DOCUMENT RESUME

ED 069 528 SE 015 339

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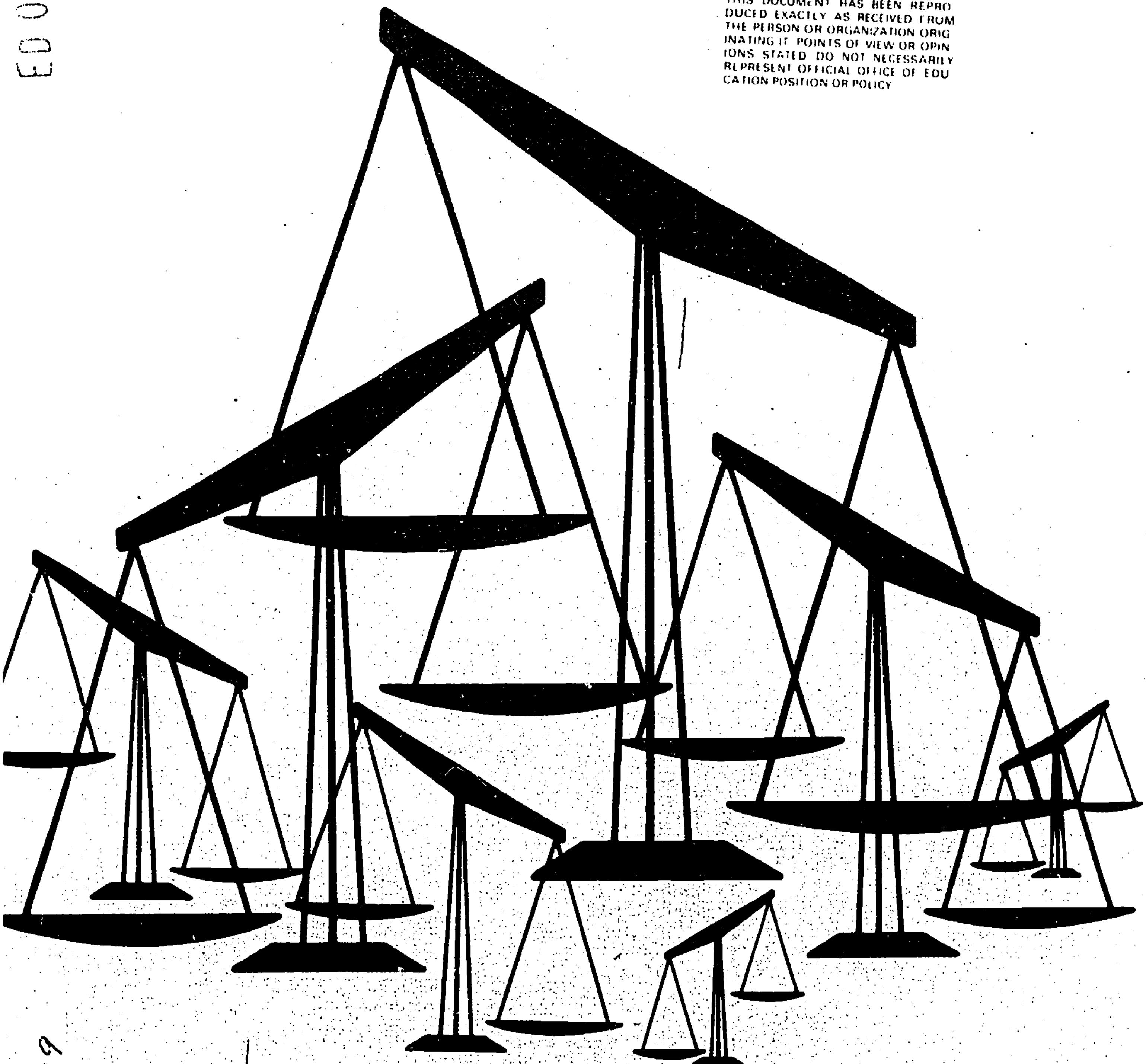
TITLE	Equa-formu-alities (Equations - Formulas - Inequalities), Teacher's Guide.
INSTITUTION	Oakland County Schools, Pontiac, Mich.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	Nov 70
GRANT	OEG-68-05635-0
NOTE	149p.; Revised Edition
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	Curriculum; Graphs; *Inequalities; Instruction; *Instructional Materials; Low Ability Students; Mathematics Education; Objectives; *Secondary School Mathematics; *Teaching Guides; Units of Study (Subject Fields)
IDENTIFIERS	Equations; ESEA Title III; Formulas Mathematics

ABSTRACT

This guide to accompany "Equa-formu-alities" contains all of the student information in SE 015 338 plus supplemental teacher materials. After each section there is a listing of terminal objectives, discussion questions, and suggested approaches. Also included is a list of necessary equipment and teaching aids. Related documents are SE 015 334 - SE 015 338 and SE 015 340 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

EQUA•FORMU•ALITIES



TEACHER'S GUIDE

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EQUA•FORMU•ALITIES

(EQUATIONS - FORMULAS - INEQUALITIES)

OAKLAND COUNTY MATHEMATICS PROJECT

All units have benefited from the combined attention of the entire project staff. The major writing responsibility for this unit was handled by

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REVISED EDITION — NOVEMBER, 1970

This unit was prepared under the auspices of U.S.O.E. Grant 68-05635-0, Title III, E.S.E.A.

PREFACE

BASIC ASSUMPTIONS:

The assumption is made that the student has worked with, but not achieved, the objectives of this booklet. The student has some experience with equations, inequalities, and formulas. Competence in working with these is not assumed, yet a complete developmental approach is not undertaken. The attempt has been made to start at a point where all students can succeed and move forward from there. Whole number computation skills and an interest in finding easier ways to work with mathematical ideas and rules are all that the student needs as background for the unit.

OBJECTIVES

The objectives for each lesson are listed separately, lesson by lesson. Do not limit your teaching to these objectives, but rather use them as a guide to the intent of the lessons. This booklet is meant to be an introduction to Algebra without getting into work so difficult that the students become discouraged and fear Algebra units.

OVERVIEW:

The booklet starts with true and false sentences and then moves to open sentences to introduce the use of variables. In the first two lessons, the students find the value of the variable by guessing or by trial and error. Next, open sentences are represented on balance beams. The students experiment with balanced beams to learn what can be done to an equation and still have it remain balanced. Inequalities

The information under Content and Approach provides the teacher with assistance in interpreting the emphasis and direction of the lesson. The questions and comments listed under Things to Discuss are meant to be suggestive of the type of class discussion which would be helpful in reviewing and extending the lesson. Answers are printed in proximity to the exercises.

STUDENT PROJECTS:

Lesson 11 is a collection of suggested student projects. This lesson is not designed to be taught as a single block. Rather, the teacher may choose selected projects and use them throughout the booklet to reinforce points or to vary the classroom routine.

EQUIPMENT LIST:

Item	Lesson Used
Balance Beam	3, 4, 5
Balance Beam Weights	3, 4, 5
Assorted Circular Objects	11
Flexible Tape Measure	11
Ruler or Yardstick	11
Trundle Wheel	11
Stop Watch	11
Fahrenheit Thermometer	11
Celsius (Centigrade) Thermometer	11
Ice Cubes	11

TABLE OF CONTENTS

TRUE OR FALSE

Study the following sentences.

1. Lansing is the capital of Michigan.
2. New York is the capital of England.
3. Students should have 25 hours of homework each night.
4. The moon is a planet.
5. The Statue of Liberty is a bridge.
6. Hawaii is a state.

DISCUSSION QUESTIONS

Which of these sentences are true? Which are false? How can you tell which are true?

Numbers can also be placed in sentences. Here are some examples of mathematical sentences.

$$2 + 8 = 4 + 4$$

$$3 + 4 = 6 + 2$$

$$5 + 3 = 8$$

$$3 \times 5 = 12 + 3$$

$$5 + 17 = 2 \times 11$$

$$7 < 4$$

$$2 + 6 = 2 + 6$$

$$5 < 8$$

$$3 \times 5 = 4 \times 4$$

$$15 + 4.5 < 12 + 8.3$$

$$15 \times 14 \times 0 < 1 \times 1 \times 1$$

TRUE OR FALSE

OBJECTIVES

1. The student shall be able to distinguish between true and false English and mathematical sentences.

2. The student shall be able to determine whether mathematical sentences are true or false after whole number values have been substituted for the $\square$.

3. The student will be able to recognize mathematical sentences that are true for all values of the $\square$.

CONTENT AND APPROACH

This lesson is a brief reintroduction to mathematical sentences. The students are to realize that some sentences are true, some are false and some are neither true nor false.

It has been assumed that students know the meaning of $<$ and $>$. These symbols may need to be reviewed for your class.

THINGS TO DISCUSS

Ask the students if false English sentences are ever printed in newspapers or magazines. Discuss how false advertising may use false sentences.

THOUGHT QUESTIONS

On page 1, some of the English sentences are true and some are false. Some of the mathematical sentences at the bottom of page 1 are true and some of them are false.

1) Which of the matnematical sentences are true and which are false?
2) How can you tell if one of the mathematical sentences is true?

EXERCISES

In the exercises below, you are to: (1) replace the □ with the number indicated; (2) state whether the resulting sentence is true or false. Examples A, B, and C have been worked for you.

	Sentence	□ = 5	□ = 7	□ = 2
A.	□ + 5 = 12	False	True	False
B.	□ + □ = 2 × □	True	True	True
C.	□ + 5 = 11	False	False	False
1.	□ + 3 = 8			
2.	□ × 4 = 28			
3.	1 + □ + 3 = 4 + □			
4.	13 + □ = □ + 13			
5.	54 - □ = 49			
6.	35 ÷ □ = 5			

ANSWERS TO THOUGHT QUESTIONS

True Sentences from page 1

$5 + 3 = 8$

$3 \times 5 = 12 + 3$

$15 + 4.5 < 12 + 8.3$

$5 + 17 = 2 \times 11$

$15 \times 14 \times 0 < 1 \times 1 \times 1$

$5 < 8$

$3 \times 4 = 6 \times 2$

False Sentences from page 1

$3 \times 5 = 4 \times 4$

$2 \times 6 = 2 + 6$

$2 + 8 = 4 + 4$

$7 < 4$

ANSWERS TO EXERCISES

1.	True	False	False
2.	False	True	False
3.	True	True	True
4.	True	True	True
5.	True	False	False
6.	False	True	False

EXERCISES (Continued)

	Sentence	$\square = 5$	$\square = 7$	$\square = 2$
7.	$5 + (\square + 3) = (5 + \square) + 3$			
8.	$\square = 1$			
9.	$\square \times 7 = 39$			
10.	$\frac{\square}{\square} \times 0 = 0$			
11.	$\square + \square + \square + 7 = \square + \square + \square + \square$			
12.	$5 \times (\square + 8) = (5 \times \square) + (5 \times 8)$			
13.	$\square + 5 = \square + 7$			

DISCUSSION QUESTIONS

1. Which exercises were true for all three values substituted?

2. Would these sentences be true for <u>all</u> number replacements of the $\square$?

3. Which sentences were true for the first two values and false for the third?

4. Would any of the sentences in (1) be true for values of the $\square$ other than 5, 7, or 2?

5. Which sentences were true for only one given value of the $\square$?

ANSWERS TO EXERCISES

7.	True	True	True
8.	False	False	False
9.	False	False	False
10.	True	True	True
11.	False	True	False
12.	True	True	True
13.	False	False	False

CONTENT AND APPROACH

These nine discussion questions are more difficult and many of the students may not be able to answer them. They are meant to challenge the better students. These questions should identify the students who are already able to work with mathematical sentences. These discussion questions are meant to be discussed in class and not assigned as homework.

DISCUSSION QUESTIONS

1. 3, 4, 7, 10, 12
2. Yes, except #10 is not true for $\square = 0$.
3. None.
4. Yes-exercises 3, 4, 7, 10 and 12 would be true for other values of the $\square$.
5. Exercises 1, 2, 5, 6 and 11.

DISCUSSION QUESTIONS (Continued)

6. Would any of the sentences in (5) be true for any other value of the $\square$?

7. Which sentences were false for all of the values substituted?

8. Which of the sentences in (7) would be true for some substitution for the $\square$?

9. Were any of the sentences examples of the commutative, associative, or distributive laws? If so, which ones?

6. No.
7. Sentences 8, 9 and 13.
8. Sentences 8 and 9 are true for some value of the $\square$.
   (A bright student might figure out that the value for 9 is $5\frac{4}{7}$.)
   Sentence 13 could not be true for any value of the $\square$.
9. Sentence 4 is an example of the commutative law of addition.
   Sentence 7 is an example of the associative law of addition.
   Sentence 12 is an example of the distributive law of multiplication over addition.

NOT ALWAYS TRUE OR FALSE

1. The U.S. Supreme Court is elected by the people of Washington, D.C.
2. He is governor of this state.
3. The President of the U.S. is elected every four years.
4. That one is your favorite.
5. He is a real winner.

DISCUSSION QUESTIONS

1. Which of the sentences above are true? _____
2. Which of these are false? _____
3. Which ones are neither true nor false? _____
4. Why can't you tell if some of these sentences are true or false?

$5 + \square = 13$

$5 + 5 + 5 = 15$

$54 - \square = 46$

$3 \times \square = 24$

$27 + \square = 43$

$14 + 3 = 28 - 10$

$5 + \square = 16 - 3$

$3 + \square - 5 = 6$

$27 + 56 + 44 + 0 = 1 - 1$

$15 > \square$

$27 - \square = 11 + \square$

$3 < \square$

$2 \times \square + 3 \times \square = 40 + \square$

NOT ALWAYS TRUE OR FALSE

OBJECTIVES

1. Given a set of true, false and open sentences the students will be able to identify the open sentences.
2. Given a replacement set for the variable in a simple open sentence, the students will be able to pick the subset that makes the sentence true.
3. Given a simple open sentence and a finite replacement set, the students will be able to determine, by trial and error, the members of the replacement set which make the sentence true.

DISCUSSION QUESTIONS

1. Number 3 is true.
2. Number 1 is false.
3. Numbers 2, 4, and 5 are neither true nor false.
4. Because you don't know what "He" or "That one" refers to.

CONTENT AND APPROACH

This lesson introduces the students to open sentences. Some of the first five English sentences show examples of sentences that are neither true nor false because it is not known what they refer to.

The open sentences should give the teacher some idea of the students' ability to solve open sentences by guessing or trial and error.

Examples of the commutative, associative and distributive properties are included in the exercises. However, understanding these properties is not a primary objective of the lesson.

DISCUSSION QUESTIONS (Continued)

5. Which of the mathematical sentences are true? Which are false?
6. Which of these mathematical sentences are neither true nor false?
7. If $\square$ is replaced by 10 in each sentence, could you tell if the sentence is true or false?
8. Which ones would be true if the $\square$ was replaced with the number 10?
9. In each sentence, what number does the $\square$ have to be replaced by to get a true sentence?

When we have a mathematical sentence such as $5 + \square = 14$ we call the $\square$ a <u>variable</u>. A sentence containing a variable is neither true nor false. When we substitute numbers for the variables, we get either a true sentence or a false sentence. We are primarily interested in those values which make the sentence true. These values are called the <u>solution</u> <u>set</u> of that sentence.

In $5 + \square = 14$ what number could replace the $\square$ to make a true sentence?

Right! A 9 in place of the $\square$ would make a true sentence because $5 + 9 = 14$.

In the sentence $2 \times \square + 3 = 11$, if the $\square$ could be replaced by any member of the set $\{1, 2, 3, 4, 5\}$, which number(s) would make the sentence true?

ANSWERS TO DISCUSSION QUESTIONS

5. True | False

True	False
$5 + 5 + 5 = 15$	$14 + 3 = 28 - 10$
$27 \times 56 \times 44 \times 0 = 1 - 1$	

6. All sentences containing a variable.

7. Yes.

8. $2\times\square + 3\times\square = 40 + \square$

   $15 > \square$ $\qquad$ $3 < \square$

9.

$$\left.\begin{array}{ll} 5 + \square = 13 & 54 - \square = 46 \\ 3 \times \square = 24 & 3 + \square - 5 = 6 \\ 5 + \square = 16 - 3 & 27 - \square = 11 + \square \end{array}\right\} \square = 8$$

$2\times\square + 3\times\square = 40 + \square$	$\square = 10$
$27 + \square = 43$	$\square = 16$
$15 > \square$	$\square = 0, 1, 2, 3, \ldots 14$
$3 < \square$	$\square = 4, 5, 6, \ldots$

EXERCISES

In the following mathematical sentences you are to find the value(s) for the variables (□'s, △'s or ○'s) that make the sentence true. All the values are to be selected from { 0, 1, 2, 3, 4, 5}.

1. $\square + 7 = 11$
2. $3 + \triangle + 2 = 10$
3. $3 \times \square > 9$
4. $4 \times \square + 7 = 23$
5. $5 \times \square = 30$
6. $3 \times \square < 5$
7. $6 - \square = \square - 2$
8. $\square - 19 = 2 - 2$
9. $45 > 47 - \square$
10. $\square + 7 = 7 + \square$
11. $\triangle + \triangle = \triangle \times \triangle$
12. $3 \times \square + 15 = 27$

The set from which we choose replacements for a variable is known as the <u>replacement set</u>. In the following exercises, use the replacement set {0, 1, 2, 3, 4, 5, $5\frac{1}{2}$, 6, 7, 8, 9, 10}. For each sentence, give all the values that make the sentence true.

1. $\square < 14 - 8$
2. $\square \times 1 = \square$
3. $2 \times (\square \times 3) = (2 \times \square) \times 3$
4. $\square \div \square < 2$
5. $2 \times \square + 6 = 17$

ANSWERS TO EXERCISES

1. 4
2. 5
3. 4, 5
4. 4
5. No solution
6. 0, 1
7. 4
8. No solution
9. 3, 4, 5
10. 0, 1, 2, 3, 4, 5
11. 0, 2
12. 4

Problems 5 and 8 have answers which are not members of the given replacement set.

ANSWERS TO EXERCISES

1. $\square = 0, 1, 2, 3, 4, 5, 5\frac{1}{2}$
2. $\square$ = Entire replacement set
3. $\square$ = Entire replacement set
4. $\square = 1, 2, 3, 4, 5, 5\frac{1}{2}, 6, \ldots 10$
5. $\square = 5\frac{1}{2}$

6. $\triangle + \triangle + \triangle = 3 \times \triangle$

7. $4 \times \square + 6 = 12$

8. $\frac{\square}{4} = 17 - 17$

9. $4 \times (\square + 2) = (4 \times \square) + (4 \times 2)$

10. $\frac{\square}{2} + \square = 12$

11. $3 \times \triangle > 25$

12. $4 \times \square + 1 = 18$

See if you can find the values of the variables that make these sentences true. Use only names of whole numbers to replace the variables (except in Exercise 1).

EXERCISES

1. $\square$ is governor of Michigan

2. $\triangle + \triangle + \triangle + \triangle = 45$

3. $\square + 5 = \square - 4$

4. $\square + 3 < 4$

5. $\square + 5 > \square + 5$

6. $(3 \times \triangle) + 1 = 19$

7. $\triangle + \triangle + \triangle = (2 \times \triangle) + \triangle$

8. $\frac{\square}{\square} < 3$

ANSWERS TO EXERCISES (Continued)

6. $\triangle$ = Entire replacement set
7. No solution
8. $\square$ = 0
9. $\square$ = Entire replacement set
10. $\square$ = 8
11. $\triangle$ = 9, 10
12. No solution

ANSWERS TO EXERCISES

1. William Milliken
2. No solution
3. No solution
4. $\square$ = 0
5. No solution
6. $\triangle$ = 6
7. $\triangle$ = any number
8. $\square$ = any number except 0

LESSON 3

WELL BALANCED SENTENCES

BALANCE BEAM

A balance beam is used to weigh items. The beam will show which of two weights is heavier, or if they weigh the same.

How does a balance beam show that two objects have the same weight? Suppose you have a group of small blocks that all weigh the same. Place five blocks on one end of the balance beam and four blocks on the other end. Will the beam balance? ______ If not, which end will rise? ____________

Because the end with five blocks is heavier, we can write $5 > 4$ or $4 < 5$. Add three blocks to the end with five blocks. Will the beam balance? ______

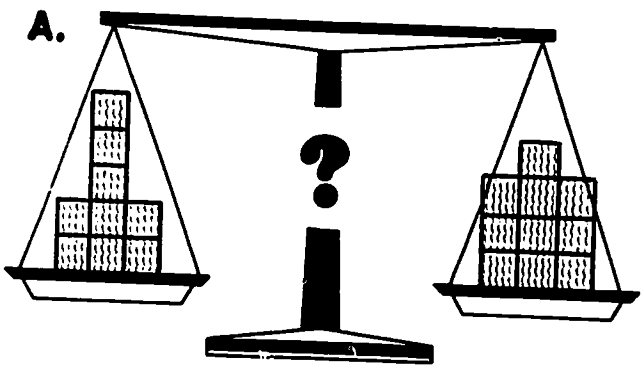
On the other end, add four blocks to the four already there. Will the beam balance now? ________

Now there are $5 + 3$ blocks on one end and $4 + 4$ on the other end. Make a mathematical sentence out of $5 + 3$ ____ $4 + 4$, by putting a $<$, $=$, or $>$ in the blank.

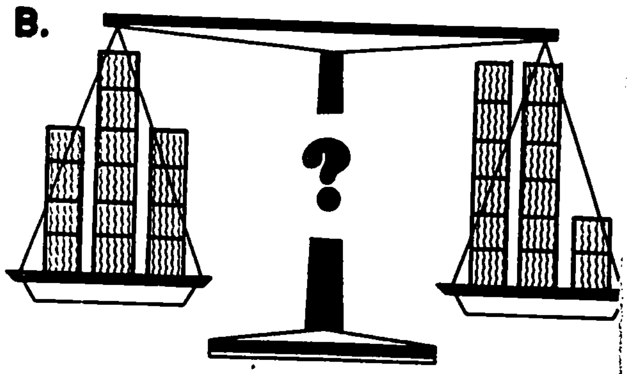
DISCUSSION QUESTIONS

In which of the following situations will the beam balance? Assume the small blocks all weigh the same.

A.



B.



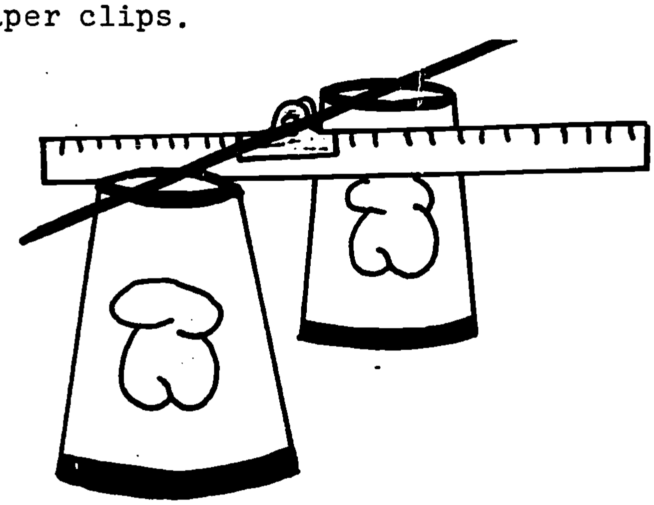
WELL BALANCED SENTENCES

OBJECTIVES

1. Given a balance beam with groups of weights equidistant from the fulcrum, the students will be able to tell if the beam will balance. If the beam does not balance, the students will be able to tell which way the beam will rotate.
2. Given a balanced beam with an unknown weight on one end, the students will be able to subtract known weights from both sides of the beam to find the value of the unknown weight.
3. Given a balanced beam with several equal unknown weights on one end, the students will be able to divide the weights on both ends of the beam until they find the value of one unknown weight.

