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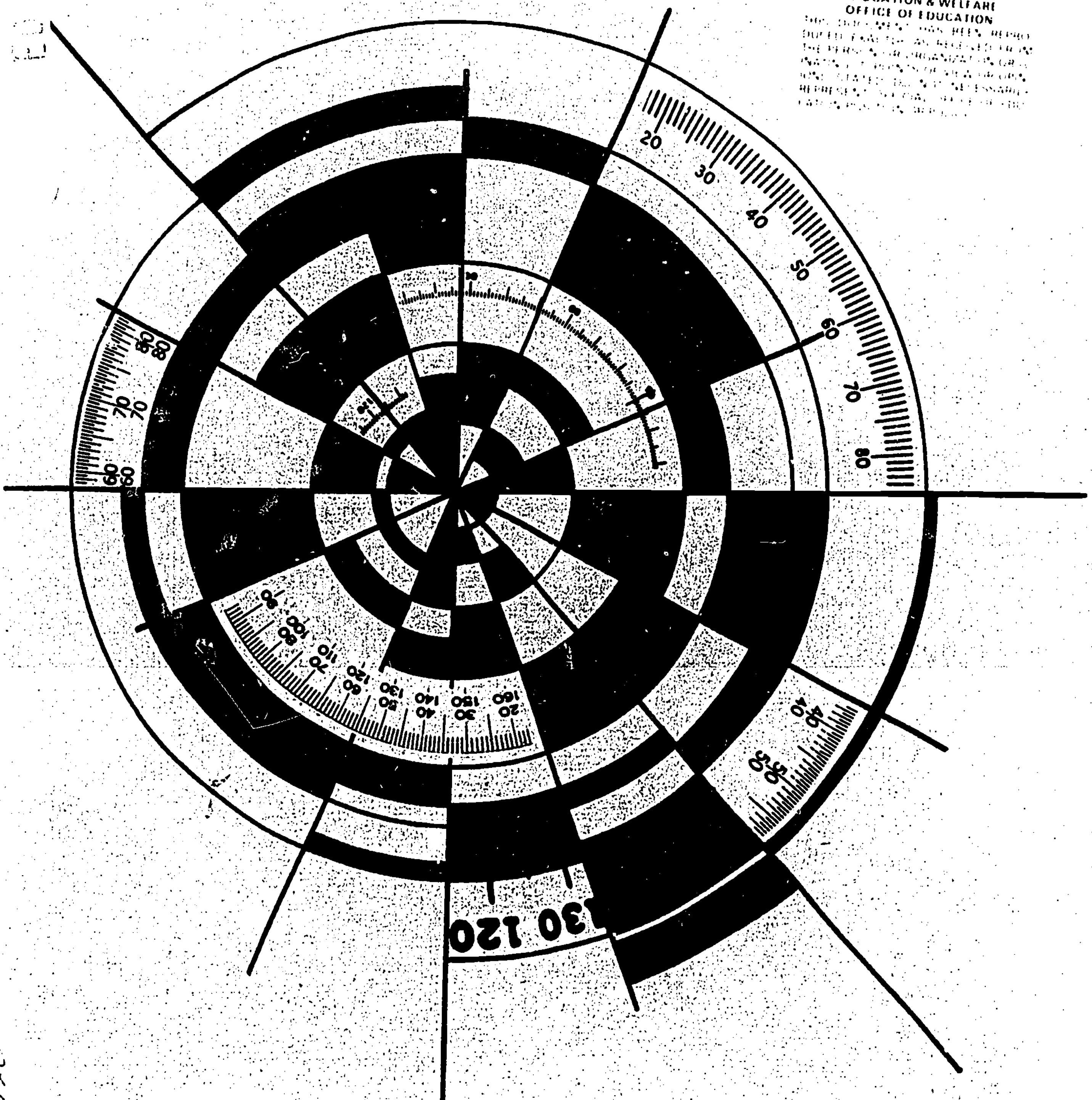
ABSTRACT

This instructional unit seeks to prepare the student to exhibit competence in the mechanics of measuring and estimating angle size and in making generalizations on the nature of measurement. Experimentation with the use of circular and semi-circular protractors is encouraged. Exercises and discussion questions are given for each section. Appendices are included which contain material for review, remediation, and enrichment. A teacher's guide is also available. Related documents are SE 015 334 - SE 015 339 and SE 015 341 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ANGLE MEASURE



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ANGLE MEASURE

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

NO HUM, NO CLICK, NO BLINK . . .

This is a teaching machine. Absolutely guaranteed against mechanical failure, it is composed of the finest printed circuits but completely non-electronic. It has been produced in its present form for more than 500 years and is still one of man's most successful devices in communicating knowledge.

EMPHASIS AND CONTENT . . .

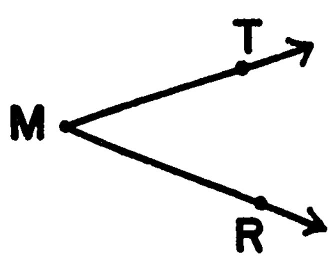
This booklet concentrates on measuring and estimating angle size. Each lesson concentrates on one or two main ideas. A variety of activities will be used to develop your ability to use a protractor properly and to make reasonable estimates.

Read the booklet carefully, work the exercises, and participate in the class activities and discussions. This material will help you learn, but only if you use it properly.

OBJECTIVES . . .

When you complete this booklet, you should be able to...

1. ... locate and name the (a) vertex and (b) sides of an angle when given a labeled representation of an angle.

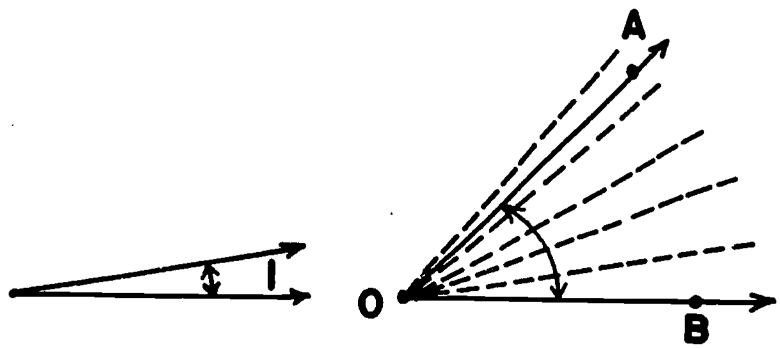


VERTEX: ____________

SIDES: ____________

2. ... demonstrate your understanding of unit angle by answering questions such as:

a.



Given $\angle 1$ as the unit angle, complete the following:

(1) _____ units $< m(\angle BOA) <$ _____ units

(2) to the nearest unit, $m(\angle BOA) =$ _____ units

b. If zips and zaps are units of measure and

$m(\angle BEF) = 6$ zips

$m(\angle BEF) = 4$ zaps

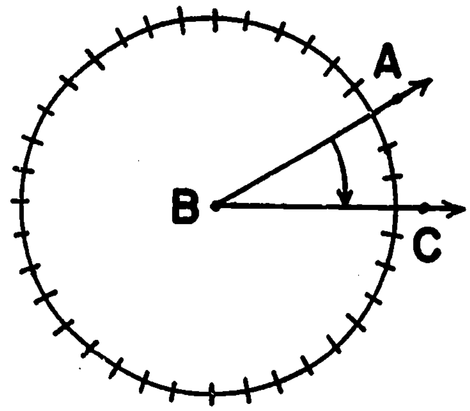
which is the larger unit of measure, a zip or a zap?

3. ... demonstrate your understanding of amount of turn as a measure of an angle through exercises such as:

a. Describe, by drawing the appropriate arrows, clockwise and counterclockwise rotation.

b. Given an angle whose vertex is at the center of a circle with the direction of rotation indicated, measure the angle. The unit of measure is a fractional amount of turn.

The circle is divided into congruent arcs as an aid in determining the measure.



$m(\angle ABC) = \frac{3}{36}$ of a turn

c. Define a degree as $\frac{1}{360}$ of a turn.

d. Convert an angle measure from a fractional amount of turn to degree measure and vice versa.

e. Recognize and describe a right angle as an angle swept out by a $\frac{1}{4}$ turn and a straight angle as an angle swept out by a $\frac{1}{2}$ turn.

4. ... estimate the size of an angle.

a. Given an angle whose measure is between 0° and 360°, you will be able to give a reasonable estimate (a 10° tolerance) of the measure in degrees.

b. Given three angles differing in measure by at least 10°, you will be able to rank them in order of size without measuring.

c. Given three angles differing in measure by at least 10°, and a given measure, in degrees, you will be able to select the angle which is closest to the given measure.

5. ... measure an angle whose measure is between 0° and 360° to the nearest degree under the following conditions:

a. When the angle is determined by...

(1) two intersecting lines

(2) two rays having a common endpoint.

b. Using either a...

(1) circular protractor, or

(2) semi-circular protractor.

c. With the protractor in...

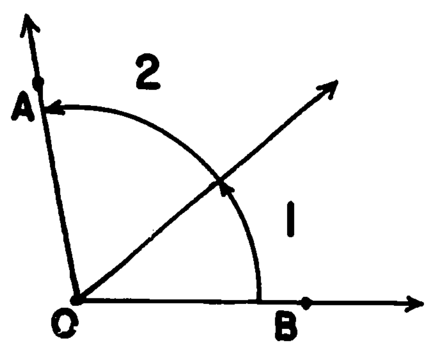
(1) standard position (one ray through the zero point), or

(2) non-standard position.

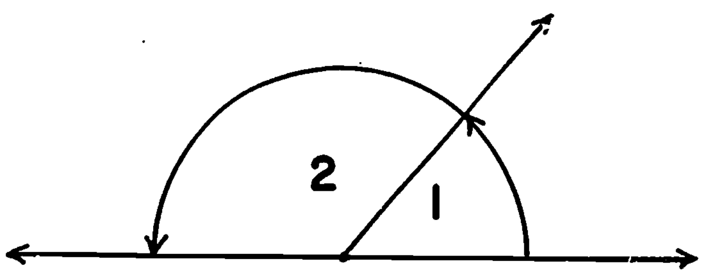
d. When the "sides" of the angle must be extended to get a reading on the protractor scale.

It is assumed that the protractor used is of sufficient quality to enable an accurate reading to the nearest degree.

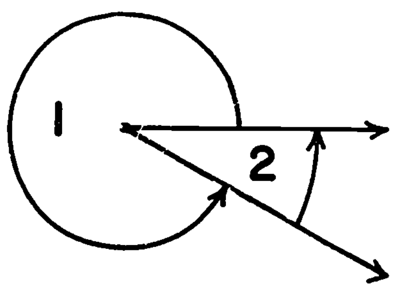
6. ... demonstrate your knowledge of the additive property of measurement and the fact that there are 360° in one complete revolution by answering questions such as the following without using a protractor.



If $m(\angle 1) = 40^{\circ}$, and
$m(\angle 2) = 60^{\circ}$, then
$m(\angle BOA) =$ _____.



If $m(\angle 1) = 50^{\circ}$, then
$m(\angle 2) =$ _____.



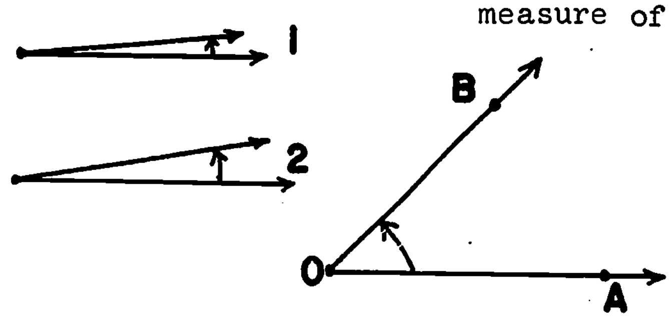
If $m(\angle 1) = 330^{\circ}$, then
$m(\angle 2) =$ _____.

7. ... use a protractor to draw an angle having a given measure, for any measure in degrees between 0° and 360°.

8. ... state that one degree equals sixty minutes and one minute equals sixty seconds.

9. ... demonstrate your understanding of precision in angle measurement by answering questions such as:

a. Which unit angle will yield the more precise measure of $\angle AOB$?



b. In Figure 1 circle A is divided into 24 congruent arcs and in Figure 2 circle A is divided into 36 congruent arcs. Which circle helps determine the more precise measure of $\angle RAT$?

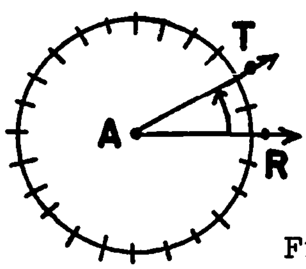


Fig. 1

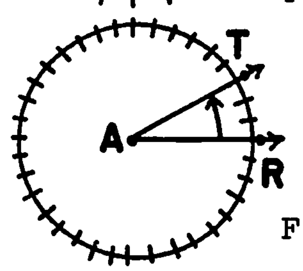


Fig. 2

c. Which unit gives the more precise measure of an angle; degree, minute, or second?

As you study this booklet, use the **EXERCISES**, **CLASS ACTIVITIES**, **DISCUSSION QUESTIONS**, and ✓ **POINTS** to evaluate your progress in achieving the objectives for this booklet.

If you get "stuck", try again. If you are still confused after careful study, ask for help.

MEASUREMENT IS APPROXIMATE

WE CAN NOT HOPE TO ELIMINATE ALL ERRORS ---

OUR GOAL IS TO REDUCE THE SIZE

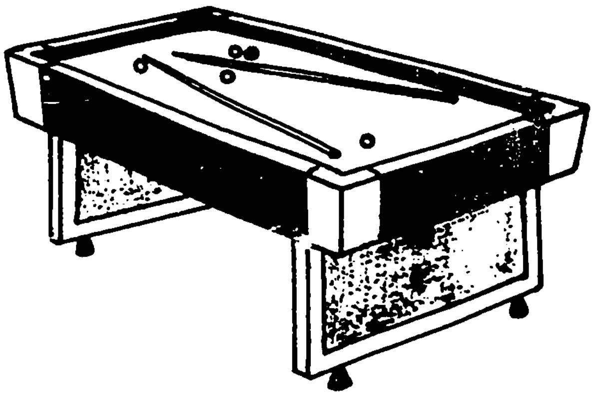
AND NUMBER OF THESE ERRORS.

TABLE OF CONTENTS

LESSONS

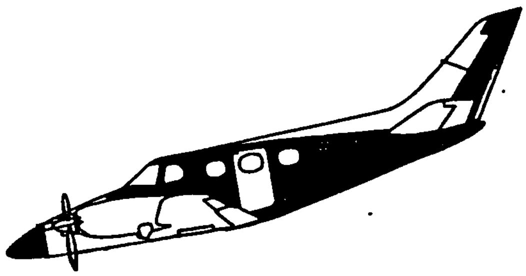
APPENDICES

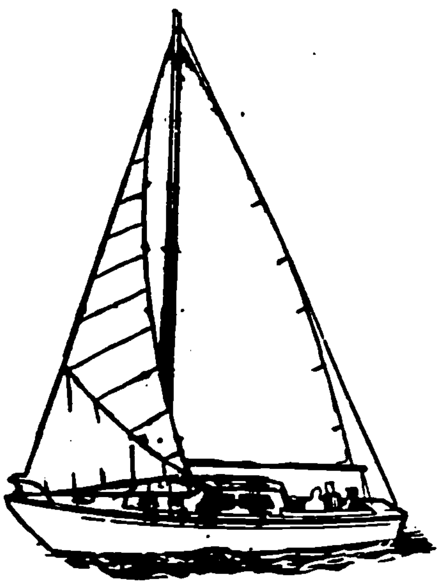
MEASURING THE OPENING



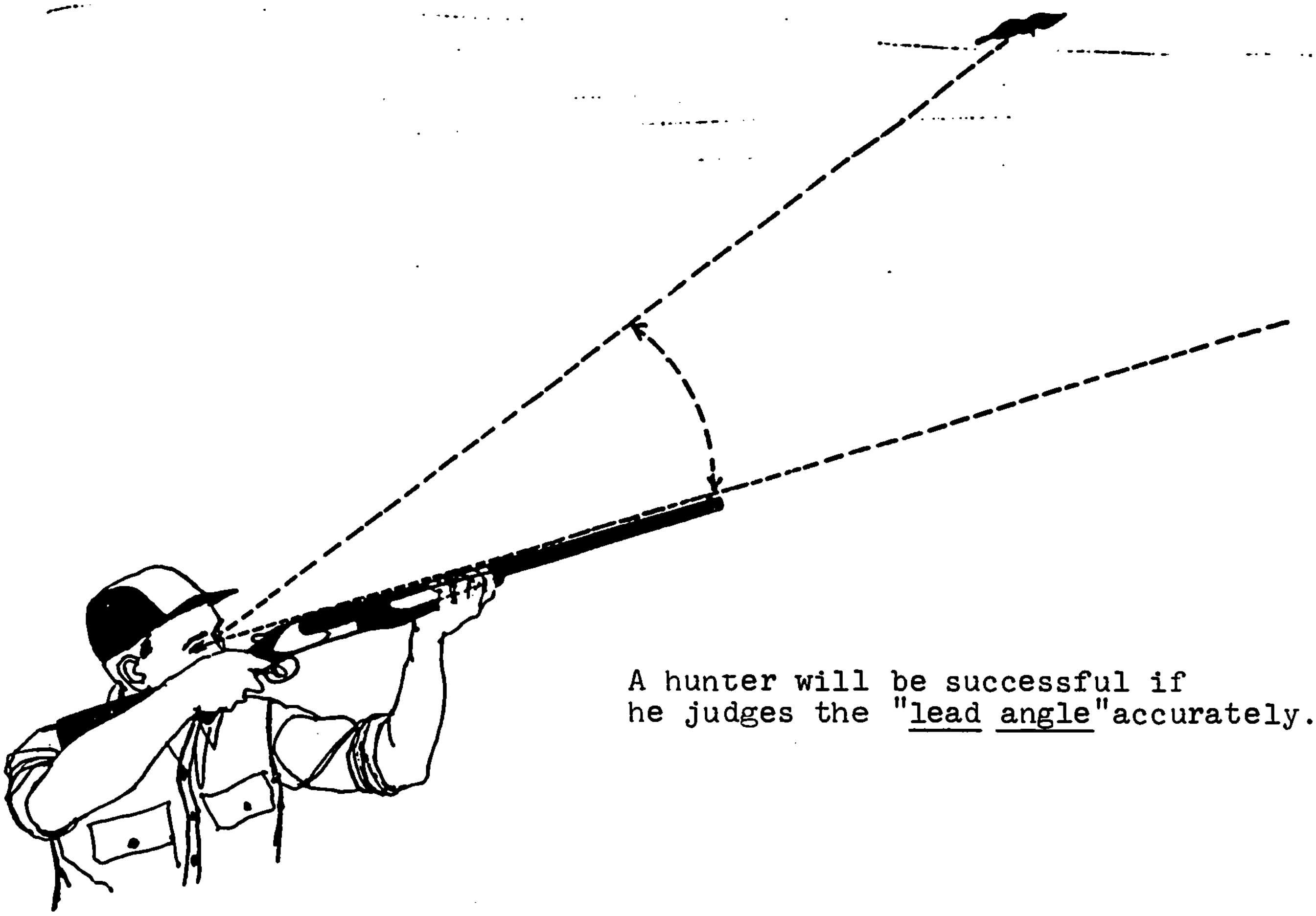
In billiards, a successful "bank shot" depends on the angle of incidence and reflection.

The smoothness of a plane's landing depends on the angle of descent.





To stay on course, the bearing (in degree measure) of a boat must be measured accurately.



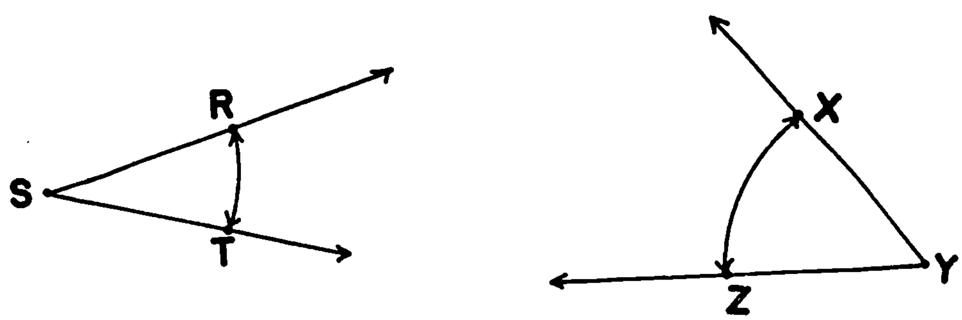
A hunter will be successful if he judges the "lead angle" accurately.

A space ship will safely re-enter the earth's atmosphere if the angle of re-entry is not too steep or too shallow.

THE SIZE OF AN ANGLE IS...

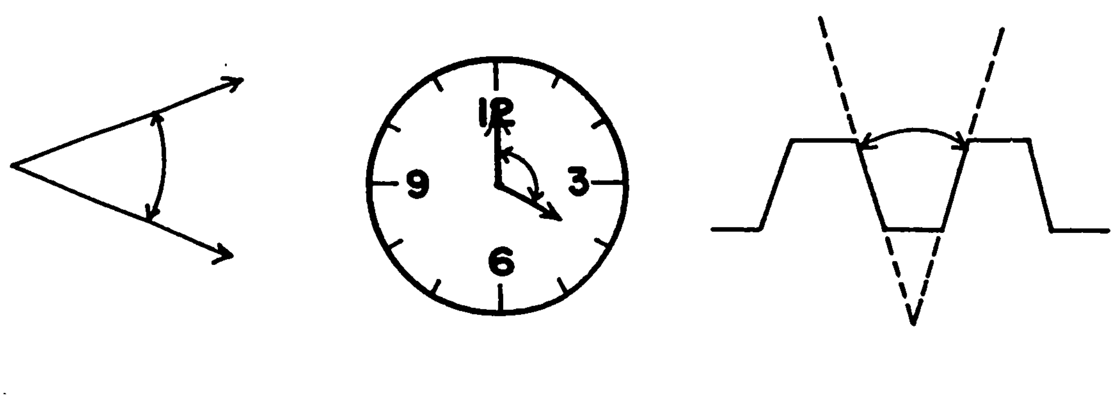
The size of an angle depends on the amount of opening between its sides. The greater the opening, the greater the size of the angle.

To illustrate, ∠XYZ has a greater size than ∠RST because "the opening between the sides of ∠XYZ is greater than the opening between the sides of ∠RST".



The size of ∠XYZ is greater than the size of ∠RST.

A double-headed arrow will be used to indicate the opening between the sides of an angle.



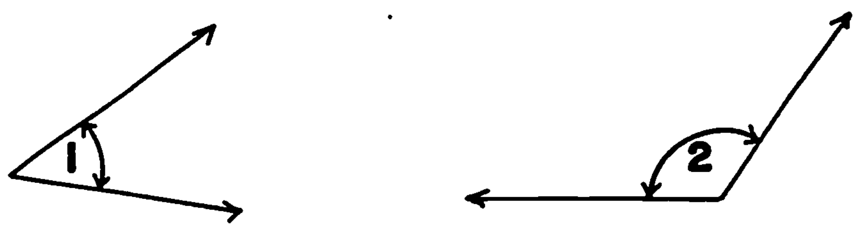
✓ **POINT**

1. What determines the size of an angle?
2. In this booklet, what symbol is used to indicate the amount of opening between the sides of an angle?

WHICH ANGLE IS GREATER?

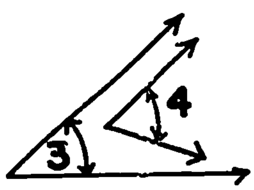
Without measuring, which angle in each pair seems to have the greater opening between the sides? Circle your response.

A.



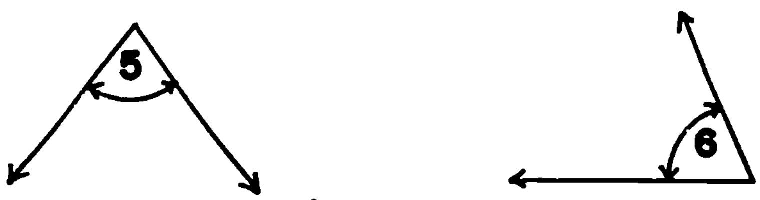
A. $\angle 1, \angle 2$

B.



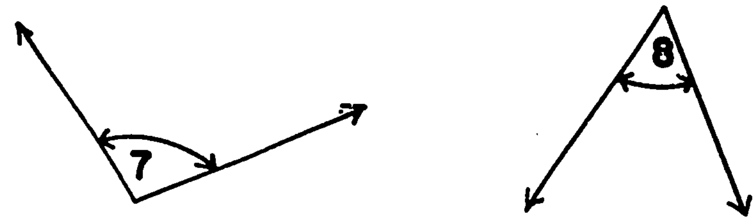
B. $\angle 3, \angle 4$

C.



C. $\angle 5, \angle 6$

D.



$\angle 7$, $\angle 8$

A METHOD IS NEEDED...

Although the angles in some of the pairs on pages 4 and 5 seem to have the same amount of opening, this is not the case. (The correct responses are $\angle 2$, $\angle 4$, $\angle 5$, and $\angle 7$.)

Before you could accurately determine which angle in each pair has the greater opening, a method is needed for measuring the opening between the sides of an angle.

MEASURING THE OPENING . . .

To measure angles, a method similar to the one for measuring length is usually used.

GIVEN AN ANGLE TO BE MEASURED . . .

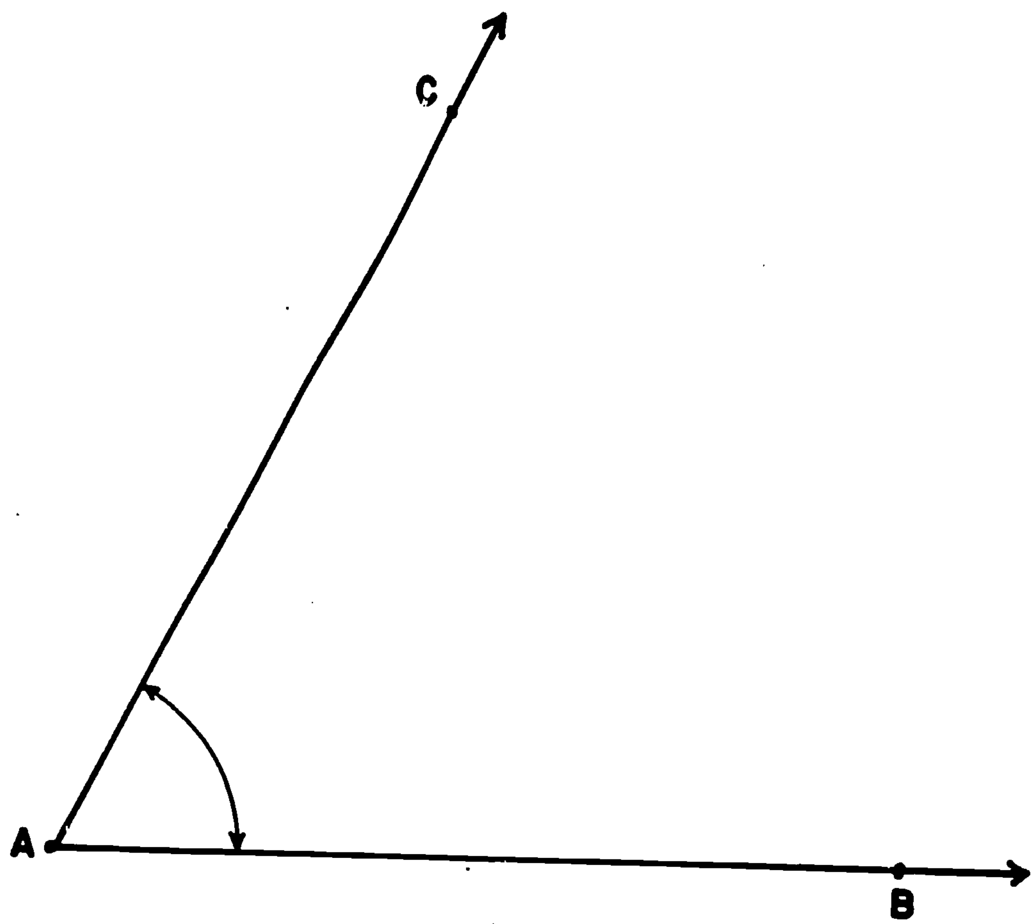
(1) Select some angle to use as a unit of measure.

(2) Compare the angle to be measured with the unit angle by counting the number of unit angles it takes to "fill up" the opening of the angle being measured.

This method of measuring angles is illustrated on pages 6 and 7.

EXAMPLE

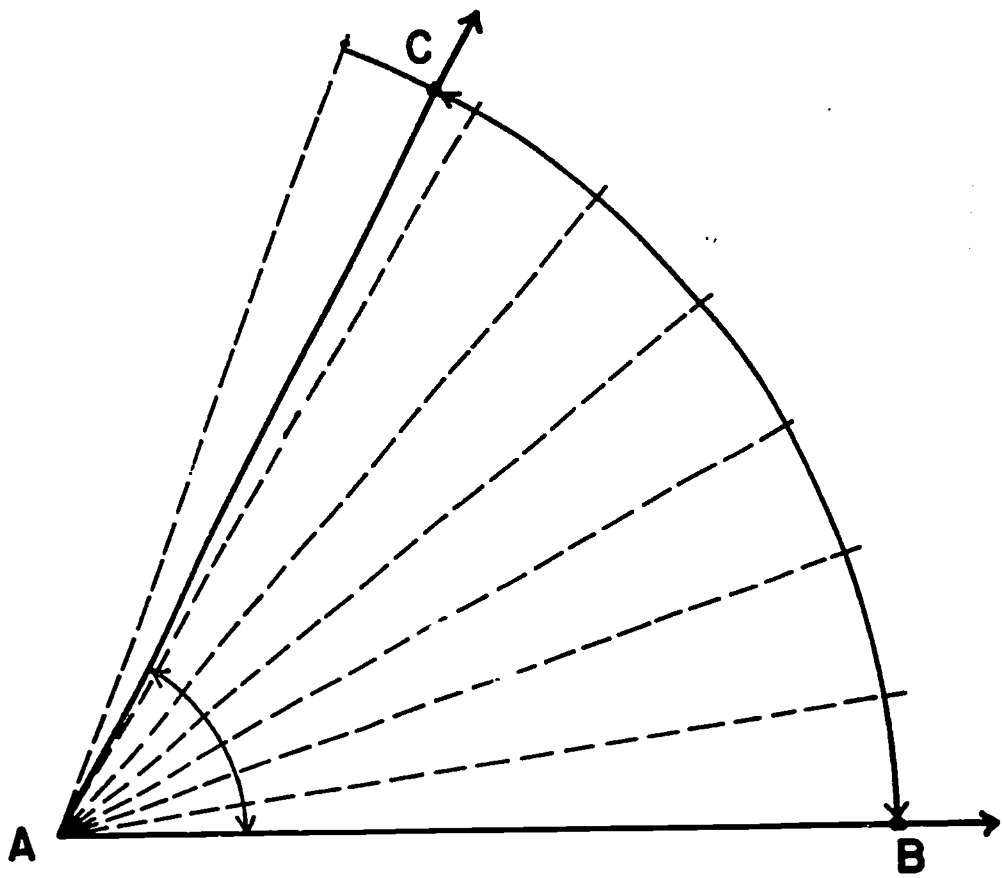
What is the measure of ∠ CAB?



SOLUTION

(1) SUPPOSE THE RED UNIT ANGLE WEDGE FROM THE PACKET OF ANGLE WEDGES IS SELECTED AS THE UNIT OF MEASURE.

(2) COMPARE THE ANGLE TO BE MEASURED WITH THE UNIT BY COUNTING THE NUMBER OF UNIT ANGLES IT TAKES TO "FILL UP" THE OPENING OF THE ANGLE BEING MEASURED.



6 units < m(∠ CAB) < 7 units

It takes more than 6 and less than 7 units to "fill up" the angle being measured. Since the measure (size) of ∠ CAB is closer to 6 units than it is to 7 units,...

m(∠ CAB) = 6 units when measured to the nearest unit.

[m(∠ CAB) is read "the measure of angle CAB".]

You will be given an envelope containing unit angle wedges of three different sizes. These will be used in the remainder of this lesson.

YET TO BE DONE . . . .

1. Verify the solution on pages 6 and 7 by using the red units from your envelope.
2. Find the measure of ∠ CAB to the nearest yellow unit. to the nearest blue unit.

CLASS ACTIVITY

1. You will be given some angles to measure. Measure each angle to the nearest whole red unit, yellow unit, and blue unit.

ANGLE	MEASURE TO THE NEAREST WHOLE...		
	Red Unit	Yellow Unit	Blue Unit

2. Extend the sides of two of the angles listed in the above table. Measure these angles again. Does the length of the sides of an angle have any effect on the measure of the angle?

CHECK AND CORRECT YOUR RESULTS BEFORE
GOING TO THE NEXT PAGE.

✓ POINT

1. Describe the process for measuring angles used in this lesson.

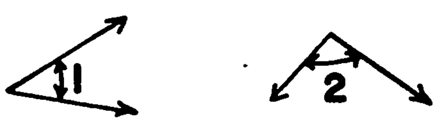
2. Look at your results for the **CLASS ACTIVITY** (p. 8). Does using a different unit of measure change the number used to express the measure of the angle? the amount of opening between the sides of the angle? Give reasons for your answers.

EXERCISES

1. Make each statement true by inserting either $>$, $<$ or $=$ in the blank provided.

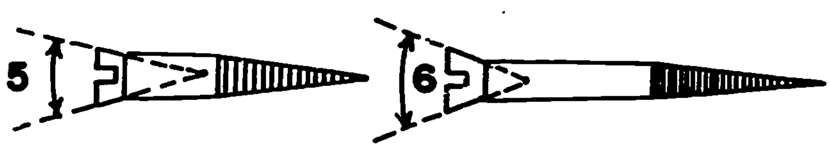
   $>$ means "is greater than"
   $<$ means "is less than"
   $=$ means "is the same as"

   Recall that the measure of an angle depends on the amount of opening between the sides of the angle.



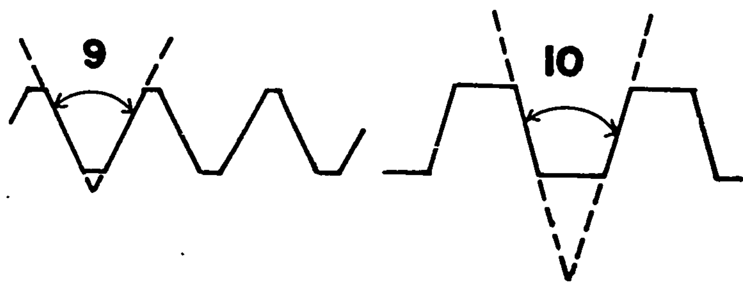
a. $\angle 1$ ___ $\angle 2$

b. $\angle 3$ ___ $\angle 4$



c. $\angle 5$ ___ $\angle 6$

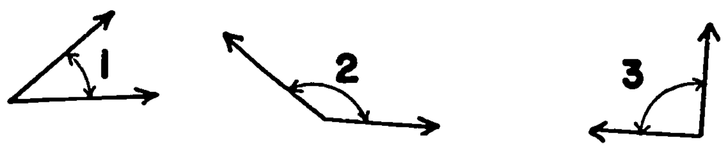
d. ∠7 ___ ∠8



e. ∠9 ___ ∠10

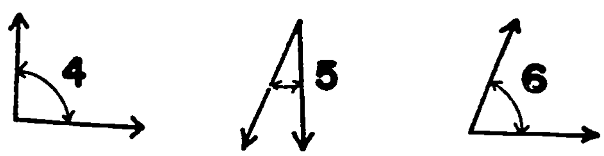
2. For each set of angles, arrange them in order from smallest to largest. Part a is done as an example. Record your answers in the blanks provided.

a.



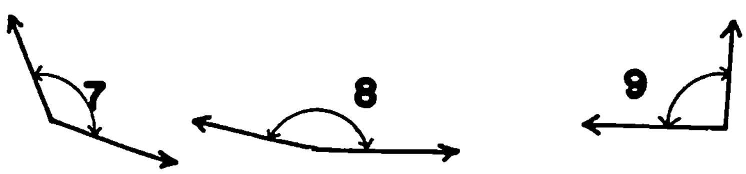
a. ∠1, ∠3, ∠2

b.



b. ____________

c.

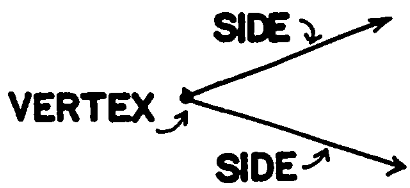


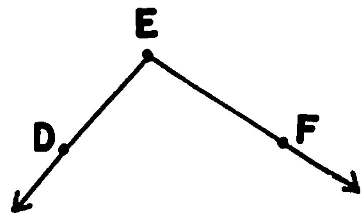
c. ____________

d. Of the 9 angles pictured in parts a-c, which angle seems to be the largest? the smallest?

d. ____________

The rays which form an angle are called the <u>sides</u> of the angle. The common endpoint of the sides is called the <u>vertex</u> of the angle.



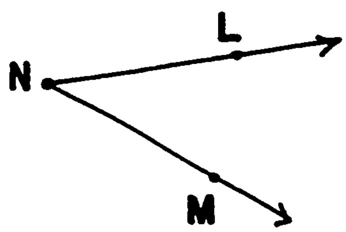


$\overrightarrow{EF}$ is read "ray $\overrightarrow{EF}$".

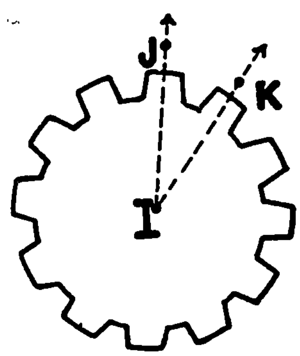
vertex: E
sides: $\overrightarrow{ED}$ and $\overrightarrow{EF}$

3. For each angle, name the (1) <u>vertex</u> and (2) the <u>sides</u> of the angle. Record your answers in the blanks provided.

a.



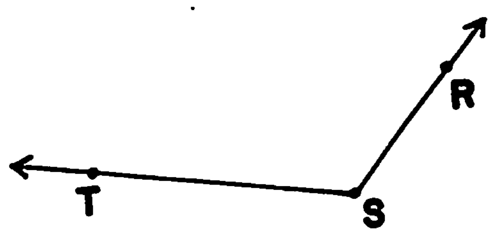
b.



a. vertex ______
sides ______

b. vertex ______
sides ______

c.



c. vertex ______
sides ______

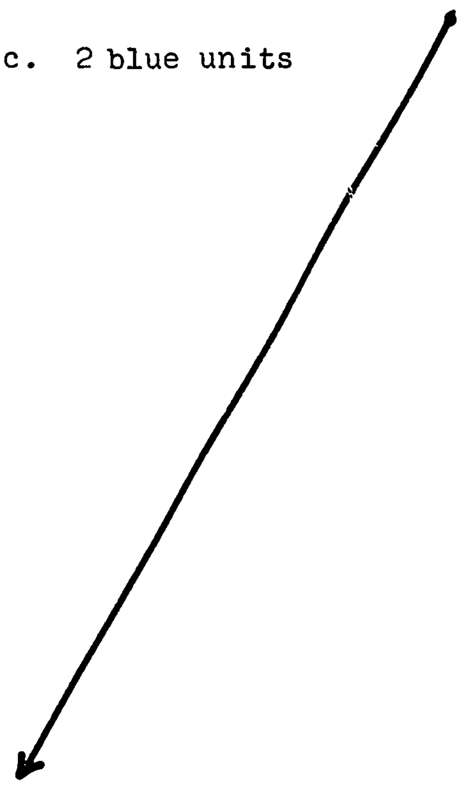
4. Using the unit angles from your envelope as guides, draw angles having the following measures.

