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ABSTRACT

This paper focuses on the relationship of verbal factors to mathematics achievement and reviews research from 1930 to the present. The effects of verbalization in the mathematics learning process are considered; mathematics functioning as a unique language in its own right is analyzed. Research on the readability of mathematics materials and research relating problem-solving abilities to verbal abilities are reviewed. An extensive bibliography is included. (Editor/DT)

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MATHEMATICS EDUCATION REPORTS

LANGUAGE FACTORS

IN LEARNING MATHEMATICS

by Lewis R. Aiken, Jr.

ERIC Information Analysis Center for
Science, Mathematics, and Environmental Education
1460 West Lane Avenue
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Mathematics Education Reports

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Foreword

The nature of the interaction between mathematical, verbal, and general intellectual abilities with achievement in mathematics has long been a primary research concern of mathematics educators. This paper focuses on the relationship of verbal factors to mathematics achievement, and reviews relevant research from 1930 to the present. The paper considers the effects of verbalization in the mathematics learning process, and analyses mathematics as a unique language in its own right. Research on the readability of mathematics materials is also reviewed. Because problems in mathematics are so often presented verbally, a separate section details the research which relates problem-solving abilities to verbal abilities.

The extensive bibliography which is attached to this review should help guide the reader to documents in this area which are available through the ERIC system.

Jon L. Higgins
Editor

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LANGUAGE FACTORS IN LEARNING MATHEMATICS

Lewis R. Aiken, Jr.

Guilford College

It is generally recognized that not only do linguistic abilities affect performance in mathematics but that mathematics itself is a specialized language. Monroe and Englehart (1931) summarized some of the earlier research on the relationship of reading ability to problem solving. More recently, the writer (Aiken 1971 b) briefly reviewed research concerned with verbal factors in mathematics learning and teaching conducted during the past four decades. As one reader of that paper pointed out, however, a review of studies pertaining to the effects of all language factors would be more useful. This is the intention of the present paper. Although many of these studies involve only a few variables, are not clearly tied to other investigations, and frequently pose more questions than they answer, a number of implications and suggestions for further research are embedded in them.

Mathematical, Verbal, and General Intellective Abilities

Reading Ability

It is not difficult to understand how reading ability could affect performance on verbal arithmetic problems, and supporting data are plentiful. Table 1 summarizes the results of a representative sample of studies in which various measures of general and specific reading abilities have been found to be correlated positively with scores on arithmetic and mathematics tests. These investigations, the majority of which have been based on children in the intermediate grades, yielded correlations between reading ability and mathematics achievement ranging between .40 and 186 (see column 3 of Table 1).

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Table 1

Correlations of Mathematics Achievement with Reading Ability and General Intelligence

Reference and Sample	Variables	Zero-order r	Partial r[a]
Balow (1964), 368 California	1. Stanford Achievement Test--Arithmetic Reasoning	$r_{12} = .46$	$r_{12.3} = .13$
	2. Stanford Achievement Test Reading		
	3. California Short-Form Test of Mental Maturity		
Chase (1960), 119 California sixth graders	1. Gates Reading to Note Details	$r_{14} = .40$	$r_{14.23} = .27$
	2. Primary Mental Abilities--Verbal	$r_{24} = .34$	$r_{24.13} = .21$
	3. Primary Mental Abilities--Number		
	4. Iowa Every Pupil Test--Arithmetic (problems section)		
Cleland & Toussaint (1962), 172 intermediate-grade pupils in Western Pennsylvania	1. Gates Reading Survey--Form 2	$r_{14} = .49$	$r_{14.5} = .45$
	2. STEP Listening Comprehension	$r_{24} = .41$	$r_{14.6} = .46$
	3. Durrell-Sullivan Reading Capacity	$r_{34} = .46$	$r_{24.5} = .36$
	4. American School Arithmetic		$r_{24.6} = .37$
	5. Primary Mental Abilities-- Total		$r_{34.5} = .45$
	6. Stanford-Binet Intelligence Scale		$r_{34.6} = .42$

Table 1 (cont'd)

Cottrell (1968), 18 New York third-grade arithmetic underachievers	1. Stanford Achievement Test--Arithmetic	$r_{12} = .86$	
	2. Stanford Achievement Test--Reading		
Erickson (1958), 269 Indiana sixth graders	1. ITBS (sixth-grade arithmetic section)	$r_{12} = .62$	
	2. Iowa Silent Reading Test (Vocabulary)	$r_{13} = .67$	
	3. Iowa Silent Reading Test (Reading Comprehension)		
Ivanoff, DeWane & Praem (1965), 286 Wisconsin males in ninth-grade algebra and 162 males in general mathematics	1. High School Placement Test--IQ Rating	$r_{12} = .69$	$r_{12.6} = .56$
	2. High School Placement Test--Reading	$r_{13} = .59$ $r_{14} = .61$	$r_{13.2} = .37$ $r_{13.4} = .37$
	3. High School Placement Test--Arithmetic	$r_{23} = .52$ $r_{25} = .50$	$r_{14.6} = .41$ $r_{23.1} = .19$
	4. High School Placement Test--Language	$r_{26} = .59$	$r_{25.1} = .22$
	5. Eighth-grade mathematics mark	$r_{34} = .53$ $r_{45} = .55$	$r_{26.1} = .41$ $r_{34.1} = .31$
	6. Successful completion of general mathematics vs. algebra	$r_{46} = .53$	$r_{45.1} = .34$ $r_{46.1} = .32$

Table 1 (cont'd)

Muscio (1962), 206 California sixth-grade boys and girls	1. California Vocabulary Tests, Mathematics	$r_{17} = .78$	$r_{17.6} = .50$
	2. Stanford Reading Tests, Paragraph Meaning	$r_{27} = .78$	$r_{27.6} = .48$
	3. Stanford Reading Tests, Word Meaning	$r_{37} = .78$	$r_{37.6} = .47$
	4. Gates Basic Reading Tests, Test C	$r_{47} = .68$	$r_{47.6} = .34$
	5. Gates Basic Reading Tests, Test D	$r_{57} = .55$	$r_{57.6} = .27$
	6. California Test of Mental Maturity (mental ages)		
	7. Functional Evaluation in Mathematics, Test 1: Quantitative Understanding		
Pitts (1952), 210 Georgia eleventh-grade Negro girls	1. Davis Test of Functional Competence in Mathematics	$r_{12} = .53$	$r_{12.3} = .31$
	2. Iowa Silent Reading Tests (reading grade levels)	$r_{13} = .46$	$r_{13.2} = .12$
	3. Otis Gamma IQs		

[a]In many of the papers, the zero-order and/or first-order partial correlation coefficients were not reported in the papers but were computed by the present writer to serve as a basis for comparing the results of different investigations.

