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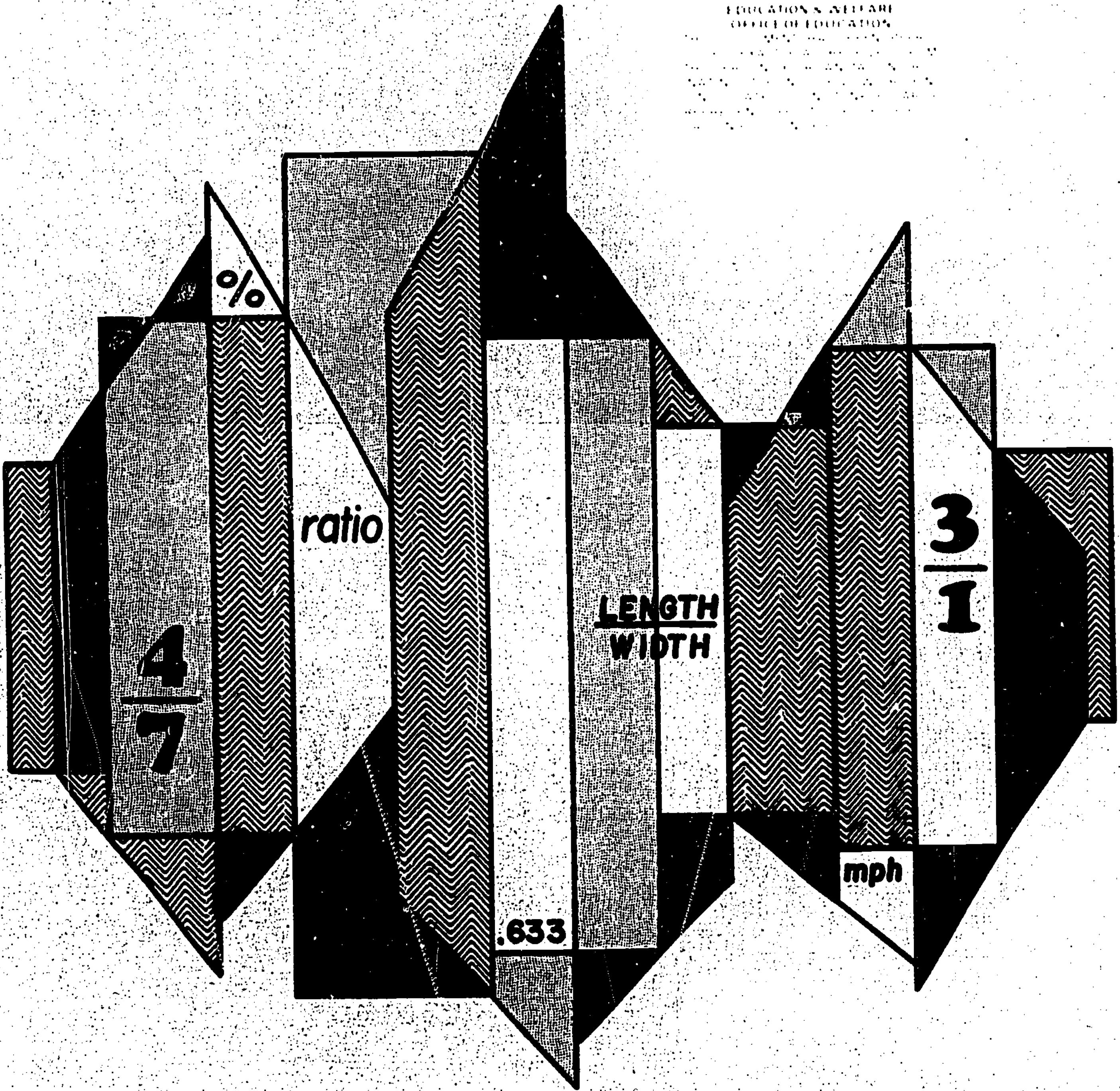
ABSTRACT

This instructional unit focuses on writing ratios and proportions in problem situations, solutions by means of proportions, and determination of percentages. A number of experiments are suggested and worksheets and discussion questions are included. The activities are oriented toward situations in which the students would probably have had some previous experience. A teacher's guide is also available. Related documents are SE 015 334, SE 015 335, and SE 015 337 through SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ACTIVITIES WITH RATIO & PROPORTION



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ACTIVITIES WITH RATIO AND PROPORTION

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

Ratios and proportions are useful tools for problem solving. They can be used in many situations involving numerical relations. Some examples are comparing winning records of baseball teams, predicting the weight of an object suspended on a spring, and increasing the size of a recipe to feed a large number of people. It is because of the problem-solving power of ratio and proportion that this booklet was written.

If a technician working for a manufacturing firm knows the weight and volume of a metal object, he can tell what it is made of. This identification process (known as finding specific gravity) can be used by a detective (What is this piece of metal?), a dairy worker (Does the milk meet the state butterfat content requirement?), a service station attendant (Is there enough antifreeze in the radiator? Is this battery fully charged?), as well as in many other areas.

Each lesson centers on one or two main ideas. This booklet contains a variety of activities to help you develop and apply the ideas of ratio and proportion. Read, listen, and above all, participate in the discussions and class activities.

After actively <u>participating</u> in the activities of this booklet, you should be able to:

1. Write ratios and draw diagrams which represent statements like "7 out of 9 tires are whitewalls" and "For every 3 squares there are 5 triangles."

   (LESSON 1)

14. Estimate a whole number per cent of a given number.

(LESSON 14)

15. Use proportions to calculate a whole number per cent of a given number.

(LESSON 15)

16. Recognize whether or not a problem situation requires an inverse proportion.

17. Solve problems requiring inverse proportions.

(LESSON 16)

You should be able to apply what you have learned about ratio and proportion to other booklets this year as well as in your future mathematics and science classes.

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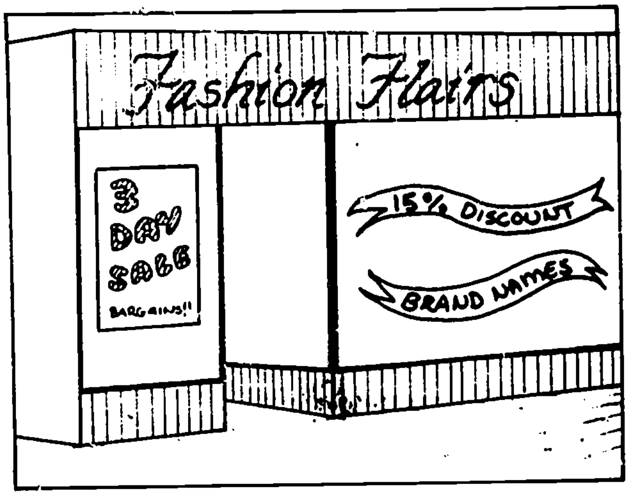
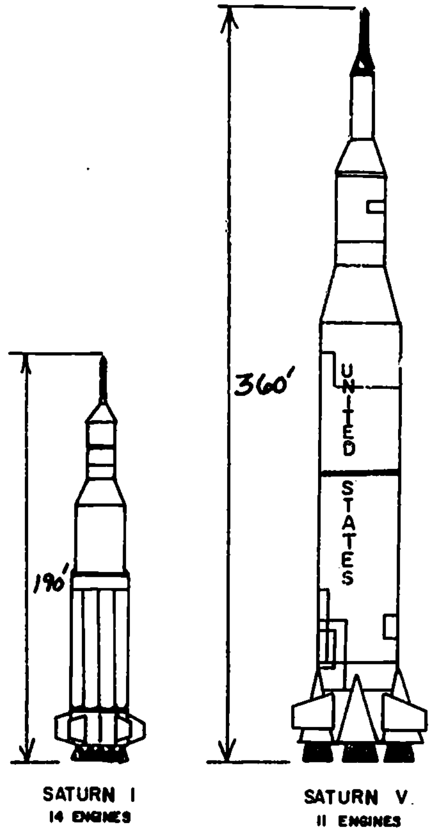
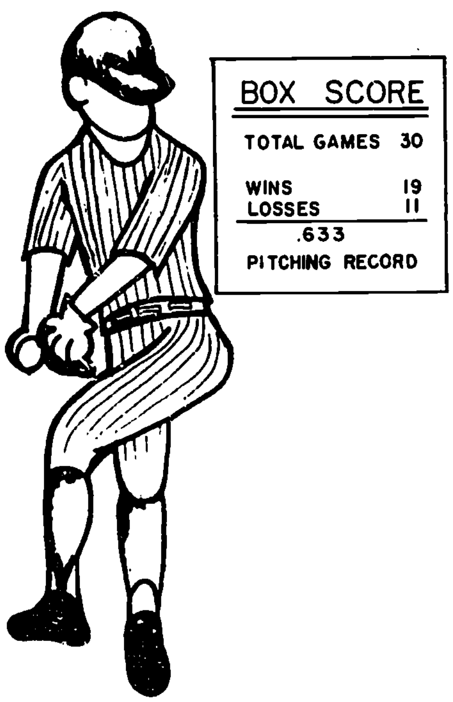
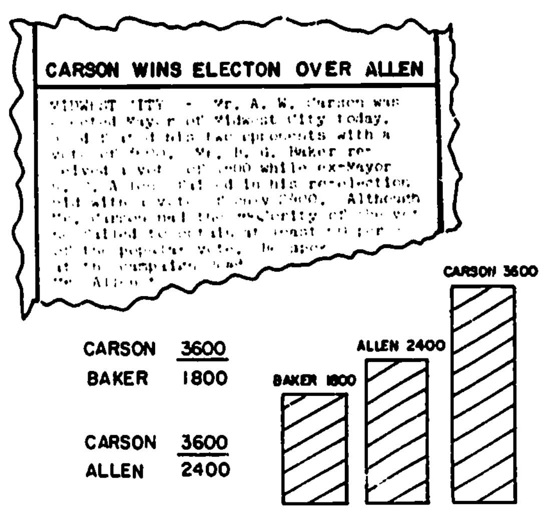
THREE OUT OF EVERY FOUR. . .


Fashion Flairs
3 DAY SALE
BARGAINS!!
15% DISCOUNT
BRAND NAMES


190'
360'
UNITED STATES
SATURN I
14 ENGINES
SATURN V
11 ENGINES


BOX SCORE
TOTAL GAMES 30
WINS 19
LOSSES 11
.633
PITCHING RECORD


CARSON WINS ELECTON OVER ALLEN
CARSON 3600
BAKER 1800
CARSON 3600
ALLEN 2400
BAKER 1800
ALLEN 2400
CARSON 3600

A baseball pitcher's record can be written as a ratio. The ratio is $\frac{19}{30}$ where the numerator is the number of wins and the denominator is the total of wins and losses. The decimal .633 is found by dividing 19 by 30. This is the way a pitcher's record is reported in the newspaper.

Stores offer sales to clear out goods they have had on hand in order to make room for new merchandise. The amount items are reduced is usually figured by using per cent. What does it mean when a store advertises 15% off? How are per cents written as ratios?

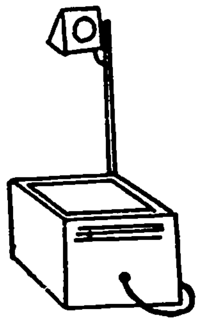
The results of an election are reported using the number of votes each candidate receives. The person receiving the highest total wins the election. Who was the winning candidate? By how many votes did he beat his nearest rival? Ratios can be used to compare votes. The ratio $\frac{3600}{1800}$ shows that people voted 2 to 1 in favor of Mr. Carson over Mr. Baker. What ratio indicates the comparison of votes for Mr. Carson to votes for Mr. Allen? What per cent of the vote did Mr. Carson get?

Saturn I was one test rocket used to explore space before an actual moon flight was attempted using the Saturn V. What is the ratio of the height of Saturn V to the height of Saturn I? About how many times longer is Saturn V? What is a ratio that compares the number of engines? Does it seem reasonable that a longer space rocket uses fewer engines?

Ratios are used in many situations, some of which are indicated on the previous two pages. Performances of cars, athletes, students, and home appliances can be compared using ratios.

PART I

The first activity of this unit contains a review of the meaning of ratio. Your teacher will use the overhead projector to illustrate several ways ratios can be interpreted.



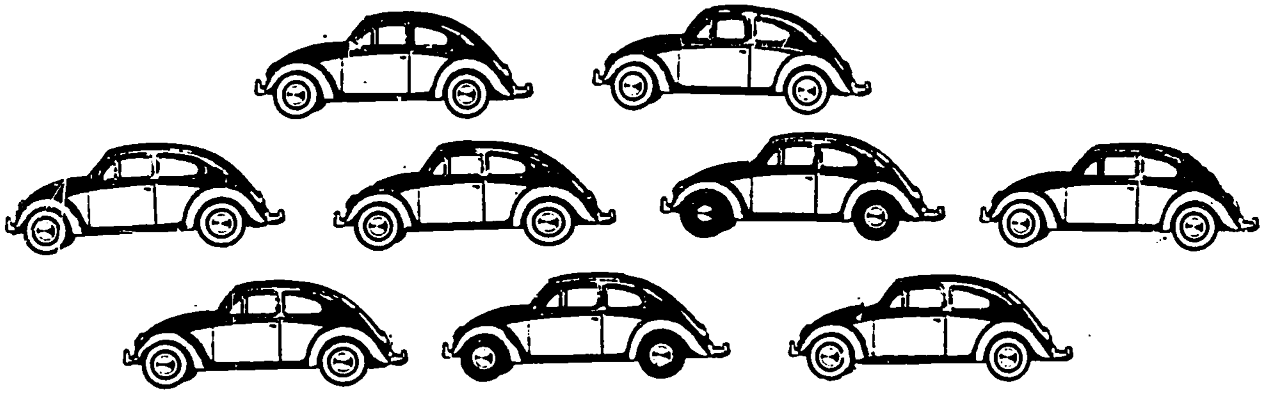
Watch carefully and participate in the discussion. If you have questions during the discussion, ask them.

PART 2

There are several kinds of statements indicating that a ratio could be written. In this part, you will work with one kind of statement.

LESSON 1

EXAMPLE 1:



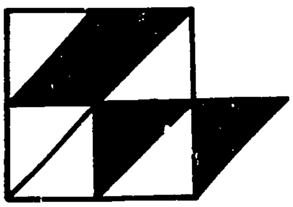
7 out of the 9 cars have white-walled tires.

$\frac{7}{9}$ expresses the statement in ratio form.

The denominator of the ratio represents the entire set.

The numerator represents the part of the set referred to.

