DOCUMENT RESUME

ED 069 505 SE 015 196

AUTHOR Holland, Bill
TITLE Learning Activity Package, Algebra-Trigonometry.
INSTITUTION Ninety Six High School, S. C.
PUB DATE 72
NOTE 150p.

EDRS PRICE MF-$0.65 HC-$6.58
DESCRIPTORS *Algebra; Curriculum; *Individualized Instruction; *Instructional Materials; Mathematics Education; Objectives; *Secondary School Mathematics; Teacher Developed Materials; Teaching Guides; *Trigonometry; Units of Study (Subject Fields)

ABSTRACT

A series of ten teacher-prepared Learning Activity Packages (LAPs) in advanced algebra and trigonometry, the units cover logic; absolute value, inequalities, exponents, and complex numbers; functions; higher degree equations and the derivative; the trigonometric function; graphs and applications of the trigonometric functions; sequences and series; permutations, combinations, and probability; descriptive statistics; and special theorems and functions. The units each contain a rationale for the material being covered; lists of behavioral objectives; a list of reading assignments, problem sets, tape recordings, and filmstrips that go with the unit; a student self-evaluation problem set, suggestions for advanced study, and references. For other documents in this series, see SE 015 193, SE 015 194, SE 015 195, and SE 015 197. (DT)

$[(P \rightarrow Q) \wedge P] \rightarrow Q$

LEARNING
ACTIVITY
PACKAGE

LOGIC

Ninety Six High School · Ninety Six, S.C.

ALGEBRA-TRIG. 124

REVIEWED BY

LAP NUMBER 42A

WRITTEN BY

TABLE OF CONTENTS

RATIONALE

From your past experience, you should now realize that your learning of new mathematics is dependent on your understanding of previous mathematical concepts. This LAP is predicated on the idea that you have successfully completed LAP 1 in Algebra C-1.

In this LAP you will take the basic concepts of LAP 1 (Logic) and utilize them in the performance of proofs. The main content of this LAP is geared to one end--the demonstration of proofs.

This material is necessary not only for achieving a solid understanding of the logic underlying mathematics, but also giving a good introduction to the analysis of arguments whether they appear in mathematics, the sciences, debates or business.

Since this LAP is an extension of LAP 1, it is necessary for you to be able to recall certain definitions and concepts.

We realize that some of you may have forgotten some of the necessary information. To help you decide how well prepared you are, we have provided a review section for you. Use this section wisely.

DO YOU REMEMBER?

1. The connectives and their meanings: negation "$\sim$," conjunction "$\wedge$," disjunction "v," conditional "$\rightarrow$" and biconditional "$\leftrightarrow$"?

Explanation:

negation "$\sim$" (read not) - when the statement P is true $\sim P$ is false and when P is false $\sim P$ is true.

conjunction "$\wedge$" (read and) $P \wedge q$ is only true when P is true and q is true, otherwise it is false.

disjunction "v" (read or) $P \vee q$ is only false when P is false and q is false, otherwise it is true.

conditional "$\rightarrow$" (read if, then) $P \rightarrow q$ is only false when P is true and q is false, otherwise it is true.

biconditional "$\leftrightarrow$" (read if and only if) $P \leftrightarrow q$ is true only when P and q have the same truth value, otherwise it is false.

2. What is meant by equivalent sentences?

Explanation: P is equivalent to q ($P \Leftrightarrow q$) if and only if P and q have the same truth values. (Another way of saying this is P is equivalent to q if and only if the statement $P \leftrightarrow q$ is a tautology).

3. Dr. Morgan's laws of disjunction and conjunction?

Explanation: the following are equivalent

a) $\sim(P \wedge q) \Leftrightarrow \sim P \vee \sim q$

b) $\sim(P \vee q) \Leftrightarrow \sim P \wedge \sim q$

4. What is meant by a tautology?

Explanation: A tautology is a sentence which is always true.

Example: $P \vee \sim P$.

5. The Law of Syllogism?

Explanation: This is a tautology you will find very helpful in doing proofs.

$$\left[(P \rightarrow q) \wedge (q \rightarrow R)\right] \rightarrow (P \rightarrow R)$$

6. The following relatives of a conditional sentence and their meaning?

a) its converse

b) its inverse

c) its contrapositive

Explanation: Given the conditional sentence $P \rightarrow q$,

its converse is $q \rightarrow P$

its inverse is $\sim P \rightarrow \sim q$

its contrapositive is $\sim q \rightarrow \sim P$.

The conditional and its contrapositive are equivalent

$P \rightarrow q \Leftrightarrow \sim q \rightarrow \sim P$.

The inverse and converse are equivalent

$\sim P \rightarrow \sim q \Leftrightarrow q \rightarrow P$.

7. Modus Ponens or the Rule of Detachment.

Explanation: Given $p \rightarrow q$
p
we can conclude q

Knowledge of all these concepts are necessary for success in this LAP.

Study these pages carefully:

a) If you feel you understand these concepts, turn to the review test as an additional check.

b) If you have forgotten some of these concepts, go back to LAP 1 in Algebra C-1 for the necessary review and then take the review test.

Note: The questions on the review test are coded to the behavioral objectives in LAP 1 in Algebra C-1.

REVIEW TEST

OBJECTIVES	I. TRUE or FALSE:
11	1. When a conditional sentence is true, its converse is true.
8	2. If "$p \to q$" is true, we may infer "p" is true.
8	3. If "$p \to q$" is true, we may infer "p" is false.
8	4. If "$p \to q$" is false, we may infer "p" is true.
11	5. If a conditional sentence is true, its contrapositive is true.
8	6. $p \to q \Leftrightarrow \sim p \vee q$.
11	7. If a conditional sentence is true, then its inverse is true.
8	8. If "$p \to q$" is true and p is true, then "q" must be true.
9	9. $\lfloor p \vee (q \wedge r) \rfloor \leftrightarrow \lfloor (p \vee q) \wedge (p \vee r) \rfloor$ is a tautology.
9	10. $p \to p$ is a tautology.
9	11. $p \to (P \wedge q)$ is a tautology.
9	12. $p \to (p \vee q)$ is a tautology.

OBJECTIVES

II. Match the statement in Column A with the equivalent statement in Column B.

5,6,8,12

	Column A		Column B
1.	$\sim(\sim p)$	(a)	$\sim(\sim p \wedge \sim q)$
2.	$p \leftrightarrow q$	(b)	$\sim(p \wedge \sim q)$
3.	$p \wedge q$	(c)	$p \rightarrow q$
4.	$p \vee q$	(d)	p
5.	$p \rightarrow q$	(e)	$\sim p \vee \sim q$
6.	$\sim(p \rightarrow q)$	(f)	$\sim p \Leftrightarrow \sim q$
7.	$\sim(p \wedge q)$	(g)	$\sim(p \vee q)$
8.	$\sim p \wedge \sim q$	(h)	$\sim(\sim p \vee \sim q)$
9.	$\sim p \vee q$	(i)	$p \wedge \sim q$
10.	$p \Leftrightarrow q$	(j)	$(p \rightarrow q) \wedge (q \rightarrow p)$

III. By use of Modus Ponens, what conclusions can we make for each of the following:

14

1. p

   $p \rightarrow q$

   $\therefore$ (?)

2. $p \wedge r$

   $(p \wedge r) \longrightarrow (q \wedge s)$

   $\therefore$ (?)

3. $t \rightarrow w$

   t

   $\therefore$ (?)

4. $(p \rightarrow r) \longrightarrow (r \rightarrow s)$

   $(p \rightarrow r)$

   $\therefore$ (?)

5. If roses are red then violets are blue

   Roses are Red

   $\therefore$ (?)

6. If it snowed yesterday, the schools were closed.

   It did snow yesterday

   $\therefore$ (?)

KEY TO REVIEW TEST

I.

1. False
2. False
3. False
4. True
5. True
6. True
7. False
8. True
9. True
10. True
11. False
12. True

II.

1. d
2. j
3. h
4. a
5. b (or c) (b is the preferred response)
6. i
7. e
8. g
9. c (or b) (c is the preferred response)
10. f

III.

1. $\therefore q$
2. $\therefore q \wedge s$
3. $\therefore w$
4. $\therefore r \rightarrow s$
5. $\therefore$ violets are blue
6. $\therefore$ the schools were closed

If you have missed more than three problems in any one of the three sections on the preceding test, you are to report to your teacher for additional direction; if not, turn to the list of behavioral objectives for this LAP.

LIST OF BEHAVIORAL OBJECTIVES

At the end of this LAP, you should be able to:

1. given the statements for a proof, supply the correct reasons for each statement, such as:
   a) Substitution
   b) Rule of Conditional Proof
   c) Rule P
2. given the necessary theorems, postulates and definitions, perform a proof using these as reasons.
3. demonstrate your ability to recall the summary of rules of inference and the important tautologies, as outlined on pages 2 and 3 of this LAP by using them for reasons in a proof when the statements are given.
4. given the problem to perform an indirect proof, choose the statement to assume as true.
5. given a set of premises and a required conclusion, prove or disprove the conclusion by using an indirect proof, either symbolically or in paragraph form.

I BEHAVIORAL OBJECTIVES

(from pages 12 to 22)

You should be able to:

1. given the statements for a proof, supply the valid reasons for each statement, such as:
   a) Substitution
   b) Rule of Conditional Proof
   c) Rule P
2. given the necessary theorems, postulates and definitions, perform a proof using these as reasons
3. demonstrate your ability to recall the summary of rules of inference and the important tautologies as outlined on pages 2 and 3 of this LAP by using them for reasons in a proof when the statements are given.

THE RULE OF SUBSTITUTION AND THE RULE OF CONDITIONAL PROOF

If a given sentence P contains the sentence s and $s \Leftrightarrow t$, then the sentence Q obtained by substituting t for s is equivalent to P. That is, $P \Leftrightarrow Q$. For instance, if we have the sentence

$$P: \quad q \wedge s$$

and $s \Leftrightarrow t$

then we obtain $Q: \quad q \wedge t.$

Hence, $P \Leftrightarrow Q$

or $(q \wedge s) \Leftrightarrow (q \wedge t).$

The concept discussed in this paragraph is called the rule of substitution. We will accept this rule without proof since the proof is beyond our discussion here and the truth of this rule should appear quite clear to you.

Example 1: $[p \wedge (p \rightarrow q)] \Leftrightarrow [p \wedge (\sim p \vee q)]$
since $(p \rightarrow q) \Leftrightarrow (\sim p \vee q)$

RULE OF CONDITIONAL PROOF

The theorems which we prove can be represented as conditional sentences. That is,

$$p \rightarrow q.$$

In making a proof we generally "accept" p as true. For if p were false then the conditional $p \rightarrow q$ would already be true. Now that we have accepted p as true some chain of steps should bring us to the fact that q is true. Hence, if p is true and q is true, we know $p \rightarrow q$ is true and the proof in question is complete. The rule by which we infer p is true is called Rule P. The rule by which we infer $p \rightarrow q$ is true is called the Rule of Conditional Proof. It is abbreviated "Rule C. P."

Suppose we wished to prove the following theorem:

If a triangle is equilateral, then the triangle is equiangular.

Consider,

P: A triangle is equilateral

Q: A triangle is equiangular.

Thus we wish to prove $P \rightarrow Q$. Our steps within the proof may involve the following with Q as our conclusion:

P (by Rule P)

Postulates

Definitions

Previously proved theorems

$\therefore$ Q

Having Q deduced in this argument we now obtain

$P \rightarrow Q$ (by Rule C. P.)

Some people incorrectly believe that the objective in proving a theorem, $P \rightarrow Q$, is simply to deduce the consequent Q as the conclusion. However, this is not the case. We are actually trying to prove the conditional $P \rightarrow Q$. You should notice, nevertheless, that once we arrive at the fact Q is true we are basically finished since we can assume P is true and therefore $P \rightarrow Q$ is true. Hence, the last step of a proof is many times omitted. That is, occasionally the Rule of Conditional Proof is not used.

Example 2: Let us suppose we have the following information at our disposal:

Postulate	Theorems	Definitions
p_1	T_1: $m \rightarrow n$	D_1: $n \Leftrightarrow c$
	T_2: $(a \wedge p_1) \rightarrow q$	D_2: $q \Leftrightarrow m$

Prove: $a \rightarrow c$

Proof:

True Statements	Reasons
1. a	1. Rule P
2. p_1	2. Postulate
3. $(a \wedge p_1) \to q$	3. T_2
4. $a \wedge p_1$	4. Conjunctive Inference (steps 1 and 2)
5. q	5. Modus Ponens (steps 3 and 4)
6. $q \Leftrightarrow m$	6. D_2
7. m	7. "Definition of Equivalence"
8. $m \to n$	8. T_1
9. n	9. Modus Ponens
10. $n \Leftrightarrow c$	10. D_1
11. c	11. Definition of Equivalence
12. $a \to c$	12. Rule C. P. (steps 1 and 11)

IMPORTANT:

We use this method of proof (Rule of Conditional Proof) only when proving a conditional sentence.

Problem Set 1

1. Supply the reasons for the assertions in the following proof.

   Theorems: T_1: $(a \vee c) \to b$, T_2: $d \to (a \vee c)$

   Prove: $d \to b$

Proof:

True Statements	Reasons
1. d	1.
2. $d \rightarrow (a \vee c)$	2.
3. $a \vee c$	3.
4. $(a \vee c) \rightarrow b$	4.
5. b	5.
6. $d \rightarrow b$	6.

2. Give the reasons for the following assertions in this proof:

Theorems	Definitions
T_1: $a \rightarrow b$	D_1: $b \Leftrightarrow c$
	D_2: $c \Leftrightarrow d$

Prove: $a \rightarrow d$

Proof:

True Statements	Reasons
1. a	1.
2. $a \rightarrow b$	2.
3. b	3.
4. $b \Leftrightarrow c$	4.
5. c	5.
6. $c \Leftrightarrow d$	6.
7. d	7.
8. $a \rightarrow d$	8.

3. Furnish a complete proof for the following:

(Hint: you could use the Law of Syllogism)

Theorems	Definitions
T_1: $a \rightarrow b$	D_1: $f \Leftrightarrow g$
T_2: $b \rightarrow (a \wedge c)$	
T_3: $(a \wedge c) \rightarrow f$	

Prove: $a \rightarrow g$

4. Can you define "$\vee$," "$\rightarrow$" and "$\leftrightarrow$" using the connectives "$\wedge$" and "$\sim$."

5. Prove that if $(P \wedge q)$ is true then P is true (this is called Law of Conjunctive Simplification. (Hint: is $(P \wedge q) \rightarrow P$ a tautology?).

Let us now review the Law of the Syllogism and the Law of Contrapositive Inference which have been already established in LAP 1.

Law of the Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Law of Contrapositive Inference: $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

Example 1: Given: $\sim s \rightarrow c$

$c \rightarrow l$

$\sim l$

Prove: s

Proof:

True Statements	Reasons
1. $\sim s \rightarrow c$	1. Given
2. $c \rightarrow l$	2. Given
3. $(\sim s \rightarrow c) \wedge (c \rightarrow l)$	3. Conjunctive Inference
4. $[(\sim s \rightarrow c) \wedge (c \rightarrow l)] \rightarrow (\sim s \rightarrow l)$	4. Law of the Syllogism
5. $\sim s \rightarrow l$	5. Modus Ponens 3 and 4
6. $\sim l$	6. Given
7. $[\sim l \wedge (\sim s \rightarrow l)]$	7. Conjunctive Inference
8. $[\sim l \wedge (\sim s \rightarrow l)] \rightarrow \sim(\sim s)$	8. Law of Contrapositive Inference
9. $\sim(\sim s)$	9. Modus Ponens 7 and 8
10. $s \Leftrightarrow \sim(\sim s)$	10. Theorum of Double Negation
11. s	11. "Definition of Equivalence"

It seems quite important that you realize that the conditional, $[(\sim s \rightarrow c) \wedge (c \rightarrow l) \wedge \sim l] \rightarrow s$ is a tautology and this is the case of any conditional you are able to prove true. Can you tell why? We could have used a truth table for the proof, but this is not very instructive. We've spent quite a lot of time developing our axioms and theorems so let's use them. Besides a truth table becomes cumbersome if we have four variables to deal with, because we would then have to be concerned with sixteen rows.

Our development has brought us many rules of inference and it seems appropriate to summarize many of them at this point. See Problem 5, Problem Set 1 for Conjunctive Simplification.

Summary of the Rules of Inference

I. Rule P

II. Conjunctive Inference

III. Conjunctive Simplification

IV. Rule of Conditional Proof (Rule C. P.)

V. Modus Ponens

VI. Rule of Substitution

VII. Tautologies

1.	$[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$	Law of Contrapositive Inference
2.	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Law of the Syllogism
3.	$\sim(\sim p) \Leftrightarrow p$	Theorem 1 (Law of Double Negation)
4.	$(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$	Law of Contraposition
5.	$\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$	De Morgan's Law
6.	$\sim(p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$	De Morgan's Law
7.	$(p \rightarrow q) \Leftrightarrow (\sim p \vee q)$	Equivalence for Implication and Disjunction
8.	$\sim(p \rightarrow q) \Leftrightarrow (p \wedge \sim q)$	Negation of the Implication
9.	$(p \vee q) \Leftrightarrow (q \vee p)$	Commutative Law for Disjunction
10.	$(p \wedge q) \Leftrightarrow (q \wedge p)$	Commutative Law for Conjunction
11.	$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$	Associative Law for Disjunction
12.	$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative Law for Conjunction
13.	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributive Law
14.	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive Law

Problem Set 2

1. Give the reasons for each step in the following proof:

   Prove: $(p \to q) \wedge \sim(q \vee r) \to \sim p$

   Proof:

True Statements	Reasons
1. $(p \to q) \wedge \sim(q \vee r)$	1.
2. $p \to q$	2. Conjunctive Simplification
3. $\sim(q \vee r)$	3.
4. $\sim(q \vee r) \Leftrightarrow (\sim q \wedge \sim r)$	4.
5. $\sim q \wedge \sim r$	5.
6. $\sim q$	6.
7. $\sim q \wedge (p \to q)$	7.
8. $[\sim q \wedge (p \to q)] \to \sim p$	8.
9. $\sim p$	9.
10. $[(p \to q) \wedge \sim(q \vee r)] \to \sim p$	10.

2. Given: (1) If wages increase, then there will be inflation.

   (2) If there is inflation, then the cost of living will increase.

   (3) Wages will increase.

   Prove: The cost of living will increase.

   Consider, W: Wages increase.

   I: There will be inflation.

   C: The cost of living will increase.

   Thus, we are given:

   (1) $W \to I$

   (2) $I \to C$

   (3) W

   Prove: C

3. Prove the following. Use the given letters as symbols.

Given: (1) If the market is free, then a single dealer cannot affect prices. (F, D)

(2) If a single dealer cannot affect prices, then there are a large number of dealers. (D, N)

(3) There are not a large number of dealers.

Prove: The market is not free.

4. Given: (1) $w \rightarrow d$

(2) $p \rightarrow \sim d$

Prove: $w \rightarrow \sim p$

5. Prove: $\left[(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow s)\right] \rightarrow (p \rightarrow s)$.

(This is called a "chain" of syllogisms). Do not use a truth table).

