This paper examines the fiscal impacts on public schools of some of the federal aid proposals under consideration by Congress. The four major discussion sections focus on (1) an outline of the overall analytic framework, including a conceptual model of federal, state, and local relationships; (2) a brief presentation of the theoretical analysis on which the statistical work is based; (3) the calculation results of the impacts of various aid alternatives pertaining to local school districts and states; and (4) a discussion of the overall policy implications, especially with respect to the two most widely discussed federal aid alternatives—general revenue sharing and direct federal aid to school districts channeled through the states. (RA)
THE IMPACT OF FEDERAL AID TO STATES
ON FINANCIAL SUPPORT FOR THE PUBLIC SCHOOLS

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INTRODUCTION

During the last few years, both the Congress and the Executive Branch have been approaching a decision on the federal response to the fiscal problems of state and local government. According to many observers, some form of new, large scale federal financial aid to states is imminent. Until very recently, it seemed that the vehicle would be a variant of revenue sharing—either completely unrestricted aid or block grants for education and other major functions. Now, since the President's State of the Union Address in January, attention has shifted to the possibility of direct federal aid for the public schools, to be financed, at least according to the initial Administration announcements, by a new value-added tax. It is not yet clear what the outcome will be as the Congress deals with the variety of plans now before it. But whatever federal funding mechanism ultimately emerges—whether it provides direct aid to school districts or general aid to states—one thing is certain: Any form of federal intergovernmental aid on the scale now being contemplated will have major effects on financial support for the public schools.

The purpose of this paper is to examine the fiscal impacts on public schools of some of the federal aid proposals now being discussed and debated in Washington. "Fiscal impact," in this context, means changes in expenditures and taxes of school districts and school-related expenditures and taxes of states that result from the institution of new forms of federal aid. These fiscal consequences of federal policy are, or should be, important to federal policymakers and to various groups and segments of the population who are affected by levels of support for the schools and by state and local tax burdens.

Presently available information on the consequences of alternative federal funding schemes is not sufficient to meet the needs of interested parties. With respect to revenue sharing, for example, no one is able to say what percentage of shared revenues would be used for state tax reduction or what percentage would be made available to local school districts. Nor is it known how much of
the latter would be allocated by local officials to increased school spending and how much to local property tax relief. If that information were available, not only for revenue sharing, but for other alternatives, it might be instrumental in molding the attitudes of school administrators, teachers' organizations, and taxpayers' groups—not to mention state and local legislators—toward rival proposals. Moreover, the same information could well affect the perceptions and assessments of federal decisionmakers and thus have both a direct and indirect bearing on the final federal decision.

The main reason that information on fiscal impacts of aid is not available and is difficult to obtain is that so much depends on the behavior of the state or local aid recipient. Results can rarely, if ever, be predicted from specifications of the aid scheme along. If a state, for example, receives a federal grant "earmarked" for the improvement of education, but responds by reducing state taxes or not raising them as much as it would have otherwise, the net impact of aid on education outlays may be much less than the amount of the grant. In general, adaptive behavior by aid recipients can produce impacts that differ substantially from both the effects intended by the grantor and the nominal effects reported in the account books. Only by constraining aid recipients very tightly—in effect, dictating state-local fiscal behavior in detail—would it be possible for the federal government to guarantee fiscal outcomes fully in line with federal objectives. Since such an alternative is neither feasible nor desirable, it becomes important to concentrate on aid formula design, using information about state-local responses to allocate funds in a way that is likely to accomplish the national purpose.

The central analytical question, therefore, is whether fiscal responses of state governments and local school districts can be predicted. Are those behaviors sufficiently systematic that stable relationships between spending and taxing and such external factors as federal aid can be discovered? So far, the answer from this study is a highly qualified "yes". As will be shown, the fiscal
behavior of local school districts, using statewide aggregates or averages as units of observation, is fairly well "explained" (in the statistical sense) by economic and demographic variables including state and federal aid. The statistical model that derives from that analysis can be used to make at least rough predictions of effects of infusions of outside aid on school spending and local school tax aggregates. Moreover, certain threats to the credibility of statistical expenditure determinant models—especially the lack of consistency that was found in earlier studies when the same model was applied to data for different years—have been partly eliminated by basing the models more closely on results of a theoretical analysis.

The fiscal behavior of state governments themselves is much less well explained. This is not surprising since, in studying the states, we are dealing with individual decisionmaking units rather than aggregates, multi-function rather than single-purpose governments, and governments that vary greatly in structure and responsibilities. Nevertheless, it is disappointing because it means that state aid to school districts, which is one of the important determinants of local educational spending, is itself not very well explained. This does not mean that consequences of federal policies cannot be analyzed, since it is still possible to examine implications of alternative assumptions about state responses, using the available results to delimit the relevant range of behavior. If anything, it points up the importance of aid formula design in assuring that state behavior will be at least generally compatible with the objectives for which federal money is being provided.

The remainder of the paper is organized into four sections: first, an outline of the overall analytical framework; second, a condensed presentation of the theoretical analysis on which the statistical work is based; third, presentation of the results pertaining to local school districts and states; and finally, a discussion of the overall policy implications, especially with respect to the two most widely discussed federal aid alternatives, general revenue sharing and direct federal aid to school districts channeled through the states.
FRAMEWORK FOR ANALYSIS

STATE-LOCAL INTERACTIONS

The effects of any form of federal aid on financial support for the public schools will depend on fiscal decisions by both state governments and local school districts. The role of each level is most clearly seen in the case of unrestricted federal aid to states. Given complete freedom, each state would determine for itself how much federal aid will be channeled to school districts, how much will be expended by the state for other purposes (either directly or as aid to cities and counties), and how much will translate into reductions in state taxes. Local decisions would then determine how much of the additional money that does become available to districts will go for increased school spending, as opposed to local property tax relief.

It might seem from the above that the state is involved only when it is the direct recipient of federal funds; however, that is not the case. Suppose, for example, that the federal government aids districts directly, completely bypassing the states. It would be reasonable, in that case, to expect state officials, seeing the additional funds available to districts, to revise their own estimates of local needs downward and to respond to the influx of federal funds with a partially offsetting reduction in state aid. That response and its effect on the local districts would have to be considered in measuring the net impact of federal grants.

Moreover, there are other complexities. Even if a state allocated none of its shared revenues to education, local school districts could still benefit. For example, a decision to provide additional state aid to cities or counties could affect school finance by reducing horizontal competition among local governments for property tax revenues. Similarly, a reduction in state income or sales taxes could benefit school districts by making local taxpayers more able and willing to support higher school levies. Because of these multiple interactions, it is necessary to work with a two-sector model of government that is capable of dealing with the stimulus-response relationships in both directions.
SCHEMATIC VIEW OF THE MODEL

The federal-state-local system to be modeled is depicted in Figure 1, which shows the main fiscal variables that enter into an analysis of the impacts of federal aid. Note that "other local governments" (cities, counties, etc.) are acknowledged, but not treated as a separate sector as they would be in a larger model. Instead, their expenditures and taxes are aggregated with those

Figure 1. Schematic Diagram of the Federal-State-Local System
of the state. This means that all references to "state" expenditures and taxes should be understood as applying to the whole state-local sector, except for local school districts.