Obviously, as Monroe and Englehart (1931) concluded some 40 years ago about related investigations, these findings are open to various interpretations. One recommendation made in that classic paper was that further research on the relationships between reading abilities and mathematics achievement should be directed toward specific reading skills rather than general reading ability. In response to this recommendation, a number of studies have been concerned with specific reading skills in mathematics (e.g., Johnson, 1949; Henney, 1969). Unfortunately, the results have not consistently demonstrated superior predictive validity for measures of specific reading abilities, either singly or in combination, when compared to measures of general reading ability. For example, Henney (1969) reported that specific reading abilities were no more highly correlated than general reading ability with arithmetic problem solving in a large sample of fourth graders.

Perhaps what is required is an extensive cross-sectional study of the relationships of various aspects of verbal (linguistic) ability to performance on a variety of mathematical tasks. Some of the data included in the 1963 Technical Report on the California Achievement Tests are representative of findings cited in various sources. These data, presented in Table 2, show that Reading Vocabulary, Reading Comprehension, Mechanics of English, and Spelling have higher correlations with Arithmetic Reasoning than with Arithmetic Fundamentals at all elementary grade levels. However, the correlations of these four linguistic tests with Arithmetic Fundamentals are also sizable.

The findings of other investigations (e.g., Martin, 1964; Wallace, 1968; Harvin & Gilchrist, 1970) underscore the relationships between problem solving and reading ability. Thus, Martin (1964) obtained the following result from administering the Iowa Tests of Basic Skills to fourth and eighth graders.

Table 2

Correlations of California Achievement Tests--Arithmetic

With CAT Reading, English, and Spelling Tests[a]

Test	Lower Primary (Grade 1)		Upper Primary (Grade 2)		Upper Primary (Grade 3)		Elementary (Grade 4)		Elementary (Grade 5)	
	AR	AF	AR	AF	AR	AF	AR	AF	AR	AF
Arithmetic Reasoning (AR)										
Arithmetic Fundamentals (AF)	.63		.65		.53		.56		.76	
Reading Vocabulary	.63	.59	.76	.58	.71	.44	.69	.46	.69	.64
Reading Comprehension	.56	.44	.77	.57	.65	.35	.60	.34	.75	.68
Mechanics of English	.65	.59	.79	.62	.65	.43	.61	.49	.75	.65
Spelling	.58	.49	.60	.49	.62	.37	.56	.41	.61	.59

[a]From Technical Report on the California Achievement Tests, 1963 Norms. Monterey, Calif.: California Test Bureau, 1967.

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The partial correlation[1] between reading comprehension and problem solving abilities, with computational ability partialed out, was higher at both grade levels than the partial correlation between computational ability and problem solving ability, with reading comprehension partialed out. Finally, Murray (1949) cited evidence that performance on a geometry test, which one might suspect to depend greatly on spatial ability, was also closely related to the verbal abilities of certain students.

General Intelligence

In addition to the fact that they are related to each other, scores on tests of mathematical and verbal abilities are also correlated with general intelligence. Consequently, the positive correlation between the first two variables may be explicable in terms of their common correlation with the latter variable. For example, as the partial correlation coefficients in column 4 of Table 1 reveal, the correlation between reading (or other measures of verbal ability) and mathematical achievement may decrease substantially when the joint relationship of these two variables with general intelligence is partialed out.

Underlying many of these studies is the recognition that mathematical ability is not a unitary concept. Thus, certain researchers (Coleman, 1956; Skemp, 1961) have reported some factor-analytic evidence for the existence of the mathematics educator's "computation and structure" dimensions. And both Werdelin (1966) and Kline (1960) have cited evidence for as many as five different factors involved in mathematics performance. The results of different methods of factoring and rotation, however, do not always agree, and neither do the conclusions of different interpreters of the same factor

[1]The partial correlation coefficient is a measure of that part of the relationship between two variables which cannot be explained by their joint relationship to a third variable (the variable partialed out).

structure. In addition, the tests used and the characteristics of the particular sample of examinees-- sex, grade level, nationality, educational background, social class, etc.-- influence the results of factor analysis.

Especially relevant to this discussion is Wrigley's (1958) argument for the existence of separate mathematical and verbal factors. On the basis of several factor analyses, Wrigley concluded that high general intelligence is the first requirement for success in mathematics, and that the positive correlations between measures of verbal and mathematical abilities can be explained by the joint relationship of these two variables to general intelligence. Therefore, he argued, the portion of verbal ability not included in general intelligence does not contribute to achievement in mathematics.

A comparison of the zero-order correlations in column 3 of Table 1 with the corresponding first-order partial coefficients (general intelligence test scores being partialed out) in column 4 supports the hypothesis that the relationship between verbal and mathematical ability is affected by their common correlation with general intelligence. Nevertheless, statistical tests also show that most of these first-order partial correlations are significantly greater than zero. Thus, Wrigley's conclusion is not a completely accurate picture of the situation. General intelligence can account for a substantial portion of the variability shared by verbal and mathematical abilities, but a significant degree of overlap between the last two variables remains unexplained.