6. Determine which of the following arguments are valid.

a. All politicians are honest.

Jenkins is a politician.

∴ Jenkins is honest.

b. All politicians are honest.

All lawyers are politicians.

∴ All lawyers are honest.

c. Only men are mortal.

This person is a man.

∴ This man is mortal.

d. The diagonals of a rectangle are congruent.

All squares are rectangles.

∴ Any square has congruent diagonals.

7. Negate the following:

   a. $\sim p \wedge \sim q$

   b. $p \rightarrow \sim q$

   c. $\sim p \rightarrow q$

   d. $p \vee (p \rightarrow q)$

   e. Frank is old and bald.

   f. $x^3 = 8$ or $x^4 = 13$

   g. If snow falls, then it will be cold.

8. What do you think the negation would be for each of the following:

   a. For all numbers x, $x^2 = 4$

   b. For some number x, $2x + 3 = 5$

   c. Some mice are meek.

   d. No cats are black.

   e. All rabbits are not bunnies.

   f. Some people do not like dogs.

SELF-EVALUATION

1. Give the reason for each assertion in the following proof.

D_1: $f \Longleftrightarrow \sim b$

D_2: $\sim q \Longleftrightarrow h$

D_3: C

T_1: $h \rightarrow f$

T_2: $(d \wedge c) \rightarrow \sim q$

Prove: $a \rightarrow \sim b$

	Assertion	Reason
1.	a	
2.	c	
3.	$(d \wedge c) \quad q$	
4.	$\sim q$	
5.	$\sim q \Longleftrightarrow h$	
6.	h	
7.	$h \rightarrow f$	
8.	f	
9.	$f \Longleftrightarrow \sim b$	
10.	$\sim a$	
11.	$a \rightarrow \sim b$	

2. Give the reason for each assertion in the following proof:

Given: $\sim m \vee n$

$n \Longleftrightarrow \sim c$

$\sim c \rightarrow x$

Prove: $m \rightarrow x$

	Assertion	Reason
1.	$\sim m \vee n$	

SELF-EVALUATION (cont')

	Assertion	Reason
2.	$\sim m \vee n \Longleftrightarrow m \rightarrow n$	
3.	$m \rightarrow n$	
4.	m	
5.	n	
6.	$n \Longleftrightarrow \sim c$	
7.	$\sim c$	
8.	$\sim c \rightarrow x$	
9.	x	
10.	$m \rightarrow x$	

3. Prove the following argument valid:

   a. We will not reduce our military budget or the Russians will surpass us as a nuclear power.

   b. If the red Chinese do not invade Russia, then Russia will not surpass us as a nuclear power.

   c. Therefore, if we reduce our military budget, the Red Chinese will invade Russia.

   Symbolize as follows:

   We will reduce our military budget - B
   The Russians will surpass us as a nuclear power - S
   The Red Chinese will invade Russia - I

4. Prove the following argument valid.

   a. If Jack studies hard, then he will pass his test.

   b. If Jack passes his test, then his parents will let hem use the car.

   c. If Jack uses the car, he will ask Sue for a date.

   d. Jack does not ask Sue for a date.

   e. Therefore Jack does not study hard.

   Symbolize as follows:

SELF-EVALUATION (cont')

Jack studies hard - H
He passes his test - P
His parents let him use the car - C
He asks Sue for a date - D

5. Prove the following argument valid.

   a. The world monetary crisis will be resolved or there will be a depression.

   b. If there is a depression, then 98% of the labor force will not have jobs.

   c. If the President is happy, the 98% of the labor force has jobs.

   d. The President is happy.

   e. Therefore, the world monetary crisis will be resolved.

   Symbolize as follows:

   The world monetary crisis will be resolved - r
   There will be a depression - d
   98% of the labor force will have jobs - j
   The President is happy - h

STOP! SEE YOUR TEACHER. A PROGRESS TEST IS SCHEDULED HERE.

..ementary Problems to Problem Set 2

The following problems give a total of 50 points toward depth study. These problems should not be attempted until all the problems in Problem Set I have been completed.

1. $o \rightarrow (p \rightarrow q)$ — 30 points toward depth study

   $(p \wedge q) \rightarrow (o \wedge r)$

   p

   Prove: $(\sim o \vee \sim q) \rightarrow (\sim o \wedge \sim q)$

2. $t \rightarrow w$ — 20 points toward depth study

   $(x \rightarrow \sim y) \rightarrow \sim w$

   Prove: $t \rightarrow x$

When you accumulate a total of 60 points toward depth study, you will receive a 6 depth study grade (it increases your average by .1)

70 points is equivalent to a 7 toward depth (increase .2)

80 points is equivalent to a 8 toward depth (increase .3)

90 points is equivalent to a 9 toward depth (increase .4)

After you have finished all the material up to this point, make an appointment with one of your instructors.

On the basis of the conference you will either take Quiz 1, or be recycled through additional materials. In either case, Quiz 1 <u>must be</u> taken before you continue on in this LAP.

II BEHAVIORAL OBJECTIVES

(from pages 24 to 31)

The student should be able to:

(a) Given the problem to perform an indirect proof, choose the statement to assume as true.

(b) Given a set of premises and a required conclusion, prove or disprove the conclusion, by using an indirect proof, either symbolically or in paragraph form.

INDIRECT METHOD OF PROOF

The indirect method of proof, as a logician thinks of it, is generally based upon the rule of contrapositive inference.

$$[\sim q \wedge (p \to q)] \to \sim p.$$

This type of proof is fundamental to all branches of mathematics.

> In an indirect proof of a statement A, it is customary to show that the assumption ~A leads to a "contradiction," so that ~A must be false and thus A is true. A contradiction is understood to be the denial of an axiom or of a previously proved theorem.

In addition, any statement known to be false because of its form alone can serve as a contradiction. Perhaps the simplest such form is "$p \wedge \sim p$," for whatever the truth value taken for p, the truth value of $p \wedge \sim p$ will clearly be false. Therefore, any statement in the form "$p \wedge \sim p$" is false, and can serve as the contradiction in an indirect argument.

We shall take as the basic form of an indirect proof of A, any argument that starts with ~A and leads to a statement which we know is false.

In carrying out an indirect proof, one must be careful to start the argument with the negation of the statement to be proved, or with some statement equivalent to it. In most cases, the statement, A, to be proved is a conditional of the form "$P \to Q$," so that the argument must start with

$$\sim(P \to Q)$$

which is equivalent to

$$P \wedge \sim Q.$$

Thus, if we want to prove

$P \rightarrow Q$: If Jane is smart, then she gets good grades,

we consider $\sim(P \rightarrow Q)$ or

$P \wedge \sim Q$: Jane is smart and she does not get good grades.

Now by conjunctive simplification we have

(1) Jane is smart

(2) Jane does not get good grades.

These last few remarks are very important.

Example 1: Given: (1) $w \rightarrow d$

(2) $p \rightarrow \sim d$

Prove: $w \rightarrow \sim p$

Proof: Since we're using an indirect proof we'll assume

$\sim(w \rightarrow \sim p)$.

True Statements	Reasons
1. $\sim(w \rightarrow \sim p)$	1. Given
2. $\sim(w \rightarrow \sim p) \Leftrightarrow (w \wedge p)$	2. Negation for Implication
3. $w \wedge p$	3. Definition of Equivalence
4. w	4. Conjunctive Simplification
5. p	5. Conjunctive Simplification
6. $w \rightarrow d$	6. Given
7. d	7. Modus Ponens (Steps 4 and 6)
8. $p \rightarrow \sim d$	8. Given
9. $\sim d$	9. Modus Ponens (Steps 5 and 8)
10. $d \wedge \sim d$	10. Conjunctive Inference (Steps 7 and 9)

At this point we may stop and write "Contradiction," since we know $d \wedge \sim d$ is always false. Our assumption has allowed us to prove a false statement, therefore $\sim(w \rightarrow \sim p)$ must be false and its negation true, but its negation is what we want, that is

$$w \rightarrow \sim p.$$

Hence, when we established $d \wedge \sim d$, the contradiction, we were basically finished with proof. Informally, we could say that all assertions we make in the left-hand column of our proof should be true. However, $d \wedge \sim d$ certainly is not true and therefore our assumption $\sim(w \rightarrow \sim p)$ is false, consequently $w \rightarrow \sim p$ is true. This was the statement to be proved.

<u>In Summary, once we arrive at a conjunction of the form $p \wedge \sim p$, we'll consider the proof to be finished</u>.

<u>Example 2</u>: Given: (1) $P \rightarrow q$

(2) $\sim R \rightarrow \sim q$

(3) $\sim R$

Prove: $\sim P$

Proof: Since we're using an indirect proof, we'll assume $\sim(\sim P)$.

True Statement	Reasons
1. $\sim(\sim P)$	1. Given
2. $\sim(\sim P) \Leftrightarrow P$	2. Double negation
3. P	3. Definition of equivalence
4. $P \rightarrow q$	4. Given
5. q	5. Modus Ponens 3, 4
6. $\sim R \rightarrow \sim q$	6. Given
7. $\sim R \rightarrow \sim q \Leftrightarrow q \rightarrow R$	7. Contrapositive
8. $q \rightarrow R$	8. Definition of equivalence

9.	R	9. Modus Ponens 5, 8
10.	$\sim R$	10. Given
11.	$R \wedge \sim R$	11. Conjunctive Inference

"Contradiction" therefore $\sim(\sim P)$ is false, so $\sim P$ must be true.

Logic will be a great aid to us in our future studies of mathematics and this is the primary purpose for devoting so much time to the subject. Our future proofs will not be as formal as the proofs we have presented so far. However, this is not to say they will be informal. We'll not show such great concern over the Rule of Syllogism, Modus Ponens, Conjunctive Inference, etc. Once you understand what really comprises a proof to fill in every rule of inference adds very little to your understanding of the argument. Proofs also become very cumbersome when written in all detail, however, you should realize that it is possible to furnish these details. If we encounter arguments or negations which are questionable we shall call upon our study of logic to help us through the difficulty.

Our proofs, up to this point, have been two-column proofs. The next example will demonstrate a paragraph proof. We'll not give every rule of inference, only the essential ones. In the future this is the type of proof we'll generally use. The requirements for such a proof will only come with experience.

Example 3: Use an indirect proof.

Given: (1) Jim is funny, then he laughs.

(2) Jim doesn't laugh or he doesn't cry.

Prove, P: If Jim is funny, then he does not cry.

Proof: Using an indirect proof we assume ~P: Jim is funny and he cries. Using Conjunctive Simplification we have,

(3) Jim is funny

(4) Jim cries.

From (1) and (3) and Modus Ponens we can assert

(5) Jim laughs.

Now, the disjunction in (2) is true, but the first disjunct is false by (5). Thus, the second disjunct must be true. Therefore, we conclude

(6) Jim doesn't cry.

Finally, (4) and (6) are a contradiction. Now the proof is finished.

Example 4: Use an indirect proof.

Given: (1) If lambs growl, then dogs bleat.

(2) If chickens sing, then dogs do not bleat.

Prove, P: If lambs growl, then chickens don't sing.

Proof: Since we're using an indirect proof we assume ~P or equivalently, "Lambs growl and chickens sing."

Thus, by Conjunctive Simplification, we know

(3) Lambs growl.

(4) Chickens sing.

Now, using (2) and (4) with Modus Ponens we can conclude

(5) Dogs do not bleat.

Taking the contrapositive of (1) we have "If dogs do not bleat, then lambs do not growl." By using (5) and Modus Ponens we assert that

(6) Lambs do not growl.

Noting (3) and (6) we have a contradiction and is complete.

Problem Set 3

Use an indirect proof, similar to Examples 3 and 4, for problems 1 - 6.

1. Given: (1) If Henry is bad he won't play baseball.

   (2) If it isn't raining, then Henry will play baseball.

   Prove: If Henry is bad, then it rains.

2. Given: (1) If John is tall and fast, then John is an athlete.

   (2) John is tall and not an athlete.

   Prove: John is not fast.

3. Given: (1) If $4 + x = 5$, then ($x = 1$ and $2x = 2$)

   (2) $4 + x = 5$ and $x = 1$

   Prove: $2x = 2$

4. Given: (1) If Joan is sick, then Joan is at home.

   (2) If Joan is at school, then Joan isn't home.

   Prove: If Joan is sick, then Joan isn't at school.

5. Given: (1) If a cat is hungry, then it drinks milk.

   (2) If a cat isn't hungry, then the cat sleeps.

   Prove: If a cat doesn't drink milk, then the cat sleeps.

6. Given: (1) Every integer is either odd, or else even. (Not both)

   (2) If an integer is odd, then its square is odd.

   (3) If an integer is even, then its square is even.

   Prove: If the square of an integer is even, then the integer itself is even.

7. Write a negation for each of the following:

a) Chicago is the Windy City.

b) John is not at his home.

c) All natural numbers are integers.

d) Some cats are black.

e) No irrational number is rational.

f) All courses overlap.

g) Some triangles are not isosceles.

h) $\forall x(4x + 9 = 6)$.

i) Some unpleasant statements are true.

j) $\sim[\exists x(6x^2 \neq 3)]$.

k) None of us is perfect.

l) No employees are dishonest.

8. Negate each of the following:

a) The bus stops at Bay Street or Water Street.

b) $\triangle$ ABC is isosceles or equilateral.

c) The pay telephone takes nickels, or dimes, or quarters.

d) If $a \neq b$ and $c \neq d$, then $a + c \neq b + d$.

e) If $a > b$ and $b > c$, then $a > c$.

f) If two triangles are congruent, their corresponding sides are congruent.

g) $a > b$.

h) If n is an even number, then n is divisible by 2.

i) $\exists_x(x + 4 = 6)$ and "$\frac{7}{8}$" is a fraction.

j) Sam is not at home and he is at church.

k) $a > b$ and $b < c$.

l) a is perpendicular to c and b is perpendicular to c, and a is not parallel to b.

9. "If $6x + 4 = 22$, then $x = 3$" is true, can you immediately conclude that 3 solves the equation "$6x + 4 = 22$"? Why or why not? Is "checking your answer" a logical necessity when you are solving equation? Why or why not?

10. "$6x + 4 = 22$ if and only if $x = 3$" is true. Do you have to "check" that $x = 3$ actually solves $6x + 4 = 22$? Why or why not?

At this point you have finished LAP 17 To prepare yourself for the Unit Test we have a student self-evaluation test on the next page. The answers are supplied in this package. Grade your test and then make an appointment with your instructor.

STUDENT SELF-EVALUATION TEST

I.

TRUE or FALSE

1. Given a set of premises and a valid conclusion following from those premises, there is only one correct method to prove that conclusion.

2. If "p" is a true statement and you are able to infer "q" is true, then "$p \rightarrow q$" is true by the Rule of Conditional Proof.

3. There are some tautologies which can never be used as a rule of inference.

4. In formalizing an indirect proof the law of contrapositive inference is most important.

5. The two connectives we have used to define the other connectives in logic have been "v" and "~."

II.

1. Given: (1) $(p \wedge q) \rightarrow r$

   (2) $p \wedge \sim r$

   Which of the following conclusions is true if the argument is valid:

   a) r

   b) $\sim p$

   c) q

   d) $\sim q$

   e) $\sim p \wedge \sim q$

2. Given: (1) If Paul doesn't slice, then he hooks.

   (2) If Paul hooks, then he is sad.

   (3) Paul isn't sad.

Which of the following conclusions is true if the argument is valid:

a) Paul hooks.

b) Paul is sad.

c) Paul slices.

d) Paul hooks and Paul slices.

e) None of the above.

3. Given: (1) Babies are illogical.

(2) Nobody is despised who can manage a crocadile.

(3) Illogical persons are despised.

Which of the following conclusions is true if the argument is valid:

a) Babies are not despised.

b) Illogical persons are not despised.

c) Babies can manage crocodiles.

d) Babies cannot manage crocodiles.

e) Babies are not illogical.

III.

Give a two-column, valid argument, if the conclusion which you are trying to prove is true. Otherwise give a counter-example. For problem 1 only give the missing reasons and missing statements.

1. Given: (1) $H \to D$

(2) $\sim H \to S$

Prove: $\sim D \to S$

Statement	Reasons
1. $\sim D$	1.
2.	2. Given
3.	3. Given
4. $\sim D \rightarrow \sim H$	4.
5.	5. Modus Ponens Step__and__
6. S	6.
7.	7.

2. Given: (1) $P \rightarrow q$

   (2) $R \rightarrow \sim q$

   Prove: $P \rightarrow \sim R$

3. Given: (1) $\sim(p \vee q)$

   (2) $R \rightarrow q$

   (3) $\sim R \rightarrow S$

   Prove: S

4. Given: (1) $p \rightarrow q$

   (2) $\sim q$

   Prove: $\sim p$

IV.

Give an indirect proof for the following:

1. Given: (1) $(p \wedge q) \rightarrow R$

   (2) $p \wedge \sim R$

   Prove: $\sim q$

L E A R N I N G

A C T I V I T Y

P A C K A G E

ABSOLUTE VALUE, INEQUALITIES, EXPONENTS, and COMPLEX NUMBERS

Ninety Six High School

Ninety Six, S.C.

Algebra 124

IEWED BY

LAP NUMBER 43

WRITTEN BY Bill Holland

RATIONALE

Much of the world of mathematics is a world of numbers, and in order to work with numbers effectively, we must know the rules that govern their use. You already have had a good deal of experience with the basic rules of algebra; in fact, you are probably so familiar with them that you apply them mechanically without thinking about them. For this reason, we are going to use the rules of algebra in an unfamiliar setting. We are going to use these rules to solve equations involving absolute values and also inequalities. Then we are going to undergo a thorough review of the Laws of Exponents and radicals. Finally we are going to study a set of numbers that allow us to determine the solution set of the relatively simple equation $x^2 + 1 = 0$: the set of Complex Numbers.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define the absolute value of a real number and answer questions and solve problems relating to this definition.
2. Determine the solution set of any given inequality.
3. Demonstrate your understanding of the laws of exponents by solving problems relating to these laws that have integral and/or rational exponents.
4. Take any expression written with a rational exponent and change it into an equivalent radical and vice-versa.
5. Simplify any given radical by applying the appropriate laws of exponents.
6. Change the order of any given radical.
7. Simplify any expression involving radicals with the same indices by performing the four fundamental operations on the expression as inidcated.

RESOURCES I

I. Readings:

1. White: Advanced Algebra - #1 ____; #2 p. 107; #3 pp. 3-4; #4 ____; #5 - #7 pp. 6-8.
2. Rees: Algebra and Trigonometry - #1 p. 14; #2 pp. 75-76, pp. 80-87, pp. 214-218; #3 pp. 98-100, pp. 101-105, pp. 106-108; #4 pp. 108-109; #5 pp. 110-112; #6 p. 112; #7 pp. 114-115.
3. Vance: Modern Algebra and Trigonometry - #1 pp. 76-77; #2 pp. 185-189; #3 pp. 59-61; #4 ____; #5 pp. 62-63; #6 ____; #7 pp. 64-66.