   One side (ray) is given for each angle as a starting point. Use the endpoint of the ray as the vertex of the angle.

   a. 4 yellow units

   b. 4 red units

c. 2 blue units



NAMING ANGLES . . .

An angle may be named in several ways. The more common methods of naming angles are listed in **APPENDIX B** at the back of this booklet.

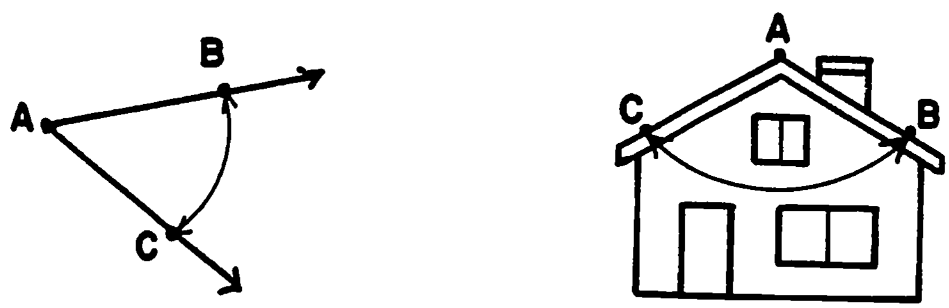
BECOMING MORE PRECISE

Measurement is done by <u>comparison</u>. In the previous lesson, the measure of an angle was found by comparing the opening of the angle being measured with the opening of the unit angle.

This lesson will review and formalize some of the ideas studied in Lesson 1.

ANGLES ARE . . .

Angles are formed by two rays having a common endpoint.



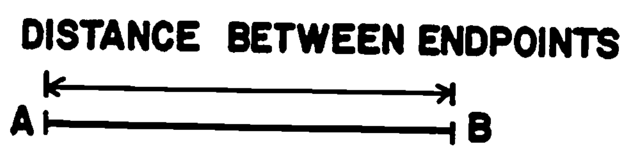
$\angle$ BAC or $\angle$ CAB

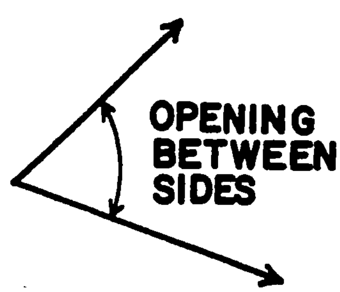
In both cases above, $\angle$ BAC (or $\angle$ CAB) is formed by the rays, $\overrightarrow{AB}$ and $\overrightarrow{AC}$, having a common endpoint, A.

DISTANCES AND OPENINGS . . .

A segment is measured by the distance between its endpoints.

An angle is measured by the amount of opening between its sides.



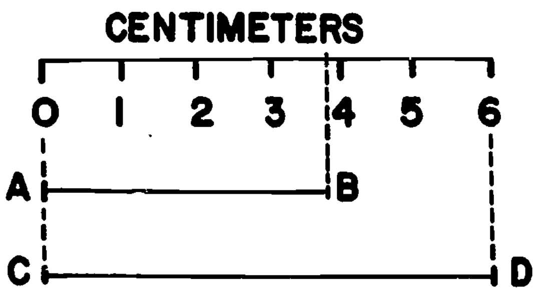


MEASURING LENGTH . . .

When measuring length,...

(1) a length (foot, pace, centimeter,...) is selected to use as a unit of measure, and...

(2) the length to be measured is compared with the unit of measure by counting the number of unit lengths it takes to "fill up" the length being measured.



Suppose 1 cm. is selected as the unit of measure. The measures of $\overline{AB}$ and $\overline{CD}$ to the nearest whole unit are...

m($\overline{AB}$) = 4 cm.
m($\overline{CD}$) = 6 cm.

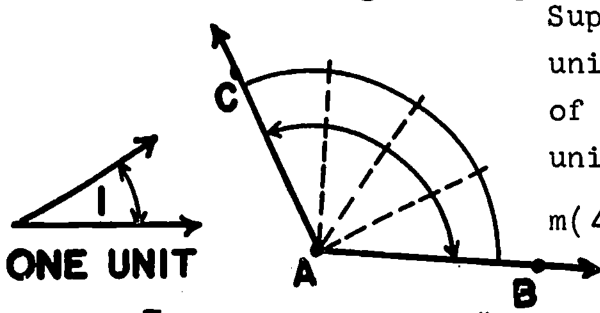
[m($\overline{AB}$) is read "the measure of segment AB".]

MEASURING ANGLE SIZE . . .

When measuring an angle,...

(1) an angle is selected to use as a unit of measure, and...

(2) the angle to be measured is compared with the unit of measure by counting the number of unit angles it takes to "fill up" the opening of the angle being measured.



Suppose ∠1 is selected as the unit of measure. The measure of ∠CAB to the nearest whole unit is...

m(∠CAB) = 4 units.

[m(∠CAB) is read " the measure of angle C-A-B".]

The examples on page 15 illustrate that the measure of a length or angle...

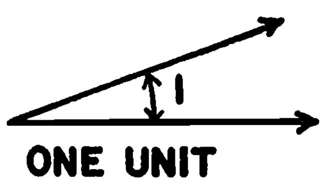
(1) depends on the unit of measure selected, and...

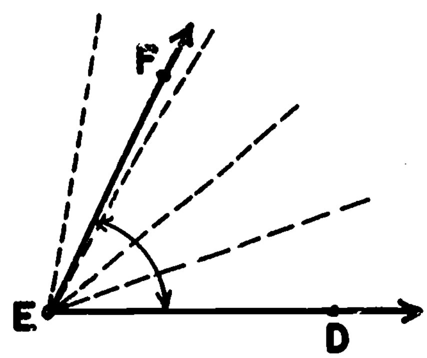
(2) is the number of unit angles or lengths it takes to "fill up" the length or opening of the angle being measured.

SOME DON'T COME OUT EVEN . . .

When measuring length, most measurements don't "come out even". For example, $m(\overline{AB})$ on page 15 was not exactly 3 centimeters nor 4 centimeters but was somewhere between 3 and 4 centimeters.

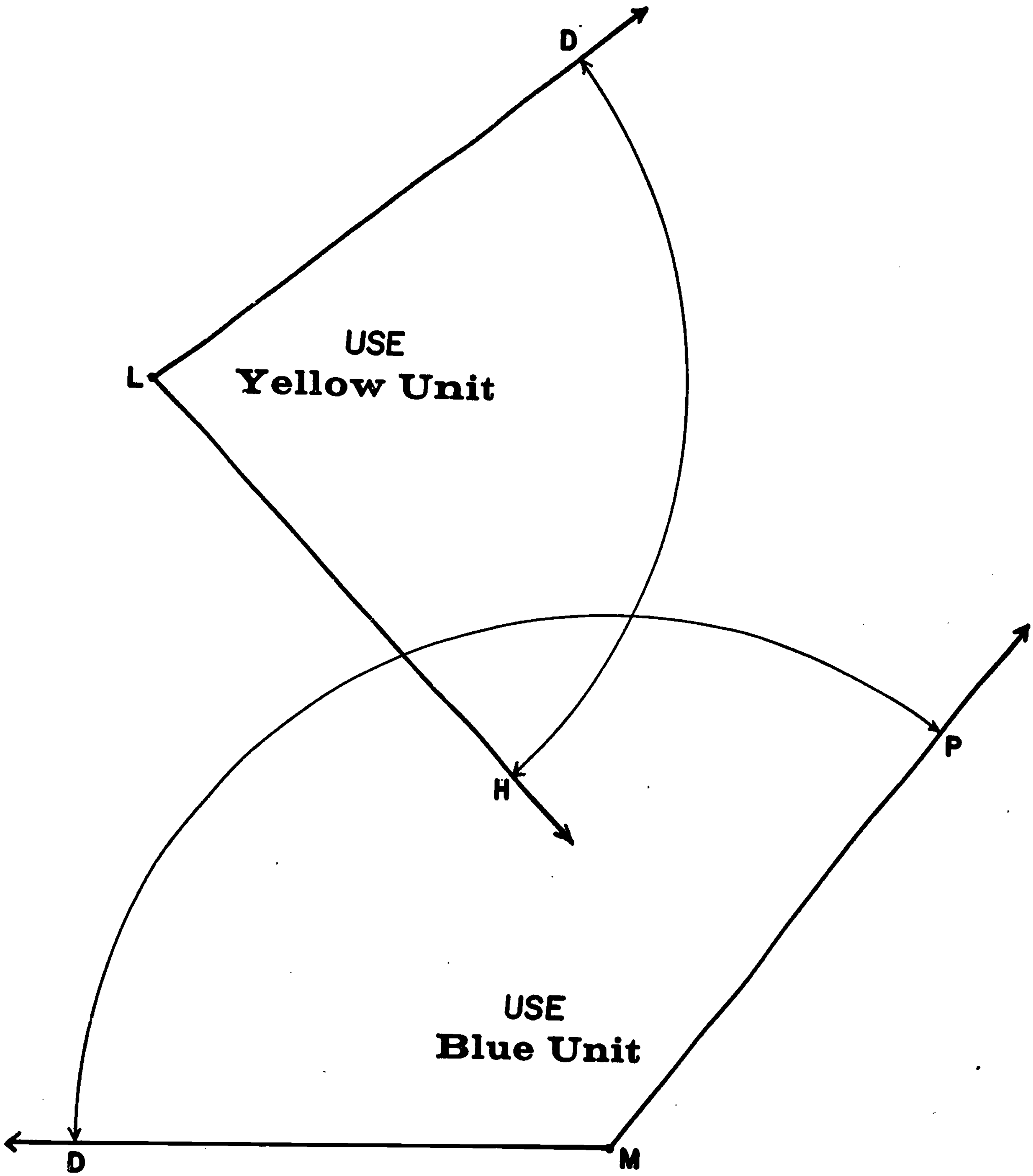
The same is true of angle measure. Using $\angle 1$ below as the unit of measure, $m(\angle DEF)$ is between 3 and 4 units.



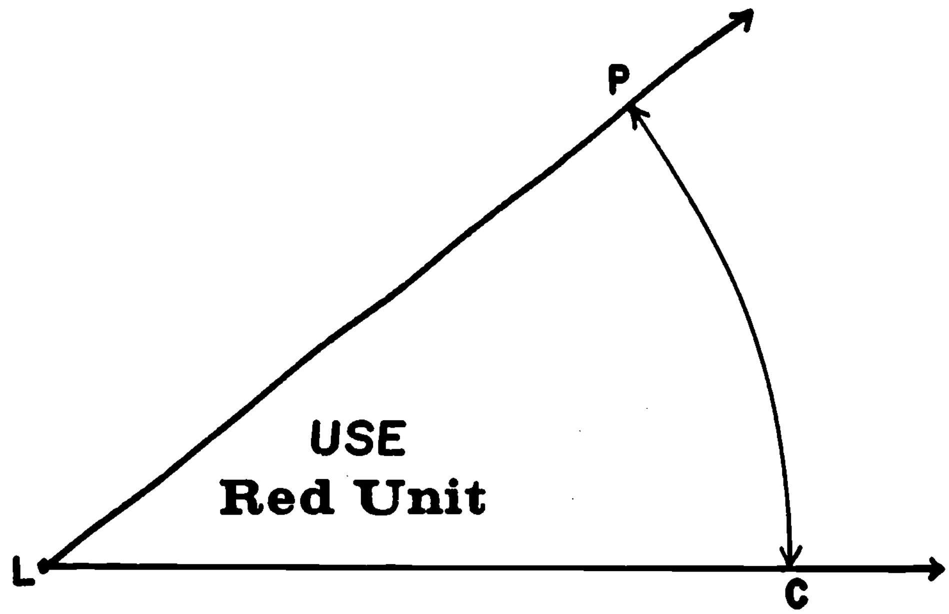


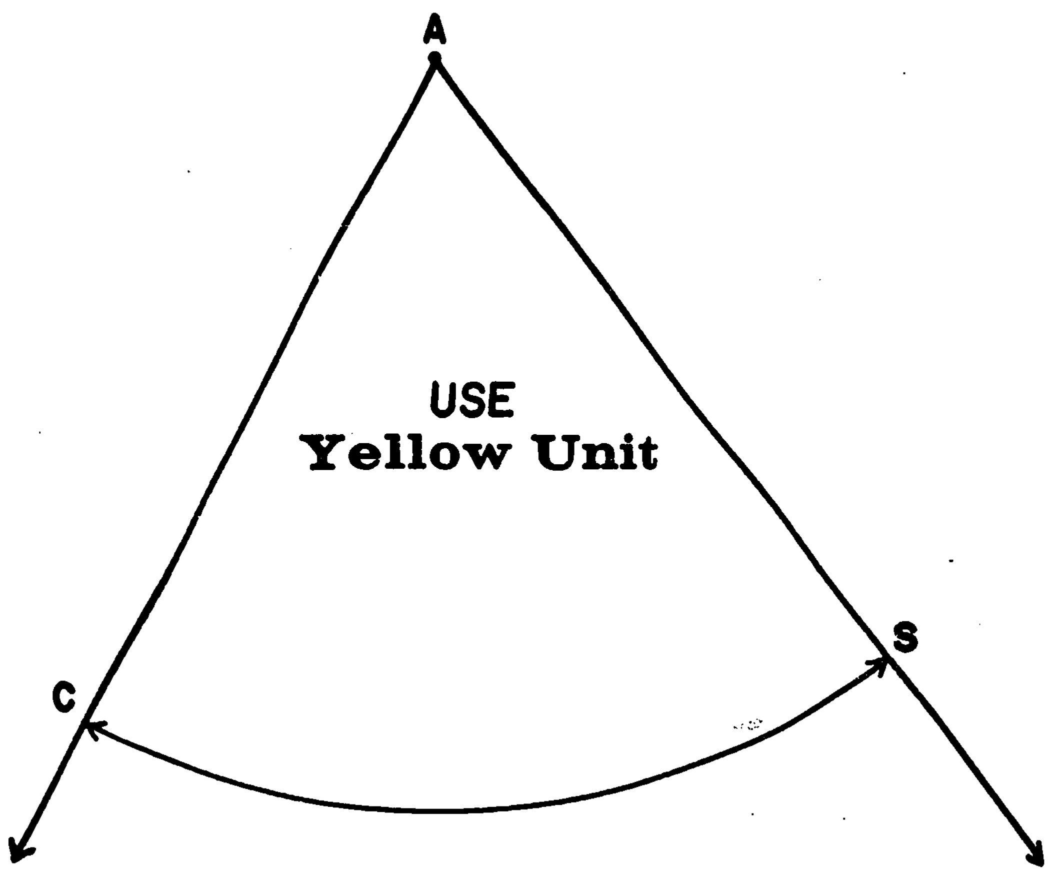
We write this: $3 \text{ units} < m(\angle DEF) < 4 \text{ units}$. This statement is read "the measure of $\angle$ DEF is greater than 3 and less than 4."

What is $m(\angle DEF)$ when measured to the nearest whole unit? (Use $\angle 1$ as the unit of measure.) You have to decide whether $m(\angle DEF)$ is closer to 3 units or 4 units. Determining to which of the two units $m(\angle DEF)$ is closer involves sighting, estimating, and judgment . In order to measure accurately, you must be able to make accurate guesses as to which of two units the measure is closer.



For use with CLASS ACTIVITY , p. 17.





For use with CLASS ACTIVITY , p. 17.

CLASS ACTIVITY

For this activity use the unit angle wedges that were used in lesson 1.

INSTRUCTIONS:

Measure the angles pictured on both sides of the insert between pages 16 and 17, using the unit given for each angle.

Record your results in the table below.

ANGLE MEASURED	UNIT OF MEASURE	COMPLETE THE STATEMENTS
1. $\angle$ DLH	Yellow	___ units < m($\angle$ DLH) < ___ units. To the nearest whole unit, m($\angle$ DLH) = ___ units.
2. $\angle$ DMP	Blue	___ units < m($\angle$ DMP) < ___ units. To the nearest whole unit, m($\angle$ DMP) = ___ units.
3. $\angle$ PLC	Red	___ units < m($\angle$ PLC) < ___ units. To the nearest whole unit, m($\angle$ PLC) = ___ units.
4. $\angle$ SAC	Yellow	___ units < m($\angle$ SAC) < ___ units. To the nearest whole unit, m($\angle$ SAC) = ___ units.

BECOMING MORE PRECISE . . . .

All measurements are approximations. A more precise measure of an angle is one in which a better approximation of the angle's size is obtained.

EXAMPLE

$\angle$ CAB has been measured to the nearest whole unit, using two different units of measure. Which unit gives the more precise measure (the better approximation of the size of $\angle$CAB)?

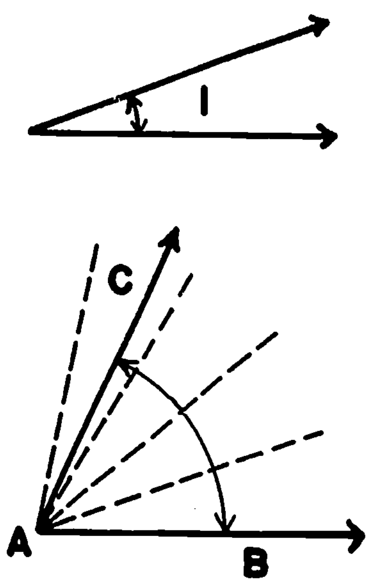


Figure 1

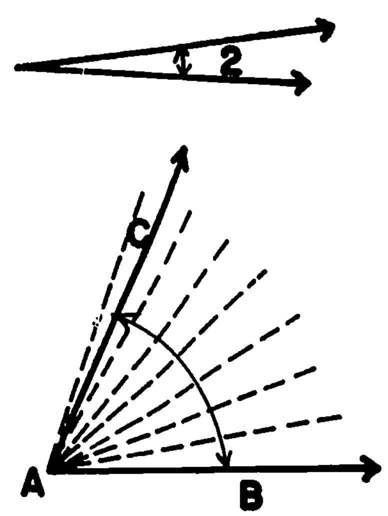


Figure 2

Using $\angle$ 1 as the unit of measure,...

3 units $<$ m($\angle$CAB) $<$ 4 units

Using $\angle$ 2 as the unit of measure,...

6 units $<$ m($\angle$CAB) $<$ 7 units

In Figure 1:

m($\angle$CAB) lies somewhere in the interval between 3 units and 4 units. The measure is located in an interval equal to the size of $\angle$ 1, the unit of measure.

In Figure 2:

m($\angle$CAB) lies somewhere in the interval between 6 units and 7 units. The measure is located in an interval equal to the size of $\angle$ 2, the unit of measure.

Which of the two measurements are more precise? The more precise measurement will give the better approximation of the angle size.

Since the interval is smaller in Figure 2, the measure obtained for $\angle$CAB, in Figure 2, is a better approximation of its actual size.

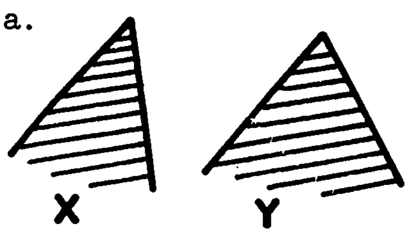
Therefore $\angle$ 2 gives a more precise measure than $\angle$ 1 as it locates m($\angle$ CAB) in a smaller interval.

PRECISION	— depends on the unit of measure. The smaller unit of measure, the more precise the measurement.

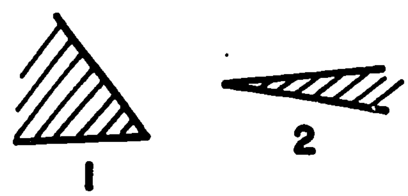
✔ **POINT**

1. Which unit in each pair will give the more precise measurement?

a.



b.



2. Suppose blobs and gobs are units for measuring angles. When $\angle$ 1 is measured to the nearest unit,...

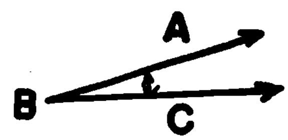
   2 blobs < m($\angle$ 1) < 3 blobs
   4 gobs < m($\angle$ 1) < 5 gobs

   a. Which is the larger unit of measure, a blob or a gob?

   b. Which unit will give the more precise measurement?

EXERCISES

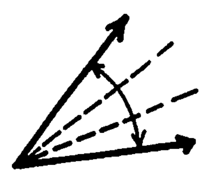
1. Suppose an angle with an opening as large as that of ∠ ABC has a measure of 1 unit.



ONE UNIT

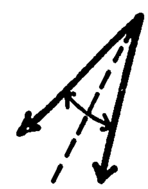
If the measure of ∠ ABC is one unit, what is the measure of the following angles?

a.



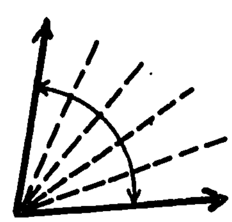
_____ **UNITS**

d.



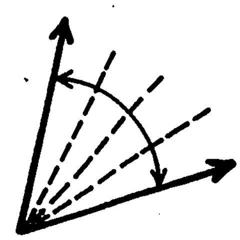
_____ **UNITS**

b.



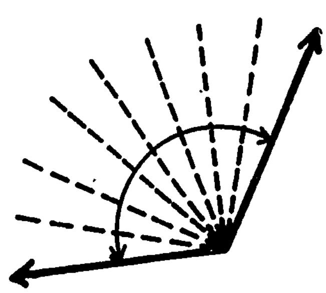
_____ **UNITS**

e.



_____ **UNITS**

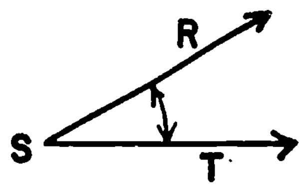
c.



_____ **UNITS**

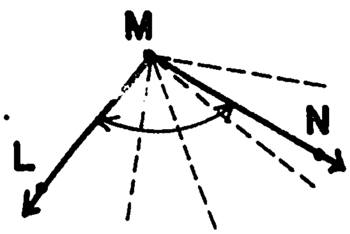
2. The measure of the angle in 1(c) is how many times the measure of the angle in 1(d)? _____ How many times the measure of the angle in 1 (e)? _____

3. Use $\angle RST$ as the unit angle. For each angle pictured, complete the blanks to make true statements.



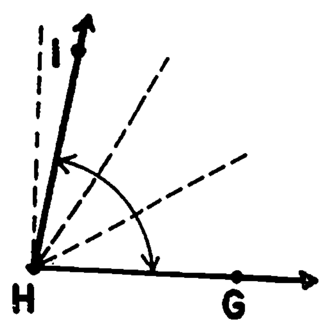
$m(\angle RST) = 1$ UNIT

a.



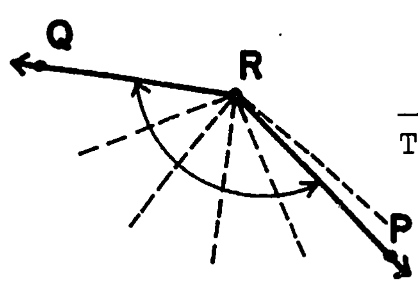
___ units $< m(\angle LMN) <$ ___ units.
To the nearest whole unit,
$m(\angle LMN) =$ ___ units.

b.



___ units $< m(\angle GHI) <$ ___ units.
To the nearest whole unit,
$m(\angle GHI) =$ ___ units.

c.



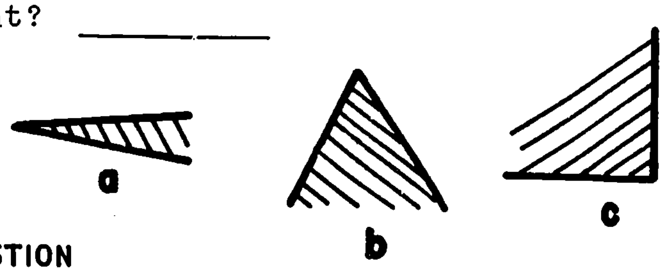
___ units $< m(\angle QRP) <$ ___ units.
To the nearest whole unit,
$m(\angle QRP) =$ ___ units.

4. Suppose zips and zaps are units for measuring angles. When $\angle TGC$ is measured,...

$$m(\angle TGC) = 10 \text{ zips}$$
$$m(\angle TGC) = 6 \text{ zaps}$$

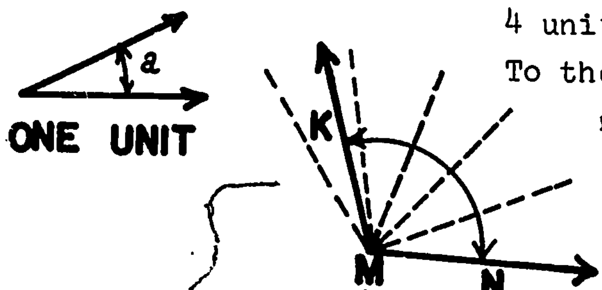
a. Which is the smaller unit of measure?
b. Which unit will give a more precise measurement?

5. Which of the following units will give the most precise measurement? ________



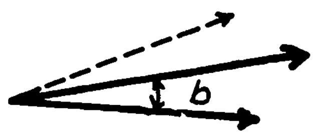
DISCUSSION QUESTION

1. Using ∠a as the unit of measure,



4 units < m(∠ NMK) < 5 units.
To the nearest whole unit,
m(∠ NMK) = 4 units.

Suppose a new unit of measure (∠ b) is selected which is $\frac{1}{2}$ the size of ∠ a.



Would the following statements be true for ∠ NMK? Why or why not?

8 units < m(∠ NMK) < 10 units.
To the nearest whole unit,
m(∠ NMK) = 8 units.

USING OTHER METHODS . . .

This booklet describes the most common method of measuring angles, using a unit angle as the unit of measure.

Another possibility for measuring angle size is discussed in the section entitled **CAN A RULER BE USED TO MEASURE ANGLES?** in **APPENDIX C** at the back of this booklet.

ANGLES FORMED BY ROTATIONS

In Lessons 1 and 2, angles were thought of as the union of two rays having a common endpoint. Another way to think of angles is to consider the amount of rotation that is made when an object turns.

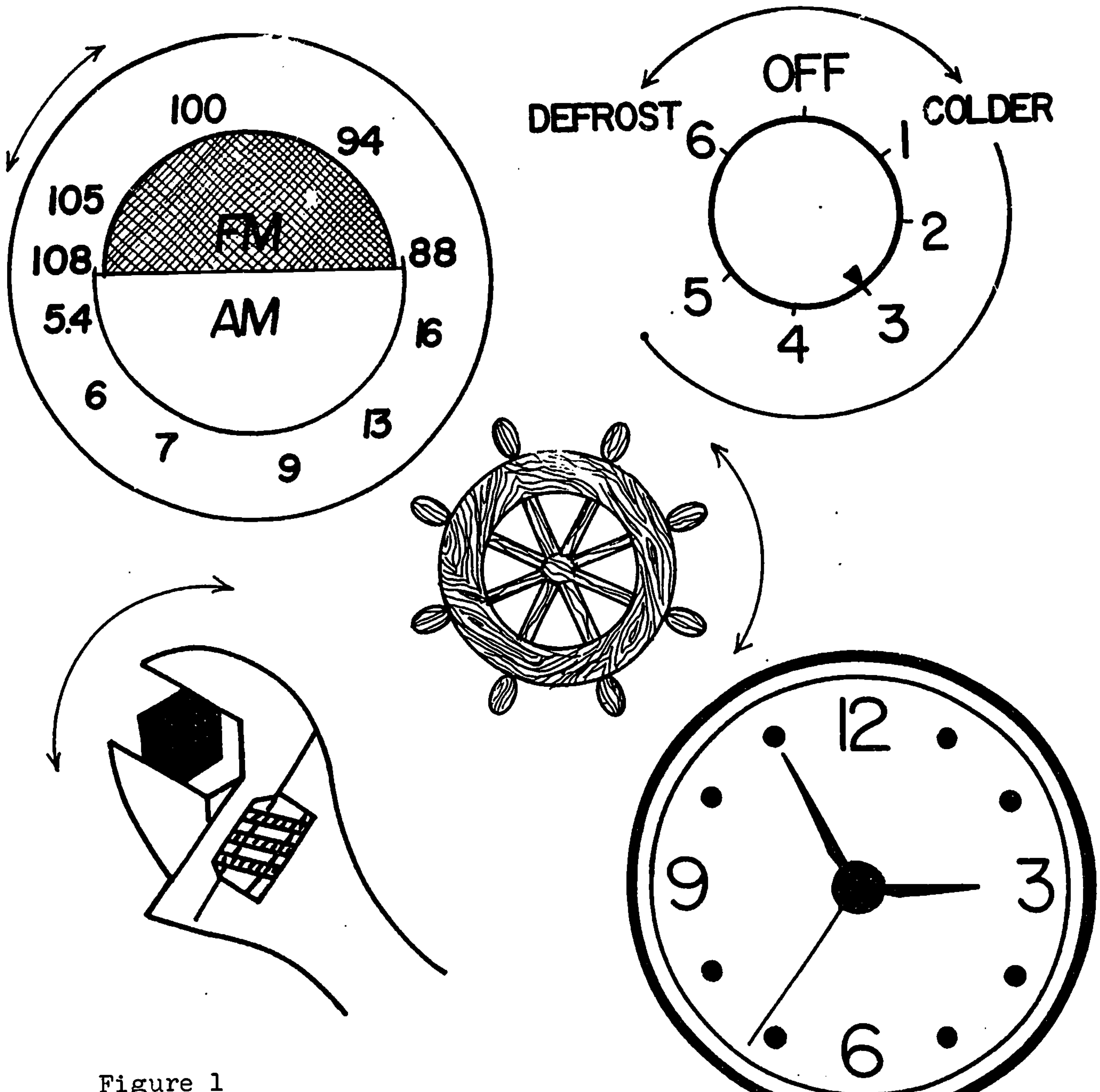


Figure 1

Lay a pencil down on your desk. Hold the eraser end firm and rotate the free end. Notice the amount of turn.

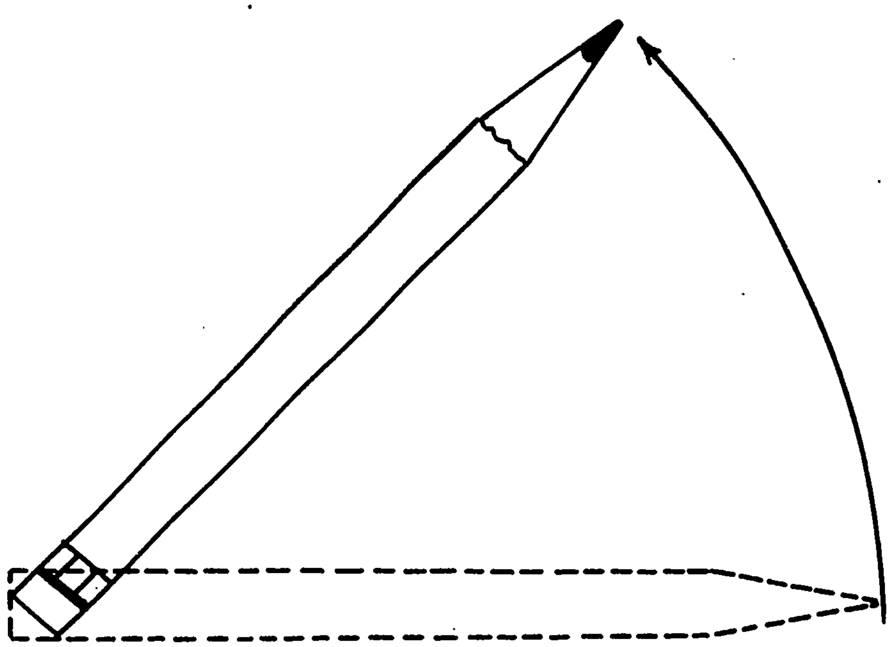


Figure 2

The diagram in Figure 2 indicates that the pencil has made about one-eighth of a turn.

A turn is one complete revolution.

The second hand on a clock makes <u>one turn</u> every 60 seconds. What part of a turn does it make in 15 seconds? In 30 seconds?

The measure of an angle swept out by a rotation is the amount of turn.

Thus the measure of the angle swept out by the second hand of a clock in 20 seconds is $\frac{1}{3}$ of a turn.

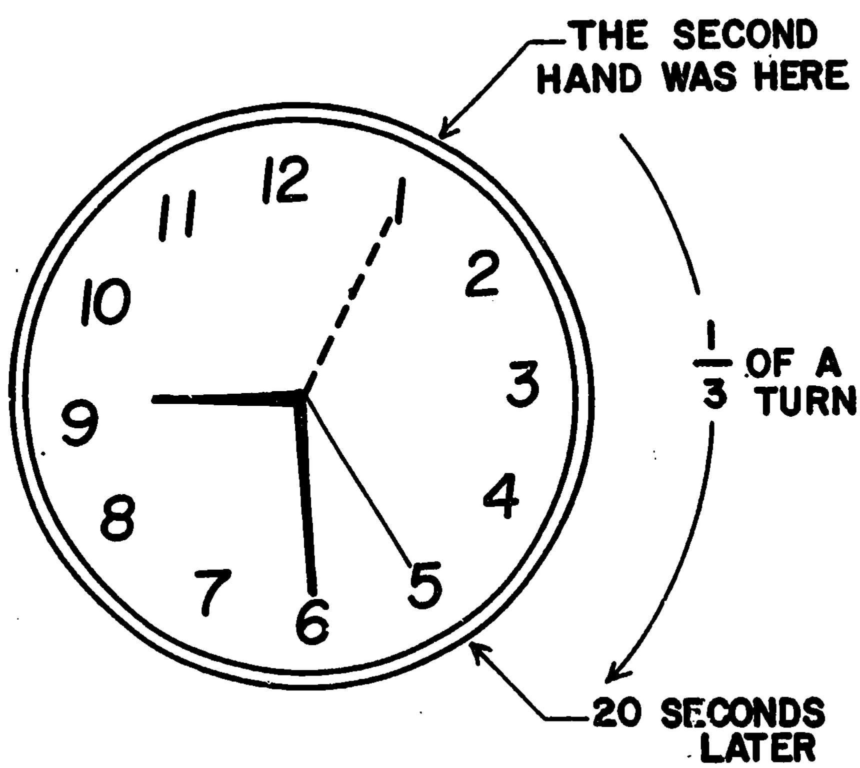


Figure 3

To determine the amount of turn you must keep track of the starting position.

What is the measure of the angle swept out by the minute hand in 45 minutes?

The circle provides a useful way of studying the angle swept out by an amount of turn.

A scale is provided in cases where the amount of turn must be measured carefully.

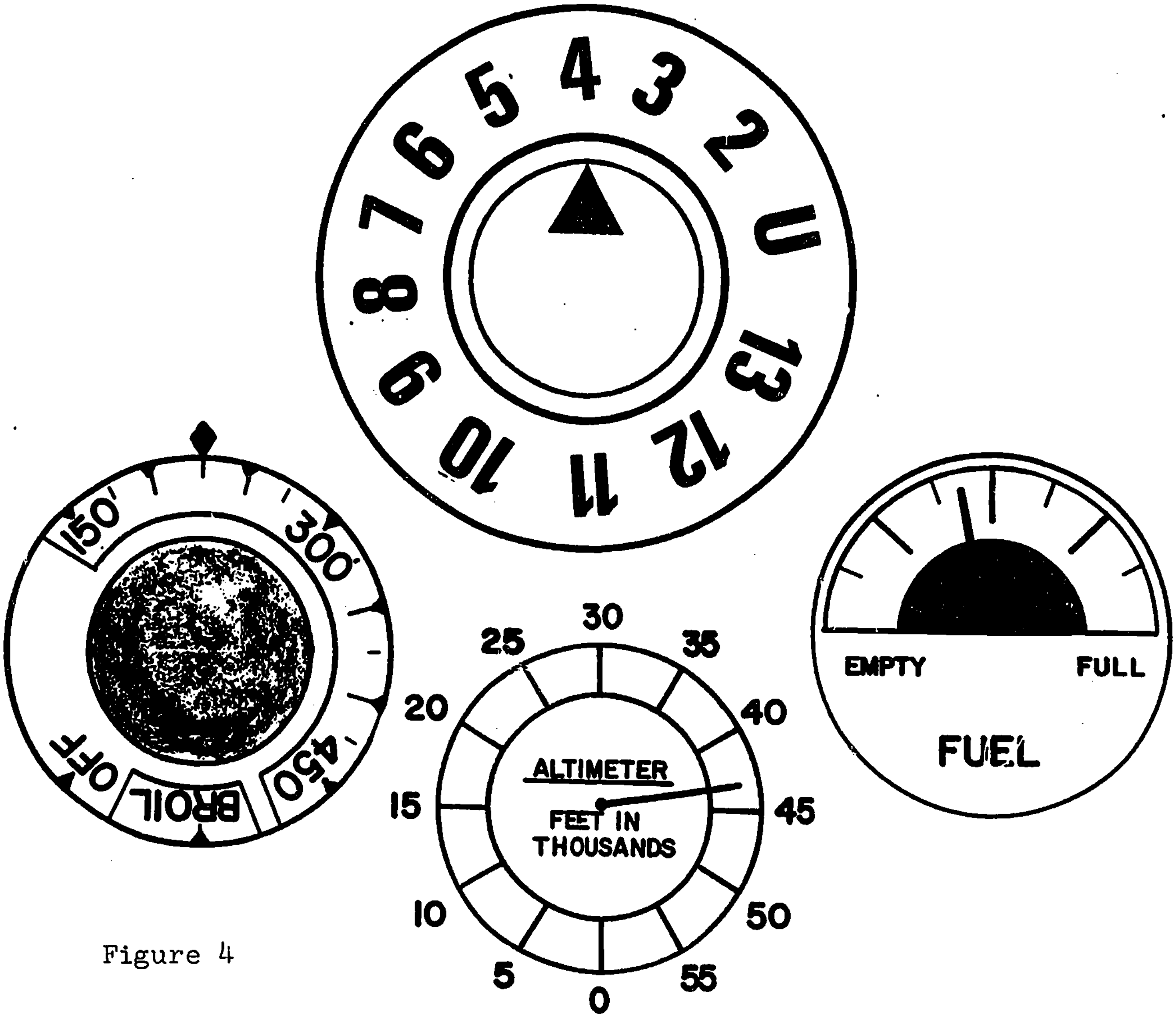


Figure 4

What part of a turn is necessary to change the T.V. from channel 2 to 7? Use both directions. What do you notice about the sum of the two fractions?

The circle in Figure 5 has been separated into 8 <u>congruent arcs</u>.

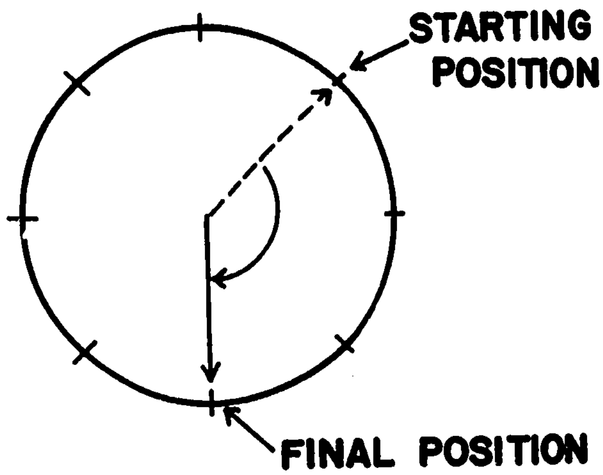


Figure 5

Each arc is what part of the circle?

The measure of the angle swept out is $\frac{3}{8}$ of a turn. Why?

The circle in Figure 6 has been separated into 12 congruent arcs.

