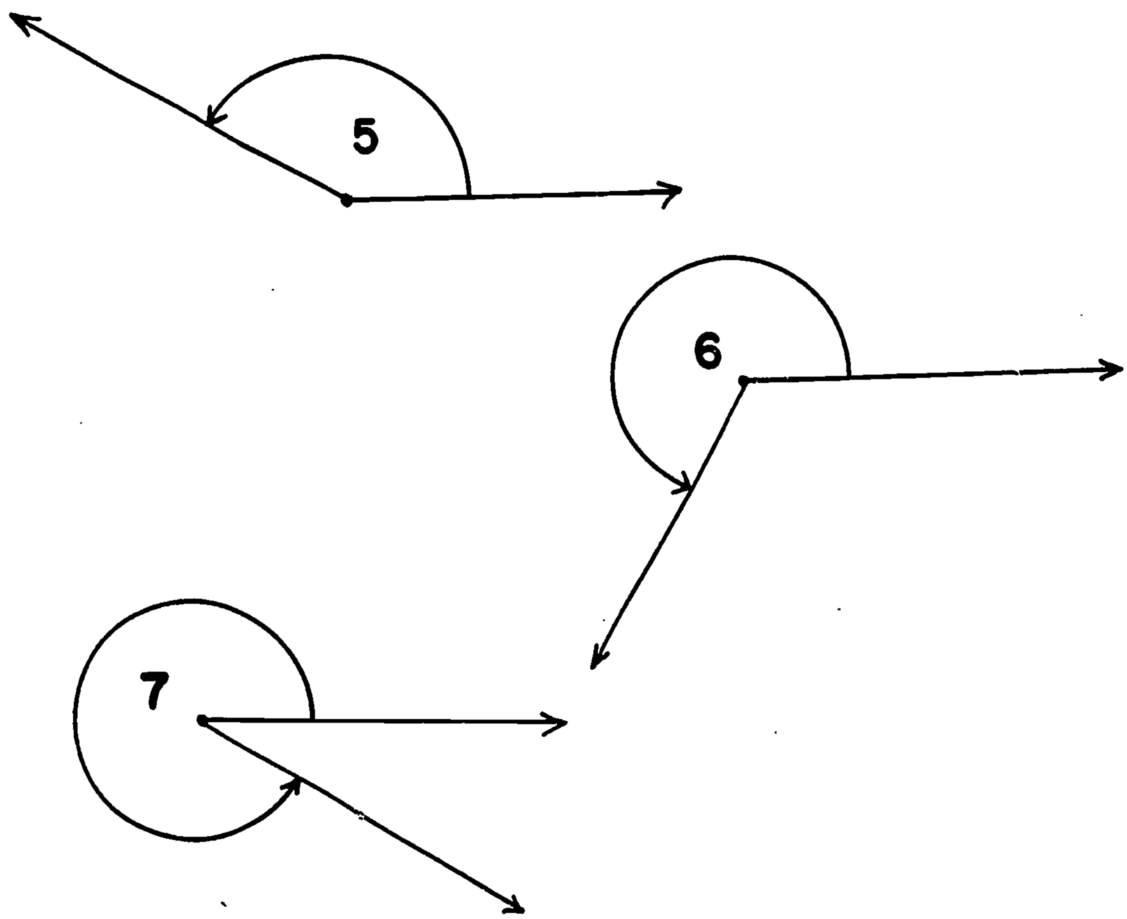
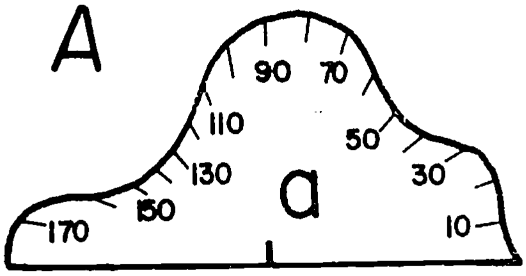