SECTION I

RESOURCES I (cont')

4. Fisher: Integrated Algebra and Trigonometry - #1 pp. 26-28; #2 pp. 22-25, pp. 29-31; #3-#7 pp. 32-39.

II. Problems:

1. White: Advanced Algebra - #1 ____; #2 pp. 108-109 ex. A(14-18), B(13-16); #3 p. 5 A(1-24) even numbered exercises; #4 ____; #5-#7 p. 9 A(1-20), B(1-10) even numbered exercises.
2. Rees: Algebra and Trigonometry - #1 ____; #2 p. 77 ex. 1-20 even numbered exercises; pp. 87-88 ex. 1-20 even numbered exercises; #3 pp. 100-101 ex. 1-44 even numbered exercises, pp. 105-106 ex. 1-44 even numbered exercises, p. 109 ex. 17-36 odd numbered exercises; #4 p. 109 ex. 1-16; #5 pp. 112-113 ex. 1-20, 45-48; #6 pp. 113-114 ex. 49-60; #7 p. 113 ex. 21-44, pp. 115-116 ex. 1-40 even numbered exercises.
3. Vance: Modern Algebra and Trigonometry - #1 pp. 77-78 ex. 6-7; #2 p. 188 ex. 1-20; #3 p. 60 ex. 1-12, pp. 61-62 ex. 1-28 even numbered exercises; #4 ____; #5 pp. 63-64 ex. 1-30 even numbered exercises; #6 ____; #7 pp. 64-65 ex. 1-20 even numbered exercises, pp. 66-67 ex. 1-41 even numbered exercises.
4. Fisher: Integrated Algebra and Trigonometry - #1 p. 29 ex. 1, 2, 4, 8; #2 pp. 25-26 ex. 2, pp. 30-31 ex. 1-3, 5-7; #3 - #7 pp. 35-36 ex. 1-5, pp. 39-40 ex. 1-6.

SELF-EVALUATION I

1. a. If m is any real number, define the absolute value of m.

   b. Solve the following equations for all real values of x:

   1) $|x^2| = -3$ 2) $|7x - 2| = 5x - 3$

2. Find the solution set of the following inequalities:

   a. $9 - 7x < 4$ b. $\frac{x - 1}{x + 2} < 4$ c. $|x - 9| \leq 5$

3. Simplify the following expressions as far as possible. Express your answer without negative or zero exponents:

   a. $\frac{3x^{-2}y}{27xy^{-5}}$ b. $\left(\frac{3^{\frac{1}{2}}a^5b^{\frac{7}{2}}c^{-\frac{3}{4}}}{9^{\frac{1}{4}}a^{-\frac{3}{2}}b^{-\frac{1}{2}}c^0}\right)^2$

4. a. Change the following rational expressions into radicals.

   1) $7^{\frac{1}{2}}$ 2) $\left(\frac{xy}{z}\right)^{\frac{3}{4}}$ 3) $x^{\frac{1}{2}}y^{\frac{1}{3}}z^{\frac{1}{4}}$

   b. Change the following radical expressions into expressions with rational exponents:

   a. $\sqrt{9}$ b. $\frac{\sqrt{x^2yz^{-2}}}{\sqrt{xyz^{-4}}}$ c. $\sqrt[4]{a^{-1}bc^{-5}}\ \sqrt[3]{a^2bc^4}$

5. Simplify the following as completely as possible:

   a) $\sqrt{54}$ b) $\sqrt[4]{m^5n^{-10}p^0}$ c) $ab^2\ \sqrt{b^2c^3}$

6. Reduce the following radicals to the lowest order possible then simplify:

   a) $\sqrt[4]{36a^6y^8}$ b) $\sqrt[12]{64x^{18}y^{14}}$

7. a. Combine the following radicals by simplifying:

   1) $\sqrt{48} + \sqrt{12} - \sqrt{75}$

   2) $a\sqrt[3]{x} + b\sqrt[3]{x^4} + c\sqrt[3]{x^{13}}$

   b. Determine the following products and quotients and simplify as completely as possible.

   1) $\sqrt{x} \cdot \sqrt{x^2y} \cdot \sqrt{xz^2}$

   2) $\sqrt{x^{-3}yz^{-1}} \div \sqrt{x^4y^{-5}z^2}$

   3) $(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})$

   4) $\sqrt[3]{3xy^{-5}z^{-2}} \div \sqrt[3]{81x^{-5}yz^4}$

   c. Rationalize the denominators in the following problems:

   1) $\dfrac{\sqrt{6} + 3}{\sqrt{7}}$

   2) $\dfrac{3x + 5}{6 + \sqrt{x}}$

8. Answer the following true or false:

   ________ a. $(3x)^2 = 9x^2$

   ________ b. $(3a)^3\ 5(2b)^2 = 540a^3b^2$

   ________ c. $\sqrt{3} + \sqrt{4} = \sqrt{7}$

   ________ d. $\dfrac{a^2}{b^2} = \left(\dfrac{a}{b}\right)^2$

   ________ e. $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$

   ________ f. $\dfrac{x^2 + y^2}{x^2 - y^2} = 1$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. The integers $a = 0$ and $b = 0$ satisfy the equation $ab = a + b$. Are there any others?

2. Given x and y are positive numbers. Show that $\sqrt{x + y} < \sqrt{x} + \sqrt{y}$.

3. Prove: If $\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$ and $a^2 = b^2 + c^2$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. Is 6 the only value of $(x^2 + 6x + 9)^{\frac{1}{2}} - (x^2 - 6x + 9)^{\frac{1}{2}}$? Justify your answer.

5. Rationalize the denominator in the fraction $\frac{1}{5 - \sqrt[3]{5}}$.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

8. Define the set of complex numbers and answer questions relating to this definition.

9. Simplify powers of the imaginary unit.

10. Define the complex conjugate of a complex number and solve problems relating to this definition.

11. Perform any of the four fundamental operations on any given pair of complex numbers.

12. Graph any complex number.

13. Define the modulus (absolute value) of any given complex number and solve problems relating to this definition.

RESOURCES II

I. Readings:

1. Vannatta: Advanced High School Mathematics - #8, #9, #11 pp. 23-24; #12, #13 pp. 25-28.

2. White: Advanced Algebra - #8 - #11 pp. 76-80; #12 pp. 83-84; #13 ____.

3. Rees: Algebra and Trigonometry - #8, #10, #11 pp. 315-318; #12 ____; #13 ____.

4. Vance: Modern Algebra and Trigonometry - #8, #11 pp. 163-164; #9 ____; #10 p. 166; #12 ____; #13 ____.

5. Fisher: Integrated Algebra and Trigonometry - #8, #9, #11 pp. 179-182; #10 pp. 183-185; #12 ____; #13 ____.

II. Problems:

1. Vannatta: Advanced High School Mathematics - #8, #9, #11 pp. 24-25 ex. 1-3, 20-22; #12, #13 p. 28 ex. 1-3.

2. White: Advanced Algebra - #8 - #11 pp. 78-79 ex. A(1-9), B(1-10), pp. 81-82 ex. A(1-16), B(1-10); #12 pp. 84-85 ex. 1-13, 14-15; #13 ____.

3. Rees: Algebra and Trigonometry - #8, #10, #11 pp. 319-320 ex. 1-52 even numbered exercises; #12 ____; #13 ____.

4. Vance: Modern Algebra and Trigonometry - #8, #11 p. 164 ex. 1-4, 6; #9 ____; #10 p. 167 ex. 1-6; #12 ____; #13 ____.

5. Fisher: Integrated Algebra and Trigonometry - #8, #9, #11 p. 183 ex. 1, 2, 4, 10; #10 pp. 185-186 ex. 1-4, 8, 9; #12, #13 ____.

SELF-EVALUATION II

1. a. What is a relationship between the set of complex numbers and the set of real numbers?

   b. Which of the following are not examples of complex numbers:

      1) 3 2) $-\pi$ 3) $(2,-5)$ 4) $3^5 - 2i$ 5) $\sqrt{17}$

2. Simplify the following:

   1) i^{19} b) $3 + 2i^{16}$ c) $9i^{37}$

3. If z_1 and z_2 are two complex numbers, show that $\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$.

4. Simplify the following:

   a) $\left[(3,2) - (8,9)\right] + \left[(-8,1)(1,-8)\right]$

   b) $\left[\dfrac{6i - 9}{2 - 3i}\right] - \left[(7i - 4)(6 + 5i)\right]$

   c) $\left[\dfrac{2 + \sqrt{-3}}{2 - \sqrt{-1}}\right] - \left[(\sqrt{-4} - 5)(2 - \sqrt{-9})\right]$

5. Graph the following complex numbers:

   a) $-4i$ b) $2 - \sqrt{-25}$ c) $(-5,1)$ d) $-3i + 2$

6. a. Define the modulus of a complex number.

   b. Determine the modulus of the following complex numbers:

      1) $-2 + 3i$ 2) $(-5,0)$ 3) $-6i$ 4) $\sqrt{-36} - 3$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR LAP TEST.

ADVANCED STUDY II

1. We say that the set of real numbers is ordered since for any two real numbers a and b we can determine whether $a < b$, $a > b$, or $a = b$. Is the set of complex numbers ordered? Justify your answer. If your answer was no, try to determine a way to set up an order relationship for the set of complex numbers.

2. Suppose z_1 and z_2 are two complex numbers such that $3z_1 + 5z_2 = 3 - 6i$ and $5z_1 - 3z_2 = 3i - 5$. Determine a value for z_1 and z_2.

3. Let z_1 and z_2 be two complex numbers that are not real. If z_1z_2 and $\frac{z_1}{z_2}$ are real numbers, what can you say about z_1 and z_2? Justify your answer.

REFERENCES

I. Textbooks:

1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).

2. White: Advanced Algebra (Allyn and Bacon, Inc., 1962).

3. Rees, Sparks: Algebra and Trigonometry, 2nd Edition, (McGraw-Hill, Inc., 1969).

4. Vance: Modern Algebra and Trigonometry, 2nd Edition, (Addison-Wesley Publishing Co., Inc., 1968).

5. Fisher, Ziebur: Integrated Algebra and Trigonometry, 2nd Edition, (Prentice-Hall, Inc., 1967).

LEARNING ACTIVITY PACKAGE

FUNCTIONS

$F(X) = X$

Ninety Six High School · Ninety Six, S.C.

ALGEBRA 124

LAP NUMBER 44

WRITTEN BY William Holland

REVIEWED BY

5372

3½

RATIONALE

Functions are one of the most important mathematical tools we have. They are used in Physics, Chemistry, and Biology. They are used in Statistics, all types of engineering, and in computer programming. But they are not restricted to use in the sciences. Such diverse fields as Economics and Music make extensive use of functions.

In this LAP you will analyze several different functions, study the relations between them, and some of their applied uses.

SECTION I: General Functions

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define a function and demonstrate your understanding of this definition by determining whether a given relation is a function.

2. Determine the complete domain and range of any given function.

3. Compute f(a) for any given function f and any real variable a.

4. Given two functions f and g be able to determine the domain, range, and a formula for:

   a. $f(x) + g(x)$
   b. $f(x) \cdot g(x)$
   c. $\frac{f(x)}{g(x)}$
   d. $-f(x)$
   e. $\frac{1}{f(x)}$

5. Sketch a graph of any given function for an indicated domain.

6. Determine the zeroes of any given function.

7. State and prove the distance formula, and apply this formula to solve problems relating to finding the distance between any two given points.

8. Define a direct variation relationship and solve problems involving direct variation.

9. Derive a formula that will enable you to determine the slope of a line and solve problems relating to this formula.

10. Determine the x and y intercepts of the graph of any linear function.

11. Determine the equation of a line given two points on the line or a point on the line and the slope of the line.

12. Define an inverse variation relationship and solve problems relating to inverse variation.

RESOURCES I

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #1, #2, #3, pp. 36-38; #4 ____; #5 pp. 38-39; #6 p. 54.; #7 pp. 39-41; #8 ____; #9 pp. 42-44; #10 pp. 45-46; #11 pp. 46-50; #12 ____.

2. Rees: Algebra and Trigonometry - #1, #2, #3 pp. 117-123; #4 ____; #5, #6 pp. 126-128; #7 p. 300; #8 pp. 291-293; #9 ____; #10 pp. 128-130; #11 ____; #12 pp. 291-293.

3. Fisher: Integrated Algebra and Trigonometry - #1, #2, #3 pp. 43-51; #4 ____; #5 pp. 58-61; #6 p. 69; #7 pp. 51-56; #8 pp. 62-64; #9, #10, #11 pp. 65-69; #12 pp. 70-72.

4. Vance: Modern Algebra and Trigonometry - #1, #2, #3 pp. 93-97; #4 pp. 99-100; #5 pp. 100-103; #6 p. 100, pp. 169-171; #7 pp. 85-86; #8 pp. 198-199; #9 ____; #10 ____; #11 ____; #12 pp. 198-199.

5. Dolciani: Modern Introductory Analysis - #1, #2, #3 pp. 222-223; #4 pp. 226-229; #5 ____; #6 ____; #7 pp. 167-169; #8 ____; #9, #10, #11 pp. 203-206; #12 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #1, #2, #3 p. 38 ex. 1-10; #4 ____; #5 p. 39 ex. 1-6; #6 p. 54 ex. 1-12; #7 p. 41 ex. 1-10; #8 ____; #9 pp. 44-45 ex. 1-10; #10 p. 46 ex. 1-10; #11 pp. 50-51 ex. 1-20; #12 ____.

2. Rees: Algebra and Trigonometry - #1, #2, #3 pp. 123-124 ex. (every third exercise); #4 ____; #5, #6 ex. 1-4, 5-24 (every third exercise); #7 ____; #8 p. 296 ex. 1-2, 4, 13, 15, 18, 20; #9 ____; #10 ____; #11 ____; #12 p. 296 ex. 14, 17, 19.

3. Fisher: Integrated Algebra and Trigonometry - #1, #2, #3 pp. 47-48 ex. 1-5, 9-10; pp. 50-51 ex. 1-10; #4 ____; #5 pp. 61-62 ex. 1-5, 7, 9(c,d,e); #6 ____; #7 p. 57 ex. 1-11; #8 pp. 64-65 ex. 1-10, 11-12; #9, #10, #11 pp. 69-70 ex. 1-4, 6-11; #12 pp. 73-74 ex. 1-4, 7-12.

4. Vance: Modern Algebra and Trigonometry - #1, #2, #3 pp. 97-98 ex. 1-8, 10-17; #4 p. 100 ex. 1-5; #5 pp. 103-104 ex. 1-11, 14, 16; #6 p. 171 ex. 1-4; #7 pp. 86-87 ex. 1-12, 14-15; #8 pp. 199-200 ex. 1-2, 5, 7, 9, 12, 13-14, 16; #9 ____; #10 ____; #11 ____; #12 pp. 199-200 ex. 4, 8, 10, 17, 20.

5. Dolciani: Modern Introductory Analysis - #1, #2, #3 pp. 224-225 ex. 1-34, 39-48 (all even numbered exercises); #4 pp. 229-230 ex. 1-20; #5 ____; #6 ____; #7 pp. 169-170 ex. 1-14, 19-24; #8 ____; #9, #10, #11 pp. 207-208 ex. 1, 3-6, 8-42(even numbered exercises); #12 ____.

(cont')

RESOURCES I (cont')

III. AUDIO:

1. Wollensak Teaching Tape C-3852: Graphing Linear Functions

2. Wollensak Teaching Tape C-3854: The Slope of a Line

3. Wollensak Teaching Tape C-3855: Slope Intercept Form

SELF-EVALUATION 1

1. a. Define a function?

   b. Determine whether the following relations are functions:

      (1) $\{(3,1), (4,2), (4,3), (5,4)\}$

      (2) $f(x) = \frac{3x - 9}{x - 5}$

      (3) $y \geq x + 5$

      (4) $f(x) = [x]$

2. Find the domain and range of the following functions.

   a. $f(x) = x$

   (b) $y = \frac{3x - 2}{x + 5}$

   (c) $f(x) = |x|$

   (d) $y = \frac{x}{x^2 - 4}$

3. a. (1) Given $f(x) = |x|$ Compute $f(0)$, $f(-1)$, and $f(-9)$

      (2) Given $f(x) = [x]$ Compute $f(1)$, $f(3\frac{1}{2})$, and $f(-5\frac{1}{2})$

      (3) Given $f(x) = \frac{7x - 9}{x + 3}$ Compute $f(-3)$, $f(0)$, and $f(5)$

   b. Given $f(x) = [x]$, which of the following relations hold for f?

      (1) $f(x^2) = (f(x))^2$ (2) $f(x + y) = f(x) = f(y)$

      (3) $f(4x) = 4f(x)$

4. a. If $f(x) = 2x - 7$ and $g(x) = \frac{7}{x}$, determine:

      1. $h(x)$ if $h(x) = f(x) + g(x)$
      2. $h(x)$ if $h(x) = \frac{1}{-g(x)}$

   b. What is domain and range of each of the functions determined in part a?

5. Sketch a graph of the following functions.

   (a) $f(x) = [x]$
   (b) $f(x) = -2x + 5$

6. Find the zeroes of the functions given in problem 5.

7. a. State and prove the distance formula.

   b. Find the distance between the following pairs of points.

      1) (0,3) and (-1,-2)

      2) (-2,-5) and (4,2)

8. a. What does it mean to say y is directly proportional to x.

   b. The point (1,-3) belongs to the graph of the equation $y = f(x)$ and y is directly proportional to x. Find the formula for $f(x)$.

   c. Given y is directly proportional to x and $y = f(x)$. Does $f(a+b) = f(a) + f(b)$ for any two numbers a and b. Justify your answer.

9. a. Find the slope of the line given by the equation:

      1) $f(x) = -\frac{1}{2}x - 6$

      2) $y = 11x + 17$

   b. Find the slope of the line passing through the given points.

      1) (-1,0) and (3,2)

      2) (4,3) and (1,-1)

10. Find the x and y intercepts of the graphs.

    a. $f(x) = -3x - 2$

    b. $f(x) = -x + 7$

11. a. Find the equation of the line passing through the points (3,1) and (-2,0).

    b. Find the equation of the line passing through the points (-1,4) with slope -3.

12. a. What does it mean to say x is inversely proportional to y?

    b. If y is inversely proportional to x and the graph of the equation $y = f(x)$ contains the point (-2,1), what is the formula for $f(x)$?

c. If y is inversely proportional to x and $y = f(x)$ does $f(ab) = a \cdot f(b)$ where a and b are any non-zero numbers? Justify your answer.

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES IN THIS SECTION, TAKE THE PROGRESS TEST.

ADVANCED STUDY I

1. Sketch a graph of the following relations:

   a) $f(x) = [\,x\,] + x$ $\qquad -4 \le x \le 4$

   b) $y^2 = 4 - x^2$

   c) $|y| = |x|$

   d) $y = 3[\,x\,] - 5$ $\qquad -5 \le x \le 5$

2. Let $f(x) = 7x - 3$ and $g(x) = -2x + 4$. Let functions p and q be defined by $p(x) = f(g(x))$ and $q(x) = g(f(x))$. What is a relationship between the graphs of p and of q?