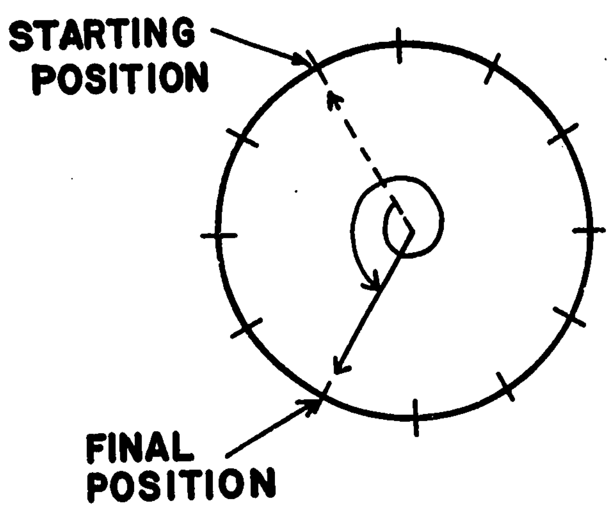


Figure 6

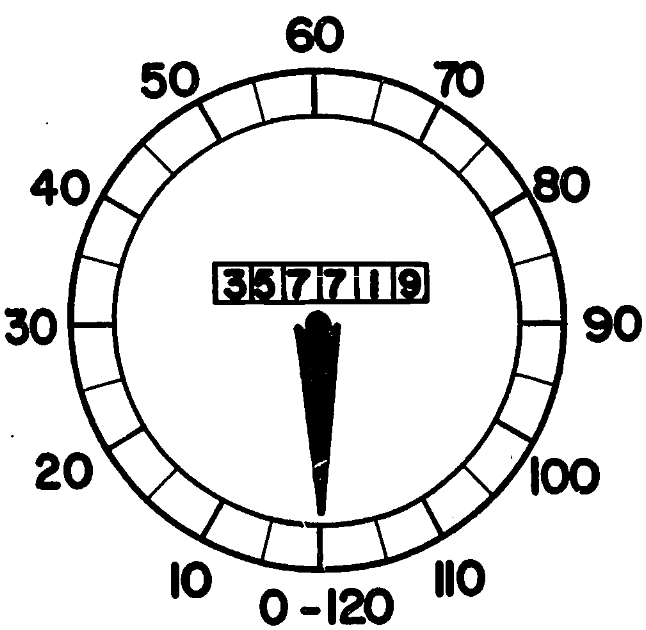
The measure of the angle swept out is $\frac{16}{12}$ turns. Why?

✓ POINT

1. What is a turn?
2. What is the measure of an angle swept out by a rotation?
3. What was the measure of an angle as described in Lessons 1 and 2?
4. Each arc of the circle in Figure 6, page 27, is what part of the circle?

EXERCISES

1. The hours marked on a circular clock face divide the circle into 12 congruent arcs. Each arc is what part of the circle?
2. What is the measure of the angle you turn your T.V. channel selector to go from channel 2 to channel 4? Use the shorter direction. (See Figure 4, page 26.)
3. What is the measure of the angle swept out by the hour hand in 7 hours?
4. Suppose an automobile has a speedometer with a circular dial.

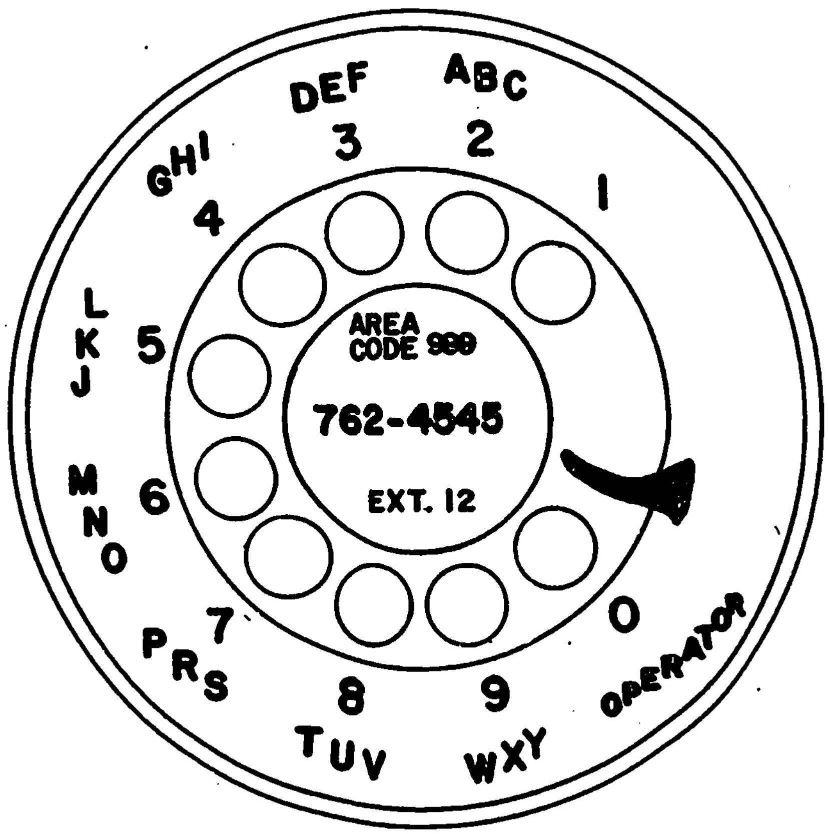


The dial is scaled from 0 mph to 120. Each mark represents 5 mph.

a) Each small arc is what part of the circle?

b) What is the measure of the angle swept out when accelerating from 0 to 40 mph?

5. Through approximately what size angle do you turn your phone dial when dialing the number 2? The number 5?



6. Through what size angle does the minute hand of a clock turn in:

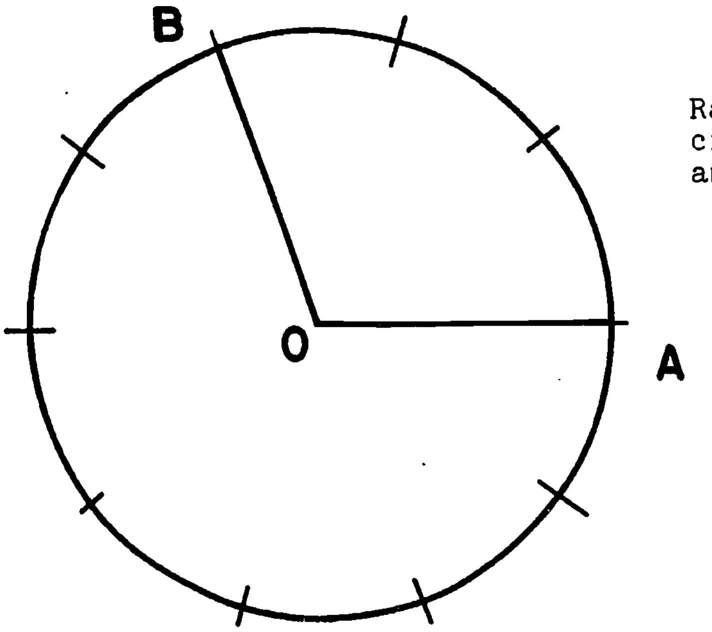
a) 1 hour b) 30 min. c) 20 min.
d) 3 hours e) 15 min. f) 30 seconds

7. One complete turn of the pedals (with no coasting) makes John's bike travel nearly 14 feet. What angle do the pedals turn through if the bike travels:

a) 42 feet b) 10 feet c) 21 feet

CENTRAL ANGLES

An angle determined by two radii (pronounced ray-dee-i) of a circle is called a central angle.



Radii OA and OB of circle O form central angle AOB.

Figure 1

The measure of the angle determined by the two radii is the amount of turn necessary to make the two radii coincide.

What is the measure of $\angle$ AOB? This question cannot be answered until there is an indication of the direction and amount of rotation that is intended.

For Example: (See Figure 1, page 30.)

Case 1. Hold OB fixed and turn OA in a counterclockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{3}{10} \text{ of a turn.}$$

Case 2. Hold OB fixed and turn OA in a clockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{7}{10} \text{ of a turn}$$

Case 3. Hold OA fixed and turn OB in a clockwise direction to meet OB. Then

$$m(\angle AOB) = \frac{3}{10} \text{ of a turn.}$$

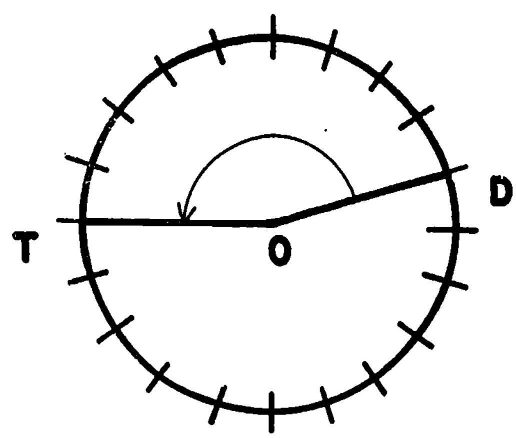
Case 4. Hold OA fixed and turn OB in a counterclockwise direction to meet OA. Then

$$m(\angle AOB) = \frac{7}{10} \text{ of a turn.}$$

What do you notice about the sum of the fractions in Case 1 and Case 2?

To avoid the confusion caused by the various ways in which a rotation can be made to make the radii coincide, we use an arrow.

The arrow in Figure 2 indicates how the rotation is made.



The arrow indicates how OD is rotated to meet OT. The direction is counterclockwise and the amount of turn is $\frac{9}{20}$.

Figure 2

$$m(\angle DOT) = \frac{9}{20} \text{ of a turn}$$

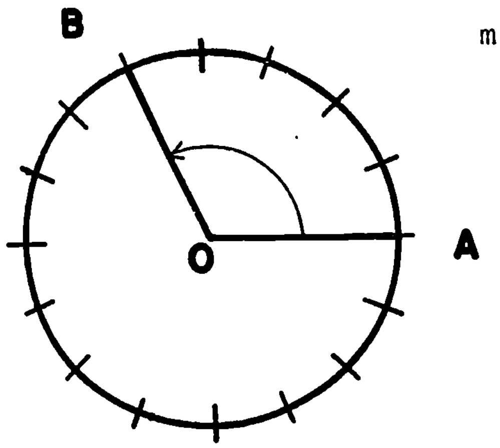
✓ POINT

1. What are the sides of a central angle?
2. Where is the vertex of a central angle?
3. How is the measure of a central angle determined?

EXERCISES

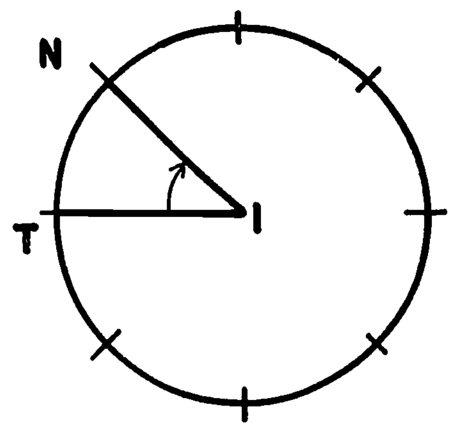
1. Imagine that you are looking down at the Earth from the North Pole. Is the Earth turning in a clockwise or a counterclockwise direction?

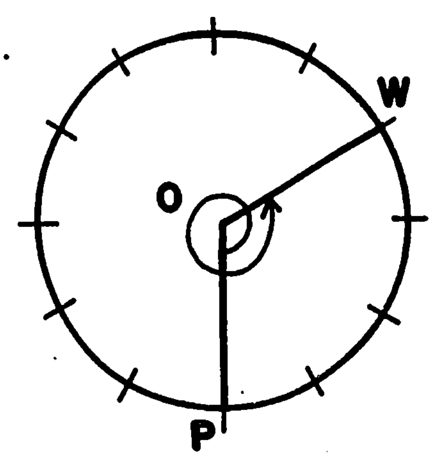
2-6. Determine the measure of each of the following central angles.

2. 

m(∠AOB) = ___ of a turn.

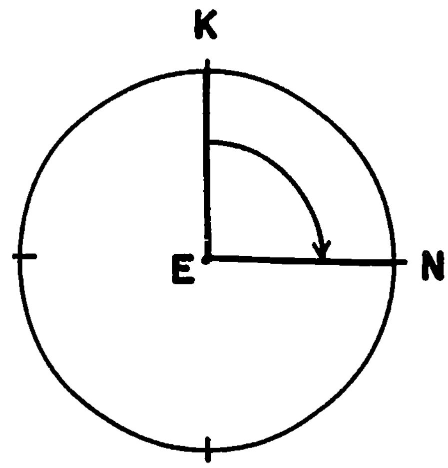
3. m(∠TIN) = ___



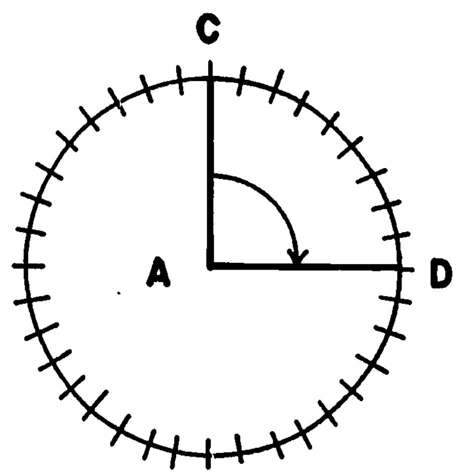
4. 

m(∠POW) = ___

5. m(∠ KEN) = ____



6.

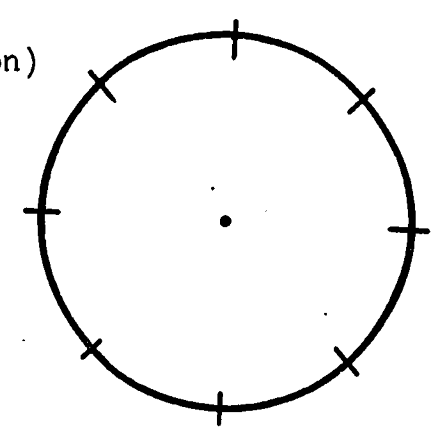


m(∠CAD) = ____

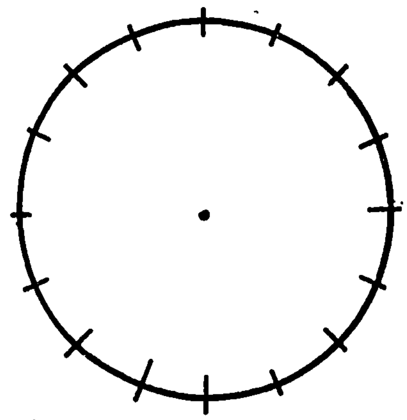
7. Which of the Exercises 2-6, show a central angle formed by a clockwise rotation?

8-10. Draw a central angle having the given measure and direction.

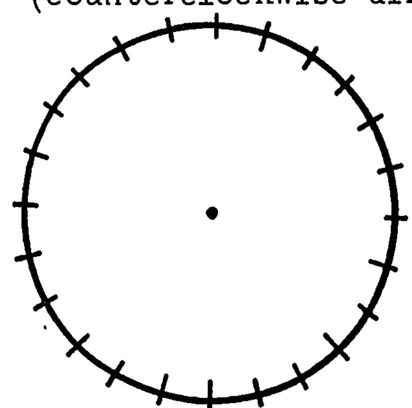
8. $\frac{3}{8}$ of a turn. (clockwise direction)



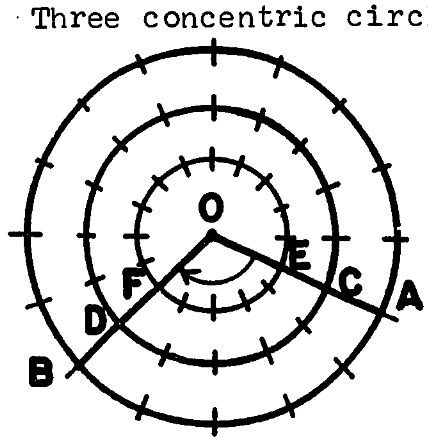
9. $1\frac{5}{16}$ turns. (counterclockwise direction)



10. $\frac{3}{12}$ of a turn. (counterclockwise direction)



11. Three concentric circles are shown below.



a) How many congruent arcs are marked off on each circle?

b) How do the lengths of the arcs on the three circles compare?

c) Give the measure of each of the three central angles.

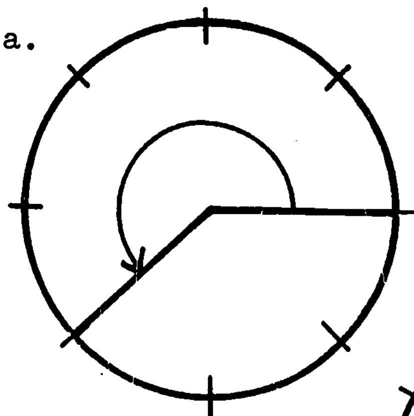
m(∠AOB) = ____

m(∠COD) = ____

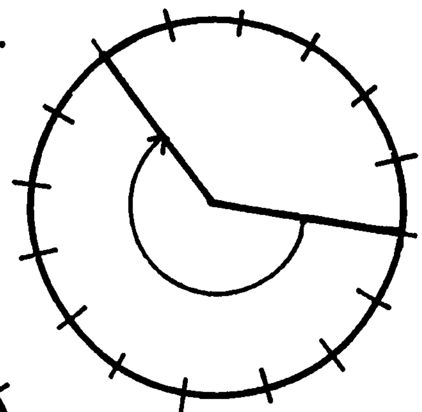
m(∠EOF) = ____

12. Three central angles are shown below. Give the measure of each angle. What do you conclude about the measures of these three angles?

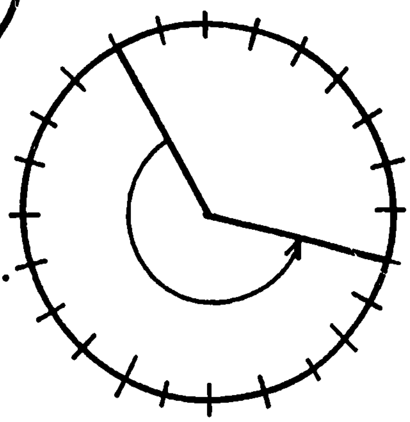
a.



b.



c.



13. A set of fractions is given in each exercise below. In each set, circle the fractions which are equivalent.

a) $\frac{1}{2}, \frac{6}{10}, \frac{16}{32}, \frac{40}{20}$.

b) $\frac{2}{7}, \frac{2}{6}, \frac{4}{12}, \frac{5}{15}$.

c) $\frac{15}{20}, \frac{1}{3}, \frac{9}{12}, \frac{1}{8}$.

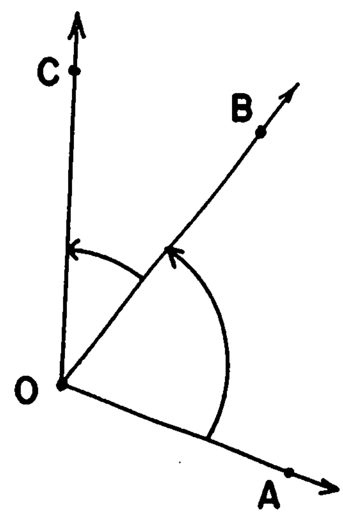
d) $\frac{1}{2}, \frac{6}{48}, \frac{2}{3}, \frac{5}{40}$.

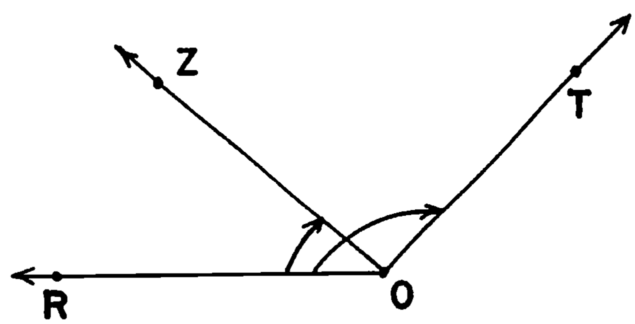
e) $\frac{10}{15}, \frac{2}{6}, \frac{2}{3}, \frac{6}{9}$.

Adding and Subtracting Angles

1. The measure of angle AOB is $\frac{1}{5}$ of a turn.
   The measure of angle BOC is $\frac{1}{10}$ of a turn.

   The measure of angle AOC is _____ of a turn.



2. 

$m(\angle ROT) = \frac{3}{8}$ of a turn

$m(\angle ROZ) = \frac{1}{9}$ of a turn

$m(\angle ZOT) =$ __ of a turn

3. Ray MN is rotated $\frac{11}{24}$ of a turn in a counterclockwise direction. Then MN is rotated $\frac{1}{6}$ of a turn in a clockwise direction. How much farther will MN have to be rotated in a clockwise direction before it returns to its original position?

4-10 Solve for N

4. $\frac{2}{5} + \frac{2}{3} = N$
5. $1\frac{1}{6} + \frac{1}{2} = N$
6. $\frac{1}{3} + \frac{4}{5} = N$
7. $\frac{3}{4} - \frac{1}{8} = N$
8. $1 + N = \frac{13}{9}$
9. $2 + N = \frac{17}{6}$
10. $1\frac{3}{8} + 1\frac{1}{3} = N$

THE SIZE OF AN ANGLE

In Lesson 1 it was made clear that the size of an angle does not depend on the length of its sides. The five angles pictured in Figure 1 are all the same size.

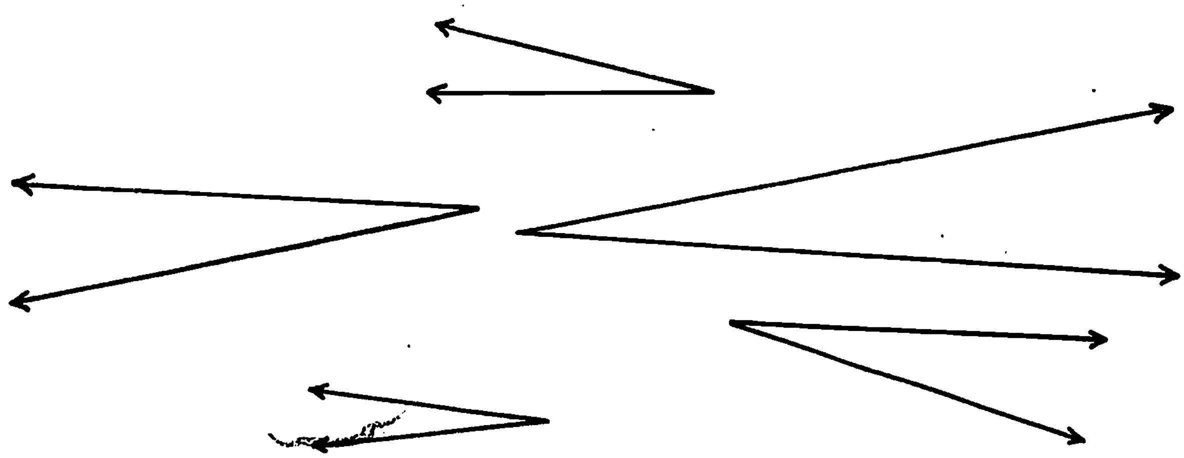
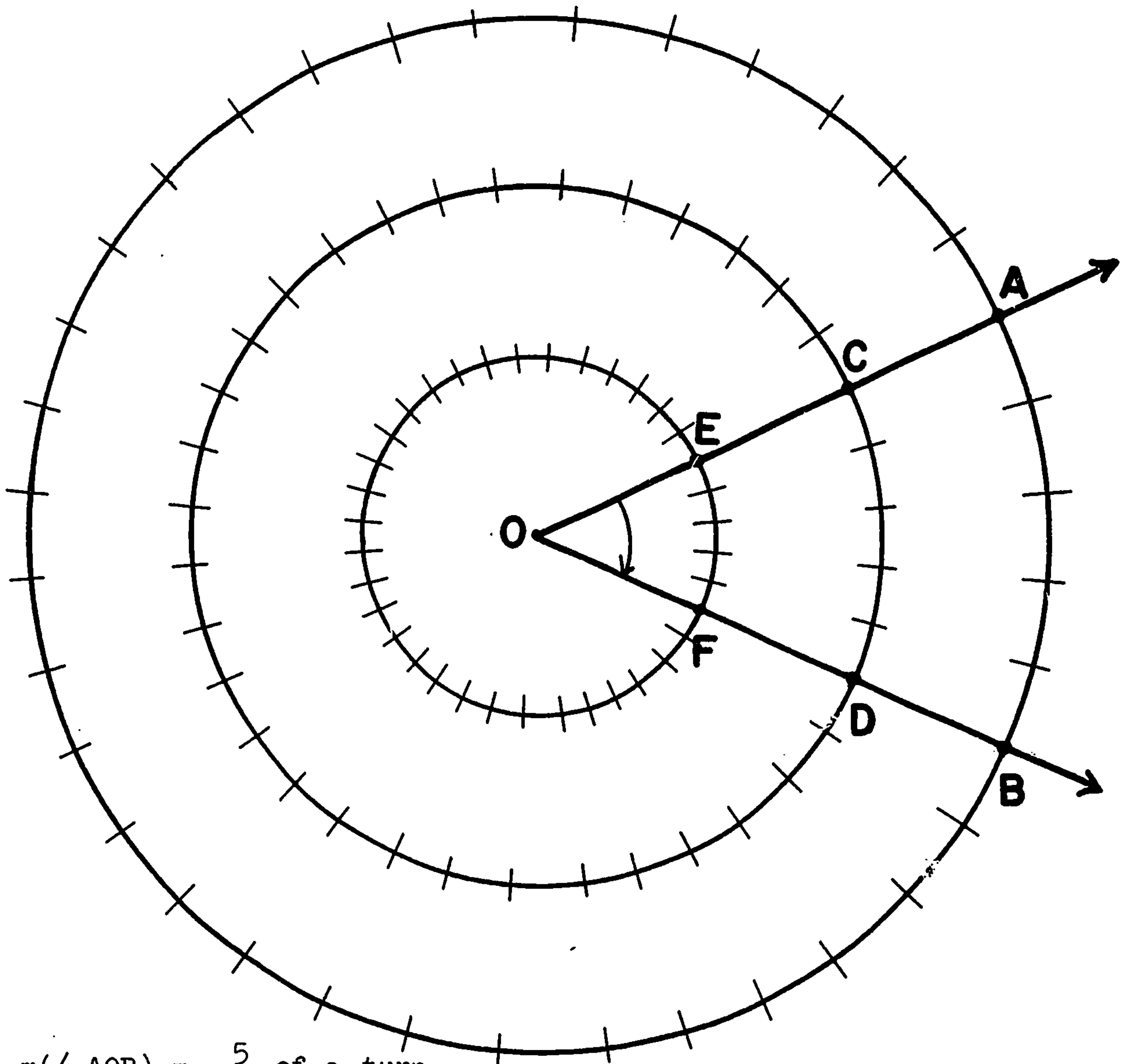


Figure 1

Remember that the sides of an angle can be thought of as rays. A ray can be extended indefinitely.

In Lessons 3 and 4 circles divided into congruent arcs were used to help measure the size of angles. Does the measure of an angle depend on the size of the circle used to measure it?

Each of the three circles in Figure 2 have been divided into 36 congruent arcs. Notice that the larger the circle--the longer the arc.



$m(\angle AOB) = \frac{5}{36}$ of a turn

$m(\angle COD) = \frac{5}{36}$ of a turn

$m(\angle EOF) = \frac{5}{36}$ of a turn

Figure 2

Notice that, in each circle, the angle cuts off the same number of arcs.

THE SIZE OF AN ANGLE IS DETERMINED BY THE AMOUNT OF TURN

The size of an angle does not depend on the size of the circle used to measure it.

The circle in Figure 3 has been separated into 20 congruent arcs. The sides of the angle, in the figure, do not lie on any of the marks. To measure this angle we must make an approximation.

Example 1. Measure $\angle$ TOM to the nearest whole unit.

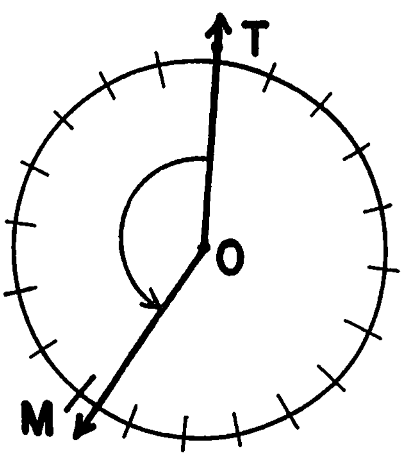


Figure 3

Since the circle is divided into 20 congruent arcs, the unit is $\frac{1}{20}$ of a turn.

$\angle$ TOM cuts off 8 units plus a little more.

$$m(\angle TOM) \approx 8(\tfrac{1}{20} \text{ of a turn})$$
$$\approx \frac{8}{20} \text{ of a turn.}$$

But to the nearest unit

$$m(\angle TOM) = \frac{8}{20} \text{ of a turn.}$$

Example 2. Measure $\angle$RAT.

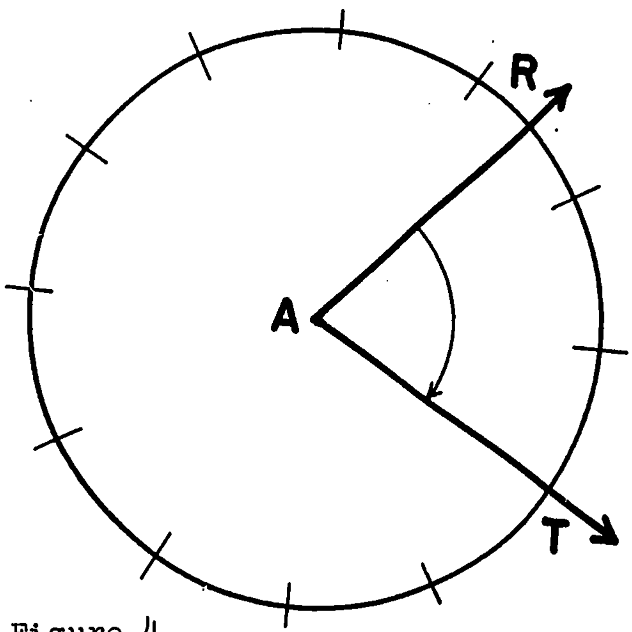


Figure 4

$\angle$RAT cuts off less than 3 of the arcs. We write

$$m(\angle RAT) < \frac{3}{12} \text{ of a turn.}$$

$\angle$RAT cuts off more than 2 of the arcs. We write

$$m(\angle RAT) > \frac{2}{12} \text{ of a turn.}$$

The measure of $\angle$ RAT lies somewhere in an interval between $\frac{2}{12}$ of a turn and $\frac{3}{12}$ of a turn.

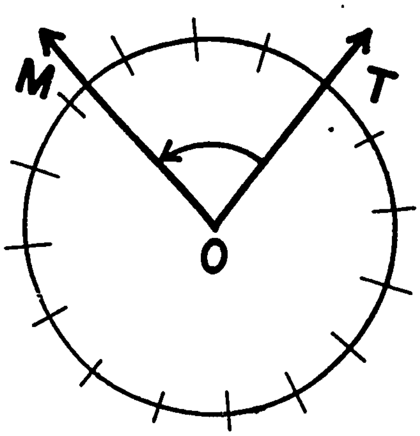
$$\frac{2}{12} \text{ of a turn} < m(\angle \text{RAT}) < \frac{3}{12} \text{ of a turn.}$$

Notice that the length of this interval is the unit used to measure $\angle$ RAT.

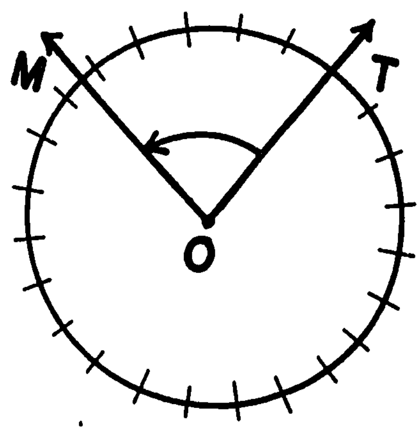
It appears (looking at Figure 4) that the measure of $\angle$ RAT is nearer $\frac{3}{12}$ of a turn. The measure of $\angle$ RAT to the nearest whole unit is $\frac{3}{12}$ of a turn.

All measurements are approximations. A more precise measure of an angle is one in which a better approximation is obtained.

To have a more precise measure of an angle means the measure lies within a smaller interval.



5a



5b

Figure 5

In Figure 5a:

$$\frac{3}{16} \text{ of a turn} < m(\angle \text{TOM}) < \frac{4}{16} \text{ of a turn.}$$

The interval is $\frac{1}{16}$ of circle A.

In Figure 5b:

$$\frac{5}{24} \text{ of a turn} < m(\angle \text{TOM}) < \frac{6}{24} \text{ of a turn.}$$

The interval is $\frac{1}{24}$ of circle A.

The smaller the interval that the measure lies in, the better the approximation.

Thus, $\frac{5}{24}$ of a turn is a better approximation of the measure of $\angle$TOM than $\frac{4}{16}$ of a turn.

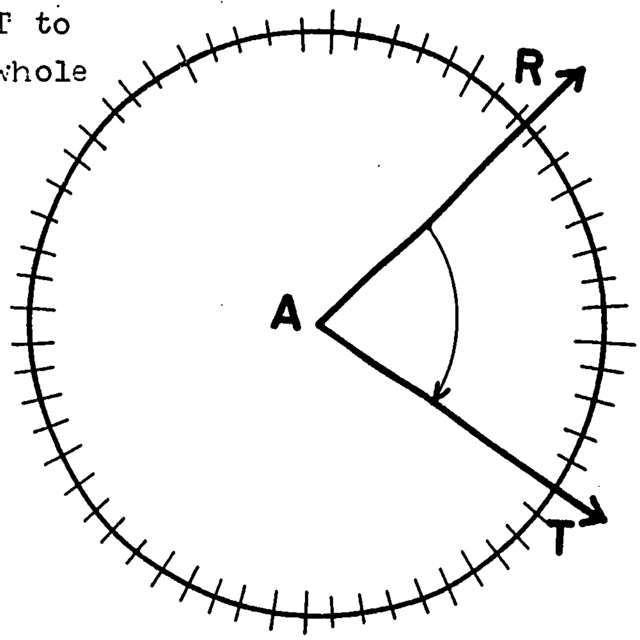
The measure of an angle can always be placed in an interval whose length is equal to the size of the unit of measure.

Dividing a circle into more and more congruent arcs makes the size of the unit of measure smaller.

The smaller the unit, the more precise the measure.

What can be done to the circle in Figure 4, page 40, to get a more precise measure of $\angle$RAT?

Example 3. Measure $\angle RAT$ to the nearest whole unit.



Solution:

$m(\angle RAT) = \frac{13}{60}$ of a turn.

Figure 6

The unit used in Example 2, page 40, is $\frac{1}{12}$ of circle A. The unit used in Example 3 above is $\frac{1}{60}$ of circle A.

$$\frac{1}{60} \text{ of circle A} < \frac{1}{12} \text{ of circle A.}$$

thus, $\frac{13}{60}$ of a turn is a better approximation of $\angle RAT$ than $\frac{3}{12}$ of a turn.

Dividing the same circle into more congruent arcs gives a more precise measure of the angle.

✓ **POINT**

1. What can be done to circle O in Figure 3, page 40, in order to obtain a more precise measure of $\angle TOM$?

2. The "size" of an angle depends on which of the following?

   a. The length of its sides.

   b. The amount of opening between its sides.

   c. The "size" of the circle used to measure it.

   d. The "size" of the unit angle used to measure it.

3. A circle used to measure an angle is divided into 40 congruent arcs. What is the unit being used to measure the angle?

4. Explain how dividing a circle into more congruent arcs gives a more precise measure of the angle.

EXERCISES

1. Circle R is divided into 18 congruent arcs. In using Circle R to measure an angle to the nearest unit, what is the unit? (Use amount of turn.)

2. Circle O is divided into 360 congruent arcs.

   a) In using circle O to measure an angle to the nearest unit, what is the unit?

   b) Besides a part of a turn, what other name is given to this unit?

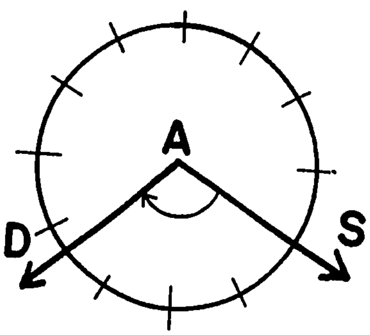
3. Which will give the more precise measure of an angle:

   a) A circle divided into 50 congruent arcs or

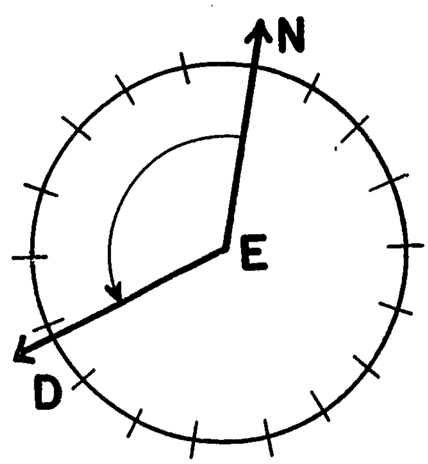
   b) the same circle divided into 100 congruent arcs?

4-6. Measure each of the following angles to the nearest whole unit. Use the indicated unit.

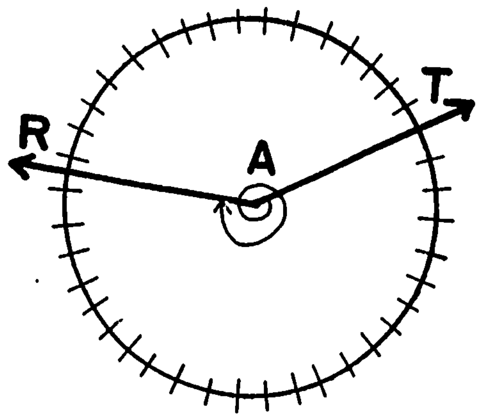
4. m(∠SAD) = ____ of a turn



5. m(∠NED) = ____

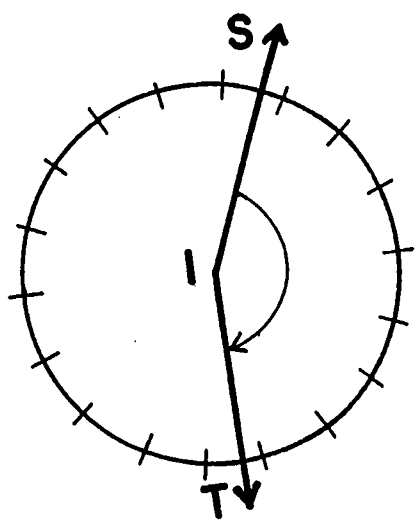


6. m(∠TAR) = ____

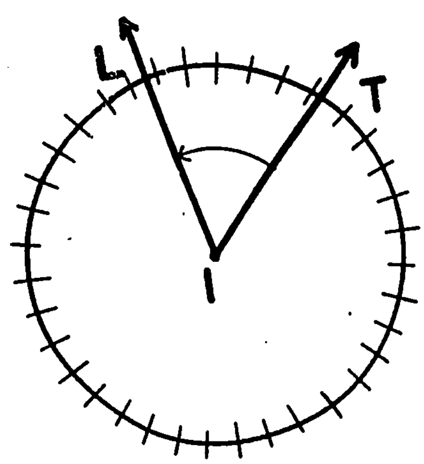


7-9. Estimate the measure of each of the following angles to the nearest whole unit.

7. m(∠SIT)= ____ of a turn.

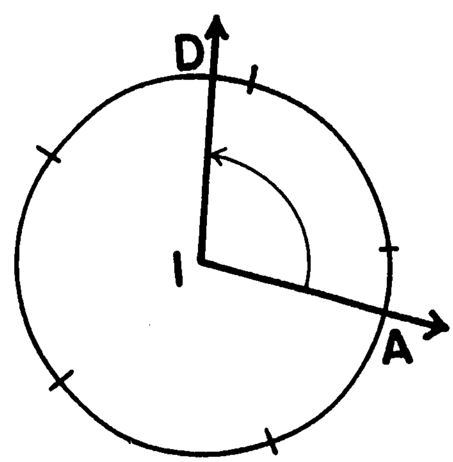


8.



m(∠TIL)= ____

9. m(∠AID) = ____



LESSON 6

WHAT IS A DEGREE?