TABLE D-2

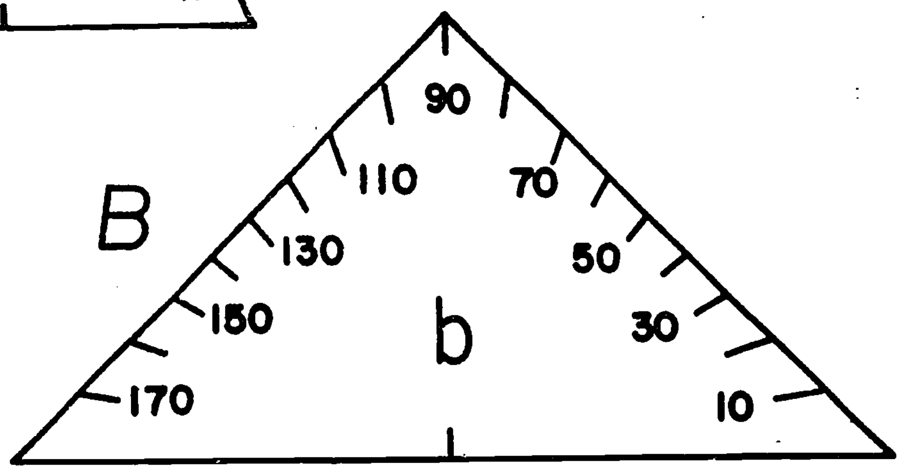
ANGLE	ESTIMATED MEASURE	MEASURING USING PROTRACTOR	
		F	G
5			
6			
7			