On the other hand, inspection of the data in columns 3 and 4 of Table 1 also reveals that the correlation between general intelligence and mathematical ability is appreciably reduced when reading ability or scores on other linguistic tests are partialed out. This result might lead one to argue that the pivotal variable is not general intelligence but rather reading (linguistic) ability. Differences in reading ability may serve to explain the positive correlation between scores on mathematics tests and general intelligence tests. Evidence supporting this hypothesis was provided by Muscio (1962), who, arguing against Wrigley's (1958) thesis of a general mathematical factor, maintained that arithmetic achievement depends on both general intelligence and verbal ability. Furthermore, Cottrell (1968) interpreted the positive correlations which he observed among reading, psycholinguistic, general mental ability, and arithmetic factors as being due to a general linguistic ability.

Mathematical Vocabulary, Syntax,[2] and Readability

Among investigations of the relationships between mathematical abilities and specific aspects of linguistic ability, particular attention has been directed toward vocabulary, and, to a lesser extent, syntax. The results of studies conducted some years ago (e.g. Hanson, 1944; Treacy, 1944) indicate that knowledge of vocabulary is important in solving mathematics problems and consequently should be a goal of mathematics instruction. A fairly comprehensive correlational study reported a few years later (Johnson, 1949) involved administering six tests of arithmetic problems and six of the Primary Mental Abilities tests to several hundred Chicago school children. The following correlations between PMA Vocabulary and the standardized achievement tests were obtained: Stanford Arithmetic Reasoning (.51), Chicago Survey Test in Arithmetic (.50), Stone Reasoning Test (.45). Furthermore, PMA Vocabulary correlated more highly with scores on a non standardized test composed of problems with numbers (.40) than with scores on a test of problems without numbers (.26).

[2] As used in this context, _syntax_ refer to the way in which words and symbols are put together to form the phrases and sentences of mathematics problems.

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Difficulty Level of Vocabulary and Syntax

Although one recent report (Olander & Ehmer, 1971) suggests that understanding of mathematical terms on the part of elementary school pupils has significantly improved during the past 40 years, it is reasonable to assume that difficult vocabulary and syntax continue to interfere with effective problem solving. This hypothesis was confirmed in a study by Linville (1970). Four arithmetic word-problem tests, each consisting of the same problems but varying in difficulty of syntax and vocabulary, were prepared: 1. Easy Syntax, Easy Vocabulary; 2. Easy Syntax, Difficult Vocabulary; 3. Difficult Syntax, Easy Vocabulary; 4. Difficult Syntax, Difficult Vocabulary. The four tests were randomly assigned and administered to 408 fourth-grade students in 12 schools. Analysis of variance of the results revealed significant main effects in favor of both the easy syntax and easy vocabulary tests. The authors concluded that both syntactic structure and vocabulary level, with vocabulary level perhaps being more crucial, are important variables in solving verbal arithmetic problems. A secondary finding of the study was that, regardless of treatment condition, pupils of higher general ability and/or higher reading ability made significantly higher scores on the arithmetic problems than pupils of lower ability.

Training in Vocabulary and Syntax

Another approach to studying the relationship of knowledge of vocabulary and/or syntax to achievement in mathematics is to determine whether specific training in vocabulary has any effect on mathematical performance. For example, both Dresher (1934) and Johnson (1944) found gains in problem-solving ability when pupils were given specific training in mathematics vocabulary. More recently, in an investigation which unfortunately suffered from the lack of a separate control group and certain other shortcomings.[3] Lyda and Duncan (1967) found that direct study of quantitative vocabulary contributed to growth in reading, arithmetic computation, and arithmetic reasoning by 25 second graders. Also concerned with arithmetic vocabulary in the early grades was a recent survey by Wilmon (1971) of selected primary arithmetic books. This survey revealed that children are introduced to approximately 500 new technical words and phrases by the time they reach the fourth grade. These findings led Wilmon to conclude that teachers need to reinforce the textbooks by concentrating more on specialized mathematics reading vocabulary in the first three grades.

Somewhat more carefully designed than the Lyda and Duncan (1967) investigation was VanderLinde's (1964) experiment with nine fifth-grade classes matched with nine control classes on IQ and scores on achievement tests in vocabulary, reading comprehension, arithmetic concepts, and arithmetic problem solving. The experimental classes studied a different list of eight quantitative terms each week for 20-24 weeks, after which the achievement tests were readministered. Analysis of results revealed significantly greater gains by the experimental than by the control classes on both arithmetic concepts and problem-solving. There were no sex differences in gains on the achievement tests, but students with low IQs showed smaller gains than students with average or above-average IQs.

Parallel to Linville's (1970) demonstration that the difficulty level of the syntax in which verbal problems are phrased affects the ease with which they are solved, Sax and Ottina (1958) found that specific training

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[3] For example, reported achievement gains of 1.4, 2.3, and 2.9 months, assessed over a period of two months, may be statistically significant but hardly surprising!

in syntax can also improve performance. It was shown that training in syntax elevated the mathematics achievement of seventh-graders who had no arithmetic training in the early grades, when compared with seventh-graders who had arithmetic in the early grades.

Measuring the Readability of Mathematics Materials

Concerning the relationships of vocabulary and syntax to ease of reading, several types of readability formulas have been applied to mathematics texts and problems. The most popular are the Dale-Chall formula, the Spache formula, and the Cloze technique. Kane (1968, 1970) has given detailed reasons why readability formulas for ordinary English prose are usually inappropriate for usewith mathematics materials. According to Kane, ordinary English and mathematical English differ in that: (1) letter, word, and syntactical redundancies are different; (2) in contrast to ordinary English, in mathematical English the names of mathematical objects usually have a single denotation; (3) adjectives are more important in mathematical English than in ordinary English; (4) the grammar and syntax of mathematical English are less flexible than in ordinary English.

In spite of Kane's disclaimer, the Dale-Chall formula (Dale & Chall, 1949), which requires counting the number of unfamiliar words in passages to be rated, has been employed in a number of investigations. For example, Shaw (1967), using both the Dale-Chall and Spache formulas, found a wide range of readability levels in selected California public school mathematics textbooks. And Thompson (1968), also applying the Dale-Chall and Spache formulas in California, studied the effects of the readability level of arithmetic problems on the mathematical performance of 368 sixth graders. He found that readability affected performance at both of the IQ levels studied (above 110 and below 100), but it had a greater effect with pupils whose IQs were below average.