When an angle is formed by the rotation of a ray, the measure of the angle is the amount of turn.

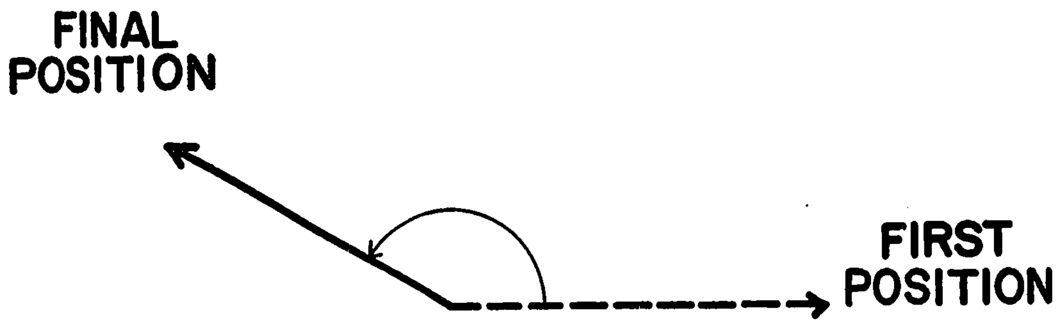


Figure 1

In Lessons 3, 4 and 5, the circle was an aid in determining the amount of turn.

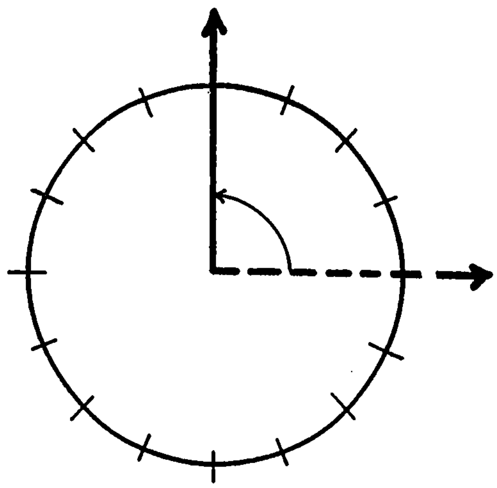
A REVIEW (Complete the following statements.)

(1) The vertex of the angle was located at ______________.

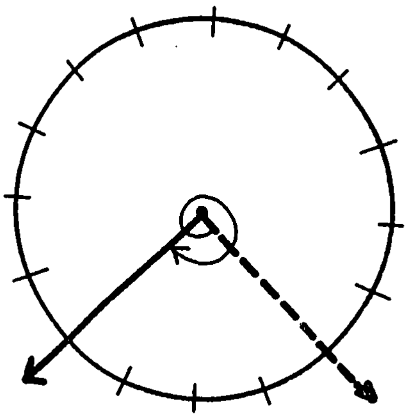
(2) The circle was divided into ______________________.

(3) An arrow indicated ___________________________.

(4) A ratio of the number of arcs "swept out" by one side of the angle to ______________________ is a measure of the angle.



2a



2b

Figure 2

Example 1. Figure 2a shows an angle formed by a rotation in a counterclockwise direction. The amount of rotation is less than a turn. Since the circle is divided into 16 congruent arcs, the unit for measuring the amount of rotation is $\frac{1}{16}$ of a turn. The measure of the angle is $\frac{4}{16}$ of a turn.

Example 2. Figure 2b shows an angle formed by a rotation in a clockwise direction. The amount of rotation is greater than a turn. The measure of the angle is $\frac{20}{16}$ or $1\frac{4}{16}$ turns.

We call a quarter turn a <u>right angle.</u> How many right angles are there in a complete turn?

The angle swept out in Figure 2a is a right angle. How many right angles were swept out by the rotation shown in Figure 2b?

The angle swept out by a half turn also has a special name. It is called a straight angle. How many quarter turns are there in a straight angle?

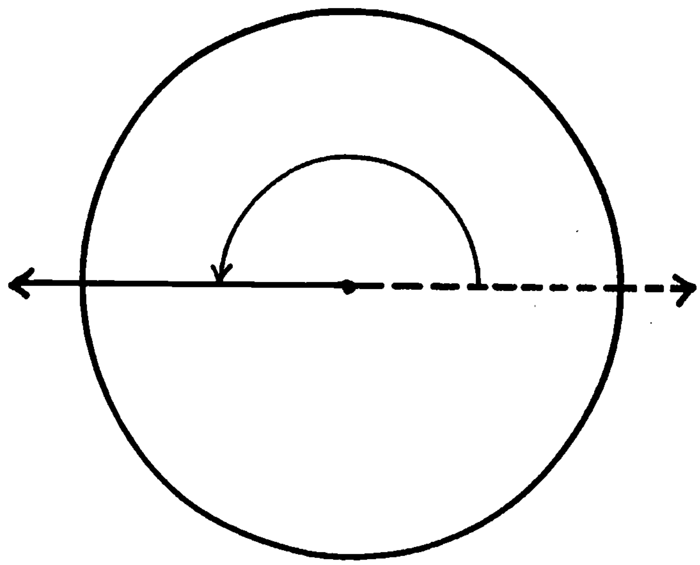


Figure 3

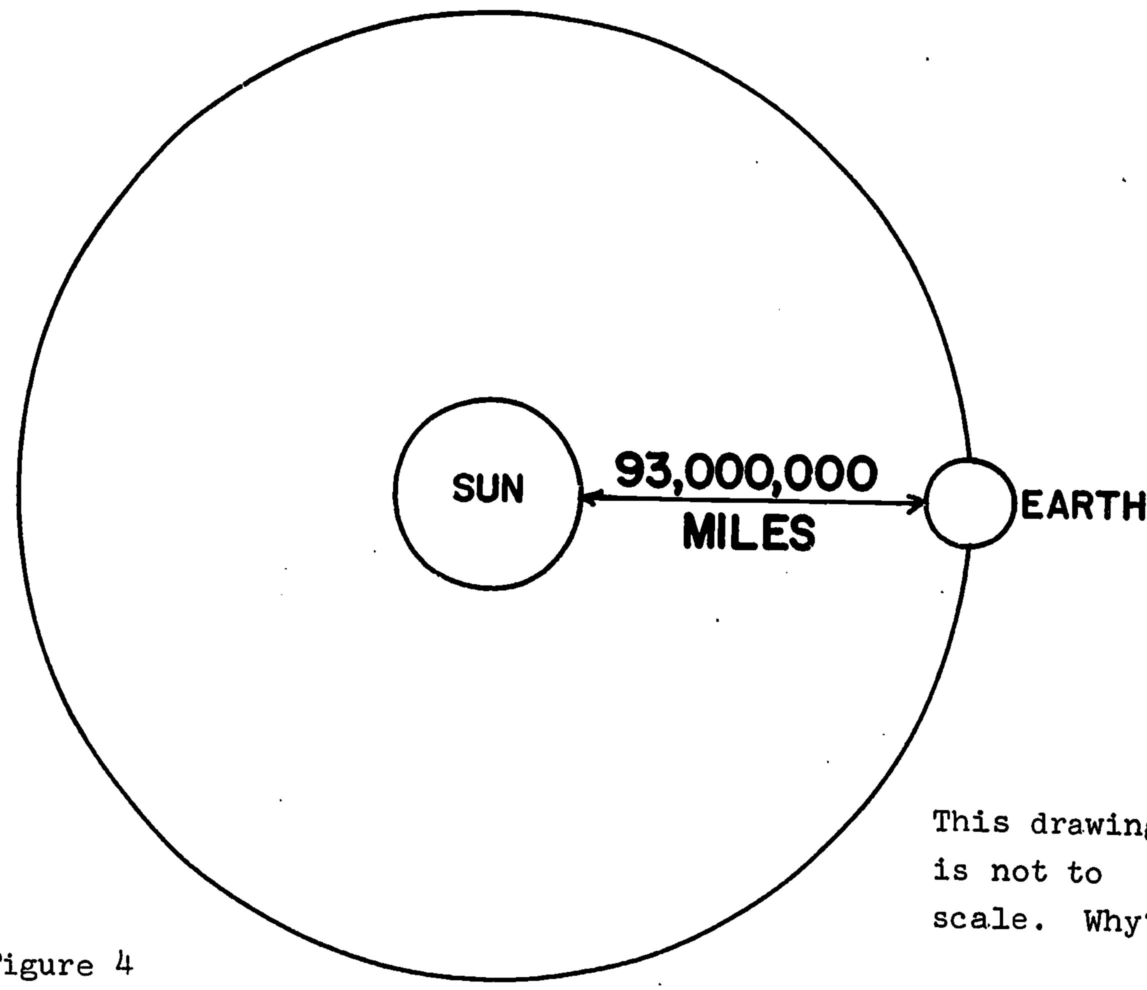
From previous work in mathematics you may remember that the measure of a right angle is 90 degrees.

A degree is the most commonly used unit for measuring angles.

What is a degree and where did it come from?

The degree is related to arcs of a circle. We inherited the degree from the ancient Sumerians who lived near the Tigris and Euphrates Rivers several thousand years ago. The Sumerians used a calendar of 12 months, each month having 30 days.

The Earth travels around the sun once each year. The path of its travel is almost circular.



This drawing is not to scale. Why?

Figure 4

If the year were exactly 360 days (every so often the Sumerians had to add extra days to their year), the Earth in its annual passage around the sun would pass through one degree per day.

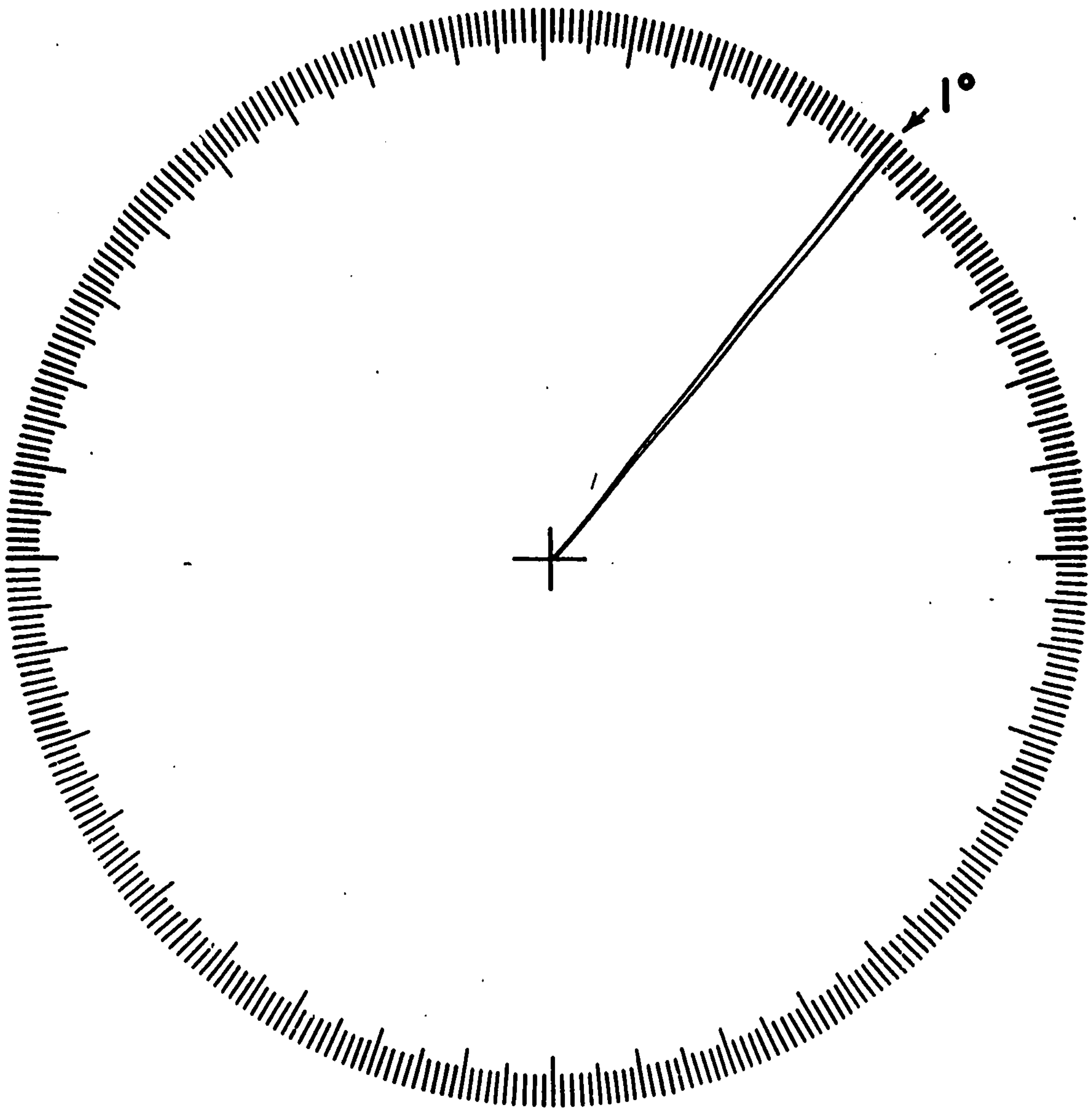
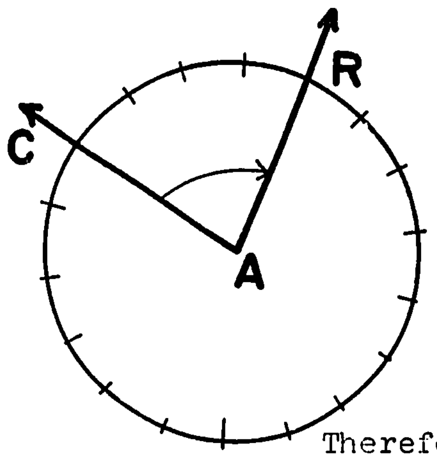


Figure 5

The circle in Figure 5 is divided into 360 congruent arcs. Each arc determines a central angle of one degree.

$$\frac{1}{360} \text{ of a turn} = 1 \text{ degree}$$

Example 3. Give the measure of $\angle CAR$ as a part of a turn and in degrees.



$m(\angle CAR) = \frac{4}{18}$ of a turn.

$$\frac{4}{18} = \frac{x}{360}$$

$$\frac{4 \cdot 20}{18 \cdot 20} = \frac{80}{360}.$$ Thus,

$x = 80$.

Therefore, $m(\angle CAR) = \frac{80}{360}$ of a turn and

$\frac{80}{360}$ of a turn $= 80^\circ$.

The symbol "$^\circ$" is read degrees.

Example 4. How many degrees is $\frac{1}{3}$ of a turn?

There are 360° in one turn.

$\frac{1}{3}(360^\circ) = 120^\circ$

Example 5. Convert 135° to the corresponding fractional amount of turn.

$$\frac{135^\circ}{360^\circ} = \frac{3 \cdot 45^\circ}{8 \cdot 45^\circ} = \frac{3}{8}$$

Answer: $135^\circ = \frac{3}{8}$ of a turn.

✓POINT

1. What special name is given to angles formed by
   (a) one-quarter of a turn?
   (b) one-half turn?

2. What makes a central angle different from angles in general?

3. A degree is what part of a turn?

4. Explain how a degree is determined.

EXERCISES

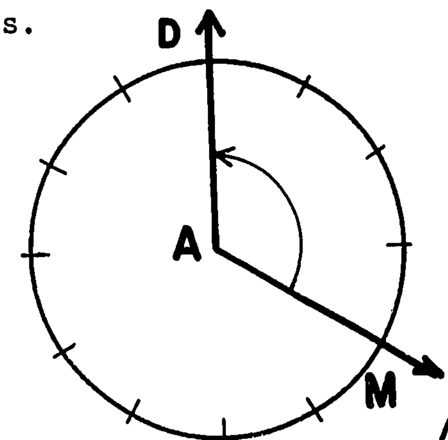
1. How many degrees in:

   a) $\frac{1}{4}$ of a turn   b) $\frac{1}{2}$ of a turn   c) 1 turn

   d) $\frac{1}{3}$ of a turn   e) $1\frac{3}{8}$ turns   f) $3\frac{1}{4}$ turns

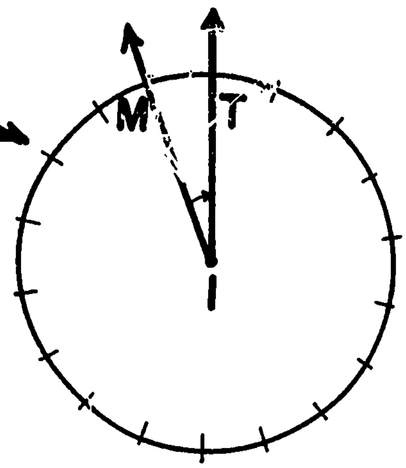
2. How many degrees in
   a) a right angle?
   b) a straight angle?

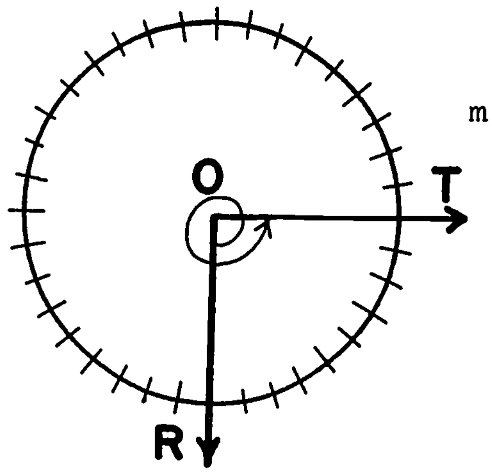
3. Give the measure of each of the following angles in degrees.

   a) 

   m(∠MAD) = ___°

   b) m(∠MIT) = ____



c) 

m(∠ROT) = ______

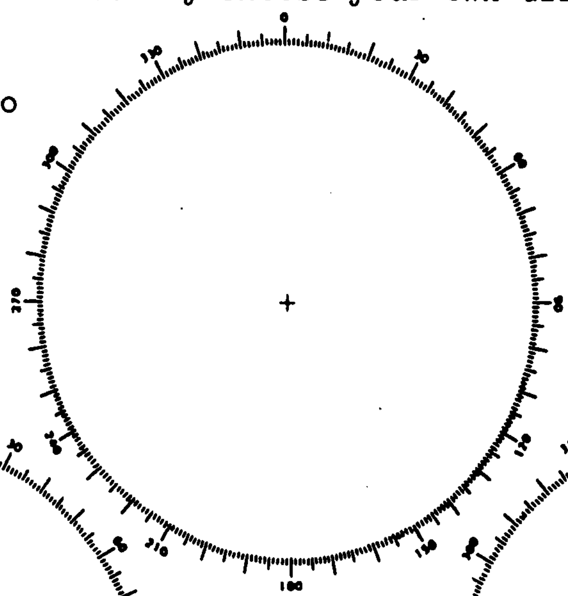
4. Convert the given angle measure from degrees to the corresponding fractional amount of turn.

a) 180° b) 30° c) 270°

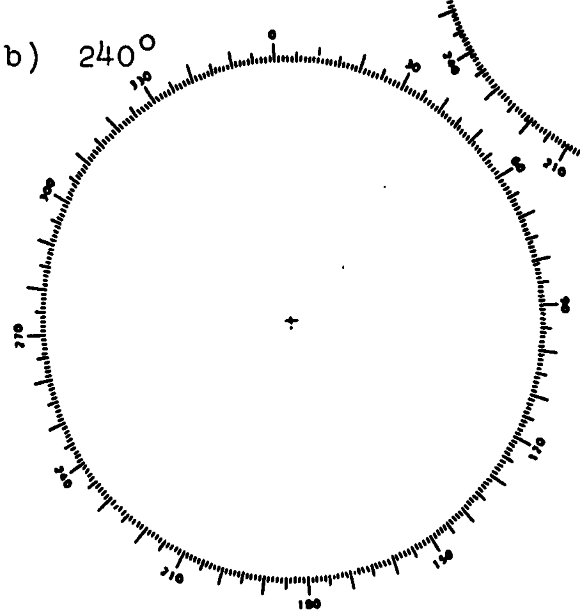
d) 45° e) 720° f) 1°

5. For each of the following, draw a central angle of the size given. You may choose your own direction of rotation.

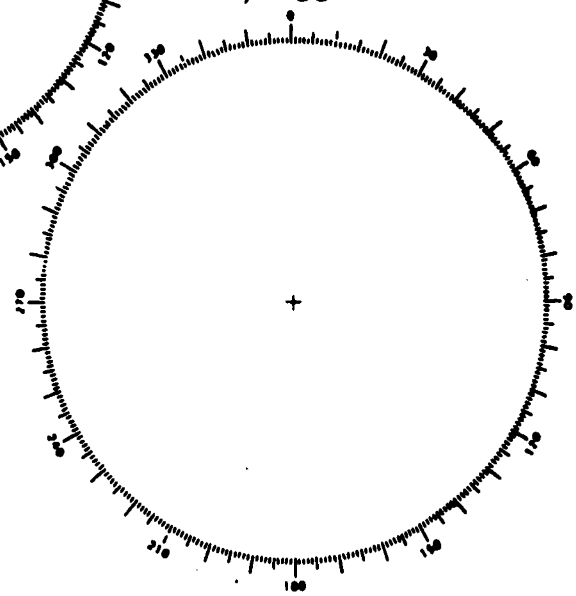
a) 60°



b) 240°



c) 330°



6. A circle is divided into 8 congruent arcs.

   a) How many degrees will be measured by each arc?

   b) A central angle of this circle cuts off three of the eight arcs. What is the measure of this angle in degrees?

7. A circle is divided into 24 congruent arcs.

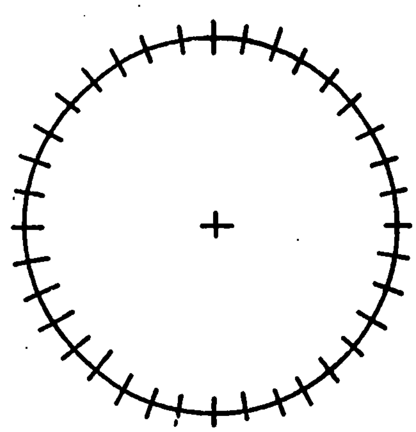
   a) How many degrees will be measured by each arc?

   b) A central angle of this circle cuts off 18 of the 24 arcs. What is the measure of this angle in degrees?

8. A circle is divided into 36 congruent arcs.

   a) How many degrees will be measured by each arc?

   b) Central angle STU of this circle has a measure of $2\frac{3}{36}$ turns in a counterclockwise direction. Draw $\angle$STU.

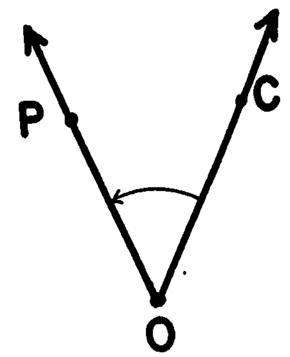


   c) What is the measure of $\angle$STU in degrees?

9-12. Estimate the measure of each of the following angles in degrees by:

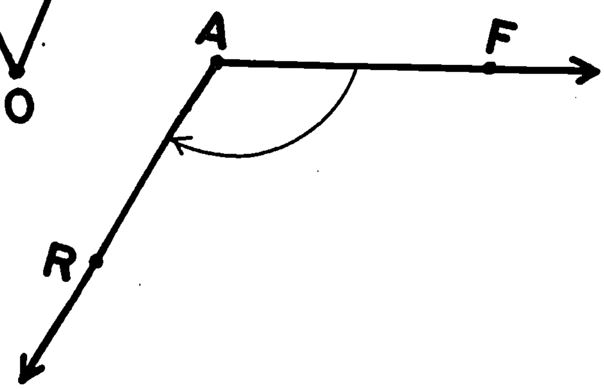
(1) Use a drawing compass to draw a circle using the vertex of the angle as the center. You decide what radius to use.

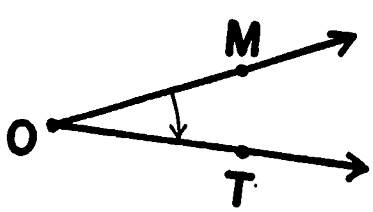
(2) Divide the circle into a number of congruent arcs. Use your judgment as to how many arcs to use. Mark off the arcs by estimating their positions.

9. 

$m(\angle COP) \approx$ _____$^\circ$

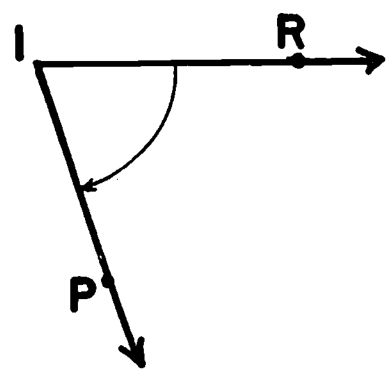
10. $m(\angle FAR) \approx$ ____



11. 

$m(\angle MOT) \approx$ ____

12. $m(\angle RIP) \approx$ ____



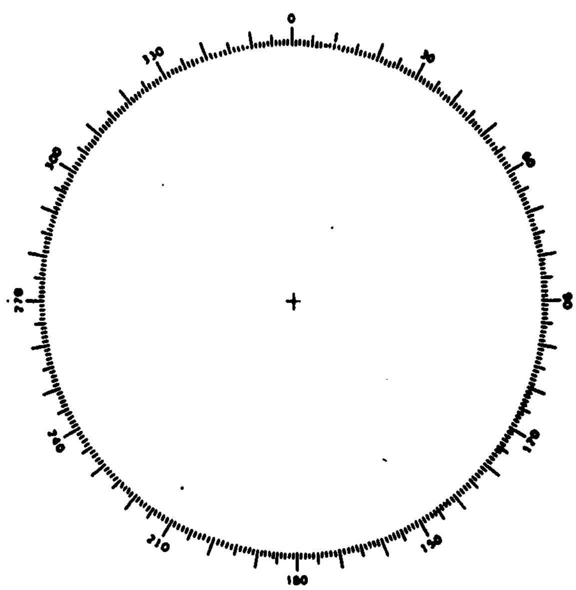
DISCUSSION QUESTIONS

1. What is a degree?

2. How are a degree and an inch alike?

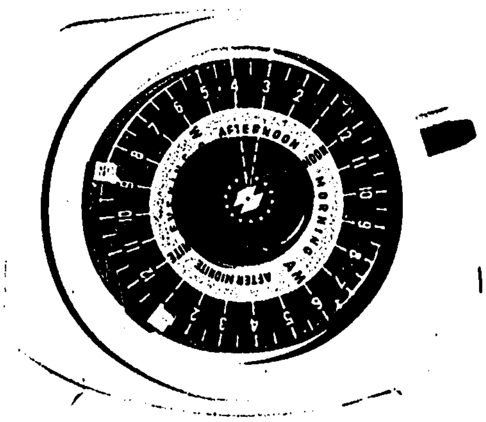
3. Discuss applications where an angle whose measure is greater than 360° might be found.

4. The circle in Figure 5, page 51, is divided into 360 congruent arcs. The circle drawn below is also divided into 360 congruent arcs. Discuss the following statement: Each arc determines a central angle of one degree <u>regardless of the size of the circle.</u>



DIALS AND GAUGES

Many dials and gauges are either (1) circular or (2) have a scale which is based on a circular design. Some examples are included in the following exercises.



1. An electronic timer such as the one pictured to the left will turn appliances and lights on (and off) automatically.

   Using the settings shown in the picture, the lights will go "on" at 9 p.m. and "off" at 1 a.m.

When the markings are extended, the smaller angle formed by each numbered marking with the numbered markings on either side has a measure of 15°. (The 15° angle formed by the 3 p.m. and 4 p.m. markings is indicated by dotted rays.)

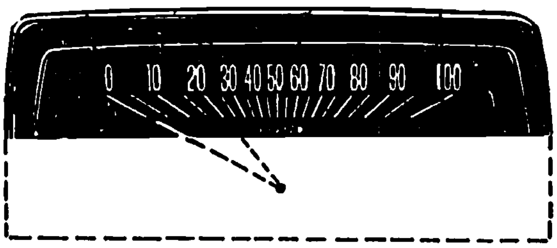
On the setting shown above, the smaller angle formed by the 9 p.m. and 1 a.m. markings has a measure of 60° ($4 \times 15^\circ$).

(a) What will be the angle formed by the markings for the following settings? (Move from the 1st to the 2nd setting in a counter-clockwise direction.)

First Setting	Second Setting	Angle Measure
9 p.m.	5 a.m.	120° ($8 \times 15^\circ$)
7 p.m.	12 midnight	____
6 p.m.	2 a.m.	____
8 p.m.	6 a.m.	____

(b) For each angle measure, give a pair of settings whose markings, when extended, will form an angle of that size. (There are several for each part.)

Angle Measure	Settings
135°	on at _____, off at _____
90°	on at _____, off at _____
210°	on at _____, off at _____
15°	on at _____, off at _____



2. On the speedometer to the left, part of the needle which indicates speed is hidden from view (indicated by dotted line).

The entire needle pivots on a point and the end of the needle follows a circular path as it moves. In the above speedometer as the needle moves from 0 to 20 m.p.h., the degree measure of the angle formed is approximately 20°.

(a) As the car accelerates or decelerates from the first speed to the second speed, give the measure of the angle formed by the needle as it moves from the first speed to the second speed.

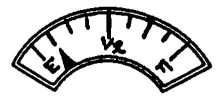
First Speed	Second Speed	Angle Measure
0 m.p.h.	40 m.p.h.	40°
20 m.p.h.	80 m.p.h.	_____
30 m.p.h.	50 m.p.h.	_____

(b) Using the speedometer needle, describe three different ways for determining an angle of 40°.

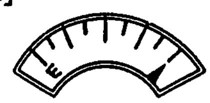
First Speed _____ _____ _____

Second Speed _____ _____ _____

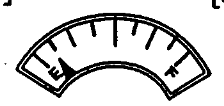
3. The gasoline gauge on a car looked like this when there were 2 gallons of gasoline in the tank. Estimate the number of gallons of gasoline in the tank for each gauge-reading below.



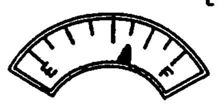
[A]



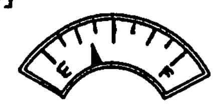
[B]



[C]



[D]



(a) ______ (b) ______ (c) ______ (d) ______

4. The timer pictured at the right will time intervals up to 60 minutes long.

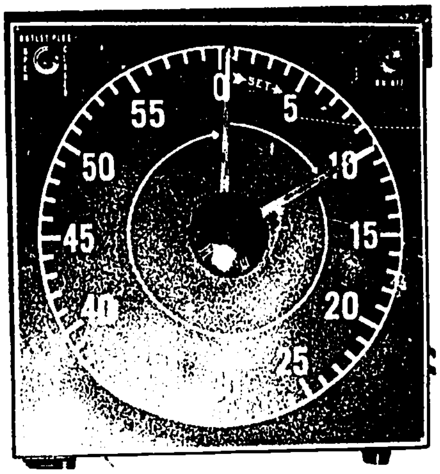
The timer shown is set for an interval of 40 minutes. (Assume the vertical pointer is on "0".)

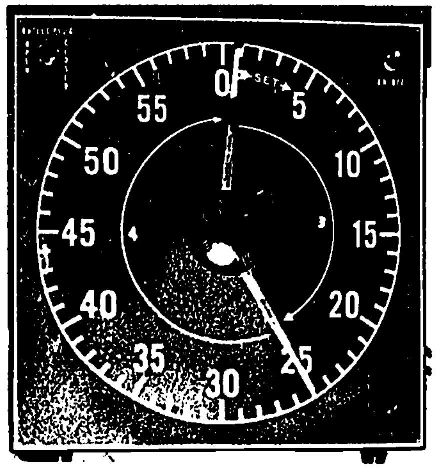
(a) The circular dial on the timer has been divided into how many congruent arcs? __________

(b) Each arc is what fraction of the circle? __________

(c) What is the degree measure of each arc? __________

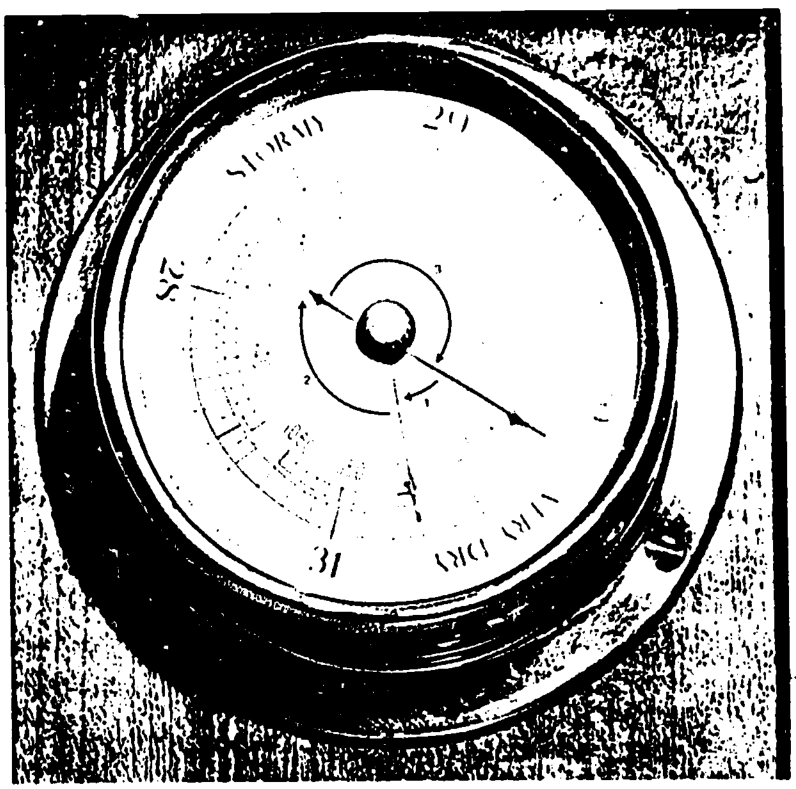
(d) Give the degree measure of each angle indicated.





m (∠1) = __________
m (∠2) = __________

m (∠3) = __________
m (∠4) = __________



5. The outer dial of the barometer pictured to the left is divided into 40 congruent arcs. (Using numbered markings only.)

(a) What is the degree measure of the smaller angle (∠1) pictured?

m(∠1) = ________

(b) Using the settings pictured above, . . .

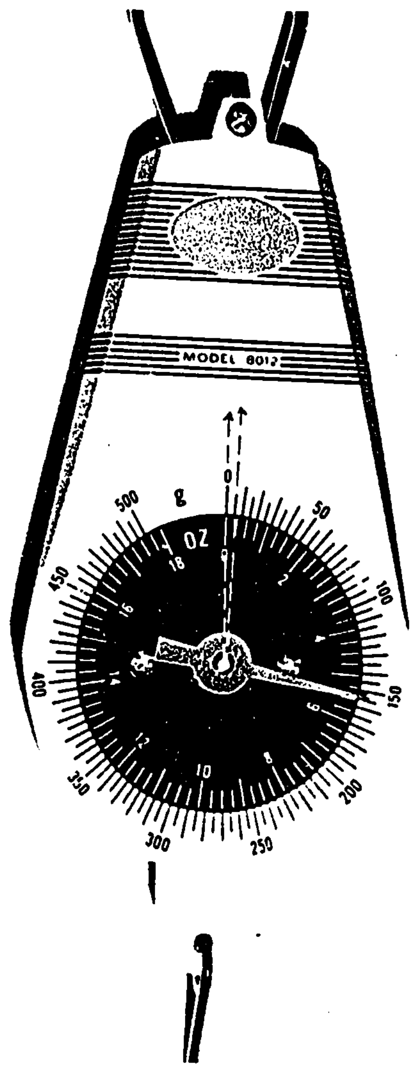
m (∠2) = ________ m (∠3) = ________

6. The outer dial (0 to 500g.) of the spring scale pictured to the right is divided into 100 congruent arcs.

As the pointer moves from 0 to 500g., it moves through an angle of 330°. Therefore, the small angle indicated by the dotted rays has a measure of 3.3°.

(a) What is the measure of the angle formed as the pointer moves from 0g. to each of the following settings?

Setting	Degree Measure
155 g.	102.3° ($31 \times 3.3^{\circ}$)
100 g.	_____
250 g.	_____
400 g.	_____



The pointer above indicates a weight of 155 g.

(b) Using the dials of the scale, give the approximate weight in grams equivalent to each of the following weights in ounces. (The ounce scale is on the inner part of the circle.)

Weight in ounces	Weight in grams
6 oz.	170 g.
9 oz.	______
16 oz.	______
4 oz.	______
$9\frac{1}{2}$ oz.	______

(c) A pound is equivalent to approximately how many grams? ______

FOLLOW-UP . . .

The dials and gauges are only a sample of the dials and gauges that could have been shown. Bring in dials and gauges (or pictures of them) whose scales are based on a circular design. Describe the angles formed as the indicator or needle moves from one marking to another.

THE CIRCULAR PROTRACTOR

A circular protractor is an instrument for measuring angles. It is a copy of a circle that has been divided into congruent arcs. (Usually 360 arcs are used so that each arc corresponds to one degree.)

A circular protractor measures an angle by treating the angle as if it were a central angle.

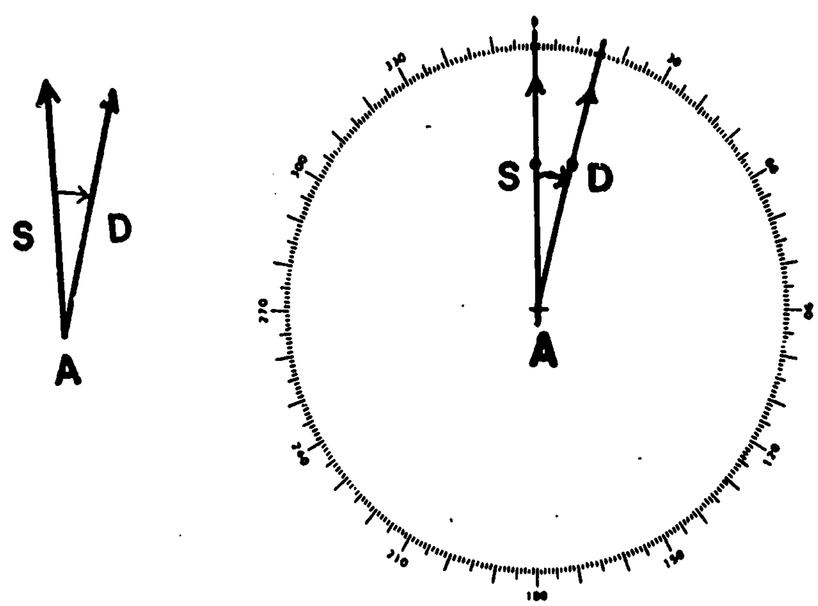


Figure 1

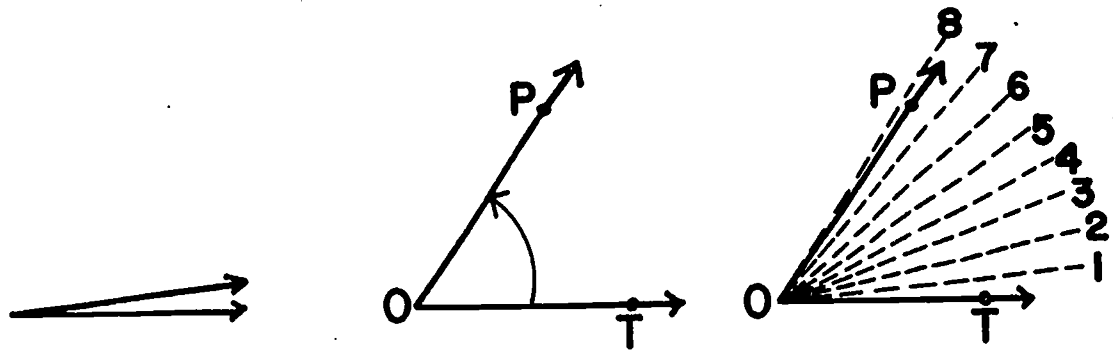
The center of the circular protractor is placed on the vertex of the angle.