INSERT A

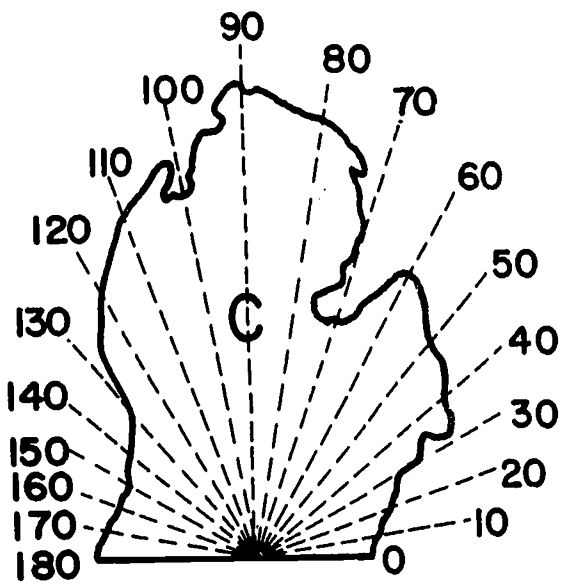
A



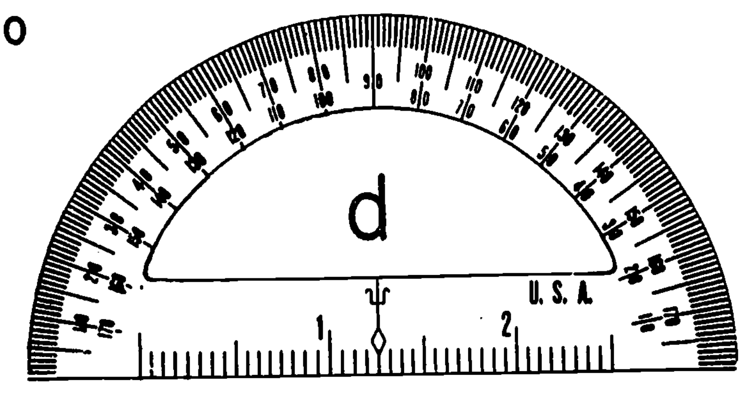
B



C

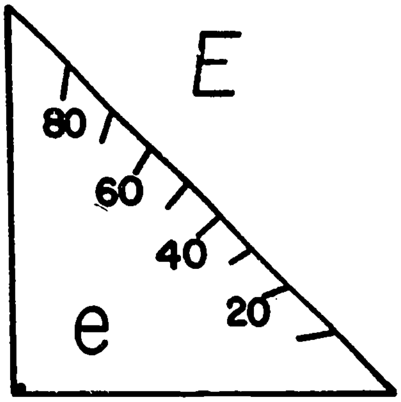


D

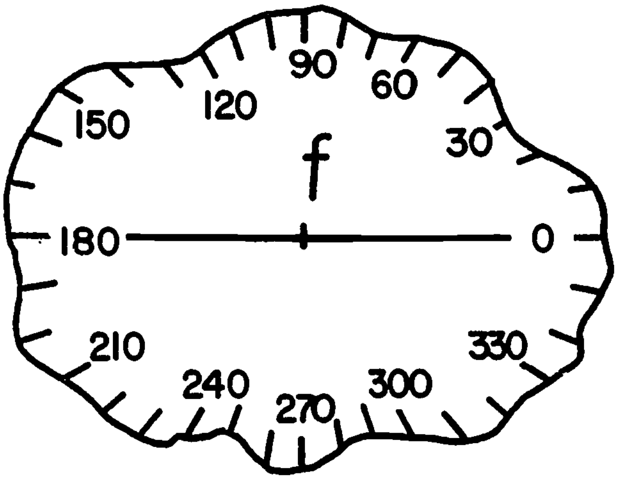


INSERT B

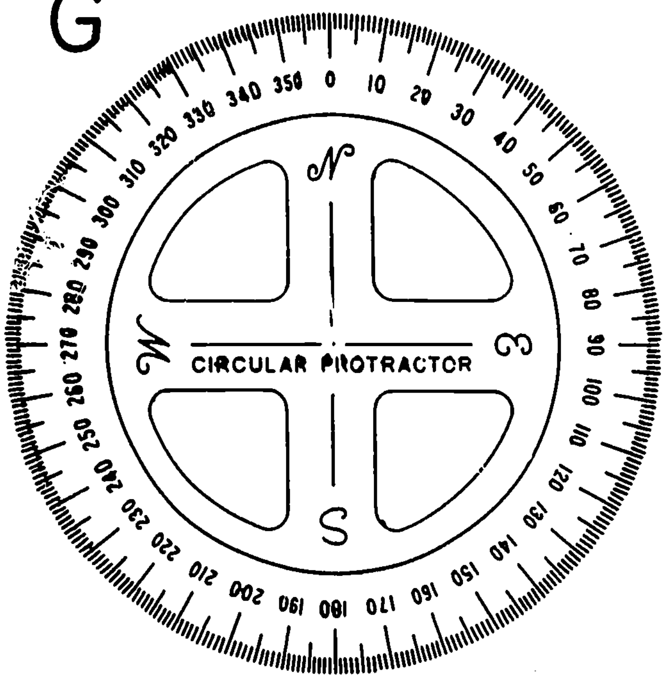
E


80
60
40
20
e

F


90
60
120
150
30
f
180
0
210
330
240
300
270

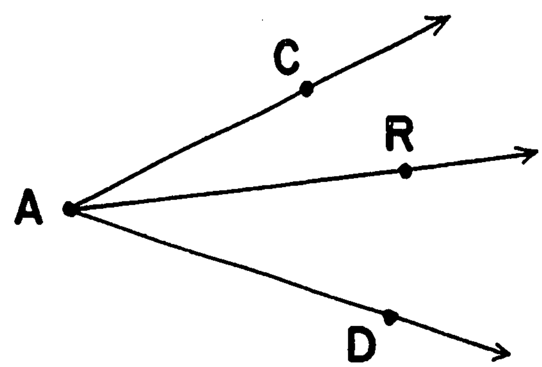
G


0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350
N
W
CIRCULAR PROTRACTOR
S

GLOSSARY

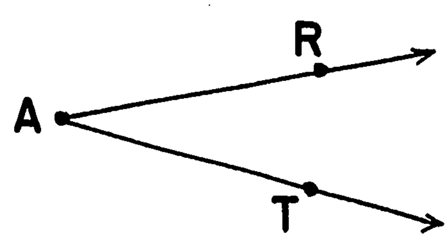
ACUTE ANGLE . An angle whose measure is greater than 0° but less than 90° is an acute angle.