Definitions of the fiscal variables represented in the diagram are as follows:

\[ e_L = \text{real expenditure per pupil by local school districts.} \]
This is used as the measure of the level of educational services. It is obtained by deflating the dollar amount of expenditure per pupil by \( P_e \), an index of prices of inputs to education.

\[ e_S = \text{real expenditure per capita by state government for all functions other than aid to public schools.} \]
It is obtained by deflating the dollar amount of direct state outlays by \( P_g \), an index of prices of inputs to government services other than education.

\[ t_L, t_S, t_F = \text{real per capita tax collections by local school districts, the state government, and the federal government, respectively.} \]
In each case, real values are obtained by deflating the dollar amount of per capita taxes by \( P_c \), an index of prices of private goods (general price index). The use of this deflator means that real taxes are measured in terms of the loss of private consumption or purchasing power.

\[ f_D = \text{real federal aid per pupil to school districts.} \]
The deflator is \( P_e \), which means that aid is measured in units of purchasing power for educational services.

\[ f_S = \text{real federal aid per capita to the state government.} \]
In this case, the deflator is \( P_g \), so that aid will be measured in units of purchasing power for other government services.

\[ s = \text{real state aid per pupil to local school districts, measured in the same units as federal aid to school districts.} \]

*Federal aid to school districts includes aid that is channeled through the state for redistribution to specific local districts according to federal rules.*
In terms of these variables, the point of the analysis is to determine how all the state-local fiscal variables, but especially the school district fiscal variables, $e_L$ and $t_L$, are likely to respond to changes in the federal aid variables and the level of federal taxes. The significance of using a two-stage model of the state-local sector is that the state and local responses will first be worked out separately. That is, we construct a state government model to determine how $e_S$, $t_S$, and $s$ respond to changes in $f_S$ and $t_F$, than a local government model to determine how the state fiscal variables and the relevant federal variables ($f_L$ and $t_F$) influence $e_L$ and $t_L$.

LIMITATIONS

Two limiting characteristics of the analysis are implicit in the flow diagrams and the enumeration of variables, but deserve to be made explicit.

One is that the analysis throughout is highly aggregative. This is so in two senses. It is aggregative, first, in that individual expenditure programs are not considered. Only undifferentiated totals of local expenditure, state aid, and direct state expenditure are included in the models. The principal loss is that important differences in the composition of spending from one state to another are neglected although they could be important in assessing effects of new federal aid. The gain, of course, is that one can work with a relatively manageable two-sector model instead of a multi-sector model for which the analytical and data requirements would be far more severe. The model is also aggregative in that the unit of observation is the state, i.e., either the state government itself or the aggregate of all local governments (school districts) within the state. This eliminates the possibility of making statements about differential effects of federal funding on various classes of school systems (e.g., urban, suburban, rural). The task of expanding the model in that direction does not 'seem particularly difficult and may be attempted as a follow-on to the initial study.
The other major limitation is that this is a "closed" rather than an "open" model of the state-local public sector. The model is closed in that it does not deal with repercussions of state-local decisions on the rest of the economy. Community characteristics are treated as "given." Possible effects of expenditure and tax shifts on aggregate income and of changes in state-local purchases on prices are neglected. Also, possible effects of state-local fiscal decisions on federal finances (e.g., changes in federal tax revenue because of changes in levels of state or local activity) are ignored. An open model would consider some or all of these linkages explicitly. However, to do so would require both a macroeconomic model of income determination and a model of factor markets to deal with demand-induced price changes. It is clearly not feasible to construct so comprehensive a model and, in any case, the effects are probably second-order compared to those that are included.
THEORETICAL MODELS

Behavior of both states and school districts is analyzed within the framework of the general economic model of constrained maximizing behavior. It is assumed that decision-makers at each level have consistent preferences with respect to different combinations of expenditures and taxes. These preferences are expressed in terms of a governmental unit's willingness to "trade off" marginal increments in expenditures for increments in taxes and, if the unit is a state, willingness to trade off increments in one category of expenditures for increments in another. Government decision-makers are assumed to seek an optimal balance among the fiscal variables they control, subject to a budget constraint that reflects the availability of grants-in-aid (utility maximization). By working out the implications of maximization, one is able to derive a set of empirically testable expenditure and tax equations.*

THE MODEL OF SCHOOL DISTRICT BEHAVIOR

Although we seek a set of equations to represent the behavior of all school districts within a state, it is convenient to develop the theory with reference to a single school district. Aggregation can be handled in a straightforward manner and poses no special theoretical problems.**

*Several writers have used the constrained maximization approach to analyze responses to grants-in-aid, both of state-local governments in general and school districts in particular. See, for example, James A. Wilde, "The Expenditure Effects of Grants-in-Aid Programs," National Tax Journal, 21, September 1968, pp. 340-49; and S.R. Wilensky, "State Aid to Education," Ph.D. dissertation, University of Michigan, Ann Arbor, Michigan, 1968. Usually, these expositions use indifference curve diagrams directly analogous to those used in developing the theory of consumer demand. The assumptions and results of this analysis differ from those earlier studies in some important respects. See below, especially p. 14 in.

We think of the decisionmakers in a school district as seeking to establish an optimal balance between the educational services they provide and the local property tax burdens they impose on citizens. It is assumed that preferences among alternative combinations of educational services and tax burdens are consistent in a given district at a given time, but that these preferences may be influenced by community characteristics ("taste variables") that vary both over time and among districts. Educational services, as measured by real expenditures per pupil, are valued positively. Tax burdens are valued negatively. The burden of local school taxes is assumed to depend on both the real dollar amount of taxes per capita and on per capita disposable income. A given level of taxes is assumed to be more burdensome when income is lower.

Preferences

The above assumptions are embodied in a school district preference function of the form

$$U = U(e_L, b_L(t_L, y_D), z),$$

where $e_L$ and $t_L$ are real per pupil spending and real per capita school taxes, defined as before; the function $b_L$ represents the burden of local school taxes; $y_D = y - t_S - t_F$ is real per capita disposable income, where $t_S$ and $t_F$ are state and federal taxes; and $z$ represents unspecified taste variables that may enter into the preference function. First derivatives of the local preference function have the following signs:

$$\frac{\partial U}{\partial e_L} > 0, \quad \frac{\partial U}{\partial d_L} < 0, \quad \frac{\partial b_L}{\partial e_L} > 0, \quad \frac{\partial b_L}{\partial y} < 0.$$

It is convenient to work with a quantity analogous to the marginal rate of substitution in consumer demand theory, which we call the marginal rate of trade-off between expenditures and taxes and denote by $m$. The marginal rate of trade-off may be interpreted as the maximum increment in school taxes per capita the district would be willing to impose to obtain one additional dollar of educational spending per pupil. The definition of $m$ in terms of the utility function is
In developing the model, we assume that $M$ has the following properties:

1. $\frac{\partial m}{\partial e_L} < 0$: As educational outlay per pupil, $e_L$, increases, each marginal increment becomes less urgent than the preceding one and the district becomes willing to impose a progressively smaller tax to obtain it.