The sides of the angle are extended, if necessary, so that they cut the circle.

What is the measure of ∠SAD to the nearest degree?

A COMPARISON

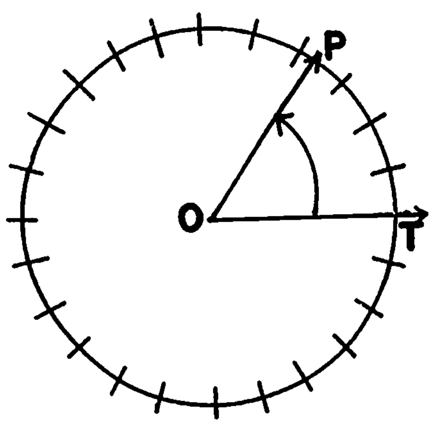
In Lessons 1 and 2 an angle was measured by first selecting a unit angle, then determining how many copies of this unit angle would "fill up" the opening of the other angle.



Unit Angle The Angle to be Measured

Figure 2 $m(\angle TOP) = 8$ unit angles (To the nearest whole unit.)

In Lessons 3-7 we have been looking at angles formed by a rotation. The measure of such an angle is the amount of turn. A circle divided into congruent arcs makes it easier to measure an amount of turn.



$m(\angle TOP) = \frac{4}{24}$ of a turn.
(To the nearest whole unit.)

Figure 3

The unit used in Figure 3 is $\frac{1}{24}$ of a circle. What is the unit used in Figure 1? Which unit is more precise? Why?

DISCUSSION QUESTIONS

1. How does the circular protractor use both the "unit angle" (Figure 2) and the "unit arc" (Figure 3) ideas for measuring angles?

2. How is the unit angle of 1^{o} related to the circle divided into 21,600 congruent arcs?

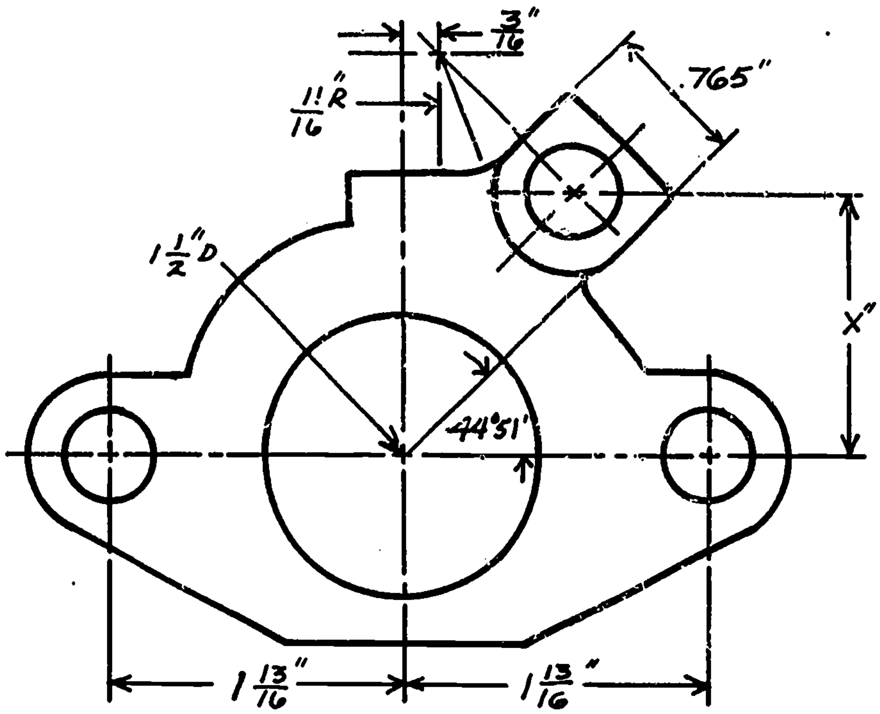
3. In what way does the circular protractor treat every angle like it was a central angle?

4. How can a circular protractor be used to measure an angle whose measure is greater than 360^{o}?

MINUTES AND SECONDS

Modern mass production methods require that parts be machined to size limitations which allow parts to be easily interchanged.

Figure 4



In many industrial applications angles must be measured precisely and the degree is not a small enough unit. In these cases the degree is divided up into smaller units.

One degree is divided into 60 parts.
Each part is called a <u>minute</u>.

$1^\circ = 60'$ (The symbol ' is read minutes.)

One minute is divided into 60 parts.
Each part is called a <u>second</u>.

$1' = 60''$ (The symbol " is read seconds.)
$1^\circ =$ __?__ "

A measurement of 38 degrees, 42 minutes, 16 seconds is written as $38^\circ\ 42'\ 16''$ and means: $38^\circ + 42' + 16''$.

Degree measures are often written in decimal form. Thus, it is sometimes necessary to convert these measures to the correct number of degrees, minutes and seconds.

Example 1. Convert 62.82° to the correct number of degrees, minutes and seconds.

Solution: $62.82^\circ = 62^\circ + .82^\circ$. We must convert $.82^\circ$ to minutes and seconds.

Since $1^\circ = 3{,}600''$ we can set up a proportion:

$$\frac{3600''}{1^\circ} = \frac{N}{.82^\circ} . \qquad N = \frac{.82^\circ \times 3600''}{1^\circ} .$$

Thus $N = 2{,}952''$. Now $1' = 60''$. Think: What number times $60''$ equals $2{,}952''$.

$60\overline{)2952.}$ By division we see that

$$\begin{aligned} 2{,}952'' &= 49 \times 60'' + 12''. \\ &= 49 \times 1' + 12'' \end{aligned}$$

Thus, $.82^\circ = 2{,}952'' = 49' + 12''$.

So $62.82^\circ = 62^\circ\ 49'\ 12''$.

Example 2. Convert $18^\circ\ 14'\ 24''$ to degrees in decimal form.

Solution: We must convert $14'\ 24''$ to degrees.

$1' = 60''$ so $14' = 14 \times 60'' = 840''$.

Thus $14'\ 24'' = 840'' + 24'' = 864''$.

$$1^\circ = 3600''. \quad \frac{1^\circ}{3600''} = \frac{N}{864''}. \quad N = \frac{1^\circ \times 864''}{3600''}.$$

Thus $N = .24^\circ$ and $18^\circ\ 14'\ 24'' = 18.24^\circ$.

✓POINT

1. How many minutes are there in one degree?
2. Is an angle measure of 2,785" greater than or less than 1°?

EXERCISES

1. How many seconds are there in one degree?
2. A circle would have to be divided into how many congruent arcs if each arc is to measure 1 second of angle?
3. Complete the following conversions:

   (a) $180' = \_\_\_\_^\circ$ (c) $\frac{1}{3}^\circ = \_\_\_\_'$

   (b) $900' = \_\_\_\_^\circ$ (d) $18{,}000'' = \_\_\_\_^\circ$
4. Convert the following degree measures to the correct amount of degrees, minutes and seconds.

   (a) 5.5° (b) 47.25°

   (c) 13.19° (Hint: Multiply .19 times 3600".)
5. Convert the following angle measures to degrees in decimal form.

   (a) $5^\circ\ 15'$ (b) $137^\circ\ 42'$ (c) $58^\circ\ 28'\ 12''$

HOW MANY DEGREES?

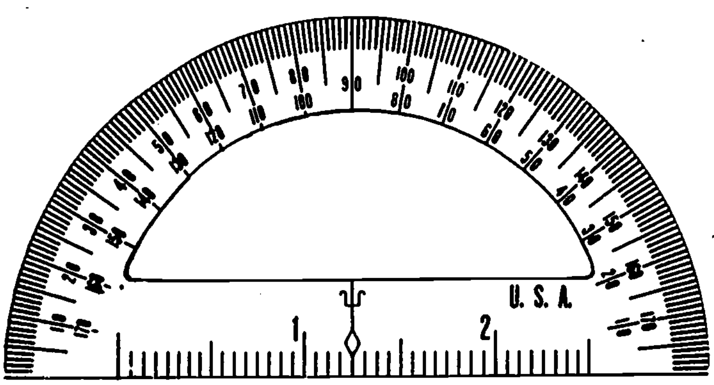
A VARIETY OF SHAPES AND SIZES

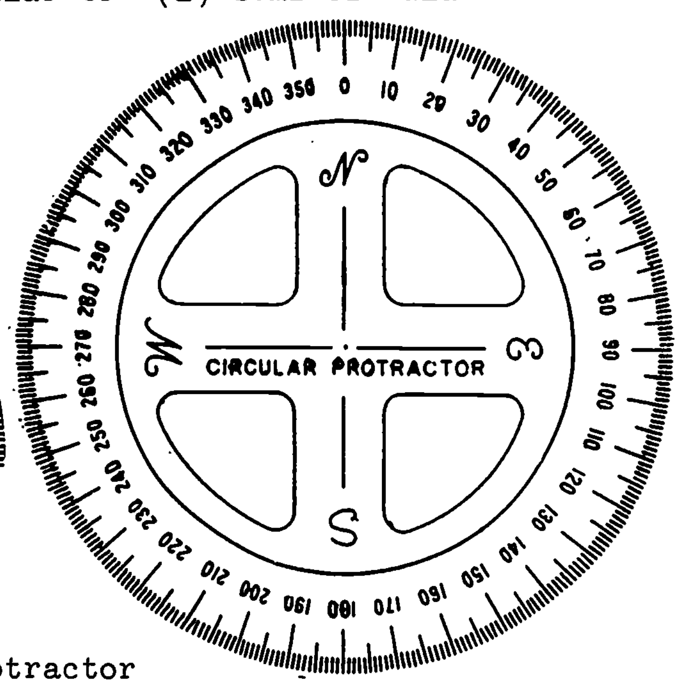
Protractors come in a variety of shapes and sizes. Some of these are pictured in the back of this booklet in **APPENDIX D** .

Regardless of shape or size, all protractors are read in a similar manner. If the protractor you are using is not like the ones pictured in this lesson and you can not determine how it is read, consult **APPENDIX D** or your teacher for help.

THE SHAPE OF A PROTRACTOR IS . . .

The protractors you will use in this booklet are for the most part either (1) circular or (2) semi-circular in shape.





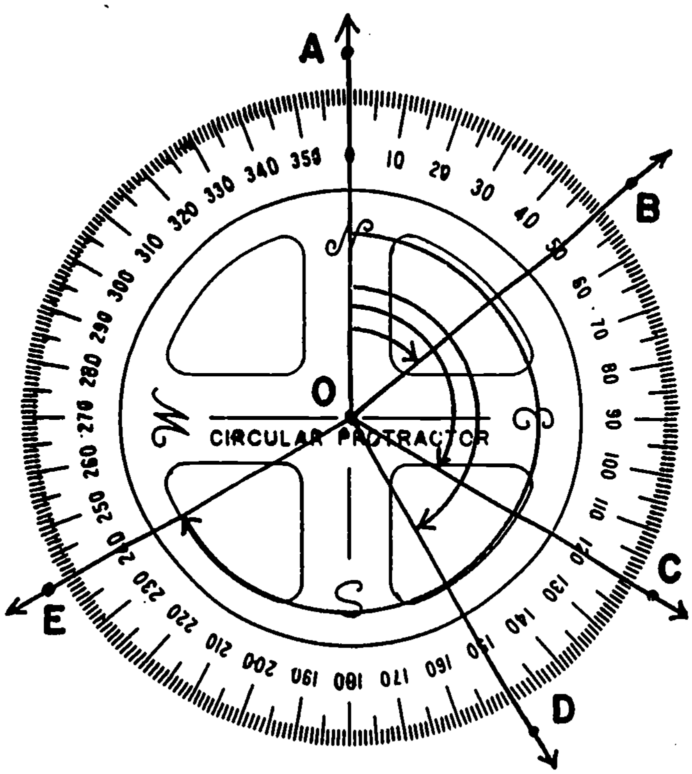
TO BE EXPLORED . . .

Is it necessary for a protractor to be circular or semi-circular?

To explore the question above, refer to **APPENDIX E** at the back of this booklet.

USING A CIRCULAR PROTRACTOR . . .

In some situations, a circular protractor may be more suitable for measuring angles.



Using the circular protractor to the left,...

$m(\angle AOB) = 50^\circ$
$m(\angle AOC) = 120^\circ$
$m(\angle AOD) = 150^\circ$
$m(\angle AOE) = 240^\circ$

When reading the angle measures listed above,...

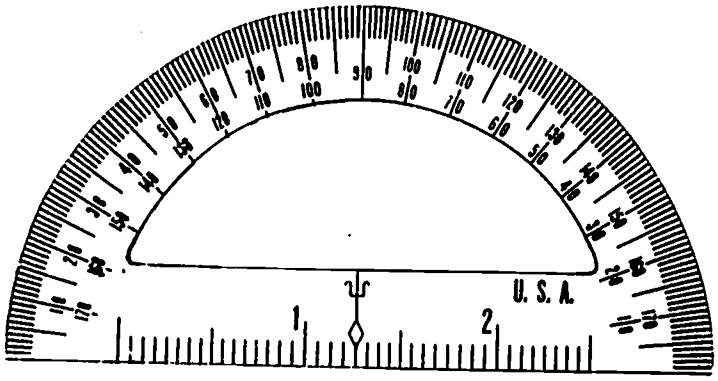
(1) the center point of the protractor is on the vertex of the angle.

(2) one side (ray) of the angle passes through the zero point on the protractor scale.

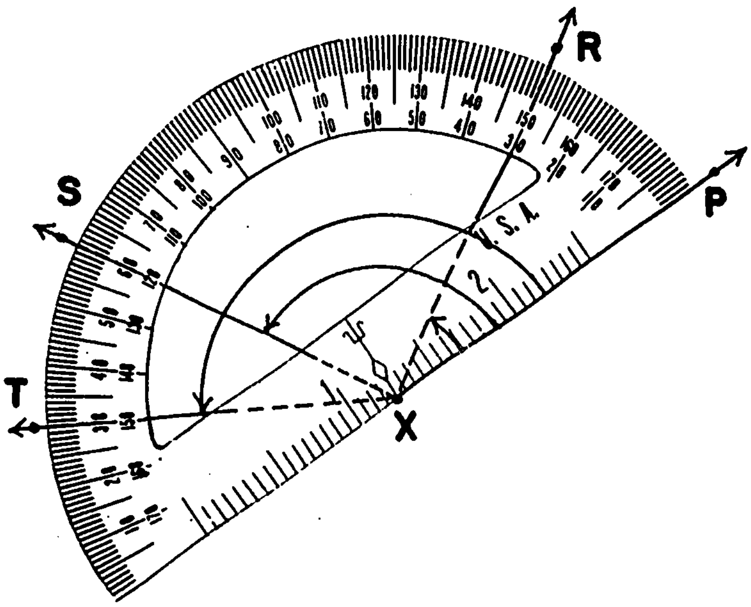
(3) the number on the protractor scale corresponding to the other ray gives the measure of the angle.

USING A SEMI-CIRCULAR PROTRACTOR . . .

Most of the protractors which you see for sale in department stores, drug stores, etc.... are semi-circular. Both the circular and semi-circular protractors have advantages in different situations.



SEMI-CIRCULAR PROTRACTOR



Using the semi- circular protractor at the left,...

$m(\angle PXR) = 30^\circ$

$m(\angle PXS) = 120^\circ$

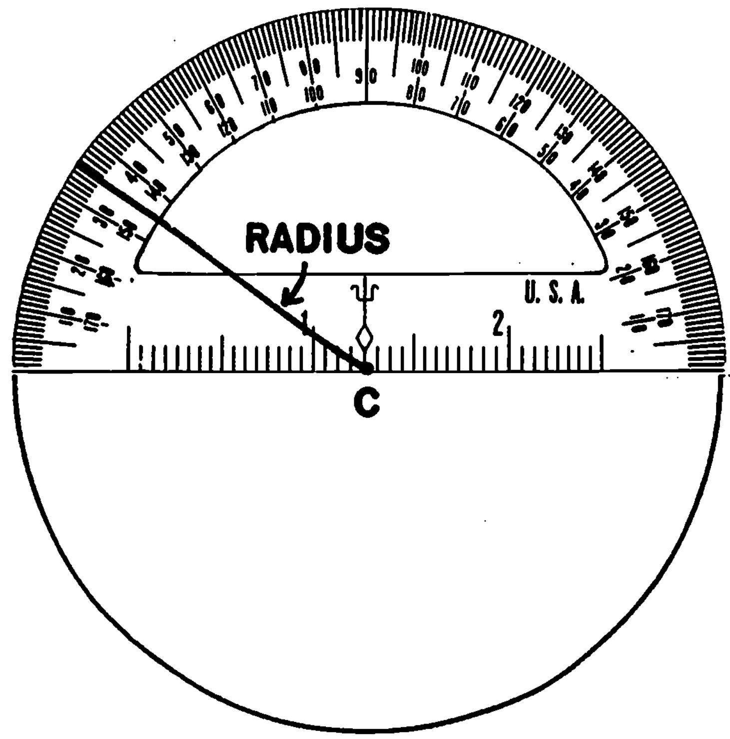
$m(\angle PXT) = 150^\circ$

Notice that the semi-circular protractor is used in the same manner as the circular protractor.

SOME SIMILARITIES . . .

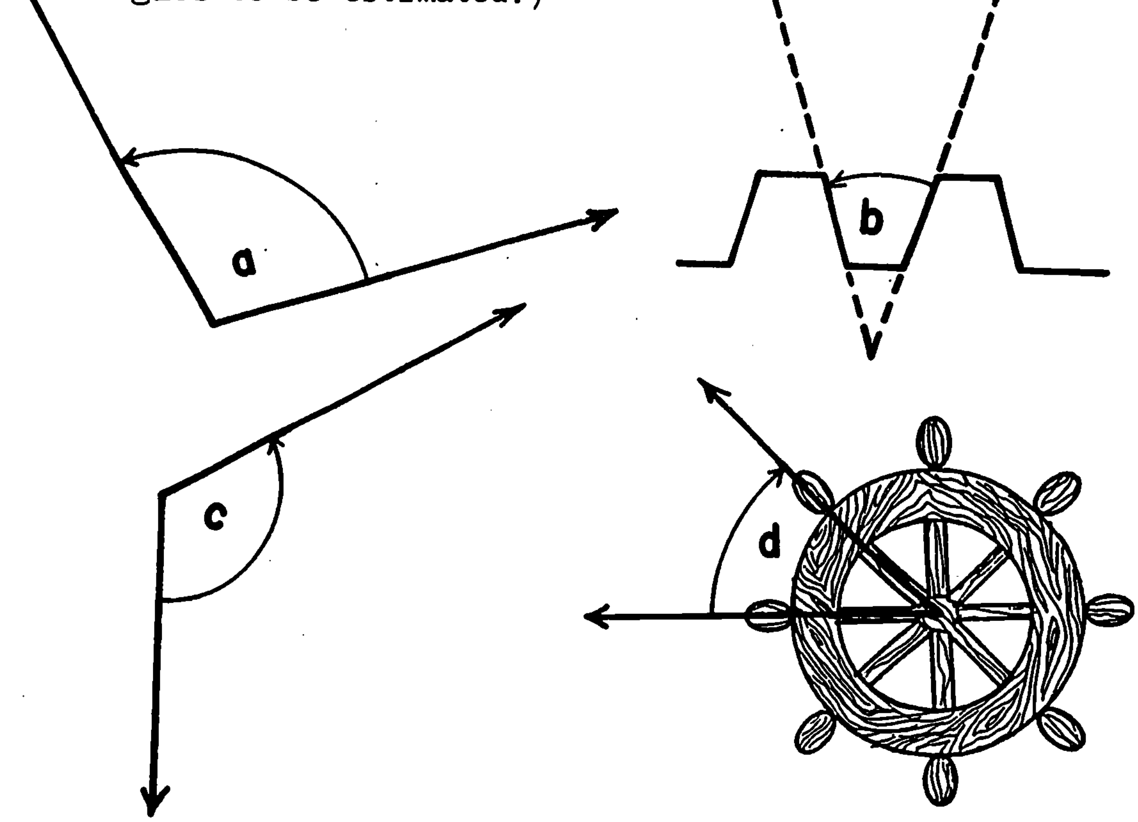
The point of a semi-circular protractor which is placed on the vertex of the angle being measured is also the <u>center</u> of a circle. (See picture to the right.)

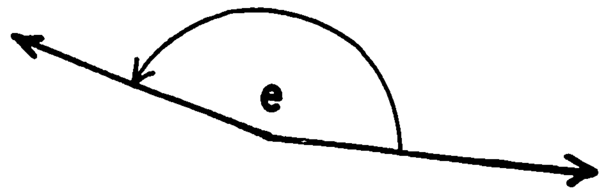
Thus, any semi- circular protractor could be considered as one-half of a circular protractor.

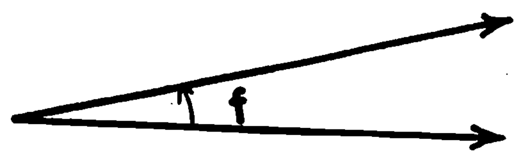


EXERCISES I

1. Estimate in degrees the measure of each angle. Record your estimates in **TABLE 9-1** . (The arrows indicate the angles to be estimated.)




e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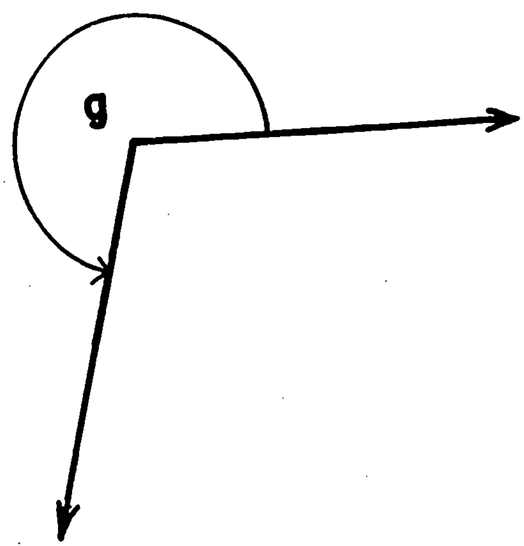
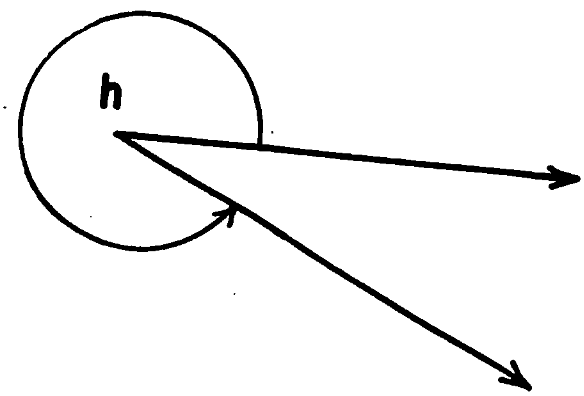

g


h

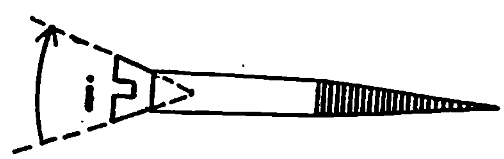

i

TABLE 9-1

ANGLE	ESTIMATE IN DEGREES...	MEASURE TO NEAREST DEGREE...
a		
b		
c		
d		
e		
f		
g		
h		
i		

2. For each angle in exercise 1, measure to the nearest degree using either ...

   a. a circular protractor, or

   b. a semi-circular protractor.

   Record your results in **TABLE 9-1**.

   If necessary, extend the sides of the angles so the measure can be read.

How close were your estimates? If your estimates were within 10° of the measured size, your estimates were very good.

DISCUSSION QUESTIONS

1. In Exercise 2, would you get the same measure regardless of which protractor was used? Why or why not?

2. Explain how to use a semi-circular protractor to measure an angle whose measure is greater than 180° and less than 360°.

3. When measuring angles, would the ability to make a reasonable estimate for the measure of an angle help you to check your work? Why or why not?

REFINING YOUR ESTIMATES...

To estimate accurately, it is necessary to develop a "feeling" or mental picture of the units of measure involved. For example, to estimate the length of a room in feet, it is helpful to be able to visualize the length of one foot or one yard (3 feet).

As a guide, an angle of 1° is probably too small to be of use when estimating angle size. (See Figure 1.)

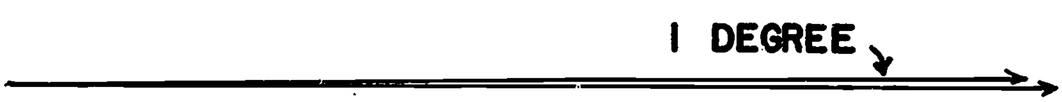
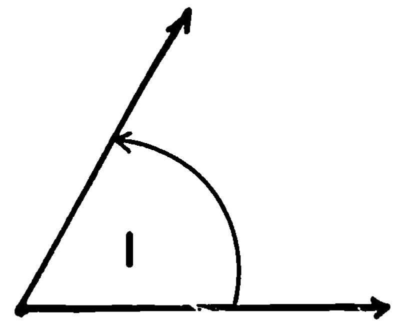


Figure 1

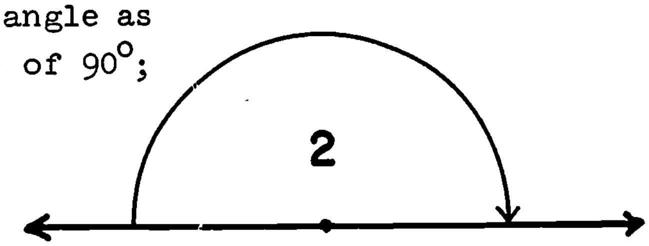
SOME USEFUL GUIDES...

Some useful estimation guides are those angles whose measures are multiples of 30° (30°, 60°, 90°, 120°.,...) and 45° (45°, 90°, 135°.,...).

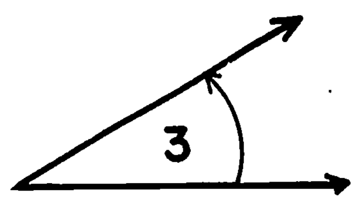
$m(\angle 1) = 60^\circ$. Using this angle as a guide, determine an angle of 30°.



$m(\angle 2) = 180^\circ$. Using this angle as a guide, determine an angle of 90°; of 45°.



$m(\angle 3) = 30^\circ$. Using this angle as a guide, determine an angle of 120°. ($120^\circ = 4 \times 30^\circ = 90^\circ + 30^\circ$)



Several angle measures can be interpreted as combinations of these angles. For example, $150^\circ = 90^\circ + 60^\circ$ or $5 \times 30^\circ$. Complete the following.

$135^\circ = 90^\circ +$ ________ $= 3 \times$ ________.

$225^\circ = 180^\circ +$ ________.

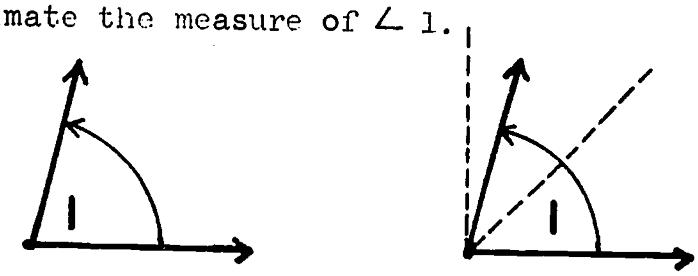
$270^\circ = 180^\circ +$ ________ $= 3 \times$ ________.

$315^\circ = 270^\circ +$ ________.

The examples on the next page will illustrate how these angles may be used as guides in estimating.

EXAMPLES:

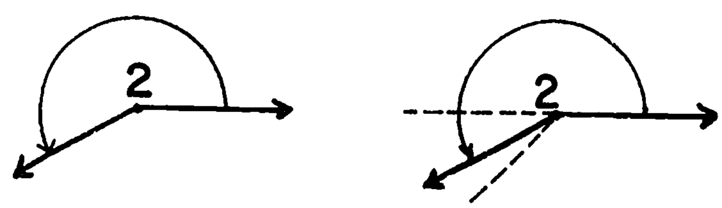
1. Estimate the measure of $\angle 1$.



SOLUTION: $m(\angle 1)$ is less than 90° and more than 45°.
$m(\angle 1)$ is a little more than half-way between 45° and 90°.
$67\frac{1}{2}^\circ$ is half-way between 45° and 90°.
<u>$m(\angle 1) \approx 70^\circ$.</u>

($\approx$ is read "approximately the same as".)

2. Estimate the measure of $\angle 2$.



SOLUTION: $m(\angle 2)$ is more than 180° and less than 225°.
$m(\angle 2)$ is closer to 225° and more than half-way between 180° and 225°.
<u>$m(\angle 2) \approx 215^\circ$.</u>

CHECK: Measure $\angle 1$ and $\angle 2$ with a protractor.
To the nearest degree,...
$m(\angle 1) = 76^\circ$
$m(\angle 2) = 210^\circ$

NOTE: The above angles could have been estimated in a variety of ways. The solutions are meant only as an example of <u>one</u> way these angle sizes could be estimated.

EXERCISES II

1. For each angle pictured,...
   (1) estimate the measure,
   (2) measure to the nearest degree using a protractor.
   (3) check the closeness of each estimate by computing the difference between your estimate and your measurement.
   (4) record your results in **TABLE 9-2** .

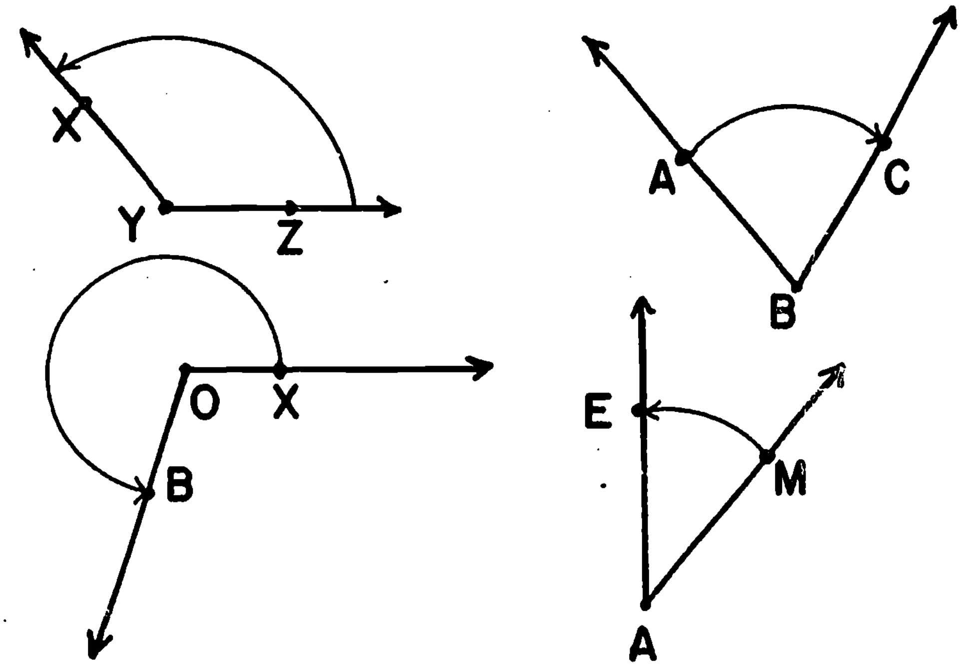


TABLE 9-2

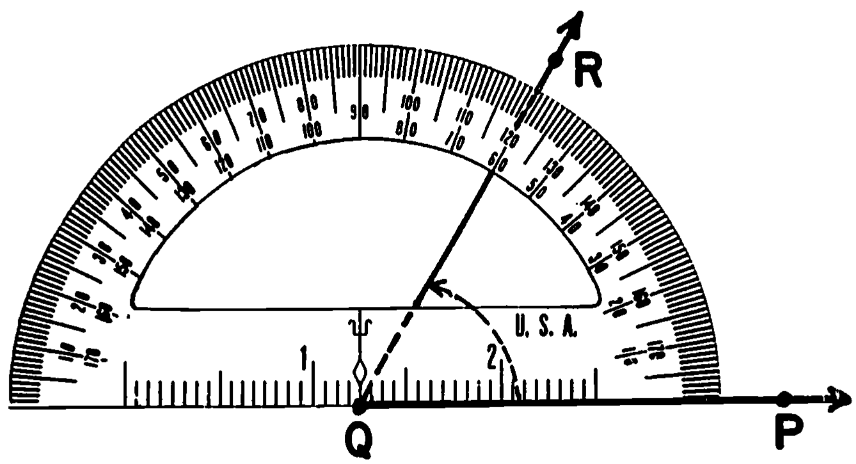
ANGLE	ESTIMATE	MEASUREMENT	DIFFERENCE
∠ ZYX			
∠ ABC			
∠ XOB			
∠ MAE			

KEEP IN PRACTICE . . .

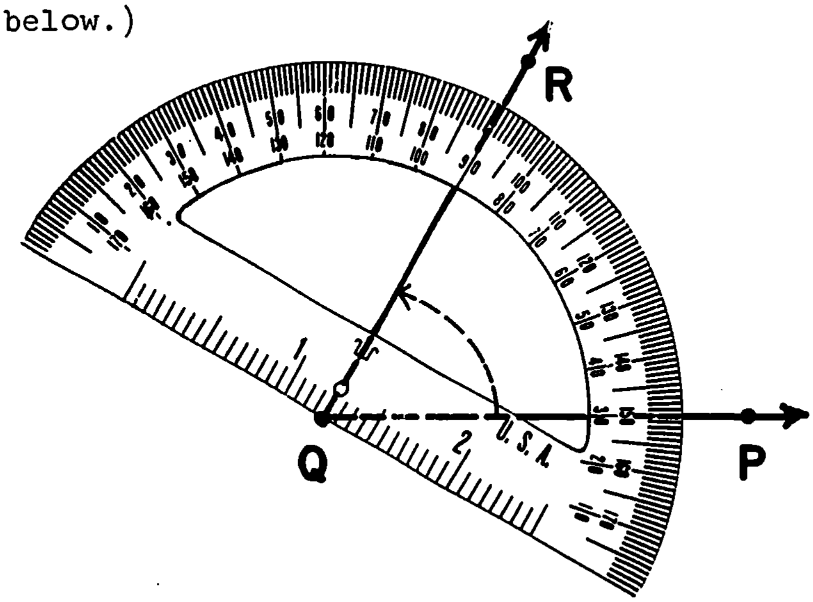
The ability to make reasonable estimates is not a skill which can be developed in one lesson. Keep in practice by estimating whenever possible. If you can make reasonable estimates, you will be able to check the reasonableness of any measuring you do.

BROKEN PROTRACTORS

ANY WAY YOU WANT TO . . .



$\angle$ PQR pictured above has a measure of 60°. Suppose a protractor was placed on $\angle$ PQR in a different manner. (See picture below.)



The measure of $\angle$ PQR as measured above is still 60° ($90^\circ - 30^\circ = 60^\circ$). In both cases the center point of the protractor is placed on the vertex of the angle (Q).

DISCUSSION EXERCISES

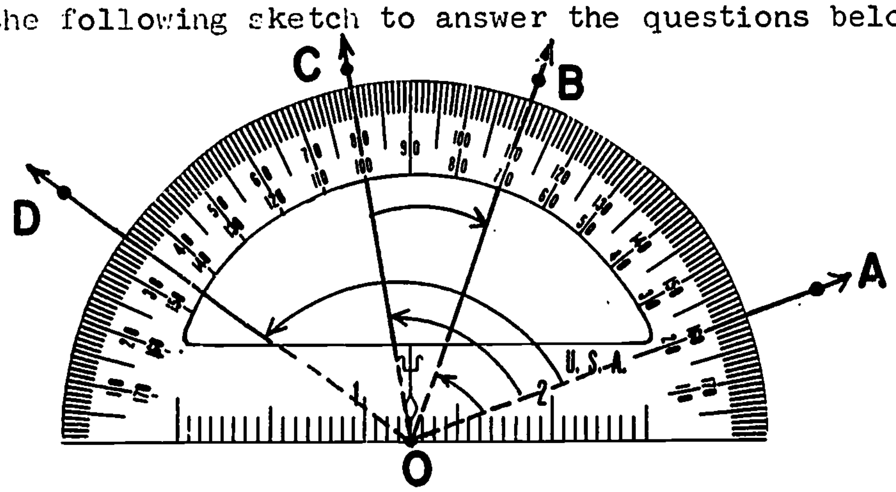
1. Name some other ways to measure ∠PQR.

2. When measuring an angle with a protractor, is it necessary for one of the rays of the angle to pass through the zero point on the protractor scale? Why or why not?

3. Are some points of the protractor scale more convenient to use as the "zero point" than others?

4. If ∠ PQR was measured with a circular protractor graduated in degrees, would m(∠PQR) = 60°?

5. Use the following sketch to answer the questions below.



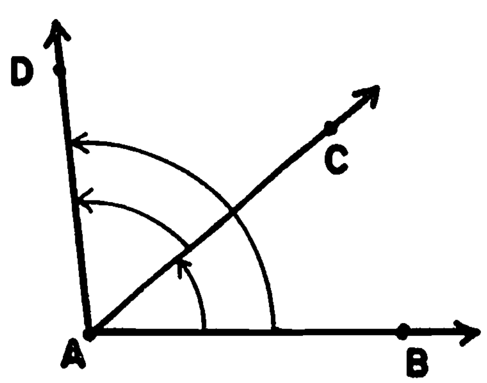
m(∠AOB) = _____

m(∠AOC) = _____

m(∠AOD) = _____

m(∠COB) = _____

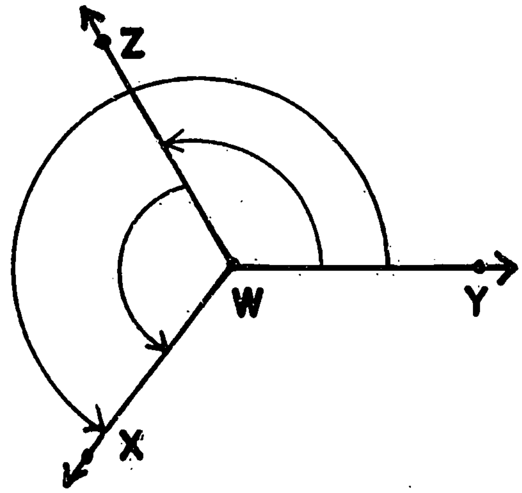
6.



If m(∠BAC) = 40°
and m(∠CAD) = 55°,

m(∠BAD) = _____

7.



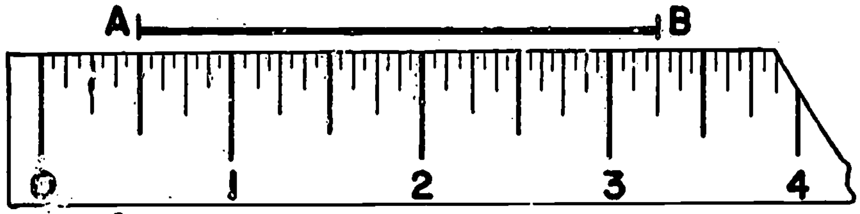
If $m(\angle YWZ) = 120^\circ$
and $m(\angle YWX) = 230^\circ$,

$m(\angle ZWX) =$ ______

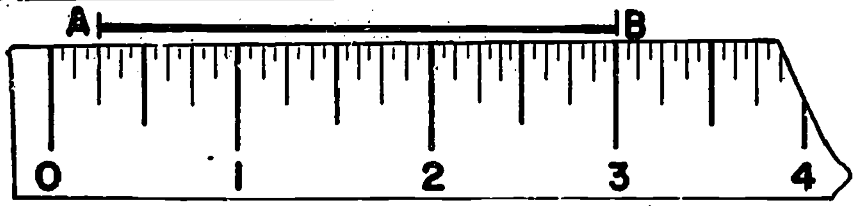
START WHERE YOU WANT TO . . .