ADJACENT ANGLE . In the diagram below, $\overrightarrow{AR}$ is a side of both $\angle$ CAR and $\angle$ DAR, and A is the vertex of both angles. Side $\overrightarrow{AR}$ is between sides $\overrightarrow{AD}$ and $\overrightarrow{AC}$.



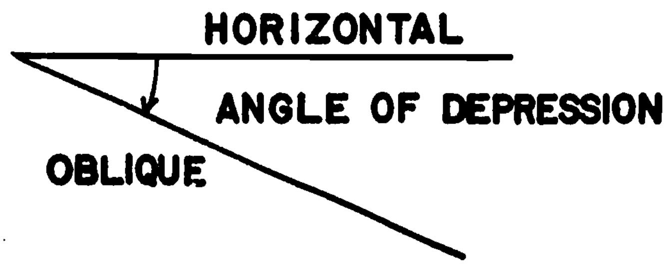
If two angles have the same vertex and a common side which lies between the other two sides, then the angles are <u>adjacent angles</u>. Notice that $\angle$ CAD is not adjacent to $\angle$ DAR or $\angle$ CAR.

<u>ANGLE</u> An angle is a plane figure that consists of two rays having a common endpoint.

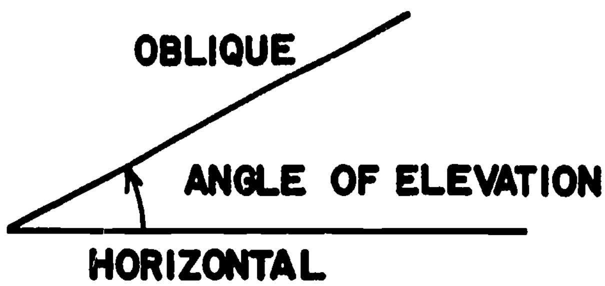


The rays are usually called the <u>sides</u> of the angle. The common endpoint is called the <u>vertex</u> of the angle.

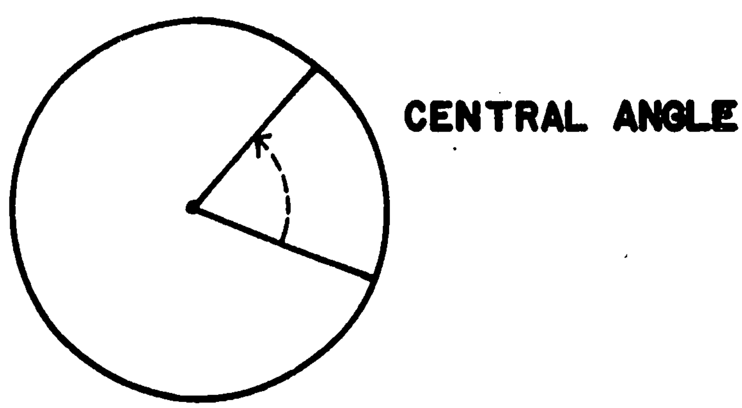
ANGLE OF DEPRESSION . The angle between the horizontal and the oblique line (neither parallel to nor perpendicular to - sloping) joining the observer's eye to some object lower than (beneath) the line of his eye.



ANGLE OF ELEVATION . The angle between the horizontal plane and the oblique line from the observer's eye to a given point above the line of his eye.



CENTRAL ANGLE . An angle determined by two radii of a circle.



COMPLEMENTARY ANGLES. Two angles whose measures sum up to 90°. The two acute angles of a right triangle are complementary.

CONGRUENT ANGLES. Two angles are congruent if and only if they have the same measure.

CONGRUENT ARCS. Two arcs are congruent if and only if they have the same size and shape.

Two congruent central angles of the same circle determine two congruent arcs of that circle.