EXAMPLE 2: 4 out of 9 triangular regions are shaded.



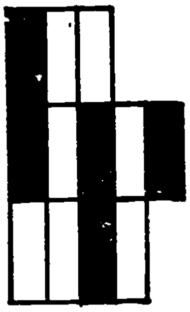
4 out of 9 is represented by $\frac{4}{9}$.

The denominator of the ratio is 9, the numerator is 4.

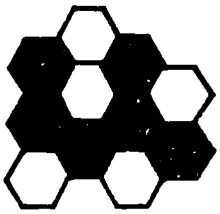
EXERCISES

1. 2 out of 5 triangular regions are shaded. Write the ratio represented by the statement.

2. 4 out of 7 square regions are shaded. Write the ratio.

3. 

Of the 12 rectangular regions, 5 are shaded. Write the ratio indicated.

4. 

a. 6 out of the 11 hexagonal regions are shaded. Write the indicated ratio.

b. 5 out of the 11 hexagonal regions are not shaded. What ratio represents this statement?

5. 

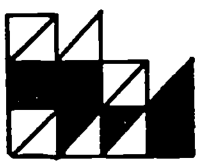
a. ____ out of 6 circular regions are shaded.

b. Write the ratio.

6. 

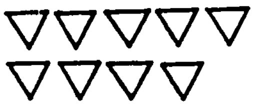
a. 4 out of ____ squares are circled.

b. Write the ratio.

7. 

a. ____ out of _____ triangular regions are shaded.

b. Write the indicated ratio.

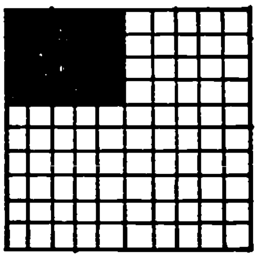
8. 

a. Circle the correct number of triangles to illustrate: 7 out of 9 triangles are circled.

b. Write the indicated ratio.

✓ **POINT**

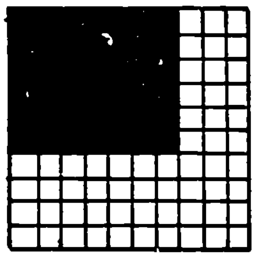
How do you know which number is the numerator and which is the denominator?

9. 

The picture contains 100 squares like this: □

a. ____ out of the 100 square regions are shaded.

b. What ratio is indicated?

10. 

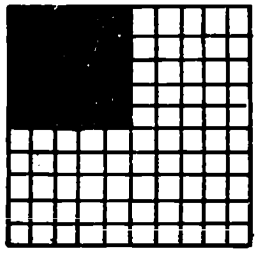
a. ____ out of 100 regions are shaded.

b. Write the ratio indicated.

c. ____ out of ____ regions are <u>not</u> shaded.

d. What ratio is represented now?

25 out of 100 regions are shaded. $\frac{25}{100}$ is the indicated ratio. Ratios having a denominator of 100 are given the special name <u>PER CENT</u>. Per cent means "out of 100" or "for each 100".



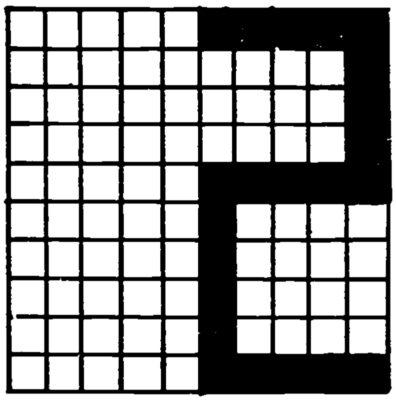
25 per cent and $\frac{25}{100}$ represent the same idea.

11. a. Write 35 per cent as a ratio.

    b. $\frac{75}{100}$ can be written as ____ per cent.

    c. If 40 out of 100 regions are shaded, the ratio indicated is ___ . Write the ratio using its special name.

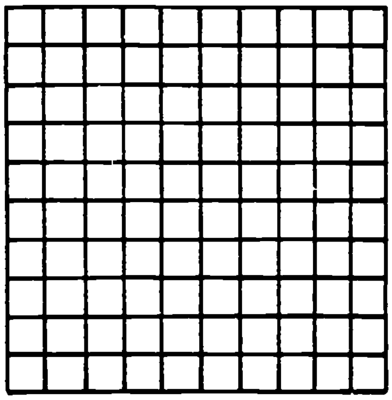
12.



a. In the picture, ____ out of 100 regions are shaded.

b. The ratio indicated is ____.

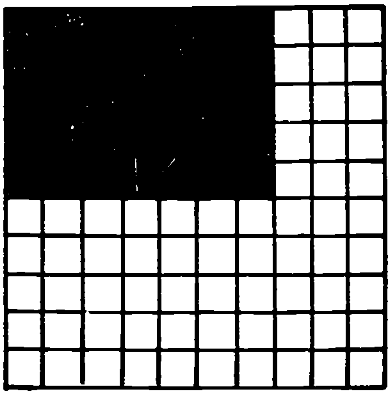
c. Express the ratio as a per cent.

13. 

a. Shade the small square regions to show that 15 out of 100 regions are shaded.

b. What ratio is represented by the picture?

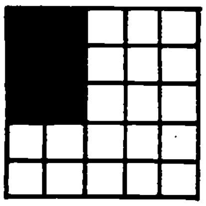
c. Write the ratio as a per cent.

14. 

35 per cent of the regions are shaded.

Circle below the ratio representing the statement:

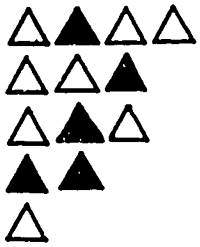
$\frac{35}{65}$ $\frac{65}{100}$ $\frac{100}{35}$ $\frac{35}{100}$

15. 

a. ___ out of ____ regions are shaded.

b. The indicated ratio is ____.

c. At the same rate, how many regions would be shaded out of a total of 50 regions?

16. 

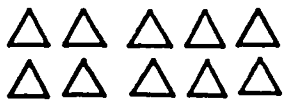
a. ____ out of ____ regions are <u>not</u> shaded.

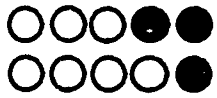
b. Write the indicated ratio.

17. 4 out of 10 triangular regions are to be shaded.

a. Complete the picture:

b. Write the ratio.



18. a. Complete the picture to represent $\frac{6}{10}$: 

b. ____ out of ____ circular regions are shaded.

19. In a picture using circles to represent $\frac{4}{11}$,

a. How many circles would be in the entire picture?

b. How many circular regions would you shade?

c. Draw a picture to represent the ratio $\frac{4}{11}$.

20. Which of the ratios represents the statement, 18 out of 53 triangular regions are shaded?

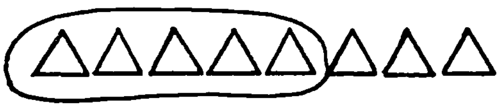
$\frac{53}{18}$ $\quad \frac{18}{100}$ $\quad \frac{18}{53}$ $\quad \frac{53}{100}$

PART 3

The ratio $\frac{5}{8}$ can be used to express the statement "For every 5 there are 8." This statement differs from the first type we wrote because it indicates that two different sets of objects are being compared. The first type of ratio was a comparison of part of a set to the whole set.

Look at the two types of ratios to note their differences:

5 out of 8 triangles are circled.



$\frac{5}{8}$

For every 5 triangles there are 8 squares.

$\frac{5}{8}$

EXAMPLE: Draw the picture and write the ratio representing: For every 11 * there are 6 #.

Answer:

Picture

```
*   *   *   *          #  #
*   *   *   *  ---->  #  #  #
*   *   *               #
```

Ratio

$\frac{11}{6}$

POINT

How do the ratio statements and pictures introduced in Part 3 differ from those you worked with in Part 2 ?

EXERCISES

1. For every 2 △ there are 5 □.
   a. Complete the picture: △△ →
   b. Write the ratio which represents the statement.

2. For every 3 △ there are 7 □.
   a. Complete the picture: → □□□□□□□
   b. Write the ratio representing the statement.

3. For every 4 △ there are 5 □.
   a. Draw a picture representing the statement.
   →
   b. Write the ratio.

4. Use the picture to fill in the blanks: For every ______ there are ______ .

   △△△△ → □□□□□□□

5. Consider a set of 5 ○ and a set of 6 ⬡.
   a. Write a ratio indicating the number of ○ compared to the number of ⬡.
   b. Draw a picture which represents the ratio.
   →
   c. Fill in: For every _____ there are _____.

6. For every 3 ○ there are 7 ⬡.

Circle the ratio which represents the above statement. $\frac{3}{7}$ $\frac{7}{3}$

7. Given 4 ⬡ and 9 ○ and the ratio $\frac{9}{4}$, fill in the blanks:

For every ______ there are ______ .

8. Draw a picture representing the statement: For every 6 □ there are 11 △ .

a. →

b. What ratio represents the statement?

9. For every 7 △ there are 4 □ .

a. Complete the picture: → □□□□

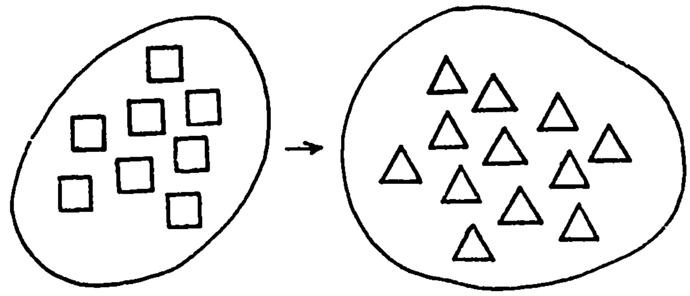
b. Write the ratio.

10. Consider the ratio, $\frac{3}{5}$.

a. Draw a picture representing the ratio.

b. For every _____ there are _____.

11. Look at these two sets:



a. For every 8 □ there are ______.

b. Write a ratio comparing □ to △ .

12. For every 8 △ there are 10 ○ .

a. Complete the picture

△△△△△△△△ →

b. Write a ratio which represents the picture.

13. Write the ratio represented by

□□□□ → △△△△△△△

14. For every 6 ○ there are 9 □ .

a. Complete the picture: → □□□□□ □□□□

b. Write a ratio which represents the picture.

c. This exercise compares the number of _____ to the number of _______.

For every 2△ there are 3⬡ is expressed by the ratio $\frac{2}{3}$.

△△ → ⬡⬡⬡

Another statement can be written for the picture. It is "2 triangles compared to 3 hexagons." The ratio $\frac{2}{3}$ also represents this.

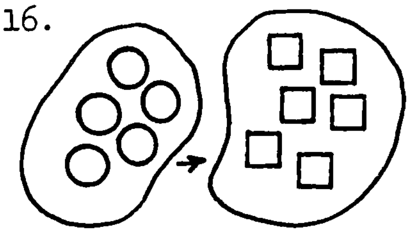
Example: There are 28 days in February compared to 31 days in March. The ratio representing this comparison is $\frac{28}{31}$.

15. There are 4 triangles compared to 7 circles.

    a. Complete the picture: △△△△ →

    b. Write the indicated ratio.

16.



    a. 5 circles are compared to ____ squares.

    b. The ratio is _____.

17. There are 7 * compared to 11 #

    a. Draw a picture representing the comparison. →

    b. Write the ratio.

    c. Complete the statement: For every ____ there are ___ .

18. 5 out of 9 triangles are shaded.

   a. Write the ratio indicated by the statement.

   b. Draw a picture representing the ratio.

19. Write the <u>ratio</u> and <u>draw</u> a picture representing each of the following:

   a. For every 9 squares there are 6 triangles.

   b. 6 out of 9 squares are shaded.

   c. There are 9 triangles compared to 6 squares.

20. Circle the ratio which represents each of the following:

   a. There are 16 squares compared to 11 triangles.

   $\frac{11}{16}$ $\frac{11}{27}$ $\frac{16}{11}$ $\frac{27}{16}$

   b. For every 8 shaded triangles there are 15 circles.

   $\frac{15}{8}$ $\frac{8}{15}$ $\frac{8}{23}$ $\frac{23}{15}$

   c. 9 out of the 17 squares are shaded.

   $\frac{17}{9}$ $\frac{9}{17}$ $\frac{9}{26}$ $\frac{8}{26}$

DISCUSSION QUESTIONS

1. Look at the two pictures: A □□□ → △△△△
   B □□□ ← △△△△

   a. How do they differ?

   b. What ratios do they represent?

   c. Are both pictures correct? Explain.

2. Look at the drawing.

   ● ● ● ●
   ○ ○ ○ ○
   ○ ○ ○ ○

   a. Write all of the different ratios and sentences that could describe the drawing.

   b. Was it possible to use a statement like

      For every ____ there are ____ , or ____ out of ____.

      to describe any of your ratios? Explain your answer.

3. a. If you were to close your eyes and imagine a picture for each of the statements, what would you see?

      2 out of 3 cars are red.
      For every 2 red cars there are 3 blue cars.

   b. Are the ratios different?

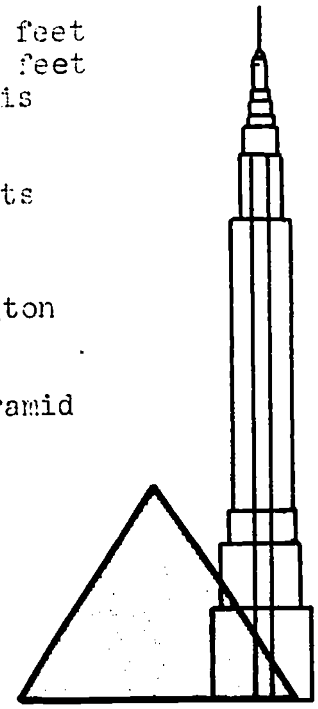
   c. Tell how the statements describe different situations.

18. The Egyptian pyramid (Cheops) is 481 feet tall; the Washington Monument is 555 feet high; and the Empire State Building is 1472 feet high.

Write ratios which compare the heights of the:

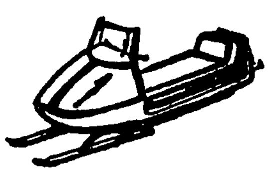
a. Pyramid of Cheops to the Washington Monument.

b. Empire State Building to the Pyramid of Cheops.



19. About how many times taller is the Empire State Building than the Egyptian Pyramid?

20. The oceans contain 97.3 per cent of the earth's water supply. Write this as a ratio.

21. 

Two-cycle engines require a mixture of oil and gasoline. Some engines require 2 quarts of oil for every 5 gallons of gasoline.