The previous section illustrates one similarity between protractor and ruler measurement.

(1) When using a ruler, any point of the ruler scale can be used as the "zero point".

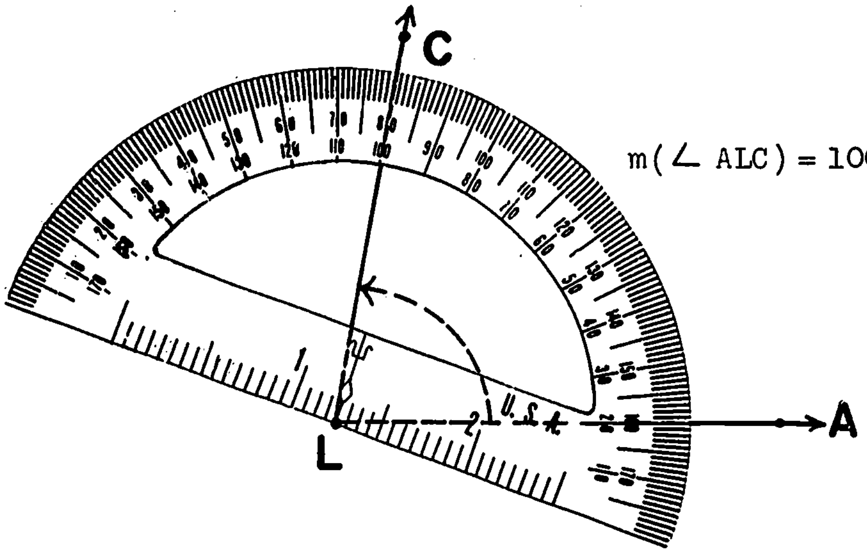


$m(\overline{AB}) = 2\frac{3}{4}$ in.



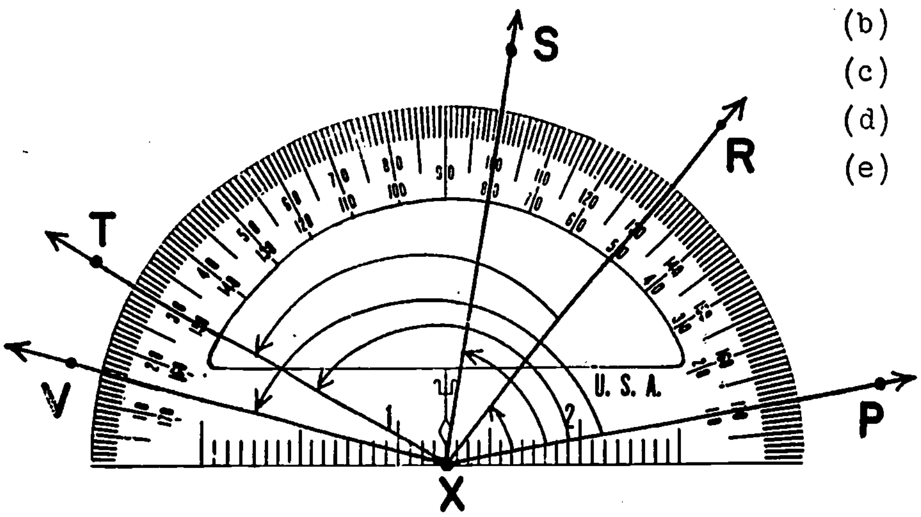
$m(\overline{AB}) = 2\frac{3}{4}$ in.

(2) When using a protractor, any point of the protractor scale can be used as the "zero point".



$m(\angle ALC) = 100^\circ - 20^\circ = 80^\circ$

EXERCISES



1. Complete the following:

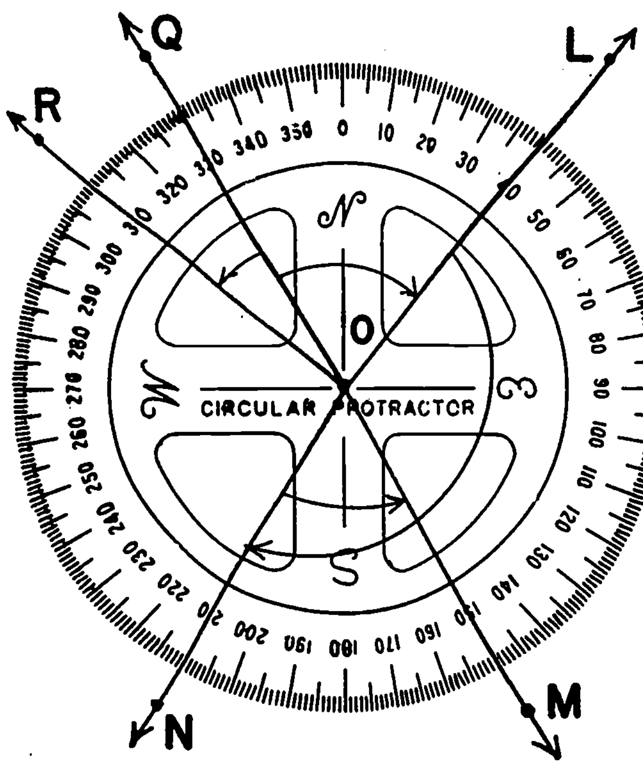
   (a) m(∠ PXR) = _____.

   (b) m(∠ PXS) = _____.

   (c) m(∠ PXT) = _____.

   (d) m(∠ PXV) = _____.

   (e) m(∠ RXT) = _____.



2. Complete the following:

   (a) m(∠ QOL) = _____.

   (b) m(∠ LON) = _____.

   (c) m(∠ QOR) = _____.

   (d) m(∠ NOM) = _____.

3. For this exercise, you will use a "broken" protractor. Use this "broken" protractor to measure each of the angles below. Record your results in **TABLE 10-1** (in column labeled **BROKEN PROTRACTOR**).

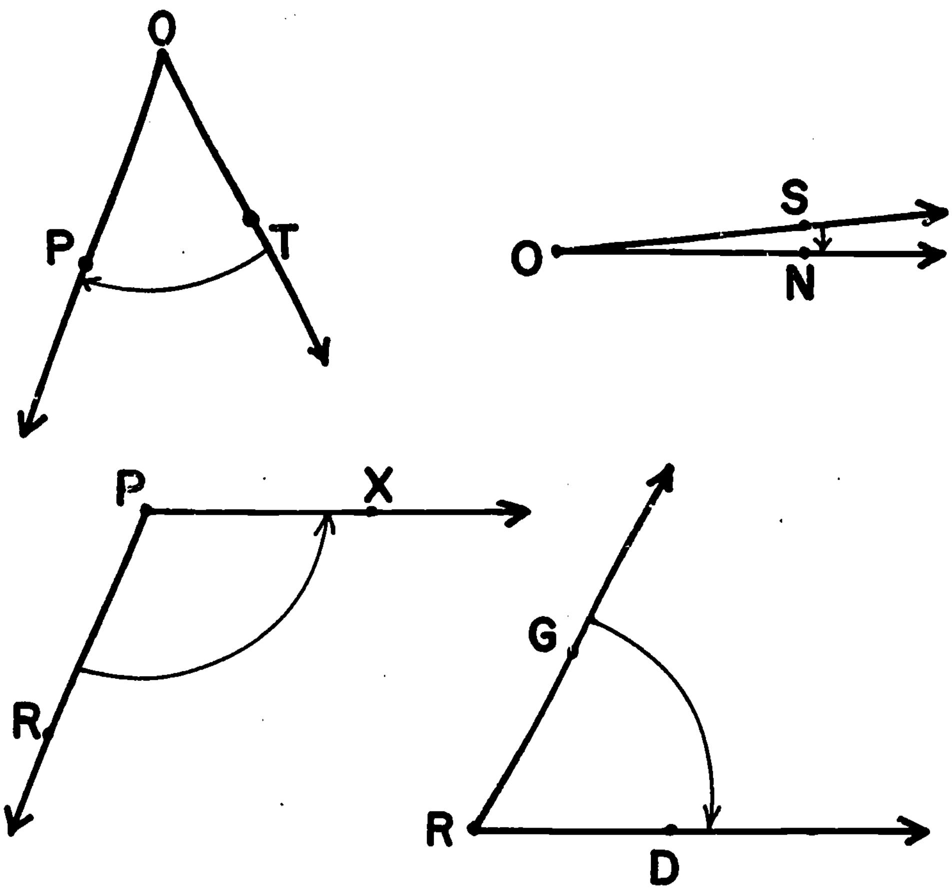
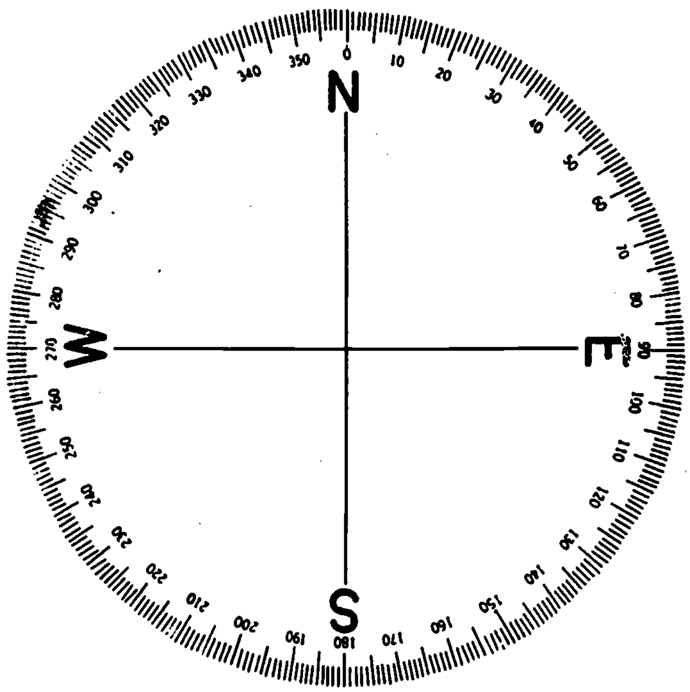
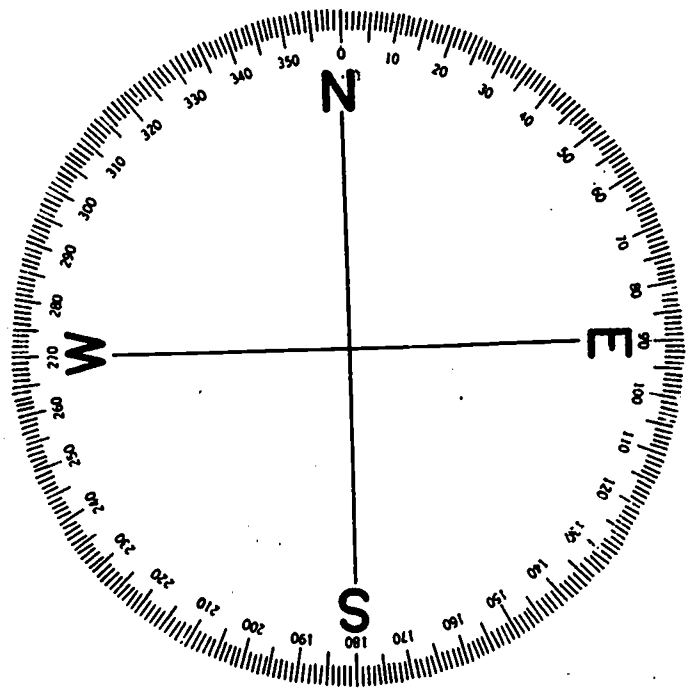
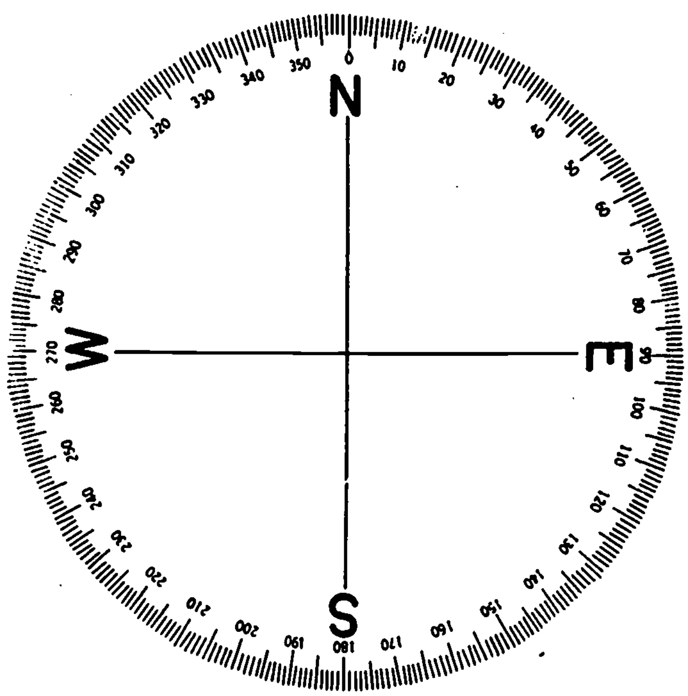


TABLE 10-1

ANGLE	MEASURE TO NEAREST DEGREE USING...	
	BROKEN PROTRACTOR	UNBROKEN PROTRACTOR
∠ TOP		
∠ SON		
∠ RPX		
∠ GRD		



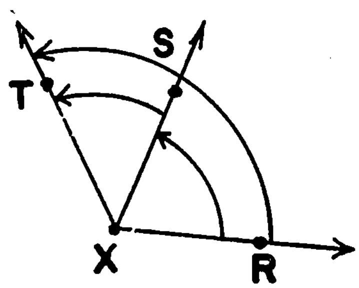




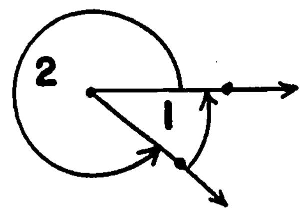
FOR USE WITH EXERCISES 3-4, pp. 84-85.

4. Measure each of the angles in exercise 3 using an unbroken protractor. Record your results in **TABLE 10-1**. (column labeled **UNBROKEN PROTRACTOR**).

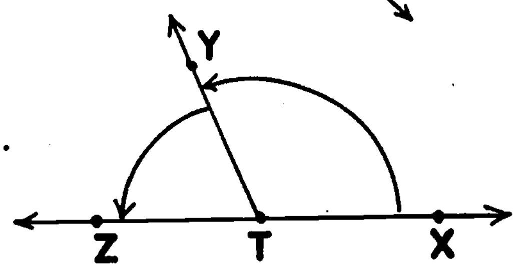
5. Complete the following:

a. 

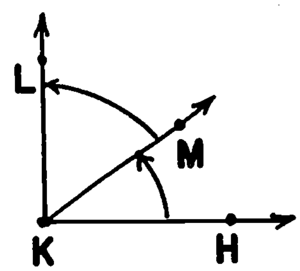
$m(\angle RXS) = 70^\circ$
$m(\angle SXT) = 50^\circ$
$m(\angle RXT) =$ ______

b. 

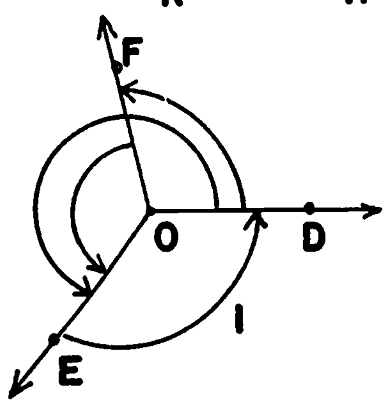
$m(\angle 1) = 40^\circ$
$m(\angle 2) =$ ______

c. 

$m(\angle YTZ) = 65^\circ$
$m(\angle XTY) =$ ______

d. 

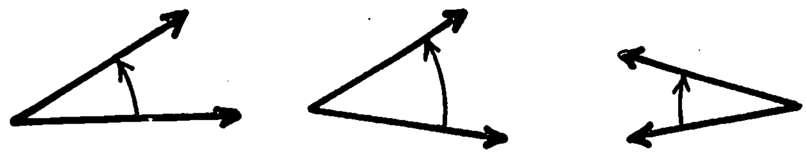
$\angle HKL$ is a right angle.
$m(\angle MKL) = 55^\circ$
$m(\angle HKM) =$ ______

e. 

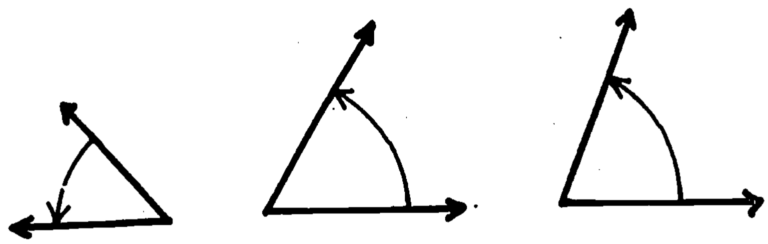
$m(\angle DOE) = 230^\circ$
$m(\angle DOF) = 105^\circ$
$m(\angle FOE) =$ ______
$m(\angle 1) =$ ______

6. Each part has three angles pictured. In each part, circle the angle which you believe to be closest to the measure given.

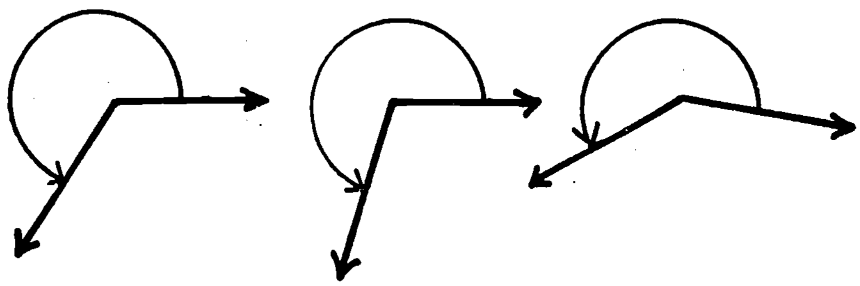
a. 25°



b. 60°

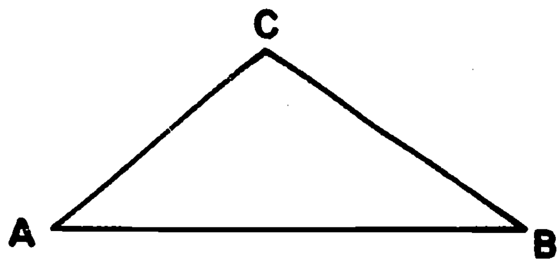


c. 240°



7. Check the accuracy of your guesses in exercise #6 by measuring each angle.

*8. (a) For each triangle pictured on page 87, measure...
(i) each side to the nearest $\frac{1}{16}$ inch, and
(ii) each angle to the nearest degree.
Record your results in the blanks provided.

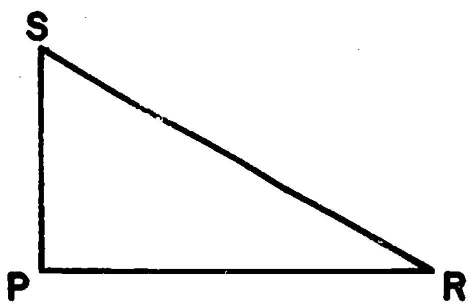


$m(\overline{AB}) =$ ____ $m(\overline{BC}) =$ ____

$m(\angle C) =$ ____ $m(\angle A) =$ ____

$m(\overline{CA}) =$ ____

$m(\angle B) =$ ____

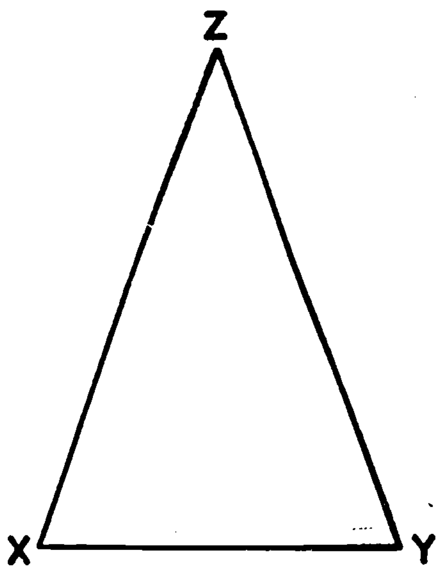


$m(\overline{SR}) =$ ____ $m(\overline{RP}) =$ ____

$m(\angle P) =$ ____ $m(\angle S) =$ ____

$m(\overline{PS}) =$ ____

$m(\angle R) =$ ____



$m(\overline{XZ}) =$ ____ $m(\overline{ZY}) =$ ____

$m(\angle Y) =$ ____ $m(\angle X) =$ ____

$m(\overline{YX}) =$ ____

$m(\angle Z) =$ ____

(b) Write a conjecture about the relationship between the size of each angle and the length of the sides opposite the angles.

(c) Write any other conjectures which seem to be true on the basis of your measurements.

DRAWING ANGLES

Given an angle, you have used protractors (both circular and semi-circular) to find its measure to the nearest degree.

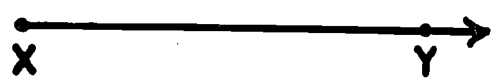
Consider the opposite of this situation. Suppose you were given an angle measure, say 67°, and were to draw an angle that size.

One such example is worked out below.

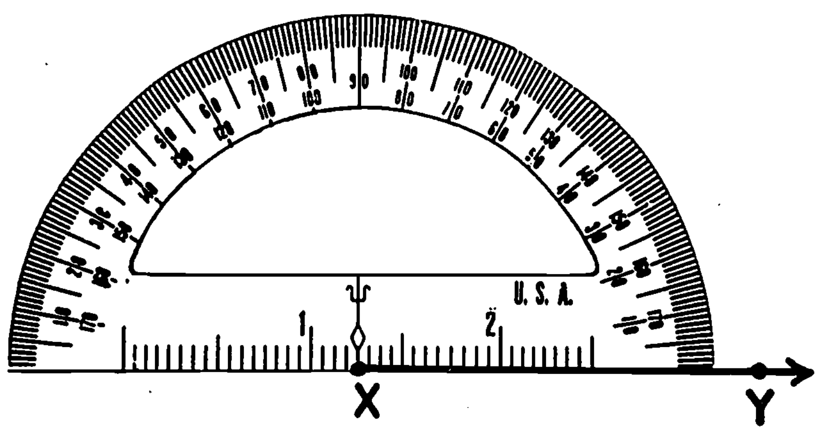
PROBLEM: Using a protractor, draw an angle whose measure is 67°.

SOLUTION:

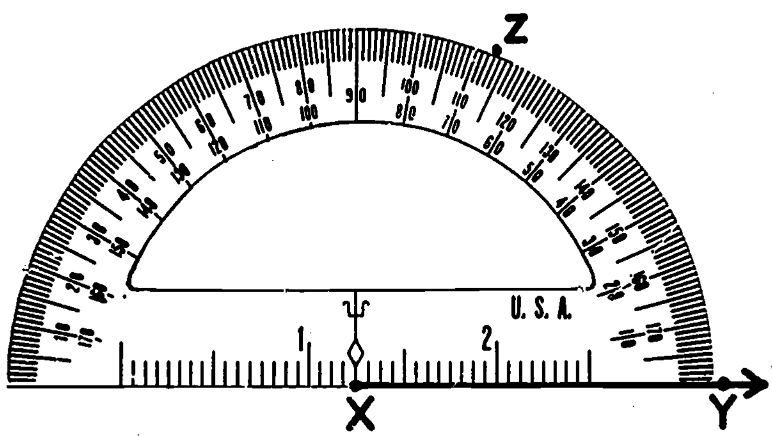
1. Draw a ray. ($\overrightarrow{XY}$)



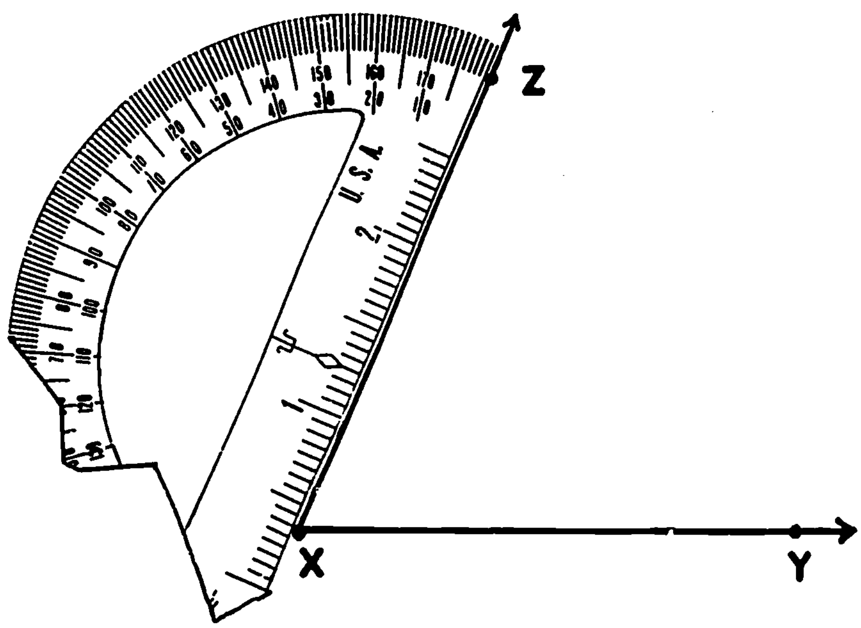
2. Place the center point of the protractor on the endpoint (X) of the ray and "line up" the ray with the zero point of the protractor scale.



3. Locate 67° on the protractor scale and make a dot (call it Z) at that point.



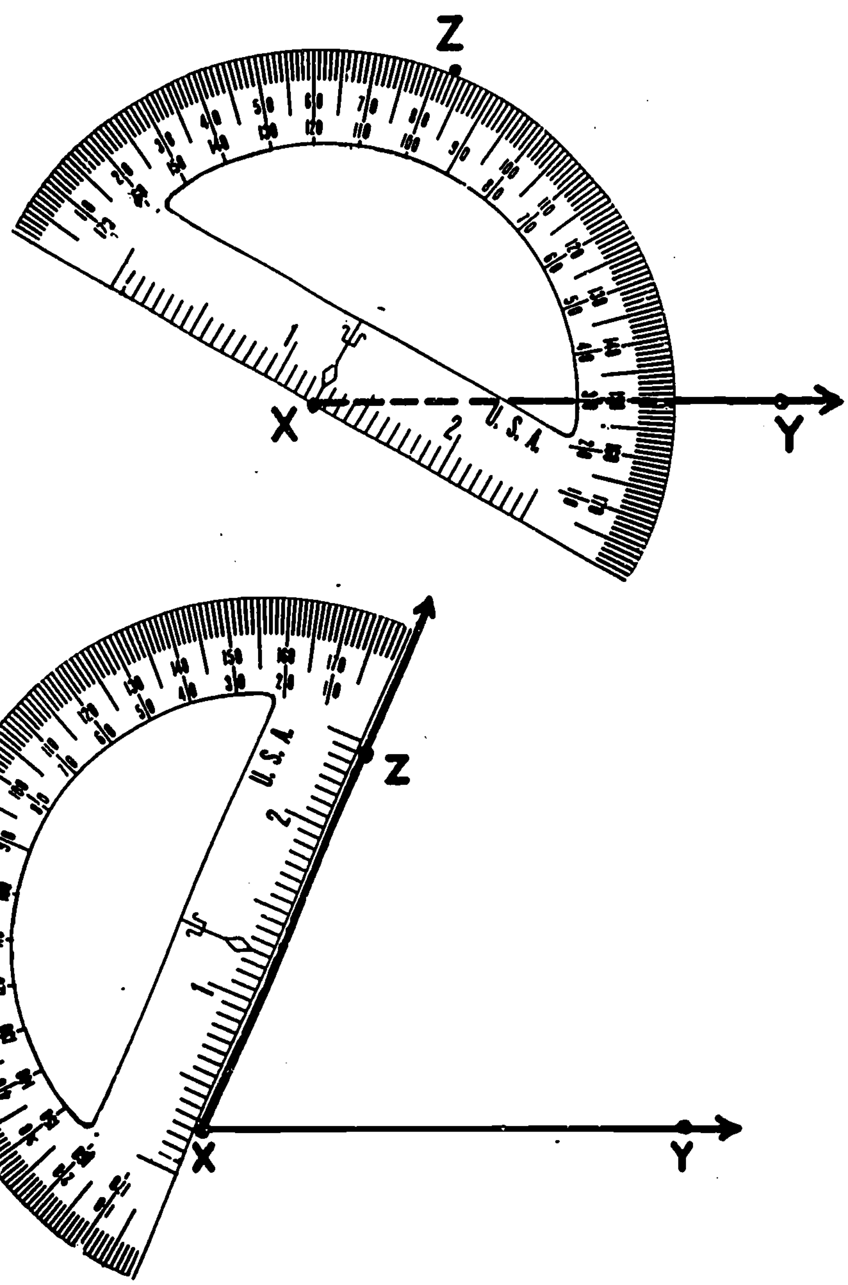
4. Remove the protractor and draw $\overrightarrow{XZ}$. $m(\angle XYZ) = 67^{\circ}$.



NOTE: There are other methods of drawing an angle of a given size. The method illustrated above is one of those more commonly used.

DISCUSSION QUESTIONS

1. After step 1, Stu "lined up" the ray with the 30° mark on the protractor scale and drew an angle as shown below.



Stu claimed that m(∠YXZ) = 67°. Was he correct? Why or why not?

2. Are there other ways of drawing an angle of 67°? If so, describe them.

3. Explain how a semi-circular protractor may be used to draw an angle whose measure is 240°.

4. Explain how a circular protractor may be used to draw an angle whose measure is (a) 121° (b) 220°.

EXERCISES

1. Using a protractor, draw angles whose measure is:

   (a) 78° (c) 127°

   (b) 323° (d) 254°

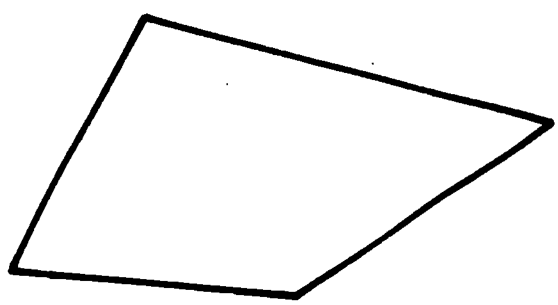
2. Using only a straightedge, draw an angle which you believe to have a measure of:

   (a) 49° (c) 153°

   (b) 300°

3. Check the closeness of your guesses in #2 by measuring each angle drawn and computing the difference between the measured size and the size you attempted to draw. (If you came within 10°, you did a very good job.)

4. Using a ruler and protractor, make a drawing which is the same size and shape as the one below.




FORTY-TWO STEPS DUE NORTH...
TWENTY-FIVE DEGREE TURN EAST...
5 STEPS AND JUMP!
155 DEGREE RIGHT TURN... TEN STEPS DUE SOUTH
STOP!
MY TREE!
YIPPEE!
AS YOU CAN SEE IT WAS WELL WORTH IT!!

COMPASS BEARINGS

The positions of ships, planes, forest fires and tornadoes, are often given in terms of their distance from a known point, and their direction.

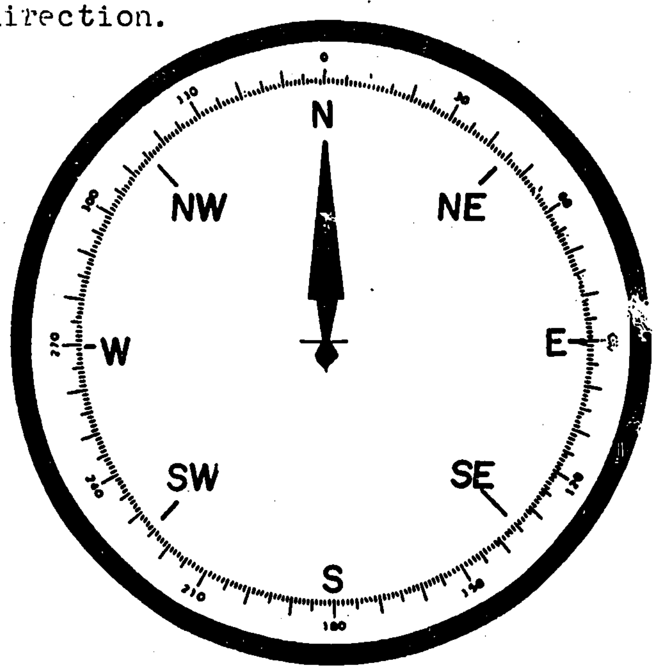
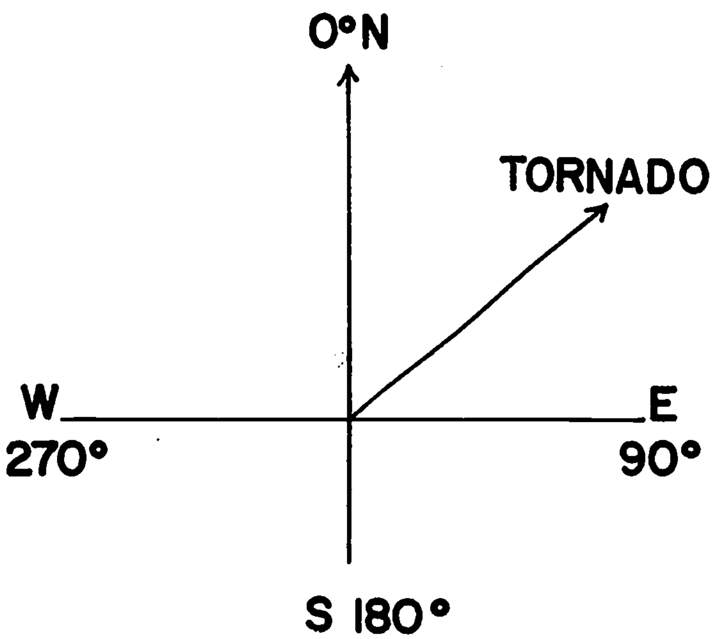


Figure 1

The magnetic compass is used to establish the north line. The direction of an object is then obtained by measuring the angle between the north line and the line joining the fixed point to the object.

Figure 2



The direction angle is measured in degrees from north in a clockwise direction. This angle is called the bearing of the object from that particular point. Thus a bearing of 90° would be due East.

Estimate, in degrees, the bearing of the tornado in Figure 2.

A large forest has several lookout stations to help guard against forest fires. When a lookout spots a fire he immediately takes its bearing from his station.

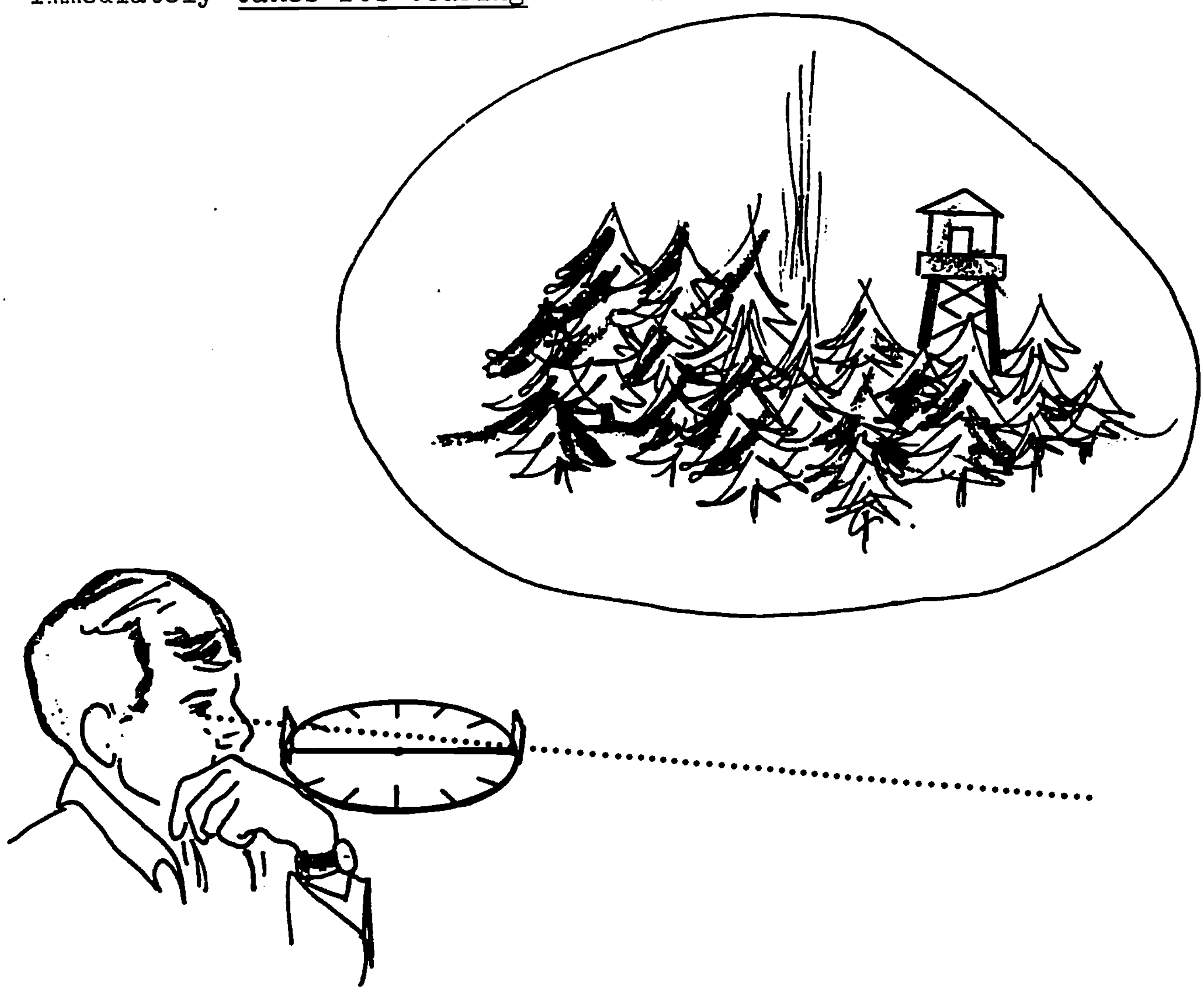


Figure 3

He then calls a second station. The second station takes the fire's bearing from his position. These two readings are plotted on a map and the fire is pinpointed.