EQUIPMENT AND TEACHING AIDS

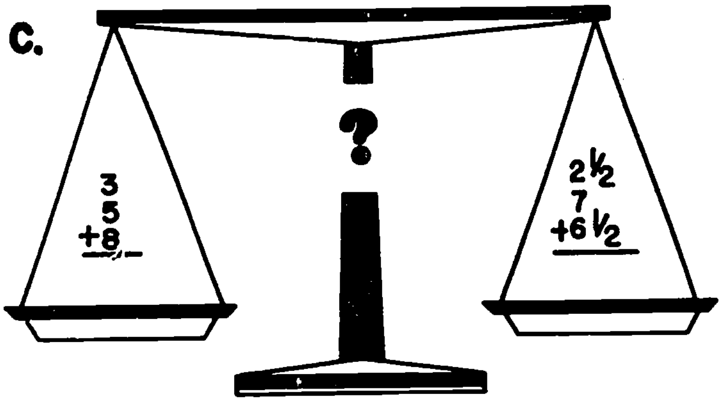
Some kind of beam balance or pan balance is desirable for this lesson. Balance beams are available commercially. The Invicta Mathematical Balance (found in many equipment catalogs) is one of these. However, a good balance beam can be made using two paper malt cups, a ruler, pencil, paper clamp and a box of paper clips.



ANSWERS

A. Beam Unbalanced
B. Beam Balanced

C.



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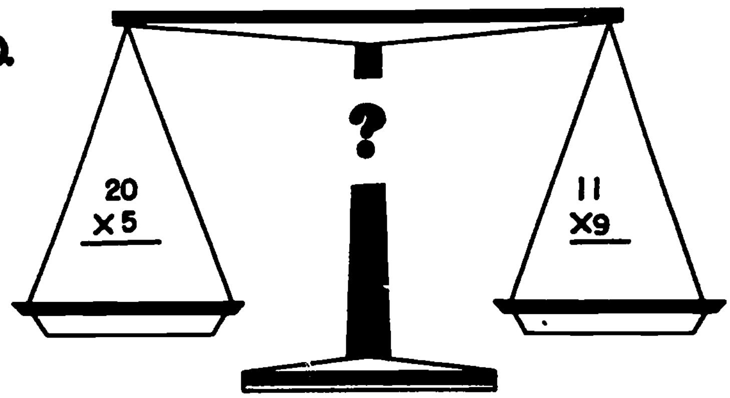
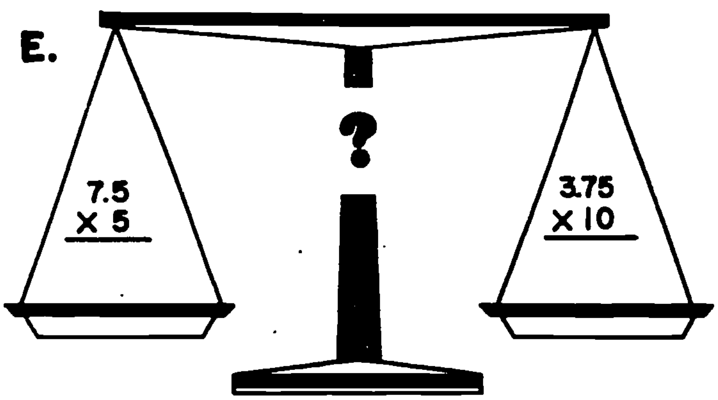


E.



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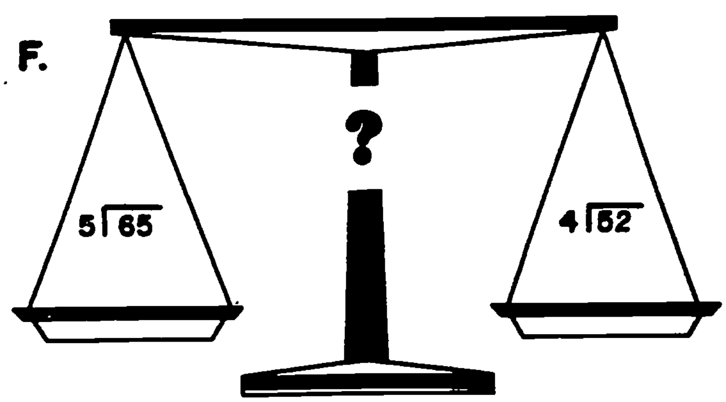


EQUIPMENT AND TEACHING AIDS (CONTINUED)

It is strongly suggested that enough balances be available so that there will be a balance for every 3-5 students.

The unknown weights can be very easily made. Several paper clips can be wrapped in tape and hung from the balance. If pan balances are used, an envelope with several paper clips sealed in it makes a good unknown weight. Don't forget to add an empty envelope on the other side of the balance to compensate for the weight of the envelope.

CONTENT AND APPROACH

The main idea the students must grasp is that each side of the balance must have the same value if it is to balance. Most students probably know this and will volunteer this information if you ask them what they know about a balance beam. Still it is important that all the students understand this simple fact. C through J emphasize this notion and give some review of basic arithmetic skills.

ANSWERS

C. Balanced

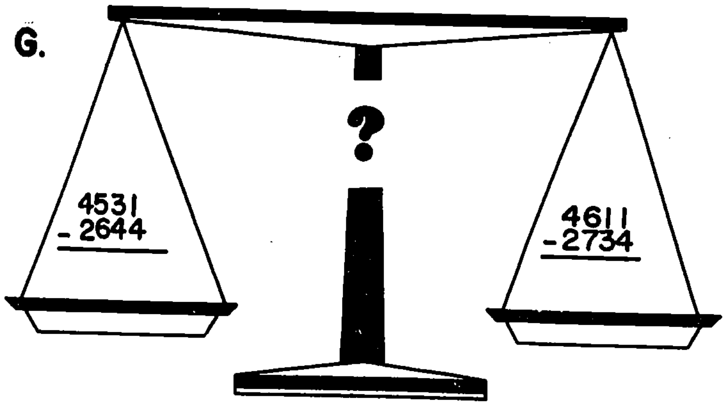
D. Unbalanced

E. Balanced

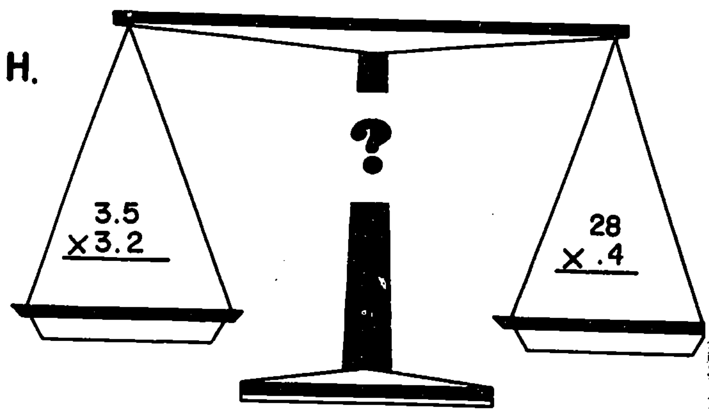
F. Balanced

LESSON 3

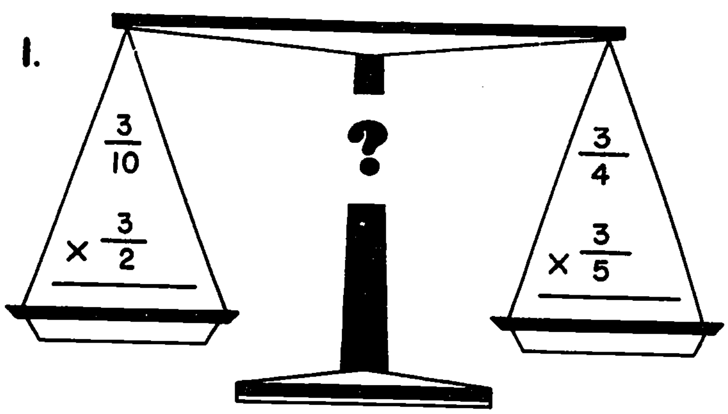
G.



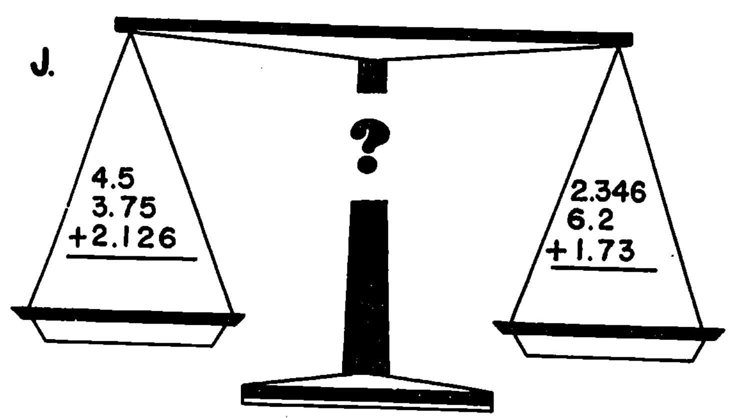
H.



I.



J.



CONTENT AND APPROACH (CONTINUED)

The real work with the balance scale begins on page 10. For this part of the lesson it is suggested that the students close their books and experiment with a balance beam.

Some experiments the class can do with balance beams:

I. Start with a balanced beam with 12 weights on each end. Do each step and observe if the beam is still balanced. After each step return to 12 weights on each end of the beam.

	Steps	Beam Balanced or Unbalanced
1.	Remove 10 weights from each side.	______________
2.	Remove $\frac{1}{2}$ the weights from each side.	______________
3.	Remove 12 weights from each side.	______________
4.	Remove 8 weights from one side and 6 weights from the other side.	______________
5.	Remove $\frac{1}{2}$ the weights from one side and 5 weights from the other side.	______________
6.	Remove two-thirds of the weights on one side and leave only 4 on the other side.	______________
7.	Add 3 weights to each side.	______________
8.	Double the number of weights on each side.	______________

ANSWERS

G. Unbalanced

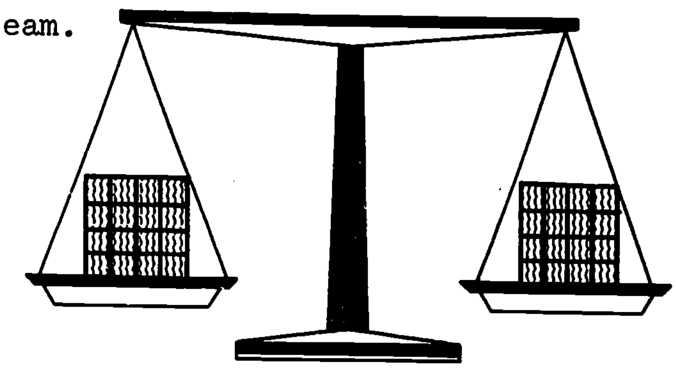
H. Balanced

I. Balanced

J. Unbalanced

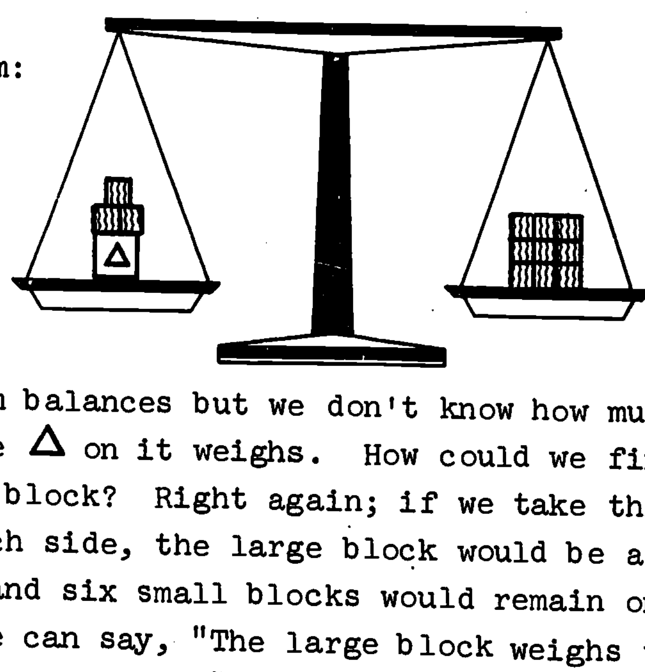
EXERCISES

Let's look at this balance beam.



1. Does it balance?
2. If we take five blocks off each side, will it balance?
3. If we take $\frac{1}{2}$ the blocks off each side, will it balance?
4. If we double the number of blocks on each side, will it balance?
5. If we add ten blocks to each side, will it balance?
6. If we take away half the blocks on one side and take away nine blocks on the other side, will the scale balance?
7. What can you do to the weights and still have the beam balance?
8. What can't you do to the weights to keep the beam balanced?

Here is another beam:



We know the beam balances but we don't know how much the large block with the Δ on it weighs. How could we find the weight of the large block? Right again; if we take three small blocks off each side, the large block would be all by itself on one side and six small blocks would remain on the other side. Then we can say, "The large block weighs the same amount as six small blocks".

CONTENT AND APPROACH (CONTINUED)

II. The next experiment on the balance beam uses both unknown and known weights. The known weights (such as paper clips) all weigh the same. Each unknown weight is equal to a whole number of known weights.

Experiments:

1. Hang an unknown weight on one end and keep adding known weights on the other end until the scale balances. Record how many known weights it took to balance that unknown weight.
   Symbol for unknown weight ________
   How many known weights does the unknown weight balance? ________
2. Find two unknown weights that will balance each other. Now put these two unknown weights on the same end and keep adding known weights to the other end until the beam is balanced. When it balances you know that two of these unknown weights balance ___ known weights. Now remove half the weights from each side. One of these unknown weights balances ___ known weights.

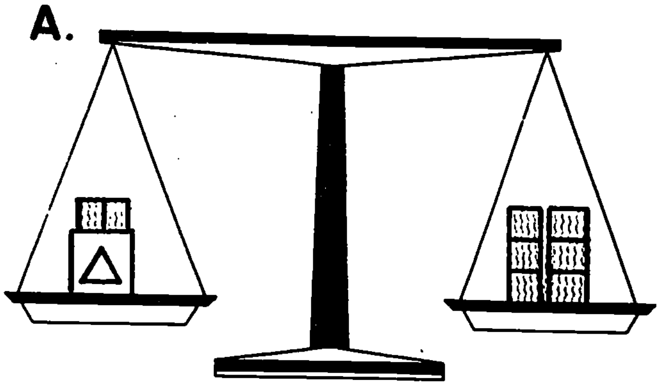
ANSWERS

1. Yes.
2. Yes.
3. Yes.
4. Yes.
5. Yes.
6. No.
7. Anything as long as you do the same thing to both sides.
8. You can't do different things to separate sides of the scale.

For each of these beams tell what you would have to do to find the weight of one of the large blocks.

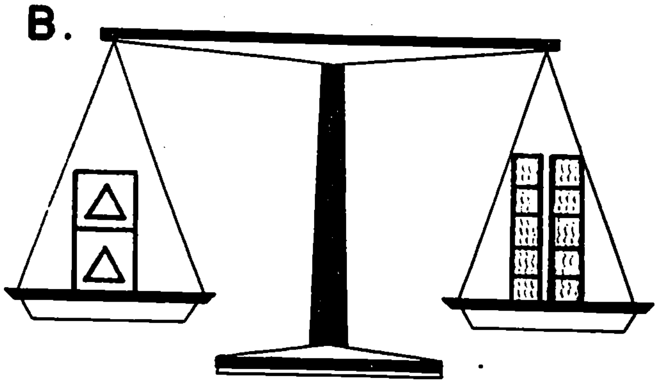
EXAMPLES:

A.



REMOVE TWO BLOCKS FROM EACH SIDE.

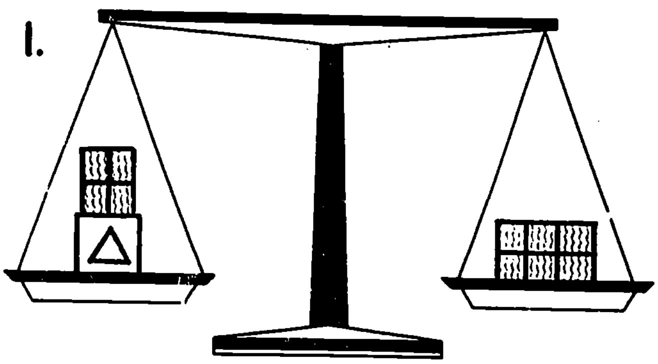
B.



REMOVE HALF THE BLOCKS FROM EACH SIDE (OR DIVIDE THE NUMBER OF BLOCKS ON EACH SIDE BY TWO.)

EXERCISES

1.



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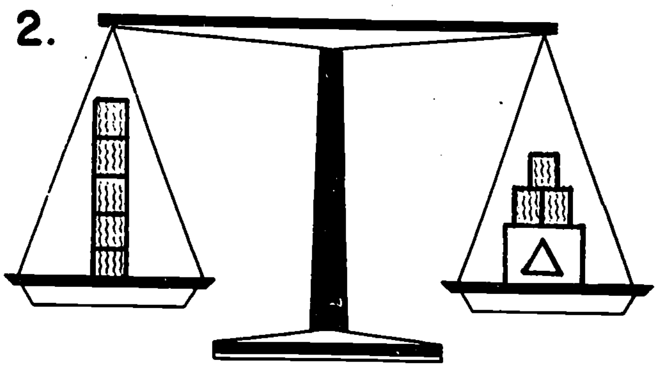
____

$\boxed{\triangle}$ = ______________

2.



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$\boxed{\triangle}$ = ______________

CONTENT AND APPROACH (CONTINUED)

Experiments with unknown weights on the balance beam (cont.):

3. Put an unknown weight and three known weights on one end of the beam and balance the beam by adding known weights on the other end.
   Now the unknown weight plus three known weights balances ______ known weights. Take three known weights from each side. Thus, the unknown weight = ___ known weights.
4. Take two different-sized unknown weights and put them on opposite ends of the balance. The beam should now be unbalanced. Now add known weights to one end of the beam until it is balanced. When it is balanced what can you say about the two unknown weights?

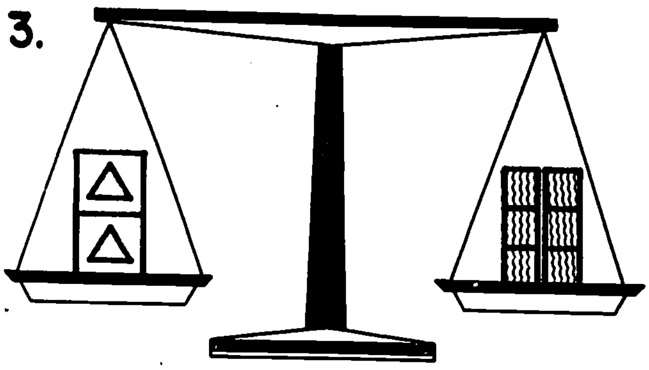
(Continued on Next Page)

KNOWN WEIGHTS: These are the many similar objects such as paper clips or bolts.
UNKNOWN WEIGHTS: These are the larger weights that are equal to some number of the KNOWN weights.

ANSWERS

1. Take four blocks from each side of the beam.
   $\boxed{\triangle} = 2$
2. Remove three blocks from each side of the beam.
   $\boxed{\triangle} = 2$

3.



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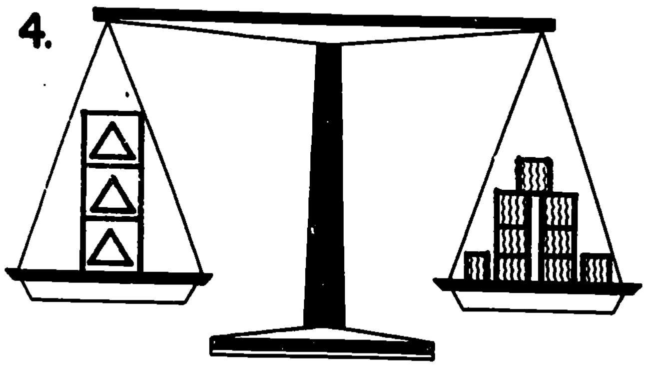
$\boxed{\triangle}$ = ________

4.



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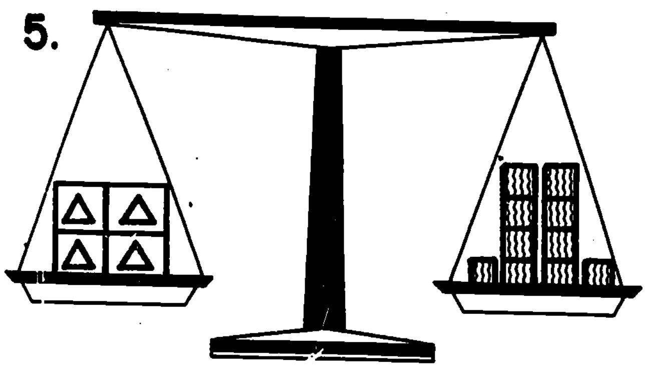
____

$\boxed{\triangle}$ = ________

5.



____

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____

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$\boxed{\triangle}$ = ________

5. Put two similar unknown and three known weights on one end of the beam and add known weights to the other end until the beam balances.

   two unknown plus three known weights = ___ known weights

   Remove three known weights from each side.

   two unknown weights = ___ known weights

   Remove half the weights from each side.

   one unknown weight = ___ known weights

6. Do the same as experiment 5, only start with three unknown and two known weights.

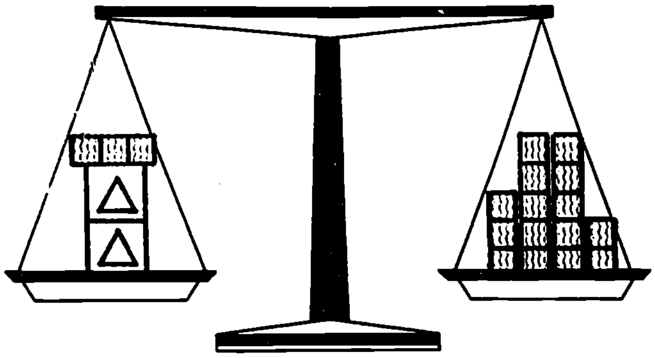
7. Have students build equations such as:

   2 unknowns + 3 paper clips = 13 paper clips

   Have them solve these equations and describe the steps they took. (This activity would probably be most useful after students have worked some of the exercises in the lesson.)

ANSWERS

3. Remove half the blocks from each side or divide each side by 2. $\boxed{\triangle} = 3$
4. Remove two-thirds of the blocks from each side or divide each side by 3. $\boxed{\triangle} = 3$
5. Remove three-fourths of the blocks from each side or divide each side by 4. $\boxed{\triangle} = 2\frac{1}{2}$



What do you have to do in this problem to find the weight of one of the large blocks? This leads us to the decision as to which to do first; take away three small blocks or take half of each amount. If we take half of each side first, we would have to take half of the three small blocks on the left side. If we take away the three blocks first, we avoid this problem.

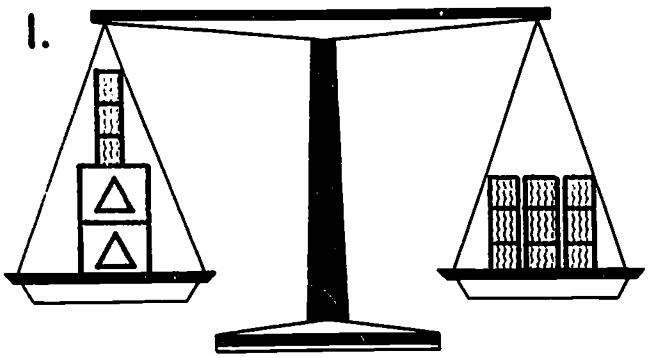
Thus, for this problem we would:

1. Take three small blocks away from each side.
2. Take half of the remaining blocks away from each side.

EXERCISES

Tell what you would do to find the weight of one large block in each of the balance scales. You should avoid removing parts of blocks if possible.

1.



____

____

____

$\boxed{\triangle}$ = __________

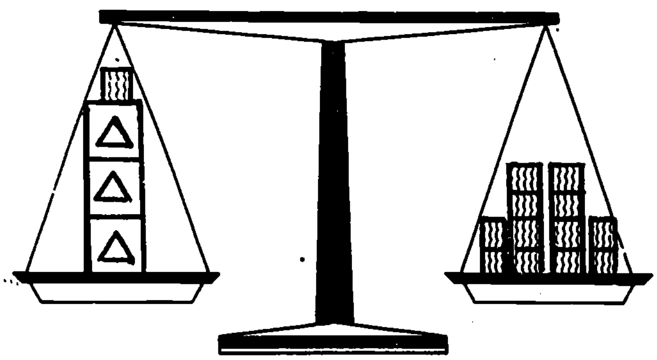
2.

____

____

____

$\boxed{\triangle}$ = __________



ANSWERS

1. (a) Remove three blocks from each side.

   (b) Remove half the blocks from each side.