2. $\frac{\partial m}{\partial t_L} < 0$: As the level of the school property tax increases, each marginal increment becomes more burdensome than the previous one and the district requires a greater increment in "education" to justify a one dollar tax increase.

3. $\frac{\partial m}{\partial y_D} > 0$: As disposable income increases, the burden of a given school tax diminishes, making the district less reluctant to impose additional taxes for a given increment in school spending.

These stipulations about properties of the marginal rate of trade-off function allow unambiguous inferences to be made about the signs of the effects of most exogenous variables on levels of school spending and taxes.

**The Budget Constraint**

Assuming that no borrowing is allowed and that districts do not accumulate cash balances, and neglecting capital outlays, the budget constraint is that total educational expenditure must equal local tax revenue plus state and federal aid. For simplicity, assume that both state and federal aid take the form of lump-sum grants to districts. An examination of state and federal grant-in-aid formulas

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show that this is realistic for all districts in all but a handful of states and for most districts in the few remaining states. Having made that assumption, the budget constraint may be written

$$P_e A e L = P_o N t L + P_e A (a + f_L),$$

where $e$ and $f_L$ are real state and federal aid per pupil to the school district, as defined earlier; $A$ is the number of pupils in the district; $N$ is the total population; $P_e$ is an index of prices of educational inputs, and $P_o$ is an index of prices of private goods. It is convenient to define two ratio variables, $a = A/N$, the pupil/population ratio, and $p_e = P_e/P_o$, the relative price of education. The budget constraint may then be rewritten more compactly as

$$p_e a (e_L - e - f_L) = t_L.$$ 

Maximization

Conditions for constrained maximization of $U$ are obtained by maximizing the Lagrangian

$$U(a_L, b_L, y_L, z) = [p_e a (e_L - e - f_L) - t_L].$$

The first order conditions are

$$\frac{\delta U}{\delta a_L} = \lambda p_e a = 0,$$

$$\frac{\delta U}{\delta b_L} \frac{\delta b_L}{\delta t_L} + \lambda = 0.$$ 

The two conditions can be combined into a single condition involving the marginal rate of trade-off:

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*An analysis of data in Public School Finance Programs, 1968-69, Office of Education, U.S. Department of Health, Education, and Welfare (Washington: 1969) shows that aid is always in the form of lump-sum grants in all but five or six states, which have matching provisions in their aid formulas. Even in these states, ceiling provisions in the aid formulas convert the grants to lump-sum amounts for most districts.*
This relationship and the budget constraint equation constitute a pair of equations in $e_L$ and $t_L$ from which responses of those two variables to changes in the exogenous variables, $y_D$, $a$, $f_L$, $p_e$, and $z$, can be deduced.

**Expenditure and Tax Implications**

A direct procedure for determining the effects on school district expenditures and taxes of changes in the exogenous variables is to solve the budget constraint equation for $t_L$, substitute the resulting expression for $t_L$ into the marginal rate of trade-off equation given above, and then differentiate totally and solve for the change in spending, $de_L$. These computations are shown in the previously cited study.* The principal effects on spending, which derive directly from the previously stated properties of the marginal rate of trade-off function, are as follows:

1. **Effect of a change in disposable income**: The model implies a positive relationship between real per pupil spending and real per capita disposable income ($de_L/dy_D > 0$). This means a negative relationship between per pupil spending and changes in state or federal personal taxes.

2. **Effect of a change in aid**: The model implies a positive relationship between real per pupil spending and real state and federal lump-sum aid ($de_L/d(z + f_L) > 0$). The magnitude of the aid effect must be less than one; i.e., lump-sum aid is always partly additive to local spending, partly substitutive for local school taxes. The aid effect and the income effect are not the same, and under reasonable assumptions about local preferences, it can be shown that the aid effect will be considerably greater than the

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income effect.* This has the important implication that the positive effect of an increase in aid will not be offset by the negative effect of taxes even when an increment in aid is fully financed, in effect, by taxes on residents of the aid-receiving district.**

3. Effects of changes in the pupil/population ratio and the relative price of education: The model implies that spending will depend negatively on the product of the two variables $p_e$ and $a$  \[ \frac{de_L}{d(p_e a)} < 0 \]. This means that, other things being equal, lower real per pupil spending is expected where a greater fraction of the population attends public school or where prices of inputs to schooling are high in relation to prices in general. The price effect will have a direct bearing on predictions whenever fiscal alternatives involve either matching grants to districts or taxes on private spending (e.g., state sales taxes or a federal value-added tax).

4. Effects of "taste" variables. The response of real per pupil spending to a change in an exogenous taste variable will be positive \[ \frac{de_L}{dz} > 0 \] if that variable is positively associated with the district's marginal rate of trade-off between taxing and spending.

Effects of the exogenous variables on real school taxes per capita may be derived in a similar manner. Very briefly, the income effect, \( \frac{dt_L}{dP} \), is positive; the aid effect, \( \frac{dt_L}{d(a + f_L)} \), is negative, with the decrease in taxes per dollar increase in aid less than one; and the price effect, \( \frac{dt_L}{d(p_e a)} \), is of indeterminate sign—positive if the price elasticity of per pupil expenditures is greater than minus one, negative otherwise.

Other Effects on Local Spending

*It has been shown elsewhere*** that effects of several other

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*Barro, op.cit., pp. 15-18. This result distinguishes the model used here from one that is more directly analogous to a consumer demand model. The latter would simply treat outside aid as an addition to the income that the community has available to allocate between education and other goods, which would imply equal income and aid effects.

**Ibid., pp. 40-41.

***Ibid., p. 33 ff.
Determinants of school spending and taxes can be incorporated into the foregoing basic model. Some of these (e.g., differences in the composition of local tax bases and equalization provisions of state aid formulas) are relevant only to studies of interdistrict expenditure variations. However, the following two factors are relevant to both interdistrict and interstate studies and are included in the empirical models:

1. Real per pupil expenditures should be negatively associated with the rate of enrollment growth in a district or state. This is because the property taxes required to pay for new facilities compete with property taxes for operating expenses.

2. Per pupil spending will respond more strongly to categorical than to unrestricted aid if the restrictions attached to categorical aid force districts to spend more on specific programs or activities than they would have chosen to spend themselves. Since a greater portion of federal than state aid is likely to be categorical, this suggests that the federal aid and state aid coefficients in the model may be different.

Hypotheses

The results of the theoretical analysis of school district behavior can be stated as a set of hypotheses about relationships between local spending and taxes and a number of exogenous variables. Taking the last two propositions stated above into account, we expect to obtain expenditure and tax equations.

\[ e_L = e_L(Y_D, s, f_L, p_ea, \Delta A/A, n) \]

and

\[ t_L = t_L(Y_D, s, f_L, p_ea, \Delta A/A, n) \]

that satisfy the conditions

a. \[ \frac{de_L}{dy_D} > 0 \]

b. \[ 1 \geq \frac{1}{p_ea} \frac{det}{dy_L} > \frac{1}{p_ea} \frac{de_L}{dy_D} > \frac{de_L}{dy_D} \]

c. \[ \frac{de_L}{d(p_ea)} < 0 \]
Further specification of the hypotheses depends on additional assumptions about the functional form of the expenditure and tax equations. Discussion of that issue is delayed until after presentation of the state-level theoretical model.