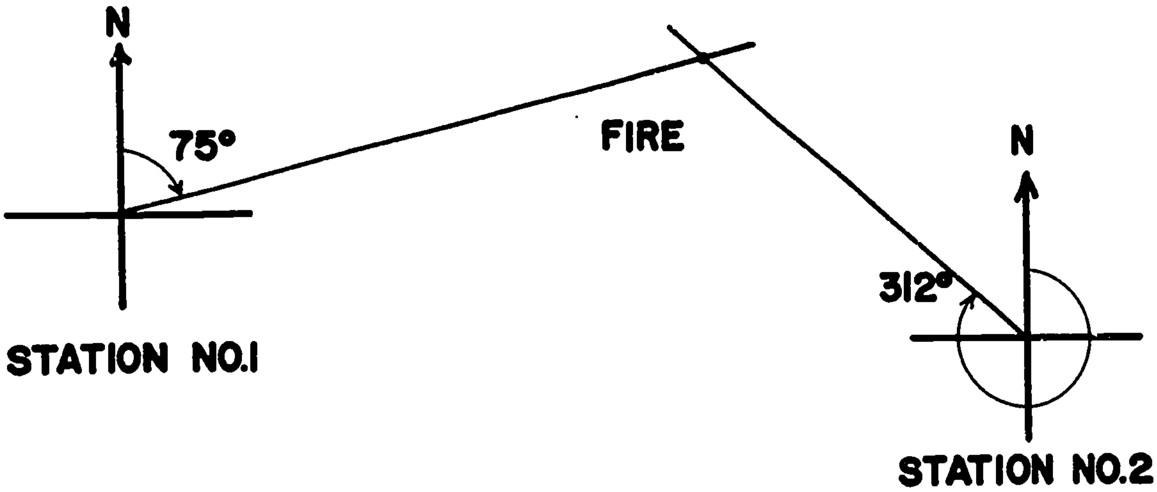
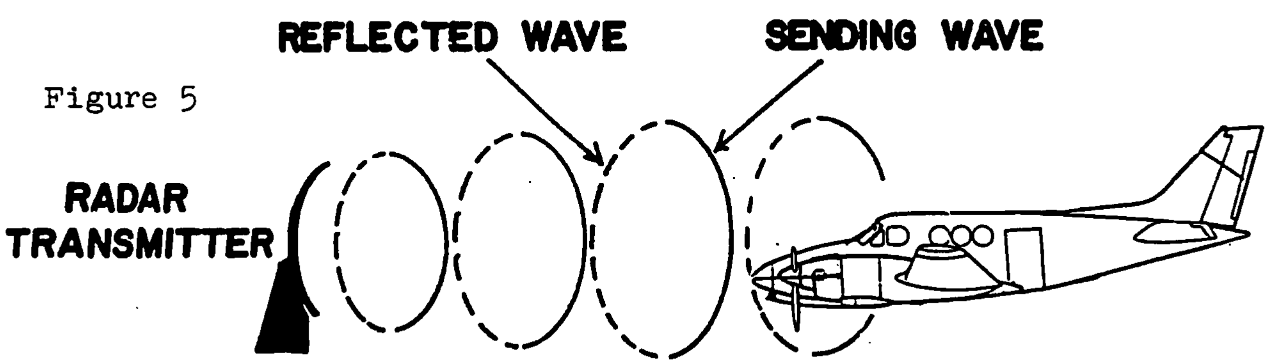


Figure 4

A radar set is used to obtain distance and direction of objects like ships, planes or tornadoes. The radar set sends out radio waves which bounce off objects they meet and return to the set.

Figure 5



The bearing is found from the direction of the radar antenna when the waves are returned. The distance is computed from the time taken by the wave to reach the object and return to the set. (Radio waves travel 186,272 miles per second.)

An object picked up by radar will appear as a bright spot on the radar screen in a position corresponding to its actual position.

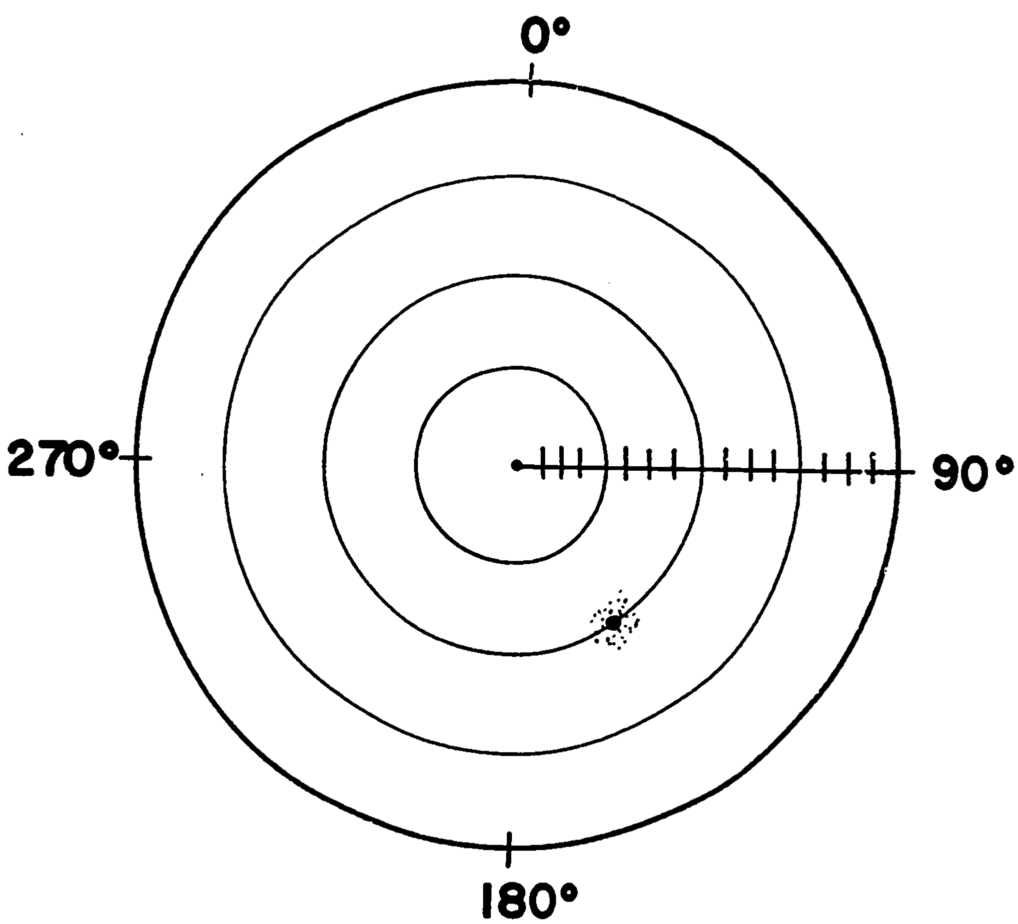


Figure 6

✓ POINT

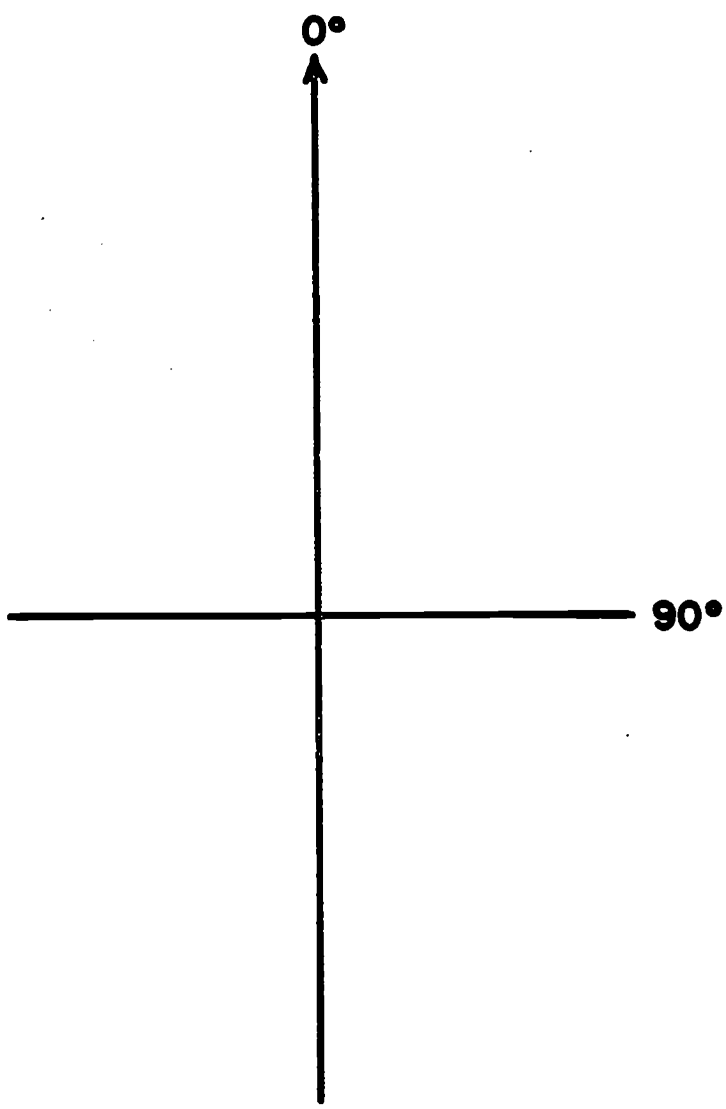
1. What is a bearing?

2. What is the measure of the angle corresponding to the south-east direction?

EXERCISES

1. Draw two perpendicular lines to represent the directions N., S., E., and W. as in Figure 2, page 93. On this diagram draw arrows to indicate the following bearings.

   (a) 135° (b) 68° (c) 317° (d) 275°



2. What is the final bearing after:

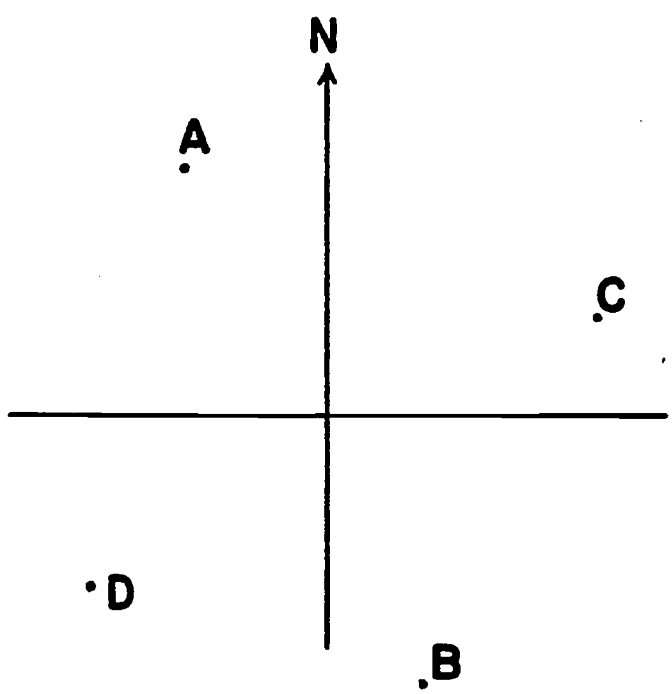
   (a) a clockwise turn of 100° from the direction S.;

   (b) a clockwise turn of 300° from the direction E;

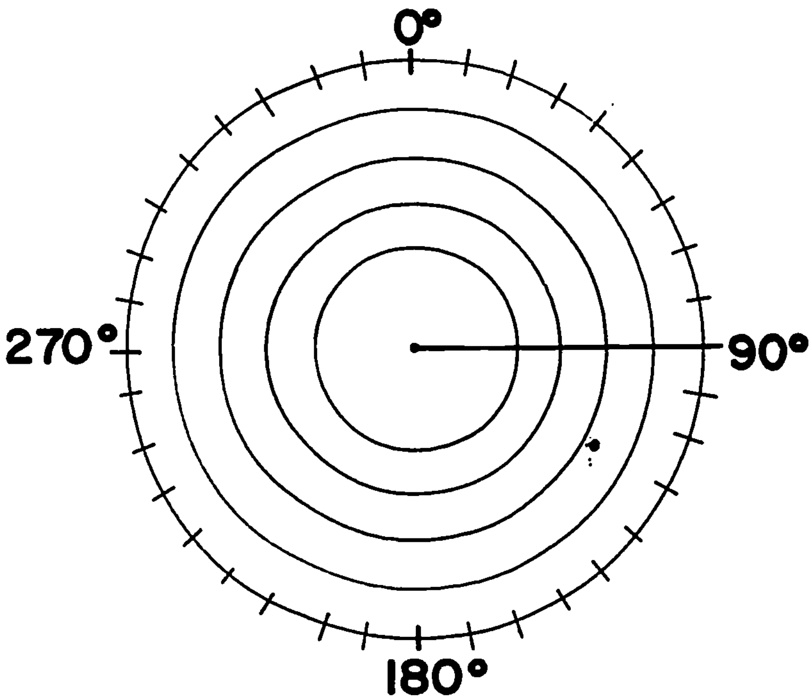
   (c) a counterclockwise turn of 43° from the direction S.W.;

   (d) a counterclockwise turn of 80° from the direction N.E.?

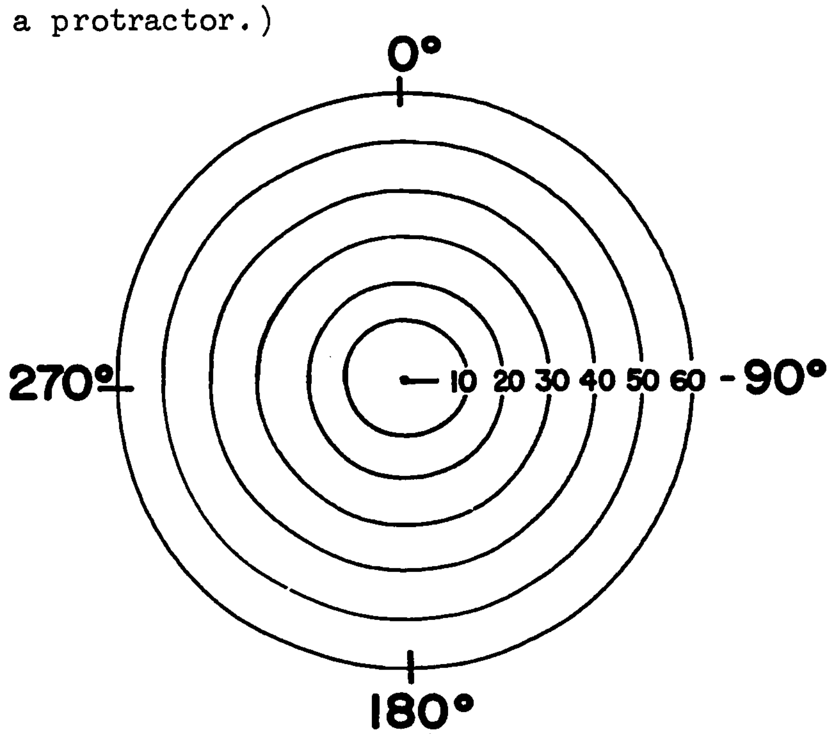
3. Use your protractor to obtain the bearings of points A, B, C and D on the diagram below.



4. A tornado has been picked up on the radar screen shown below. What is its bearing?



5. The radar set has picked up a ship. The ship's distance is 50 miles, bearing 230°. Draw a dot on the radar screen below to show the ship's position. (Use a protractor.)



6. Two lookout towers have phoned in information on a forest fire. Station No. 1 reports the fire's bearing as 110°. Station No. 2 reports the fire's bearing as 263°. Use a protractor and straightedge to plot the position of the fire on the diagram below.

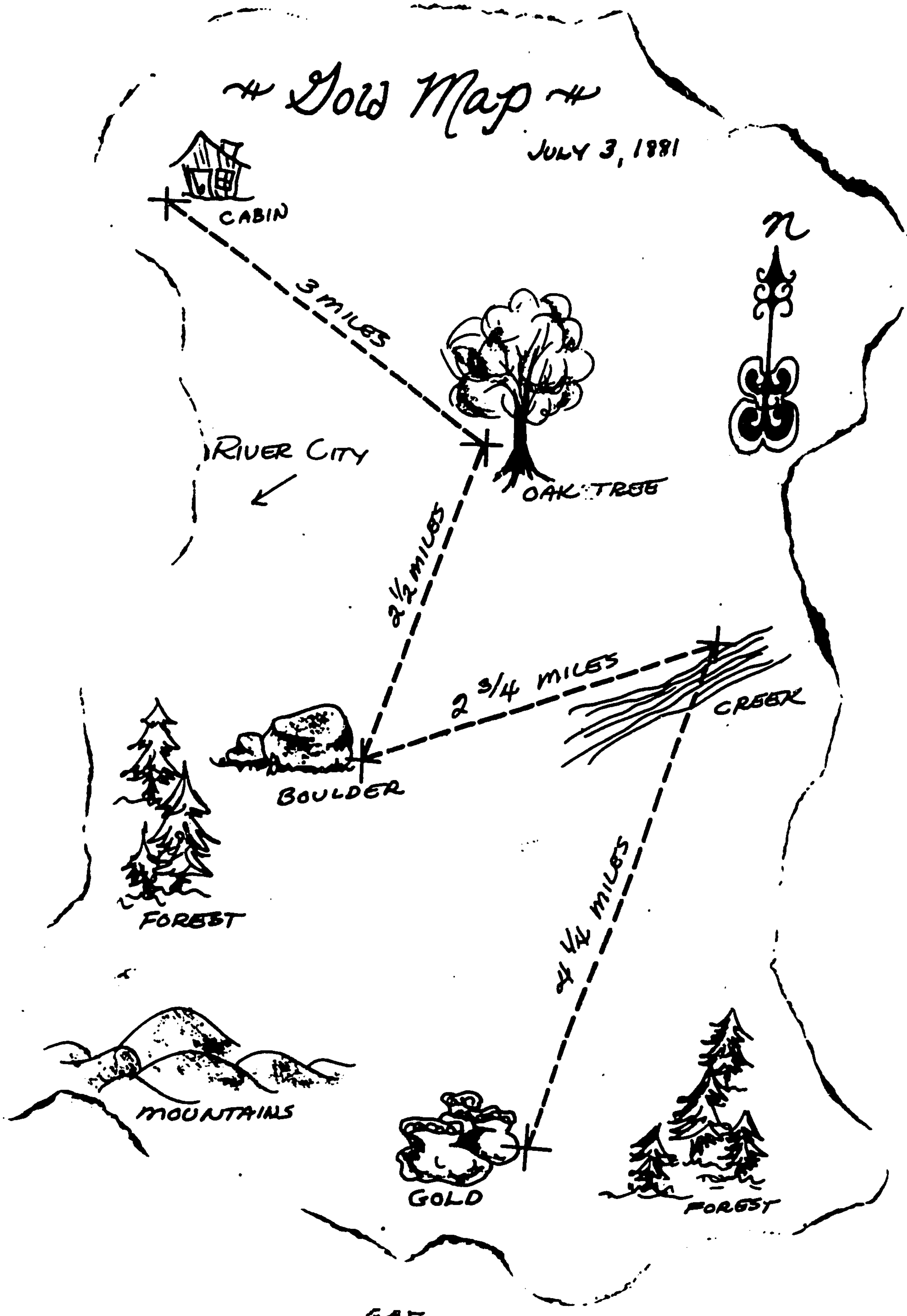
7. Old Miner Tom buried his gold and drew a map of its location. He put the distances on the map, but left off the bearings. Use your protractor to find the bearing of each point along the path.

   (a) From the Miner's cabin to the large Oak tree.
   Bearing __________

   (b) From the large Oak tree to the Boulder.
   Bearing __________

   (c) From the Boulder to the Creek.
   Bearing __________

   (d) From the Creek to the Gold.
   Bearing __________

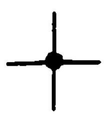

Gold Map
JULY 3, 1881
CABIN
3 MILES
n
OAK TREE
RIVER CITY
2½ MILES
2 3/4 MILES
CREEK
BOULDER
FOREST
4 1/4 MILES
MOUNTAINS
GOLD
FOREST

8. Plot the following course starting with point A below.

   (1) From point A move 3" at a bearing of 45°. Label this point B.

   (2) From point B move $1\frac{1}{2}$" at a bearing of 350°. Label this point C.

   (3) From point C move $2\frac{1}{4}$" at a bearing of 130°. Label this point D.

A 

9. In Exercise 8, above, what is the bearing of point D from point A?

APPENDICES

A. SUPPLEMENTARY EXERCISES

B. NAMING ANGLES

C. CAN A RULER BE USED TO MEASURE ANGLES?

D. USING OTHER PROTRACTORS

E. DO PROTRACTORS HAVE TO BE CIRCULAR?

F. GLOSSARY

SUPPLEMENTARY EXERCISES

The Supplementary Exercises enclosed in this appendix are designed to help measure the student's strengths and weaknesses on some objectives relevant to work with fractional numbers.

The student is encouraged to keep track of his own progress on the enclosed chart. The teacher is encouraged to oversee the individual progress of the students.

Suggestions for when to use the Supplementary Exercises are given in the Teacher's Guide Section of Lessons 3-6, 12. Students should do these exercises on an individual basis.

These exercises are not meant for drill work. They will serve the teacher and the student best by highlighting the ideas, skills and concepts with which the student needs additional help.

PERFORMANCE CHART

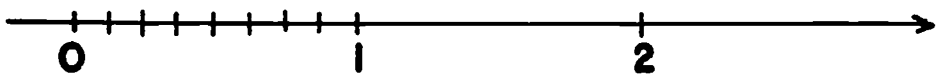
Worksheet Number	Objective	Rating: Achieved — Needs Work
1	The student will be able to demonstrate his understanding of the following meaning of fractions: $\frac{a}{b}$ means the unit interval is divided into "b" congruent parts and "a" tells the number of parts taken.	
2	The students use "other names" for one to change a fraction to higher terms.	
3	The student can apply the principle: multiplying or dividing the terms of a fraction by the same number leaves the value of the fraction unchanged.	
4	The student can apply the cross product rule to determine whether two fractions are equivalent or not.	
5	The student can apply a rule to change a whole number into a fraction with a given denominator.	

Worksheet Number	Objective	Rating: Achieved — Needs Work
6	The student is able to apply a rule to determine which of two fractions has the larger value.	
7	The student is able to apply a rule to change an improper fraction to a mixed number.	
8	The student is able to apply a rule to change a mixed number to an improper fraction.	
9	The student is able to apply a rule to add or subtract fractions.	

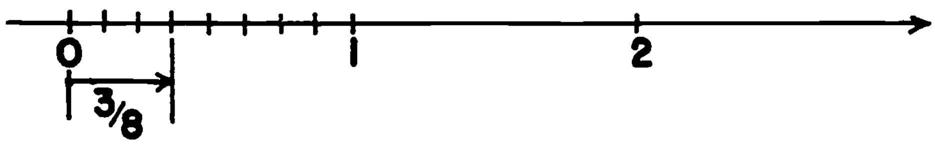
ANGLE MEASURE SUPPLEMENTARY EXERCISES

Have you ever wondered what is meant by a fraction? How do you picture the fraction $\frac{3}{8}$?

One way to look at $\frac{3}{8}$ is to take a unit length on a number line and subdivide it into 8 equal parts.



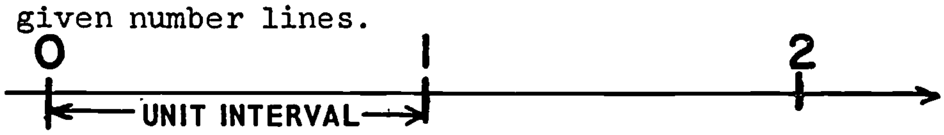
Mark off the first three parts from 0.

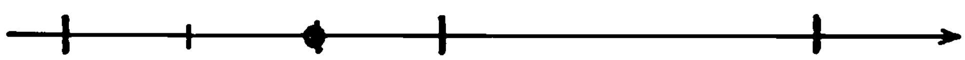


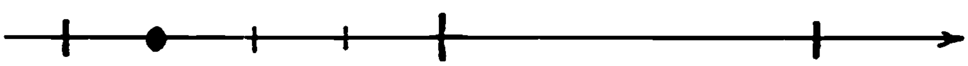
$\frac{3}{8}$ **Numerator** tells the number of equal parts from 0.
Denominator tells the number of equal parts in the unit length.

EXERCISES

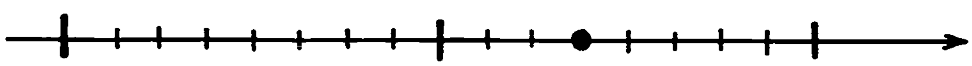
1-5. Name the fraction indicated by the mark on each of the given number lines.

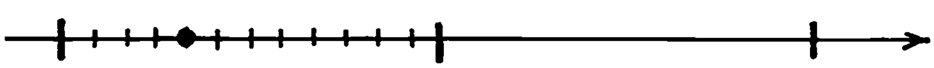


1. 

2. 

3. 

4. 

5. 

ANSWERS

1.

2.

3.

4.

5.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. Suppose you have two fractions that have different numerators and different denominators. Can these two fractions have the same value?

2. $\frac{\square}{9}$ is another name for one. What number does $\square$ represent?

3. $\triangle$ represents a "magic" number. If you take any number and multiply it by $\triangle$ you get the number you started with for your product. What number does $\triangle$ represent?

4. How many different names can you write for the number one?

You can change the numerator and denominator of a fraction and not change its value by multiplying the fraction by another name for one.

Example: $\frac{2}{3} \cdot 1 = \frac{2}{3}$ and $1 = \frac{4}{4}$

so $\frac{2}{3} \cdot \frac{4}{4} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$.

Thus $\frac{2}{3} = \frac{8}{12}$.

EXERCISES

5-7. Fill in the blanks.

5. $\frac{1}{4} \cdot \frac{?}{?} = \frac{3}{12}$

6. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{?}{15} = \frac{12}{?}$

7. $\frac{?}{5} = \frac{?}{20} = \frac{9}{?} = \frac{18}{30} = \frac{?}{10}$

8. What name for one can be used to change $\frac{3}{8}$ to 48ths?

ANSWERS

1.

2.

3.

4.

5. ___ ___

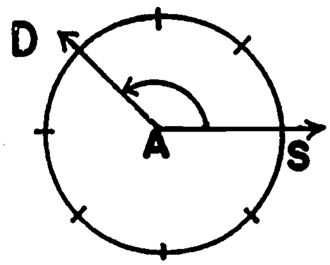
6. ___ ___

7. ___ ___
___ ___

8.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. Suppose that the circle shown below were divided into 24 congruent arcs. What would then be the measure of $\angle SAD$?



Principle for changing fractions:

Multiplying or dividing the terms of a fraction by the <u>same number</u> leaves the value of the fraction unchanged.

Examples:

(a) Change $\frac{63}{72}$ to lower terms.

$$\frac{63}{72} = \frac{63 \div 9}{72 \div 9} = \frac{7}{8}.$$

(b) Change $\frac{2}{3}$ to 24ths.

$$\frac{2}{3} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{16}{24}.$$

EXERCISES

2-10. Find the missing term so that the value of the fraction is unchanged.

2. $\frac{6}{30} = \frac{}{5}$ 3. $\frac{16}{24} = \frac{}{3}$ 4. $\frac{45}{108} = \frac{}{12}$

5. $\frac{2}{5} = \frac{}{35}$ 6. $\frac{1}{7} = \frac{8}{}$ 7. $\frac{3}{4} = \frac{}{36}$

8. $\frac{4}{6} = \frac{}{42}$ 9. $\frac{36}{64} = \frac{}{8}$ 10. $\frac{4}{7} = \frac{52}{}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

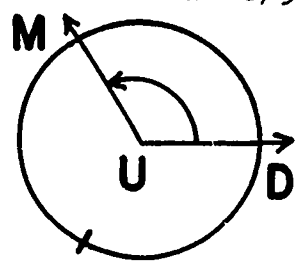
8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. John said that the measure of $\angle DUM$ was 20/30 of a turn. Sally said that its measure was 6/9 of a turn. Who was right?



The cross product rule enables you to determine whether two fractions are equivalent or not.

Examples:

(a) $\frac{12}{8}$ equals $\frac{6}{4}$ because $12 \times 4 = 8 \times 6$

(b) $\frac{3}{4}$ does not equal $\frac{7}{12}$ because 3×12 is not equal to 4×7.

EXERCISES

2-10. Determine which pairs of fractions are equivalent. Answer with $=$ or $\neq$.

2. $\frac{4}{8} \bigcirc \frac{6}{9}$
3. $\frac{8}{12} \bigcirc \frac{4}{6}$
4. $\frac{2}{3} \bigcirc \frac{5}{6}$
5. $\frac{4}{9} \bigcirc \frac{32}{72}$
6. $\frac{7}{12} \bigcirc \frac{35}{60}$
7. $\frac{3}{4} \bigcirc \frac{7}{12}$
8. $\frac{5}{8} \bigcirc \frac{3}{4}$
9. $\frac{10}{15} \bigcirc \frac{6}{9}$
10. $\frac{7}{10} \bigcirc \frac{9}{12}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Can you score 100% on this test? Try and see.

TEST. Write the following whole numbers as fractions using the given denominator.

1. $6 = \frac{\quad}{3}$ 2. $7 = \frac{\quad}{5}$ 3. $3 = \frac{\quad}{9}$

4. $13 = \frac{\quad}{11}$ 5. $9 = \frac{\quad}{9}$

If you didn't score 100%, you probably need some practice.

Try this: $6 = \frac{?}{3}$. Ask what divided by 3 gives 6 and set up this proportion:

$\frac{6}{1} = \frac{N}{3}$. Use the cross-product to obtain the answer.

EXERCISES

6-10. Write the whole number as a fraction using the given denominator.

6. $7 = \frac{\quad}{6}$ 7. $11 = \frac{\quad}{9}$ 8. $23 = \frac{\quad}{12}$

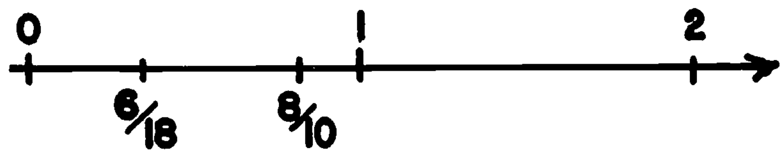
9. $15 = \frac{\quad}{8}$ 10. $13 = \frac{\quad}{5}$

ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Can you always tell which of two fractions has the larger value? How do you do it?



Remember that the larger number is always to the right of the smaller number on the number line?

EXERCISES

1-7. Answer with <, =, or >.

1. $\frac{3}{8} \bigcirc \frac{5}{8}$ 2. $\frac{9}{12} \bigcirc \frac{7}{12}$ 3. $\frac{8}{9} \bigcirc \frac{8}{5}$

4. $\frac{5}{3} \bigcirc \frac{5}{4}$ 5. $\frac{6}{8} \bigcirc \frac{18}{24}$ 6. $\frac{25}{36} \bigcirc \frac{3}{4}$

7. $\frac{5}{8} \bigcirc \frac{9}{12}$

8. Suppose two fractions have the same denominator and different numerators. Which fraction has the larger value?

9. Suppose two fractions have the same numerator and different denominators. Which fraction has the larger value?

10. If $a \cdot b > r \cdot t$, then which fraction is larger; a/r or t/b?

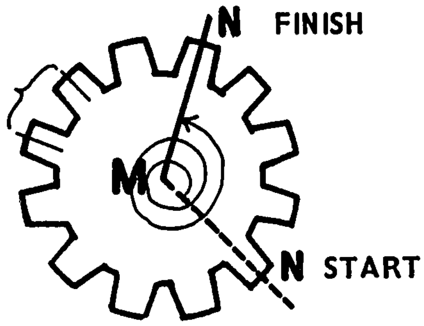
ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

Number 7

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. What angle has been swept out by segment $\overline{MN}$?



Can you change $\frac{28}{12}$ to a mixed number?

(a) $\frac{28}{12}$ means $28 \div 12$.

$$\begin{array}{r} 2 \\ 12\overline{)28} \\ \underline{24} \\ 4 \end{array}$$

Thus $\frac{28}{12} = 2\frac{4}{12}$. OR ...

(b) $\frac{28}{12} = \frac{24+4}{12} = \frac{24}{12} + \frac{4}{12} = 2 + \frac{4}{12}$

$= 2\frac{4}{12}$.

EXERCISES

2-10. Change each of the following improper fractions to mixed numbers.

2. $\frac{4}{3}$	3. $\frac{49}{16}$	4. $\frac{53}{8}$
5. $\frac{5}{2}$	6. $\frac{20}{3}$	7. $\frac{34}{5}$
8. $\frac{47}{4}$	9. $\frac{61}{7}$	10. $\frac{100}{6}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

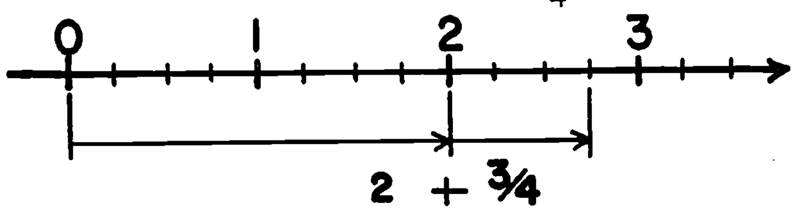
8.

9.

10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

1. How many quarter inches is $2\frac{3}{4}$ inches?



Can you change $2\frac{3}{4}$ to an improper fraction?

Examples

(a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$.

Or...

(b) $2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{8+3}{4} = \frac{11}{4}$.

EXERCISES

2-10. Change each of the following mixed numbers to improper fractions.

2. $6\frac{2}{3}$	3. $5\frac{3}{4}$	4. $13\frac{2}{5}$
5. $5\frac{1}{8}$	6. $7\frac{1}{2}$	7. $21\frac{1}{3}$
8. $4\frac{7}{12}$	9. $3\frac{9}{16}$	10. $15\frac{3}{5}$

ANSWERS

1.

2.

3.

4.

5.

6.

7.

8.

9.

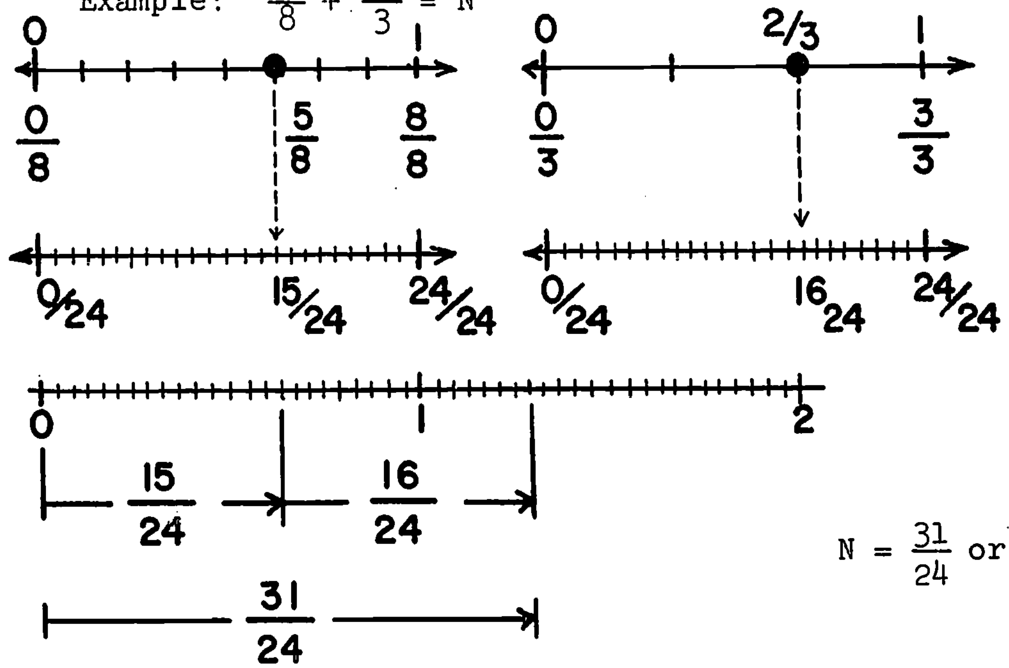
10.

ANGLE MEASURE SUPPLEMENTARY EXERCISES

Do you ever have trouble adding or subtracting fractions?

Fractions cannot be added or subtracted unless they have the same denominator.

Example: $\frac{5}{8} + \frac{2}{3} = N$



$N = \frac{31}{24}$ or $1\frac{7}{24}$

EXERCISES

1-10. Find the value of N.

1. $\frac{1}{2} + \frac{1}{3} = N$
2. $\frac{5}{8} - \frac{1}{4} = N$
3. $\frac{3}{4} + \frac{2}{5} = N$
4. $\frac{6}{8} - \frac{2}{3} = N$
5. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = N$
6. $\frac{15}{16} - \frac{1}{2} = N$
7. $\frac{5}{4} + \frac{3}{8} = N$
8. $\frac{7}{16} - \frac{3}{12} = N$
9. $\frac{9}{12} + \frac{1}{4} = N$
10. $\frac{8}{15} + \frac{1}{5} = N$

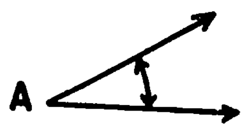
ANSWERS

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

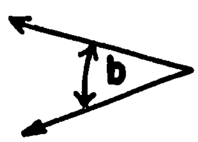
NAMING ANGLES

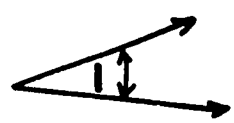
An angle may be named by...

1. ... a capital letter at the vertex, read "∠ A".

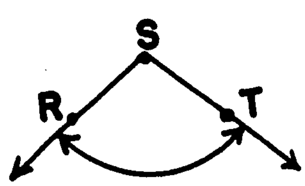


2. ... a small letter or number written inside the angle, read "∠ b or ∠ 1".

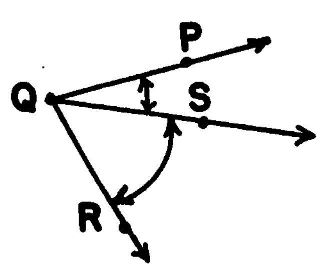




3. ... three capital letters, read "∠RST or ∠TSR". (Note that the letter at the vertex is always read in the middle.)



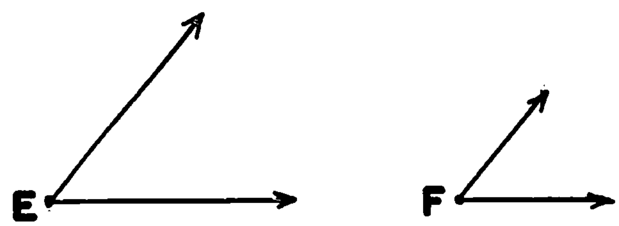
The following picture shows why three letters are sometimes needed to name an angle. If you say "∠Q", it is not clear which angle is meant. If you say "∠PQS" or "∠RQS", it is clear.



CAN A RULER BE USED TO MEASURE ANGLES?

A mathematics class was discussing the following question.

Which angle in the pair below has the greater measure?

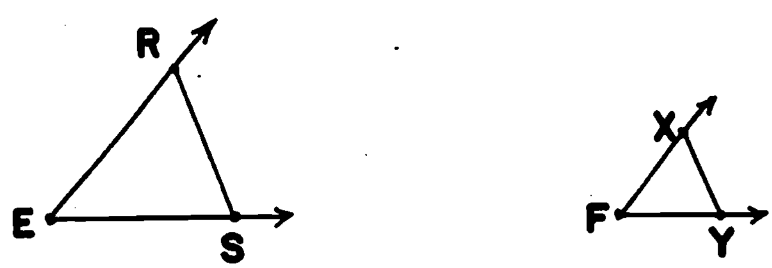


$\angle$ E or $\angle$ F

There was a disagreement when they compared $\angle$E and $\angle$F. Al said that m($\angle$ E) > m($\angle$ F) because it "opened up" more. He drew the following picture to back up his argument.

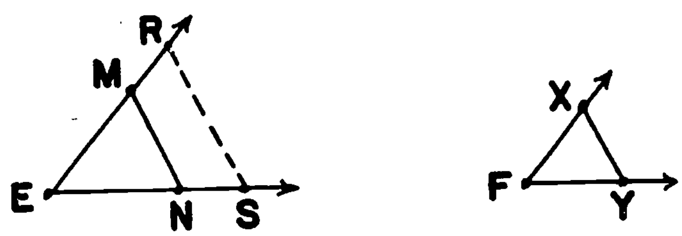
<u>Al's Argument</u>

$\overline{RS}$ is longer than $\overline{XY}$. Therefore, $\angle$ E "opens up" more than $\angle$ F and m($\angle$ E) > m($\angle$ F).



Dan disagreed with Al. Dan said that Al's drawing was unfair because he did not measure the "opening" between the sides at the "same place" on the sides.

Dan changed Al's drawing by erasing $\overline{RS}$ and drawing $\overline{MN}$ such that $m(\overline{EN}) = m(\overline{FY})$ and $m(\overline{EM}) = m(\overline{FX})$.



Dan's Argument

I measured the opening between the sides of the angles at the "same place". $\overline{MN}$ is the same length as $\overline{XY}$. Therefore, $\angle E$ "opens up" the same amount as $\angle F$ and $m(\angle E) = m(\angle F)$.