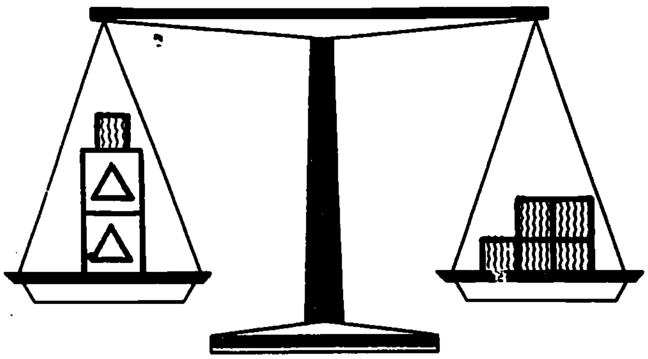
   $\boxed{\triangle} = 3$

2. (a) Remove one block from each side.

   (b) Divide each side by 3.

   $\boxed{\triangle} = 3\frac{2}{3}$

3.



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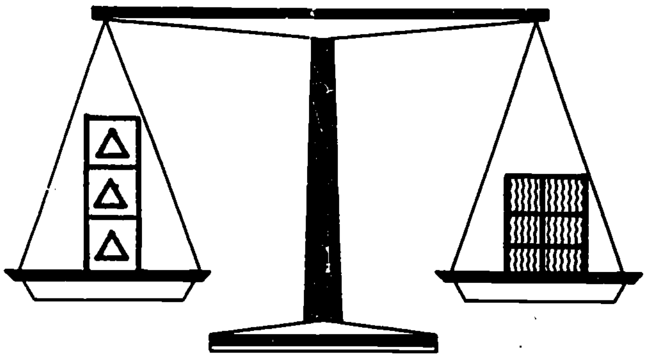
____

$\boxed{\triangle}$ = __________

4.



____

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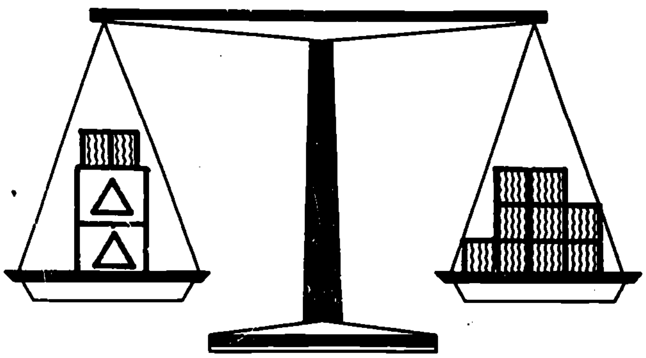
____

____

$\boxed{\triangle}$ = __________

5.



____

____

____

____

$\boxed{\triangle}$ = __________

ANSWERS TO EXERCISES

3. (a) Remove one block from each side.

   (b) Divide the remaining blocks by 2.

$\boxed{\triangle} = 2$

4. (a) Divide the number of blocks on each side by 3.

$\boxed{\triangle} = 2$

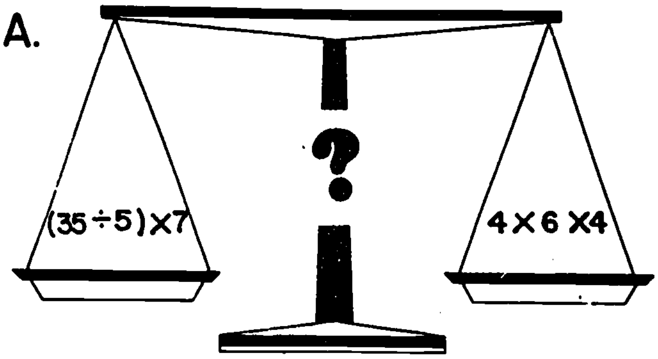
5. (a) Remove two blocks from each side.

   (b) Divide the remaining number of blocks by 2.

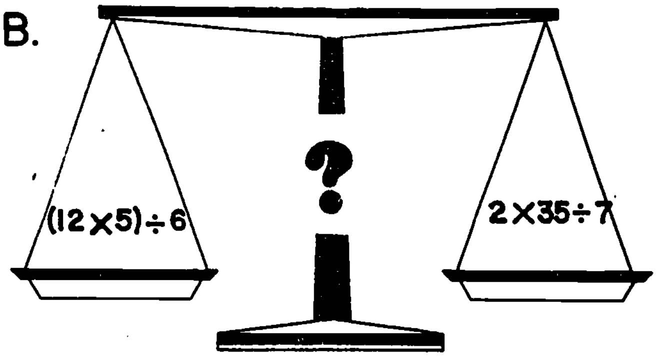
$\boxed{\triangle} = 3\frac{1}{2}$

NOT SO WELL BALANCED SENTENCES

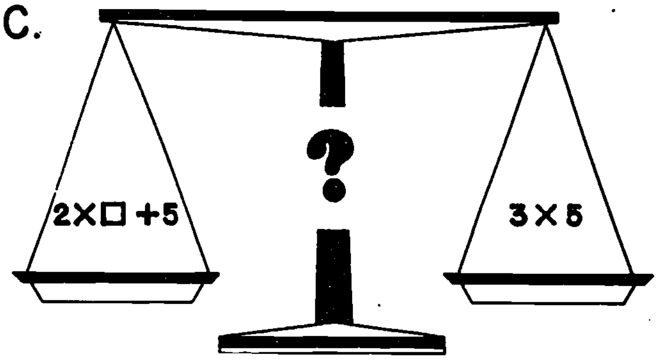
A.



B.



C.



1. Which will balance? _______

2. Which will not balance? _______

3. For some of the equations above, it is impossible to tell whether or not they balance. Which?

NOT SO WELL BALANCED SENTENCES

OBJECTIVES

1. Given numerical exercises on the balance beam, the students will be able to identify the scales that are balanced, and after identifying scales that are unbalanced will be able to tell which way the scale should tip.
2. Given problems on the balance beam such as $\boxed{} + 5 > 13$ the students will be able to solve for the unknown.
3. Given problems on the balance beam such as $3 \times \boxed{} < 24$ the students will be able to solve for the unknown.

EQUIPMENT AND TEACHING AIDS

The same equipment that was used for lesson 3 is also useful for lesson 4. The same balances are used again for this lesson, only for most of this lesson the beam will be in an unbalanced position. If you are using an Invicta Mathematical Balance, it seems to work best unbalanced when the weights are in the first or second peg away from the fulcrum.

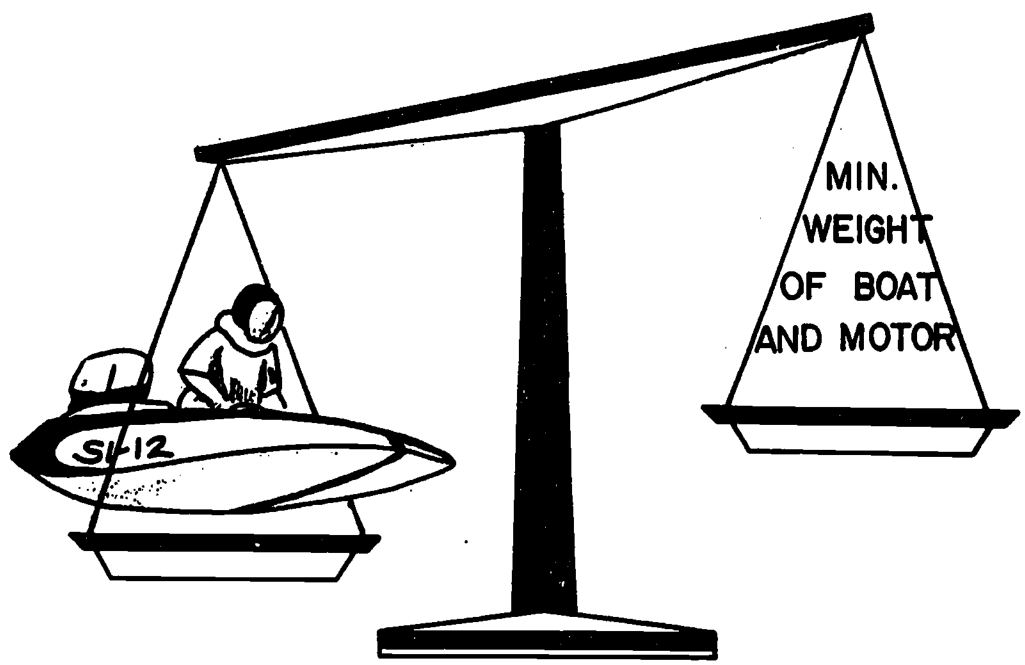
ANSWERS

1. B balances.
2. A does not balance.
3. C.

In practically all cases so far the beams have been balanced. However, there are many situations where beams are unbalanced. For example, many things have to weigh more than a certain minimum or less than a certain maximum.

In many forms of automobile and boat racing, the car or boat must weigh at least a certain amount. When the car or boat is inspected by the race officials, they weigh it. As long as it weighs more than the minimum weight, the inspectors are satisfied. The car or boat could weigh 1 lb., 100 lbs., or 500 lbs. more than the minimum and still meet the weight requirements.

Below is a picture showing the beam if the boat weighs more than the minimum weight requirements.



The American Power Boat Association rules state that a class DU Stock Outboard Hydroplane must weigh a minimum of 435 pounds. This weight includes the boat, motor, and driver.

CONTENT AND APPROACH

This lesson closely parallels lesson 3 except that the beam is unbalanced, representing an inequality. The main idea which all the students must understand is that the larger side will sink. The next main idea the class should discover is what happens to an unbalanced beam when equal amounts are removed from both sides. Through experimentation they should understand that an inequality remains in the same state after the same thing is done to both sides. Pages 16 through 20 introduce unbalanced scales and at the same time give the students some review of whole number skills. (Continued on next page.)

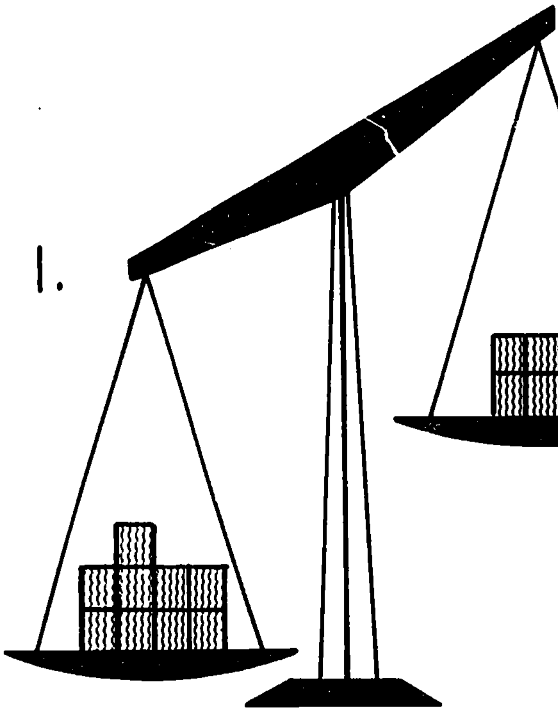
THINGS TO DISCUSS

The 1968 Rule Book of the American Power Boat Association states that: Overall weights shall include driver, hull, motor steering bar, steering wheel with cables and pulleys, motor controls, propeller, permanently attached speedometer and tachometer, permanently attached cushions and hardware, securely fastened fuel tank with remaining fuel, helmet, goggles and life jacket. The weight shall not include tools, fire extinguishers, water, or loose equipment in the boat or in the driver's clothing.

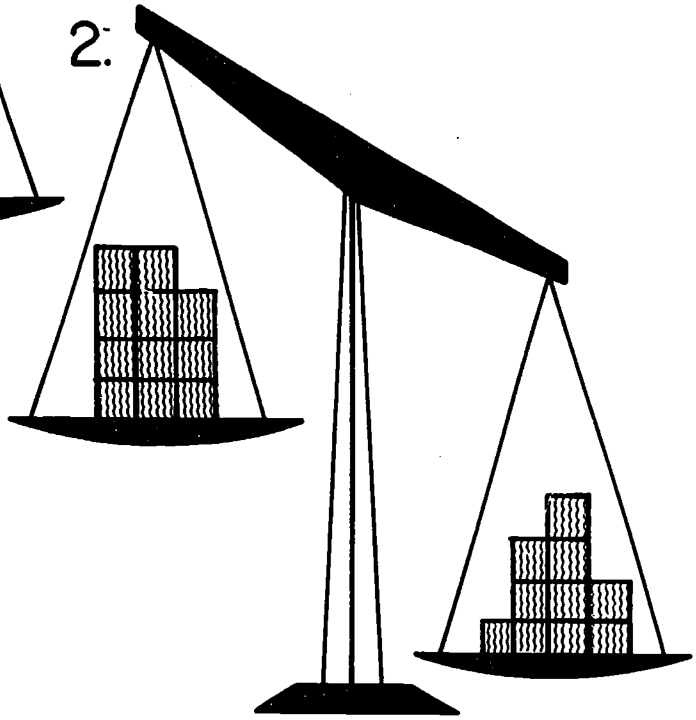
EXERCISES

Which of these scales shows the balance tipped the correct way?

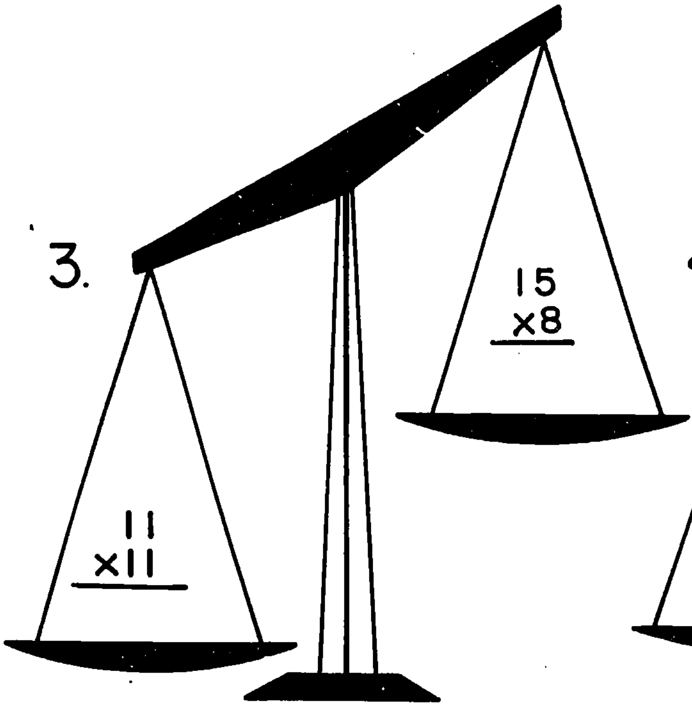
1.



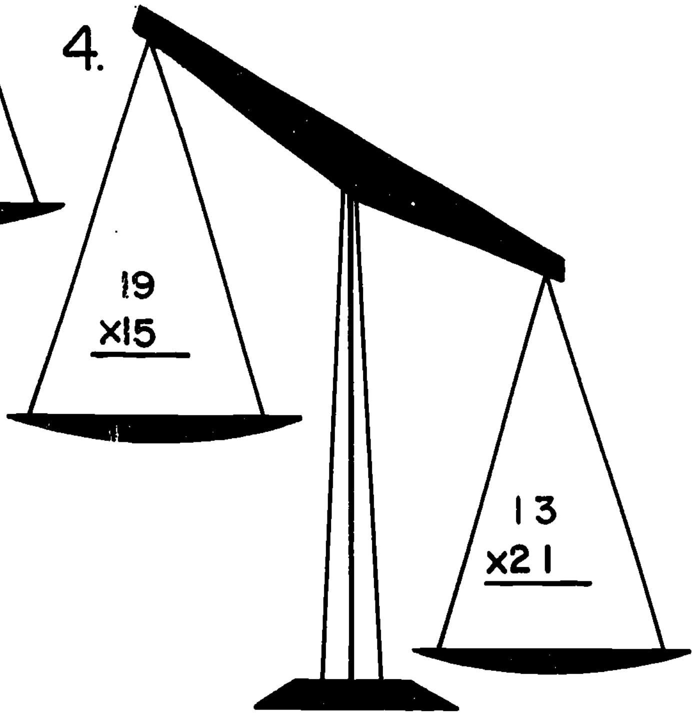
2.



3.



4.



CONTENT AND APPROACH (CONTINUED)

When the students work up to page 20 it's time for them to experiment with unbalanced scales. One series of experiments with balance beams they could do would be:

Start with eight weights on one side and seven weights on the other side. After each step tell if the beam is still unbalanced as it was when you started, then go back to the original eight and seven weights.

Steps:

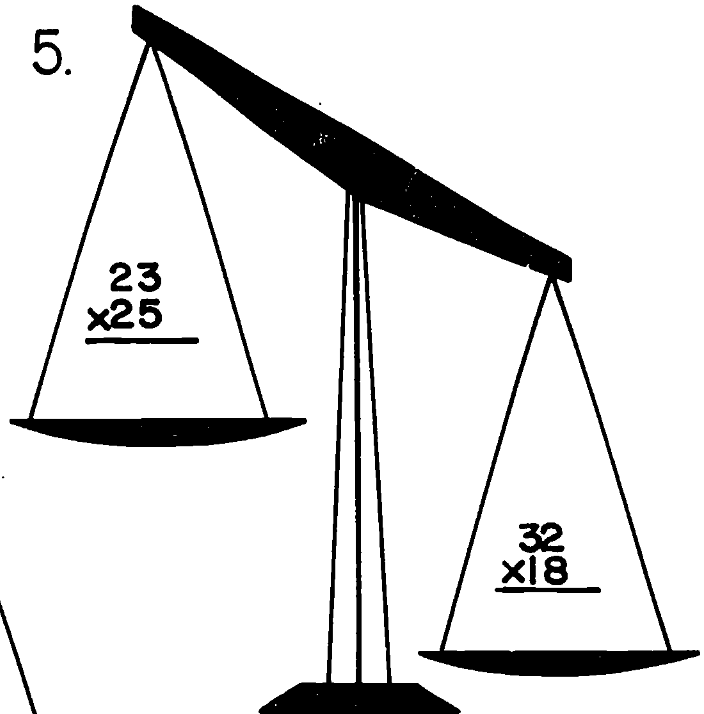
1. Remove five weights from each side.
2. Remove seven weights from each side.
3. Add two weights to each side.
4. Remove one weight from each side.
5. Add any number of weights to each side.
6. Remove three weights from the eight and remove one weight from the seven.
7. Double the number of weights on each side.

(Continued on next page)

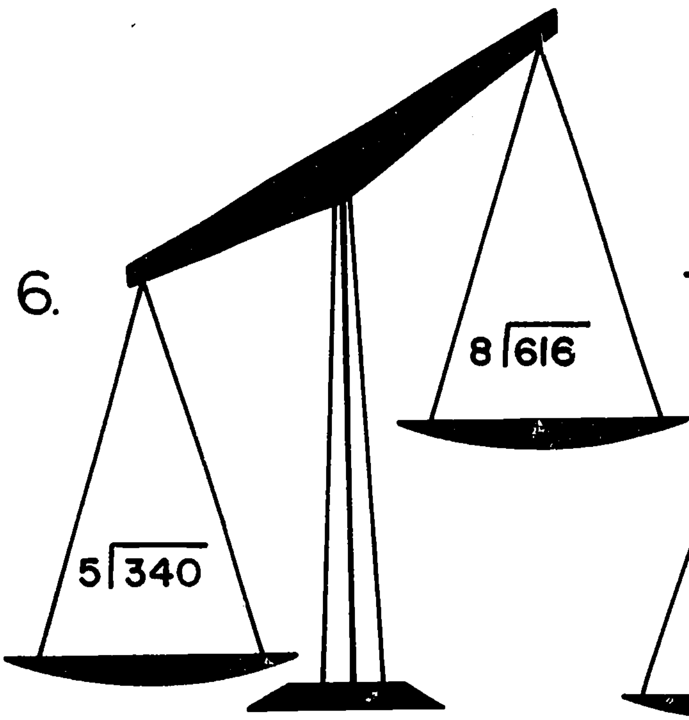
ANSWERS TO EXERCISES

1. The balance is tipped the right way. $9 > 6$
2. Balance tipped the wrong way. $11 > 10$
3. Balance tipped the right way. $121 > 120$
4. Balance tipped the wrong way. $285 > 273$

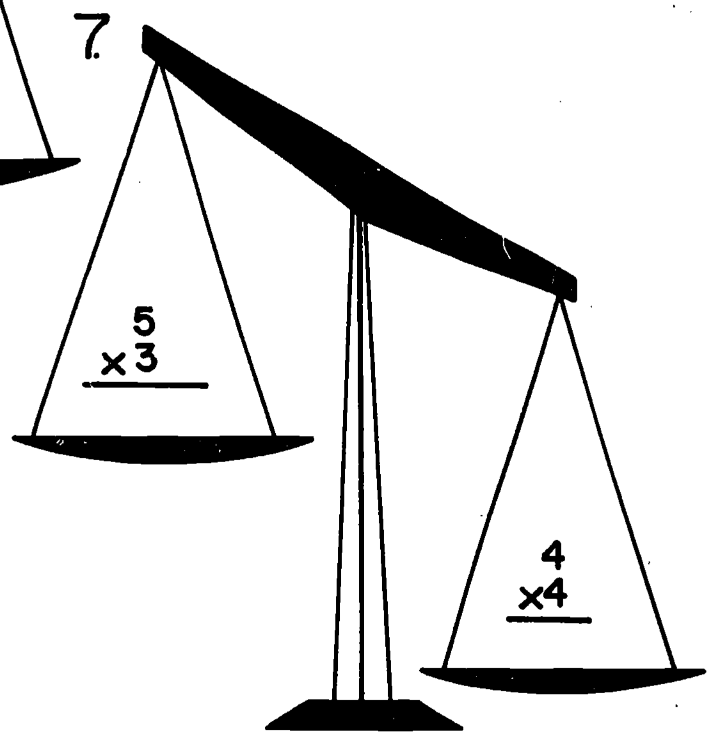
5.



6.



7.



We can write this last relationship as a mathematical sentence: $5 \times 3 < 4 \times 4$.

CONTENT AND APPROACH (CONTINUED)

More experiments on the balance beam:

1. Start with an unknown weight on one end and keep adding known weights on the other end until the scale tips towards the known weights. Thus,
   The unknown weight < _____ known weights
2. Start with two equal unknown weights on one end and keep adding known weights to the other end until the scale tip towards the known weights.
   Two unknown weights < _____ known weights
   Now take half the weights off each side. Thus,
   One unknown weight < _____ known weights
3. Start with an unknown plus three known weights on one side and again add known weights to the other side until the scale tips towards the known weights you have been adding.
   The unknown plus three known weights < _____ known weights
   Subtract three known weights from each side.
   The unknown weight < _____known weights

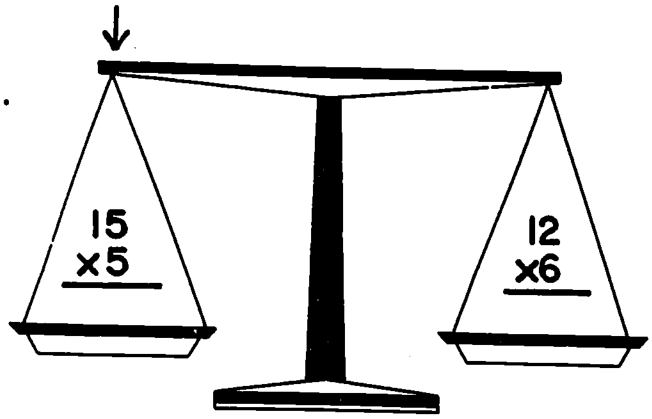
(Continued)

ANSWERS

5. Balance tipped the right way. 575 < 576
6. Balance tipped the wrong way. 68 < 77
7. Balance tipped the right way. 15 < 16

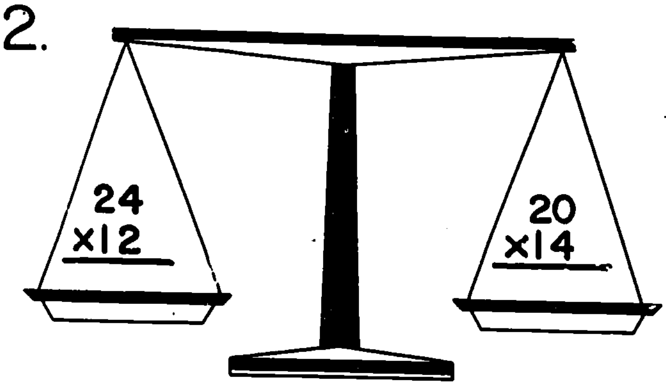
Write a mathematical sentence using =, <, or > signs for each of these balances. Also draw an arrow over the end of the beam that would move downward (as done in number 1).