THE MODEL OF STATE FISCAL BEHAVIOR

The state model is similar in many respects to the local model. Each state is viewed as seeking an optimal balance between expenditures and state tax burdens subject to a budget constraint that reflects the level and form of outside (federal) aid. Expenditure and tax implications of maximizing behavior are derived, and used to specify a set of empirically testable expenditure and tax equations. But the state model has two features that make it different from and more complicated than the local model. First, there are two categories of state expenditure to consider—aid to local school districts and "other" state spending. Only one category had to be treated in the local model. Second, in order to explain state aid to local governments, it is necessary to assume that the state is responsive to fiscal conditions at the local level. Each of these features leads to the inclusion of arguments in the state preference function that have no counterparts at the local level.

The State Preference Function

Two approaches can be taken in formulating a state government
preference function. In both, state taxes and "other state spending (direct state outlay plus aid to other local governments) appear as arguments. The difference is that in one, state aid to school districts enters directly into the preference function (i.e., aid is assumed to be valued "in itself" by the state), while in the other, local school spending and local school taxes enter the state's preference function while state aid enters the model elsewhere as an intermediate variable. The two yield similar results, but only the first approach is followed here.

Assume an additively separable state preference function of the form

\[ V = V_1(e_S) + V_2(t_S, t_L, y-t_F) + V_3(e, a^s), \]

where \( t_L \) is the expected level of local taxes and \( a^s \) is a measure of the "needed" amount of state aid per pupil (see below). State aid, \( e \), and other state spending, \( e_S \), are assumed to be positive goods to the state government. State taxes, \( t_S \), is a negative good. The marginal disutility to the state of a given increment in state taxes is assumed to decrease as income increases, but to increase as the level of expected local taxes increases. The marginal utility of a given level of state aid, \( a \), is assumed to increase as the local "need" for state aid, \( a^s \), increases. Other exogenous variables, which were previously denoted by \( e \), may enter into the preference function, but they have been omitted for simplicity.

Since there are three state fiscal variables in the preference function, we need to define two marginal rates of trade-off among them. The marginal rate of trade-off between other state expenditures and state taxes is defined as

\[ M_1(e_S, t_S, t_L, y-t_F) = -\frac{\partial V_1}{\partial e_S} \frac{\partial V_2}{\partial t_S} \]

The marginal rate of trade-off between state aid to school districts and state taxes is defined as

\[ M_2(e, t_S, t_L, a^s, y-t_F) = -\frac{\partial V_3}{\partial a^s} \frac{\partial V_2}{\partial t_S} \]

when appropriate assumptions are made about second derivatives of \( V \), it can be shown that the two functions have the following properties:
From these it will be possible to deduce the signs of the effects of exogenous variables on state aid, other state spending, and state taxes.

The Budget Constraint

The state budget constraint is that total state aid to school districts plus the total amount of other state expenditures must equal total state taxes plus federal aid. We will assume that all federal aid to states consists either of lump-sum grants or fully-utilized matching grants; i.e., that there is no matching by the federal government at the margin. This is not completely true, but does seem to be roughly correct for most federal aid programs for most states. While it would be possible to allow for matching provisions in the budget constraint, this introduces serious measurement and statistical problems. On balance, the simpler assumption seems preferable.* Noting carefully the deflators that were used in defining each variable (p. 6), we can write the constraint as

\[ P_eA_o + P_gN_eS = P_oN_tS + P_gN_fS. \]

Define the relative price of other public services, \( \varphi_g = P_g/P_o \). Then, using the previously defined relative price of education, \( P_{e_o} \), and the pupil/population ratio, \( a \), the budget constraint may be rewritten:

\[ \frac{dM_1}{da} < 0 \quad \frac{dM_1}{dY} < 0 \quad \frac{dM_1}{dL} > 0 \]

\[ \frac{dM_2}{da} < 0 \quad \frac{dM_2}{dY} < 0 \quad \frac{dM_2}{dL} > 0 \quad \frac{dM_2}{dY} > 0 \]

\[ p_e a_s + p_g g_s = t_s + p_g g_s. \]

**Maximization**

The first-order conditions derived from maximizing \( V \), subject to the budget constraint, are

\[
\frac{\partial V_1}{\partial \sigma} - \lambda p_g = 0
\]

\[
\frac{\partial V_2}{\partial \tau} + \lambda = 0
\]

\[
\frac{\partial V_3}{\partial \varepsilon} - \lambda p_\varepsilon = 0.
\]

Combining the first with the second and the third with the second, we obtain the two marginal rate of trade-off equations:

\[ M_1(e_s, t_s, t_L, y-t_P) = p_g \]

\[ M_2(\varepsilon, t_s, t_L, \varepsilon, y-t_P) = p_\varepsilon \]

These two relationships together with the budget constraint are the basic relationships of the state model.

**Measurement of \( t_L \) and \( \varepsilon \)**

An important part of this model is a set of definitions for expected local taxes, \( t_L^* \), and "needed" state aid, \( \varepsilon^* \). Consideration of the roles of these variables in the model suggests several possibilities. The purpose of including \( t_L^* \) is to allow for the influence of local tax burdens on the state's willingness to impose taxes of its own. The assumption is that the state will be more reluctant to tax when it anticipates a higher level of local taxes. Two possible specifications of \( t_L^* \) are as follows:

(a) Let \( t_L^* = t_L^{*L} - \varepsilon^* \) the level of real local school taxes per capita in the preceding year. i.e., the state expects local taxes to remain at prior year levels, adjusted for changes in the general price level.

(b) Let \( t_L^* \) be the level of school taxes per capita necessary to maintain the prior year's level of real per pupil spending,
assuming no change in state aid, i.e.,

\[ t_L^* = p_a^* \left[ a_{L-1}^* - s_{L-1}^* - f_L^* \right] \]

The role of \( s^* \) in the model is to represent the urgency of state aid to local school districts from the point of view of the state. It is assumed that, other things being equal, the state prefers to have local districts as favorably situated as possible in terms of the combinations of per pupil spending and per capita taxes accessible to them. For this reason, the state is willing, to a certain extent, to impose state taxes or curtail other state spending in order to provide education grants-in-aid. A reasonable measure of "need," therefore, is the amount of state aid per pupil required to prevent any worsening of the existing (prior year) local budget constraint. We define \( s^* \), accordingly, as the amount of state aid per pupil needed to maintain both the prior year level of real per pupil spending and the prior year real per capita local tax, or

\[ s^* = e_{L-1} - f_L^* \cdot \left( t_{L-1} / p_a^* \right) \]

**State Aid and State Tax Equations**

Effects of the various exogenous variables on state aid to school districts, state taxes, and other state expenditures can be determined by differentiating the three equations of the model totally and solving for \( d e_s, dt_g, \) and \( ds \). Details of these computations are available elsewhere. As with the local model, the results can be translated into a set of hypotheses about signs of the effects of individual exogenous variables on the dependent variables. Focusing on the two fiscal variables of direct interest, state aid and state taxes, the analysis leads us to expect equations of the general form,

\[ s = s(y - t_{p}, f_{s}, t_{L}, s^*, p_a^*, p_g) \]

and

\[ t_g = t_g(y - t_{p}, f_{s}, t_{L}, s^*, p_a^*, p_g) \]

Expected signs of the various effects, are shown in Table 1.