Write a ratio representing the oil-gas mixture.

A PICTURE OF EQUIVALENCE

EQUIVALENT RATIOS

There are many ratios which can represent the same picture. These ratios are said to be equivalent. Two questions that might be asked are: (1) How do you know when two ratios are equivalent? (2) How do you know if two ratios represent the same picture?

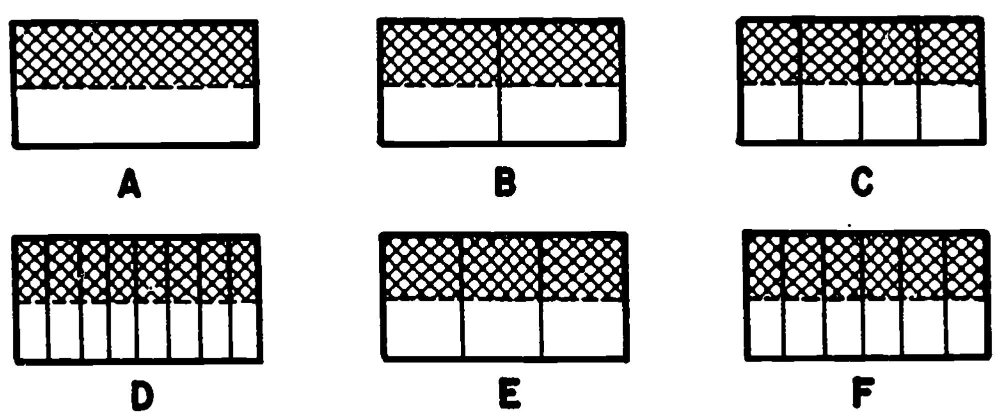
CLASS ACTIVITY

Your teacher will pass out some sheets of paper and discuss the activity. The purpose of the activity is to fold paper to illustrate equivalent ratios.

EXERCISES

The rectangles drawn below are like the rectangles you just worked with while paper-folding.

The rectangles are the same size overall, and the shaded regions are of equal size.



Example:

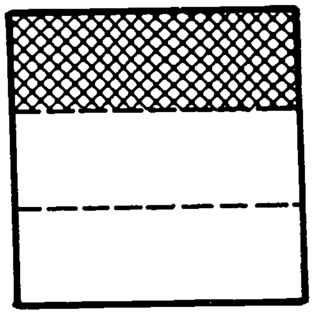

M

Figure M has been separated into 3 regions and one of them is shaded.

The ratio represented here is $\frac{1}{3}$.

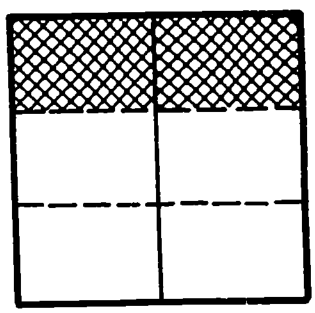

N

Figure N has an additional line segment.

The total number of regions has been doubled, 6 in all.

Also the number of shaded regions has been doubled, 2.

The ratio represented here is $\frac{2}{6}$. $\frac{2}{6}$ is equivalent to $\frac{1}{3}$ because the overall figures and the shaded portions are equal in area.

MORE ABOUT EQUIVALENCE

SIMPLIFYING RATIOS

Simplified ratios are used mainly because they help people communicate. Gear, pulley, recipe, oil-gas, and other ratios are written in simplest form. Otherwise any number of equivalent ratios could be used and a person might not recognize them as equivalent.

DIVISION METHOD

This method depends upon the idea that division by 1 does not change the value of a number; and that a number divided by itself (except 0) is 1.

For instance: $15 \div 1 = 15$

$$\frac{5}{5} = 1$$

Example 1:

Simplify $\frac{12}{18}$

$$\frac{12}{18} \div 1 = \frac{12}{18} \div \frac{6}{6}$$

(Since 12 and 18 are both divisible by 6 and $\frac{6}{6} = 1$)

$$= \frac{12 \div 6}{18 \div 6}$$

$$= \frac{2}{3}$$

Therefore $\frac{12}{18} = \frac{2}{3}$

FACTORING METHOD

The factoring method depends upon your ability to factor numbers into primes and your understanding of multiplying fractional numbers.

For instance, if $\frac{4}{5} \cdot \frac{3}{3} = \frac{4 \cdot 3}{5 \cdot 3}$,

then we must agree that

$$\frac{4 \cdot 3}{5 \cdot 3} = \frac{4}{5} \cdot \frac{3}{3}.$$

Example 1:

Simplify $\frac{12}{18}$.

by factoring: $\frac{12}{18} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3}$

$$\frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3} = \frac{2 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 3}$$

$$= \frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{2}{3}$$

$$= 1 \cdot \frac{2}{3}$$

$$= \frac{2}{3}$$

Therefore $\frac{12}{18} = \frac{2}{3}$.

2. [illegible] ratios in this way:

a. [illegible] form by dividing the [illegible] numerator.

[illegible]:

[illegible] written in [illegible] form by dividing

$$\begin{array}{r} 1.25 \\ 4\overline{)5.00} \\ \underline{4} \\ 10 \\ \underline{8} \\ 20 \end{array}$$

Leaf	a	b	c	d
Ratio				
Decimal				

b. Add [illegible] values = _____

c. Average [illegible] _____

3. Measure [illegible] other leaves from the same plant [illegible].

Leaf	e	f	g	h
Width (cm.)				

4. Predict [illegible] leaves using the "average ratio."

Length = width × [illegible] ratio.

Leaf	e	f	g	h
Predicted Length				
Measured Length				

$$12 \div 3 = 4, \quad \text{so } \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\boxed{4}}{\boxed{4}} = \frac{2 \times \boxed{4}}{3 \times \boxed{4}} = \frac{8}{12}.$$

Therefore, N = 8.

Example 2: $\frac{4}{7} = \frac{M}{21}$. Since $21 \div 7 = 3$, multiply $\frac{4}{7}$ by $\frac{3}{3}$.

Then $\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$, so M = 12.

Example 3: If a weight of 5 ounces stretches a spring 3 centimeters, and some unknown weight W stretches the spring 12 cm., how much is W?

A proportion expressing this problem is $\frac{5}{3} = \frac{W}{12}$.

Here 1 is expressed by $\frac{4}{4}$, and W is 20.

Example 4: A weight of 20 oz. stretches a spring 15 cm. What weight W stretches the same spring 3 cm.?

$\frac{20}{15} = \frac{W}{3}$ is the proportion to be solved.

In this case division by 1 can be used.

$$\frac{20}{15} \div \frac{5}{5} = \frac{4}{3}$$

Since $15 \div 5$ is 3 and dividing by 1 does not change the value of a number, we must divide 20 by 5 and find out that W = 4.

EXERCISES 2

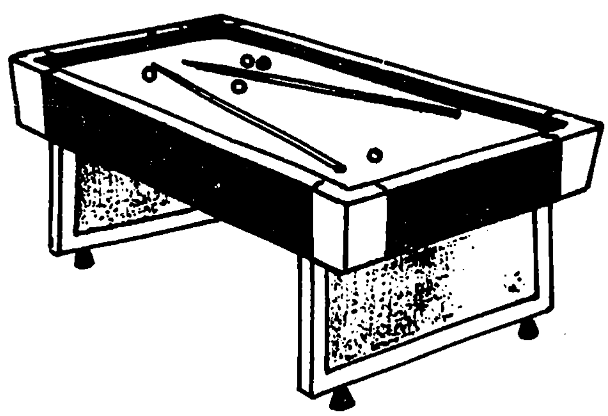
Solve each proportion for the missing member.

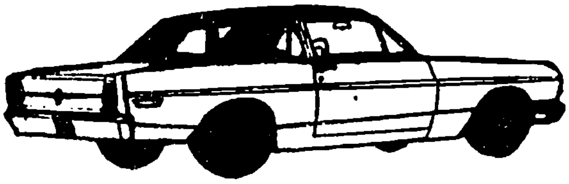
1. $\frac{3}{8} = \frac{N}{24}$
2. $\frac{4}{5} = \frac{M}{15}$
3. $\frac{24}{36} = \frac{H}{12}$
4. $\frac{15}{20} = \frac{30}{E}$
5. $\frac{20}{x} = \frac{5}{3}$
6. $\frac{R}{10} = \frac{32}{40}$
7. $\frac{7}{9} = \frac{W}{27}$
8. $\frac{8}{6} = \frac{32}{T}$
9. $\frac{5}{7} = \frac{N}{35}$
10. $\frac{12}{20} = \frac{N}{5}$
11. $\frac{N}{8} = \frac{25}{40}$
12. $\frac{9}{6} = \frac{N}{36}$

PROPORTIONS AND POOL TABLES

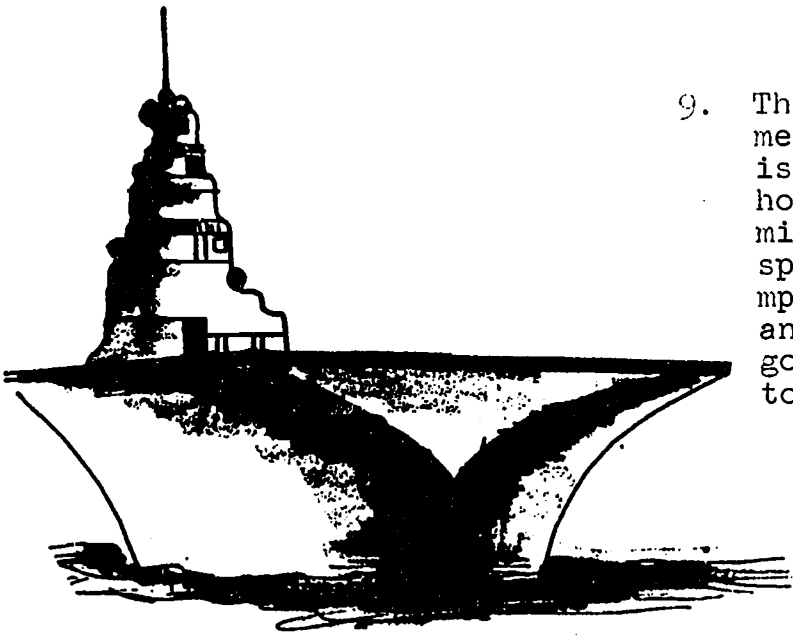
The length and width of a regulation pool table are in the ratio of $\frac{2}{1}$.

A regulation table that is 88" long is 44" wide.





8. Most countries use the metric system for measuring. This means they measure speeds in kilometers per hour. A car traveling 50 mph is going at the rate of 80 km. per hour. What speed in mph would a car be going if it traveled at 120 km. per hour?



9. The speed of ships is measured in knots. A knot is one nautical mile per hour. Nautical and land miles are different. A speed of 20 knots is 23 mph. How many mph will an aircraft carrier be going if it travels at top speed, 30 knots?

10. The marine weather bureau gives wind speed in knots. If they forecast wind speeds up to 55 knots, how many mph would this be?

DISCUSSION QUESTION

Jim Ryan can run the mile in 4 minutes. How long will it take him to run 7 miles? What do you think of the reasonableness of this question?

✓POINT

1. Write the proportion that would be used to solve each problem.

   a. The Imperial gallon is used in Canada. It is larger than the U.S. gallon. For every 5 Imperial gallons, there are 6 U.S. gallons. 17 gallons of gasoline purchased in Canada would be equal to how many gallons in the U.S.?

   b. United States Nickels are made of copper and nickel. For every 3 oz. of copper, 1 oz. of nickel is used. If 14 pounds of copper are to be used for making Nickels, how much nickel is required?

2. Circle the pairs of ratios that are equivalent.

   a. $\frac{4}{9} = \frac{20}{45}$ b. $\frac{15}{10} = \frac{18}{12}$ c. $\frac{14}{18} = \frac{35}{45}$

3. Circle the proportions that are true.

   a. $\frac{9}{6} = \frac{12}{10}$ b. $\frac{12}{18} = \frac{8}{12}$ c. $\frac{7}{11} = \frac{28}{42}$

4. Simplify the following ratios:

   a. $\frac{10}{18}$ b. $\frac{24}{16}$ c. $\frac{28}{36}$

5. Solve these proportions for N.

   a. $\frac{5}{8} = \frac{N}{40}$ b. $\frac{6}{10} = \frac{24}{N}$ c. $\frac{12}{N} = \frac{9}{6}$

7. Use proportions to predict the missing values in the table. Note: In this part you are given the height of the bounce, and you are predicting where the ball was dropped from.

height after first bounce	30	45	60
Predicted H			

8. Check your results by performing the experiment.

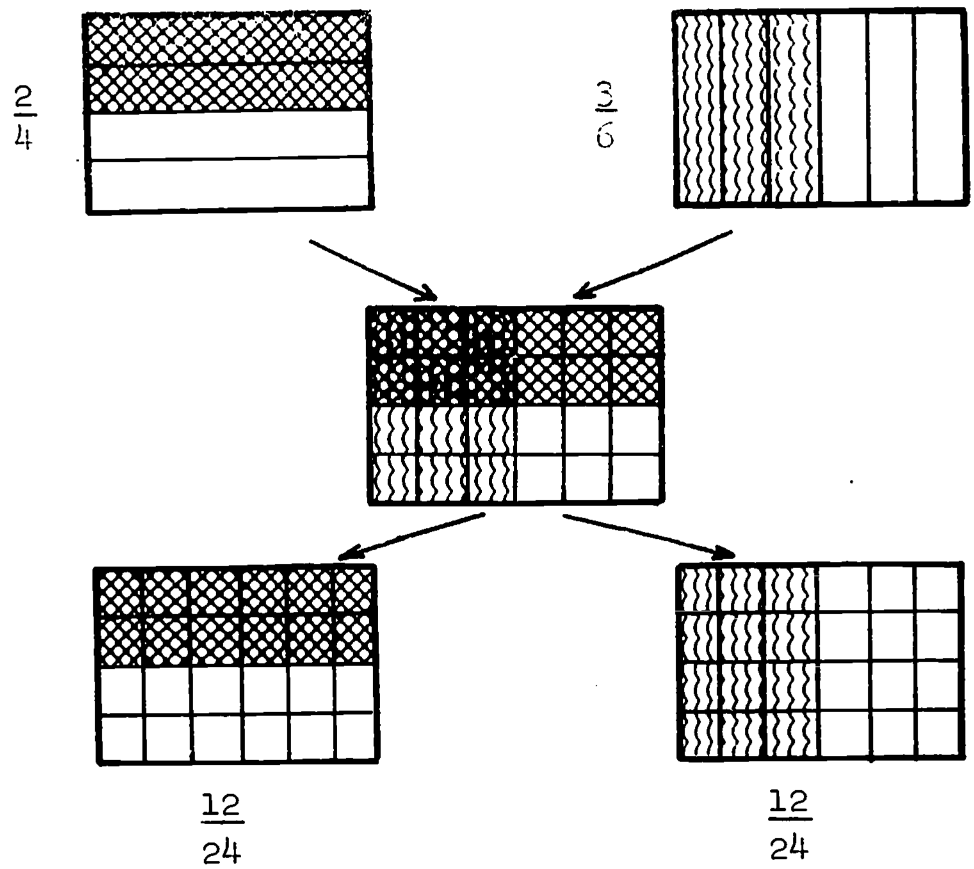
height h, inches	30	45	60
Experimental results, H			