1.



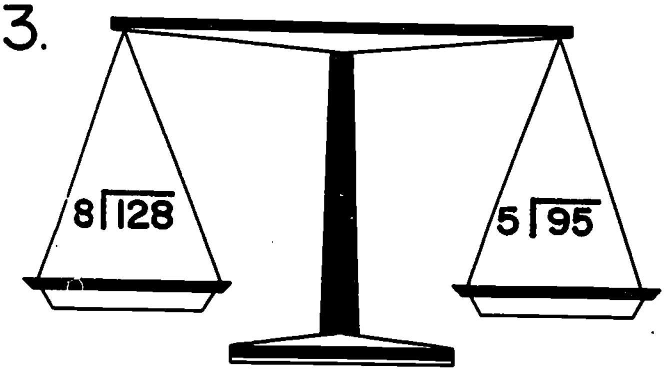
15 × 5 ________ 12 × 6

2.



24 × 12 ________

3.



________ ________ ________

CONTENT AND APPROACH (CONTINUED)

Experiments involving unknown weights on the scale:

4. Put two equal unknown weights along with three known weights on one side of the scale and add weights to the other side as in experiments 1, 2, and 3.

   Two unknown plus three known weights < _____ known weights

   Remove three known weights from each side.

   Two unknown weights < _____ known weights

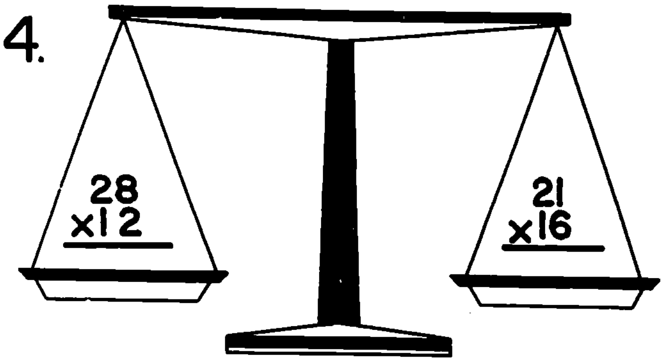
   Remove half the weights from each side.

   One unknown weight < _____ known weights

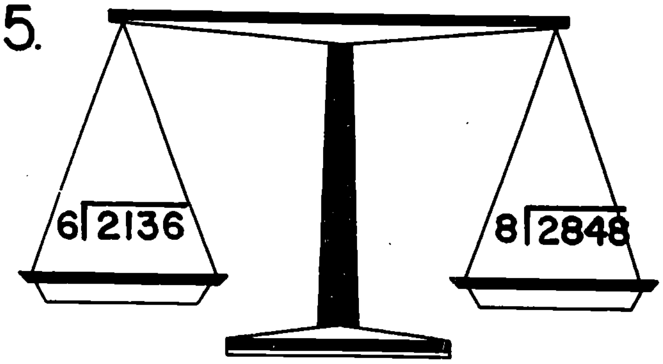
5. Do the same as experiment four but with three unknown and two known weights on one side.

ANSWERS

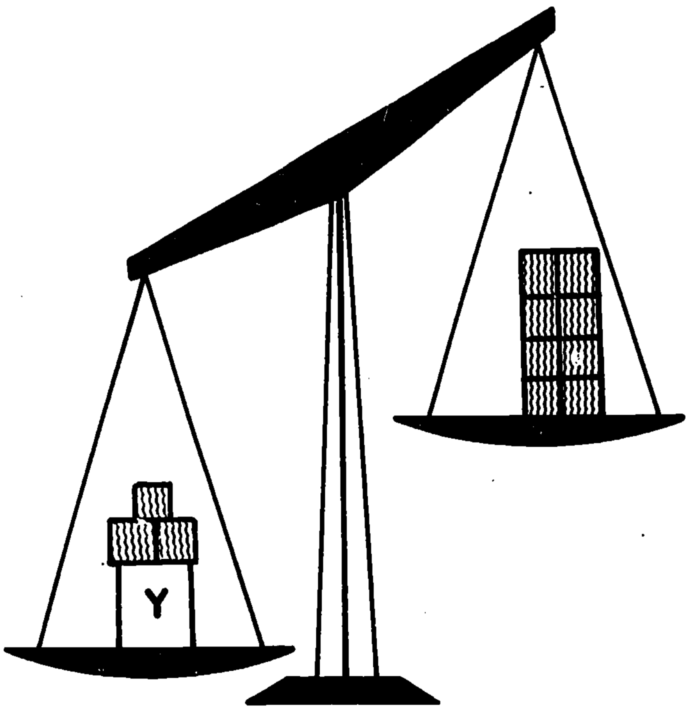
1.	$15 \times 5 > 12 \times 6$	$(75 > 72)$
2.	$24 \times 12 > 20 \times 14$	$(288 > 280)$
3.	$8\overline{)128} < 5\overline{)95}$	$(16 < 19)$

4. 

______ ___ ______

5. 

______ ___ ______



From this drawing you can see that 1 large block plus 3 small blocks is heavier than 8 small blocks. A shorter way of writing this is: $\boxed{Y} + 3 \times$ ▦ $> 8 \times$ ▦

If we remove 3 small blocks from each side, will the beam still remain unbalanced ________? If it does, the beam will show 1 large block is heavier than _____ small blocks, or $\boxed{Y} >$ _____ $\times$ ▦.

ANSWERS TO EXERCISES

4. $28 \times 12 = 16 \times 21$ $\qquad$ $(336 = 336)$
5. $6\overline{)2136} = 8\overline{)2848}$ $\qquad$ $(356 = 356)$

yes.

5 $\qquad \boxed{Y} > 5 \times \blacksquare$

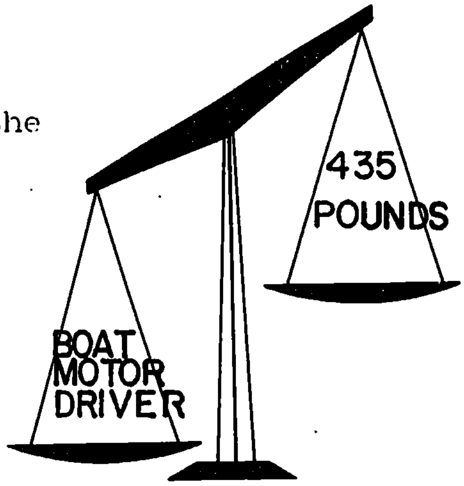
THINGS TO DISCUSS

Have the students give examples of situations where a minimum or maximum amount is involved.

Examples:

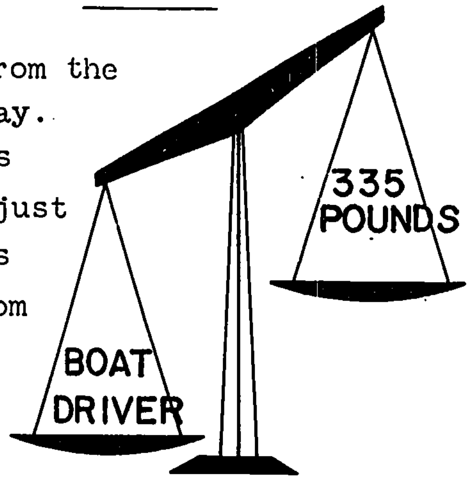
1. Amount of money it takes to buy something.
2. Weight of a boxer to box in a certain class.
3. Amount of material it takes to make something (Cloth for a dress, etc.).

Let's consider the boat problem from page 16. As the problem stated the boat, motor, and driver must weigh more than 435 pounds.

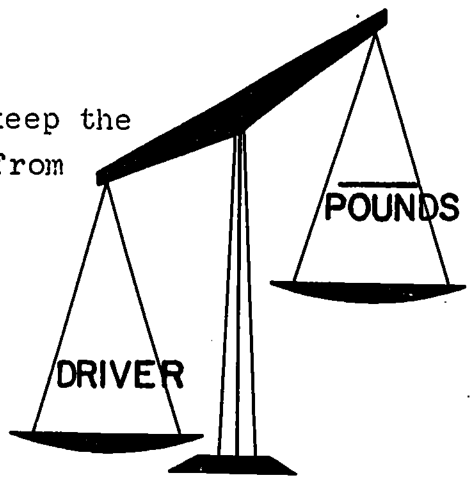


Does the combination of boat, motor, and driver above pass the weight requirement? ______

Now if the 100 lb. motor is removed from the scale, the beam would tip the other way. To prevent this, the 435 pound side is reduced until the beam is unbalanced just like it was originally. This requires 100 pounds of weight to be removed from the right side of the beam. Now the beam shows this:



Now the 200 lb. boat is removed. To keep the beam the same, 200 pounds are removed from the other side. Now the scale shows:



In other words the driver must weigh > _____.

The American Power Boat Association rules for Stock Runabouts and Hydroplanes list their minimum weights for the various class boats.

Class	Stock Runabouts	Class	Stock Hydroplanes
AU	345 pounds	ASH	330 pounds
BU	395	BSH	355
CU	470	CSH	435
36	565	DSH	435
DU	525		

The rules for the Unlimited class include a minimum weight of 5,000 pounds. Unlimited class boats are the inboard hydroplanes that race in the Gold Cup Races.

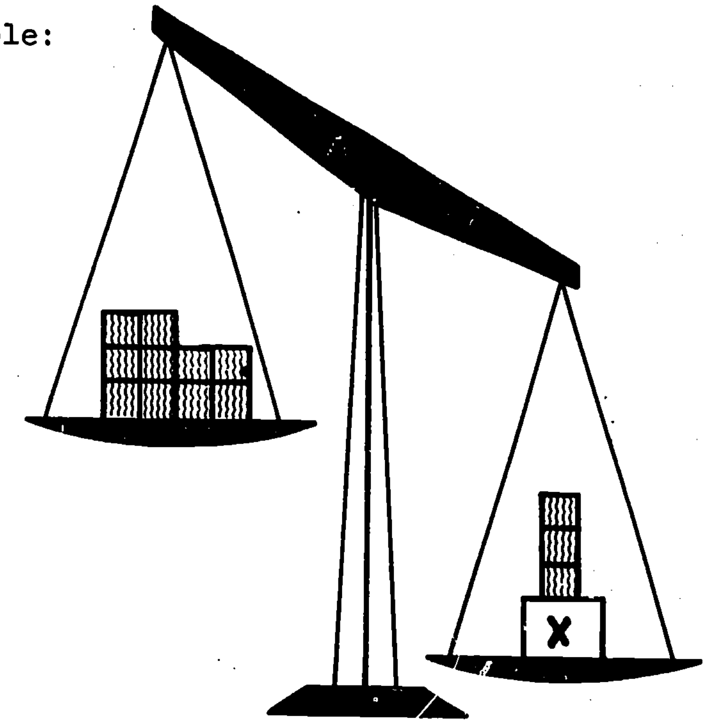
ANSWERS

135 Pounds.

Driver must weigh > 135 Pounds.

Let's look at another example:

<u>Remove 3 small blocks from each side.</u>

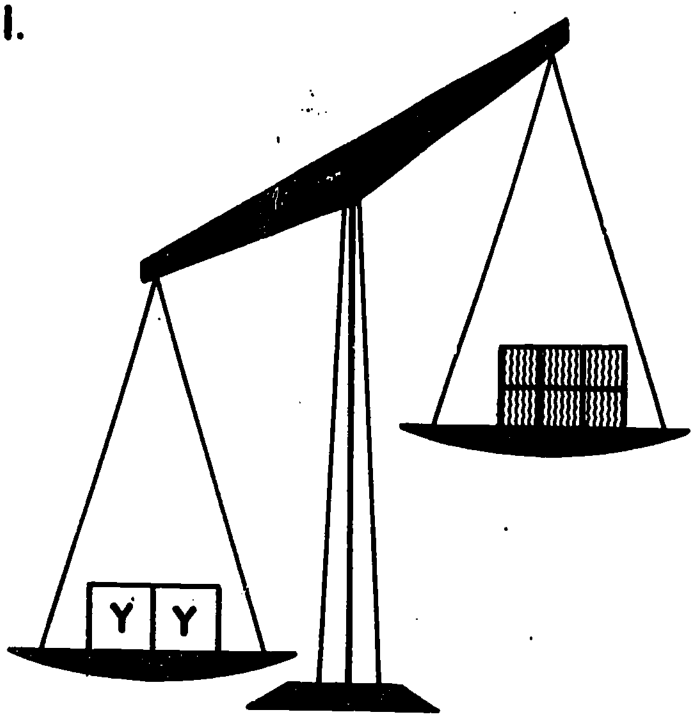


$$10 \times \square < \boxed{X} + 3\,\square$$
$$7 \times \square < \boxed{X}$$

Thus: Seven small blocks weigh less than one large block.

Try these problems. In each exercise tell what you would do to find the weight of the large blocks.

1.



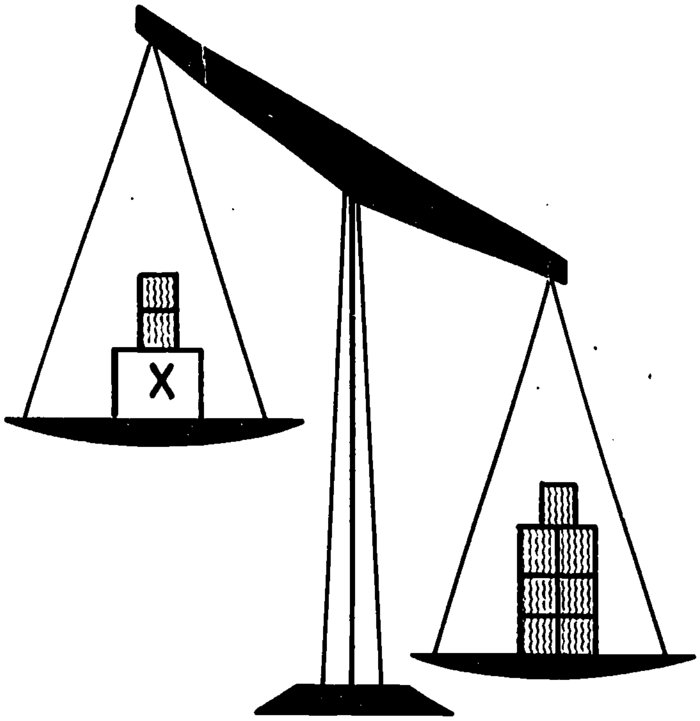
$$2 \times \boxed{Y} > 6 \times \square$$
$$\boxed{Y} > \_\_\_\_\_\_$$

____

ANSWERS

$\boxed{Y} > 3$ ____ Remove half the weights from each side.

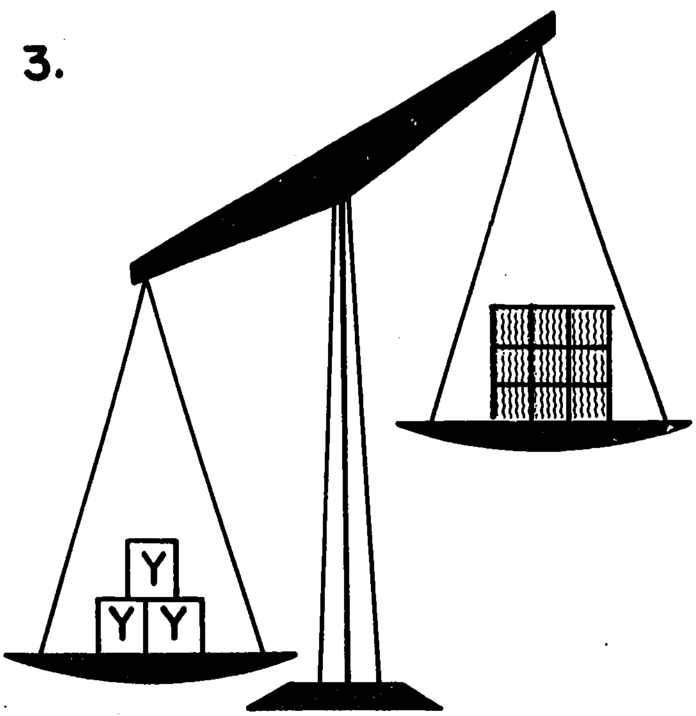
2.



$\boxed{X} + 2 \times \square < 7 \times \square$

$\boxed{X} <$ ________

3.



$3 \times \boxed{Y} >$ ________ $\square$

$\boxed{Y} >$ ________ $\square$

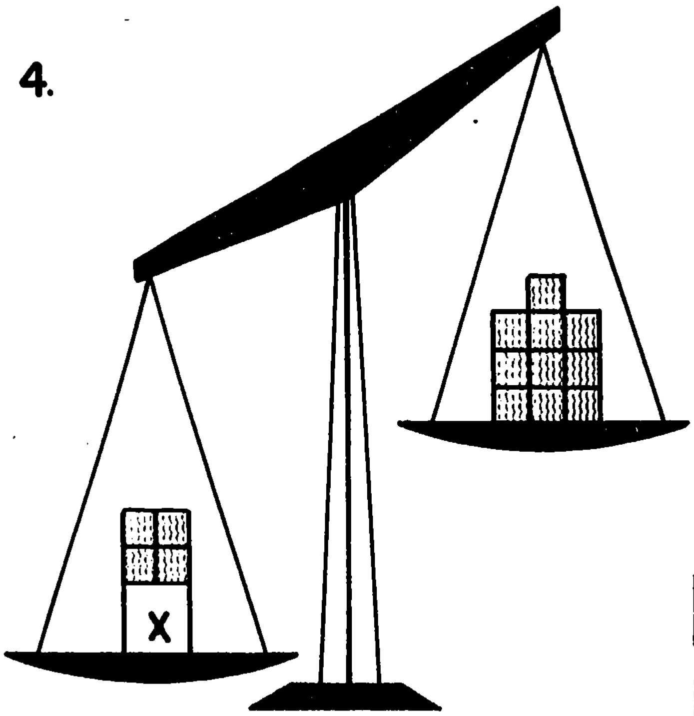
2. $\boxed{X} < 5$

<u>Remove two small weights from each side.</u>

3. $\boxed{Y} > 3$

<u>Remove two-thirds of the weights from each side</u> OR <u>Divide each side by three</u>.

4.

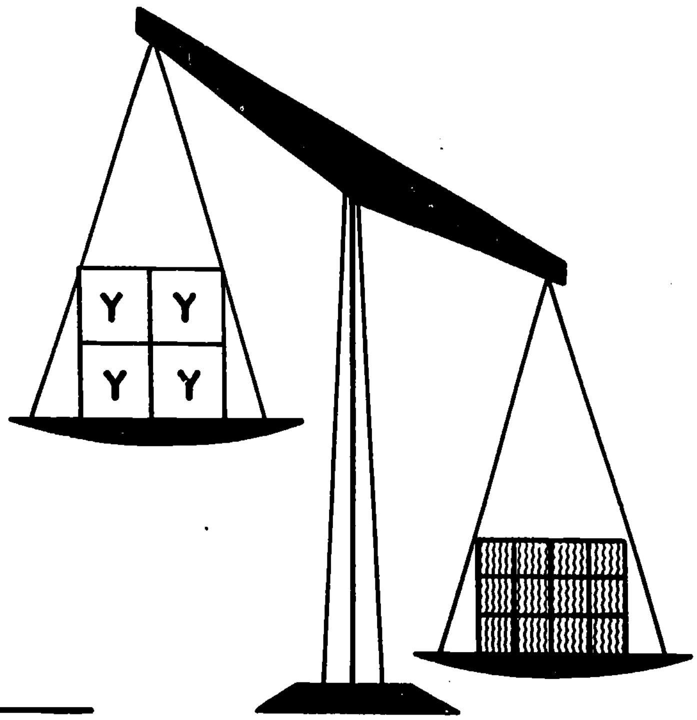


$\boxed{X} + 4 \times$ ▥ $> 10 \times$ ▥

$\boxed{X} >$ ________

____

5.



$4 \times \boxed{Y} < 12 \times$ ▥

$\boxed{Y} <$ ________

____

4. $\boxed{X} > 6$

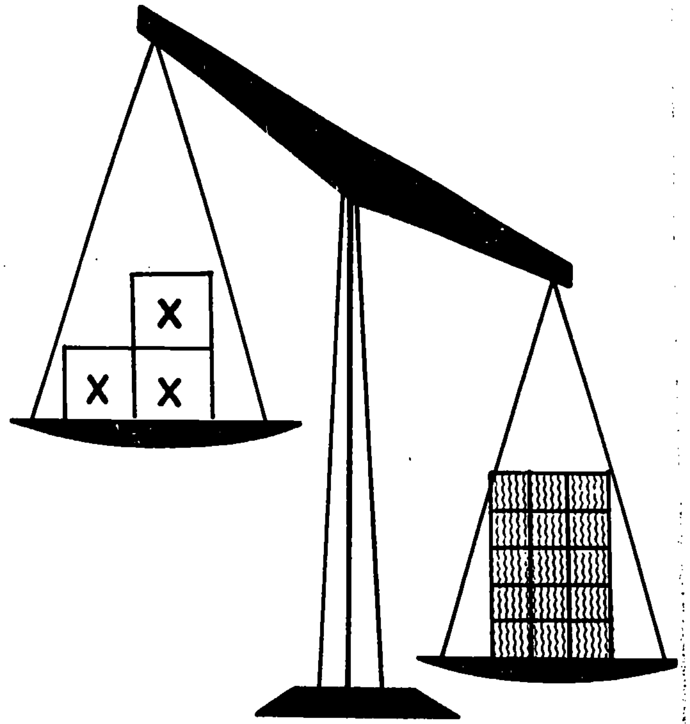
Remove four small weights from each side.

5. $\boxed{Y} < 3$

Remove three-fourths of the weights from each side OR

Divide each side by 4.

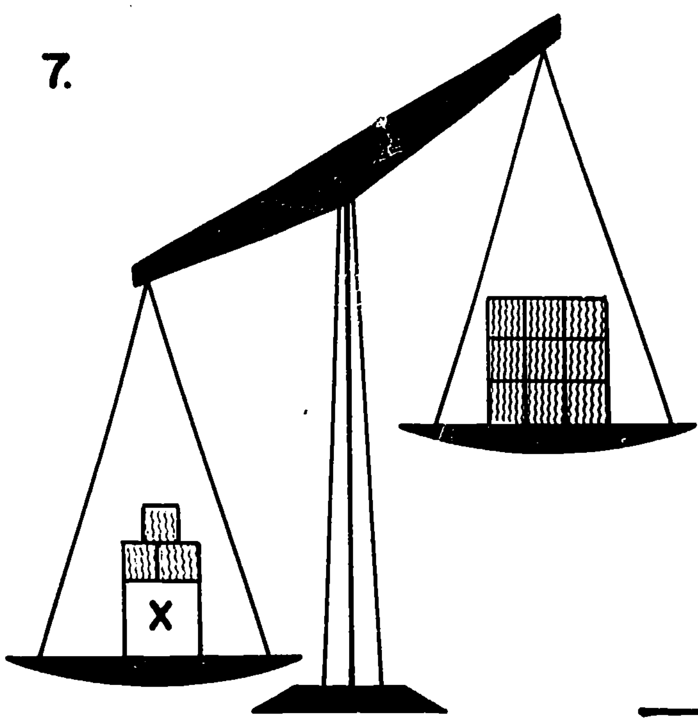
6.



$3 \times \boxed{X} > 15 \times \square$

$\boxed{X} >$ ____________

7.



$\boxed{X} + 3 \times$ ▥ $< 9 \times$ ▥

$\boxed{X} <$ ____________

6. $\boxed{X} > 5$

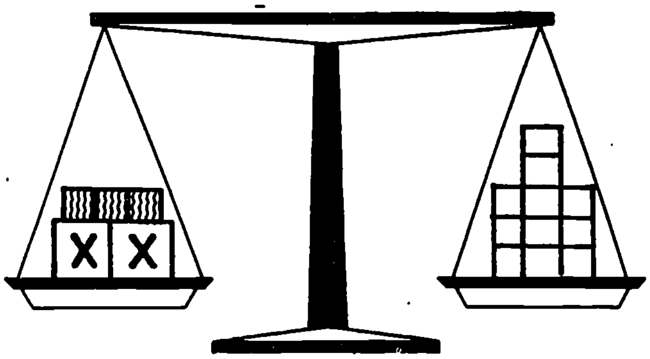
Remove two-thirds of the weights from each side OR Divide each side by three.