Although Kane (1970) states that there is no logically defensible approach to assessing the readability of mathematics textbooks, a recent study by Hater and Kane (1970) of the Cloze technique suggests that this procedure can be quite useful. In Hater and Kane's analysis, Cloze tests were found to be highly reliable and valid predictors of the comprehensibility of mathematical English passages designed for grades 7-12. But whatever readability formula is employed, in a synthesis of the literature on reading in mathematics Earp (1969) noted that the vocabulary of arithmetic texts is frequently at a higher readability level than the performance level of students in the grades where the texts are used. In addition, the vocabulary of arithmetic texts does not greatly overlap that of reading texts. However, in a survey of the readability (as measured by the Dale-Chall formula) of sixth-grade arithmetic textbooks, Smith (1971) reported that the average readability of the problems fell within bounds for the grade level. Although the results indicated wide variation from problem to problem of the same text, the reading levels of the texts were also generally comparable to those of related mathematics achievement tests. Since the readability of the sixth-grade texts and tests were at an average level generally considered appropriate for that grade, Smith concluded that readability may not be the primary cause of low scores on arithmetic problem-solving in the sixth grade.

Reading Instruction and Mathematics Learning

Clearly, understanding of the meanings of words and syntax is essential in learning to read all types of materials.[4] This is especially true in regard to modern mathematics programs, which emphasize concepts that require more verbal explication than traditional mathematics programs (see Lovell, 1971, p. 15). But as Henney (1971) explains, students find reading mathematics to be different from reading other materials and often quite difficult. Several reasons why students experience difficulty in reading arithmetic are given by Spencer and Russell (1960): (1) the names of certain numerals are confusing; (2) number languages which are patterned differently from the decimal system are used: (3) the language of expressing fractions and ratios is complicated: (4) charts and other diagrams are frequently confusing; (5) the reading of computational procedures requires specialized skills. Therefore, the question arises as to whether detailed instruction in reading, and especially reading in mathematics, can improve mathematical achievement.

Experiments on Reading in Mathematics

It is difficult to conduct controlled experiments in educational settings, but in recent years several experiments or quasi-experiments concerned with

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[4]Training in reading is not invariably an important prerequisite to understanding particular aspects of mathematics. For example, Symmes and Rapoport (Report on Educational Research, Aug. 18, 1971), noting that difficulty in reading is sometimes related to a child's talent for space visualization, hypothesized that the correlation is due to a sex-linked recessive gene (also see Garron, 1970). The result, they maintain, is better space visualization but poorer reading ability in boys than in girls. Consequently, it is suggested that such children might do better in school if they studied geometry before reading.

the effects of instruction in reading on achievement in mathematics have been reported (see Earp, 1970b for a brief review). Gilmary (1967) compared two groups of elementary school children in a six weeks summer school program in remedial arithmetic. The experimental group had instruction in both reading and arithmetic, whereas the control group had instruction in arithmetic only. On the Metropolitan Achievement Test-Arithmetic the experimental group gained one-third of a grade more than the controls. Furthermore, when differences in IQ were statistically controlled by covariance analysis the experimental group gained one-half of a grade more on the test than the control group.

In a study of the effects of special reading instruction, Henney (1969) divided 179 fourth graders into two groups. Over a period of nine weeks, Group 1 (N=88) received 18 lessons in reading verbal problems. On alternate days during this time period, Group 2 (N=91) studied and solved verbal problems in any way that they chose under the supervision of the same instructor as Group 1. Although both groups improved significantly from pretest to posttest on a verbal problems test, the difference between the mean posttest scores of the groups was not significant. However, the girls in Group 1 made a higher mean score on the verbal problems posttest than the boys in that group.

In an experiment with high school students, Call and Wiggin (1966) investigated the effects of two different methods (or rather, two different teachers) on the teaching of second-year algebra. The experimental group was taught by an English teacher (Wiggin) with some training in teaching reading but no experience in teaching mathematics. The control group was taught by an experienced mathematics teacher (Call). The major difference between the two instructional methods was the fact that the English teacher stressed understanding the meanings of words in mathematics problems and translating the English statements into mathematical symbols. This teacher's procedure was more like that used in teaching reading rather than mathematics. The outcome of this quasi-experiment was that the experimental group did better on the criterion test in mathematics than the control group, even when initial differences in reading and mathematics test scores were statistically controlled.

Specific Techniques and Teaching Procedures

A number of specific techniques for motivating students and helping them to understand mathematics have been described in recent articles. Some of these procedures are directly implied by the findings of empirical research, whereas others have been derived from informal classroom observations. For example, Strain (1969) and Phillips (1970) discuss the use of children's literature as a means of motivating elementary school pupils and conveying mathematical ideas to them. Phillips suggests that the teacher keep books such as The Dot and the Line (by Norton Juster, Random House, 1963) on hand and periodically read a story, part of a book, or a poem related to mathematics to the class. She feels that this procedure can affect not only mathematical understanding but also pupils' attitudes toward mathematics. Strain (1969) maintains that using children's literature in mathematics classes can clarify ideas, illustrate practical applications of mathematical ideas, stimulate creative expression, and develop vocabulary skills. Both authors give examples of children's books that are appropriate for these purposes.

It is a truism that the best teaching starts with what the pupil already knows and proceeds from there. Illustrative of this principle is Capps'

(1970) observation that mathematical concepts such as commutativity, associativity, and the distributive property have their analogies in language arts. By pointing out these analogies to students and challenging them to find others for themselves, improved motivation and understanding in mathematics and language arts may develop.