3. Let $f(x)$ be a function that is defined to be the distance between x and the nearest even integer. Then $f(9) = 1$, $f(17\frac{1}{2}) = \frac{1}{2}$ and so on. Sketch a graph of $f(x)$ for x such that $-5 \le x \le 5$ and determine a formula for $f(x)$.

4. Let f be a function such that $f(a+b) = f(a) - f(b)$.

   a) Show $f(0) = 0$
   b) Show $f(1) = f(0)$
   c) What can you say about $f(b)$ if b is any real number? Justify your answer.

5. a) For the function $f(x) = x^4$, show $f(-x) = f(x)$. Such a function is called an even function. Give another example of such a function and show that it is an even function.

   b) For the function $f(x) = x^5$, show that $f(-x) = -f(x)$. Such a function is called an odd function. Give another example of such a function and show that it is an odd function.

6. For each of the following functions:

   a) Determine the domain and range of the function.

   b) Determine the inverse function.

   c) Determine the domain and range of the inverse function.

   d) Sketch a graph of each function and its inverse on the same set of coordinate axes.

      1) $f(x) = 3x - 7$

      2) $f(x) = \dfrac{x - 3}{x + 2}$

      3) $f(x) = x^2 - 9$

SECTION II: Exponential and Logarithmic Functions

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

13. Define an exponential function and solve problems relating to this definition.
14. Sketch the graph of a given exponential function for an indicated domain.
15. Define a logarithmic function and solve problems relating to this definition.
16. State and/or prove the following fundamental properties of logarithms and solve problems relating to these properties:

    a) If M and N are positive numbers and b is any positive base, then

    $$\text{Log}_b MN = \log_b M + \log_b N$$

    b) If N is a positive number, p is any real number, and b is any positive base, then

    $$\text{Log}_b N^p = p\text{Log}_b N$$

    c) If M and N are positive numbers and b is any positive base, then

    $$\text{Log}_b \frac{M}{N} = \text{Log}_b M - \text{Log}_b N$$

    d) If a and b are two positive bases, and N is any positive number, then

    $$\text{Log}_a N = \frac{\text{Log}_b N}{\text{Log}_b a}$$

17. Determine the characteristic and Mantissa of a common logarithm and use tables to solve problems relating to common logarithms.
18. Sketch the graph of any given logarithmic function for an indicated domain.

RESOURCES II

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #13 ____; #14 ____; #15, #16, #17 pp. 101-102; #18 ____.

2. Rees: Algebra and Trigonometry - #13 p. 377, #14 pp. 374-376; #15 pp. 354-355; #16 pp. 355-357, 373-374; #17 pp. 363-372, pp. 377-378; #18 pp. 374-376.

3. Fisher: Integrated Algebra and Trigonometry - #13, #14 pp. 77-80; #15 pp. 81-83; #16 pp. 84-86; #17 pp. 91-103; #18 pp. 87-90.

4. Vance: Modern Algebra and Trigonometry - #13, #14 pp. 305-306; #15, #16 pp. 313-316; #17 pp. 316-328; #18 pp. 313-314.

5. Dolciani: Modern Introductory Analysis - #13, #14 pp. 327-331; #15, #16 pp. 353-355; #17 pp. 356-362; #18 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #13 ____; #14 ____; #15, #16, #17 pp. 102-103 ex. 1-6.

2. Rees: Algebra and Trigonometry - #13 ____; #14 p. 377 ex. 25-28; #15 pp. 357-358 ex. 1-52 (every third exercise); #16 pp. 358-359 ex. 53-68, p. 377 ex. 5-16; #17 pp. 367-368 ex. 1-67(every third exercies), p. 369 ex. 1-20, 33-47 (even numbered exercises); pp. 372-373 ex. 1-64 (every fourth exercise), p. 379 ex. 1-28 (even numbered exercises); #18 p. 377 ex. 17-24.

3. Fisher: Integrated Algebra and Trigonometry - #13, #14 pp. 80-81 ex. 1-4, 7, 10; #15 pp. 83-84 ex. 1-3, 5-10, 12; #16 pp. 86-87 ex. 1-5, 9; #17 pp. 93-94 ex. 1-9, pp. 97-98 ex. 1(a-3), 2-9, p. 100 ex. 1-2, pp. 103-104 ex. 1-4; #18 pp. 90-91 ex. 1-2, 7-9.

4. Vance: Modern Algebra and Trigonometry - #13, #14 p. 306 ex. 1-6; #15, #16 p. 316 ex. 1-21, 26-35; #17 p. 318 ex. 1-24 (odd numbered exercises), pp. 319-320 ex. 1-20 (even numbered problems), p. 321 ex. 1-16 (even numbered exercises), pp. 324-325 ex. 1-8, pp. 327-328 1-23, 26-29 (odd numbered exercises); #18 p. 316 ex. 22-25.

5. Dolciani: Modern Introductory Analysis - #13, #14 pp. 332-333 ex. 1-14, 22-24; #15, #16 pp. 355-356 ex. 1-14, 31-34; #17 pp. 359-360 ex. 1-46 (even numbered exercises); #18 ____.

SELF-EVALUATION II

1. a. Define an exponential function.

   b. Find the bases of the exponential functions whose graphs contain the following points:

      1) (2,8)

      2) $(-2,\frac{1}{2})$

   c. Why is the base b of the exponential functions restricted to positive numbers?

2. Sketch the graphs of the following functions.

   a. $y = 2^x \qquad -3 \le x \le 3$

   b. $y = 5^x \qquad -2 \le x \le 2$

3. a. Define a logarithmic function?

   b. Write the following equations in logarithmic form:

      1. $9^2 = 81$

      2. $a^3 = B$

   c. Solve the following equations:

      1. $\log_3 x = 5$

      2. $\log_{16} 4 = x$

      3. $\log_x a = 0$

      4. $2^{\log_2 x} = 5$

4. a. Prove the following:

   If M and N are positive numbers and b is any base, then

   $\log_b M \cdot N = \log_b M + \log_b N$

   b. Given that $\log_b 3 = .7$ $\log_b 10 = 1.35$ and $\log_b 7 = .95$,

   find the number $\log_b \left(\frac{70}{3}\right)^{\frac{11}{4}}$

   c. Solve for x:

      1. $\log_2 (x + 1) + \log_2 (x - 1) = 3$

      2. $\log_{10} \sqrt{x} = 2$

5. a. Approximate the following logarithms:

   1. log 3.634

   2. log 7.675

   b. Find N if:

   1. log N = 2.1652

   2. log N = 1.3511

   c. Find the number x

   1. $10^x = 7.22$

   2. $10^x = .3976$

   d. Using logarithms solve the following problems.

   1. $\dfrac{632 \times 1.35}{4{,}611 \times .005}$

   2. $\left(2\frac{1}{3}\right)^{\frac{1}{5}}$

   e. Determine which number is larger, $25^{\frac{5}{4}}$ or $28^{\frac{4}{3}}$

   f. Solve for x

   1. $5^x = 17$

   2. $x^{-3} = 4$

   3. $3 \cdot 4^x = 5 \cdot 6^{2x}$

   4. $\log(x + 1) - \log(x - 1) = 1$

   g. The volume of a sphere is a given by the formula $V = \frac{4}{3}\pi r^3$ where V is the volume, r is the radius of the sphere and π is a constant whose approximate value is 3.14. if the volume of a sphere is 472 cubic inches, what is the radius of the sphere correct to three decimal places?

6. Sketch the graphs of the following functions:

   a. $y = \log_2 x$

   b. $y = \log_{10} 3x$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

ADVANCED STUDY II

1. Sketch a graph of $f(x) = 5^x$ for x such that $-5 \leq x \leq 5$, and a graph of $g(x) = \log x$ for x such that $0 < x \leq 5$ on the same set of coordinate axes. What relationship exists between these two functions.

2. Let $f(x) = \left|\log |x|\right|$

   a) Sketch the graph of f.
   b) Show that $f(ab) _ f(a) + f(b)$ for all real numbers a and b.
   c) Show that $f(x^p) = |p|\ f(x)$ for every real number p.

3. Determine all pairs of integers that satisfy the equation $\log(a + b) = \log a + \log b$.

4. Let f be an exponential function. Is the equation $\dfrac{f(x + .9) - f(x)}{.9}$ directly prooprtional to f(x)? Justify your answer.

5. Prove: If a, b, c are positive real numbers none of which is equal to one, the

$$\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$$

6. Solve for x in each of the following:

   a) $\ln 3.71 = x$
   b) $\ln 27.2 = x$
   c) $\ln .375 = x$
   d) $\ln 26.72 = x$

REFERENCES

I. TEXTBOOKS:

1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).
2. Rees and Sparks: Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).
3. Fisher and Ziebur: Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall,
4. Vance: Modern Algebra and Trigonometry, 2nd Edition (Addison-Wesley Publishing Co., 1968).
5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton: Modern Introductory Analysis (Houghton Mifflin Company, 1970).

II. AUDIO:

1. Wollensak Teaching Tape C-3852: Graphing Linear Functions
2. Wollensak Teaching Tape C-3854: The Slope of a Line
3. Wollensak Teaching Tape C-3855: Slope Intercept Form

LEARNING

ACTIVITY

PACKAGE

HIGHER DEGREE EQUATIONS AND THE DERIVATIVE

Ninety Six High School - Ninety Six, S.C. -

Algebra 124

.EWED BY

LAP NUMBER 45

WRITTEN BY Bill Holland

67 51072

RATIONALE

In previous math courses you have learned how to determine the roots of linear equations, quadratic equations and selected higher degree equations. In this LAP you will study methods that will enable you to determine whether a given equation has any real roots. If an equation has real roots and they are rational, you will learn a method to determine their value and if they are irrational, you will learn to approximate their value.

Also, in this LAP you will be introduced to the underlying foundations of the calculus - the limit and the derivative.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define a polynomial of degree n and solve problems relating to this definition.
2. Derive the quadratic formula and determine the roots of a second degree equation by factoring the equation or by using the quadratic formula.
3. Use the discriminant to determine the nature of the roots of a second degree equation.
4. Use the method of synthetic division to divide a first degree polynomial into any higher degree polynomial.
5. State and prove the Remainder Theorem and solve problems relating to this theorem.
6. State and prove the Factor Theorem and its converse and solve problems relating to these two theorems.
7. State the Fundamental Theorem of Algebra and solve problems relating to this theorem and its corollary.
8. State and prove the Rational Root Theorem and solve problems relating to this theorem.
9. Demonstrate your understanding of the statement of Descartes Rule of Signs by being able to determine the possible numbers of positive and negative roots of a given polynomial equation.
10. Isolate the real roots of any given polynomial equation between two consecutive integers.
11. Determine integral upper and lower limits (bounds) for the real roots of any given polynomial equation.
12. Solve problems involving imaginary roots of polynomial equations.

RESOURCES I

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 ____; #2 p. 54; #3 ____; #4 pp. 54-56; #5, #6 pp. 56-59; #7 pp. 59-60; #8 pp. 60-62; #9 pp. 62-63; #10, #11 pp. 64-67; #12 pp. 67-78.

2. Rees: Algebra and Trigonometry - #1 p. 11; #2 pp. 191-193; #3 pp. 206-207; #4 pp. 334-336; #5 pp. 332-333; #6 pp. 333-334; #7 p. 340; #8 pp. 345-348; #9 ____; #10 p. 343; #11 pp. 341-343; #12 ____.

3. Fisher: Integrated Algebra and Trigonometry - #1 p. 198; #2, #3 pp. 206-210; #4 pp. 200-201; #5, #6 pp. 217-218; #7 pp. 220-221; #8 pp. 227-230; #9 ____; #10 pp. 231-232; #11 ____; #12 pp. 223-226.

4. Vance: Modern Algebra and Trigonometry - #1 p. 246; #2 pp. 49-50, pp. 179-183; #3 p. 192; #4 pp. 45-46; #5, #6 pp. 246-248; #7 pp. 250-251; #8 pp. 253-256; #9 ____; #10 ____; #11 p. 249; #12 pp. 260-261.

5. Dolciani: Modern Introductory Analysis - #1 pp. 230-231; #2 pp. 267-270; #3 p. 276; #4 p. 236; #5 p. 235; #6 pp. 238-240; #7 pp. 269-270; #8 pp. 241-243; #9 pp. 245-246; #10 pp. 289-291; #11 pp. 244-245; #12 pp. 275-276.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #1 ____; #2 p. 54 ex. 4-12; #3 ____; #4 p. 56 ex. 1-8; #5 p. 59 ex. 1-10; #6 p. 59 ex. 11-20; #7 ____; #8 p. 62 ex. 1-10; #9 p. 63 ex. 1-10; #10, #11 p. 67 ex. 1-12; #12 pp. 68-69 ex. 1-10.

2. Rees: Algebra and Trigonometry - #1 ____ #2 pp. 198-199 ex. 1-12, 25-48 (every third exercise); #3 ex. 1-20 (even numbered exercises); #4 p. 337-338 ex. 25-40; #5, #6 pp. 336-337 ex. 1-24; #7 ____; #8 pp. 348-349 ex. 1-32 (even numbered exercises); #9 ____; #10, #11 p. 344 ex. 21-32; #12 ____.

3. Fisher: Integrated Algebra and Trigonometry - #1 p. 201 ex. 1-4; #2, #3 pp. 210-211 ex. 1, 3, 4, 5, 11; #4 p. 202 ex. 8; #5, #6 p. 219 ex. 1-3, 6-8; #7 pp. 22-223 ex. 1-2, 6,10; #8 p. 230 ex. 1-3; #9 ____; #10 p. 233 ex. 1-2; #11 ____; #12 p. 226-227 ex. 1-3,5,6.

4. Vance: Modern Algebra and Trigonometry - #1 ____; #2 p. 50 ex. 3, 11, 12, 23, 24, 26, 27, 29, 30, 37, 38, 40-44, pp. 183-184 ex. 1-4, 7-9, 13, 19, 21; #3 ____; #4 pp. 46-47 ex. 1-12(even numbered exercises); #5, #6 ex. 1-14; #7 p. 251 ex. 1-5; #8 p. 256 ex. 1-14 (even numbered exercises); #9 ____; #10 ____; #11 p. 251 ex. 1-15; #12 p. 262 ex. 17-26.

RESOURCES I (cont')

5. Dolciani: Modern Introductory Analysis - #1 p. 232 ex. 1-6; #2 ex. 1-5, 6-8; #3 ____; #4 p. 237 ex. 9-14; #5 p. 237 ex. 23; #6 p. 240 ex. 1-8, 13-14; #7 ____; #8 pp. 243-244 ex. 1-10; #9, #11 p. 246 ex. 1-12; #10 pp. 291-292 ex. 1-8; #12 pp. 276-277 ex. 1-8.

SELF-EVALUATION I

1. a. Define a polynomial of degree n.

   b. For each of the following polynomials state the degree of the polynomial and tell the constant term.

      1. $9x^5 - 7x^3 + 3x - 5 = 0$

      2. $7 - 9x + 16x^3 = 0$

      3. $8t^4 - 7t^2 - 5t = 0$

2. Determine the roots of the following second degree equation.

   a. $x - 7 + \frac{12}{x} = 0$

   b. $-3x^2 - 5x - 1 = 0$

   c. $\frac{3}{x - 1} + 7x = -2$

3. What is the nature of the roots in the following equations.

   a. $x^2 - x - 2 = 0$

   b. $\frac{x - 2}{x - 1} + 5x - 7 = 0$

4. Simplify the following using synthetic division:

   $(4x^5 + 1) \div (x + 3)$

5. Use the Remainder Theorem to determine P(r) for the given values of P(x) and r.

   a) $P(x) = x^4 - 3x^2 + 6x - 8 \quad r = 2$

   b) $P(x) = x^{17} + 1 \quad r = 1$

6. a. State and prove the Factor Theorem.

   b. Use the Factor Theorem to show that $x - a$ is a factor of $x^{17} - a^{17}$.

7. Form the equation which has $(x - 3)$, $(x - 4)$, and $(x + 1)$ as linear factors.

8. Find the rational roots of the following equations:

   a. $x^3 + 5x^2 + x - 7 = 0$

   b. $12x^3 + 16x^2 - 7x - 6 = 0$

SELF-EVALUATION I (cont')

9. Determine the maximum and minimum number of positive and negative roots of the following equations.

   a) $5x^3 - 7x^2 + 6x - 9 = 0$

   b) $10x^5 - x^3 - x^2 - 5x + 1 = 0$

10. Isolate the real roots of the following equation between two consecutive integers.

   a) $x^3 - 3x^2 - x + 2 = 0$

   b) $x^4 - 2x^3 - 10x^2 - 8x + 4 = 0$

11. For the equations listed in exercise 10, determine the least upper limit and the greatest lower limit of the real roots.

12. a. Must the equation $ax^3 + bx^2 + cx + d = 0$ have any real roots if a, b, c, and d are all real numbers? Justify your answer.

   b. Form the cubic equation two of whose roots are 3 and 1ti.

If you have mastered all the Behavioral Objectives, take your Progress Test.

ADVANCED STUDY I

1. For the following equations, determine all values of k for which the solutions are real numbers.

   a. $2x^2 + 2kx + 10 = 0$

   b. $3kx^2 - 6x + 9k = 0$

2. Suppose $P(x) = ax^2 + bx + c$ where $a < 0$ and b and c are real numbers. What conditions are necessary for both roots to be positive? Both roots to be negative? Both roots to be of opposite signs.

3. Find a quadratic polynomial whose zeroes are the cubes of the zeroes of $x^2 + 7x - 9$.

4. The square of twice a certain number is larger than the sum of the number and 1. Which numbers possess this property?

5. Solve the equation $4^{3X} - 2^{3X + 1} + 1 = 0$.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

13. Determine the slope of a line tangent to a curve at a point on the curve.
14. Determine the equation of a line tangent to a curve at a point on the line.
15. Determine the derivative of a given polynomial.
16. Find the extreme points of a given polynomial and determine whether they are maxima or minima.
17. Sketch a graph of any given equation for an indicated domain.
18. Approximate a real root of a polynomial equation to a specified degree of accuracy.
19. Use the method of iteration to determine a real root of a given polynomial equation.
20. Determine the given number of roots of any given real numbers.

RESOURCES II

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #13 pp. 69-72; #14 pp. 72-73; #15 ____; #16 pp. 73-76; #17 pp. 77-78; #18 pp. 79-80; #19 pp. 81-84; #20 pp. 85-86.
2. Rees: Algebra and Trigonometry - #17 pp. 338-339.
3. Fisher: Integrated Algebra and Trigonometry - #17 pp. 209-210.
4. Vance: Modern Algebra and Trigonometry - #17 pp. 251-252.
5. White: Advanced Algebra - #13, #14 pp. 268-272; #15 pp. 272-274; #16 pp. 275-280, 283-287; #17 ____; #18 pp. 294-295; #19 ____; #20 ____.

RESOURCES II (cont')

6. Dolciani: Modern Introductory Analysis - #13, #14 pp. 293-295, 297-298; #15 pp. 300-302; #16, #17 pp. 304-307; #18 ____; #19 pp. 313-316; #20 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #13 p. 72 ex. 1-10; #14 p. 73 ex. 1-8; #15 ____; #16 pp. 76-77 ex. 1-6; #17 p. 79 ex. 1-10; #18 p. 80 ex. 1-8; #19 p. 84 ex. 1-6; #20 p. 86 ex. 1-4.