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Table 2. Effects of Changes in Exogenous Variables on State Aid and State Taxes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Effect on State Aid</th>
<th>Effect on State Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y - t_F$</td>
<td>$(?)^a$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$f_S$</td>
<td>$&gt; 0, &lt; 1$</td>
<td>$&lt; 0; &gt; -1$</td>
</tr>
<tr>
<td>$t_L^*$</td>
<td>$(?)^a$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$s^*$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$p_e^*$</td>
<td>$&lt; 0$</td>
<td>?</td>
</tr>
<tr>
<td>$p_g$</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

See text.

Note that the signs of certain effects are ambiguous: The income effect will be greater than zero provided that neither $y$ nor $s$ is an inferior good and that the positive effect of an increase in income on the state's willingness to tax outweighs the negative effect of a reduction in local tax burdens. The effect of $t_L^*$ on $s$ will be positive under the same conditions. Signs of the ambiguous price effects depend on whether certain price elasticities are greater or less than one.

Empirical analyses of the state-level data are aimed at testing the hypotheses implied by the table, including the general hypothesis, where the expected sign is uncertain, that the indicated variable "makes a difference."
Empirical Results

The Local Model

The local model was tested using cross-section and pooled time-series cross-section data. The principal source of data on expenditures, revenues, and numbers of pupils in each state was the biennial U.S. Office of Education (USOE) report, Statistics of State School Systems.* Complete information was obtained for 48 states (Alaska and Hawaii were excluded) for nine alternate school years from 1951-52 to 1967-68. Items obtained from this source include (a) total current expenditure for education, (b) total school district revenues from state sources, (c) total revenues from federal sources, and (d) numbers of pupils in average daily attendance. Data on disposable personal income per capita were obtained from a special tabulation published in the Survey of Current Business.** Population data consisted of Census estimates included in the biennial USOE reports.

Price Variables

The only serious measurement problem that arises in testing the local model concerns the two price variables, $P_e$, the index of prices of inputs to schooling, and $P_o$, the general price index. The former enters into the model as the deflator of per pupil spending and state and federal aid. The latter enters as the deflator of per capita income. The two also appear as a ratio, the relative price of education ($P_e$), in the composite price term, $p_e a$.

*U.S. Department of Health, Education, and Welfare, Washington, D.C. (In earlier years, these data were published in the USOE Biennial Survey of Education in the United States.)

Ideally, we would want to compute the education price index as a weighted average of prices of individual inputs into education, holding input quality constant across states and from year to year. By far the largest component of the index is professional salaries—the "prices" paid by school districts for teachers, administrators, and other professional education personnel. Such salaries average about 70 percent of current educational expenditures. Therefore, the possibility of constructing a satisfactory interstate index hinges on the availability of data that would permit identification of the quality component in interstate salary variation.

Such data are not now available. In fact, the concept of "quality" of educational personnel is not well defined, even in principle. Therefore, construction of a "true" interstate price index is not an available option. We must rely, instead, on some more sweeping assumption about quality, recognizing the biases that will inevitably result.

Dresch* has suggested the following three alternatives:

(a) Assume constant input quality in all areas (states), thereby assuming that all observed salary differences are price differences. This implies that the salary component of expenditures in each state is to be deflated by the ratio of the salary index in that state to the salary index in the nation.

(b) Assume that all observed salary differences reflect quality rather than price variations. The appropriate deflator then becomes the average salary index in the nation (i.e., an identical deflator for all states).

(c) Assume that observed salary differences are price differences only to the extent that they correspond to interstate variations in all wages and salaries (or in income levels), but that any variation in excess of the general wage and salary

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variation is attributable to quality differences. The appropriate deflator would be the same index as in (a), multiplied by the ratio of general wages and salaries (or incomes) in the area to the same magnitude for the nation.

Since all of these assumptions are unrealistic and none seems clearly preferable a priori, we have estimated our equations according to assumption (b), which is the simplest to handle. In other words, we use an education price index that varies from year to year, but not from state to state. The salary component of this national price index (base year = 1965) is computed from data on salaries of instructional staff published in the previously cited USOE biennial reports. This component is given a weight of 70 percent. Prices of other inputs into schooling are measured by the services (less rent) component of the consumer price index and given a weight of 30 percent.

Much the same situation prevails with respect to the general price index, $P_o$, except that there are fewer options. Lacking a consumer price index for each state, we are forced to use the national index, thereby implicitly assuming that there is variation over time only, but not across states. Some data are available that could be used to construct crude state-by-state indexes, but we have not undertaken that task in this study.*

Our incentives to develop interstate indexes have been substantially reduced by the realization that little is to be gained from

*Comparative data on family budgets for different U.S. cities compiled by the Bureau of Labor Statistics (e.g., in City Worker's Family Budget for a Moderate Standard of Living, U.S. Department of Labor, Bulletin 1570-I, Washington, D.C., 1967) could conceivably be used to develop rough indexes of general price variation across states. Another possibility would be to use the "iso-prop" indexes developed by Harold W. Watts ("The Iso-Prop Index: An Approach to the Determination of Differential Poverty Income Thresholds," Journal of Human Resources, Vol. II, No. 1, Winter 1967). Both types of data, however, are available for only one year and could only be applied to a single cross-section analysis.
developing either a $P_e$ or a $P_o$ index unless both are developed simultaneously. This is because the two appear in the model as a ratio in the price term, $p_e \alpha$. While it is unrealistic to assume that $P_e$ and $P_o$ are constant across states, it is probably much less unrealistic to assume that the relative price of education, $P_e/P_o$ is constant. Therefore, using a true interstate index of either $P_e$ or $P_o$, while continuing to treat the other as a constant, could actually result in poorer measurement of the relative price of education.

Cross-sectional Results

Two forms of the model were tested using cross-section data: (a) a conventional linear form containing all the variables identified under "hypotheses" (pp. 15 – 16), and (b) an alternative form, linear in the same variables or ratios thereof, based on the Linear Expenditure System used in studying consumer demand.* In addition, we tested variants of the models containing additional state characteristics (urbanization, population density, and regional identity) and forms allowing for interactions among the independent variables. Both elaborations are compatible with the theoretical model.

The conventional linear form was found to be superior to the form based on the Linear Expenditure System according to all the usual measures ($R^2$, $F$, and $t$-values of coefficients) whether or not additional state characteristics or interaction terms were included. Therefore, the latter form was eliminated. The linear model, without either type of elaboration, generally satisfied the expectations regarding signs and magnitudes of coefficients, except that no significant difference between federal and state aid effects could be detected.

In addition, examination of the residuals of the cross-sectional regressions revealed that a South vs. non-South regional difference remained unaccounted for. A series of tests led to the inclusion of both an additive regional (Southern) dummy variable and a term representing a regional influence on the aid coefficient. There was also found to be a significant difference in spending between states with especially low population densities (population per square mile less than 30) and the other states. The final set of cross-section equations, therefore, includes the income, aid, price \((p_a)\), and enrollment growth terms, the Southern region and low density dummy variables, and the regional-aid interaction term.