Particularly troublesome to pupils are verbal problems in mathematics. As might be expected, the word clues or verbal hints included in a problem may facilitate finding the solution (see Early, 1968 and Wright, 1968). But in such cases children are frequently able to find the answer to the problem without really trying to understand it. What is needed are procedures that assist children in analyzing the problem, setting it up, and arriving at the answer without literally having the solution given to them. In this regard, helpful suggestions have been made by Schoenherr (1968), Pribnow (1969), and Henney (1971); more detailed approaches are described by Earp (1970a), Taschow (1969) and Dhamus (1970).

Earp (1969) notes that verbal arithmetic problems, which have a high conceptual density factor, include three types of symbolic meanings — verbal, numerical, and literal — in a single task. Consequently, three kinds of reading adjustment are required: (1) adjustment to a slower rate than that for narrative materials; (2) varied eye movements, including some regressions; (3) reading with an attitude of aggressiveness and thoroughness. In addition, particular attention must be paid to special uses of common words. Earp (1970a) lists five steps in reading verbal problems: (1) read first to visualize the overall situation; (2) read to get the specific facts; (3) note difficult vocabulary and concepts and get the teacher's assistance when needed; (4) reread to help plan the solution; (5) reread the problem to check the procedure and solution.

Some approaches to teaching verbal problem-solving have been christened with symbols. For example, Taschow (1969) describes a remedial-preventive procedure in mathematical reading. This includes administration of a Group Informal Reading Inventory designed to identify students who do not know how to read and think through mathematics problems, followed by the Directed Reading Activity in Algebra, or DRA. The DRA consists of five phases: (1) readiness, (2) guided silent reading, (3) questions (4) oral reading (only when needed), and (5) application.

Dahmus (1970) discusses a "direct-pure-piecemeal-complete," or DPPC, method of solving verbal problems. The characteristics of this method are concrete translation of all facts and the recognition that the best way to become a good problem solver is to solve many problems. Using the DPPC method, by concentrating on a few words at a time the student first learns to convert English statements into mathematical statements. Next, by writing down everything in piecemeal fashion he learns to solve equations and finally systems of equations. Clearly, the DPPC is a concrete, non-Gestalt method that leaves little to sudden discovery or insight.

One conclusion to be drawn from studies reviewed thus far is that instruction in reading in general or the reading of mathematics in particular improves performance in the latter subject. It seems reasonable that attempting to cultivate the skill of reading carefully and analytically in order to note details and understand meanings, thinking about what one is reading, and translating what is read into special symbols would improve performance on many types of mathematics problems. Of course, there are other skills, such as logical reasoning and the ability to discover and formulate mathematical generalizations, that might also be stressed as a means of improving mathematical abilities. In any case, the findings of research on reading in mathematics underscore the importance of a particular language factor, that of verbal reading ability, to mathematics achievement. This "verbal" theme will be pursued a bit further before shifting the focus of the paper to mathematics as a language in more general terms.

Verbalizations, Verbal Interaction, and Mathematics Learning

Verbalization vs. Nonverbalization

Another aspect of the relationship between language and mathematics learning is the effect of student and/or teacher verbal behavior. An important principle in the psychology of learning is that learning with awareness ("insight" or understanding) is more permanent than learning without awareness. In addition, it seems reasonable to suppose that requiring the learner to verbalize a mathematical principle or concept after he appears to understand it might increase his degree of awareness of that particular abstraction and help to fix it in his mind. However, not all of the findings of research and informal observation are consistent with this supposition. Consequently, whereas certain mathematics educators are proponents of verbalization, others maintain that having to verbalize a mathematical discovery either adds nothing to one's understanding of a generalization or even interferes with his ability to apply the generalization.

Some investigators have reported positive effects of verbalization on learning and problem solving at the elementary school level. For example, in a study by Irish (1964), fourth-grade teachers spent part of the class time that was usually spent on computation in helping pupils state generalizations about number problems. The results were that these pupils made greater yearly gains on the STEP Mathematics Test than other pupils in the school system. Several projects on improving ability to verbalize mathematical generalizations have also been conducted (Elder, 1969; Retzer, 1969). Elder (1969) demonstrated that explicit instruction in certain topics of logic improved the ability of college algebra students to verbalize three generalizations which they discovered while working through a programed unit on vectors. Furthermore, Retzer (1969) found that teaching certain concepts of logic not only had differential effects on eighth graders' abilities to verbalize mathematical generalizations, but that students with high verbalization abilities could better transfer learned mathematical generalizations. However, these results may be partly accounted for by other intellectual abilities that are related to verbalization ability.

Unfortunately, being able to verbalize a concept does not guarantee better performance on problem-solving tasks. Thus, in a study of concept learning in third graders, Stern (1967) found that requiring children to say the concept aloud was no more effective in improving problem solving than not requiring the children to make overt verbal responses. Also, Palzere (1968) found a non-significant difference between the posttest problem-solving scores of secondary students who were required to verbalize a concept after demonstrating awareness of it and those who did not verbalize it. To be sure, modes of thinking other than the verbal one are also involved in problem solving, and level of verbal awareness varies with the individual. In addition, a great deal of covert talking to oneself undoubtedly occurs during the problem-solving process, and this may be sufficient verbalization to facilitate problem solving. Therefore, it is perhaps an oversimplification to expect overt verbalization to be consistently effective in improving understanding and ability to solve problems.

One aspect of the relationship between verbalization and problem solving that has not been studied adequately in mathematics learning is the effect of verbalization on long-term retention of problem solutions and conceptual understanding. It is possible that the really important influence of verbalization is on more permanent retention of learned material. For example, the results of research on the relationship between language and cognition indicate that the process of linguistic encoding, as in giving something a name, improves both recall and recognition of that thing (see Brown and Lenneberg, 1954). The question arises as to whether this is primarily the result of increased attention or whether linguistic encoding acts as a kind of advance organizer (see Ausubel, 1960) that gives greater meaning to the learned material.