2. Rees: Algebra and Trigonometry - #17 p. 344 ex. 1-12 (even numbered exercises).

3. Fisher: Integrated Algebra and Trigonometry - #17 p.210 ex. 9.

4. Vance: Modern Algebra and Trigonometry - #17 p. 253 ex. 1-15 (odd numbered exercises).

5. White: Advanced Algebra - #13, #14 p. 272 ex. 1-6; #15 ____; #16 p. 280 ex. 1-3, p. 287 ex. 1-6; #17 ____; #18 p. 295 ex. 1-3; #19 ____; #20 ____.

6. Dolciani: Modern Introductory Analysis - #13, #14 p. 296 ex. 1-12, 15-16, p. 299 ex. 1-8 (part b only), 9-12; #15 pp. 302-303 ex. 1-16; #16, #17 p. 307 ex. 1-12 (parts a, b, and d only); #18 ____; #19 p. 316 ex. 7-12; #20.

SELF-EVALUATION II

1. Determine the slope of a line tangent to the graph of the curve at a point with given abscissa:

   a. $y = x^2 - 9x + 3$ abscissa = 2

   b. $y = 2x^3 - 7x$ abscissa =-1

2. Determine the equation of the line tangent to the graph of the equation $y = x^3 - 7x + 5$ at the point $(1,-1)$.

3. a. Determine the derivatives of the following polynomials:

      1. $y = 7x - 11$

      2. $y = 3x^2 - 10$

   b. Determine the derivatives of the polynomials you found in part a.

4. Determine the coordinates of the maxima and minima of the equation $y = x^3 - 3x - 1$.

5. Sketch a graph of the equation $y = x^3 - 6x - 4$ for the domain $-3 \leq x \leq 3$.

6. Find the value of a root of the equation $y = x^3 + 4x - 1$ correct to one decimal place.

7. Use the method of iteration to approximate a root of the equation $y - 3x^3 - 4x^2 - 7$ to three decimal places.

8. Find the three cube roots of 2 graphically.

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

ADVANCED STUDY II

1. In a 220-volt circuit having a resistance of 20 ohms, the power W in watts when a current I is flowing is given by $W = 220I - 20I^2$. Determine the maximum power that can be delivered by this circuit.

2. What is the minimum velocity of a particle whose velocity with respect to time is given by the equation $y = t(t - 3)$?

3. Determine the derivatives of the following equations with respect to x.

   a) $y = \sqrt[3]{x^3 + 5x^2 - 7x + 2}$

   b) $y = (x^3 - 9x^2 + 5x)(3x^4 - 7x^2 - 8x + 6)$

   c) $y = \dfrac{2x^2 - 8}{7x - 5}$

4. Evaluate each of the following integrals.

   a) $\int \frac{1}{2} dx$

   b) $\int (7x - 2)dx$

   c) $\int 2x(3 - 4x^2)dx$

   d) $\int \sqrt{x^3}\, dx$

   e) $\int \dfrac{x - 1}{x^5}\, dx$

   f) $\int \dfrac{1}{\sqrt{7x - 9}}\, dx$

5. The slope of a tangent line to a curve at the point $(-1,3)$ is $2x^2 - 5x$. Determine the equation of this curve. Also, write the equations of the tangent and normal line to this curve at the point with abscissa 4.

RESOURCES

I. Textbooks:

Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).

Rees and Sparks: Algebra and Trigonometry (McGraw-Hill, Inc., 1969).

Fisher and Ziebur: Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).

Vance: Modern Algebra and Trigonometry (Addison-Wesley Publishing Company, Inc., 1968).

White: Advanced Algebra (Allyn and Bacon, Inc., 1962).

Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton: Modern Introductory Analysis (Houghton Mifflin Company, 1970).

LEARNING

ACTIVITY

PACKAGE

THE TRIGONOMETRIC FUNCTIONS

Ninety Six High School

Ninety Six, S.C.

Algebra 124

WED BY

LAP NUMBER 46

WRITTEN BY Bill Holland

80 51572 3½

RATIONALE

Trigonometry is perhaps the most easily applied branch of mathematics studied on the secondary level. In short order, you will be able to easily solve problems that without trigonometry would be extremely challenging or impossible; problems dealing with subjects ranging from civil engineering to ballistics, from biology to automotive engineering.

In this LAP you will be introduced to the trigonometric functions and will learn to perform basic trigonometric analysis.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Define the following functions* and solve problems relating to these definitions:

   a) sine
   b) cosine
   c) tangent
   d) cotangent
   e) secant
   f) cosecant

2. Write the value of any function of the following angles without using tables: 0^0, 30^0, 45^0, 60^0, 90^0.
3. Use tables to determine the value of a function of a given acute angle.
4. State and prove any cofunction relation and solve problems relating to these relations.
5. State and prove any reciprocal relation and solve problems relating to these relations.
6. State and prove any quotient (ratio) relation and solve problems relating to these relations.
7. State and prove any Pythagorean relation and solve problems relating to these relations.

* Unless otherwise stated, the word function in this LAP refers only to trigonometric functions.

RESOURCES I

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #1 pp. 106-109, 111-112; #2 pp. 109-111; #3 pp. 113-114; #4 p. 115; #5 p. 116; #6, #7 pp. 117-119.

2. Hooper: A Modern Course in Trigonometry - #1 pp. 6-8, 14-15, 16-17, 18-19, 20-21, 24; #2 pp. 25-26; #3 pp. 9-10, 12-13, 15, 19; #4 - #7 pp. 21-22, 102-105.

3. White: Advanced Algebra - #1 pp. 18-21, 26-29; #2 p. 30; #3 ____; #4 p. 27; #5 - #7 pp. 36-39.

4. Rees: Algebra and Trigonometry - #1 pp. 141-142, 147-154; #2 pp. 155-158; #3 pp. 173-175; #4 ____; #5 - #7 pp. 160-162.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #1 p. 109 ex. 1-6, p. 112 ex. 3-8; #2 p. 111 ex. 1-8; #3 pp. 114-115 ex. 1 - 18; #4 pp. 115-115 ex. 1-8; #5 p. 116 ex. 1-13; #6 - #7 p. 119 ex. 1-12.

2. Hooper: A Modern Course in Trigonometry - #1 p. 24 ex.1-12; #2 pp. 27-28 ex. 1-33 (odd numbered exercises); #3 p. 11 ex. 1-2, p. 14 ex. 1-10, p. 16 ex. 1-3 p. 17 ex. 2-6, p. 20 ex. 2-5; #4-7 p. 23 ex. 12-45 (even numbered exercises), p. 106 ex. 1-4.

3. White: Advanced Algebra - #4 - #7 p. 42 ex. 1-10.

4. Rees: Algebra and Trigonometry - #1 pp. 154 ex. 11, 13, 21, 25, 29, 32-33, 40; #2 p. 159 ex. 26-27, 29-32; #3 pp. 175-176 ex. 1-8, 13-24, 29-36; #4 ____; #5 - #7 ____.

III. AUDIO:

1. Wollensak Teaching Tape C-3711: The Sine Function

2. Wollensak Teaching Tape C-3712: The Cosine Function

3. Wollensak Teaching Tape C-3713: The Tangent Function

IV. VISUAL:

Filmstrip 514: Introductory Trigonometry

SELF-EVALUATION I

1. a) Define the following functions:

   1. tangent

   2. secant

   3. sine

   b) 1. Given that (3,4) is a point on the terminal side of angle α, find the value of the six functions of α.

   2. If $\sin \theta = \frac{2}{3}$, then what are the values of the other functions of θ?

2. Find the following without using tables:

   a. $\sin 30^{0}$

   b. $\tan 90^{0}$

   c. $\frac{\sec 0^{0} + \cos 90^{0}}{\tan 30^{0} \cdot \csc 45^{0}}$

   d. $\frac{\tan 60^{0} + \cot 30^{0}}{1 - \sin 60^{0} \cdot \sin 30^{0}}$

3. a. Evaluate the following using tables:

   1. $\sin 40^{0}30' =$

   2. $\tan 53^{0}13' =$

   3. $\cos 73^{0}49' =$

   b. Find the following angles from the given numerical value.

   1. $\cot x = 1.4019$

   2. $\cos x = .8066$

   3. $\sin x = .7465$

4. a. If $\sin 39^0 = .6293$, then $\cos 51^0 =$

   b. If $\tan 21^0 = .3839$, then $\cot 69^0 =$

5. a. If $\sin A = \frac{\sqrt{3}}{2}$, then $\csc A =$

   b. If $\tan B = \frac{1}{2}$, then $\cot B =$

6. a. Prove $\tan \Theta = \frac{\sin \Theta}{\cos \Theta}$

   b. If $\cos A = .2$ and $\sin A = .5$, then $\cot A =$

7. a. Prove $\sin^2 \Theta + \cos^2 \Theta = 1$

   b. If $\cos A = \frac{2}{3}$, then $\sin A =$

   c. If $\csc T = 1$, then $\cot T =$

If you have mastered all the Behavioral Objectives, take the Progress Test.

ADVANCED STUDY I

1. Evaluate the following using logarithms:

   a) $\sqrt{\sin 33^{\circ}20' \tan 57^{\circ}40'}$

   b) $(\cos 39^{\circ}10')^{\frac{4}{3}} \cdot (\sin 77^{\circ}40')^{\frac{5}{8}}$

   c) $\left(\dfrac{\tan 37^{\circ}40' \cot 49^{\circ}30'}{\sin 23^{\circ}50' \cos 88^{\circ}10'}\right)^{\frac{1}{2}}$

   d) $(\sec 27^{\circ}40')^{8} \cdot (\csc 78^{\circ}50')^{9}$

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

8. Define a radian and solve problems involving radians and degrees.

9. Determine the value of a positive or negative angle of any size.

10. a. Determine the values of the functions of the following quadrantal angles without using tables: 0^o, 90^o, 180^o, 270^o, 360^o.

    b. Prove and/or apply statements relating to functions of multiples of the quadrantal angles given above.

11. State and prove formulas relating to the sum or difference of two angles and solve problems relating to these formulas: e. g. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

12. State and prove formulas relating to double angles and half angles and solve problems relating to these formulas: e.g. $\sin 2\Theta = 2\sin\Theta\cos\Theta$, $\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$.

13. State and prove formulas that transform a sum or difference of functions into a product of functions and solve problems relating to these formulas: e.g. $\sin x + \sin y = 2\sin\frac{x + y}{2}\cos\frac{x - y}{2}$.

RESOURCES II

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #8 pps. 123-124; #9 pp. 126-130; #10 p. 131; #11 pp. 133-135; #12 pp. 136-138; #13 p. 139.

2. Hooper: A Modern Course in Trigonometry - #8 pp. 70-73; #9 pp. 36-49; #10 pp. 53-55; #11 pp. 183-186; #12 pp. 187-190; #13 pp. 192-195.

3. White: Advanced Algebra - #8 pp. 22-23; #9, #10 pp. 30-34; #11 pp. 63-69; #12 pp. 70-73; #13 ____.

4. Rees: Algebra and Trigonometry - #8 pp. 142-144; #9 pp. 168-172; #10 pp. 157-158; #11, #12 pp. 300-313; #13 ____.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #8 pp. 124-125 ex. 1-6; #9 p. 130 ex. 1-12, 23-31; #10 pp. 131-132 ex. 1-5; #11 pp. 135-136 ex. 1-8; #12 p. 138 ex. 1-9; #13 pp. 139-140 ex. 1-10.

2. Hooper: A Modern Course in Trigonometry - #8 pp. 73-74 ex. 1-39; #9 p. 44 ex. 1-5, pp. 45-46 ex. 1-2, p. 48 ex. 1-24, p. 50 ex. 1-22, p. 74 ex. 56-70; #10 p. 55 ex. 1-10; #11 p. 74 ex. 40-55, pp. 184-185 ex. 1-10, 15-16, 26-29, pp. 186-187 ex. 1-10, 15-16; #12 p. 188 ex. 1, 5-9, p. 191 ex. 1-4, 6-7, 9, 11, 13; #13 p. 193 ex. 1-16, pp. 195-196 ex. 1-16, 23-26.

3. White: Advanced Algebra - #8 p. 25 ex. 1-27; #9, #10 pp. 35-36 ex. A (1,2,4), B (1-3); #11 p. 35 ex. 3 pp. 69-70 ex. 1-15; #12 p. 73 ex. 1-14; #13 ____.

4. Rees: Algebra and Trigonometry - #8 pp. 144-145 ex. 1-28; #9 p. 172 ex. 1-32; #10 pp. 158-159 ex. 1-28; #11, #12 pp. 303-304 ex. 1-8, 13-16, p. 307 ex. 1-8, 13-16, pp. 311-312 ex. 1-8, 13-16; #13 ____.

SELF-EVALUATION II

1. a. Define a radian.

   b. Change the following degrees to radians.

   1. 61^{0} 2. 153^{0} 3. 333^{0}

   c. Change the following radians to degrees.

   1. $\frac{\pi}{9}$ 2. $\frac{7\pi}{10}$ 3. 1.9

2. a. Prove: If $A > 0$, then $\cos(-A) = \cos A$.

   b. Find the value of the following:

   1. $\sin 334^{0}$
   2. $\tan 179^{0}$
   3. $\csc(-327^{0})$
   4. $\sec(-100^{0})$
   5. $\cot 385^{0}$
   6. $\cos(-450^{0})$

3. a. Write the value of the following without using tables:

   1. $\sin 0 \cdot \cos 270^{0} - \sec 180^{0} \cdot \tan 0^{0}$
   2. $\cot 90^{0} \cdot \sin 360^{0} + \csc 270^{0} \cdot \cos 0^{0}$

   b. Prove: $\sin(k \cdot 90^{0}) = 0$ if k is an even integer.

   c. For what values of k does $\cos(k \cdot 90^{0}) = 0$?

4. a. Prove: $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

   b. If $\sin x = \frac{2}{3}$ and $\cos y = \frac{4}{5}$, then $\sin(x + y) =$

   c. Find $\cos \frac{5\pi}{12}$ without using the tables.

   d. Evaluate $\tan(\pi + \frac{\pi}{4})$ without using tables.

5. a. Prove $\tan 2x = \dfrac{2 \tan x}{1 - \tan x}$.

   b. Find $\tan 22^{0}30'$ without using tables.

   c. If x is acute and $\cos x = \frac{2}{3}$, find the value of cos 2x. What quadrant does the terminal side of angle 2x lie in?

6.a)Prove $\sin x + \sin y = 2 \sin \dfrac{x + y}{2} \cdot \dfrac{x - y}{2}$.

b)Express $\cos 21^{0} - \cos 15^{0}$ as a product of functions.

c)Express $- 2 \sin 6x \sin 3x$ as a sum of functions.

7. Mark the following true or false.

______ a) $\tan (x + y) = \tan x + \tan y$

______ b) $2 \cos \frac{x}{2} = \cos x$

______ c) $\tan (-x) = -\tan x$

______ d) $\sin 2x - 2 \sin x$

______ e) $2 \tan \frac{x}{2} = \tan x$

______ f) $2 \left(\frac{\sin x}{2}\right) = \sin x$

______ g) $\sin x + \sin y = \sin (x + y)$

______ h) $\cos x = \sin (90^{0} - x)$

______ i) $\tan 2x = 2 \tan x$

______ j) $\cos (-x) = -\cos x$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST.

ADVANCED STUDY II

1. Describe the variation of the functions sin x, cos x, tan x, cot x, sec x, and csc x as x varies in the following manner:

   a) $0^0 \leq x \leq 90^0$

   b) $90^0 < x \leq 180^0$

   c) $180^0 < x \leq 270^0$

   d) $270^0 < x \leq 360^0$

2. Determine all values of x such that $0^0 \leq x \leq 360^0$ and for which the following relations hold:

   a) $\tan(x + y) - \tan x + \tan y$

   b) $\cos(x + y) = \cos x + \cos y$

   c) $\sin(x + y) = \sin x + \sin y$

3. Work the following problems:

   a) A plane is 2000 ft above the sea when it is 5 miles from the shore. Then it climbs steadily at an angle of 15^0 with the horizontal, flying in a straight line toward the shore. What height above sea level, to the nearest foot, will its altimeter record as it passes over the coast?

   b) To determine the width of a river, a spot directly opposite a tree on the farther bank is chosen on a straight stretch of the river. An observer then walks 50 yards along the bank and finds that the angle between the bank and the direction of the tree is 32^0. To the nearest foot, how wide is the river at the point where the tree stands?

   c) If a man 5 feet 8 inches tall casts a shadow 20 feet long, what is the angle of elevation of the sun?

SECTION III

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

14. Verify identities and/or prove statements relating to the identities given in objectives 4 - 7 and the formulas given in objectives 11 - 13.
    (a)
    e.g. Develop a formula for sin 3A in terms of sin A and cos A.

    (b) $\frac{2 \cot x}{1 + \cot^2 x} = \sin 2x$

RESOURCES III

I. READINGS:

1. Vannatta: Advanced High School Mathematics - #14 pp. 119-121, pp. 140-141.
2. Hooper: A Modern Course in Trigonometry - #14 pp. 107-108.
3. White: Advanced Algebra - #14 pp. 40-41, pp. 73-74.
4. Rees: Algebra and Trigonometry - #14 p. 162, pp. 163-165, p. 313.
5. Vance: Modern Algebra and Trigonometry - #14 pp. 124-126.

II. PROBLEMS:

1. Vannatta: Advanced High School Mathematics - #14 pp. 122-123 ex. 1-30, p. 136 ex. 9-12, p. 138 ex. 10-15, p. 140 ex. 11-15, p. 141 ex. 1-14.
2. Hooper: A Modern Course in Trigonometry - #14 pp. 108-109 ex. 1-28, p. 185 ex. 17-25, p. 187 ex. 11-16, pp. 188-189 ex. 10, 22-34, pp. 191-192 ex. 5, 8, 10, 12, 14-20, pp. 195-196 ex. 17-22, 31-32.
3. White: Advanced Algebra - #14 p. 42 ex. 11-30, p. 75 ex. 1-13.
4. Rees: Algebra and Trigonometry - #14 pp. 162-163 ex. 1-24, pp. 165-166 ex. 1-28.
5. Vance: Modern Algebra and Trigonometry - #14 pp. 126-127 ex. 7-50 (odd numbered exercises), pp. 133-134 ex. 18-26, 28-32, pp. 140-141 ex. 12-30, p. 143 ex. 17-24.

SELF-EVALUATION III

1. Derive a formula for csc (x + y) in terms of csc x, csc y, sec x, and sec y.

2. Show that $\sin(45^\circ + x) - \sin(45^\circ - x) = \sqrt{2}\sin x$.

3. Verify the following identities:

   a) $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$

   b) $\dfrac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$

   c) $\cot^2 A = \dfrac{\cos A}{\sec A - \cos A}$

   d) $\dfrac{\sin 3B - \sin B}{\cos^2 B - \sin^2 B} = 2 \sin B$

   e) $\dfrac{\sin x}{1 + \cos x} + \dfrac{\cos x}{\sin x} = \sec x$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES IN THE LAP, TAKE THE LAP TEST.