The nine cross section equations are shown in Table 2 (t-values in parentheses). Some of the major results are as follows:

1. The equations are generally consistent from year to year with respect to both "fit" \((R^2\) and coefficient of variation) and values of coefficients of the major variables. The degree of consistency is greater than has generally been found in comparative cross section studies of state-local expenditures. Experiments with alternative forms of the model showed that this consistency is at least partly attributable to inclusion of the variables \(p_a\) and \(a\) in the product form called for by the theory. There are, however, perceptible time trends in income and price coefficients. These may reflect either improper measurement of prices or actual structural changes over the 17 year period spanned by the data (see the discussion of pooled results below).

2. The magnitude of the income coefficient, which varies roughly from .14 to .20, means that 3.3 to 5.0 percent of each marginal dollar of income translates into school spending, holding the level of state aid constant. The figure varies from state to state.

*Gramlich, op.cit.

**Note that the model relates spending per pupil to income and outside aid per capita. Since there is roughly one pupil for every four or five persons in the population, the coefficients must be multiplied by a factor on the order of 0.20 to 0.25 to determine the fraction of each dollar increment in income or aid that is used for educational spending.
## Table 2  SCHOOL DISTRICT EXPENDITURE

### EQUATIONS, LINEAR MODEL: CROSS-SECTIONAL RESULTS

\[
e = \beta_0 + \beta_1 R + \beta_2 y + \beta_3 a(s + f_L) + \beta_4 p_e(a(s + f_L)R + \beta_5 p_e + \beta_6 \Delta^A + \beta_7 LD
\]

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<th>School Year</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( \beta_6 )</th>
<th>( \beta_7 )</th>
<th>( R^2 ) (adj.)</th>
<th>Coefficient of Variation</th>
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<td>(2.6)</td>
<td>(3.3)</td>
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<td>(5.5)</td>
<td>(1.9)</td>
<td>(2.9)</td>
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</tr>
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</table>

**Variables:**
- \( e \) = real current educational expenditure per pupil
- \( y \) = real per capita disposable income
- \( a \) = real state aid per pupil
- \( f_L \) = real federal aid per pupil
- \( p_e \) = relative price of education
- \( a \) = ratio of pupils to population
- \( R \) = Southern region dummy variable
- \( \Delta A/A \) = percentage increase in average daily attendance during a two-year period
- \( LD \) = dummy variable identifying states with low population densities
in any one year because of interstate variations in the ratio of pupils to population. The state aid coefficient, which averages about 1.3 for non-Southern states (but is less stable over time than the income coefficient) signifies that roughly 25 to 30 cents out of each marginal dollar of outside aid goes for increased school spending. The amount varies according to the pupil/population ratio in each state. The larger fraction of aid funds, therefore, goes for local property tax relief. Note that the aid coefficient is always substantially higher than the income coefficient (by a factor of six to ten), as hypothesized. This means that if the state or federal government chose to provide additional aid and to finance it entirely out of additional taxes on residents of the recipient districts, the increased state or federal taxes would offset only a fraction of the expenditure effect of the increased aid.

(3) Values obtained for the price coefficient translate into estimated price elasticities of per pupil spending ranging from -0.5 to -0.9, depending on the year. This means that somewhat less than half of an increase in costs of education, e.g., teachers' salaries, will be compensated for by increased spending, and the remainder by reductions in staffing or other input ratios. The sign of the effect of the price variables on real school taxes per capita, which was left ambiguous by the theory, must therefore be positive; i.e., tax burdens rise with costs of education.

(4) Until about 1961, Southern states appeared to differ from the rest of the nation in two respects: (a) they spent less per pupil, even after effects of lower than average per capita income and other variables had been taken into account; (b) southern districts appeared to be more responsive than districts elsewhere to changes in outside aid. After 1961-62, however, both regional effects become insignificant, suggesting that over time Southern tastes for education have become more like those in the rest of the country.

(5) As to the remaining variables, the enrollment growth effect is negative, as expected, but is significant \( (t > 2) \) in only four of the nine equations. The dummy variable designating states with low population densities has a positive coefficient as hypothesized.
Pooled Results

The equation obtained by simply pooling the nine sets of cross-section data, without allowing for structural changes over time, is the following:

\[ e = 218 - 109R + .202y + (1.15 + 1.01R)P_ga(e + f_L) \]
\[ (-12.1) \quad (-7.1) \quad (24.9) \quad (9.7) \quad (4.1) \]
\[ -949P_gA - 372A/A + 34.4LD \]
\[ (-11.5) \quad (-6.3) \quad (5.4) \]

\[ R^2 = .85, \text{ Coeff. of Var.} = 9.1 \text{ percent} \]

Note that a number of the coefficient values do not seem compatible with those obtained by fitting single cross-sections. For example, the price coefficient is much lower than that obtained in any cross-sectional equation, the income coefficient is just outside the range of values obtained cross-sectionally, and the constant term is much lower than in any of the equations in Table 2. This suggests that some structural changes have taken place over time that are not properly reflected by simple pooling.

We were not able to discover other variables or interaction terms that accounted for structural shifts. Having noticed, however, that there seemed to be trends in values of certain coefficients over the nine years, we tested for systematic time trend effects on coefficients in the pooled model. The following equation, with three significant time trend effects, was obtained (the time trend variable, \( T \), is 0 for school year 1951-52, 1 for 1953-54, and so forth):

\[ e = 477 - 163R + (.123 + .0074T)y + (1.35 + 3.09R - .22FT)P_ga(e + f_L) \]
\[ (13.1) \quad (-8.1) \quad (9.7) \quad (4.5) \quad (5.2) \quad (5.1) \quad (-3.8) \]
\[ -(.2218 + 43.2T)P_gA - 241A/A + 32.1LD \]
\[ (-9.9) \quad (1.6) \quad (-4.1) \quad (8.4) \]

\[ R^2 = .87, \text{ Coeff. of Var.} = 8.5 \text{ percent} \]

The values of income, aid, and price coefficients in this equation are all consistent with values obtained in the cross-section regressions. Also, the regional effect on the aid coefficient diminishes over time,
as appeared to be the case in comparing cross-sections. No significant trend term, however, was found to be associated with the additive Southern region dummy variable.

Extrapolation to school year 1971-72 yields the following expenditure equation:

\[ e = 477 - 163R + .197y + (1.35 + .89R)p_e(a + f_L) - 1786p_ea - 241AA/A + 52.1LD \]

The values of these coefficients imply, for the "average" non-Southern state:

a. an increase of roughly $0.05 in real school spending for each one dollar increase in real per capita income (or a corresponding decrease in spending per dollar increase in state or federal taxes);

b. an increase of about $0.33 in real school spending per pupil for each dollar of per pupil state aid;

c. a price elasticity of per pupil spending of approximately -0.6.

These values (with state to state variations, as called for by the model) will be used to estimate effects at the local level of various federal funding alternatives.