Among the proponents of nonverbal awareness is Gertrude Hendrix (1961). Hendrix admits that communication plays an important role in setting the stage for discovery in mathematics, but that early verbalization of a discovery may actually decrease the ability to apply that generalization. Similarly, Ahlfors et al. (1962) and Wirtz[5] have criticized the wordiness of mathematics programs that overemphasize deduction and language at the expense of inductive processes involving experimenting with objects and reporting what happens. These educators maintain that language is frequently an obstacle rather than a help in understanding mathematics, and there is some supporting evidence for this point of view. For example, in studies of the mental processes employed by high school students in setting up algebraic equations, Paige and Simon (1966) found that contradictions in problems were detected less often by students who used verbal rather than internal physical representations of the problems.

Regarding nonverbal approaches to mathematics instruction, Block (1968) devised and successfully applied a relatively nonverbal remedial mathematics

[5]From the Greensboro Record (Greensboro, N. C.), July 29, 1971.

learning program with college students having both poor verbal and mathematical skills. And Wirtz (footnote 5) has used his nonverbal approach in teaching mathematics to deaf children, and to both Japanese and English children at the same time.

With respect to the debate over the relative efficiency of verbal and nonverbal instruction, it is possible that both approaches have merit, depending on the aptitudes of the learner, the special skills of the teacher, and the type of material to be learned or solved. The writer is not aware of any studies concerned with the interaction between aptitudes and the relative effectiveness of verbalization and nonverbalization on learning mathematics, but they should not be difficult to design. Perhaps children with higher verbal abilities would learn mathematics more easily if the verbal aspect were emphasized. On the other hand, students with poor verbal language backgrounds and abilities might find a nonverbal approach more rewarding. In any case, the fact that school learning in general is primarily verbal in nature would tend to lend a "Hawthorne effect" advantage to an approach emphasizing experiments and discovery rather than rote learning and abstract verbal concepts.

The results of certain investigations of aptitude-treatment interactions suggest that achievement in mathematics taught by a verbal approach varies with the learner's abilities. Thus, in a study of 90 sixth graders Bracht (1970) found some evidence that a "verbal" approach to instruction in adding positive and negative numbers was more effective with pupils having low spatial abilities, whereas a "spatial" approach was superior with pupils having low verbal abilities. Peters' (1970) results concerning the importance of verbal and cue-discrimination training in developing the concept of conservation also indicate that the relative effectiveness of training method depends on the aptitudes of the learners.

Teacher-Student Verbal Interactions

Even proponents of nonverbal awareness (see Hendrix, 1961) realize that communication between teacher and pupil is important in the initial stages of mathematics learning. Lovell (1971) also refers to the need for constant discussion between teacher and pupil and among pupils themselves in mathematics classrooms. Therefore, it may be instructive to analyze the types of teacher-student interactions in mathematics classrooms and their consequences. One procedure involving systematic observations of verbal interactions between teachers and students is known as the Wright-Proctor Observational Instrument. Use of this instrument in an investigation of four types of high school mathematics classes revealed distinctive verbal interaction patterns in the areas of context, process, and attitude (Proctor & Wright, 1961). Another instrument for describing the components of teacher-student verbal communication in mathematics classes was designed by Fey (1969) to analyze verbal interactions according to source, pedagogical purpose, duration, mathematical content, mathematical activity, and logical process. The results of applying Fey's procedure to teacher-student interactions in four sessions of five secondary school mathematics classes point to much greater verbal activity on the part of the teacher. Thus, the teacher made more "moves" and "lines" than all of his students combined and dominated the pedagogical structuring functions of the classroom. Fey reported that fifty per cent of the verbal "moves" made by teachers and students were statements or questions of fact, 25% were evaluations (mostly by teachers), and 25% consisted of "justifying" and "analytic process."

Related findings indicative of the high verbal activity of mathematics teachers were reported by Kysilka (1970). In a comparison of mathematics teachers with social studies teachers, it was noted that the former talked more often and their students volunteered less frequently. Specifically, the mathematics teachers asked more covergent and procedure-positive questions, made more directing and describing statements, but also rejected fewer student responses than the social studies teachers.

The method employed by Lamanna (1969) for studying the verbal communications of 11 teachers with their 258 sixth-grade mathematics pupils was Flanders' System of Interaction Analysis. The behavior of the teachers was classified as "indirect and direct verbal behavior, extended indirect and extended direct verbal behavior, and supporting or rejecting student talk." Although teacher behavior in general had a non-significant effect on students' problem-solving skills and mathematical concepts, there were several significant findings with respect to achievement in mathematical computations. Thus, teachers who used "direct verbal behavior" or "rejected student talk" increased the computational achievement of students who were average in intelligence. This last finding points out again the importance of considering the interactions of aptitudes and treatments in their effects on performance.

Although the analysis of teacher-student verbal interactions is relatively new, this type of research has increased knowledge of the kinds, frequency, and effects of verbal exchanges between students and teachers in mathematics classrooms. More attention should now be given to ways in which teachers' verbal behavior helps students to learn and organize their knowledge. The results of an initial investigation along these lines (Cooney, 1970) demonstrate how deduction, induction, classification, and analysis of cognitive knowledge

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on the part of the student can be facilitated by teacher verbal behavior. Of course, communications of the nonverbal type may also have important effects on understanding and attitudes in mathematics, so nonverbal variables should also be systematically scrutinized.

The Language of Mathematics

So far in this paper the emphasis has been on the effects of verbal factors in mathematics learning. As Madden (1966), Ausubel and Robinson (1969), Cooper (1971) and other educational researchers have pointed out, however, mathematics itself is a special formalized language and should therefore be taught as such. Munroe (1963) referred to the language of mathematics as "Mathese" and indicated that it should be easier for the student to understand Occidental Mathese than other languages. But Munroe also noted that, because of the inconsistency of notation in mathematics and variations in the interpretations of symbols (especially x and y), it is impossible to construct a complete Mathese-to-English dictionary. Furthermore, the majority of mathematicians are apparently not interested in attempting to devise or agree upon a completely consistent, adequately descriptive set of mathematical notations.