ADVANCED STUDY III

1. Derive a formula for sin 5A in terms of A.

2. Verify any of the following identities:

   a) $\frac{\cot^3 x}{1 + \cot^2 x} + \frac{\tan^3 x}{1 + \tan^2 x} = \frac{\cos^4 x + \sin^4 x}{\sin x \cos x}$

   b) $\sin^3 x - \cos^3 x = \sin x\,(1 + \sin x \cos x) - \cos x\,(1 + \sin x \cos x)$

   c) $(\sin A + \cos A)^2 + (\sin B + \cos B)^2 = 2(1 + \sin A \cos A + \sin B \cos B)$

   d) $\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$

REFERENCES

I. Textbooks

1. Vannatta, Carnahan, Fawcett, Advanced High School Mathematics, expanded edition (Charles E. Merrill Publishing Co., 1965).

2. Hooper and Griswold, A Modern Course in Trigonometry, (Henry Holt, and Company, Inc., 1959).

3. White, Advanced Algebra (Allyn and Bacon, Inc., 1961).

4. Rees and Sparks, Algebra and Trigonometry, 2nd edition (McGraw-Hill, Inc., 1969).

5. Vance: Modern Algebra and Trigonometry, 2nd edition (Addison-Wesley Publishing Co., 1968).

II. Audio

1. Wollensak Teaching Tape C-3711: The Sine Function

2. Wollensak Teaching Tape C-3712: The Cosine Function

3. Wollensak Teaching Tape C-3713: Tangent Function

III. Visual

Filmstrip: Introductory Trigonometry (Colonial Films)

L EARNING

A CTIVITY

P ACKAGE

GRAPHS AND APPLICATIONS OF THE TRIGONOMETRIC FUNCTIONS

Ninety Six High School

Ninety Six, S.C.

Trigonometry

IEWED BY

LAP NUMBER 47

WRITTEN BY Bill Holland

96 3672 4

RATIONALE

When you first studied functions, you learned to do several things with them. You learned to determine their domain and range, you learned to compute f(a) for any given function f, and any real variable a in the domain of f, and you learned how to sketch a graph of any given function.

In the previous LAP you were introduced to the trigonometric functions. You learned their definitions and how to find the value of any given trigonometric function evaluated at a given angle. Also, you were taught how to change trigonometric equations using proven trigonometric identities.

In this LAP you will learn how to sketch a graph of any given trigonometric function. Also, you will learn how the trigonometric functions apply to solve problems of everyday life.

SECTION 1

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Determine the amplitude and fundamental period of any given trigonometric function.

2. Sketch a graph of the following trigonometric functions for any indicated domain:

   a) y = Asinkx   d) y = Acotkx

   b) y = Acoskx   e) y = Aseckx

   c) y = Atankx   f) y = Acsckx

3. Sketch a graph of any given compound function by the addition of ordinates or the product of ordinates method.

RESOURCES

I. Readings.

1. Vanatta - Advanced High School Mathematics: #1 pps. 172-174; #2 pps. 168-171, 174-176; #3 pps. 177-179.

2. Griswold - A Modern Course in Trigonometry: #1 p. 85; #2 pps. 80-92; #3 p. 93.

3. Vance - Modern Algebra and Trigonometry: #1,2 pps. 330-331, 332-333; #3 p. 333.

4. Rees - Algebra and Trigonometry: #1 pps. 177-180; #2 pps. 180-184; #3 pps. 185-188.

5. Fisher - Integrated Algebra and Trigonometry: #1,2 pps. 143-147; #3 ___.

II. Problems.

1. Vanatta - Advanced High School Mathematics: #1 p. 174 ex. 1-12; #2 pps. 171-172 ex. 1-7, p. 176 ex. 1-9; #3 p. 179 ex. 1-8.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2 pps. 90-91 ex. 1-3; #3 p. 93 ex. 1-8.

3. Vance - Modern Algebra and Trigonometry: #1,2 p. 331 ex. 1-8; #3 p. 334 ex. all odd problems.

4. Rees - Algebra and Trigonometry: #1 ___; #2 p. 184 ex. 1-16; #3 pps. 188-189 ex. 1-12, 29-36.

5. Fisher - Integrated Algebra and Trigonometry: #1,2 p. 147 ex. 1-3.

SELF-EVALUATION 1

1. Determine the amplitude and fundamental period of the following trigonometric functions whenever possible.

a) $y = -7 \sin \frac{x}{4}$

b) $y = \frac{3}{2} \csc \frac{x}{2}$

c) $y = -\tan \frac{x}{2\pi}$

d) $y = \pi \cos \frac{3\pi x}{5}$

e) $y = \sqrt{5} \cot \frac{6x}{5}$

f) $y = \sec 7\pi x$

2. Sketch a graph of the following trigonometric functions for the indicated domain:

a) $y = 3\sin x$ from $-\pi$ to π

b) $y = -2\tan x$ from π to 2π

c) $y = \frac{1}{2}\cos 3x$ from $\frac{-\pi}{2}$ to $\frac{3\pi}{2}$

d) $y = -\sec x$ from 0 to 2π

e) $y = 2\cot(x + 2\pi)$ from $\frac{\pi}{2}$ to $\frac{5\pi}{2}$

f) $y = \csc(-x)$ from $-\frac{\pi}{4}$ to $\frac{5\pi}{4}$

3. Sketch a graph of the following compound functions:

a) $y = x - \cos x$

b) $y = 2(2x + \sin x)$

c) $y = \sin x + \sin 2x$

d) $y = 3\sin 2x - 4\cos 2x$

e) $y = 5x - 3\sin 2x$

ADVANCED STUDY

1. Sketch a graph of the following pairs of functions. Each pair of functions should be sketched on the same graph with different color pens.

   a) cosx, sinx from -2π to 2π

   b) tanx, cotx from -2π to 2π

   c) secx, cscx from -2π to 2π

   What comparisons can you deduce about each pair of functions? Write the cosine function in such a way that it will be equal to the sine function.

2. Sketch a graph of the following functions:

   a) $y = 2x^2 - 4\cos 2x$ from -2π to 2π

   b) $y = \sin^2 x + 2\cos 2x$ from -2π to 2π

   c) $y = 2\sin 2\pi x + \cos^2 2x$ from -2π to 2π

   d) $y = \csc x + \sec x$ from -2π to 2π

   d) $y = x \cdot \sin^2 2x + 2x\cos^2 2x$ from -2π to 2π

SECTION 2

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. a. Define the composite function of two functions.

   b. Work problems relating to composite functions.

2. a. Define the inverse of a function.

   b. Work problems relating to the inverse of a function.

3. a. Define the following functions:

      1. inverse of sinx
      2. inverse of cosx
      3. inverse of tanx
      4. inverse of cotx
      5. inverse of secx
      6. inverse of cscx

   b. Work problems relating to the above definitions.

4. Determine principal and general values of the functions arcsinx, arccosx, and arctanx.

5. Sketch a graph of the relations arcsinx, arccosx, arctanx, arccotx, arcsecx, arccscx for any indicated domain.

6. Determine the solution set of any given linear trigonometric equation.

7. Determine the solution set of any given quadratic trigonometric equation.

8. Determine the solution set of a trigonometric equation which is neither linear nor quadratic.

RESOURCES 2

I. Readings:

1. Vannatta - Advanced High School Mathematics: #1 ___; #2 p. 100; #3 pps. 180-182; #4 pps. 183-184; #5 pps. 185-186; #6 p. 187; #7 pps. 188-189; #8 pps. 189-192.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2,3 pps. 94-96; #4 ___; #5, pps. 96-97; #6,7,8 pps. 109-112.

3. Vance - Modern Algebra and Trigonometry: #1 pps. 99-100; #2 pps. 265-267; #3,4,5 pps. 270-275; #6 ___; #7 ___; #8 ___.

4. Rees - Algebra and Trigonometry: #1 ___; #2 pps. 132-134; #3,4,5 pps. 445-449; #6,7 pps. 439-441; #8 pps. 441-443.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pps. 342-345; #3,4,5 pps. 347-355; #6,7,8 pps. 356-358.

II. Problems:

1. Vanatta - Advanced High School Mathematics: #1 ___; #2 ___; #3 pps. 182-183 ex. 1-26; #4 pps. 184-185 ex. 1-20; #5 pps. 186-187 ex. 1-4; #6 p. 188 ex. 1-10; #7 p. 189 ex. 1-12; #8 p. 192 ex. 1-19.

2. Griswold - A Modern Course in Trigonometry: #1 ___; #2,3 p. 96 ex. 1-20; #4 ___; #5 p. 97 ex. 1-8; #6,7,8 pps. 113-114 all even numberd exercises.

3. Vance - Modern Algebra and Trigonometry: #1 p. 100 ex. 1-12; #2 p. 268 ex. 1-12; #3,4,5 pps. 273-274 ex. 1-24, pps. 275-276 ex. 1-18, 36-45; #6 ___; #7 ___; #8 ___.

4. Rees - Algebra and Trigonometry: #1 ___; #2 pps. 139-140 ex. 9-20; #3,4,5 p. 450 ex. 1-24; #6,7 p. 441 ex. 1-24; #8 p. 444 ex. 1-30.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 p. 346 ex. 1-3, 6; #3,4,5 p. 350 ex. 1-5, p. 355 ex. 1-4; #6,7,8 p. 358 ex. 1-5.

SELF-EVALUATION 2

1. a. Define the composite function of two functions.

   b. Find gof for the following combinations:

      1) $f(x) = 3x - 9 \qquad g(x) = x^2$

      2) $f(x) = \frac{4}{x} \qquad g(x) = x^2 - 16$

      3) $f(x) = x^2 \qquad g(x) = \sqrt{x}$

      4) $f(x) = x^2 - 9 \qquad g(x) = x + 16$

2. a. Define the inverse of a function.

   b. Determine f^{-1} for the following functions and tell the domain and range of f^{-1}.

      1) $f(x) = x - 4$

      2) $f(x) = x^2 - 2$

      3) $f(x) = \frac{2x}{x - 6}$

3. a. Define the following functions:

      1) inverse of sinx

      2) arctanx

      3) $\sec^{-1}x$

   b. Find the values of the following:

      1) $x = \arccos 0 \quad 0^\circ \leq x < 360^\circ$

      2) $x = \arctan 3 \quad 0^\circ \leq x < 360^\circ$

      3) $x = \operatorname{arcsec} 4 \quad 0^\circ \leq x < 360^\circ$

      4) $x = \arcsin(-.5) \quad 0^\circ \leq x < 360^\circ$

      5) $\tan(\arctan\frac{3}{4}) = x$

      6) $\cos(\operatorname{arccot}\frac{2}{3}) = x$

      7) $\sec(\arccos\frac{4}{5}) = x$

      8) $\tan(\arcsin\frac{2}{3}) + \cot(\arccos\frac{3}{4}) = x$

4. a. Give principal values in the following exercises:

   1. Arcsin $\frac{\sqrt{2}}{2}$ 3. Arccos $\left(\frac{\sqrt{3}}{-2}\right)$

   2. Arctan$\frac{5}{6}$ 4. Arcsin(.8829)

   b. Give general values in the following exercises.

   1) arctan$\frac{1}{2}$ 3) arcsin$\left(-\frac{3}{4}\right)$

   2) aresec $\frac{\sqrt{3}}{2}$ 4) arccsc$\left(-\frac{2}{3}\right)$

5. Sketch a graph of the following relations.

   1. arcsinx $-180^\circ \leq x \leq 180^\circ$

   2. arctanx $-360^\circ \leq x \leq 180^\circ$

   3. arcsecx $-90^\circ \leq x \leq 90^\circ$

6. Solve the following equations for x in degrees.

   a. $5\sin x + 1 = 0$

   b. $\sqrt{3}\tan x + 3 = 0$

   c. $5\cos x - \sqrt{5} = 0$

   d. $\sin 3x - .5 = 0$

7. Solve the following equations for x in degrees.

   a) $2\cos^2 x - 5\cos x + 2 = 0$

   b) $\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$

   c) $3\sin^2 x - 7\sin x = 3$

   d) $\cos x \cot x - \cot x = 0$

8. Solve the following equations for principal values of x.

   a) $5\sin^2 x + 2\csc x + 4 = 0$

   b) $\cot x + 1 = \sin x$

   c) $\cos x - 1 + \tan x = 0$

   d) $6\sin x - 8\cos x = 5$

ADVANCED STUDY

1. Sketch a graph of the following relations on the same set of coordinate axes within the given values of x using a different colored pen for each one.

   a) arcsin2x d) arccot2x

   b) arccos2x e) arcsec2x $-360^\circ \leq x \leq 360^\circ$

   c) arctan2x f) arccsc2x

2. Determine the inverse of the following functions and also determine a suitable domain and range so that the inverse will be a function. Then graph the inverse function.

   a) $f(x) = |x|$

   b) $f(x) = \sqrt[3]{x - 3}$

   c) $f(x) = \dfrac{\sqrt{x^2 - 4}}{\sqrt{x^3 - 3}}$

   d) does $f(x) = [x]$ have an inverse function? Justify your answer.

3. How many times will a line parallel to the x or to the y axis intersect a function if the function has an inverse? Explain.

4. If a function is increasing (i.e. if $x_1 < x_2$, then $f(x_1) < f(x_2)$) or if a function is decreasing (i.e. if $x_1 < x_2$, then $f(x_1) > f(x_2)$) the can you say its inverse is decreasing or is increasing? Justify your answer.

SECTION 3

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Solve word problems that require construction of right triangles and use of trigonometric functions.

2. a. State and prove the Law of Sines. ($\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$)
   b. Work problems relating to the Law of Sines.

3. a. State and prove the Law of Cosines ($c^2 = a^2 + b^2 - 2ab\cos C$)
   b. Work problems relating to the Law of Cosines.

4. a. State and prove a formula that will determine the area of a triangle given two sides and the included angle.
   b. Find the area of a triangle given two sides and the included angle.

RESOURCES

I. Readings.

1. Vanatta - Advanced High School Mathematics: #1 pp. 144-147; #2 pp. 150-154; #3 pp. 154-156; #4 pp. 157-158.

2. Griswold - A Modern Course In Trigonometry: #1 ___; #2 pp. 142-151; #3 pp. 152-158; #4 pp. 158-161.

3. Vance - Modern Algebra and Trigonometry: #1,2 pp. 364-368; #3 pp. 372-376; #4 ___.

4. Rees - Algebra and Trigonometry: #1 pp. 383-389; #2 pp. 393-395, 397-399; #3 pp. 399-400; #4 pp. 395-396.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pp. 166-169; #3 pp. 170-172; #4 pp. 173-176.

II. Problems.

1. Vanatta - Advanced High School Mathematics: #1 pp. 147-149 ex. 1-21; #2 p. 154 ex. 1-8; #3 p. 157 ex. 1-10; #4 p. 158 ex. 1-8.

2. Griswold - A Modern Course In Trigonometry: #1 ___; #2 p. 145 ex. 1-8,pp.146-146 ex. 1-10, p. 150 ex. 1-10, p. 152 ex. 11-17; #3 pp. 154-155 ex. 1-10, p. 157 ex. 1-10; p. 161 ex. 6-10.

3. Vance - Modern Algebra and Trigonometry: #1,2 pp. 368-369 ex. 1-17, p. 376 ex. 1-3, 5; #3 pp. 377-378 ex. 6-9, 14-19; #4 ___.

4. Rees - Algebra and Trigonometry: #1 pp. 385-386 ex. 1-20, pp. 389-391 ex. 1-2, 5-6, 10-17, 20, 22; #2 pp.396-397 ex. 1-22, pp. 400-401 ex. 1-12; #3 p. 401 ex. 13-20; #4 p. 401 ex. 21-28.

5. Fisher - Integrated Algebra and Trigonometry: #1 ___; #2 pp. 169-170 ex. 2-3, 5-8; #3 pp. 172-173 ex. 1-3, 5-6, 8, 10; #4 p. 176 ex. 1, 3, 7.

SELF-EVALUATION 3

I. a. From a firetower 99.8 feet above the level of the ground, the angle of depression of a tree is $15^\circ 30'$. How far is the tree from a point directly under the point of observation?

b. Two poles are on horizontal ground and a person is standing between them. He is 104 feet from one pole and finds the angle of elevation to its top to be $15^\circ 13'$. He is 55 feet from the other pole and finds the angle of elevation to its top to be $29^\circ 50'$. Which pole is taller and by how much?

c. A 5.3 inch chord subtends a central angle of $11^\circ 50'$ in a circle. What is the diameter of the circle?

d. To determine the height of a tree two points A and B were located on level ground in line with the tree and the angles of elevations were measured at each point. The angle at A was $55^\circ 10'$ and the angle at B was $105^\circ 40'$. The distance from A to B was 320 feet. How tall is the tree?

II. a. State and prove the Law of Sines.

b. In triangle ABC, $A = 77^\circ 24'$, $a = 344$ feet, and $c = 406$ feet. Find B, C, b.

c. An apartment building stands on the side of a ravine that has a uniformly sloped side. At a time when the sun has an angle of elevation of $55^\circ 12'$ the shadow of the building extends down the side of the ravine. If the side of the ravine has an angle of $9^\circ 10'$, find the length of the shadow.

III. a. State and prove the Law of Cosines.

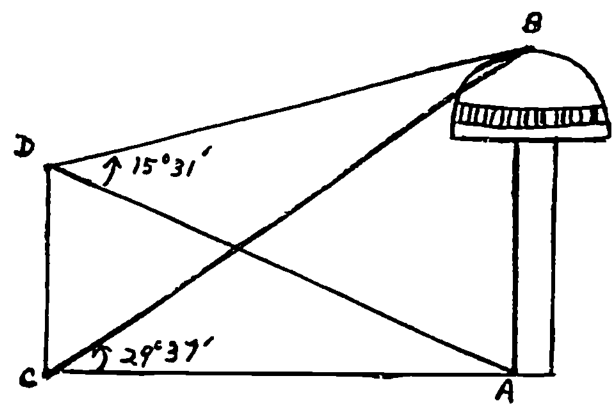
b. The sides of a parallelogram are 13 in. and 55 in. Find the length of each diagonal if the smaller angle is 32°.

IV. a. State and prove an area formula for triangles.

b. In triangle ABC, $a = 38.84$ cm, $c - 27.2$ cm, and $B = 62^\circ 32'$. Find the area of triangle ABC.

ADVANCED STUDY

1. In order to find the height of watertower AB, the angle of elevation to the top B is measured by means of a transit from point C, whose distance from the watertower is not known. Then the transit is turned through a horizontal angle of 90° and point D is located. At D the angle of elevation of the top of the watertower is again measured. Find the height of the watertower if $\angle ACB = 29^{\circ}37'$ and $\angle ADB = 15^{\circ}31'$, and CD = 200.0 feet.



2. Find a derivation of Hero's formula for the area of a triangle

$(A = \sqrt{s(s - a)(s - b)(s - c)})$, study it, and then derive this formula for your teacher.