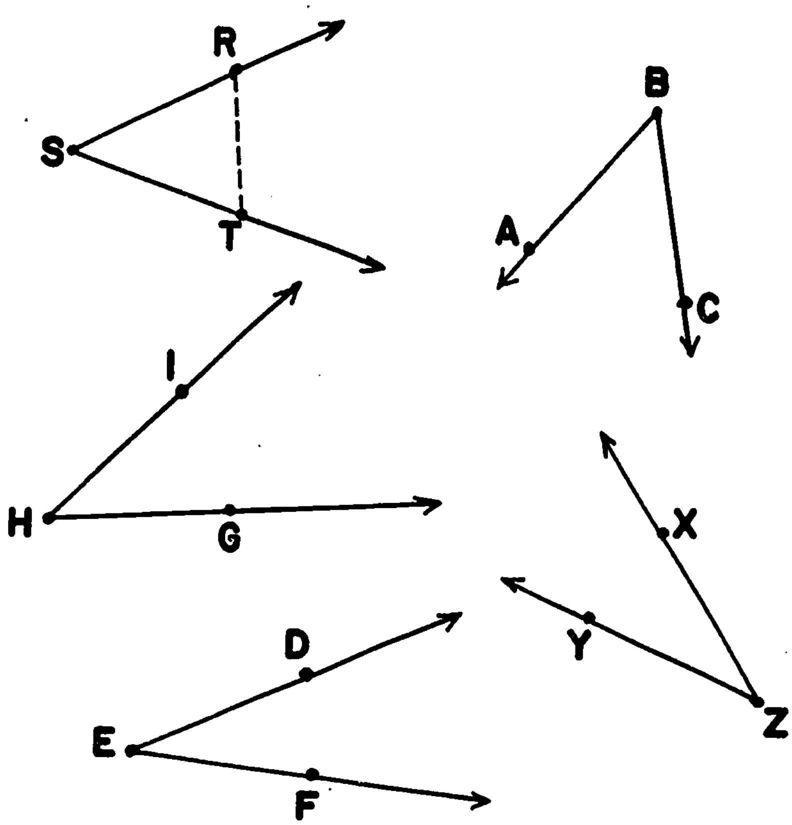
EXERCISES

1. Measure both $\angle E$ and $\angle F$ with a protractor. Make the following statement true by inserting either $>$, $<$, or $=$.

   $m(\angle E)$ ____ $m(\angle F)$

   Whose argument do you think is correct - Al's, Dan's, or neither? Why?

2. (a) Using the labeled points on the sides of the angles as endpoints, measure the "opening" of each angle on page 130 by measuring the linear distance between the sides to the nearest $\frac{1}{16}$ in. Record your results in **TABLE C-1** . (page 130).



(The linear distance to be measured for $\angle$RST is indicated by the dotted segment $\overline{RT}$.)

TABLE C-1

ANGLE	OPENING TO NEAREST $\frac{1}{16}$ in.
$\angle$RST	
$\angle$ABC	
$\angle$DEF	
$\angle$GHI	
$\angle$XZY	

(b) Arrange these five angles in order from smallest to largest.

____, ____, ____, ____, ____

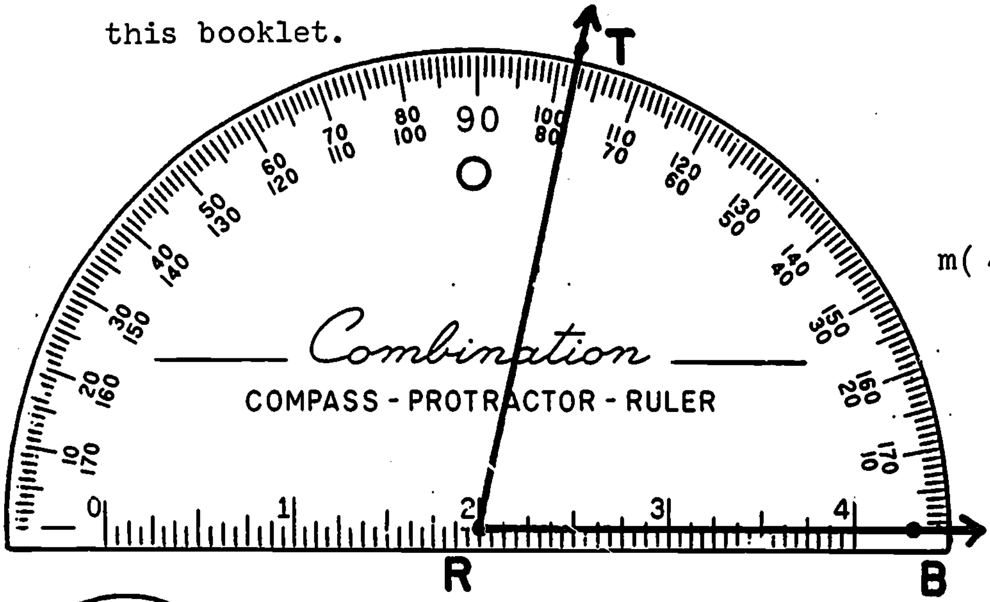
(c) Use a protractor to measure each of the angles to the nearest degree. Will these measurements give you the same order as in 2 (b) ?

3. Is it possible to measure angle size with a ruler? If your answer is yes, the measurements must be done under what conditions?

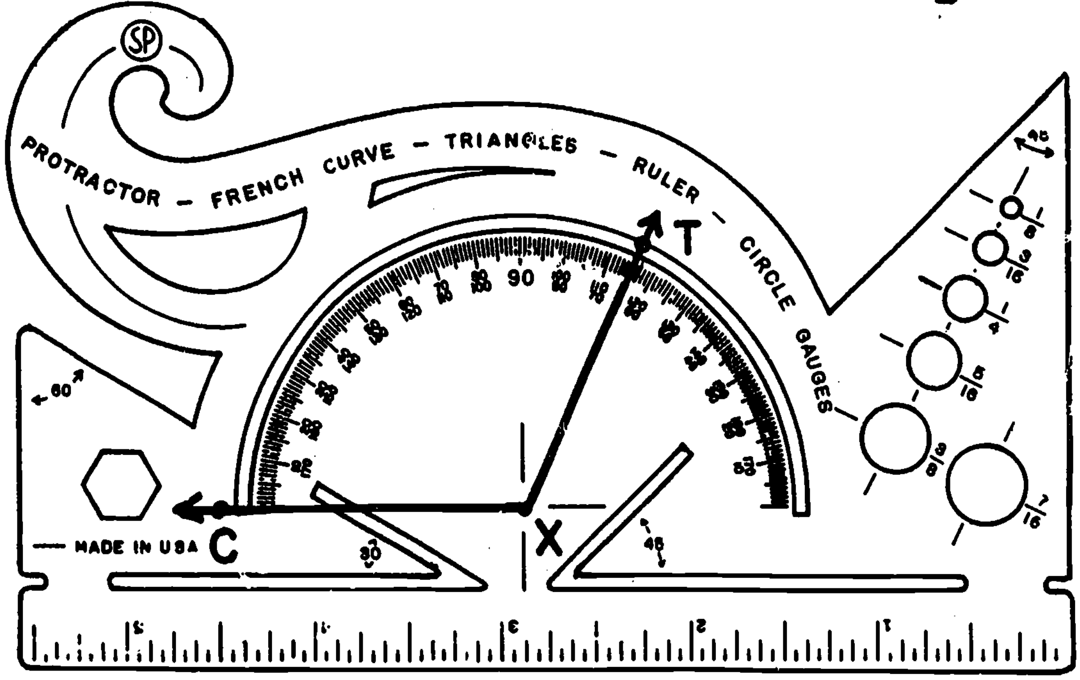
USING OTHER PROTRACTORS

Although the scale: are usually marked the same way, protractors often differ in the manner the center point of the protractor (to be placed on the vertex of the angle) is indicated.

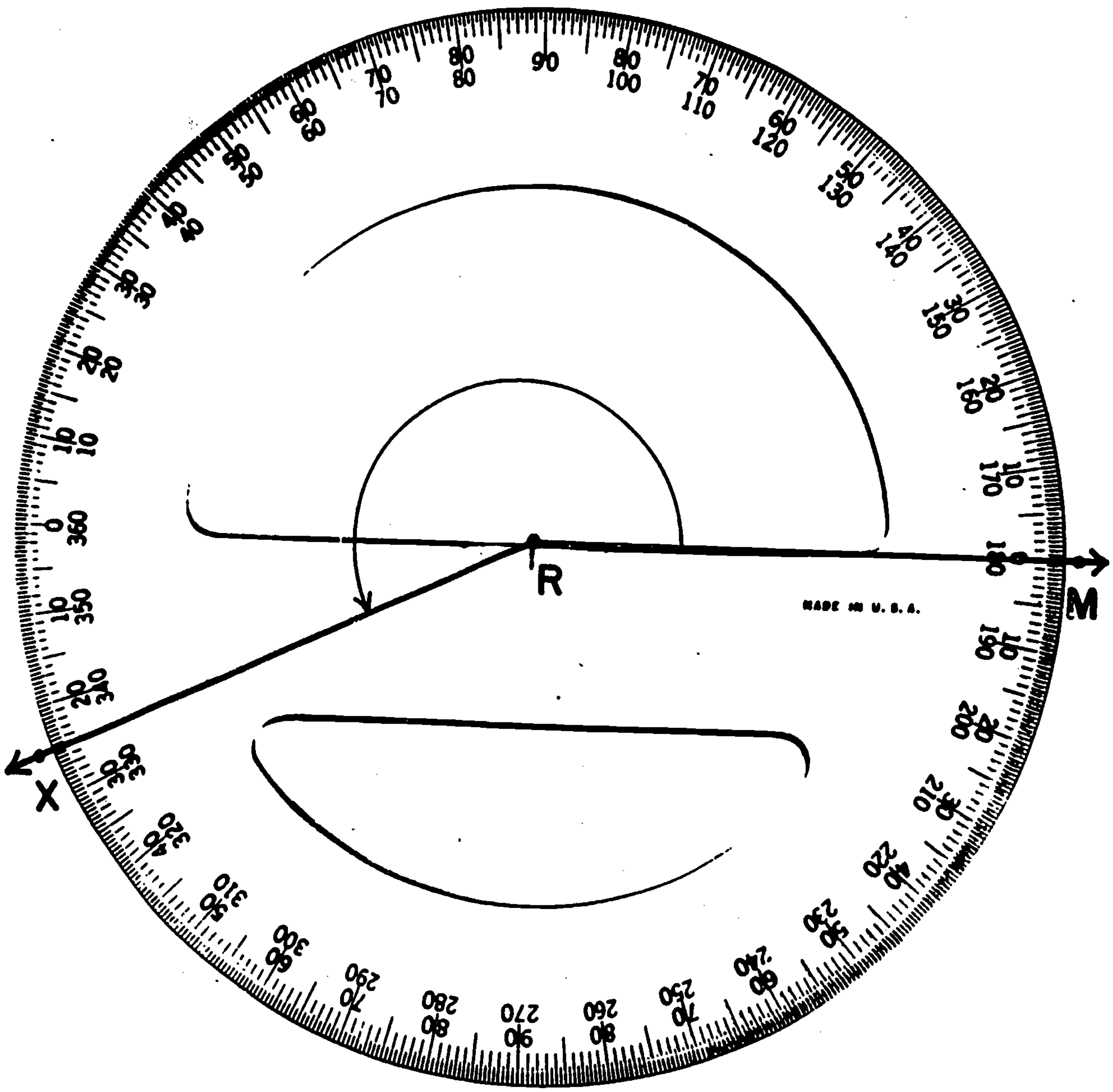
The diagrams below and on page 132 illustrate the use of some commonly used protractors whose center point is indicated in a different manner than the ones pictured in Lessons 9-11 of this booklet.



m(∠ BRT) = 78°



m(∠ CXT) = 115°



m(∠ MRX) = 205°

DO PROTRACTORS HAVE TO BE CIRCULAR?

The protractors you have used in this booklet have been either (1) circular or (2) semi-circular in shape.

TO BE EXPLORED . . .

Is it necessary for a protractor to be circular or semi-circular?

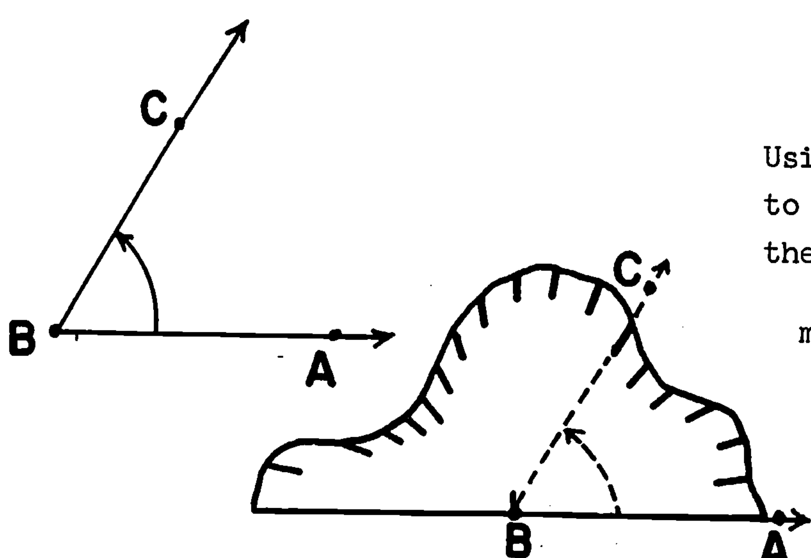
INSTRUCTIONS . . . . .

The protractors (A through G) which are to be cut out and used in Parts I and II of this activity are printed on the two inserts following page 138.

Angles 1 through 4 and **TABLE D-1** are on page 137. Angles 5 through 7 and **TABLE D-2** are on page 138.

PART I

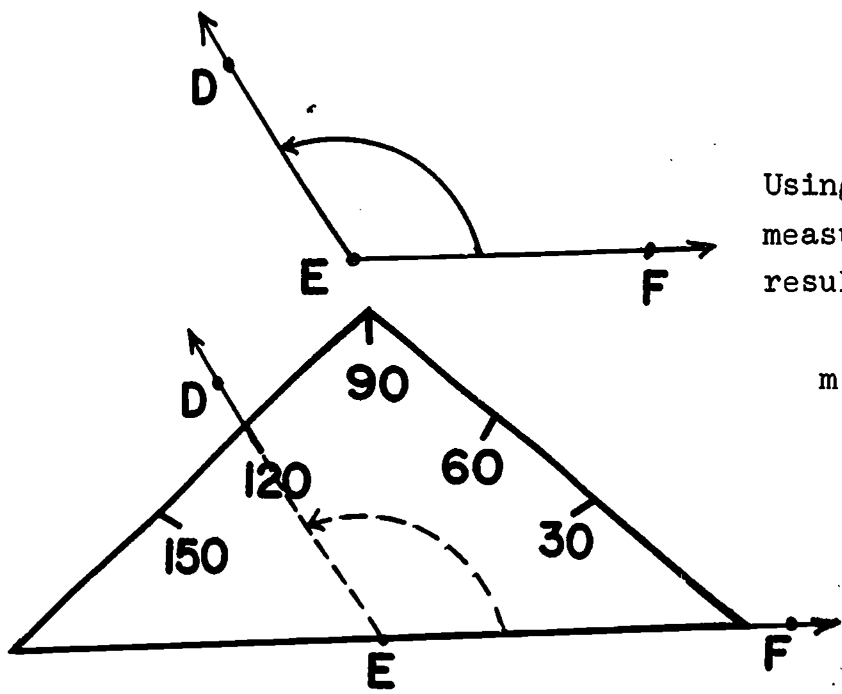
1. Look at ∠1, ∠2, ∠3 and ∠4. Estimate the measure of each angle in degrees. Record your estimates in **TABLE D-1** . (Under the heading **ESTIMATED MEASURE** .)
2. Cut out protractor A. Use this protractor to measure ∠1, ∠2, ∠3 and ∠4. Record your measurements in **TABLE D-1** .



Using protractor A to measure ∠ABC, the result is:

$m(\angle ABC) = 60°$

3. Cut out protractor B. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1**.



Using protractor B to measure $\angle DEF$, give the results:

$m(\angle DEF) = 120°$

4. Cut out protractor C. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1**.

5. Cut out protractor D. Use this protractor to measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Record your measurements in **TABLE D-1**..

Was your estimate close to the measure of each angle? Assuming your measurements were correctly done, your estimates were close if you were within 10 degrees of the measured size.

POWER QUESTIONS I

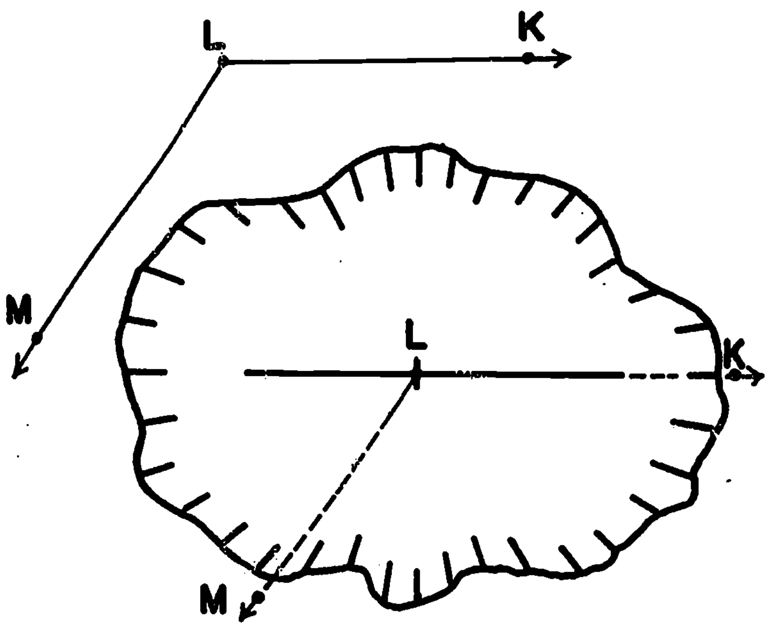
1. Were your measurements for each angle the same, regardless of the protractor used? Why or why not?

2. Explain how protractor E (with scale from 0° to 90°) could be used to measure $\angle 2$ and $\angle 4$.

PART II

1. Look at ∠5, ∠6 and ∠7. Estimate the measure of each angle in degrees. Record your estimates in **TABLE D-2**. (Under the heading **ESTIMATED MEASURE**.)

2. Cut out protractor F. Use this protractor to measure ∠5, ∠6 and ∠7. Record your measurements in **TABLE D-2**.



Using protractor F to measure ∠ KLM, the result is:

$m(\angle KLM) = 235°$

3. Cut out protractor G. Use this protractor to measure ∠5, ∠6 and ∠7. Record your measurements in **TABLE D-2**.

Was your estimate close to the measure of each angle? Assuming your measurements were accurate, your estimates were close if you were within 10 degrees of the measured size.

POWER QUESTIONS II

1. Were your measurements for each angle the same, regardless of the protractor used? Why or why not?

2. Explain how protractors A-D could be used to measure any angle from 0° to 360°.

PART III

ON YOUR OWN . . . .

Design a protractor of arbitrary shape which can be used to measure an angle whose measure is from 0° to 360°.

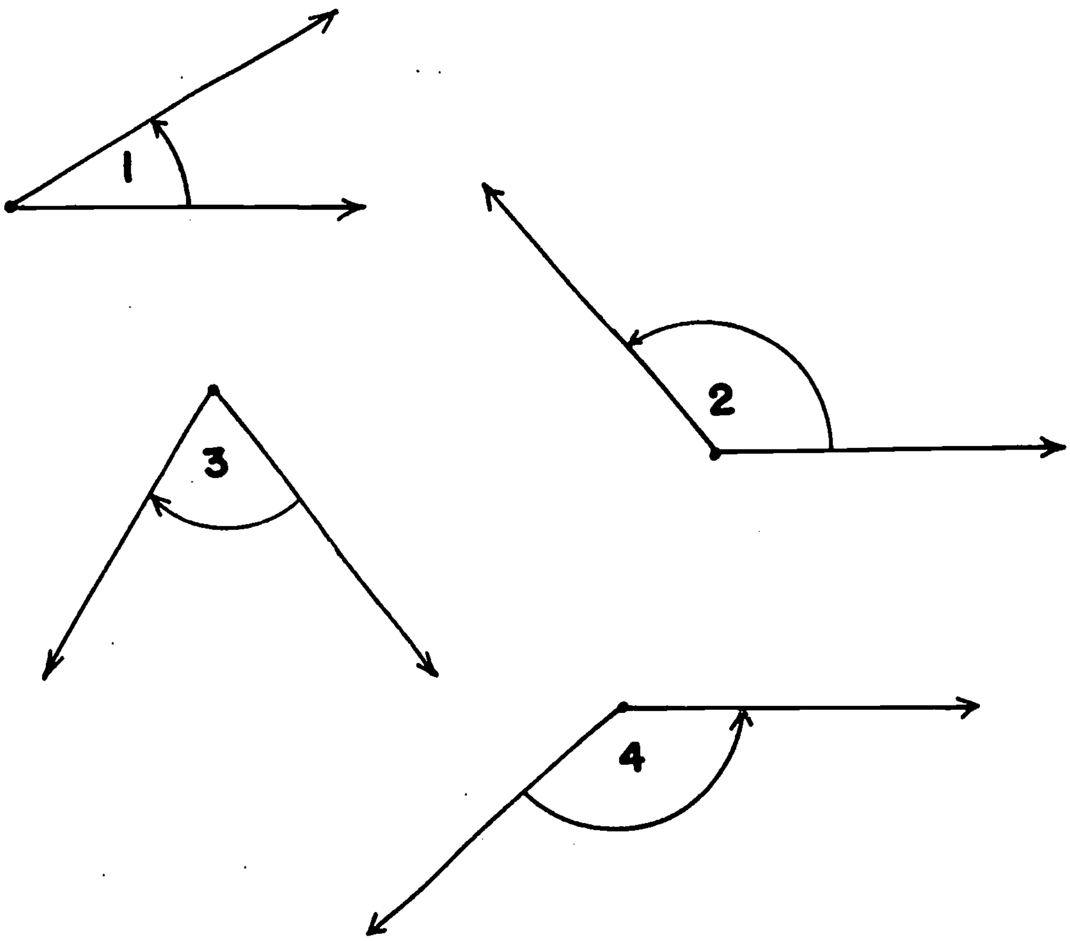


TABLE D-1

	MEASURE USING PROTRACTOR:				ESTIMATED MEASURE
ANGLE	A	B	C	D	
1					
2					
3					
4					

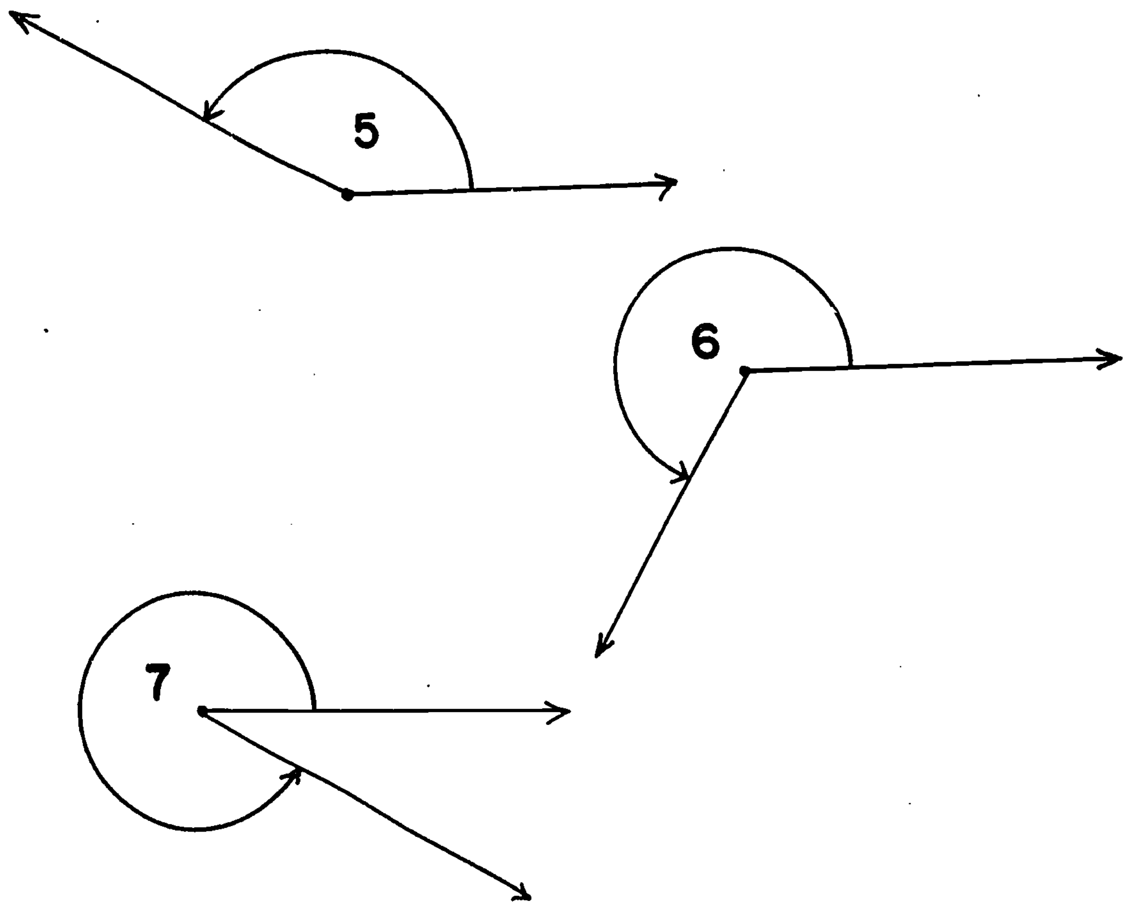
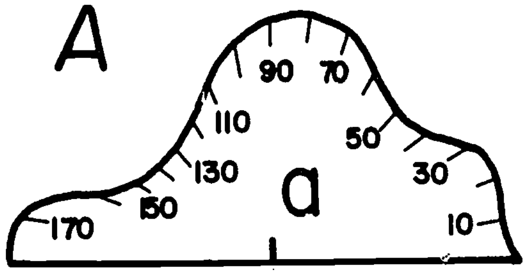


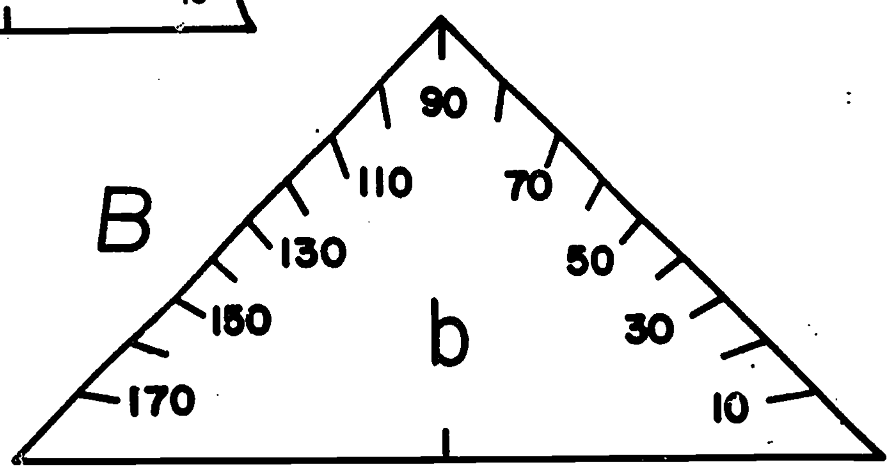
TABLE D-2			
ANGLE	ESTIMATED MEASURE	MEASURING USING PROTRACTOR	
		F	G
5			
6			
7			

INSERT A

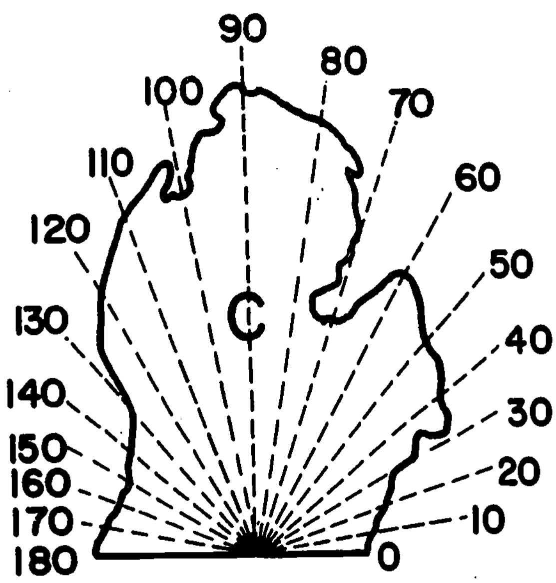
A



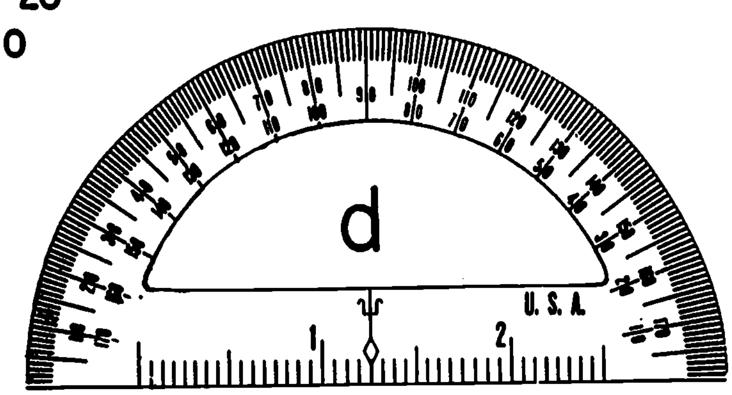
B



C

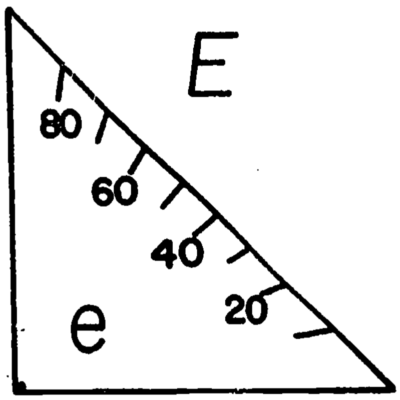


D



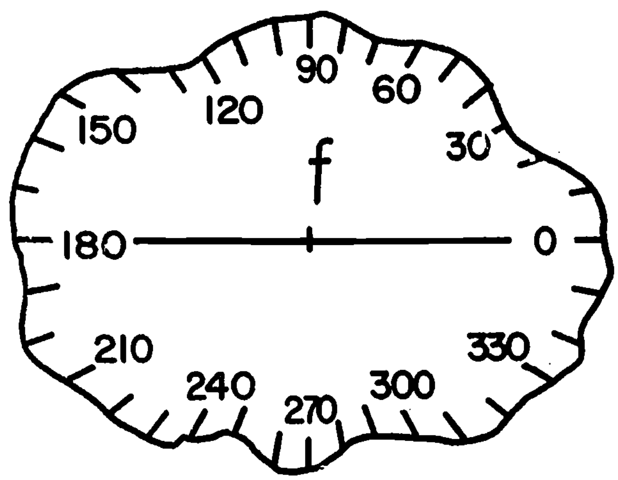
INSERT B

E



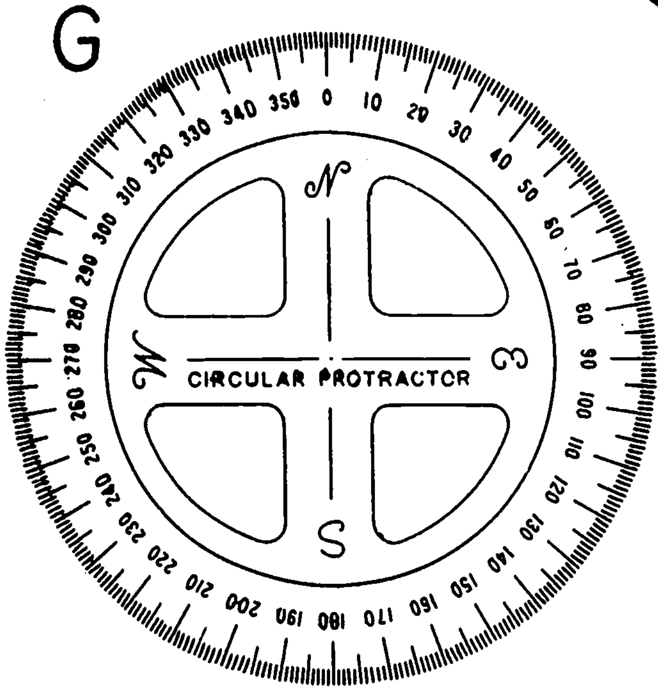
80
60
40
20
e

F



90
60
120
150
30
f
180
0
210
330
240
270
300

G

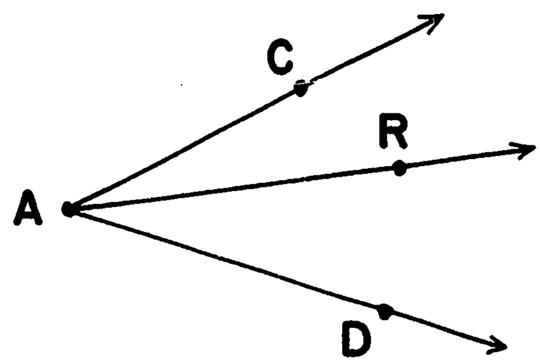


N
W
CIRCULAR PROTRACTOR
S

GLOSSARY

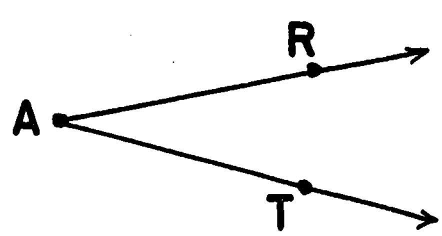
ACUTE ANGLE. An angle whose measure is greater than 0° but less than 90° is an acute angle.

ADJACENT ANGLE. In the diagram below, $\overrightarrow{AR}$ is a side of both $\angle$ CAR and $\angle$ DAR, and A is the vertex of both angles. Side $\overrightarrow{AR}$ is between sides $\overrightarrow{AD}$ and $\overrightarrow{AC}$.



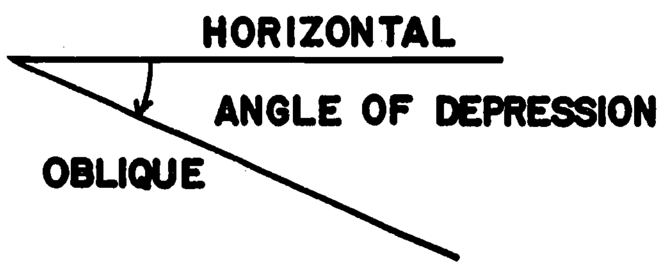
If two angles have the same vertex and a common side which lies between the other two sides, then the angles are <u>adjacent angles</u>. Notice that $\angle$ CAD is not adjacent to $\angle$ DAR or $\angle$ CAR.

ANGLE An angle is a plane figure that consists of two rays having a common endpoint.

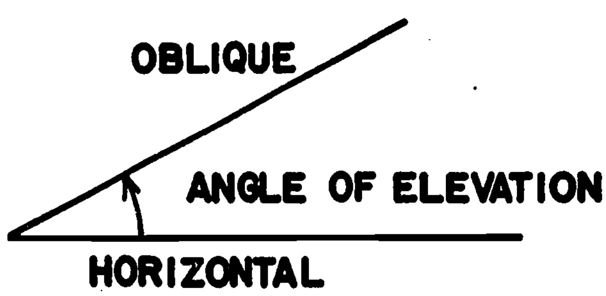


The rays are usually called the <u>sides</u> of the angle. The common endpoint is called the <u>vertex</u> of the angle.

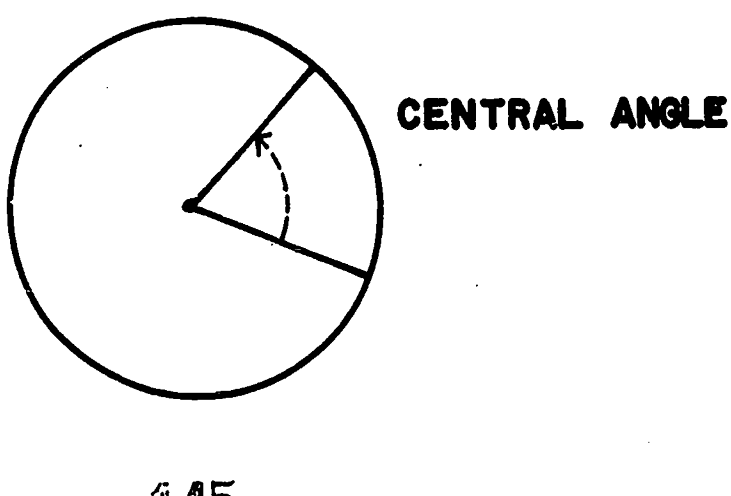
ANGLE OF DEPRESSION. The angle between the horizontal and the oblique line (neither parallel to nor perpendicular to - sloping) joining the observer's eye to some object lower than (beneath) the line of his eye.



ANGLE OF ELEVATION. The angle between the horizontal plane and the oblique line from the observer's eye to a given point above the line of his eye.



CENTRAL ANGLE. An angle determined by two radii of a circle.



COMPLEMENTARY ANGLES. Two angles whose measures sum up to $90°$. The two acute angles of a right triangle are complementary.

CONGRUENT ANGLES. Two angles are congruent if and only if they have the same measure.

CONGRUENT ARCS. Two arcs are congruent if and only if they have the same size and shape.

Two congruent central angles of the same circle determine two congruent arcs of that circle.

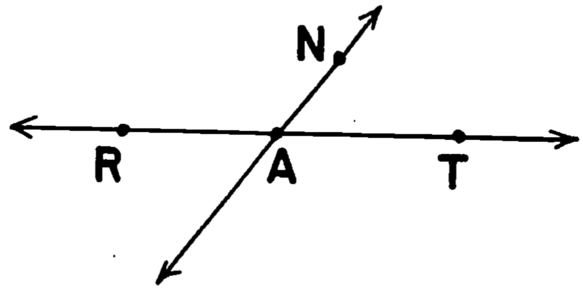
OBTUSE ANGLE. An angle whose measure is greater than $90°$ but less than $180°$ is an <u>obtuse</u> angle.

RAY. A <u>ray</u> is a subset of a line, consisting of an endpoint and all the points on the line that are on one side of the endpoint.

RIGHT ANGLE. When two lines intersect so as to form four congruent angles, then each angle is called a <u>right angle</u>. The measure of a right angle is $90°$.

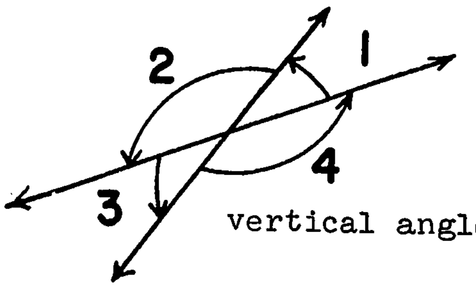
STRAIGHT ANGLE. An angle whose sides lie on the same straight line, but extend in opposite directions from the vertex. The measure of a straight angle is $180°$.

SUPPLEMENTARY ANGLES. Two angles whose sum is 180°. If two lines intersect in a point, the adjacent angles formed are supplementary.



$$m(\angle RAN) + m(\angle TAN) = 180^\circ$$

VERTICAL ANGLES. Nonadjacent angles formed by two intersecting lines are called vertical angles. Vertical angles are congruent.



vertical angles: $\angle 1$ and $\angle 3$

$\angle 2$ and $\angle 4$