7. $\boxed{X} < 6$

Remove three small weights from each side.

BALANCED SENTENCES NOT ON THE BEAM

Equations and inequalities represented on a balance beam can be solved without using the beam. For example:



This would be written $2 \times \boxed{X} + 3 \times$ ▦ $= 11 \times$ ▦ .
However there is a much shorter way to write this mathematical sentence. In place of the unknown weights $\boxed{X}$ we can just write the X. Thus, the X will stand for the weight of the unknown box $\boxed{X}$. Thus our equation becomes

$2 \times X + 3 \times$ ▦ $= 11 \times$ ▦

As you can see, there is a possibility of confusing the letter X and the multiplication sign. To prevent teachers from getting your answers confused, Algebra does away with the use of the multiplication sign $\times$ in many cases. When two symbols (not numerals) are written together without any sign between them, it means they are multiplied together. Thus $2\ell w$ means 2 times ℓ times w. This doesn't work when used between two numerals, of course. Does 45 mean 4×5? ________ Does 3 ▦ mean $3 \times$ ▦ ? __________

Our equation becomes: $2X + 3$ ▦ $= 11$▦.

Next we can make things even shorter by leaving out the ▦ The number itself can stand for the number of blocks.

The equation is now simply: $2X + 3 = 11$

BALANCED SENTENCES NOT ON THE BEAM

OBJECTIVES

1. Given an equation represented on a balance scale, the students will be able to write the equation in algebraic form.
2. Given simple algebraic equations such as $4x = 24$ or $x + 5 = 23$, the students will be able to solve them without the use of a balance beam.

EQUIPMENT AND TEACHING AIDS

The balance scale might be useful to start off this lesson.

As a transition from the balance beam, a possible sample activity is to have students write two or three equations, such as

2 Unknowns + 3 paper clips = 11 paper clips

Have them: (1) Solve these equations;
(2) Tell what they did;

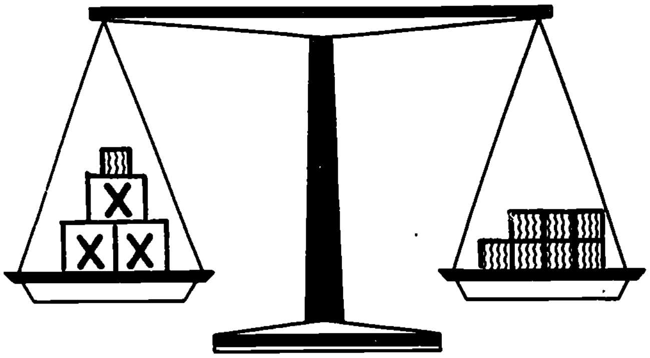
as:

(a) Remove 3 paper clips from each side;
(b) Take $\frac{1}{2}$ of each side;
(c) 1 unknown = 4 paper clips.

CONTENT AND APPROACH

A good way to start the lesson might be to start with the balance beam. Start representing equations on it that use larger numbers. For example: Ask one student to come up and put on the balance beam: $3x + 42 = 132$. When the students complain that it takes too long to count out 132 weights, you have a perfect chance to introduce solving equations without the beam. In other words, let them think they are getting away with something. Tell them they now get to learn an easier method.

EXERCISES



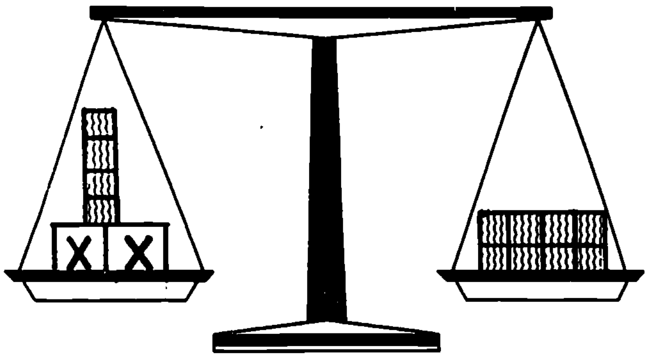
What is the simplest mathematical sentence you can write to represent this problem?

_______ X + _____ = _______

The answer is upside down at the bottom of the page.

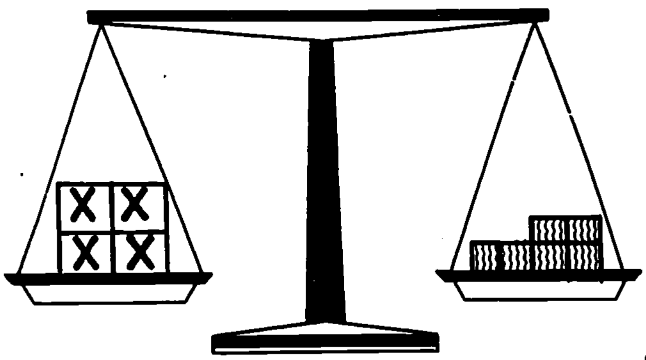
Do the same for the following exercises.

1.



_______ + _____ _______

2.



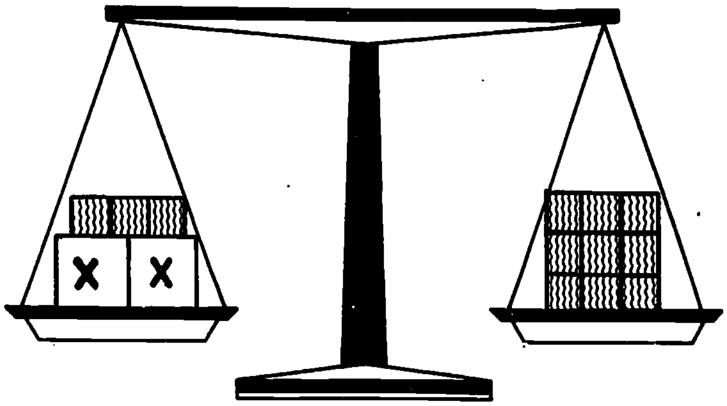
_______ _______

3X + 1 = 7

ANSWERS

$3x + 1 = 7$

1. $2x + 4 = 8$
2. $4x = 6$



A shorter way to write 1X is X.

Now consider the equation

$$2X + 3 = 9$$

a. Draw a balance beam to represent it.

b. Tell what you would do to the beam to end up with just the X on one side of the scale.

Now apply the same steps to the equation itself.

$$\begin{array}{r} 2X + 3 = 9 \\ \underline{\quad - 3 = -3} \\ 2X = 6 \end{array}$$ Remove 3 from each side.

$$\frac{2X}{2} = \frac{6}{2}$$ Divide each side by 2.

$$X = 3$$ This means $1X = 3$.

This means that X has the same weight or value as the 3. This process is known as solving the equation. Solving the equation is a fancy way of saying "Finding the value of the unknown in this equation".

EXERCISES

Draw X + 5 = 12 on a balance beam. Tell what you would do on the beam to find the weight of the X.

Now demonstrate the process with the equation.

$$X + 5 = 12$$

______________ ______________

X =

CONTENT AND APPROACH

Tell the students that all they have to do is think of how they would solve the equation on a balance beam, then apply these same steps to the equation directly.

ANSWERS TO EXERCISES

$$\begin{array}{r} X + 5 = 12 \\ -5 = -5 \\ \hline X = 7 \end{array}$$

Remove five from each side.

Example 1

Now consider $X + 135 = 163$

If you drew these blocks on the scale, what would you do to both sides to have the X remain by itself? ______

Now instead of drawing the beam, just do that step to the equation itself.

$$\begin{array}{r} X + 135 = 163 \\ - \qquad - \\ \hline X = \end{array}$$

______

Example 2

Consider $3X = 123$

On the balance beam, what would you do to both sides so that one side contained a single X? ______

Do the same for the equation.

$$\frac{3X}{} = \frac{123}{}$$

$X =$ ______

______

EXERCISES

Solve these equations: In each, write what you are going to do to both sides of the equation, then show this. Finally show the answer as X = 15 or Y = 27 rather than 15 or 27.

1. $X + 5 = 18$

$$\begin{array}{r} - \qquad = - \\ \hline X = \underline{\qquad} \end{array}$$

______

2. $\frac{4X}{} = \frac{20}{}$

______

$X =$ ______

ANSWERS

Example 1

$$\begin{array}{rcl} X + 135 & = & 163 \\ -\ 135 & = & -135 \\ \hline X & = & 28 \end{array}$$

Remove 135 from each side or (Subtract 135 from each side).

Example 2

$$\frac{3X}{3} = \frac{123}{3}$$

$$X = 41$$

Divide each side by 3.

ANSWERS TO EXERCISES

1. $$\begin{array}{rcl} X + 5 & = & 18 \\ -\ 5 & = & -\ 5 \\ \hline X & = & 13 \end{array}$$

Remove five from each side or Subtract five from each side.

2. $$\frac{4X}{4} = \frac{20}{4}$$

$$X = 5$$

Divide each side by 4.

3. $X + 17 = 49$

______ ______ ______________________________

$X =$ ______

4. $X + 145 = 355$

$X =$ ______ ______________________________

5. $\underline{5X} = \underline{60}$

____

$X =$ ______

6. $12X = 60$

____

7. $X + 18 = 36$

____

8. $X + 143 = 296$

____

ANSWERS

3. $X + 17 = 49$
$\underline{ - 17 = -17}$
$X = 32$

Remove (or subtract) 17 from each side.

4. $X + 145 = 355$
$\underline{ - 145 = -145}$
$X = 210$

Remove (or subtract) 145 from each side.

5. $\frac{5X}{5} = \frac{60}{5}$
$X = 12$

Divide each side by 5.

6. $\frac{12X}{12} = \frac{60}{12}$
$X = 5$

Divide each side by 12.

7. $X + 18 = 36$
$\underline{ - 18 = -18}$
$X = 18$

Remove (or subtract) 18 from each side.

8. $X + 143 = 296$
$\underline{ - 143 = -143}$
$X = 153$

Remove (or subtract) 143 from each side.

9. $2X + 15 = 41$

10. $3X + 8 = 44$

CHALLENGE EXERCISES

11. $4 + 10X = 64$

12. $3X + 7 = 12$

ANSWERS

9.

$$2X + 15 = 41$$

$$\underline{\quad - 15} = \underline{-15}$$ Subtract 15 from each side.

$$2X = 26$$

$$\frac{2X}{2} = \frac{26}{2}$$ Divide each side by 2.

$$X = 13$$

10.

$$3X + 8 = 44$$

$$\underline{\quad - 8} = \underline{-8}$$ Subtract 8 from each side.

$$3X = 36$$

$$\frac{3X}{3} = \frac{36}{3}$$ Divide each side by 3.

$$X = 12$$

ANSWERS TO CHALLENGE EXERCISES

11.

$$4 + 10X = 64$$

$$\underline{-4 \qquad} = \underline{-4}$$ Subtract 4 from each side.

$$10X = 60$$

$$\frac{10X}{10} = \frac{60}{10}$$ Divide each side by 10.

$$X = 6$$

12.

$$3X + 7 = 12$$

$$\underline{\quad - 7} = \underline{-7}$$ Subtract 7 from each side.

$$3X = 5$$

$$\frac{3X}{3} = \frac{5}{3}$$ Divide each side by 3.

$$X = \frac{5}{3}$$

13. $129 = 8 + 11X$

13. $129 = 8 + 11X$

$$\begin{array}{rcl} 129 &=& 8 + 11X \\ \underline{-\ 8} &=& \underline{-8 \qquad\ } \\ 121 &=& 11X \\ \frac{121}{11} &=& \frac{11X}{11} \\ 11 &=& X \end{array}$$

Subtract 8 from each side.

Divide each side by 11.

OPERATING ON BALANCED SENTENCES

See if you can solve these two equations. In each, show what you do to both sides of the equation.

A. $5X = 25$

____

$X =$ _____

B. $5 + X = 25$

____

$X =$ _____

DISCUSSION QUESTIONS

1. How do your answers compare?
2. If you got different answers, explain why this happened.
3. If you got the same answer both times, explain why this happened.

Solve these two equations:

C. $7 + X = 28$

____

$X =$ _____

OPERATING ON BALANCED SENTENCES

OBJECTIVES

1. Given equations such as $7x = 35$ and $5 + x = 37$ the students will be able to tell what method they would use to solve the equations and will be able to solve them.

2. Given equations such as $3x + 4 = 19$ the students will be able to find the value of the variable.

ANSWERS

A. $\frac{5X}{5} = \frac{25}{5}$ Divide each side by 5.

$X = 5$

B. $5 + X = 25$

$\underline{-5 \quad\quad = -5}$ Subtract 5 from each side.

$X = 20$

ANSWERS TO DISCUSSION QUESTIONS

1. The answers are different.
2. The two equations are different. In the first equation the X is multiplied by 5 and in the second equation, 5 is added to the X.
3. You should not get the same answer both times.

C. $7 + X = 28$

$\underline{-7 \quad\quad = -7}$ Subtract 7 from each side.

$X = 21$

D.

$$7X = 28$$

____

X = _____

THOUGHT QUESTIONS

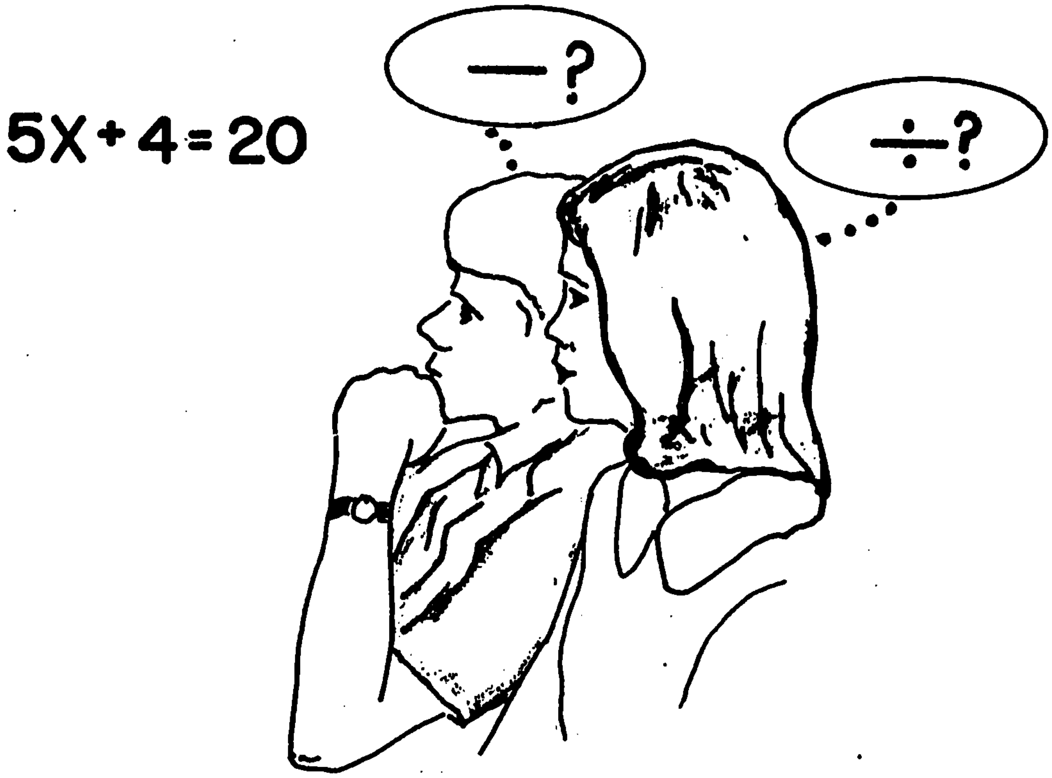
This time, did you get the same answer for parts C and D? _____
Why did this happen?

Did you use the same method to solve both equations?

If you used different methods to solve equations C and D explain why.

If you used the same method on both C and D explain why.

So far you have used two methods to solve equations; Subtraction and Division. Sometimes you subtracted the same number from both sides of the equation and other times you divided both sides of the equation by the same number.



EQUIPMENT AND TEACHING AIDS

No specific equipment is needed unless the students are still having trouble visualizing the equations on a balance beam. If this happens you might want to use the balance beam again.

CONTENT AND APPROACH

Pages 30 and 31 might best be handled orally in class, giving the students a chance to discuss the answers.

ANSWERS

D. $\frac{7X}{7} = \frac{28}{7}$ Divide each side by 7.

$X = 4$

ANSWERS TO THOUGHT QUESTIONS

No.

Same as for A and B. The two equations were different.

No.

In one equation 7 was added to the X and in the other equation the X was multiplied by 7.

Guessing which of the two methods to use doesn't work out too well because at most you could only expect to be right 50% of the time. There is an easier way to determine which method to use. Look at the variable (the letter) and determine what operations were done to it.

In 3+X=15 three was added to the X. To solve the equation you do the opposite. The opposite of adding 3 is subtracting 3.

In 3X=15 the X was multiplied by 3. The opposite of multiplying by 3 is dividing by 3. To solve the equation you must divide both sides of the equation by 3.

✓ POINT

Decide whether you would subtract or divide to solve each of these equations.

1. 3X = 25 ______________
2. 102X = 2040 ______________
3. 52 + X =75 ______________
4. X + 76 = 1776 ______________
5. 210X = 6300 ______________

CONTENT AND APPROACH

The main rules to stress with the students in working with these equations are:

1. Look at the equation and determine what operation was done to the variable.
2. To solve the equation, you perform the opposite operation.

ANSWERS

1. Divide.
2. Divide.
3. Subtract.
4. Subtract.
5. Divide.

✓ **POINT**

In each of the previous exercises, what number would you subtract from each side or divide both sides of the equation by?

1. ____________
2. ____________
3. ____________.
4. ____________
5. ____________

For 2X + 3 = 13 do you divide or subtract? Think how you would solve this if it were represented on a balance beam. What would you do then? On the balance beam you first removed 3 blocks from each side. On the equation you do that operation first also. After you have removed everything but the X's from one side of the scale on the equation, then you divide to get a single X or variable.

Your work should show the solution as:

$$\begin{aligned} 2X + 3 &= 13 \\ \underline{\quad - 3} &= \underline{-3} \\ \frac{2X}{2} &= \frac{10}{2} \\ X &= 5 \end{aligned}$$

EXERCISES Solve these equations.

1. 3X + 7 = 37
2. 12 + 5X = 22
3. 1 + 4Y = 11
4. 5Z = 37.5

ANSWERS TO ✓POINT

1. Divide by 3.
2. Divide by 102.
3. Subtract 52.
4. Subtract 76.
5. Divide by 210.

ANSWERS TO EXERCISES

1. $$\begin{aligned} 3X + 7 &= 37 \\ -7 &= -7 \\ \hline \frac{3X}{3} &= \frac{30}{3} \\ X &= 10 \end{aligned}$$

2. $$\begin{aligned} 12 + 5X &= 22 \\ -12 \qquad &= -12 \\ \hline \frac{5X}{5} &= \frac{10}{5} \\ X &= 2 \end{aligned}$$

3. $$\begin{aligned} 1 + 4Y &= 11 \\ -1 \qquad &= -1 \\ \hline \frac{4Y}{4} &= \frac{10}{4} \\ Y &= 2\tfrac{1}{2} \end{aligned}$$

4. $$\begin{aligned} \frac{5Z}{5} &= \frac{37.5}{5} \\ Z &= 7.5 \end{aligned}$$

5. $X + 7 = 49$

6. $5X + 5 = 55$

7. $10X + 12 = 1042$

8. $15 + 12X = 159$

Some mathematical sentences have a special meaning: $60D = rt$ relates the speed, time and distance traveled by any moving object, such as a car or boat.

In $60D = rt$

D stands for the distance in miles.
r stands for the speed in miles per hour(mph).
t stands for the time in minutes.

If you know any two of the three quantities, you can find the third one.

9. For example, if a car travels 30mph for 10 minutes the equation $60D = rt$ becomes

$$60D = 30 \times 10 \text{ or}$$
$$60D = 300. \text{ Find D.}$$

Therefore a car traveling at 30 mph for 10 minutes travels __________ miles.

ANSWERS

5.
$$\begin{array}{rcl} X + 7 &=& 49 \\ -\ 7 &=& -7 \\ \hline X &=& 42 \end{array}$$

6.
$$\begin{array}{rcl} 5X + 5 &=& 55 \\ -5 &=& -5 \\ \hline \frac{5X}{5} &=& \frac{50}{5} \\ X &=& 10 \end{array}$$

7.
$$\begin{array}{rcl} 10X + 12 &=& 1042 \\ -\ 12 &=& -\ 12 \\ \hline \frac{10X}{10} &=& \frac{1030}{10} \\ X &=& 103 \end{array}$$

8.
$$\begin{array}{rcl} 15 + 12X &=& 159 \\ -\ 15 &=& -15 \\ \hline \frac{12X}{12} &=& \frac{144}{12} \\ X &=& 12 \end{array}$$

9.
$$\frac{60D}{60} = \frac{300}{60}$$
$$D = 5$$

The car traveled five miles.

Point out the role of the units in $60D = rt$: The 60 stands for 60 minutes in 1 hour, thus allowing the time to be given in minutes rather than hours.

10. If a boat travels 24 mph for 10 minutes, how far does it travel?

$$60D = rt$$

Using 24 in place of r and 10 in place of t, the equation becomes

$$60D = 24 \times 10$$

Multiplying 24×10 getting 240 gives

$60D = 240.$ Find the value of D.

The boat traveled _________ miles.

Use the formula $60D = rt$ in solving the following exercises.

11. How many minutes will it take a car traveling at 30 miles an hour to go 10 miles?

12. If a boat is traveling at 15 miles an hour for 48 minutes, what distance will it have covered?

13. How fast is a plane traveling if it covers 40 miles in 4 minutes?

14. An elevator covers 52.8 feet vertically in $\frac{1}{2}$ minute. What is the rate in miles per hour that the elevator is traveling? (There are 5280 feet in one mile. D must be given in miles.)

10. $60D = rt$

$$\frac{60D}{60} = \frac{240}{60}$$

$$D = 4$$

The boat traveled <u>four</u> miles.

Exercises 11, 13 and 14 provide practice in solving for unknowns on the right-hand side of the equal sign. Some examples should be given before these exercises are assigned.

11.
$$60D = rt$$
$$60 \times 10 = 30t$$
$$600 = 30t$$
$$\frac{600}{30} = \frac{30t}{30}$$
$$20 = t$$

It will take 20 minutes.

12.
$$60D = rt$$
$$60D = 15 \times 48$$
$$60D = 720$$
$$\frac{60D}{60} = \frac{720}{60}$$
$$D = 12$$

The boat will have covered 12 miles.

13.
$$60D = rt$$
$$60 \times 40 = r \times 4$$
$$2400 = 4r$$
$$\frac{2400}{4} = \frac{4r}{4}$$
$$600 = r$$

The plane is traveling 600 miles per hour.

14.
$$60D = rt$$
$$60 \times \frac{52.8}{5280} = r \times \tfrac{1}{2}$$
$$60 \times \frac{1}{100} = \tfrac{1}{2}r$$
$$\frac{60}{100} = \tfrac{1}{2}r$$
$$\frac{60}{100} = \frac{r}{2}$$
$$\frac{120}{100} = r$$
$$r = 1.2$$

The elevator travels 1.2 miles per hour.

OPERATING ON UNBALANCED SENTENCES

EXERCISES

Consider these two mathematical sentences: Draw a balance scale to represent each one.

A. $3 + X = 8$ B. $3 + X > 8$

THOUGHT QUESTIONS

How could you solve equations like A on the balance beam?
How could you solve inequalities like B on the balance beam?
How can you solve A without using the beam?
How could you solve B without using the beam?

EXERCISES

Solve these:

C. $3X = 16$ D. $3X < 16$

Describe the step you used to solve each mathematical sentence.

Solve these two exercises:

E. $3X + 5 = 17$ F. $3X + 5 > 17$

OPERATING ON UNBALANCED SENTENCES

OBJECTIVES

1. Given equations or inequalities such as $3 + x = 8$ or $3 + x > 8$ the students will be able to solve for the unknown.
2. Given a formula such as $d = 16t^2$ the students will be able to find d if they are given a value of t.