9. What effect might a different kind of flooring or different ball have on the amount of bounce?

Example 2:

$$\frac{2}{4} \stackrel{?}{=} \frac{3}{6}$$

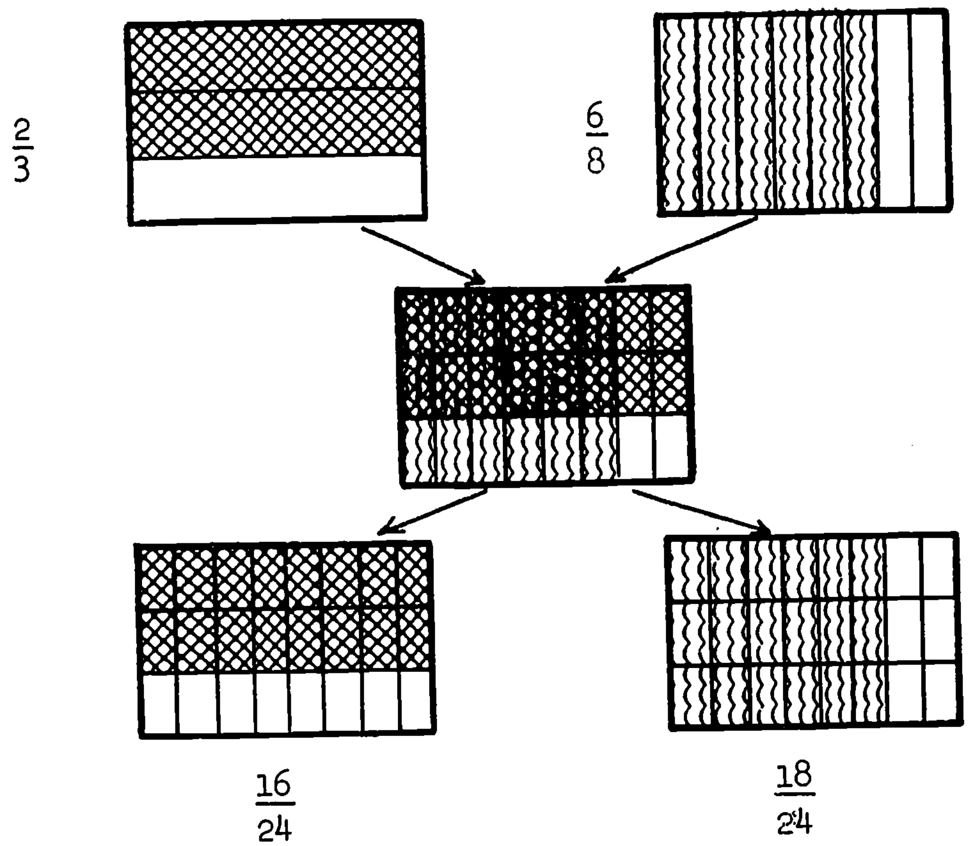
Draw rectangles to represent each of the ratios, slide them together, and pull them apart with each rectangle containing the horizontal and vertical lines.



Then $\frac{2}{4} = \frac{3}{6}$

Example 3:

$\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$



Since $\frac{16}{24} \neq \frac{18}{24}$, the proportion $\frac{2}{3} = \frac{6}{8}$ is false.

EXERCISES I

Draw diagrams like those above to determine whether the proportions are true or false. A worksheet can be found on page 129 for you to show your work.

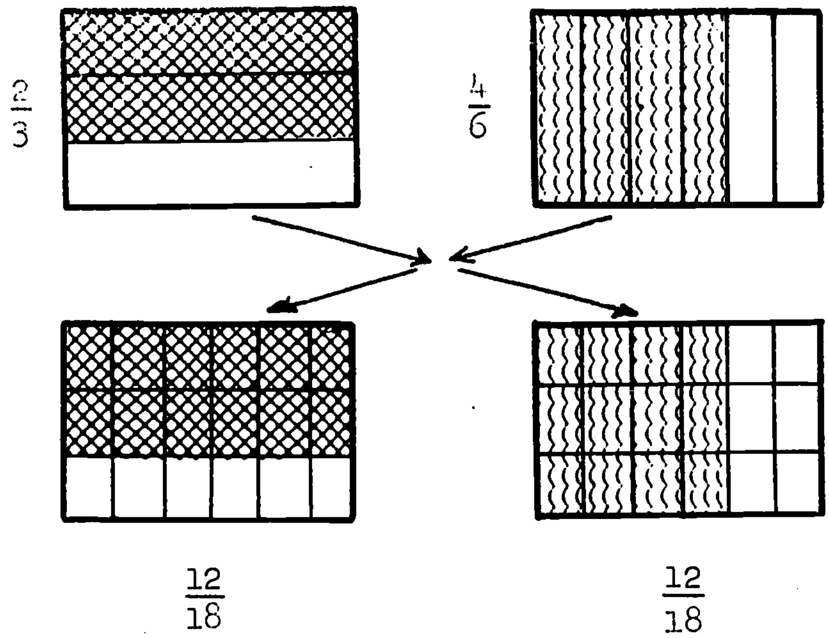
1. $\frac{2}{3} = \frac{6}{9}$
2. $\frac{2}{3} = \frac{4}{6}$
3. $\frac{4}{6} = \frac{5}{8}$
4. $\frac{3}{4} = \frac{6}{8}$
5. $\frac{3}{6} = \frac{4}{8}$

THE CROSS-PRODUCT MODEL

The rectangle model can be used to illustrate the idea of cross-product. The cross-product is a quick way to calculate whether or not a proportion is true.

Example 1:

You found out in Exercises 1 that the proportion $\frac{2}{3} = \frac{4}{6}$ is true. The rectangles looked like this.



The shaded portion of the left hand rectangle is 2 units by 6 units (2×6). The shaded portion of the right hand rectangle is 3 units by 4 units (3×4).

Notice that $2 \times 6 = 12$ and $3 \times 4 = 12$. Twelve is the numerator of both ratios above.

Also notice

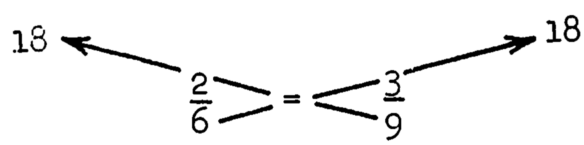
$$12 \leftarrow \frac{2}{3} = \frac{4}{6} \rightarrow 12$$

The length and width of the shaded portion of one rectangle. $\leftarrow \frac{2}{3} = \frac{4}{6} \rightarrow$ The length and width of the shaded portion of one rectangle.

Multiplying as shown by the arrows is a quick way to find the numerators of the ratios, which are both $\frac{12}{18}$.

Example 2:

Look at example 1 on page 62. $\frac{2}{6} \stackrel{?}{=} \frac{3}{9}$

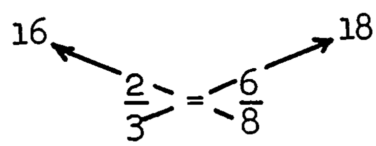


Since both products are 18, the ratios are equivalent and the proportion is true.

18 is the numerator of the ratio $\frac{18}{54}$.

Example 3:

Look at example 3 on page 65. $\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$



Since the products are unlike, 16 and 18, $\frac{2}{3} \neq \frac{6}{8}$.

16 and 18 are the numerators of $\frac{16}{24}$ and $\frac{18}{24}$.

Example 2:

Solve for N. $\frac{3}{5} = \frac{N}{32}$

$$5 \cdot N = 3 \cdot 32$$

$$5N = 96$$

$$N = \frac{96}{5}$$

$$N = 19\frac{1}{5} \text{ or } 19.2$$

EXERCISES 3

Solve these proportions for N:

1. $\frac{2}{6} = \frac{N}{15}$
2. $\frac{5}{7} = \frac{N}{20}$
3. $\frac{8}{3} = \frac{N}{20}$
4. $\frac{7}{10} = \frac{16}{N}$
5. $\frac{18}{8} = \frac{N}{3}$
6. $\frac{N}{12} = \frac{10}{18}$
7. $\frac{5}{6} = \frac{N}{20}$
8. $\frac{10}{18} = \frac{45}{N}$
9. $\frac{25}{75} = \frac{N}{300}$
10. $\frac{4}{8} = \frac{N}{10}$
11. $\frac{N}{35} = \frac{40}{100}$
12. $\frac{6}{8} = \frac{N}{75}$
13. $\frac{18}{27} = \frac{54}{N}$
14. $\frac{7}{22} = \frac{24.5}{N}$
15. $\frac{12}{25} = \frac{N}{20.5}$
16. $\frac{6.5}{20} = \frac{N}{110}$

EXERCISES 4

Go back to page 55 and solve the problems completely by solving the proportions you wrote there.

HUMAN MEASURES

CLASS ACTIVITY

For this activity you will be working in small groups (3 or 4 students per group).

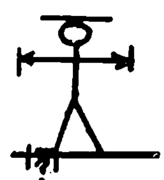
Read over the activity before beginning to do any measuring and check your equipment against the inventory.

Use proportions when making your predictions. This will let you know how well you can use proportions.

Equipment Inventory: 7 feet of brown wrapping paper, 1 meter stick, you, and masking tape.

Activity: The purpose of this activity is to measure humans and find ratios among the measures. Then the ratios are to be used to predict other human measures.

1. Use masking tape to attach the wrapping paper to the wall and floor. Attach it so you can measure your height, arm span, and foot.



2. Measure to the nearest centimeter your:

   a. height ______

   b. arm span ______

   c. length of foot ______

3. Write the ratios:

   a. comparing length of foot to height. ______

   b. comparing arm span to height. ________

4. Write these ratios in decimal form.

   Example: $\frac{5}{4}$ can be written in decimal form by dividing

$$\begin{array}{r} 1.25 \\ 4\,\overline{)5.00} \\ \underline{4} \\ 10 \\ \underline{8} \\ 20 \end{array}$$

   a. Length of foot to height. ______

   b. Arm span to height. ________

CLASS DISCUSSION

1. Write on the chalkboard the ratios obtained by the boys in parts 4a and 4b above. How do the ratios compare? (Are they close to the same value?)

2. Write the girls' ratios on the chalkboard. How do they compare?

3. Find the average ratio for the boys' and girls' measurements.

	Ratio	
	Length of foot compared to height	Length of arm span compared to height
Boys		
Girls		

EXERCISES

1. A boy has an arm span of 178 cm. How tall do you think he is?

2. How tall do you think a girl is if her foot measures 23 cm?

3. Measure your teacher's arm span, then predict his or her height. Check your prediction by measuring your teacher's height directly.

✓ POINT

1. Use the cross-product method to test whether or not these proportions are true:

   a. $\frac{[illegible]}{12} = \frac{15}{20}$ b. $\frac{7}{13} = \frac{21}{52}$ c. $\frac{12}{18} = \frac{16}{24}$

   d. $\frac{20}{15} = \frac{16}{12}$ e. $\frac{24}{36} = \frac{16}{20}$ f. $\frac{21}{15} = \frac{27}{17}$

2. Solve the following proportions for N:

   a. $\frac{5}{9} = \frac{N}{72}$ b. $\frac{6}{14} = \frac{N}{50}$ c. $\frac{22}{18} = \frac{64}{N}$

   d. $\frac{N}{12} = \frac{8}{32}$ e. $\frac{4}{N} = \frac{26}{40}$ f. $\frac{7}{12} = \frac{N}{84}$

3. Write the proportions that would be used to solve the following:

   a. A 28-inch bicycle wheel has a circumference of about 88 inches. What would be the circumference of a 20-inch bicycle wheel?

   b. In a certain pulley system, a mechanic gives a 24-pound pull to lift a weight of 120 pounds. How much pull would be needed to lift an engine weighing 480 pounds?

   c. For a pair of mother and daughter dresses, the girl's pocket measured $3\frac{1}{2}$ inches wide and $4\frac{1}{2}$ inches long. The width of the mother's pocket was $4\frac{1}{2}$ inches. How long was it?

SPRINGS AND BOLTS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.

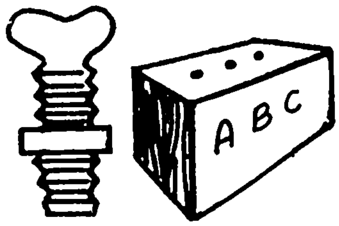
2. Follow the instructions for the activity. They begin on pages 76 and 78.

3. Use the equipment inventory to check your equipment.

4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

NUTS AND BOLTS



Equipment Inventory:

1 bolt and nut, 1 block of wood containing three holes, 1 strip of tagboard one inch long.

Activity: This activity centers around the question:

How can a bolt and nut be used to find the depth of a hole in a block of wood?

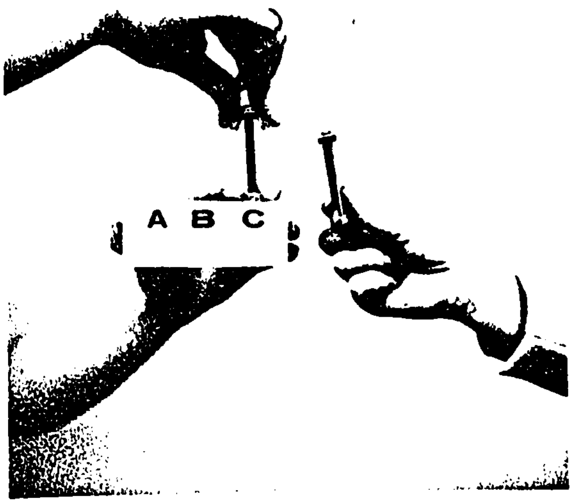
1. Use the tagboard strip which is one inch long and count the number of threads in one inch of the bolt. ______

2. Turn the bolt in the nut and count the turns as one inch of bolt passes through the nut. ______

3. How far does the bolt move through the nut in one turn? ______

4. If the bolt is turned 6 times, how far does it move? ______

5. Find the depth of holes A, B, C in the wooden block.



a. Start with the end of the bolt even with the nut.

b. Set the nut and bolt over the hole and hold with your fingers.

c. Turn the bolt and count the turns until the bottom of the hole is reached.