THE STATE MODEL

Data

In testing the state model, it was possible to work with a larger data base, not limited to biennial observations only as was the data base used for the local expenditure analysis. The principal data source was the U.S. Bureau of the Census series, State Government Finances.* Data for 18 fiscal years, from FY 1953 to FY 1971, were

obtained on the following variables: (a) state intergovernmental expenditure for education (used as the measure of state aid to local schools); (b) total intergovernmental revenue from the federal government (federal aid to states); (c) total state taxes; and (d) total state expenditures for all functions. By having annual data on the fiscal magnitudes, it became possible to calculate year-to-year increments in variables of interest and to examine time series for individual states, neither of which was feasible with the USOE biennial data.

It was necessary to adjust the data in two respects: First, regarding the data on federal aid to states, it is Census practice to count federal aid to local governments that is channeled through or distributed by states as part of federal aid to states.* To make the data compatible with the model, it was necessary, therefore, to subtract federal aid to local school districts from the federal-to-state aid totals. Data on aid to school districts was not available from the Census documents, but was obtained from the previously cited USOE reports and National Education Association estimates.**

The second adjustment pertains to data on numbers of pupils in public schools in each state. It was necessary to interpolate figures on average daily attendance for years between the USOE biennial surveys. These interpolations were based, however, on an annual series on "opening fall enrollment" in elementary and secondary schools, also published by USOE.*** Therefore, the interpolation is likely to be quite accurate. In addition, the data were adjusted

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*Ibid., p. 55.


to account for students enrolled in locally operated public junior colleges in several states. Such students are not counted in the USOE average daily attendance or fall enrollment figures, but state aid for the schools they attend is included in the Census' intergovernmental expenditure data. The adjustments were based on incomplete data on junior college enrollment published by USOE.

The state model requires one additional price variable that did not appear in the local model, namely, the price index for state-local services other than education \((P)\). This is measured by the implicit price deflator for purchases of goods and services by state and local governments.* For reasons discussed earlier, we used a price index that varies from year to year, but not across states.

Problems of Structure and Timing

In attempting to model the behavior of state governments in supporting their public schools, one encounters major problems that did not arise or were negligible at the local level. Perhaps the most important is that there are major differences in fiscal structure among states that do not appear amenable to economic or econometric explanation. For reasons that may lie deep in history and tradition, states vary substantially in the scope of their responsibilities vis-à-vis local governments and in the institutional arrangements, e.g., tax structures, for carrying out their fiscal roles.

With respect to public elementary and secondary education, in particular, there are very large interstate differences in the division of responsibility for school support between the state and the localities. A few states, such as North Carolina and Delaware, have assumed dominant roles in school finance, providing 78 and 77 percent, respectively (in 1970-71), of school revenues directly from state funds.

Several other states, e.g., New Hampshire (10 percent) and South Dakota (17 percent) leave school finance almost entirely to localities.* The others occupy positions everywhere in between. There is little apparent connection between either the absolute level of state support or the relative size of the state contribution (state support as a fraction of total state plus local support) and such likely state characteristics as per capita income, industrialization, and urbanization.** Therefore, the prospects for an adequate explanation of interstate variation in state aid to local schools do not seem high just from preliminary examination of the data and simple correlation analysis.

Another serious problem is that changes in levels of real state aid per pupil are highly discontinuous. A typical pattern is that a state will set a particular level of aid in one year, then more or less maintain it with "cost of living" increases over the next few years, perhaps with some erosion in real per pupil amounts, until finally deciding on a new level several years after the last major decision. The result is that any cross-sectional analysis is likely to catch different states at different stages of these cycles, making comparisons very difficult.*** Moreover, analysis of year-to-year increments in state aid—an otherwise promising approach, given the structural differences that exist—is virtually ruled out by this pattern of behavior.

Undoubtedly, very similar patterns would have been found at the local level if individual districts were the units of observation.


** One exception is that being a Southern state appears to be strongly associated with a high state share of state plus local support.

*** On this point, see Gramlich, op.cit., p. 579.
However, the effect of working with aggregates of local spending in each state is that such discontinuities tend to be smoothed out. Similar aggregation of state observations is not an available option. It is necessary to work directly with the individual decision making units, which means that all the idiosyncracies, cyclical patterns, and singular events affecting state aid are reflected directly in the dependent variables of the equations. Thus, the probability of discovering an equation with high explanatory power is further reduced.

State Aid Equations and Results

Our low expectations regarding ability to explain state aid were borne out by the results. Cross-sectional tests proved to be almost worthless, with most coefficients insignificant and the "significant" ones highly inconsistent from one cross-section to the next. This was true whether the cross-sectional dependent variables was the level of state aid in a given year of the state aid increment. Only pooled time-series, cross-section estimates showed any promise of yielding a meaningful state aid equation.

A state aid equation obtained from the full pooled sample (18 years x 48 states = 864 observations) and including only the income, aid, and price variables is as follows:

\[ s = 189 + 0.062y_d + 0.34f_G + 628p_e - 283p_d \]

\[ (3.5) \quad (6.2) \quad (2.2) \quad (3.0) \quad (-2.6) \]

\[ R^2 = .10, \quad \text{Coeff. of Var.} = .50 \]

This equation obviously accounts for very little of the interstate and intertemporal variance. Values of the income and state aid coefficients seem plausible, however, when it is recalled that both variables are stated in per capita terms, while \( s \) is measured in dollars per pupil, and that there are roughly 0.25 pupils per capita. I.e., the equation implies that approximately $0.015 out of each dollar increment in personal income and approximately $0.08 out of each dollar increment in federal aid to states will be used to increase state aid to local schools. If the latter value seems
low, the reason may well be that most federal aid to states in the past has been categorical aid for state programs other than education. To the extent that categorical restrictions have been enforced, shares of such aid going to other than the stipulated purposes (e.g., to state school aid or state tax reduction) would be smaller than the corresponding shares under a system of unrestricted grants. This point is discussed further below in connection with the state tax and other state expenditure equations.

The price of education term, $p_e a$, enters positively in the above equation, contrary to hypothesis. However, since the "expected tax" and "needed" state aid variables, $t^*_L$ and $s^*$, are omitted (see below), this is not ruled out. The reason is that an increase in the price of education would have the effect of increasing $s^*$, the level of aid needed to maintain prior year levels of services and tax burdens, and $t^*_L$, the tax level needed to maintain prior year values of per pupil spending, both of which would tend, according to the theory, to stimulate more state aid. The two effects could offset the pure price effect and result in a positive $p_e a$ coefficient.

It appeared, from cursory inspection of the residuals, that important regional differences were not accounted for by the variables in the equation. Consequently, we added regional dummy variables for the Northeast, Midwest, and Southern states, denoted by $R1$, $R2$, and $R3$, respectively. When the three variables were included additively in the model the following equation was obtained:

$$s = 8.5 + 0.95 y_D + 0.52 y_S - 549 p_e a + 78 p_g - 93 R1 - 78 R2 + 39 R3$$  
$$R^2 = 0.29, \text{ Coeff. of Var. } = 0.44$$

In this version considerably more variance is accounted for, albeit by regional dummies; the income and federal aid coefficients are larger than before (this reflects systematic regional differences in both federal aid allotments and per capita incomes); and the price coefficients have changed signs. The behavior of the $p_e a$ and $p_g$ coefficients seems to be erratic and not readily interpretable because
of the high colinearity (simple correlation about 0.88) between the two variables. The Eastern and Midwestern states provide lower state aid per pupil, while Southern states provide higher aid (taking differences in per capita income and other variables into account) than those in the West.