3. Derive the following:

   a. Law of Cosecants

   b. Law of Secants

   c. Law of Tangents

   d. Law of Cotangents

BIBLIOGRAPHY

1. Vanatta, Carnahan, Fawcett - Advanced High School Mathematics, Expanded Edition, (Charles E. Merrill Publishing Co., 1965)

2. Griswold, Hooper - A Modern Course in Trigonometry, (Henry Holt and Company, 1959)

3. Vance - Modern Algebra and Trigonometry, 2nd edition, (Addison-Wesley Publishing Company Inc., 1968)

4. Rees, Sparks - Algebra and Trigonometry, 2nd edition, (McGraw-Hill, Inc., 1969)

5. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd edition, (Prentice-Hall, Inc., 1967)

L E A R N I N G

A C T I V I T Y

P A C K A G E

SEQUENCES AND SERIES

Ninety Six High School - Ninety Six, S.C. -

Algebra 124

IEWED BY

LAP NUMBER 48

WRITTEN BY Bill Holland

Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

RATIONALE

Ancient Greek philosophers argued that in a race involving a hare and a tortoise the hare could not catch the tortoise if the tortoise were allowed a head start. Their reasoning was as follows: suppose the hare is ten times as fast as the tortoise and also suppose that the tortoise is allowed to start one foot head of the hare. Then when the hare travels the one foot that was between him and the hare, the tortoise has traveled one-tenth of a foot; when the hare travels the one-tenth of a foot that was between himself and the tortoise, the tortoise has traveled one-hundredth of a foot. Each time the hare travels the distance that was between himself and the tortoise, the tortoise has traveled one-tenth of that distance. Hence, the hare will never catch the tortoise.

This argument puzzled philosophers for ages. They knew that the hare would catch the tortoise, but they could not see any flaw in the above agrument.

In this LAP we will learn what sequences and series are. We will learn how to find the sum of selected finite series. Finally we will study a topic that will enable us to show directly that the hare does in fact catch the tortoise--convergent infinite series.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Determine for any given arithmetic sequence
   a. the first term of the sequence
   b. the common difference of the terms of the sequence
   c. any term in the sequence which is not given
2. Determine the means of any arithmetic sequence and/or work problems relating to the means of an arithmetic sequence.
3. Derive a formula that will enable you to determine the sum of the terms in an arithmetic series and work problems relating to this formula.
4. Determine for any given geometric sequence:
   a. the first term of the sequence
   b. the common ratio of the terms of the sequence
   c. any term in the sequence which is not given
5. Determine the means of any geometric sequence and/or work problems relating to the means of a geometric sequence.
6. Derive a formula that will enable you to determine the sum of the terms in a geometric series and work problems relating to this formula.
7. Determine the amount earned when a principal is invested at a certain interest rate r and is compounded k times per year.

SECTION I

Resources

I. READINGS:

1. Vanatta - #1 pp. 209-211; #2 p. 211; #3 p. 212; #4 pp. 213-214; #5 pp. 214-215; #6 p. 215; #7 pp. 218-219.

2. Rees - #1 pp. 409-411; #2 pp. 422-423; #3 pp. 411-414; #4 pp. 415-416; #5 pp. 423-424; #6 pp. 416-419; #7 ____.

3. Vance - #1 pp. 173-175; #2 ____; #3 p. 175; #4 pp. 307-308; #5 ____; #6 p. 308; #7 pp. 322-324.

4. Fisher - #1 pp. 324-325; #2 p. 332; #3 pp. 325-327; #4 pp. 328-329; #5 pp. 333-334; #6 pp. 329-331; #7 ____.

5. Dolciani - #1 pp. 75-80; #2 pp. 80-81; #3 p. 81; #4 pp. 83-84; #5 pp. 84-85; #6 pp. 86-87; #7 ____.

II. PROBLEMS:

1. Vanatta - #1 p. 210 ex. 1-15 (odd numbered problems), p. 211 ex. 1-10; #2 p. 212 ex. 1-7; #3 p. 213 ex. 1-5; #4 p. 213 ex. 1-5, p. 214 ex. 1-5; #5 p. 215 ex. 1-5; #6 p. 216 ex. 1-5; #7 p. 219 ex. 1-3.

2. Rees - #1 pp. 414-415 ex. 1-10, 13, 15; #2 p. 424 ex. 1-4; #3 pp. 414-415 ex. 11-12, 14, 16-28; #4 p. 421 ex. 1-10, 13, 15; #5 p. 424 ex. 5-8; #6 pp. 421-422 ex. 11-12, 14, 16-24, 29-36; #7 ____.

3. Vance - #1, #3 p. 176 ex. 1-16; #2 p. 176 ex. 17-20; #4, #6 p. 309 ex. 1-16, 21-27; #5 ex. 16-18; #7 p. 324 ex. 1-3.

4. Fisher - #1 p. 137 ex. 1; #2 p. 335 ex. 1-2; #3 pp. 327-328 ex. 2-6, 9, 11; #4, #6 p. 331 ex. 1-5; #5 p. 335 ex. 3, 4; #7 ____.

5. Dolciani - #1 p. 82 ex. 1, 4-16, 19-20; #2 p. 82 ex. 2; #3 p. 82 ex. 3, 17-18, 21-22, 24-25; #4 p. 86 ex. 1, 3, 5, 7, 13, 19-21, 24; #5 p. 86 ex. 2, 4; #6 pp. 86-87 ex. 6, 8-12, 14, 16, 22-23; #7 ____.

SELF-EVALUATION I

1. Determine the 20th term in the sequence -5, 3, 11, ...

2. Insert five arithmetic means between -2 and 4.

3. Find the sum of 31 terms of the series -2 + 1 + 4 + ...

4. Determine the 9th term of the sequence -2, 4, -8, ...

5. Insert two geometric means between 8 and 64.

6. Find the sum of 11 terms of the series $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$

7. Find the compound amount of $1500 invested for 2 years at 5%, interest being compounded quarterly.

ADVANCED STUDY

1. a. Extend the harmonic progression $1 + \frac{5}{6} + \frac{5}{7} + \text{---}$ to four more terms.

   b. Insert two harmonic means between ½ and $\frac{1}{3}$.

   c. Show that x, y, z is a geometric progression if y - x, 2y, y - z is a harmonic progression

2. Find the compound amount at the end of fourteen years on an original principal of $1,000 compounded continuously.

3. Does the series 1 - 1 + 1 - 1 + 1 ... have a sum. If so, what is it? Justify your answer.

4. A series of squares is drawn by connecting the midpoints of the sides of a four-inch square, then the midpoints of the sides of the second square, and so on. Find the approximate sum of the areas of the square.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

8. Derive a formula that will enable you to determine the sum of an infinite geometric series and work problems relating to this formula.

9. Determine whether any given geometric series is convergent or divergent.

10. Demonstrate your understanding of the comparison test for convergence of series by using it to determine whether a given series is convergent or divergent.

11. Demonstrate your understanding of the ratio test for convergence of series by using it to determine whether a given series is convergent or divergent.

RESOURCES

I. Readings:

1. Vanatta - #8 pp. 216-217; #9 pp. 219-221; #10 pp. 221-224; #11 p. 225.

2. Rees - #8 pp. 419-421; #9 ____; #10 ____; #11 ____.

3. Vance - #8 pp. 310-312; #9 ____; #10 ____; #11 ____.

4. Fisher - #8 pp. 334-335; #9 ____; #10 ____; #11 ____.

5. Dolciani - #, #9 pp. 101-103; #10 ____; #11 ____.

II. Problems:

1. Vannatta - #8 p. 218 ex. 1-8; #9 p. 221 ex. 1-5; #10 pp. 224-225 ex. 1-8; #11 p. 226 ex. 1-6.

2. Rees - #8 p. 421 ex. 21-24; #9 ____; #10 ____; #11 ____.

3. Vance - #8 pp. 312-313 ex. 1-13, 17-18; #9 ____; #10 ____; #11 ____.

4. Fisher - #8 p. 335 ex. 6, 8; #9 ____; #10 ____; #11 ____.

5. Dolciani - #8, #9 pp. 104-106 ex. 5-22, 29-30, 37, 39-43; #10 ____; #11 ____.

SELF-EVALUATION II

1. a. Determine the sum of the following series:

$$\frac{1}{3} + \frac{2}{9} + \frac{6}{27} + ---$$

b. A ball is dropped from a height of one foot. It then hits the floor and rebounds to one-half its original height, hits the floor again and then rebounds to one-half the height it had bounced the first time, etc. Neglecting external forces, how far will the ball travel while it bouncing?

2. Are the following series convergent or divergent? Justify your answer:

a. $\frac{1}{2} + \frac{3}{8} + \frac{9}{24} + ---$

b. $\frac{1}{9} + \frac{3}{18} + \frac{9}{36} + ---$

c. $\frac{1}{3} - \frac{4}{9} + \frac{16}{27} - \ldots$

3. Use the comparison test to determine whether the following series are convergent or divergent.

a. $1 + \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} + \ldots$

b. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ---$

c. $\frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 3} + \ldots$

d. $\frac{1}{3} + \frac{1}{3 \cdot 2} + \frac{1}{3 \cdot 3} + \ldots$

4. Use the ratio test to determine whether each of the following series are convergent or divergent.

a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

b) $\frac{3}{2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 5} + \frac{3}{2 \cdot 3 \cdot 5 \cdot 7} + \ldots$

c) $\frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + ---$

d) $\frac{1}{5} + \frac{3}{5^3} + \frac{5}{5^5} + \ldots$

ADVANCED STUDY

1. If a_1, a_2, a_3, ... is an infinite sequence such that the series $a_1 + a_2 + a_3 + \ldots$ converges, does the series $|a_1| + |a_2| + |a_3| + \ldots$ also converge. Justify your answer.

2. If b_1 , b_2, b_3 is an infinite sequence such that the series $|b_1| + |b_2| + |b_3| + \ldots$ converges, does the series $b_1 + b_2 + b_3 + \ldots$ also change? Justify your answer.

3. If c_1 , c_2 , c_3 , ... is an infinite sequence such that the series $c_1 + c_2 + c_3 + \ldots$ converges, does the series $(-c_1) + (-c_2) + (-c_3) + \ldots$ also converge? Justify your answer.

4. We have studied two tests that will enable us to determine whether an infinite series converges. You are to state and prove a theorem that will give another test to determine whether an infinite series converges, and then give an example of this test.

SECTION III

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

12. Use the binomial formula to write the expansion of any given binomial and/or determine any given term of a binomial in the form $(x + y)^n$.
13. Solve problems relating to factorial notation.
14. Demonstrate your understanding of the principle of mathematical induction by proving statements using mathematical induction.
15. Demonstrate your understanding of Pascal's triangle by being able to write it for any given value of n and/or using it to write an expansion of any given binomial.

RESOURCES

I. READINGS:

1. Vanatta - #12 pp. 226-228, 229; #13 p. 228; #14 pp. 230-234; #15 p. 235.
2. Rees - #12, #13 pp. 431-433, 435; #14 pp. 426-429; #15 ____.
3. Vance - #12 pp. 287-289; #13 p. 232; #14 pp. 295-297; #15 ____.
4. Fisher - #12 pp. 285-288; #13 pp. 275-276; #14 pp. 320-323; #15 ____.
5. Dolciani - #12 ____; #13 p. 89; #14 pp. 69-73; #15 p. 90.

II. PROBLEMS:

1. Vanatta - #12 p. 228 ex. 1-9, p. 229 ex. 1-6; #13 pp. 228-229, ex. 1-22; #14 p. 233 ex. 1-8, p. 235 ex. 1-2; #15 p. 235 ex. 1-4.
2. Rees - #12, #13 pp. 435-436 ex. 1-28; #14 pp. 429-430 ex. 1-24; #15 ____.
3. Vance #12 p. 290 ex. 1-24; #13 ___; #14 pp. 297-298 ex. 1-15, 18-20; #15 ____.
4. Fisher - #12 p. 289 ex. 1-3, 6-7; #13 pp. 276-277 ex. 1-2; #14 pp. 323-324 ex. 1,5,8,9; #15 ____.
5. Dolciani - #12 ___; #13 pp. 92-93 ex. 1-4, 9-16, 21-26; #14 pp. 73-74 ex. 13-24; #15 ____.

SELF-EVALUATION III

1. a. Use the binomial formula to write the expansion of $(2x - 3y)^7$.

   b. List the fifth term of $(x - 4y)^6$.

2. a. Give a numerical value for each of the following:

   1) $\frac{2!}{3!}$ 2) $(4!)$ 3) $\frac{16!}{15!}$

   b. Simplify each of the following where a and b are positive integers, $a > b$, and $a > 1$.

   1) $\frac{a!}{(a-1)!}$ (2) $\frac{(a-b)!}{(a-b+1)!}$ (3) $\frac{(a-2)!(a+1)!}{(a-1)!\,a!}$

3. Prove: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

4. Use Pascal's triangle to write an expansion of $(x - 3)^7$.

ADVANCED STUDY III

1. If $a_1, a_2, a_3, ---, a_n$ is a sequence, determine what is meant by $\sum_{i=1}^{n} a_i$. Then use mathematical induction to prove the following statements.

   a) If $a_1, a_2, a_3, ---, a_n$ and $b_1, b_2, b_3, ---, b_n$ are two sequences then $\sum_{i=1}^{n} (a_i+b_i) = \sum_{i=1}^{n} a_1 + \sum_{i=1}^{n} b_1$

   b) If $a_1, a_2, a_3, ---, a_n$ is a sequence and k is any fixed number, then $\sum_{i=1}^{n} ka_i = k \sum_{i=1}^{n} a_i$

2. Use mathematical induction to prove that

   a) $n^2 - 3n + 4$ is an even number

   b) $2n^3 - 3n^2 + n$ is divisible by 6

REFERENCES

1. Vanatta, Carnahan, Fawcett - Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., 1965).

2. Rees, Sparks - Algebra and Trigonometry, 2nd Edition (McGraw-Hill, Inc., 1969).

3. Vance - Modern Algebra and Trigonometry, 2nd Edition (Addison-Wesley Publishing Co., Inc., 1968).

4. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).

5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton - Modern Introductory Analysis (Houghton Mifflin Co., 1970).

LEARNING ACTIVITY PACKAGE

PERMUTATIONS, COMBINATIONS, AND PROBABILITY

Ninety Six High School - Ninety Six, S.C. -

Algebra 124

REVIEWED BY

LAP NUMBER 49

WRITTEN BY Bill Holland

RATIONALE

In the theory of probability, in statistics, industry, and in the sciences, it is frequently necessary to calculate the number of ways that the elements in a set can be arranged or to determine the number of ways the elements of a set can be combined into subsets. For example, a telephone company must provide each subscriber with a unique number, and a state government has a similiar problem in assigning license numbers for vehicles. We shall be concerned with problems of this nature in this LAP and proceed from them to some of the very useful concepts of the theory of probability.

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Solve problems relating to the Fundamental Principle of Choice.
2. Determine the number of permutations of n elements of a set taken r at a time where $r \leq n$.
3. Determine the number of distinct permutations of n elements of a set taken r at a time if two or more of these elements are alike and $r \leq n$.
4. Determine the number of combinations of n elements of a set taken r at a time where $r \leq n$.
5. Solve word problems involving simple event probability in which the occurrence of any event is equally likely.
6. Solve word problems involving the probability of mutually exclusive events.
7. Solve word problems involving the probability of independent or dependent events.
8. Use the Binomial Theorem to determine the probability of events or to prepare a binomial probability table.

RESOURCES

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 pp. 238-239; #2 pp. 240-241; #3 pp. 242-243; #4 pp. 243-245; #5 pp. 246-248; #6, #7 pp. 248-251; #8 pp. 253-257.

2. Rees: Algebra and Trigonometry - #1 pp. 455-456; #2, #3 pp. 457-459; #4 pp. 461-462; #5 pp. 464-465; #6 pp. 469-470; #7 pp. 470-472; #8 ___.

3. Vance: Modern Algebra and Trigonometry - #1 pp. 278-279; #2, #3 pp. 280-282; #4 pp. 284-285; #5-#8 ____.

4. Fisher: Integrated Algebra and Trigonometry - #1 pp. 273-276; #2, #3 pp. 277-281; #4 pp. 282-284; #5 pp. 294-297; #6, #7 pp. 298-302; #8 pp. 304-308.

5. Dolciani: Modern Introductory Analysis - #1 ___; #2, #3 pp. 610-612; #4 pp. 613-615; #5 pp. 599-601; #6 pp. 602-605; #7 pp. 607-608; #8 pp. 621-623.

II. Problems:

1. Vannatta: Advanced High School Mathematics - #1 p. 239 ex. 1-5; #2 pp. 241-242 ex. 1-20; #3 p. 243 ex. 1-5; #4 pp. 245-246 ex. 1-20; #5 p. 248 ex. 1-5; #6, #7 pp. 251-253 ex. 1-12; #8 pp. 257-258 ex. 1-10.

2. Rees: Algebra and Trigonometry - #1 ___; #2, #3 pp. 459-460 ex. 1-28 (even numbered exercises); #4 pp. 462-463 ex. 1-28 (odd numbered exercises); #5 ex. 1-20; #6, #7 pp. 473-475 ex. 1-32 (even numbered exercises); #8 ___.

3. Vance: Modern Algebra and Trigonometry - #1 pp. 279-280 ex. 1-12; #2, #3 p. 283 ex. 1-20; #4 pp. 285-286 ex. 1-19; #5-#8 ___.

4. Fisher: Integrated Algebra and Trigonometry - #1 pp. 276-277 ex. 1-10; #2, #3 pp. 281-282 ex. 1, 3-10; #4 pp. 284-285 ex. 1-12; #5 pp. 297-298 ex. 1-14; #6, #7 pp. 303-304 ex. 1-12, 14; #8 pp. 308-309 ex. 1-10.

5. Dolciani: Modern Introductory Analysis - #1 ___; #2, #3 pp. 612-613 ex. 1-16; #4 pp. 616-617 ex. 1-18; #5 pp. 601-602 ex. 1-12; #6 pp. 605-606 ex. 1-16; #7 pp. 608-609 ex. 1-12; #8 pp. 623-624 ex. 1-8.

SELF-EVALUATION

1. a. If the first digit cannot be equal to zero, how many five digit numbers can be formed?

   b. A penny, nickel, dime, and quarter are flipped simultaneously. How many different ways can the coins land?

2. If a coach has fifteen football players, how many different lineups can he make (one lineup is different from another if one player is at a different position in one than he is in another)?

3. Show that $r! \cdot P(n,n-r) = P(n,n)$.

4. How many distinct permutations can be made from the letters of the word COMBINATION?

5. If a convex polygon has 10 vertices, how many diagonals can be drawn?

6. How many different committees of 6 Americans, 5 Chinese, and 7 Negroes can be selected from a group of 17 Americans, 10 Chinese, and 12 Negroes?

7. How many football games are played in the Big Eight if each team plays all the other teams once?

8. If 11 coins are tossed at the same time, what is the probability that 5 of them will come up tails?

9. In a drawer a man has 7 blue socks and 9 green socks. What is the probability he will get a pair that matches if he selects 3 socks from the drawer at random?