ANSWERS TO EXERCISES

A.
$$\begin{array}{rl} 3 + X &= 8 \\ \underline{-\ 3} &\underline{=-3} \\ X &= 5 \end{array}$$

B.
$$\begin{array}{rl} 3 + X &> 8 \\ \underline{-\ 3} &\underline{=-3} \\ X &> 5 \end{array}$$

ANSWERS TO THOUGHT QUESTIONS

Remove 3 from each side of the balance.
Remove 3 from each side of the balance.
Subtract 3 from each side of the equation.
Subtract 3 from each side of the inequality.

ANSWERS TO EXERCISES

C.
$$\frac{3X}{3} = \frac{16}{3}$$
$$X = 5\tfrac{1}{3} \text{ or } \frac{16}{3}$$

D.
$$\frac{3X}{3} < \frac{16}{3}$$
$$X < 5\tfrac{1}{3} \text{ or } \frac{16}{3}$$

Divide each side by 3.

E.
$$\begin{array}{rl} 3X + 5 &= 17 \\ \underline{-\ 5} &\underline{= -5} \\ \frac{3X}{3} &= \frac{12}{3} \\ X &= 4 \end{array}$$

F.
$$\begin{array}{rl} 3X + 5 &> 17 \\ \underline{-\ 5} &\underline{= -5} \\ \frac{3X}{3} &> \frac{12}{3} \\ X &> 4 \end{array}$$

Explain any differences in the way you solve these two exercises.

EXERCISES Solve these mathematical sentences.

1. $7X = 91$

2. $X + 7 > 91$

3. $X + 3.75 = 42.85$

4. $3.8X < 57$

5. $3X + 5 > 15.5$

6. $10X + 17 = 85$

$d = 16t^2$ is a formula for the distance a falling object travels in a given number of seconds.

d stands for the distance in feet.

t stands for the number of seconds.

If a stone is dropped from a cliff and falls for three seconds before it hits the ground, how high is the cliff?

$d = 16t^2$

Of course you remember t^2 means $t \times t$, so if t is three seconds then t^2 is 3×3 or 9. Thus: $d = 16 \times 9$
$d = 144$

CONTENT AND APPROACH

Pages 41 and 42 show the students that equations and inequalities are solved in the same way. If the students have trouble doing these exercises, they might first draw a balance beam to represent the problem. Then have them apply the same steps to the equation or inequality that they would apply to the balance. Formulas are introduced in this lesson and will be covered in greater detail in lesson 10.

ANSWERS TO EXERCISES

1.
$$\frac{7X}{7} = \frac{91}{7}$$
$$X = 13$$

2.
$$\begin{array}{r} X + 7 > 91 \\ \underline{-7 = -7} \\ X > 84 \end{array}$$

3.
$$\begin{array}{r} X + 3.75 = 42.85 \\ \underline{-3.75 = -3.75} \\ X = 39.10 \end{array}$$

4.
$$\frac{3.8X}{3.8} < \frac{57}{3.8}$$
$$X < 15$$

5.
$$\begin{array}{r} 3X + 5 > 15.5 \\ \underline{-5 = -5} \\ \frac{3X}{3} > \frac{10.5}{3} \\ X = 3.5 \end{array}$$

6.
$$\begin{array}{r} 10X + 17 = 85 \\ \underline{-17 = -17} \\ \frac{10X}{10} = \frac{68}{10} \\ X = 6.8 \end{array}$$

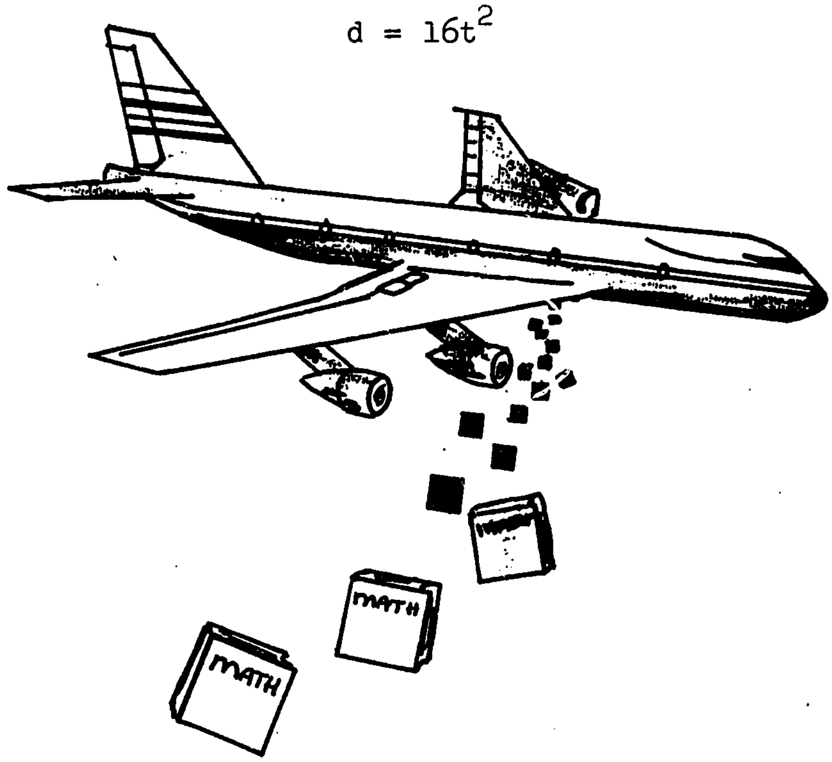
7. Find the height from which an object was dropped. It fell five seconds before hitting the ground.

$$d = 16t^2$$

Notice that in this problem you didn't have to subtract or divide both sides of the equation by anything. All you had to do was multiply: $16 \times 5 \times 5$. This happened because the letter you were to solve for (d) was already all by itself on one side of the equation. This often happens when working with formulas.

8. If a plane drops something and it takes ten seconds for it to hit the ground, what is the altitude of the plane?

$$d = 16t^2$$



ANSWERS TO EXERCISES

7. $d = 16t^2$ $\quad t = 5$

   $d = 16 \times 5 \times 5$

   $d = 400$ feet

8. $d = 16t^2$

   $d = 16 \times 10 \times 10$

   $d = 1600$ feet

$60D = rt$ can be used to find either distance, speed or time if the other two are known.

For example: If D is five miles, and t is six minutes, find r.

$$60D = rt$$
$$60 \times 5 = r \times 6$$
$$\frac{300}{6} = \frac{6r}{6}$$
$$50 = r$$

The speed is 50 mph.

9. Use $60D = rt$ to find t if D is eight miles and r is 40 mph.

$$60D = rt$$
$$60 \times \_\_\_\_\_ = \_\_\_\_\_ \times t$$

Solve for t.

10. Use $60D = rt$ to find D if r is 20 mph and t is 21 minutes.

$$60D = rt$$
$$60D = \_\_\_\_\_ \times \_\_\_\_\_$$

Solve for D.

Page 40, Exercises 11, 13 and 14 laid the groundwork for this page. Those exercises could be reviewed before this page is assigned, or this page can be used to see if students understand how to solve sentences with the variable on the right.

ANSWERS TO EXERCISES

9.
$$60D = rt$$
$$60 \times 8 = 40 \times t$$
$$\frac{480}{40} = \frac{40t}{40}$$
$$12 = t$$

time is 12 minutes.

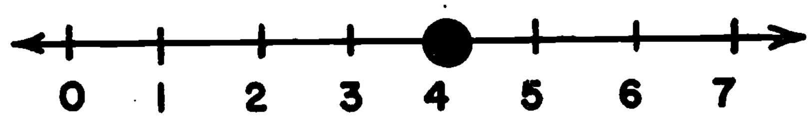
10.
$$60D = rt$$
$$60D = 20 \times 21$$
$$\frac{60D}{60} = \frac{420}{60}$$
$$D = 7$$

D is 7 miles.

SOLUTION PICTURES

Draw a picture of the solution. Example:

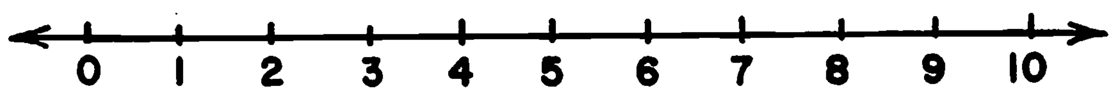
$$\begin{array}{r} X + 5 = 9 \\ \underline{-\ 5 = -5} \\ X = 4 \end{array}$$



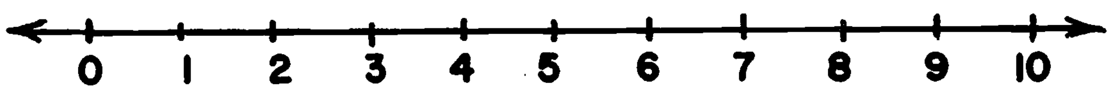
X = 4 is the solution because if X is replaced by 4 the sentence X + 5 = 9 becomes a true sentence, 4 + 5 = 9. We show the graph of X = 4 as a dot on the number line at 4.

EXERCISES

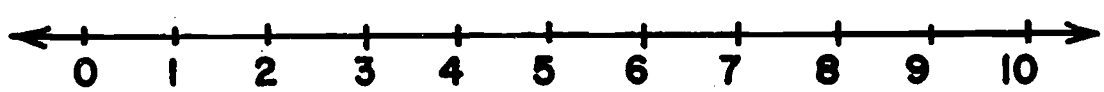
1. X + 12 = 17



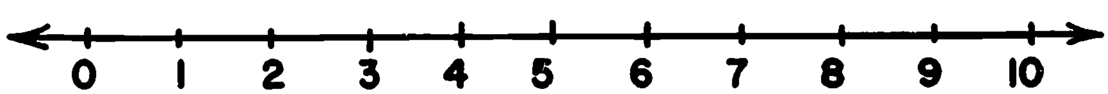
2. 3X = 15



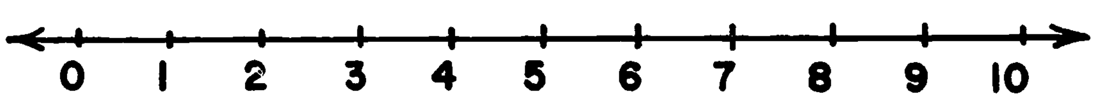
3. 5X = 15



4. X + 147 = 152



5. 12X = 36



SOLUTION PICTURES

OBJECTIVES

1. Given an equation such as $2x + 5 = 14$, the students will be able to solve it and graph the solution on a number line.

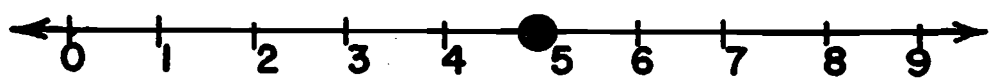
2. Given an inequality such as $3x + 4 \geq 16$ or $2x + 1 < 9$ the students will be able to solve it and graph the solution on a number line.

CONTENT AND APPROACH

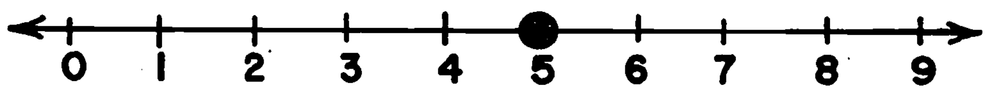
Page 45 gives the students more experience in solving equations and introduces finding points on a number line.

Continued on page T-47

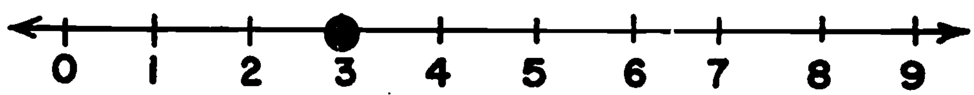
1. $X + 12 = 17$



2. $3X = 15$

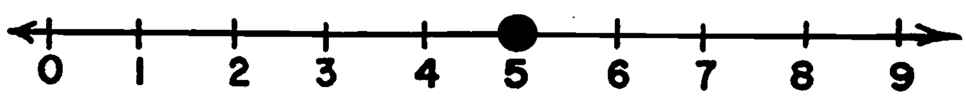


3. $5X = 15$



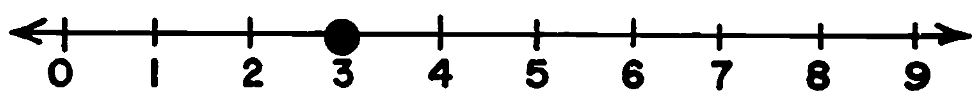
4. $X + 147 = 152$

$$\begin{array}{r} X + 147 = 152 \\ -\ 147 = -147 \\ \hline X = 5 \end{array}$$



5. $\frac{12X}{12} = \frac{36}{12}$

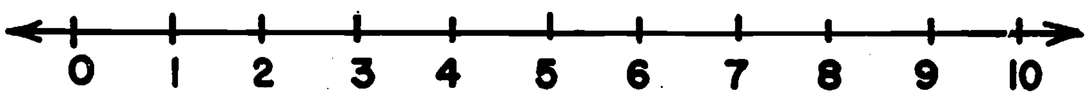
$X = 3$



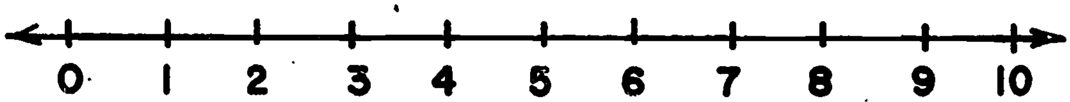
EXERCISES (continued)

6. $X + 1435 = 1441$

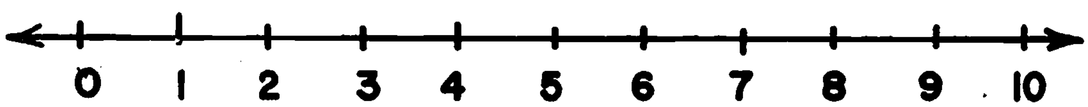
7. $2X + 5 = 13$



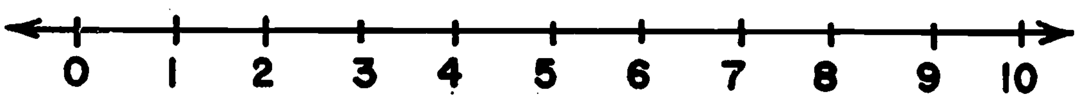
8. $4X + 4 = 16$



9. $5X + 19 = 29$



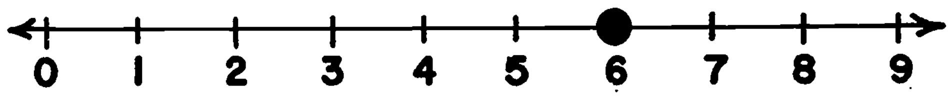
10. $3X + 9 = 21$



6. $X + 1435 = 1441$

$\underline{\quad - 1435 = -1435}$

$X = 6$

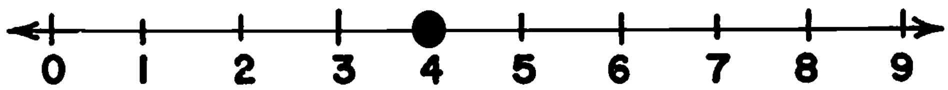


7. $2X + 5 = 13$

$\underline{\quad - 5 = -5}$

$\frac{2X}{2} = \frac{8}{2}$

$X = 4$

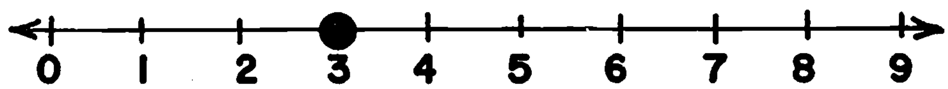


8. $4X + 4 = 16$

$\underline{\quad - 4 = -4}$

$\frac{4X}{4} = \frac{12}{4}$

$X = 3$

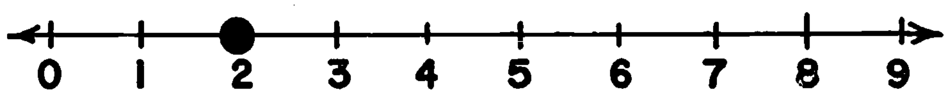


9. $5X + 19 = 29$

$\underline{\quad - 19 = -19}$

$5X = 10$

$X = 2$

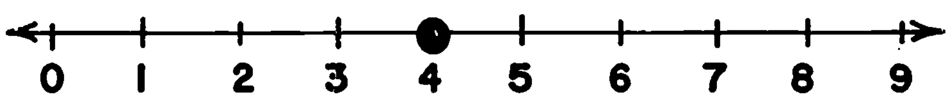


10. $3X + 9 = 21$

$\underline{\quad - 9 = -9}$

$3X = 12$

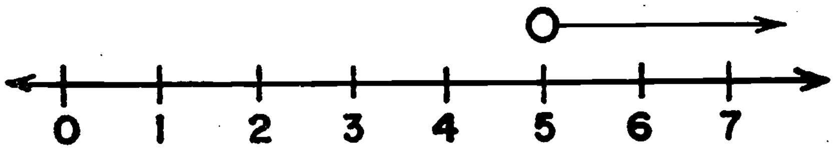
$X = 4$

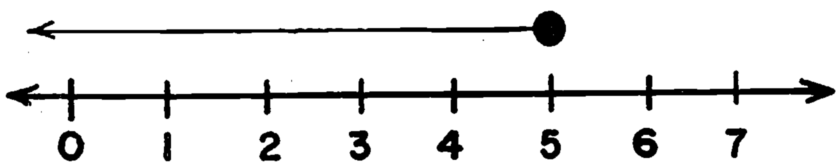


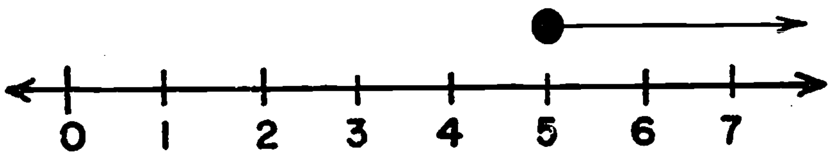
LESSON 8

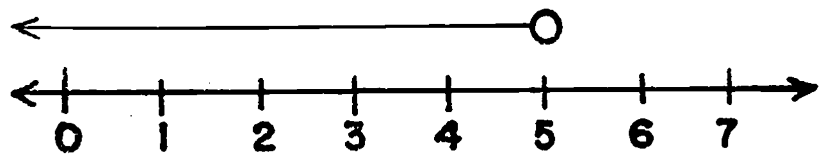
PICTURING SOLUTIONS OF NUMBER SENTENCES

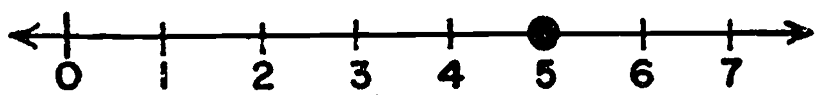
The number line is more helpful in picturing solutions of mathematical sentences that have more than one number for a solution. Here are some examples of inequalities using $\geq$, $>$ $<$, and $\leq$ graphed on number lines. As you remember, $\leq$ means less than or equal to. Thus $x \leq 5$ means that any number that replaces the x must be smaller than <u>or</u> equal to 5. Here are some examples of these graphs.

A.  $X > 5$

B.  $X \leq 5$

C.  $X \geq 5$

D.  $X < 5$

E.  $X = 5$

DISCUSSION QUESTIONS

1. Why were the arrows drawn on some of the number lines?
2. Why did the arrows point in different directions?
3. Why do some arrows start with a ● and others start with a ○ ?
4. Why isn't there an arrow in E?

CONTENT AND APPROACH

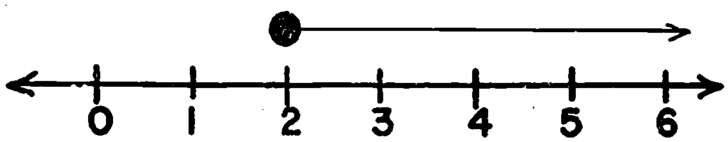
Page 47 introduces graphing inequalities on a number line. This page is meant to be discussed. It is hoped that the students can discover why some graphs start with a ○ and others with a ● by looking at the examples.

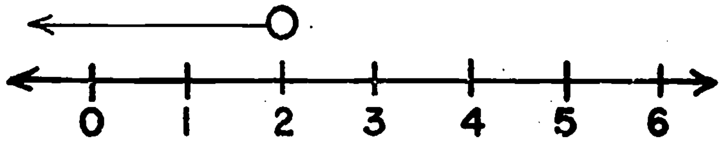
ANSWERS

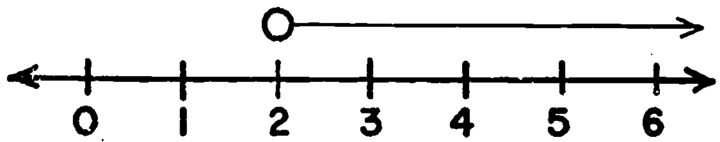
1. Because the solution to the mathematical sentence is more than one number. We represent this with an arrow because it would be impossible to draw dots to represent all the solutions to the sentence.
2. In some cases the solutions are greater than some number and in other cases are less than some number.
3. If the solution set includes that number where the arrow starts, then a ● is used at the end of the arrow. This occurs when $\leq$ or $\geq$ is used in the inequality. If the start of the arrow doesn't include that number, then a ○ is used. This occurs when $>$ or $<$ is used in the inequality.
4. There is only one solution to $x = 5$.

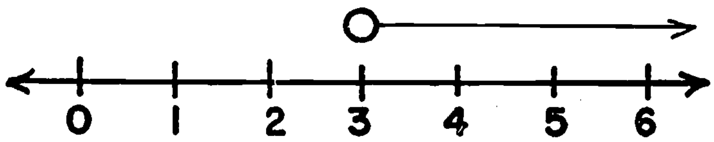
EXERCISES

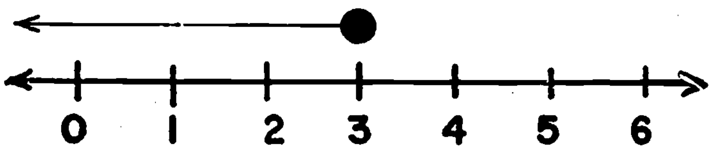
Match the mathematics sentences with their graphs.
The same sentence may be used as many times as necessary.

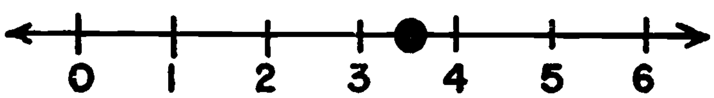
1.  ____

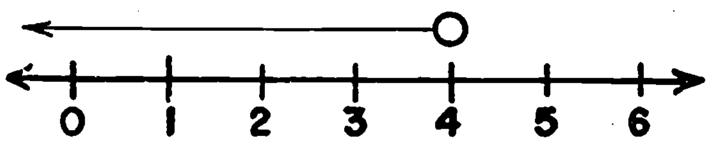
2.  ____

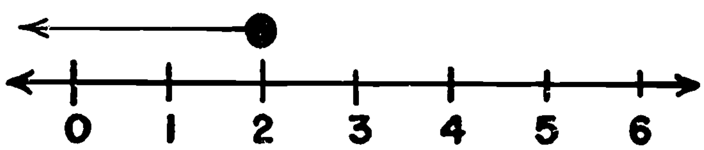
3.  ____

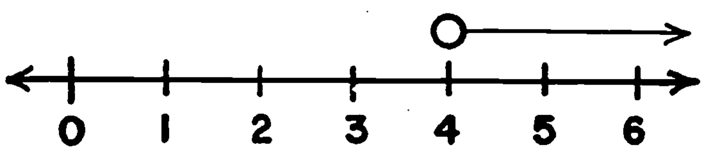
4.  ____

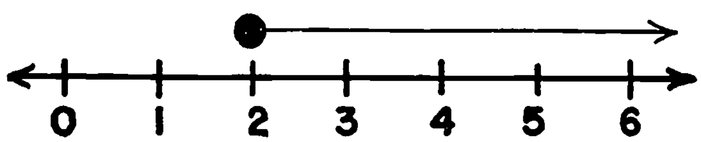
5.  ____

6.  ____

7.  ____

8.  ____

9.  ____

10.  ____

A. $X > 3$

B. $X < 4$

C. $X > 4$

D. $X \leq 2$

E. $X < 2$

F. $X \geq 2$

G. $X \geq 3$

H. $X = 3.5$

I. $X \leq 3$

J. $X > 145$

K. $X \geq 4$

L. $X > 2$

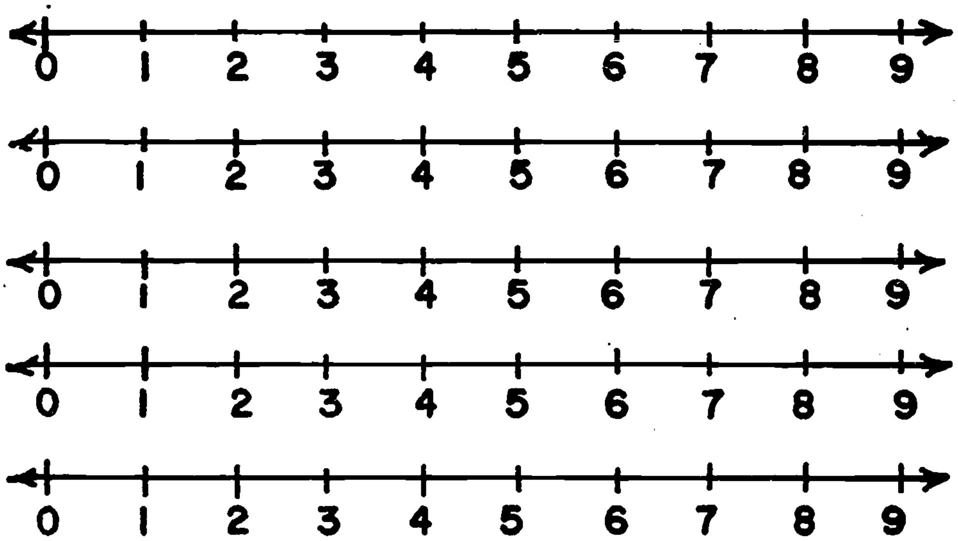
This page is meant to check to see if the students understood page 47. It is recommended that the students be given a few minutes to work this page. Then check it to see whether the students understood it.