Language Analogies to Mathematics

Although there is no one-to-one correspondence between the concepts and rules of mathematics and those of native languages, there are many similarities between verbal and mathematical languages. One teacher (Capps, 1970) has found that pointing out analogies (e.g., commutativity, associativity, distributive property) between verbal language and mathematics is a useful instructional technique. Another educator (Hickerson, 1959) has devised an experience-language approach to numbers consisting of eight overlapping

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stages: (1) engaging in multisensory problem situations; (2) acquisition of oral language to represent in complete sentence form the quantitative relations in problem situtations; (3) introduction of written arithmetic symbols as shorthand ways of writing already known spoken words; (4) acquisition of meaning of written or spoken arithmetic symbols by representing something in experience; (5) after learning to read them, the writing of numbers, number combinations, algorisms, etc.; (6) computational processes are acquired by manipulation and discovery, not by memorizing and applying math rules; (7) rules, principles, and generalizations are taught by the inductive-deductive method; (8) continuous interrelationships between first-hand quantitative experiences in life, expression of these in oral and written symbolism, and increasing consciousness and knowledge of the nature of arithmetic.

Language Influences on Mathematical Development

Many writers have referred to various aspects of the interaction between language development and the growth of mathematical understanding. Thus, Rose and Rose (1961) maintain that childhood training in precise language, resulting in a maximum of inner elaboration (H. S. Sullivan's syntaxic mode), is essential for performing well in mathematics. In observing that typically the youngest child is poorer in mathematics than his older siblings, they suggest that this is due to the parents "bombarding the youngest child with baby talk," resulting in a mode of thinking and communicating characterized by a minimum of inner elaboration (Sullivan's prototaxic mode). This explanation, Rose and Rose believe, also accounts in part for the relatively greater mathematical abilities of children from more homogeneous (e.g., upper) sociocultural backgrounds. Since such children need to spend less time and effort in socioemotional interactions than those from more heterogeneous backgrounds, they have more time to concentrate on mathematics and other abstract tasks.

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The importance to mathematical ability of language development has been considered by many psychologists, foremost among whom are Piaget (1954), Bruner (1966), and Galperin (see Lovell, 1971). Piaget maintains that growth in linguistic ability follows the development of concrete operational thought rather than preceding it, although language is important in the completion of such cognitive structures. In contrast, Bruner and his associates (Bruner, Olver, & Greenfield, 1966) maintain that the development of adequate terminology is essential to cognitive growth. Pertinent to the Piaget-Bruner debate, the finding of Gever and Weisberg (1970) that the spontaneous verbalizations of young children are unrelated to their problem-solving performance certainly casts doubt on the directive function of overt speech. On the other hand, Sollee (1969) reported that acquisition of the conservation of number and three kinds of quantity by children in the transitional stage of developing concrete operations was affected by their verbal abilities. In this study (Sollee, 1969) of 41 first and second graders, verbal competence, as measured by a composite of WISC Verbal I. Q. and other tests, was found to be related to the achievement of "stable and generalized levels of conservation, measured either nonverbally or verbally . .. even when nonverbal intelligence was held constant."

Further empirical support for the proposition that verbal ability facilitates the transition from nonconservation to conservation was obtained by Peters (1970). In a study of 131 kindergarten children of lower socioeconomic status, verbal training was found to be significantly more effective than noncued, visual-cued, or no training when the criterion was immediate learning. When the criterion was delayed retention, both verbal training and visual-cued training had greater effectiveness than the other two procedures.

Whether the acquisition of language is a cause or an effect of cognitive development, or, as appears more likely, a bit of both, needs further investigation. Carefully designed studies of the interactions among age, various measures of verbal ability (both overt and covert), general intelligence, and other organismic variables in their effects on the development of the concept of numerosity, the conservation of number and quantity, and other aspects of mathematical knowledge should provide useful information.

Stages in Learning Mathematics: Implications for Instruction

Perhaps the most cogent summary of the instructional implications of stages in mathematical learning is given by Ausubel and Robinson (1969). These writers begin by pointing out that, at least in the early stages, mathematics deals with concepts, the meanings of which are conveyed by simple explicit images. A second characteristic of mathematical learning is that its operational terms also have explicit, dynamic or kinesthetic images obtained from the child's experience. A third aspect of mathematics learning is that the child must understand systems of propositions. Ausubel and Robinson maintain that practice in mainipulating concrete objects, as in present-day arithmetic instruction, is consistent with the idea that kinesthetic images serve as a basis for understanding arithmetical ideas in particular and the inductive process of concept formation in general.

In a section on learning algebraic symbols and syntax, Ausubel and Robinson (1969) state that the same problems as in learning a second language are involved. The learner begins by translating algebraic symbols into the "native" language of arithmetic and depends on his knowledge of arithmetic syntax in order to understand the syntax of algebra. This is not so simple, because the symbols of algebra bear a one-to-many rather than a one-to-one correspondence to arithmetic symbols. Finally, with repeated application the student reaches a point where the mediational role of arithmetic is no longer needed and he can understand the meaning of an algebraic statement directly. By way of illustration, in learning to understand how the equation $2X + 3 = 11$ is solved, the learner obviously needs to know what "2X" and "equation" mean. Furthermore, he must also understand the propositions that "if equal amounts are added to, or subtracted from, each side of an equation the equality remains" and "if both sides of an equation are multiplied or divided by the same amount the equality remains." Rules such as these can be learned by induction (discovery learning) or by teacher explanation (reception learning). Ausubel, like Gagné (1968), is an advocate of careful sequencing of educational experiences. He stresses the notion that the need for discovery by the learner can be removed by the teacher's meaningful organization of the material to be learned, in addition to overlearning on the part of the student of such sequentially arranged lessons. This approach contrasts with the "discovery learning" advocated by Bruner (1966) and several other writers referred to earlier.