Hole	A	B	C
Turns			

d. Calculate how deep the hole is by using the number of turns per inch and writing a proportion.

Hole	A	B	C
depth			

6. A certain screw-type truck jack has 4 threads per inch. How many turns would it take to lift the truck 7 inches off the ground?

7. Look for some information about a micrometer in a mathematics book or an encyclopedia. How are a micrometer and the nut and bolt related in operation?

MAKING COMPARISONS

RATIOS AND COMPARISONS

Proportions are useful when making comparisons of measurements, prices, and performances. One method is to use 100 as a basis for comparison.

Example 1:

During the 1968 baseball season Willie Horton got 146 hits out of 512 at bats while Al Kaline got 94 hits for 327 at bats. A convenient way to compare their performances is to find an equivalent ratio whose denominator is 100 for each player.

Tigers

$$\frac{146}{512} = \frac{H}{100} \qquad \frac{94}{327} = \frac{K}{100}$$

$$512H = 14600 \qquad 327K = 9400$$

$$H = 28.5 \qquad K = 28.7$$

Their performances were extremely close. Batting at this rate, we could say that Kaline gets 28.7 hits for each 100 times at bat and Horton gets 28.5 hits.

Example 2:

Chef cooking oil comes in two containers like these.

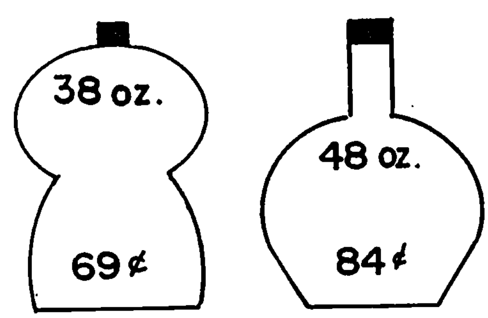
If the oils are of equal quality, which is the better buy?

$$\frac{38}{69} = \frac{X}{100} \qquad \frac{48}{84} = \frac{X}{100}$$

$$69X = 3800 \qquad 84X = 4800$$

$$X = \frac{3800}{69} \qquad X = \frac{4800}{84}$$

$X = 55.1$ oz. for 100¢ $\qquad$ $X = 57.1$ oz. for 100¢



So the 48 oz. jar is the better buy.

EXERCISES I

The following exercises ask you to compare some information. Use proportions to make the comparisons using $\frac{X}{100}$ as the second ratio for each proportion.

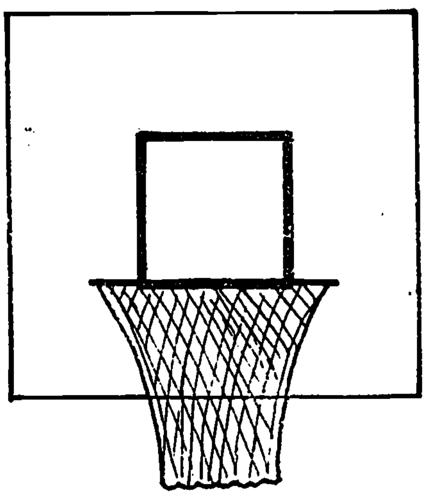
1. An instant coffee called Delight comes in several sizes. As long as spoiling is not a problem, which jar would be the best one to buy?

   A. 14 oz. for $1.79 B. 6 oz. for $.88 (use 179¢ and 88¢)

2. Compare the batting records of the 1968 Detroit Tigers (1292 hits, 5490 at bats) and the 1968 St. Louis Cardinals (1384 hits, 5561 at bats).

3. Among the free throw leaders in professional basketball during 1968 were Cazzie Russell of New York and Jerry West of Los Angeles. Russell made 282 free throws in 349 attempts while West made 391 throws out of 482 tries. Compare their performances.

4. Which would you buy? Delight Coffee: 14 oz. for $1.79, Delight Coffee: 6 oz. for $.88, or Swell Coffee: 10 oz. for $1.19?



5. In January the New York Knicks had won 18 of 47 basketball games. At the same time the Atlanta Hawks had won 17 of 44 games. Use proportions to compare these two records.

RATIOS AND MIXTURES

Nurses and doctors use proportions when preparing medicines. The chemical or drug is not generally used full-strength. Usually a certain amount of drug is mixed with a given amount of liquid to make a solution.

If an alcohol solution is made up of 70 parts alcohol and 30 parts water, the entire solution contains 100 parts. $\frac{70}{100}$ represents the fractional part of the solution that is alcohol.

Example:

Patients may need different amounts of alcohol solutions. These different amounts still have the same strength, $\frac{70}{100}$. A patient needs 30 oz. of alcohol solution. How many ounces of alcohol should the nurse use?

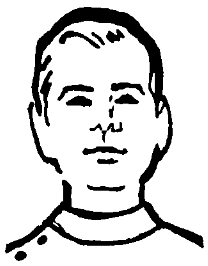
$$\frac{70}{100} = \frac{X}{30}$$

$$100X = 2100$$

$X = 21$ ounces of alcohol is to be used. Then 9 oz. of water will be added. $(30 - 21 = 9)$

EXERCISES 2

1. Twelve ounces of a $\frac{25}{100}$ solution of glycerin is to be prepared for a patient. The ratio $\frac{25}{100}$ means that 25 ounces of glycerin is to be used out of every 100 ounces of the solution. (Then 75 oz. of water will be used.) Since only 12 oz. of solution is needed, how much glycerin is required ?

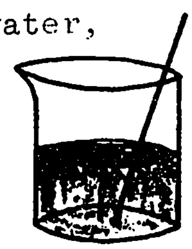


$\frac{25}{100} = \frac{X}{12}$ Finish the problem.

2. How many ounces of boric acid are in 40 oz. of solution if the strength of the solution is $\frac{1}{500}$?

3. If you have 100 oz. of a $\frac{25}{100}$ solution of glycerin,
   a) how many ounces would be glycerin?
   b) how many ounces would be water?

4. If a solution contains 2 parts of a drug and 3 parts of water,



   a) what ratio represents the strength of the solution?
   b) how many ounces of drug would be contained in 100 oz. of solution?

5. Another boric acid solution is stronger, $\frac{55}{100}$.
   a) Out of every 100 oz. of solution, how many oz. of boric acid would be needed?
   b) How many ounces of boric acid would be needed to make 80 oz. of solution?

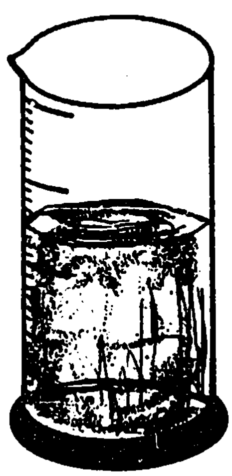
USING PER CENT

If a nurse knows the strength that is needed for a certain solution, she can use proportions to figure out the number of ounces of drug needed for any given amount of the solution. Using ratios such as $\frac{25}{100}$ or $\frac{55}{100}$ is common when comparing the amount of drug used to the amount of solution being prepared.

The ratio $\frac{55}{100}$ can be read "55 out of 100" or "55 of each hundred". It means that the drug makes up 55 out of 100 parts of the solution. You will recall the special name "per cent" that was used for this kind of comparison.

PER CENT IS A CONVENIENT METHOD OF MAKING COMPARISONS, AS WELL AS GIVING DIRECTIONS FOR MIXING INGREDIENTS (SUCH AS MEDICINES).

A boric acid solution which has a strength of $\frac{55}{100}$ is said to be a 55% solution. The symbol % represents $\frac{}{100}$.
Remember: Per cent (%) means "for each hundred". A 55% boric acid solution means 55 oz. of boric acid for each 100 oz. of solution.



If a coat is on sale for 10% off the regular price, it would sell for 10¢ off out of every 100¢ the coat cost ($\frac{10}{100}$). A coat with a regular price of $25.00 (2500¢) would sell for $22.50.

10% off

actual number of cents off

$$\frac{10}{100} = \frac{X}{2500}$$

out of each 100¢

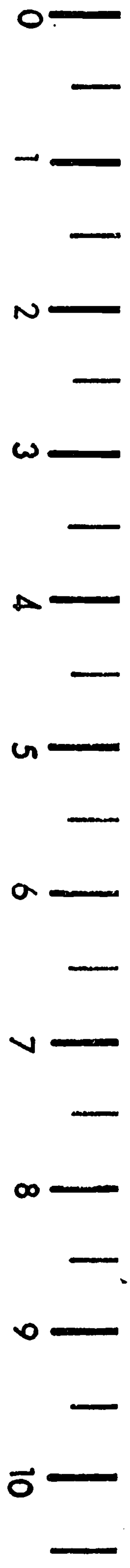
of the 2500¢ regular price.

$$100X = 25000$$

$$X = 250 \text{ cents off regular price}$$

2500¢ - 250¢ = 2250¢ or $22.50 (the sale price).

The symbol, %, is used mainly when writing about per cent and in advertising signs. When you use per cent in solving problems the fraction form is used. In calculating, 25% would be written as $\frac{25}{100}$.



STRETCHING RUBBER BANDS

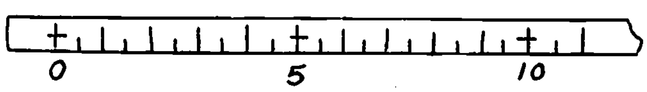
It is helpful to be able to estimate per cents such as the following: 75% of a line segment, 30% of a group of students, 25% of a length of cloth, etc. To help you <u>see</u> per cent, you are going to make a per cent calculator and work with line segments.

CLASS ACTIVITY

A rubber band can be made into a per cent calculator. With this device you can find per cents of line segments and be able to <u>see</u> what they are.

1. To make the calculator you will need a piece of rubber band.

2. Choose a partner. Stretch the rubber band the length of the ruler which is printed along the edge of this page. Then have your partner put a pencil mark on the rubber band every half inch.



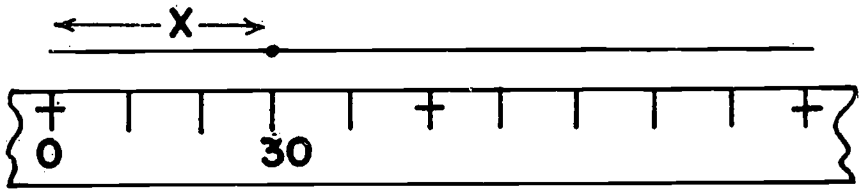
3. When you let the rubber band relax, there is a set of evenly spaced marks. They will stay evenly spaced for any length the rubber band is stretched.

   Let's try it. Mark off 30% of the segment drawn below.

Here's how.

a. Stretch the rubber band next to the segment until it is divided into 10 units of length. (Each unit represents 10%, and there will be some marks on the rubber band that will not be used.)

b. Place a dot on the segment after the third unit which will be the fourth mark on the rubber band.



c. The x on the segment indicates that the left hand part of the segment is 30% of the entire length.

4. Imagine each space having 10 smaller divisions. Then each small space represents 1 out of 100 or 1%. The fourth mark (at the end of the third unit) would then represent 30 out of 100 parts or 30%.

EXERCISES I

1. Place a mark on each segment so that the left hand part represents the given per cent.

   a. 50% ____________________

   b. 40% ______________

   c. 60% ________________________

   d. 25% ___________________

   e. 75% ______________________________

   f. 85% ________________________

   g. 90% _________________

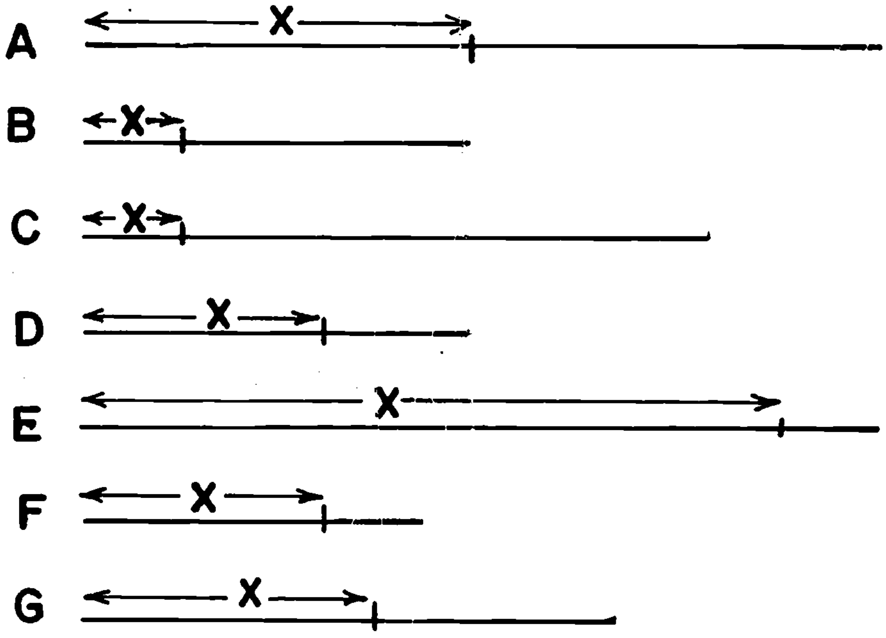
   h. 100% _________________

   i. 125% _________________

2. (I) Estimate the per cent represented by the part of the segment indicated by |← x →|.

(II) Check your estimate using the rubber band.

	A	B	C	D	E	F	G
I ESTIMATE							
II CHECK							



There are several ways in which a particular per cent can be written. Sometimes one particular form is easier to work with than others.

You know that 25% can be written as $\frac{25}{100}$.

1. If the ratio is simplified you get $\frac{25}{100} = \frac{1}{4}$. So $\frac{1}{4}$ of \$20 is the same as 25% of \$20 which is \$5.

2. To change a fraction into per cent form, use a proportion. For instance $\frac{2}{5} = \frac{x}{100}$, $x = 40$ so $\frac{2}{5} = \frac{40}{100} = 40\%$.

3. Per cent can be represented in decimal form.
$$\frac{25}{100} = 25 \div 100 = .25$$

   A simpler way is to read $\frac{25}{100}$ as "twenty-five hundredths" and then write it in decimal form as .25 .