ANSWERS TO EXERCISES

1. F
2. E
3. L
4. A
5. I
6. H
7. B
8. D
9. C
10. F

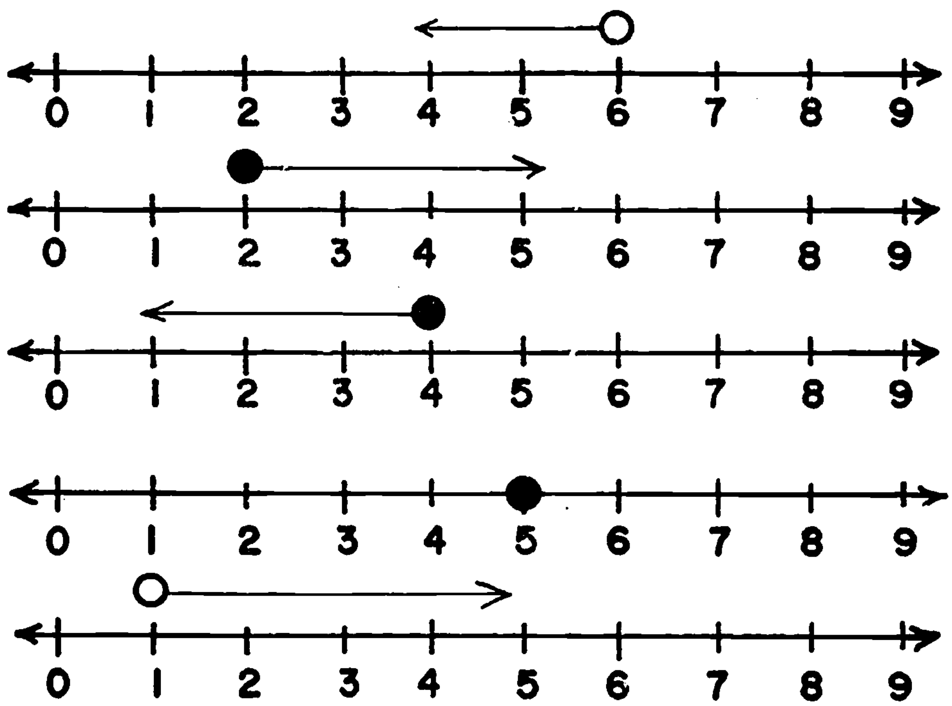
Draw the graph of each of these mathematical sentences on the number line.

1. $X \leq 3$
2. $X = 7$
3. $X > 5$
4. $5 < X$
5. $X \geq 2.5$



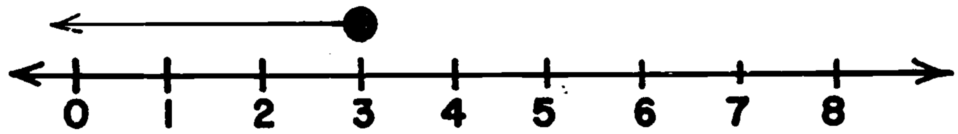
Write the mathematical sentence that is represented by these number line graphs.

6. ______
7. ______
8. ______
9.
10. ______
______

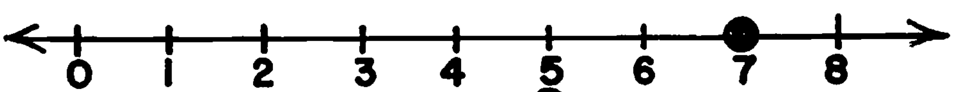


Draw the graph of each of these mathematical sentences on the number line.

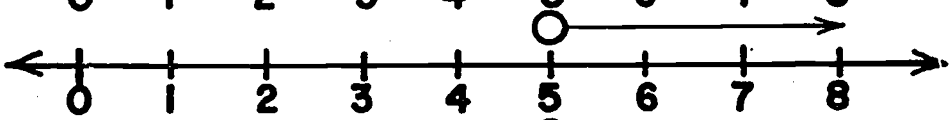
1. $X \leq 3$



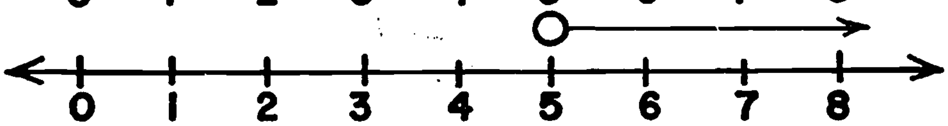
2. $X = 7$



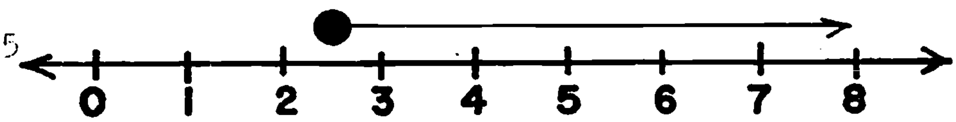
3. $X > 5$



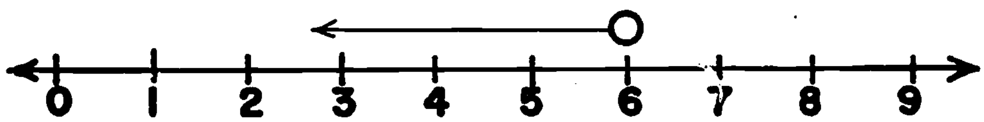
4. $5 < X$

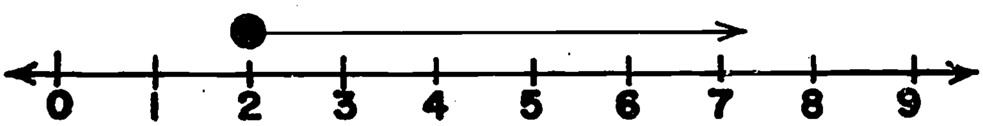


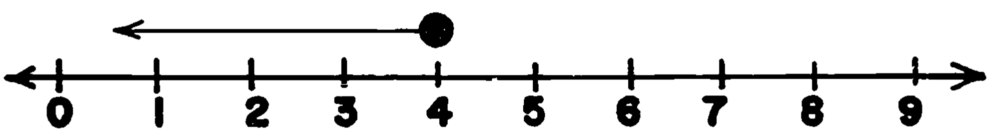
5. $X \geq 2.5$

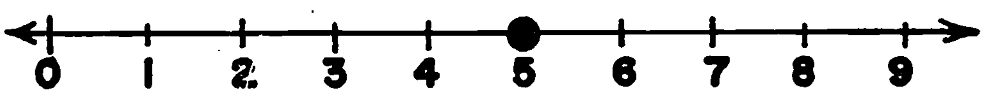


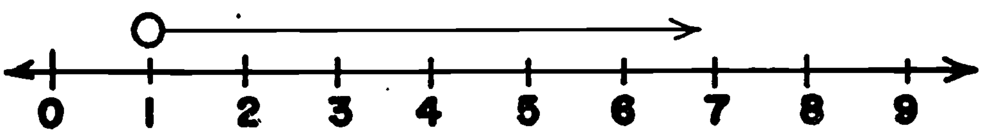
Write the mathematical sentence that is represented by these number line graphs.

6.  $X < 6$

7.  $X \geq 2$

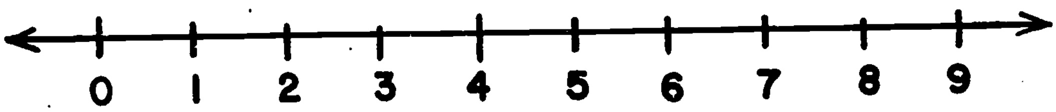
8.  $X \leq 4$

9.  $X = 5$

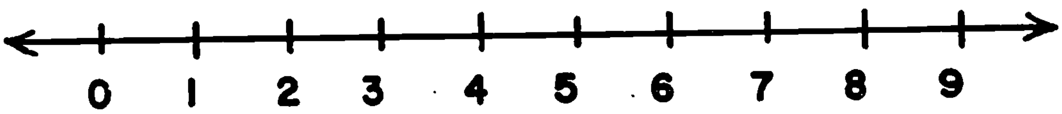
10.  $X > 1$

Solve these inequalities and graph the solutions on number lines.

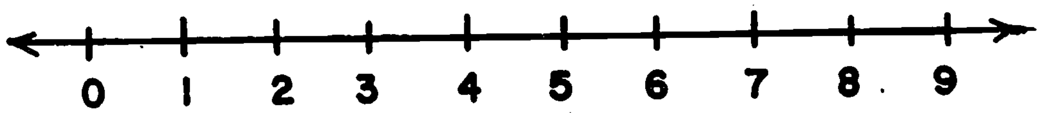
1. $5X > 35$



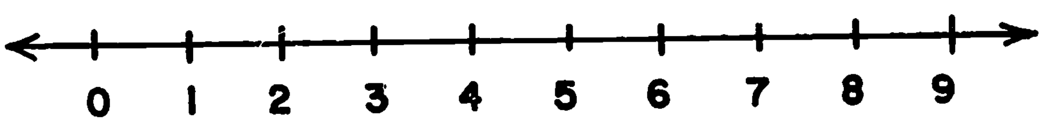
2. $X + 142 \leq 148$



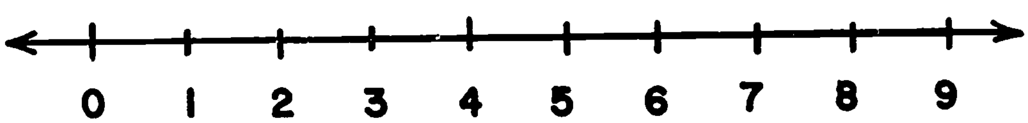
3. $5X + 22 \geq 42$



4. $10X + 17 < 87$



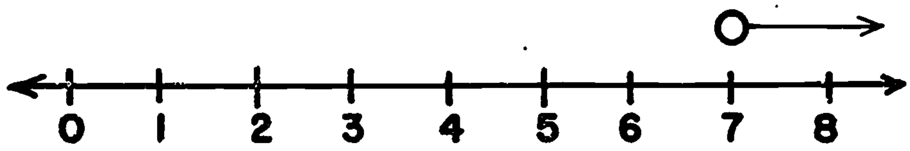
5. $25Y + 23 > 73$



ANSWERS

1. $\frac{5X}{5} > \frac{35}{5}$

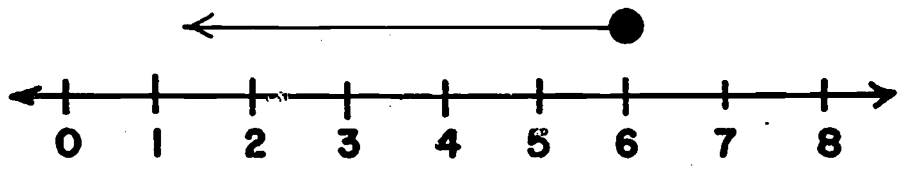
   $X > 7$



2. $X + 142 \leq 148$

   $\underline{\quad - 142 = -142}$

   $X \leq 6$

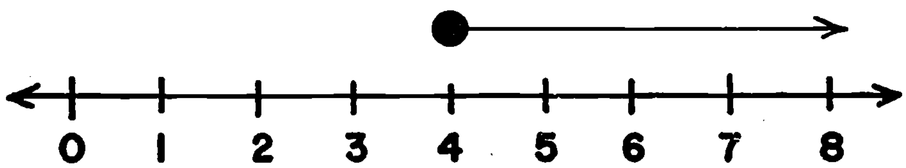


3. $5X + 22 \geq 42$

   $\underline{\quad - 22 = -22}$

   $\frac{5X}{5} \geq \frac{20}{5}$

   $X \geq 4$

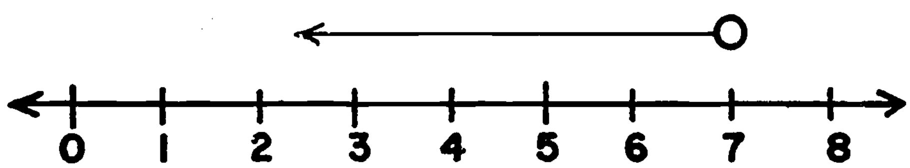


4. $10X + 17 < 87$

   $\underline{\quad - 17 = -17}$

   $\frac{10X}{10} < \frac{70}{10}$

   $X < 7$

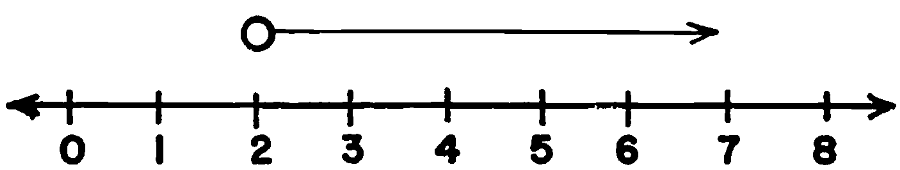


5. $25Y + 23 > 73$

   $\underline{\quad - 23 = -23}$

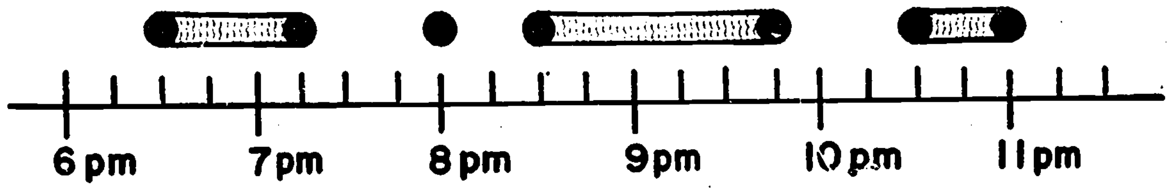
   $\frac{25Y}{25} > \frac{50}{25}$

   $Y > 2$



PHONE CALL GRAPHS

Kathy's father is keeping track of the time she talks on the phone. He is an engineer and works with graphs at work, so he made a graph of her calls one Monday evening. This is the graph:



Each unit on the scale represents 15 minutes. The letter t will be used to stand for the times that she was on the phone. The graph shows a dot at 8 p.m. (This would be written $t = 8$.) Kathy answered the phone at 8 o'clock, but hung up right away because she was mad at that boyfriend.

Her longest call was between 8:30 and 9:45. This can be written as an inequality as follows

$$8{:}30 \leq t \leq 9{:}45$$

This inequality statement is read "t is greater than <u>or</u> equal to 8:30 <u>and</u> less than <u>or</u> equal to 9:45". Because t stands for those times that she was on the phone, this inequality says that Kathy was on the phone all the time from 8:30 to 9:45. How many minutes was she on the phone during this time span? _____

How many hours is this? _____

As you remember from lesson 8, when the inequality symbols $\leq$ or $\geq$ are used, we graph the endpoints with a "closed dot".

PHONE CALL GRAPHS

OBJECTIVES

1. The students will be able to graph an inequality statement in one variable of the following types:
   a) $3 \leq X \leq 7$
   b) $X > 2$ and $X \leq 7$
   c) $X \geq 3$ or $X \leq 5$

THINGS TO DISCUSS

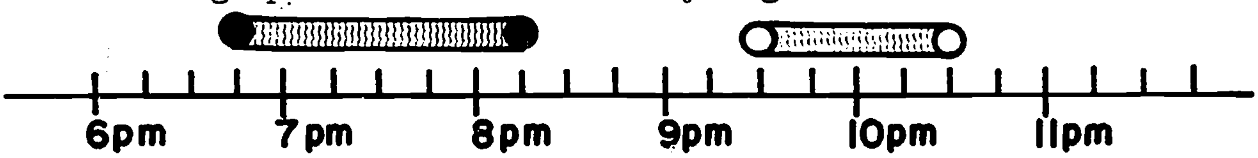
An additional assignment for this lesson might be for the students to make their own graphs of the time they spend talking on the phone, or working, or doing homework.

ANSWERS

75 minutes
$1\frac{1}{4}$ hours

Since the graph of Kathy's telephone conversation has a dot for an endpoint at 8:30 and 9:45, she was talking on the phone at 8:30 and 9:45. If the graph had showed an open circles for endpoints, it would have meant that she started talking right after 8:30 and hung up just before 9:45.

The graph of her calls Tuesday night looked like this:



EXERCISES

1. Describe the time interval she was on the phone on the first call. (For example, "She started at 5:15 P.M. and ended at 6:30 P.M.")
2. Describe the time interval she was on the phone for her second call.
3. Her first call could be written in this form:

   6:45 ______ t ______ 8:15

4. Her second call could be written as:

   9:30 ______ t ______ 10:45

5. How much time did she spend on the phone that night?
6. How much time was left for other things such as studying, watching T.V., listening to records, and washing her hair between 6:00 and 11:00 that night?

ANSWERS

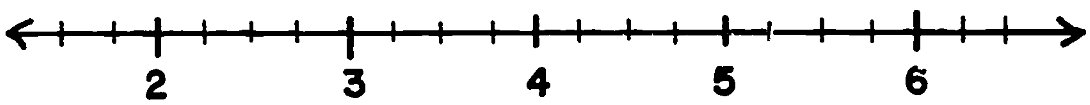
1. From 6:45 to 8:15.
2. From just after 9:30 to just before 10:45.
3. $6{:}45 \leq t \leq 8{:}15$
4. $9{:}30 < t < 10{:}30$
5. Two hours and 30 minutes or 150 minutes.
6. Two hours and 30 minutes or 150 minutes.

7. In making a two-hour lunar orbit, an astronaut is out of touch with the earth for 45 minutes. His radio contact is lost when he is behind the moon.

The table shown below gives some data on a lunar orbit.

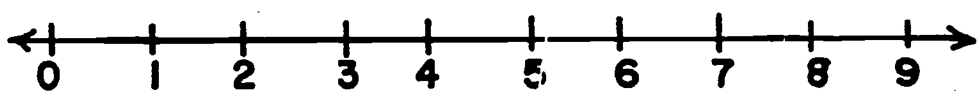
Lunar Orbit	
Time	Information
$t = 3{:}15$	begin radio blackout
$3{:}15 \leq t \leq 4{:}00$	duration of radio blackout
$4{:}30 \leq t \leq 5{:}00$	orbit correction
$5{:}45 \leq t \leq 6{:}15$	lunar photography

Graph all the times given above on a number line.

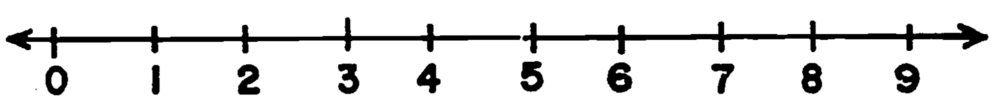


Graph the solution set for each inequality on a number line.

8. $X \geq 1$

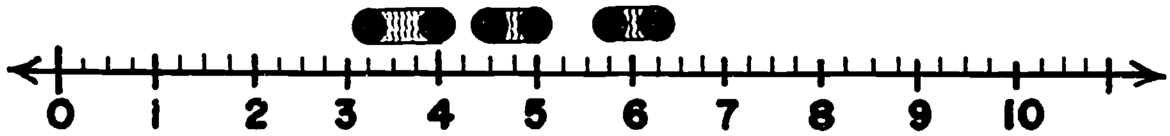


9. $X \leq 8$

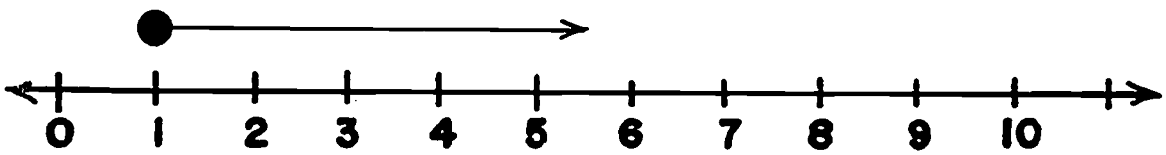


ANSWERS

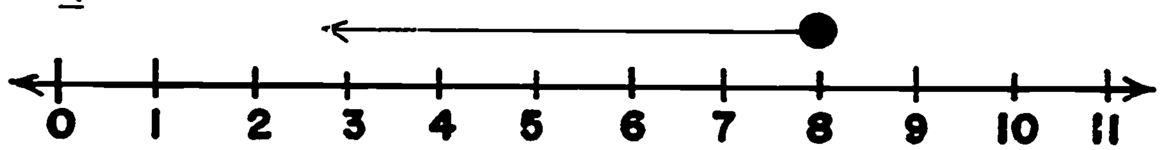
7.



8. $X \geq 1$

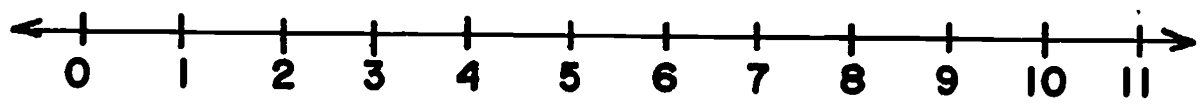


9. $X \leq 8$

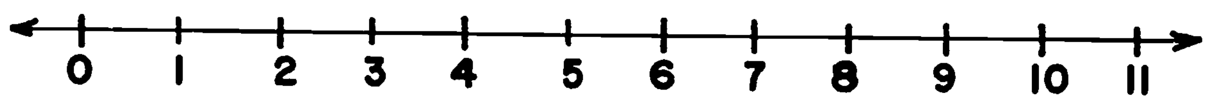


10. $X \geq 3$ and $X < 5.5$

(Hint: Graph <u>each</u> inequality. The solution set is where <u>both</u> are true.)

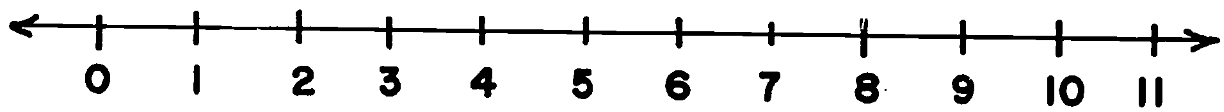


11. $5 < X < 9$



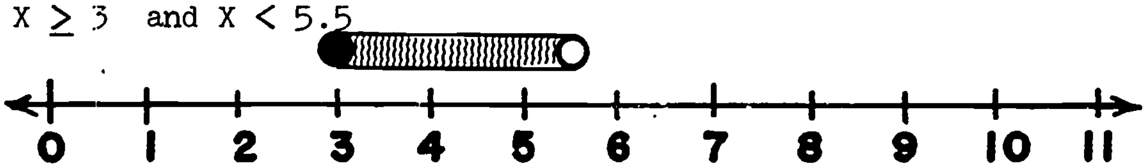
12. $X > 8$ or $X < 3$

(Hint: Graph <u>each</u> inequality. The solution set is all points on <u>either</u> graph.)