Finally, Ausubel and Robinson (1969) observe that the school is in a much better position with regard to mathematics instruction than it is with language teaching. In the case of language learning, the effectiveness of the parents' (and others') prior verbal interactions with the child plays a crucial role in the latter's understanding of vocabulary and syntax. If the parents' own command of natural language is poor, then many of the linguistic habits of the child may need revising at the outset of his school experiences. The fact that during this period the child continues to be exposed to improper linguistic models at home makes the language teacher's task an unenviable one. On the other hand, parents do not usually teach their preschool children much mathematics beyond rote counting. Therefore, teachers can build on the direct experiences of these mathematically uninstructed children with the physical environment without having to counter the effects of so much ineffective pre-school instruction in mathematics.

Research on Mathematical Thinking and Problem Solving

Studies of the symbolic processes involved in human thinking have frequently employed mathematical problems. A popular procedure (Rimoldi, 1967; Rimoldi, Aghi, & Burder, 1968) is to analyze the thinking process by requiring the subject to think aloud and noting the tactic, or sequence of questions, that he asks in reaching a solution. These kinds of investigations have revealed that the translation or encoding procedure varies with the individual. For example, Paige and Simon (1966), in an examination of the verbalizations made by students while setting up equations for algebra problems, were able to classify their subjects as "physical" and "verbal" thinkers. The "physical" thinkers constructed some kind of internal representation of the situation described by the equation, whereas the equations of the "verbal" thinkers were literal translations of the words.[6]

Gagné (1966) also recognizes that problem-solving ability and technique vary with the individual. To Gagné, amount of information stored, ease of recall, distinctiveness of concepts, fluency of hypotheses, ability to retain the solution mode, and ability to match instances to a general class are important individual difference variables. These variables affect the ease with which relevant rules and concepts are recalled, a provisional solution

[6] Of interest is the fact that written or spoken words apparently played no role in the thought processes of Albert Einstein. Einstein reported being aware of certain signs and clear images of a visual and kinesthetic type during his thinking, but the formulation of thoughts into words came only after the mental association of these nonverbal images were well established (Hadamard, 1949).

rule is derived, and a solution to the problem is verified. Gagné argues, however, that these "internal events" in problem solving can also be influenced by instructions of various kinds at different stages of the solution process. Therefore, he suggests that, in addition to further studies of individual difference variables in problem solving, research on different instructional variables, and in particular different methods of stimulating recall and making cues more distinctive, should be conducted.

Gagné's theoretical position and research program are couched in the language of stimulus-response associationism, whereas Bruner's emphasis on "learning by discovery" is more reminiscent of Gestalt theory.[7] In contrast to S-R language, Scandura (1968a, 1968b, 1969) has proposed a Set-Function Language (SFL) for formulating research questions on mathematical learning. Taking rules and principles as the basic units of behavior, Scandura states that there is no way to state <u>rules</u> in terms of associations, but in SFL they are characterized in terms of (D) stimulus properties which determine the corresponding responses, (R) covert responses or derived stimulus properties, and (O) transform or combining operations by which the covert responses are derived. The combining operation (O) indicates how mediating responses are produced by the preceding mediating stimuli. Furthermore, <u>principles</u> consist of rules plus (I) those contextual properties that identify the rule to be applied. Scandura (1968b) admits that SFL as stated deals only with idealized rules rather than actual rules (competencies) employed by people. However, he suggests that the concept of a <u>functor</u> may help bridge the gap between the real and ideal.

Scandura's formulation would appear to be a promising heuristic for

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[7]See Shulman (1970) for a summary of the viewpoints of Gagné, Ausubel, and Bruner on mathematics learning.

conceptualizing research on mathematics learning. It needs further development, however, before proving to be something other than an idiom in which research questions can be formulated, but rather a serious contender in explanatory and predictive power to behavior theory. So far, Scandura (e. g., 1969, 1970) has reported a number of investigations on rule learning, but the findings are in no way dependent on SFL.

Suggestions for Future Research

Throughout this review various evaluative comments on the investigations cited have been made, and a number of suggestions for further research have been offered. But the paper would be incomplete without a more detailed consideration of what the writer believes to be the most important and promising directions for research. To begin, multivariate studies of the relationships among selected aspects of mathematics and various linguistic variables should be undertaken. In order to be of greatest utility, separate corelational analyses need to be made for different age and sex groups.

The relationship between mathematical development and language development, and especially the degree of emphasis in the home on syntactic thinking and linguistic encoding, needs careful examination. In addition, controlled experiments concerned with the effects of instruction in vocabulary and reading on mathematics learning are called for, along with comprehensive surveys of the vocabulary levels and readabilities of mathematics textbooks. More specifically, specialized methods of teaching verbal problem solving, such as those proposed by Taschow (1969) and Dahmus (1970), should be tried out experimentally.

With respect to verbal behavior, the effects of both covert and overt verbalizations by the student, as well as nonverbal (kinesthetic) behavior, on solving mathematics problems could be studied more intensively with larger groups of children at different age levels. It may be found that verbalization has a greater effect on problem retention than on problem solution, and produces different effects with different age groups and at different stages of problem solving. Another aspect of verbal behavior, that of interaction among students and their teachers, is of concern in analyzing the events occurring in mathematics classrooms. The challenge in this area is to identify specific teacher and student responses, both verbal and nonverbal, that really make a difference in terms of achievement.

As Gagné has noted, individual differences in problem solving style continue to be of interest. In addition, instructional variables frequently interact with individual learning styles and aptitudes, and both sources of influence need to be taken into account in mathematics learning (see Aiken, 1971a for a brief review of research on this topic). Another controversy is concerned with the relative effectiveness of discovery and reception (exposition) learning. Here again there has been a mass of unreplicated, studies employing a few variables, but no serious attempt to determine what instructional and aptitude factors affect the utility of either approach.

Finally, although Scandura's Set-Function Language (Scandura, 1968a) represents an important effort to describe mathematics learning in a new linguistic medium, also needed is some scheme for analyzing mathematical language per se. Such a scheme would accomplish much more than the readability formulas discussed earlier, or simply be another way of labeling what is already known. It would provide a system or procedure for identifying and categorizing the lexical and grammatic units that are unique to mathematics, and consequently could serve as a basis for classifying and comparing mathematical materials. These, then, are merely a few of the challenges for research on language factors in mathematics learning.

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