4. The forms shown here are equivalent:
$$\frac{40}{100},\ 40\%,\ .40,\ \frac{2}{5}$$

EXERCISES 2

Complete the table by writing the various forms which represent the given per cents.

Per Cent form	50%	60%			20%		
Ratio form	$\frac{50}{100}$		$\frac{80}{100}$				
Decimal form	.50					.05	
Simplified fraction form	$\frac{1}{2}$			$\frac{3}{4}$			$\frac{1}{8}$

Some people, when estimating sizes or lengths, think of simplified fractions rather than forms such as decimal or per cent.

When estimating 25% of the length of a segment, a person would most likely think of $\frac{1}{4}$ of the segment (if only for the reason that they have worked more with fractions).

EXERCISES 3

Estimate the per cent or fractional part of the segment by placing a mark that distance from the left end. Indicate the part with an ←x→. Check your estimate using the rubber band.

Example: 50% ←x→

75%

25%

40%

90%

3/4

1/3

30%

1/4

80%

ESTIMATION AND PER CENT

SALES, ESTIMATES, AND SHOES

A sale on shoes! 15% off. Stores often advertise sales in this way because it catches your eye. Some people will buy items even if they do not need them because the idea of "money off" impresses them. Estimation can help a person decide if the amount off is really enough to make purchase worthwhile.

Estimates of per cents can be made more easily if you have some guide for making the estimation. Per cents written in fraction form can be very helpful in estimating because you are quite familiar with fractional parts of numbers. For instance:

$50\% = \frac{50}{100} = \frac{1}{2}$. So 50% of \$25 is $\frac{1}{2} \cdot \$25$ or \$12.50.

EXERCISES I

For each per cent, write a simplified fractional number.

Per Cent	Ratio: $\frac{N}{100}$	Simplified Ratio
50%	$\frac{50}{100}$	$\frac{1}{2}$
75%		
25%		
20%		
40%		
60%		
10%		
30%		

PER CENTS AS SIMPLIFIED RATIOS

As a simplified ratio, 20% = $\frac{1}{5}$ because $\frac{20}{100} = \frac{1}{5}$.

Using 20% as $\frac{1}{5}$, 20% of \$25.00 is \$5.00 because $\frac{1}{5} \times 25 = 5$.

It is not necessary to memorize all of the per cents in a table to use them. Compare 20%, 40%, 60%, and their corresponding ratios $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{5}$. Note that the per cents are multiples of 20 and the ratios are multiples of $\frac{1}{5}$. If you know that 20% = $\frac{1}{5}$, then 60% = $\frac{3}{5}$ because $20 \times 3 = 60$ and $\frac{1}{5} \times 3 = \frac{3}{5}$.

EXERCISES 2

Complete the table by applying the previous discussion.

Per Cent	Simplified Ratio
5%	
10%	
20%	
15%	
25%	
30%	
70%	
40%	
80%	

ESTIMATION...

It is frequently enough to estimate a per cent of a given number. When a sale is advertised, an estimation of the amount of savings is all that is necessary to decide if you want to buy the item or not.

In estimating, the symbol $\approx$ is used and means "approximately the same as".

Example: Estimate 15% of \$25.00.

Solution 1: $5\% = \frac{1}{20}$, so $15\% = \frac{3}{20}$

$\frac{1}{20}$ of 25 is $\frac{1}{20} \times 25 \approx 1$,

so $\frac{3}{20}$ of $25 \approx 3 \times 1 = \$3.00$

Solution 2: 15% is midway between 10% and 20% .

$10\% = \frac{1}{10}$ so $\frac{1}{10} \times 25 = \$2.50$

$20\% = \frac{1}{5}$ so $\frac{1}{5} \times 25 = \$5.00$

This means that 15% of \$25.00 is midway between \$2.50 and \$5.00 which is about \$3.50 . (\$3.75 to be exact.)

Solution 3: Find 10% of \$25.00 and then add half of the amount (5%) to it.

10% of \$25.00 = \$2.50, $\frac{1}{2}$ of \$2.50 = \$1.25.

\$2.50 + \$1.25 = \$3.75.

EXERCISES 3

Estimate the following per cent of the given number.

Given Number	Per Cent	Estimated Amount
18	50%	
24	75%	
35	20%	
20	40%	
70	10%	
90	30%	
120	60%	
120	25%	
80	45%	

Another good use of estimation and per cent is to check whether or not an answer is reasonable.

Example: A $120.00 chair is on sale for 30% off the regular price. What is the amount of savings when buying the chair on sale?

Estimating the amount of savings:

30% is close to 25%

$25\% = \frac{1}{4}$

$\frac{1}{4}$ of $120.00 is $30.00

The savings is a little more than $30.00

Calculate the actual amount of savings:

$$\frac{30}{100} = \frac{X}{120}$$

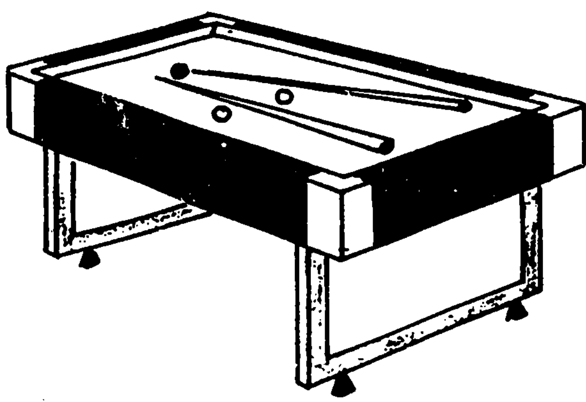
$$100X = 3600$$

$$X = \$36.00 \text{ in savings}$$

If you get an answer of $3.60 or $360.00 your estimate would tell you that these are incorrect.

EXERCISES 4

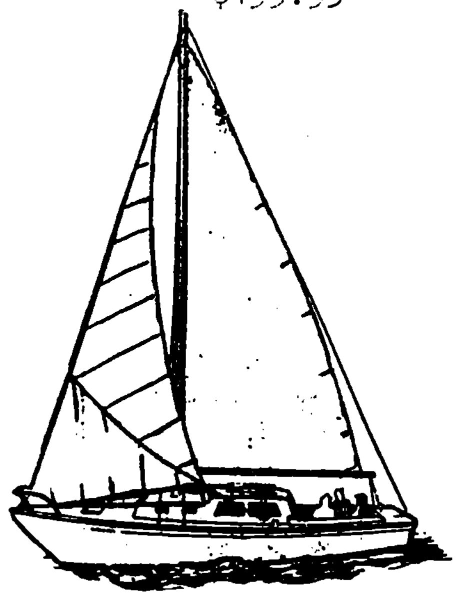
For the following problems, estimate only the answers. Do not actually calculate the correct answer.

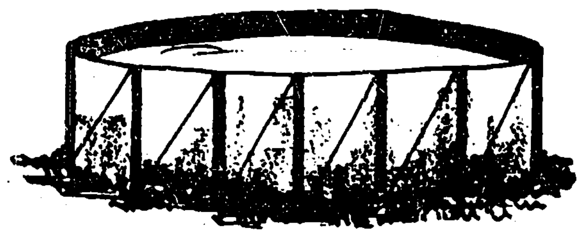


1. A pool table regularly selling for $79.95 is advertised on sale for 44% off. What is the amount of savings on this table?

2. How much less does a pool table sell for if it regularly costs \$695.00 and is reduced in price by 36%?

3. These swimming pools are selling for 26% off. How much is saved?
   a) 20 ft. pool, regularly \$399.95
   b) 24 ft. pool, regularly \$499.95





The speeds of cars and boats cannot be compared directly. Boat speeds are reported in knots while car speeds are in miles per hour.

The way boat owners convert their boat speeds to mph is to use this formula: (Number of knots) + (15% of the number of knots) = mph.

So a boat traveling 20 knots is going 23 mph. (20 knots) + (15% of 20) = 20 + 3 = 23 mph.

15% of 20: 10% of 20 = 2, 5% of 20 = 1,
then 15% of 20 = 3

4. a. Find in mph the speed of a boat traveling 30 knots.

   b. How many mph does a speed of 50 knots represent?

5. Coats! 20% off. How much would you save?

   a) Fur-trim coats: regularly $68.00 and $109.00

   b) All-weather coats: regularly $14.99 and $24.99

6. How much is saved on a one-day sale of ladies' diamond ring sets if they regularly sell for $310.00 and are advertised for 40% off?

7. A rayon instant wardrobe! A coat plus two coordinating shift dresses. They are selling for 30% off the regular price of $23.95. How much would be saved?

8.

EARTH

MARS

The pull of gravity on Mars is 39% of the pull of gravity on Earth.

If you weigh 150 pounds on Earth, what would you weigh on Mars?

9. A ladies' pleated tennis outfit and racket usually sells for $29.99. How much less would it cost when on sale for 20% off?

DISCUSSION QUESTION

When we are estimating costs of items, why doesn't it matter if we do not agree exactly on the estimate?

A CHALLENGE FOR EXPERTS

1.

Most car radiators are protected from freezing by permanent-type antifreeze. The amount of protection is calculated according to the per cent of antifreeze in the cooling system. For instance, if 40% of the coolant is antifreeze, then the car is protected to a temperature of -12° F.

This table shows the protection provided by different per cents of antifreeze.

ADAMS ANTIFREEZE

% of Antifreeze	Your car is Protected to, F
20	+16
25	+10
33	0
40	-12
45	-22
50	-34
55	-48
60	-62

Fill in the table below:

Size of Cooling System, Quarts	Protection Desired, °F	Amount of Antifreeze Required, Quarts
14	-12	
17	-34	
21	0	
15	-48	

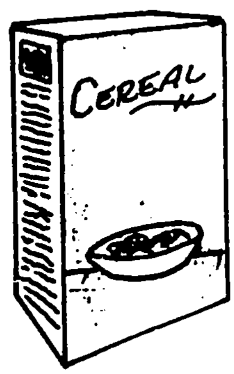
2. Adults require 10 mg. (milligrams) of iron per day in their food.

1000 mg. = 1 gram, 1 gram $\approx$ .04 ounces, so 10 mg. $\simeq$ .0004 oz.

The amount of iron per ounce of cereal varies from brand to brand.

Example: Rice Krispies gives 5% of the adult daily requirement per ounce of cereal. How many mg. of iron is in each ounce of cereal?

Solution using estimation:



10% of the requirement (10 mg.) is found first. This is the same as $\frac{1}{10}$ of that number.

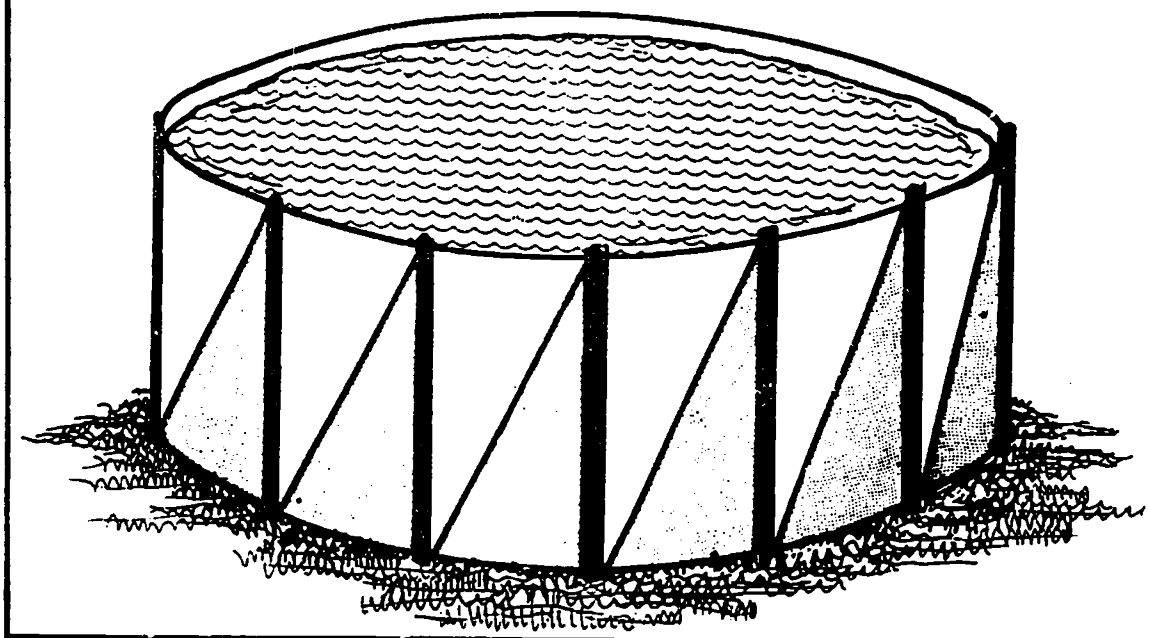
$\frac{1}{10}$ of 10 mg. is 1 mg.

Since 5% is one-half of 10%, Rice Krispies gives an adult .5 mg. of iron per ounce.

Fill in the table:

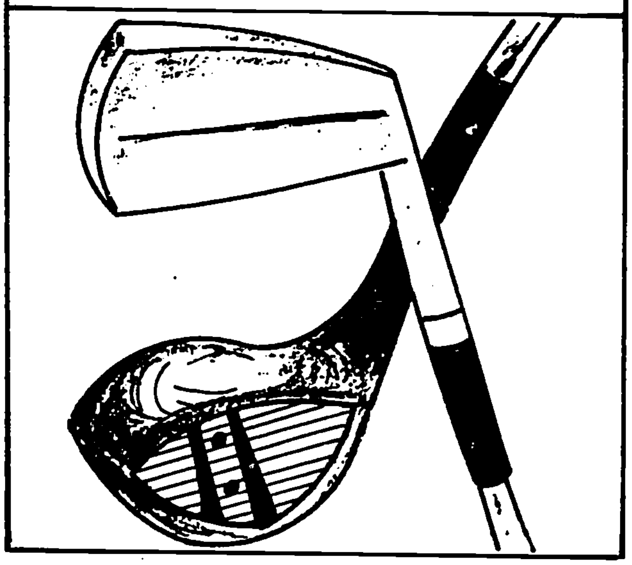
In One Ounce of Cereal	Per Cent of Adult Daily Iron Requirement	Milligrams of Iron Per Ounce of Cereal
Coco-Wheats	8.1%	
Cream of Wheat	120%	
All-Bran	30%	

PUTTERS AND POOLS



SAVE 10% ON POOLS – 5 DAYS ONLY!