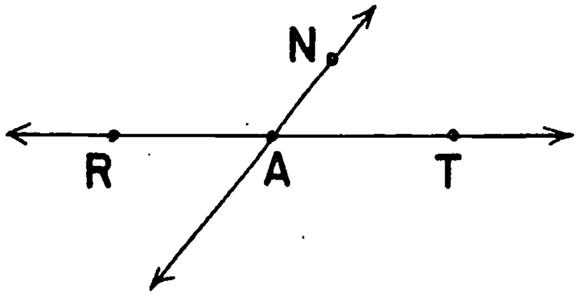
OBTUSE ANGLE. An angle whose measure is greater than 90° but less than 180° is an obtuse angle.

RAY. A ray is a subset of a line, consisting of an endpoint and all the points on the line that are on one side of the endpoint.

RIGHT ANGLE. When two lines intersect so as to form four congruent angles, then each angle is called a right angle. The measure of a right angle is 90°.

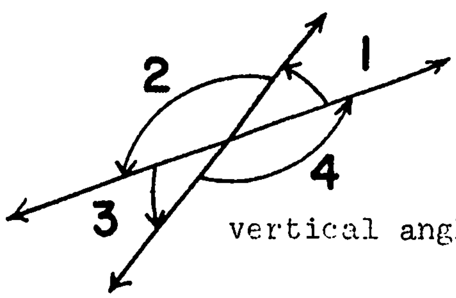
STRAIGHT ANGLE. An angle whose sides lie on the same straight line, but extend in opposite directions from the vertex. The measure of a straight angle is 180°.

SUPPLEMENTARY ANGLES. Two angles whose sum is 180°. If two lines intersect in a point, the adjacent angles formed are supplementary.



$m(\angle RAN) + m(\angle TAN) = 180^\circ$

VERTICAL ANGLES. Nonadjacent angles formed by two intersecting lines are called vertical angles. Vertical angles are congruent.



vertical angles: $\angle 1$ and $\angle 3$
$\angle 2$ and $\angle 4$

ANGLE MEASURE SUPPLEMENTARY EXERCISES

ANSWER KEY

NUMBER 1

1. 2/3
2. 1/4
3. 5/6
4. 11/8 OR 1 3/8
5. 4/12

NUMBER 2

1. yes
2. 9
3. 1
4. many or infinite
5. 3/3
6. 10, 18
7. 3, 12, 15, 6
8. 6/6

NUMBER 3

1. 9/24 of a turn
2. 1
3. 2
4. 5
5. 14
6. 56
7. 27
8. 28
9. 4 1/2
10. 91

NUMBER 4

1. they were both right
2. $\neq$
3. =
4. $\neq$
5. =
6. =
7. $\neq$
8. $\neq$
9. =
10. $\neq$

NUMBER 5

1. 18
2. 35
3. 27
4. 143
5. 81
6. 42
7. 99
8. 276
9. 120
10. 65

NUMBER 6

1. $<$
2. $>$
3. $<$
4. $>$
5. =
6. $<$
7. $<$
8. the larger numerator
9. the smaller denom.
10. a/r

NUMBER 7

1. 2 4/12 OR 2 1/3 OR 28/12 OR 7/3
2. 1 1/3
3. 3 1/16
4. 6 5/8
5. 2 1/2
6. 6 2/3
7. 6 4/5
8. 11 3/4
9. 8 5/7
10. 16 4/6 OR 16 2/3

NUMBER 8

1. 11 quarter inches
2. 20/3
3. 23/4
4. 67/5
5. 41/8
6. 15/2
7. 64/3
8. 55/12
9. 57/16
10. 78/5

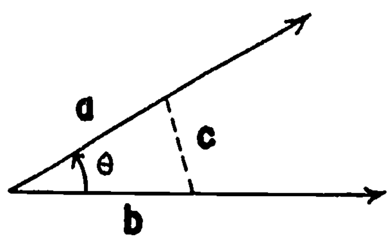
NUMBER 9

1. 5/6
2. 3/8
3. 23/20 OR 1 3/20
4. 2/24 OR 1/12
5. 23/12 OR 1 11/12
6. 7/16
7. 13/8 OR 1 5/8
8. 9/48
9. 12/12 OR 1
10. 11/15

CAN A RULER BE USED TO MEASURE ANGLES?