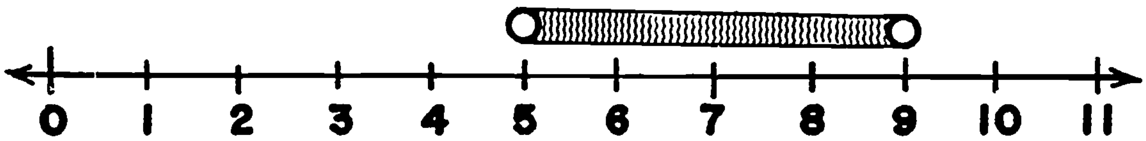


ANSWERS

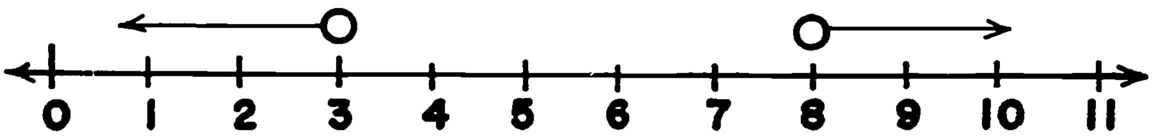
10. $X \geq 3$ and $X < 5.5$



11. $5 < X < 9$



12. $X > 8$ or $X < 3$



WRITING MATHEMATICAL SENTENCES

FORMULAS

$A = \pi r^2$

$C = 2\pi r$

$F = \frac{9}{5}C + 32$

$i = prt$

$60D = rt$

$D = rt$

$C = \frac{5}{9}(F - 32)$

$P = a + b + c$

$d = 16t^2$

$p = 4s$

$E = IR$

$A = p + prt$

$A = s^2$

$P = 2l + 2w$

DISCUSSION QUESTIONS

1. Above are some common equations. Which ones do you recognize? (Hint: if you don't recognize any of them go back and look at lesson 7.) Tell what these formulas mean.
2. Which of the above formulas do you recognize from your science classes?
3. How many more formulas are there besides the ones listed above?

Formulas are not really very mysterious. Anyone can make one. For example, if your grade in a class was based on your total score on three tests, you might write the formula $S = A + B + C$ to represent your grade.

In this formula S stands for your total score. A stands for your score on the first test, B stands for your score on the second test, and C the score on your third test.

4. What does this formula say is done with your three test scores? ______________________________

WRITING MATHEMATICAL SENTENCES

OBJECTIVES

1. Given a simple formula such as $C = 2a + b$ the students will be able to tell what it means.
2. Given an English sentence describing some simple mathematical function, the students will be able to write a formula to represent it.
3. Given a formula and some values to substitute into it, the students will be able to find the unknown value.

CONTENT AND APPROACH

Much of this lesson is meant to be a class discussion. You might tell the students that a formula is a shorthand way of writing some mathematical instructions. Because mathematicians are lazy they started writing formulas to save time. Let the students think they are getting out of work by learning to write and use formulas.

ANSWERS

1. $d = 16t^2$ and $60D = rt$ were used in lesson 7.
2. $F = \frac{9}{5}C + 32$ and $C = \frac{5}{9}(F - 32)$ are temperature conversion formulas and $E = IR$ is an electrical formula.
3. There are many thousands of other formulas.
4. The formula says <u>the three scores are added together</u>.

Which is easier to write, the description you just wrote or $S = A + B + C$? Actually, a formula is nothing more than a shorter way of stating a rule.

Examples

English Description	Formula
Twice A plus 3 times B plus 5 times C will give X amount.	$2A + 3B + 5C = X$
Centigrade temperature may be found by subtracting 32° from the Fahrenheit temperature and multiplying the result by $\frac{5}{9}$.	$C = \frac{5}{9}(F - 32^{\circ})$
The distance an object falls is found by squaring the number of seconds it falls and then multiplying by 16.	$d = 16t^2$

CONTENT AND APPROACH

Try to show the students that a formula is just a simpler and easier way to write out a mathematical thought.

EXERCISES : Fill in the blank.

English Description	Mathematical Formula
1. E added to D equals X.	________
2.	$B + C = 6$
3. G added to twice C equals 25.	________
4.	$A + B + C = 35$
5. The stopping distance of an automobile can be approximated by squaring the speed of the car and multiplying that number by .055.	Stopping distance = $\_\_\_\_^2 \times \_\_\_\_$
6. A knot is a nautical term meaning nautical mile per hour. To find mph you must multiply knots by 1.15.	mph = ____ × ____

ANSWERS

1. $D + E = X$
2. C added to B equals 6
3. $2C + G = 25$
4. The sum of A, B, and C equals 35.
5. Stopping distance = speed^2 × .055
6. mph = knots × 1.15

7.	$\ell = 1.9w$
8.	$C = \pi d$
9. One formula used in electric circuits is Volts (E) equals Current (I) multiplied by Resistance (R).	E =
10. Another electrical formula is watts (W) equals amperes (I) multiplied by voltage (E).	W =

Formulas are meant to help people solve mathematical problems. They are actually a set of instructions. For example, in #2 on page 57,

$$B + C = 6. \qquad \text{If } B = 5, \text{ find } C.$$

If we substitute the number 5 for the letter B, the equation becomes $5 + C = 6$. As you remember from working with the balance beam, all you have to do to find C is subtract 5 from each side of the equation.

ANSWERS

7. ℓ equals 1.9 times w.

8. C equals π times d.

9. $E = IR$

10. $W = IE$

$$\begin{array}{rl} B + C &= 6 \\ 5 + C &= 6 \\ -\underline{5 \quad\quad} & \underline{= -5} \\ C &= 1 \end{array}$$

11. Using the formula in exercise # 2, find B if C = 4.

12. Using the same formula, find the value of C if B = 3.5.

13. Using the formula you wrote in exercise 3, find the value of G if C has a value of 10.

14. Using formula 5 on page 57, find the stopping distance of a car traveling 30 mph.

15. Find the stopping distance of a car traveling 60 mph.

16. If an ocean liner has a top speed of 40 knots, how many mph would this be?

ANSWERS

11. $B + C = 6$

$B + 4 = 6$

$\underline{\quad - 4 = -4}$

$B = 2$

12. $3.5 + C = 6$

$\underline{-3.5 \quad = -3.5}$

$C = 2.5$

13. $2C + G = 25$

$2 \times 10 + G = 25$

$20 + G = 25$

$\underline{-20 \quad = -20}$

$G = 5$

14. Stopping distance = $\text{speed}^2 \times .055$

Stopping distance = $30^2 \times .055$

Stopping distance = $900 \times .055$

Stopping distance = 49.5 feet

15. Stopping distance = $\text{speed}^2 \times .055$

Stopping distance = $60^2 \times .055$

Stopping distance = $3600 \times .055$

Stopping distance = 198 feet

16. mph = knots × 1.15

mph = 40 × 1.15

mph = 46

17. The formula given in exercise # 7 actually gives the relation between the length and width of a United States flag. If a flag has a width of 20 units, find its length.

18. Use the formula in #8 to find C if d is 20. (Use 3.14 or $3\frac{1}{7}$ as an approximation for π.)

19. Using the formula you wrote for #9, find the voltage in an electrical circuit if the current is 10 amperes and the resistance is 17 ohms.

20. Using the formula in # 10, find the number of watts used by an electric frying pan if it uses 10 amps of electricity at 120 volts.

21. Find the stopping distance of these three cars at a dragstrip. At the end of a quarter-mile the first car has reached a speed of 100 mph the second car is going 120 mph and the third car is going 140 mph.

ANSWERS

17. $\ell = 1.9\ w$ w = width ℓ = length
    $\ell = 1.9 \times 20$
    $\ell = 38$

18. $C = \pi d$
    $C = 3.14 \times 20$
    $C = 62.8$

19. $E = IR$
    $E = 10 \times 17$
    $E = 170$ volts

20. $W = IE$
    $W = 10 \times 120$
    $W = 1200$ watts

21. The 100 M.P.H. car required 550 feet to stop.
    The 120 M.P.H. car required 792 feet to stop.
    The 140 M.P.H. car required 1078 feet to stop.
    All the cars accelerated for a quarter mile or 1320 feet.
    The 140 M.P.H. car took almost that much distance to stop.

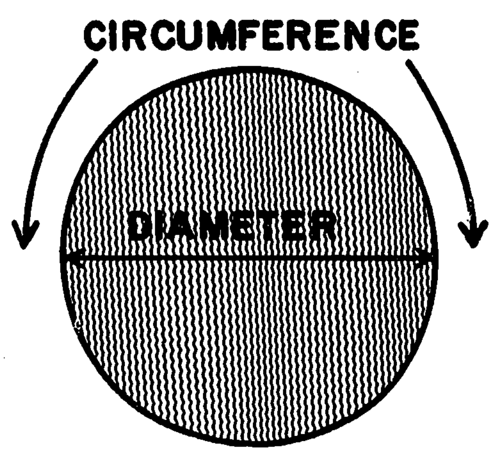
USING MATHEMATICAL SENTENCES

PROJECT #1 Instructions: Gather together several round objects such as tin cans, wastebaskets, paper cups, circles cut out of cardboard, etc. Also you will need a flexible tape measure.

Make a table like this one:

Description of object	Circumference	Diameter	Value of $\frac{C}{D}$

Using a flexible tape, measure the circumference (distance around the object) and the diameter (distance across the object at its widest point).



USING MATHEMATICAL SENTENCES

OBJECTIVES

1. Given the formula and the necessary data, the students will be able to find the unknown quantity.

CONTENT AND APPROACH

This lesson is a series of projects. It is <u>not</u> intended that each student do each project. It is suggested that the teacher assign various projects to the students depending upon the students' interests and abilities.

EQUIPMENT AND TEACHING AIDS

Project # 1 requires several round objects such as coffee cans, wastebaskets, and paper cups. Also either a flexible tape measure or a ruler and a piece of string is required.

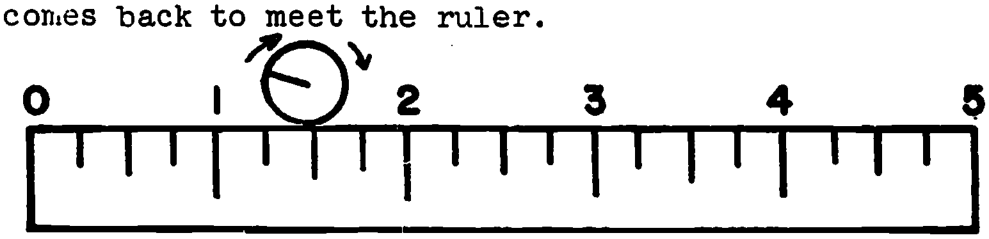
Write down these two measurements for each object in your table. Then, using the formula $C = \pi D$, find an approximation for π. Example: if C measured 31 inches and D measured 10 inches the formula would be $31 = \pi 10$.

$$\frac{31}{10} = \frac{\pi \times 10}{10}$$

$$3.1 = \pi$$

PROJECT #2

You can measure the distance around a round object by rolling it along a ruler or yardstick. To do this you must make a mark on the edge of the circle. Start with this mark at the zero mark on the ruler. Then roll the object until the mark comes back to meet the ruler.



If you knew the distance a wheel traveled making one complete turn, what would be the distance traveled in two complete turns? _____ How about four complete turns? _____ How far would the circle travel in ten turns? _____ How could you use a circular object to measure distance? _____________

An automobile odometer uses this principle to measure the number of miles a car travels. The odometer is connected to the rear wheels on most cars and counts the number of turns of the wheels. The odometer is geared so that turns of the wheel are converted to miles.

DISCUSSION QUESTIONS

How would these things affect the accuracy of a car odometer?

1. Driving fast for a while and then driving slow.
2. Changing to a much larger or smaller tire.

EQUIPMENT AND TEACHING AIDS

For Project # 2, a yard stick or meter stick is needed, along with a round object to roll along the measuring instrument.

ANSWERS TO EXERCISES

Twice as far.
Four times as far.
Ten times as far.
Measure how far it rolled during one complete revolution and count how many times it revolved in traveling a distance.

ANSWERS TO DISCUSSION QUESTIONS

1. Changing speeds should have no effect on the car odometer because the tires still cover the same distance with each revolution. (At extremely high speeds centrifugal force expands the tire and makes the circumference of the tire larger--however for this lesson this can be ignored.)
2. This affects the odometer because it is geared to work with a certain size tire. All the odometer does is count the tire revolutions. For the same number of turns a larger tire would take the car further, and a smaller tire would cover less distance.

3. Spinning the tires on ice or snow.
4. A hot rodder leaving a patch of rubber.
5. Driving slow, then fast, then slow.
6. A car backing up a great distance.
7. Leaving the engine idling for several hours with the transmission in neutral.
8. Coasting down hills with the engine turned off.
9. A wrecker towing the car with the rear wheels lifted off the highway.

MEASURING BY ROLLING

Any round object can measure distance. The formula is

$$D = nc$$

D stands for the distance the object is rolled. n stands for the number of turns the object rolled. c stands for the circumference or distance around the object.

Example #1 If a wheel with a circumference of six feet makes eight complete turns, what distance does it travel?

$$D = nc$$
$$D = 8 \times 6 \text{ feet}$$
$$D = 48 \text{ feet}$$

Example #2 If a wheel has a circumference of five feet, how many turns does it make in traveling a mile? (Don't forget a mile is 5280 feet.)

$$D = nc$$
$$5280 = n \times 5$$
$$\frac{5280}{5} = \frac{5n}{5}$$
$$1056 = n$$

The wheel must make 1056 turns to travel a mile.

ANSWERS

3. and 4. If the tires spin and the car doesn't move, the odometer still counts the turns of the wheels and registers distance. Therefore, the odometer reading would be inaccurate.
5. Has no effect on the accuracy.
6. Backing up will make the odometer turn backwards and therefore affects its accuracy.
7. As long as the wheels don't turn the odometer is accurate.
8. The wheels still turn so the odometer is accurate.
9. In practically all cars (except front wheel drive cars and 1964 Oldsmobiles) the odometer is geared to the rear wheels, thus the odometer will not record the distances the car was towed.

Part A Use a wheel to measure some distances around your classroom or school. If possible measure the same distance several times using different size wheels. Always use the formula D = nc.

Fill in the table below.

What was measured	Circumference of wheel	Number of turns	Distance

Part B Use the formula D = nc to solve these problems.

1. How many turns does a boat trailer tire make to travel a mile (5280 feet) if its circumference is four feet?
2. How many turns does an auto tire make in a mile if its circumference is seven feet?
3. If a tire turns 660 times and has a circumference of eight feet, what distance does it travel?
4. If a go-cart tire makes 25 turns in 60 feet, what is its circumference?

EQUIPMENT AND TEACHING AIDS

Some sort of trundle wheel is needed for project # 2, part A.

Part B ANSWERS

1. 1320 turns.
2. $754\frac{2}{7}$ turns.
3. 5280 feet (one mile).
4. 2.4 feet.

5. If a tire has a circumference of 54 inches and turns 16 times, what distance does it travel?

PROJECT #3 Another way to measure distance is by using the formula $D = rt$.

D stands for distance.

r stands for the rate of travel.

t stands for the time.

If the rate is given in feet per <u>second</u> then the time must be measured in seconds.

If r is given in feet per <u>minute</u> then t must be measured in minutes.

If r is given in feet per <u>hour</u> then t must be given in hours.

> $60D = rt$ is a formula you have used before.
> It is used when r is given in miles per <u>hour</u> and t is given in <u>minutes</u>.

To use either $D = rt$ or $60D = rt$ to find the distance you must know the rate of travel.

First Step: Find your rate of walking. Have someone time you while you walk off a distance you know (such as the width of your classroom). The timing should be done in seconds.

D (the distance you walked) is ______________.

t (the number of seconds it took you) is ________.

With these two values find r.

Second Step: using $D = rt$

______ = r ______

______ = r

Thus your rate of walking is ___ feet per second.

Project #2, part B #5 ANSWERS

5. 864 inches or 72 feet.

EQUIPMENT AND TEACHING AIDS

For project # 3, a stop watch is helpful but not necessary. An ordinary second hand is good enough for the timing. The other thing needed is a known distance (such as the width of the classroom).

Third Step: Now you're ready to measure other distances. For example, to measure the length of your school hall, time how long it takes you to walk this distance. Try to walk at the same speed you did in the first step of the project.

r (your rate of walking) is _____

t (number of seconds) is _____

$D = rt$

$D =$ _____ × _____

$D =$ __________

Measure four other distances around your school using this method. Fill in the table and show the mathematics you had to do to get the answer.

$D = rt$

From what point to what point did you measure?	r (your rate)	t (number of seconds it took you)	D (distance)
Example: one end of hall to the other	5 ft. per second	45 sec.	225 ft.
1.			
2.			
3.			
4.			
5.			

Five feet per second seems to be an average rate of walking.

PROJECT #4 Temperature Conversion

For this experiment you need two thermometers, one reading in Fahrenheit and the other in Centigrade. Also you need a glass of water and ice cubes. The group working on this project is divided up into two parts. One part reads the Fahrenheit thermometer and converts these readings into Centigrade degrees. The other students do just the opposite. They record the readings on the Centigrade thermometer and convert them into Fahrenheit degrees. The two groups must take their readings at the same time.

Procedure: The two thermometers are placed in a glass of ice water until the mercury stops dropping. Both groups record the reading of their thermometer at this time. Then the thermometers are removed from the ice water and placed near heat (such as a heat register or the sunlight). Every 30 seconds each group takes another reading on their thermometer.

Each group would fill in one of the tables on the next two pages.

EQUIPMENT AND TEACHING AIDS

For project # 4, two thermometers are needed, one that reads only in Centigrade and one that reads only in Fahrenheit. A container of ice water is also necessary.

For the group reading the thermometer in Fahrenheit degrees, the formula to use is

$$C = \frac{5}{9}(F - 32)$$

For example, if the temperature read was 42° F., this formula becomes

$$C = \frac{5}{9} \times (42-32)$$

$$C = \frac{5}{9} \times (10)$$

$$C = \frac{5}{9} \times 10 = \frac{50}{9} = 5\frac{5}{9}$$

Then 42° F. must be the same as $5\frac{5}{9}{}^\circ$ C.

Their table is

Time	Degrees Fahrenheit	What this would be in Centigrade degrees

For the group reading the thermometer in Centigrade degrees, the formula is

$$F = \frac{9}{5}C + 32$$

For example, if the temperature is 6° C., the formula becomes

$$F = \frac{9}{5} \times 6 + 32$$

$$F = \frac{54}{5} + 32$$

$$F = 10\frac{4}{5} + 32$$

$$F = 42\frac{4}{5}^{\circ}$$

Thus 6° C. is the same temperature as $42\frac{4}{5}^{\circ}$ F.

The table is

Time	Degrees Centigrade	Degrees Fahrenheit

PROJECT #5 The formula: class rating = $\frac{\text{weight}}{\text{H.P.}}$ is very important at the dragstrip. The National Hot Rod Association places stock cars in various classes according to this formula. To find the class rating, the title weight of the car is divided by its advertised horsepower. Then a table gives the class in which the car will race.

For example, consider these three cars:

A. A 3000 lbs. compact car with a 200 H.P. engine.
B. A 3750 lbs. intermediate car with a 250 H.P. engine.
C. A 4500 lbs. luxury car with a 300 H.P. engine.

The formula for Car A would be:

$$\text{Class rating} = \frac{3000 \text{ lbs.}}{200 \text{ H.P.}} \qquad 200\overline{)3000}^{\,15}$$

Class rating = 15 lbs. per H.P.

Car B:

$$\text{Class rating} = \frac{3750 \text{ lbs.}}{250 \text{ H.P.}} \qquad 250\overline{)3750}^{\,15}$$

Class rating = 15 lbs. per H.P.

Car C:

$$\text{Class rating} = \frac{4500 \text{ lbs.}}{300 \text{ H.P.}} \qquad 300\overline{)4500}^{\,15}$$

Class rating = 15 lbs. per H.P.

Because all three cars have the same class rating (15) they would all race in the same class. According to the table at the top of page 65, what class would this be? Do you think it would be a fair race?

Project 5 is the one in which many of the boys may be interested.

Car weight is the weight stated on the car title and license registration.

Theoretically all three cars listed would accelerate at the same speed and it would be a fair race. All three cars would race in class N/S.

A good discussion question would be: If you get a higher horsepower engine, what body weight should you get to be able to race in the same class?

Here is part of the table the National Hot Rod Association uses to classify stock cars.

A/S-7.50 to 7.99
B/S-8.00 to 8.49
C/S-8.50 to 8.99
D/S-9.00 to 9.49
E/S-9.50 to 9.99
F/S-10.00 to 10.49
G/S-10.50 to 10.99
H/S-11.00 to 11.49
I/S-11.50 to 11.99
J/S-12.00 to 12.49
K/S-12.50 to 12.99
L/S-13.00 to 13.99
M/S-14.00 to 14.99
N/S-15.00 to 15.99
O/S-16.00 to 16.99
P/S-17.00 to 18.99
Q/S-19.00 to 20.99
R/S-21.00 to 22.99
T/S-23.00 to 24.99
U/S-25.00 to 26.99
V/S-27.00 or more

Part A Classify these ten stock cars into their proper class:

1. A 3500 lb car with a 350 H.P. engine.
2. A compact weighing 2475 lbs. with a 200 H.P. engine.
3. A 4000 lb. sedan with a 200 H.P. engine.
4. A 3700 lb. hardtop with a 425 H.P. Hemi engine.
5. A 3240 lb. compact convertible with a 180 H.P. engine.
6. A 2880 lb. economy car with a 120 H.P. engine.
7. A 4100 lb. luxury sedan with a 325 H.P. engine.
8. A 3000 lb. car with a 375 H.P. engine.
9. A 3155 lb. hardtop with a 250 H.P. engine.
10. A 2400 lb. Model A with a 40 H.P. engine.

Part B Find the weight and horsepower of several cars and classify them according to NHRA rules. For each car give the make and model, title weight, horsepower, and class it would run in at the dragstrip.

In classes A through N, automatic transmission cars are put in separate classes. Stick shift cars would be classified as B/S, D/S, L/S etc. Automatic transmission cars would be classified as B/SA, D/SA, L/SA etc.

However in classes O through V both stick and automatic transmission cars run in the same class, such as P/S, R/S, and U/S.

ANSWERS

1. F/S
2. J/S
3. Q/S
4. C/S
5. P/S
6. T/S
7. K/S
8. B/S
9. K/S
10. V/S

THINGS TO DISCUSS

Car #8 runs in a faster class (B/S) than car #4 (C/S) even though it has less horsepower. This happens because it weighs less. Cars 7 and 9 run in the same class even though one has a larger engine because that car weighs more.

Part B The students can find specifications of weight and horsepower in many car magazines.

PROJECT #6 Here are some other formulas. Make up your own exercises.

#1 $W = 5.5(H - 60) + 110$

This formula is thought by some people to give the proper weight of a person.

W stands for weight in pounds.
H stands for height in inches.

Example: Find the weight of a person who is 5 feet 10 inches tall.

5 feet 10 inches equals 70 inches ($5 \times 12 + 10$)
$W = 5.5\ (70 - 60) + 110$
$W = 5.5\ (10) + 110$
$W = 5.5 \times 10 + 110$
$W = 55 + 110$
$W = 165$ lbs.

Thus the formula tells us that this person should weigh 165 lbs.

#2 Imperial gallons $\times$ 1.2 = U.S. gallons
U.S. gallon $\times$ .8327 = Imperial gallon

#3 Batting Average $= \frac{H}{B}$

H = number of hits
B = official times at bat
The division is carried out correct to three decimal places.

Example: A batter has 27 hits out of 72 official times at bat.
Batting Average $= \frac{27}{72}$ $\qquad 72 \overline{)27.000}$ = .375

Thus, his batting average is .375.

Project # 6 gives some additional formulas in which some of the students might be interested.

#4 $ERA = \frac{ER}{I} \times 9$

The earned Run Average is a way of comparing the abilities of baseball pitchers.

The formula used is $ERA = \frac{ER}{I} \times 9$

ER is the number of earned runs the pitcher has allowed.

I stands for the number of innings the pitcher has pitched.

Example: A pitcher has allowed 15 earned runs in 45 innings.

$$ERA = \frac{15}{45} \times 9$$
$$ERA = \frac{1}{3} \times 9$$
$$ERA = 3.00$$