SAVE TO 50% OFF GOLF EQUIPMENT



CALCULATING PER CENTS

Proportions can be used to find per cents of numbers.

Example: Find 56% of 283.

Estimation: Finding a per cent of a number is the same as finding a fractional part of the number. 56% is close to 50% or $\frac{1}{2}$.
$\frac{1}{2}$ of 283 is approximately 140.

Solution: $\frac{56}{100} = \frac{N}{283}$

$100N = 15848$

$N = 158.48$ This is close to the estimate.

EXERCISES I

Estimate the answer and then calculate the following percentages.

1. 25% of 88
2. 35% of 120
3. 48% of 130
4. 76% of 90
5. 88% of 64
6. 23% of 44
7. 65% of 150
8. 93% of 165
9. 18% of 58
10. 57% of 234

11. The generally accepted rate of tipping at a restaurant is 15% of the cost of the meal. If you and your date had meals totaling $4.80 at Rick's Restaurant, what tip would you leave for the waitress?

EXERCISES 2

Go back to Exercises 4 on pages 98-101 and find out the actual answers to the questions. Compare your answers to the estimates you made as a check on the reasonableness of your answers.

PER CENT IN ADVERTISING

GOLFER'S PARADISE SALE

MEN'S MATCHED SETS

MODEL	IRON-WOODS	REGULAR PRICE	SALE PRICE
'67 Spalding Silverlines	8-4	$210	$100
'67 Allied Tournaments	8-3	$150	$ 75
'68 Ram Aluminum	9-4	$325	$195
'67 Voit 300 Magnums	9-4	$313	$185
'68 MacGregor Tourney's	9-4	$284	$195
'67 Spalding Elites	8-4	$340	$225
'68 Ram Invitationals	8-3	$180	$ 95
'67 Wilson Signatures	8-4	$270	$150
'68 Wilson Signatures	8-4	$270	$185
'68 Wilson Dave Marr	8-3	$130	$ 70
'68 Spalding Dan Sikes	8-3	$150	$ 88
'68 Burke Bombers	8-3	$200	$115
'67 Slipstreams	9-4	$275	$100
'68 Fernquist-Johnson	9-4	$270	$180
'68 Paul Anthony's	8-3	$100	$ 45
'68 Wilson Shotmakers	8-3	$161	$115

SAVE TO 50% OFF BAGS, CARTS, BALLS, SHOES, etc.

Sale advertisements, especially those involving per cents, should be read carefully.

The amount an item is reduced is called the <u>amount of discount</u>.

For example, the '67 Spalding Silverlines are on sale for $100.00 while they sold regularly for $210.00. The discount is $110.00. When the discount is expressed as a per cent, it is called <u>discount rate</u> or <u>rate of discount</u>.

EXERCISES 3

Read the previous advertisement and answer the questions below.

1. How many sets of golf clubs advertised are on sale at a discount greater than 50%?

2. How many sets are on sale at a discount less than 50%?

3. How many sets are on sale for exactly 50%?

4. On which of the sets advertised will you receive the largest rate of discount? The smallest rate of discount? Be able to justify your answer.

5. On which of the sets advertised will you receive the largest amount of discount? The smallest amount of discount?

6. Compare your answers to exercises 4 and 5. Were your answers the same? Why or why not?

7. What is the meaning of the phrase "save to 50% off"?

8. Do you feel this advertisement is misleading? Why or why not?

SUMMER POOL CLOSEOUTS

ALL POOLS ARE GREATLY REDUCED!!

OPEN 9-9! COME OUT TODAY

DELUXE -- "Our Finest Pool"-- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'–6"	2	28%	$540	$390
18'x4'–5½"	19	27%	$400	$292
15'x4'–5½"	3	24%	$310	$232

CAPRI --"Lowest Priced Winterized Pool Ever"

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'–5½"	41	34%	$420	$277
18'x4'–5½"	3	26%	$310	$229
15'x4'–5½"	13	27%	$250	$182

DELUXE OVAL -- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
32'x18'x4'	6	29%	$940	$671
25'x15'x4'	2	28%	$730	$522

ARISTOCRAT OVAL -- Includes 2 Redwood Decks

SIZE	QUANTITY	SAVE	WAS	NOW
★31'x16'x4'	3	25%	$700	$526
★24'x12'x4'	11	25%	$540	$405
18'x12'x4'	9	25%	$350	$262
15'x10'x3½'	7	27%	$140	$102

Save an "extra" 10% discount off these prices on any display model pool!

EXERCISES 4

The following exercises refer to the advertisements on the previous page.

1. For the swimming pools in the chart below (they are starred in the ad) calculate the sale price of each pool.

MODEL	SIZE	RATE OF DISCOUNT	REGULAR PRICE	SALE PRICE	ACTUAL CASH SAVINGS
Capri	24' X 4'5½"				
Deluxe	24' X 4'6"				
Aristocrat Oval	31' X 16' X 4'				
Aristocrat Oval	24' X 12' X 4'				

2. Compare the sale prices you calculated to those in the advertisement. Are the prices in the ad exactly what the discount rate indicated?

3. Since the sale price you calculated and the sale they quoted are not equal, the rates of discount in the advertisement are not exact. Why does the ad use discount rates that are not exact?

4. What would be the sale price of a Capri Model 24' × 4' 5½" if the pool were a display model?

✓POINT

1. Using 100 as a basis, compare the following costs for coffee: 2 oz. for 29¢ and 6 oz. for 88¢.

2. Compare the 1968 passing records of two leading football quarterbacks:

    Len Dawson 224 attempts and 131 completions.
    Earl Morrall 317 attempts and 182 completions.

3. A patient needs 40 oz. of medication. The strength of the medicine is 30%. How many ounces of the drug are needed?

4. Write:

    a. $\frac{35}{100}$ as a per cent using the symbol, %.

    b. $\frac{40}{100}$ as a simplified ratio; in decimal form.

    c. .75 as a per cent using the symbol, %.

5. Use a mark to show your estimate of 40% of the line.

6. Estimate: a. 15% of 60

    b. 40% of 250

7. Calculate: a. 30% of 145

    b. 45% of 72

8. A store advertised that their $14.95 skirts were on sale for 12% off. How much would be saved on the skirt?

VARIATION

Most of the work you have done with ratio and proportion has involved direct proportion.

DIRECT PROPORTION

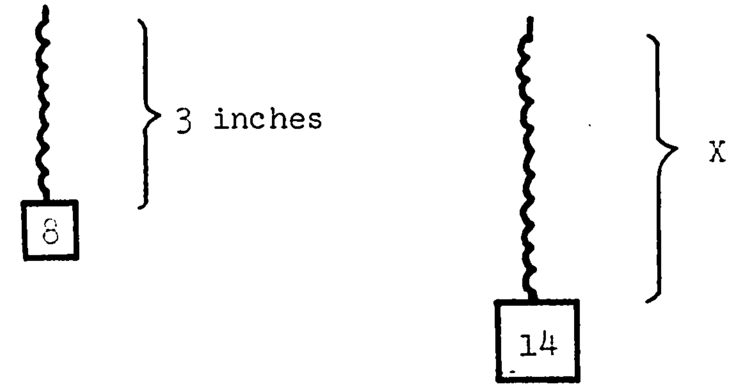
Direct proportion behaves like this: As one quantity increases, a related quantity increases.

Example 1: The price of apples is 2 for 5¢. The more apples you buy, the higher the total price.

2 apples cost 5¢, 6 apples cost 15¢, 20 apples cost 50¢, etc.

Example 2: The activity, Stretching Springs, involves direct proportion. The more weight you hang on a spring, the longer the spring stretches.

For a particular spring, a weight of 8 ounces stretches the spring 3 inches. How far would a weight of 14 ounces stretch the spring?



One proportion is $\frac{8}{3} = \frac{14}{X}$. From the problem situation you can see that the spring stretches as more weight is added.

increase in weight

$$\frac{8}{3} = \frac{14}{X}$$

increase in stretch

EXERCISES I

1. 25 pounds of a certain fertilizer is required for 5000 sq. ft. of lawn. How many pounds of fertilizer would be needed for 9500 sq. ft. of lawn?

2. A set of pulleys requires [illegible] pounds of pull to lift a weight of [illegible] pounds. How much pull is required to lift 1[illegible] pounds?

INVERSE PROPORTION

[illegible] of direct proportion, [illegible] inverse proportions.

An inverse proportion [illegible] one quantity increases [illegible] or as the first quantity [illegible].

Example: [illegible] value of the [illegible].

The key to solving proportion problems is to recognize whether the situation is direct or inverse.

The problem in solving proportion problems is that:

1. Direct proportion problems can be set up in two ways.
2. Inverse proportion problems can be set up in only one way.

Example: Direct Proportion

An alloy of copper and zinc makes brass. The ratio of the weight of copper to weight of zinc is 17 to 13. If 85 pounds of copper is used, how many pounds of zinc will be needed?

One proportion is $\frac{17}{13} = \frac{85}{N}$. $\quad \frac{\text{copper}}{\text{zinc}} = \frac{\text{copper}}{\text{zinc}}$

Another proportion is

$$\frac{17}{85} = \frac{13}{N} \qquad \frac{\text{copper}}{\text{copper}} = \frac{\text{zinc}}{\text{zinc}}$$

N = 65 in both cases.

Direct proportions allow two methods to set up the proportion, each resulting in the correct answer.

Examples: <u>Inverse Proportion</u>

1. If 24 men can do a job in 30 days, how many days will it take 36 men to do the job?

   Since more men are working on the job, it will take fewer than 30 days to complete.

   The problem involves an inverse proportion since the number of days decreases as the number of men increases.

<u>An inverse proportion can be set up in only one way. Each ratio must be a comparison of like quantities.</u> In this case men to men and days to days.

The proportion is $\frac{24}{36} = \frac{N}{30}$.

N represents the number of days the 36 men will take to do the job.

Notice that the 24 and 30 (and the 36 and N) are not both in the numerator (in the denominator) although they correspond. This is what makes the proportion inverse. The second ratio appears upside down to what you might expect.

The reason the second ratio is written upside down is explained below.

In $\frac{24}{36} = \frac{N}{30}$, the numerators are smaller than the denominator ($24 < 36$ so $N < 30$).

We know that the number of days the 36 men take will be less than 30, so N must be in the numerator to obtain this result.

One way to set up the proportion is to write the terms so they correspond $\frac{24}{36}$ and $\frac{30}{N}$, then turn the second ratio upside down and write the proportion: $\frac{24}{36} = \frac{N}{30}$.

The number of days needed by the 36 men is $N = 20$.

If a proportion is made where the terms of each ratio are not like quantities, an incorrect answer will result. (You might try it and see.)

2. At 50 mph a car can travel from Huntsville to Carson City in 5 hours. How long will it take to make the trip at 65 mph?

This is an inverse proportion because as the speed increases the time will decrease.

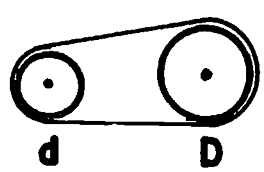
The proportion is $\frac{50}{65} = \frac{N}{5}$.

$$65N = 250$$

$$N \approx 3.8 \text{ hours}$$

EXERCISES 2

1. If the tension of a banjo string is kept constant, the frequency (vibrations per second) is inversely proportional to the length of the string. If a string 30 inches long vibrates 250 vps, how fast will it vibrate when 25 inches long?

2. 

A small drive wheel (d) is connected by a belt to a larger wheel and drives it (D is the driven wheel).

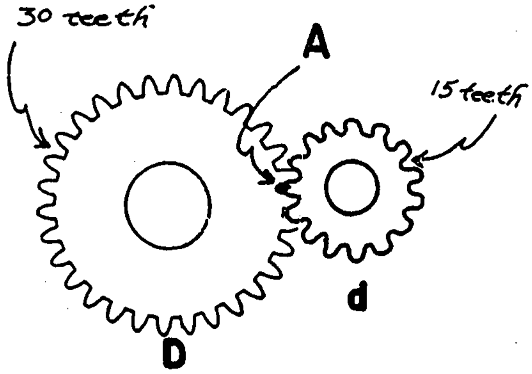
The drive wheel, d, turns faster than the driven wheel, D. The diameter of d is less than the diameter of D.

When two wheels (called pulleys) are connected in this way the number of revolutions per minute (rpm) of a wheel is inversely proportional to the diameter of the wheel.

What is the speed of pulley d if its diameter is 4 inches? D has a diameter of 10 inches and is turning 100 rpm.

3. Find the diameter of a pulley, D, having a speed of 80 rpm if it is driven by a 5 inch pulley having a speed of 200 rpm.

4. Gears behave somewhat like pulleys. Instead of being connected by a belt, they mesh using teeth.



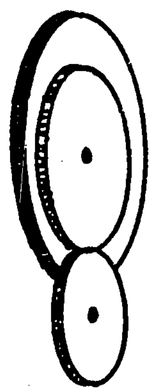
The smaller gear turns faster than the larger gear in order for all of the teeth to mesh, yet the number of teeth in the smaller gear is less than in the larger gear.

a. The teeth meet at point A. If all 30 teeth of D pass point A, how many teeth from d must pass point A?

b. For every revolution of D, how many revolutions does d make?

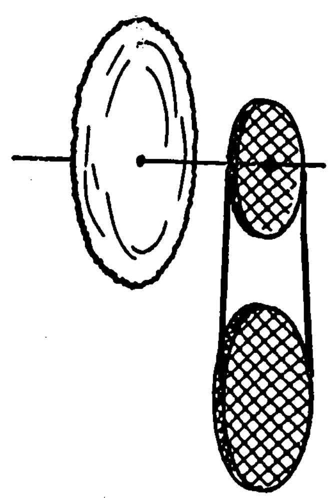
The number of revolutions per minute of two meshed gears is inversely proportional to the number of teeth in the gears.

c. If D turns 100 rpm, how many rpm will d make?

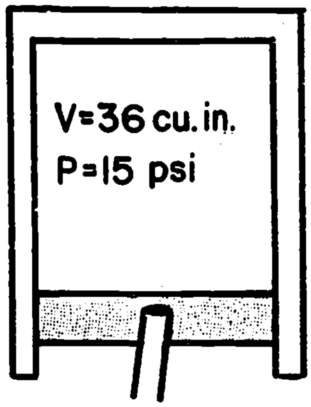
5. 

A grinding wheel turns at 850 rpm. It is attached to a 32 tooth gear and is driven by an 8 tooth gear. How fast is the drive gear turning?