CONTENT AND APPROACH (pp. 128-130)

The conclusion of this enrichment activity is that a ruler <u>can</u> be used to measure angles, providing it is done under the appropriate conditions (see Dan's Argument, p. 110). The validity of this method is based upon the Law of Cosines.



$$c^2 = a^2 + b^2 - 2\,ab\,\text{Cos}\ 0$$

The following table shows what happens to the value of c^2 when $m(\angle\theta)$ is $0°$, $45°$, $90°$, $135°$, and $180°$. The term 2ab will always be positive and the distances a and b will remain constant for all values of θ.

θ	Cos θ	c^2
$0°$	1	$a^2 + b^2 - 2ab$
$45°$	$\frac{1}{2}\sqrt{2}$	$a^2 + b^2 - \sqrt{2}\,ab$
$90°$	0	$a^2 + b^2$
$135°$	$-\frac{1}{2}\sqrt{2}$	$a^2 + b^2 + \sqrt{2}\,ab$
$180°$	$^-1$	$a^2 + b^2 + 2\,ab$

Notice that $m(\angle\theta)$ increases from $0°$ to $180°$, the value of c^2 increases. Therefore, the opening of the angle as measured by c will increase as the degree measure of the angle increases.

After the distances a and b are selected, c becomes a function of the cosine of θ. Thus if the degree measure of the one angle is twice the degree measure of another, the opening c of the larger angle will not necessarily be twice the opening of the smaller angle.

THINGS TO DISCUSS

All angles shown have degree measures between 0° and 180°. How could this method be extended to measure angles whose degree measures are between 180° and 360°?

ANSWERS (pp. 129-130)

1. $m(\angle E)$ $\underline{\ =\ }$ $m(\angle F)$.

   Dan's argument is correct.

2. (a)

TABLE C-1

ANGLE	OPENING TO NEAREST $\frac{1}{16}$ in.
$\angle$RST	3/4 in.
$\angle$ABC	7/8 in.
$\angle$DEF	1/2 in.
$\angle$GHI	11/16 in.
$\angle$XZY	5/8 in.

(b) $\angle$DEF, $\angle$XZY, $\angle$GHI, $\angle$RST, $\angle$ABC

(c) Yes

3. Yes - under the conditions specified by Dan's argument.

DO PROTRACTORS HAVE TO BE CIRCULAR?

ANSWERS

PART I (pp. 133-134)

	TABLE D-1				
	MEASURE USING PROTRACTOR:				ESTIMATED MEASURE
ANGLE	A	B	C	D	
1	30°	30°	30°	30°	*
2	130°	130°	130°	130°	*
3	70°	70°	70°	70°	*
4	140°	140°	140°	140°	*

*Estimates will vary.

POWER QUESTIONS I (p. 134)

1. Yes. Although of different shapes, all the protractors have scales based on a circular design.
2. Consider $m(\angle 2)$ as $90^\circ + m(\angle 2 - 90^\circ)$

PART II (p.135)

	TABLE D-2		
ANGLE	ESTIMATED MEASURE	MEASURING USING PROTRACTOR	
		F	G
5	*	150°	150°
6	*	240°	240°
7	*	330°	330°

*Estimates will vary.

POWER QUESTIONS II (p. 135)

1. Yes. See answer for number 1 above.
2. Consider $m(\angle \theta)$ as $180^\circ + m(\angle \theta - 180^\circ)$

STRATEGIES FOR USING INSERTS A AND B

Inserts A and B are printed on 16 lb. bond paper. Although the instructions for APPENDIX E state that the protractors are to be cut out, there are at least two alternate strategies that could be used. These are outlined below.

Strategy 1: The paper on which the inserts are printed is transparent enough for use as an overlay to measure the angles on pages 137 and 138 accurately. Pull out the inserts and superimpose the protractors on the angles to be measured.

Strategy 2: Use the inserts as masters for preparing acetate overlays. Measure the angles by superimposing the overlays on the angles to be measured.

GLOSSARY

The terminology included is associated with angle measure but not necessarily with this booklet. This section is to be used as a reference for students in instances where these terms may arise as a part of class discussion.