10. It has been determine experimentally that the success of an event is .75. What is the probability of 3 successful events in 4 trials?

ADVANCED STUDY

1. A diagonal of a polygon is a line that joins 2 non-adjacent vertices. How many diagonals does an n-sided polygon have?

2. a. Prove the relation $C(n,r+1) = \frac{n-r}{r+1} C(n,r) \quad 0 \le r < n$.

   b. Prove $C(n,r) + C(n,r-1) = C(n+1,r)$.

3. A baby has 11 letter blocks that consist of four S's, four I's, two P's, and one M. The baby places the blocks all in a row and all right side up. What is the probability he will spell the word MISSISSIPPI. If he selects 3 blocks and places them right side up in a row, what is the probability that he will spell the word IMP?

4. Three boxes each contain 5 white, 3 red, and 2 blue poker chips. One chip is selected at random from the first box and placed in the second. Two chips are then selected at random from the second box and placed in the third box. Finally 3 chips are selected at random from the third box. What is the probability that all three chips will be of a different color?

5. If you glance at your watch, what is the probability the second hand will be exactly at the 30 second mark? What is the probability it will be between the 29 second and 31 second mark?

BIBLIOGRAPHY

1. Vannatta, Carnahan, Fawcett - Advanced High School Mathematics, expanded edition (Charles E. Merrill Publishing Co., 1965).
2. Rees, Sparks - Algebra and Trigonometry, 2nd edition (McGraw-Hill Book Co., 1969).
3. Vance - Modern Algebra and Trigonometry, 2nd edition (Addison-Wesley Publishing Co., Inc., 1968).
4. Fisher, Ziebur - Integrated Algebra and Trigonometry, 2nd edition (Prentice-Hall, Inc., 1967).
5. Dolciani, Beckenbach, Donnelly, Jurgensen, Wooton - Modern Introductory Analysis (Houghton Mifflin Co., 1970).

LEARNING

ACTIVITY

PACKAGE

DESCRIPTIVE STATISTICS

Ninety Six High School - Ninety Six, S.C. -

Algebra 124

:EWED BY

LAP NUMBER 50

WRITTEN BY Bill Holland

RATIONALE

It has often been said that one cannot be an intelligent member of society today without some understanding of statistics. In the ever-increasing complexity of modern society, we have a penchant for taking data, organizing it, and drawing whatever conclusions we may from it. When a student takes the Scholastic Aptitude Test his performance is ranked by the use of statistics. Statistics can tell us what the probability of our living another ten years is. Pollsters use statistics to tell us what we do and do not like. Statistics determines the success or failure of any television program. Some presidents have even watched polls based on random sampling to help them decide on a popular course of action to follow.

In this LAP we will be concerned with the tools of statistics. We will then use these to see how Mr. Gallup can predict that 55% of the people will vote for a certain candidate.

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Compute the arithmetic mean, mode, and median of any given set of data as indicated.
2. Compute the geometric, harmonic, or quadratic mean of any given set of data as indicated.
3. Compute the mean deviation and/or the semi-interquartile range for a given set of data.
4. Compute the standard deviation for a given set of data.
5. Construct a frequency distribution for a given set of data and from this frequency distribution,
    a. Sketch a histogram and/or frequency polygon.
    b. Compute the arithmetic mean, median, and standard deviation.
6. Demonstrate your understanding of normal distribution by sketching a graph of a normal curve using formulas or binomial expansion coefficients and/or solve problems relating to normal distribution.
7. Demonstrate your ability to interpret normally distributed data using standard deviations by being able to determine the percent of the data that falls in a certain range or the probability that a datum will fall within a certain range.
8. Compute for any random sample of data the standard error of the mean and establish a level of confidence about the sample mean.

RESOURCES

I. Readings:

1. Vannatta: Advanced High School Mathematics - #1 pp. 260-264; #2 pp. 264-265; #3 pp. 266-267; #4 pp. 267-278; #5 pp. 269-276; #6 pp. 276-280; #7 pp. 280-282; #8 pp. 283-285.

2. White: Advanced Algebra - #1 pp. 302-305; #2 ___; #3 ___; #4 pp. 313-314; #5 pp. 305-312; #6, #7 pp. 315-319; #8 pp. 319-322.

II. Problems:

1. Vannatta: Advanced High School Mathematics - #1 p. 262 ex. 1-5, p. 263 ex. 1-5, p. 264 ex. 1-3; #2 p. 265 ex. 1-5; #3 p. 267 ex. 1-3; #4 p. 268 ex. 1-3; #5 p. 271 ex. 1-2, p. 272 ex. 1-4, p. 276 ex. 1; #6 p. 278 ex. 1-2, p. 280 ex. 1-2; #7 pp. 282-283 ex. 1-4; #8 pp. 285-286 ex. 1-4.

2. White: Advanced Algebra - #1, #5 pp. 308-309 ex. 1-5, pp. 312-313 ex. 1-3; #2 ___; #3 ___; #4 - #8 pp. 322-323 ex. 1-5.

SELF-EVALUATION

1. The grades scored on a test by a twelfth grade class are as follows: 85, 89, 93, 89, 95, 74, 79, 93, 89, 100, 81, 94, 76, 89, 93, 79, 81, 87.

   a) Compute the mean grade.

   b) Compute the median grade.

   c) Determine the modal grade.

2. Compute the quadratic mean of 7.5, 8.9, 4.5, 3.7, 8.3, 5.4, 6.2, and 7.1.

3. Compute the mean deviation and semi-interquartile range of the following numbers: 32, 88, 67, 72, 85, 56, 93, 81, 48, 57, 63, 79, 89, 39.

4. Compute the standard deviation of the numbers in Ex. 3. Which measure of variability is greater?

5. Make a frequency distribution of the following weights in grams of selected materials: 3.2, 5.7, 4.3, 6.8, 2.1, 2.7, 3.5, 3.9, 2.6, 4.7, 4.1, 6.3, 5.9, 2.4, 4.9, 6.5, 4.2, 3.1, 2.9, 4.3, and 6.7. From this frequency distribution,

   a) Construct a frequency polygon.

b) Compute the mean, media, and standard deviation.

6. For the function $y = ke^{-hx^2}$, let $h = \frac{3}{2}$, $k = 11$, and construct the graph.

7. The mean of a set of normally distributed numbers is 82 and the standard deviation is 6.

   a) What percent of the numbers fall in the range from 73 to 91?

   b) What percent fall in the range from 80 to 84?

   c) What is the probability that a number selected at random from the data will be greater than 82.

   d) What is the probability that a number selected at random from the data will be less than 64?

8. A random sample of 100 students from Zeer High School shows a mean heght of 66 inches and a standard deviation of 1.6 inches.

   a) Find the standard error of the mean.

   b) What is the range about the mean of the sample that will give a 90% level of confidence that the true mean will fall within it?

ADVANCED STUDY

1. Determine the scores made on the Scholastic Aptitude Test by the graduating class of 1973 at Ninety Six High School. Then

   a. Compute the mean score

   b. Compute the median score.

   c. Compute the modal score.

   d. Which one of the averages seems to give a better representation of the data? Why?

   e. Make a frequency distribution for this data.

   f. From this frequence distribution, construct a frequency polygon.

   g. Are the scores normally distributed? If they are not, give possible reasons why not.

   h. Compute the standard deviation for the scores.

   i. What is the probability that a score picked at a random will be less than 820?

   j. Find a range about the mean which will include 90% of the scores.

2. Determine the heights of 50 randomly selected individuals at Ninety Six High School.

   a. Compute the mean height.

   b. Compute the standard deviation.

   c. Compute the standard error of the mean.

   d. What is a range of heights about the sample mean of the data that will give a .95 probability that the true mean will fall within it?

   e. Select 20 more individuals at random and determine whether their heights fall within the range.

   f. Based on part (e) what conclusions can you draw about the sample mean of your data?

BIBLIOGRAPHY

1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Edition (Charles E. Merrill Publishing Co., Inc., 1965).

2. White: Advanced Algebra (Allyn and Bacon, Inc., 1962.)

L EARNING

A CTIVITY

P ACKAGE

SPECIAL THEOREMS AND FUNCTIONS

Ninety Six High School - Ninety Six, S.C. -

ALGEBRA 124

EWED BY

LAP NUMBER 51

WRITTEN BY Bill Holland

RATIONALE

In a previous LAP you learned that the set of real numbers is a proper subset of the set of complex numbers and that any complex number can be expressed in the form a + bi where a is the real part and bi is the imaginary part. This form is referred to as the rectangular form of a complex number and is sometimes expressed as (a, b). There are many applications of complex numbers that are associated with the amplitude of the complex number. One section of this LAP will be devoted to developing a form to express complex numbers using trigonometric functions.

In a LAP on trigonometric functions, you learned how to determine the function of an angle, but you did not learn how all these values were arrived. You will study series in this LAP that will enable you to compute any of the six functions to a desired degree of accuracy.

SECTION I

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

1. Take a given complex number and:

   a) plot it on a rectangular coordinate system
   b) compute its modulus and/or draw a line on a coordinate system to represent its modulus
   c) determine its amplitude correct to the nearest 10 minutes by use of trigonometric tables

2. Express a complex number given in rectangular form in trigonometric form and vice-versa.

3. Compute the product and quotient of any two complex numbers expressed in trigonometric form.

4. State and prove DeMoivre's Theorem and apply the statement of this theorem to determine the value of a complex number raised to the p^{th} power.

5. Apply the statement of DeMoivre's Theorem to determine the p^{th} root of any given complex number.

SECTION I

RESOURCES

I. Readings:

1. Vanatta: #1 pp. 25-28, pp. 183-184; #2 pp. 289-291; #3 pp. 291-293; #4 pp. 294-295; #5 pp. 295-297.

2. Rees: #1-#2 pp. 320-321; #3 pp. 321-322; #4 p. 324; #5 pp. 324-326.

3. White: #1-#2 pp. 83-84, pp. 88-90; #3 pp.91-92; #4-#5 pp. 93-96.

4. Vance: #1-#3 pp. 166, 353-358; #4-#5 pp. 361-363.

5. Fisher: #1-#3 pp. 186-189; #4-#5 pp. 190-194.

6. Dolciani: #1-#3 pp. 258, 493-496; #4-#5 pp. 498-501.

II. Problems:

1. Vannatta: #1 p. 38 ex. 1-2, p. 184 ex. 3, 7, 8; #2 p. 291 ex. 1-14; #3 p. 293 ex. 1-11; #4 p. 295 ex. 1-8; #5 p. 297 ex. 1-8.

2. Rees: #1-#3 pp. 323-324 ex. 1-48 (every third exercise); #4-#5 pp. 330-331 ex. 1-8, 17-36 (odd numbered exercises).

3. White: #1-#2 p. 84 ex. 1, 3, 5, 7, pp. 90-91 ex. 1-20 (even numbered exercises); #3 pp. 92-93 ex. 1-8; #4-#5 p. 96 ex. 1-18 (even).

4. Vance: #1-#3 p. 359 ex. 17-21, 23; #4-#5 pp. 363-364 ex. 1-19 (odd numbered exercises).

5. Fisher: #1-#3 pp. 189-190 ex. 2, 8-9; #4-#5 p. 194 ex. 1, 3.

6. Dolciani: #1-#3 p. 497 ex. 1-24 (even numbered exercises); #4-#5 p. 502 ex. 1-6, 11-18.

SELF-EVALUATION I

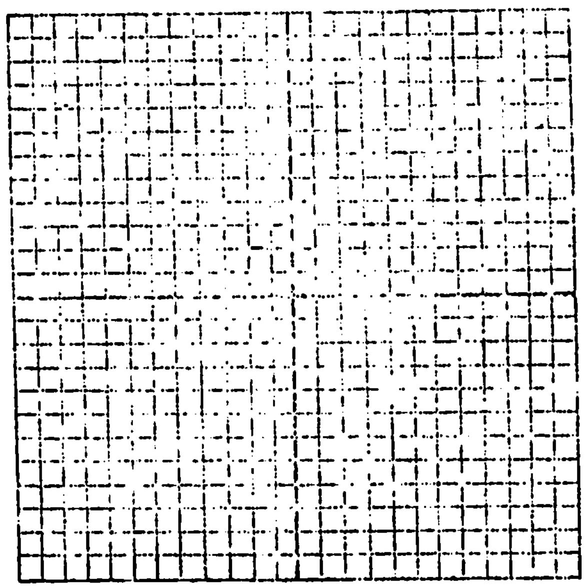
1. For the given complex numbers:

   a) plot them on the graph

   b) compute their modulus and draw a line on the graph to represent their modulus

   c) determine their amplitude.

   (1) $3 - 4i$ (2) $6i - 5$ (3) -5 (4) $8i$



2. Express the complex numbers given in problem 1 in trigonometric form.

3. Express the following complex numbers in rectangular form:

   a) $3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$ (b) $2(\cos 0 + i \sin 0)$

   c) $3\sqrt{3}\,(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$

4. Simplify the following and express your answer in the form a + bi.

(a) $2\sqrt{3}\,(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) \div \sqrt{27}\,(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})$

(b) $$\frac{6\sqrt{2}\,(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}) \cdot 5\,(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})}{7(\cos\pi + i\sin\pi) \cdot 9\sqrt{2}\,(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8})}$$

5. State and prove DeMoivre's Theorem.

6. Simplify the following and express your answer in rectangular form:

(a) $(4 + 5i)^4$ (b) $\left[7\,(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})\right]^6$

7. Determine one root in each of the following:

(a) $(1 - i)^{\frac{1}{3}}$ (b) $(-6)^{\frac{1}{4}}$ (c) $\left[3(\cos\pi + i\sin\pi)\right]^{\frac{1}{2}}$

IF YOU HAVE MASTERED ALL THE OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. a) Prove: The reciprocal of $r(\cos \theta + i \sin \theta)$ is $\frac{1}{r}(\cos \theta - i \sin \theta)$

   b) State the conditions under which the conjugate and reciprocal of a complex number are equal.

2. Suppose Z is a complex number such that $Z^6 = 1$. If $R = Z^5 + Z^4 + Z^3 + Z^2 + Z + 1$, show that $RZ = R$. What can you conclude about R?

3. Let U and V denote the points representing $u = r(\cos \theta + i \sin \theta)$ and $v = s(\cos x + i \sin x)$ where θ and x are acute angles. Let O denote the origin, A the point $(1,0)$ and P the point that represents the product uv. Show that triangle OVP is similar to triangle OAV.

4. Apply the binomial theorem and DeMoivre's Theorem to $(\cos \theta + i \sin \theta)^3$ to prove that $\cos 3\theta = 4 \cos^3\theta - 3 \cos \theta$ and $\sin 3\theta = 3 \sin \theta - 4 \sin^3\theta$.

5. Let x_1 and x_2 be real numbers. Prove:

   (a) $e^{ix_1} \cdot e^{ix_2} = e^{i(x_1 + x_2)}$ (b) $\dfrac{e^{ix_1}}{e^{ix_2}} = e^{i(x_1 - x_2)}$

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

6. Evaluate $\lim_{\alpha \to 0}$ for any quotient involving α, $\sin k\alpha$, $\cos k\alpha$, and $\tan k\alpha$.
7. Determine a value for $\sin x$ and $\cos x$ to a specified number of decimal places by using the trigonometric series.
8. Use the exponential series to compute e^x correct to a specified number of decimal places.
9. Derive a formula to determine the amount of money A you have if you invest a principal P for k number of years when the interest rate is r and the interest is compounded continuously and where $rk = 1, 2, 3, 4, 5$ and solve problems relating to this formula.
10. Derive Euler's Formulas and solve problems relating to these formulas.

Resources

I. Reading:

1. Vannatta: #6 pp. 297-299; #7 pp. 299-301; #8 pp. 301-302; #9 pp. 303-304; #10 pp. 304-305.

II. Problems:

1. Vannatta: #6 p. 299 ex. 1-14; #7 p. 301 ex. 1-6; #8 p. 302 ex. 1-2; #9 p. 304 ex. 1-3; #10 p. 306 ex. 1-11.

SELF-EVALUATION II

1. Evaluate $\lim_{\alpha \to 0} \frac{\tan 2\alpha}{\alpha}$.

2. Compute the following correct to three decimal places using the trigonometric series:

   (a) $\sin \frac{\pi}{8}$ (b) $\cos \frac{5\pi}{12}$

3. Compute e^3 correct to three decimal places.

4. If \$600 is invested at 8% compounded continuously for $12\frac{1}{2}$ years, what amount of many do you have at the end of this time?

5. Derive the formula $\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$.

6. a) Use Euler's Formulas to show that $\cos 2x = 2 \cos^2 x - 1$.

   b) Express the following in exponential form:

   $3 (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$

   c) $\log_e (-2) = ?$

IF YOU HAVE MASTERED THE OBJECTIVES, TAKE YOUR LAP TEST.

ADVANCE STUDY II

1. a) Compute $\sec \frac{\pi}{12}$ correct to four decimal places.

   b) Compute $\csc \frac{\pi}{8}$ correct to four decimal places.

   c) Compute e^4 correct to three decimal places.

   d) Compute $\tan \frac{5\pi}{6}$ correct to four decimal places.

   e) Compute $\cot \frac{7\pi}{8}$ correct to four decimal places.

2. Evaluate the following:

   a) $\lim\limits_{\alpha \to 0} \frac{\sin 2\alpha}{2\alpha}$

   b) $\lim\limits_{\alpha \to 0} \frac{\sin^2 \alpha + 2\alpha \cos^2 \alpha}{\alpha}$

   c) $\lim\limits_{\alpha \to 0} \frac{1}{\alpha \sqrt{\cot^2 \alpha + 1}}$

   d) $\lim\limits_{\alpha \to 0} \frac{\tan^2 \alpha}{\alpha^2}$

   e) $\lim\limits_{\alpha \to 0} \frac{\tan 2\alpha + \frac{1 - \cos \alpha}{\sin \alpha}}{\alpha}$

   f) $\lim\limits_{\alpha \to 0} \frac{1}{\alpha \sqrt{\csc^2 \alpha - 1}}$

3. a) Prove: $\tan \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{ie^{i\alpha} + ie^{-i\alpha}}$

ADVANCED STUDY II (cont')

b) Use Euler's Formulas to prove $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

c) Use Euler's Formulas to prove $\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$.

d) Use Euler's Formulas to prove $\cos (x - y) = \cos x \cos y + \sin x \sin y$.

BIBLIOGRAPHY

I. Textbooks:

1. Vannatta, Carnahan, Fawcett: Advanced High School Mathematics, Expanded Ed. (Charles E. Merrill Publishing Co., 1965).

2. Rees, Sparks: Algebra and Trigonometry, 2nd Edition (McGraw-Hill, Inc., 1969).

3. White: Advanced Algebra - (Allyn and Bacon, Inc., 1962).

4. Vance: Modern Algebra and Trigonometry, 2nd Edition (Addison-Wesley Publishing Co., Inc., 1968).

5. Fisher, Ziebur: Integrated Algebra and Trigonometry, 2nd Edition (Prentice-Hall, Inc., 1967).

6. Beckenbach, Dolciani, Donnelly, Jurgensen, Wooton: Modern Introductory Analysis (Houghton Mifflin Co., 1970).