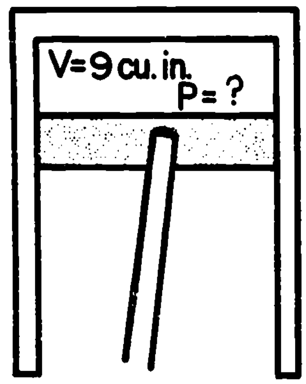
6. A circular saw is connected to a 5 inch pulley and turns 1000 rpm. How large is the drive pulley if it turns 240 rpm?



7. The pressure, P, inside an automobile cylinder is inversely proportional to the volume, V.



piston at bottom
of stroke



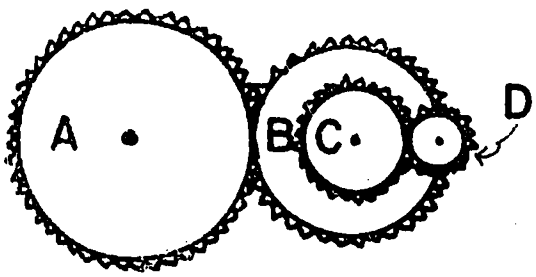
piston at top
of stroke

What is the pressure in the compressed cylinder?
(psi means pounds per square inch.)

8. The 1968 Mustang Mach I has a compression ratio of 10.50 to 1. This means that if the volume of the cylinder is 1 when the piston is up, the volume is 10.50 when the piston is at the bottom of the stroke. What is the pressure when the piston is up, if the pressure is 15 psi when the piston is at the bottom of the stroke?

A CHALLENGE FOR EXPERTS

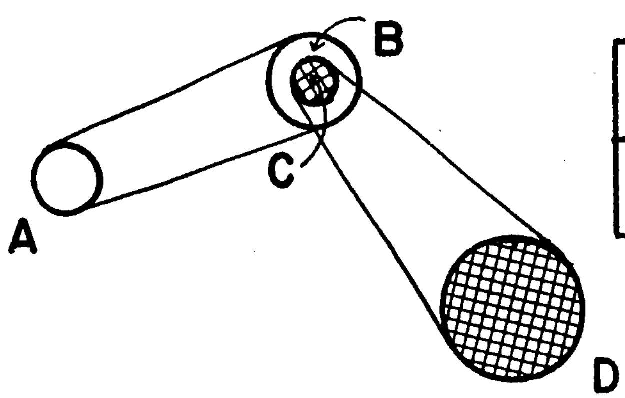
1. The diagram at the right shows a gear train (a train is a series of connected gears.) Gear A is turning at 70 rpm. How many rpm is Gear D turning?



Gear	A	B	C	D
Teeth	90	40	32	20

2. What is the effective gear ratio, $\frac{\text{gear A}}{\text{gear D}}$, in the gear train of exercise 1?

3. What diameter must pulley A have in order for it to turn 2400 rpm? Pulley D is turning 180 rpm.



Pulley	A	B	C	D
Diameter	?	5"	3"	16"

BEATERS AND BEAMS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.

2. Follow the instructions for the activities which begin on the next page.

3. Use the equipment inventory to check your equipment.

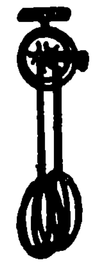
4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

THE EGG BEATER

Equipment Inventory: 1 Hand egg beater or

1 Hand speed drill



Activity:

This activity centers on the question: How many times does the beater revolve for each complete turn of the handle?

1. Turn the handle one complete turn and have your partner count the number of turns of the beater. ________

2. Turn the handle 4 times. How many beater revolutions were there? ____ For 6 handle turns, how many beater revolutions are there? ________

3. Using the information from parts 1 and 2, how many beater revolutions are there for one handle turn? ________

4. Predict the number of beater turns if the handle makes 8 turns. _______ Did you have to count the turns of the beater? Do you think there is a better way to make this prediction?

5. Write a ratio B to H, where B is the number of beater turns and H is the number of handle turns.

6. Count the teeth in each gear, and write a ratio comparing them. _____

7. How do the ratios you wrote in parts 5 and 6 compare?

8. What is the relation between the handle and beater revolutions and the number of gear teeth on each?

9. What is the precise number of turns the beater will make when the handle is turned one revolution? ________

10. If the beater turns 45 times, how many times will the handle turn? ________

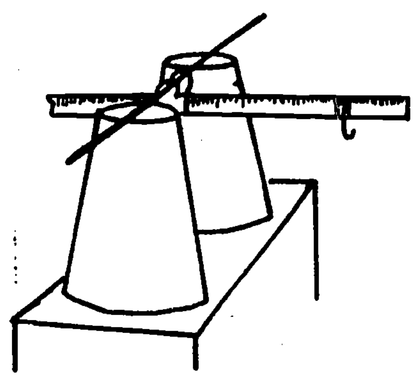
11. Write a proportion which will show how handle and beater turns and number of gear teeth are inversely proportional.

12. An egg beater has gears with 64 and 14 teeth each. If the handle is turned 28 times, how many turns will the beater make?

THE BALANCE BEAM

Equipment Inventory: 1 meter stick, 2 dixie cups, 1 metal paper-clamp, 1 wire rod, 3 known weights (W_1, W_2, W_3), 2 unknown weights (W_4, W_5), 2 metal hooks.

Activity: This activity centers on the question: How can a beam balance be used to predict the weight of an object?



1. Assemble the beam balance including the hooks and move the clamp position until the meter stick balances.

2. Place weights W_1 and W_2 on hooks and slide the hooks until the meter stick balances.

3. Write as ratios:

   a. The distance W_2 is from the center of the stick to the distance W_1 is from the center. $\frac{D_2}{D_1}$:

   b. The weight of W_1 compared to the weight of W_2. $\frac{W_1}{W_2}$:

4. Use weights W_1 and W_3 and repeat parts 2 and 3.

   $\frac{D_3}{D_1}$: $\frac{W_1}{W_3}$:

5. Are the ratios obtained in exercises 3 and 4 equivalent?

6. If a proportion is written $\frac{D_2}{D_1} = \frac{W_1}{W_2}$, what kind of a proportion is it? ________ __________

7. Place weights W_1 and W_4 on the beam and adjust until it balances. Use a proportion to predict the weight of W_4. ______

8. Check the weight of W_4 by weighing on a scale. _____

9. Predict the weight of W_5. ______

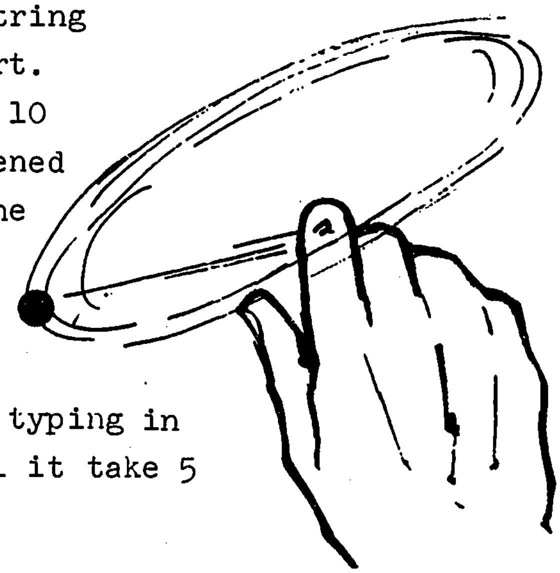
10. Check the weight of W_5. _____

✓ **POINT**

1. Which of the following situations involve inverse proportions.

   a. Hanging various weights on a spring stretches the spring various lengths. This device can be used to predict the unknown weight of an object.

   b. A ball tied to the end of a string is twirled with constant effort. First the length of string is 10 inches, and then it is lengthened to 15 inches. The speed of the ball changes as the length of string changes.



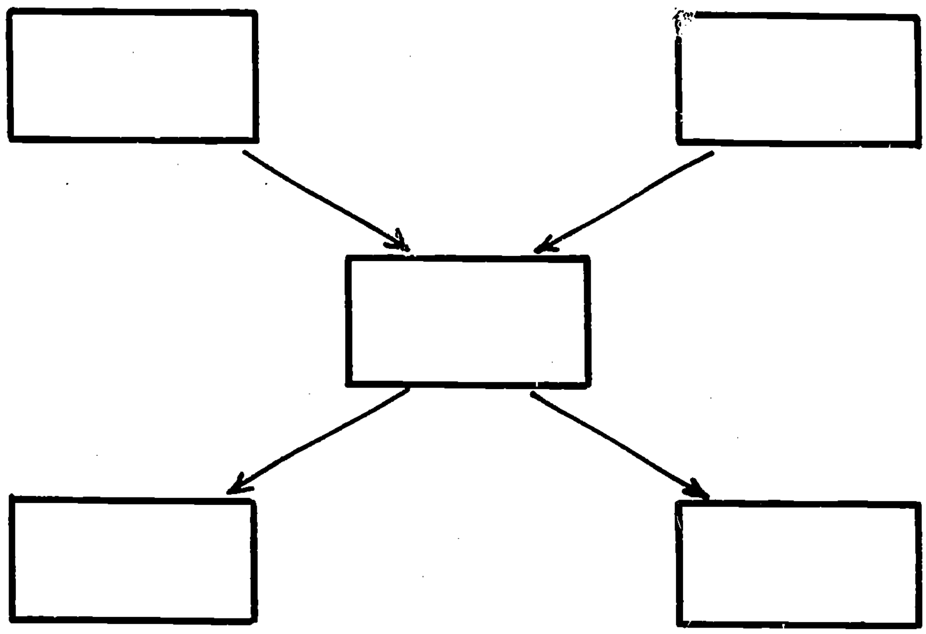
   c. Three secretaries can do some typing in 16 hours. How many hours will it take 5 secretaries to do the job?

   d. A machine can grind 35 bolts in 15 minutes. How many bolts can it grind in 100 minutes?

2. How can you tell whether a situation involves direct or inverse proportions?

Diagram outlines for Exercises 1, page 65.

1.



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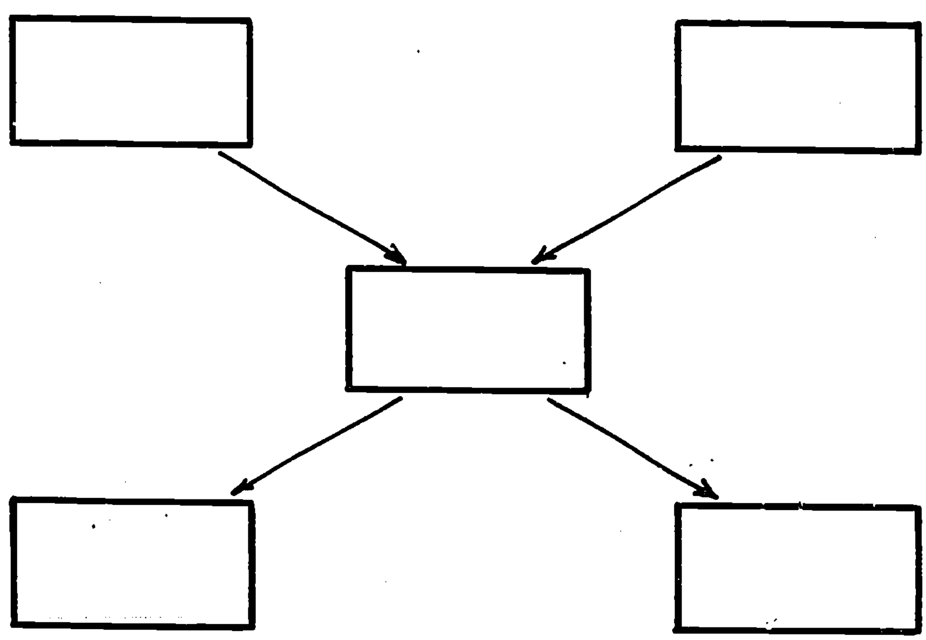

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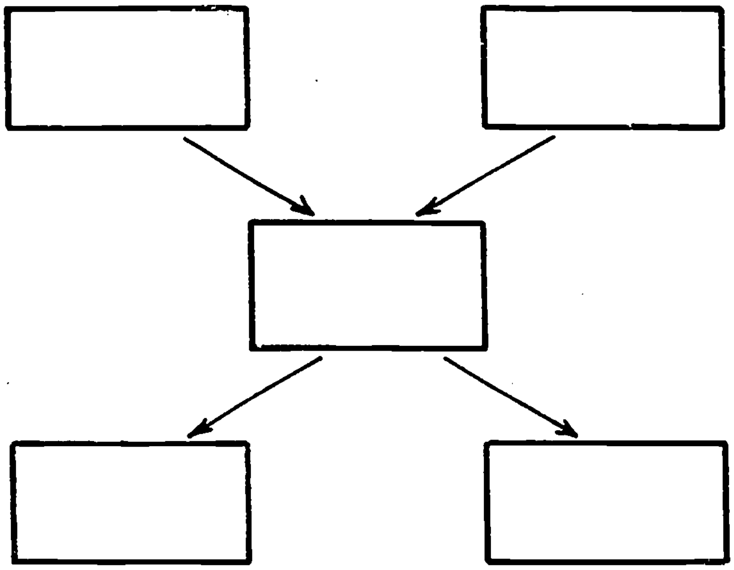

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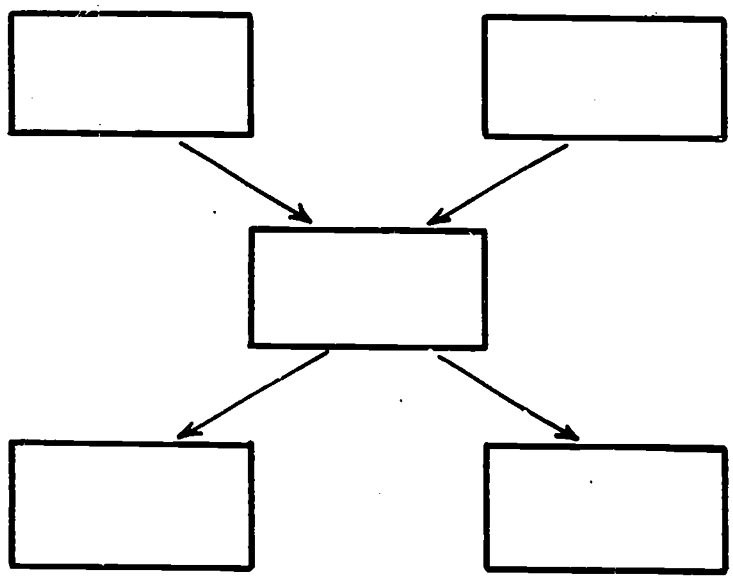

5.