Further tests for effects of urban-rural differences and for differential effects of federal aid among regions yielded the following equation (the new variable, URB, is the fraction of a state's population that is urban):

\[ s = 208 + 0.071y_D + (0.35 + 1.72R1 + 0.71R2 + 1.58R3)f_S \\
(3.4) (6.0) (2.1) (5.4) (2.2) (5.1) \\
+ 159p_\alpha - 384p_\beta - 145R1 - 87R2 - 2.6R3 + 233URB \\
(0.7) (-3.0) (-8.8) (-5.2) (-0.2) (7.0) \]

\[ R^2 = .34, \text{ Coeff. of Var.} = .42 \]

This version provides some support for the proposition that states in different parts of the country respond differentially to federal aid (or, more generally, that there are important differences in behavior patterns among states not accounted for by the variables in the model). Some of the estimates of regional parameters, however, do not seem reasonable. For example, the Northeastern region's federal aid coefficient would be more than 2.0, which would imply that more than 50 percent of all federal aid to that region would go for increased state aid to schools. Since only about 20 to 22 percent of state expenditure now goes for education aid, and since some fraction of federal aid to states would probably be allocated to state tax relief rather than increased expenditure, a response coefficient that high seems very unlikely. A more plausible explanation is that the regional dummies in this, and the preceding, equation are acting as proxies for other state characteristics or individual state factors that have been omitted from the model and that the specification of the equation is incomplete.

An additional shortcoming of the empirical models is that the variables representing expected local tax burdens and "needed" state aid failed to operate as hypothesized. When entered into a state aid
equation, they typically appeared significant, but with an incorrect sign. The reason is clear retrospectively. Given the way $t^*_L$ is measured, for example (see p. 19), a state that finances a relatively small share of school support from state funds will "expect" a greater increase in local taxes to maintain prior year expenditure levels. Other things being equal, however, the level of aid in such a state will be low. Therefore, a negative, rather than positive, correlation between $t^*_L$ and $s$ will be observed.

The existence of a state reaction to the local fiscal situation was confirmed, however, in one respect. It follows from the role of $s$ in the model that state aid to local districts should decline as direct federal aid to districts increases. The reason is that, other things being equal, the state will perceive declining local "need" for state transfers as additional outside funds become available (note that federal aid, $f^*_L$ enters negatively in the definition of $s^*$ on p. 20). To test this effect, a term in $f^*_L$ was added to the state aid equation. The result was the following:

$$ s = 164 + .073y_D = (.38 + 1.66R1 + .73R2 + 1.64R3)f^*_L $$

$$ + 92p_e - 304p_g - .41f^*_L - 151R1 - 95R2 - 7.5R3 + 224URB $$

$$ R^2 = .35, \text{ Coeff. of Var. } = .42 $$

The $f^*_L$ coefficient of -.41 represents an offsetting state aid reduction of 41 percent of the amount of new federal aid to local districts. Should such a coefficient be confirmed by further analysis it would have considerable policy significance, since it implies a much smaller effect of federal aid than would be predicted from analysis of only the local recipients' behavior. The quality of the present equation is too low, however, to justify treatment of the offset effect as anything but a suggestion and a matter for further investigation.

In an effort to eliminate individual state effects from the pooled equation, we also estimated difference forms of equations similar to those given above. The results were generally discouraging,
with many of the key coefficients either insignificant or of unreasonable magnitude. It appeared that the difference form not only failed to eliminate individual state effects, but suffered from the previously noted discontinuity in state expenditure changes over time. In hopes of minimizing the latter effect, we reestimated the equations using three-year and five-year differences rather than the single year differences used in the initial version. However, this attempt to measure "average" changes over several years was also unsuccessful.

Given these results our overall conclusion is that the state aid equation has not yet been developed in an acceptable form. The major required improvements seem to be (a) additional provisions to take account of differences in fiscal structure among states (e.g., by including structural dummy variables, or perhaps, incorporating individual state effects in the pooled cross-section, time-series analysis); (b) inclusion of time trend or other structural shift parameters in the equations to allow for changes in "tastes" over time; (c) development of better measures of state perceptions of local "needs" for aid, to avoid the ambiguities inherent in the definitions of the $s^*$ and $t^*_L$ variables, and (d) investigation of the possibility that state responses to federal aid and other variables operate with lags. Work along all these lines is proceeding.

Equations for Other Expenditures and State Taxes

Efforts were made to develop predictive equations for the remaining two state fiscal variables, "other" state expenditure, $e_s^*_S$, and state taxes, $t_S^*$. The latter, of course, is needed to provide inputs to the local expenditure model. The former, while not entering directly into the school district model, provides a useful consistency check.

Pooled time-series, cross-section equations for $e_s^*_S$ and $t_S^*$ containing income, aid, and price terms only, are as follows:
\[ e_S = 69.4 + .031y + 1.58f_S - 28p_g \]
\[ (5.3) \quad (11.9) \quad (36.7) \quad (-1.5) \]
\[ R^2 = .75, \quad \text{S.E.E.} = 30.0 \]

\[ t_S = -138 + .024y + .40f_S + 212p_g \]
\[ (-11.9) \quad (10.5) \quad (10.6) \quad (13.2) \]
\[ R^2 = .62, \quad \text{S.E.E.} = 26.7 \]

A series of tests for differential regional effects and influences of other state characteristics yielded the following more elaborate forms:

\[ e_S = 105 + .048y + (1.54 + .25R1 + .13R3)f_S - 70p_g - 60URB \]
\[ (7.3) \quad (11.9) \quad (34.4) \quad (4.1) \quad (1.9) \quad (-3.2) \quad (-5.4) \]
\[ R^2 = .76, \quad \text{S.E.E.} = 29.1 \]

\[ t_S = -121 + (.050 - .21R1 - .014R2 )y + (.20 + .64R1 + .21R2)f_S \]
\[ - 152p_g \]
\[ (-10.4) \quad (17.7) \quad (-11.9) \quad (-6.7) \quad (5.0) \quad (7.7) \quad (2.3) \]
\[ R^2 = .71, \quad \text{S.E.E.} = 23.5 \]

Despite the apparently good results in terms of "fit" and variance explained, especially in comparison with the state aid equations, the most important feature of these results is that the values of federal aid coefficients in each equation are in direct conflict with theory. In the equation for \( e_S \), the federal aid coefficient is greater than one. This, if taken literally, would imply that federal lump-sum grants do not substitute at all for states' own revenues, but actually stimulate the state to raise more revenues than would otherwise have been forthcoming. Similarly, the positive coefficient of \( f_S \) in the equation for \( t_S \) (where a negative value was called for by the theory) reinforces the implication of a net stimulative effect. Such results are very difficult to accept, since they are barred by...
relatively weak theoretical assumptions. Moreover, the results, if questionable, also call into question the validity of estimates of the effect of \( f_S \) on state aid to local schools. It would clearly be unjustified to use these state equations for predictive purposes unless the conflict with the theory can be resolved.

A possibility that has been discussed in the literature is that federal aid coefficients in equations of this type are likely to be incorrectly estimated because the simultaneity of expenditure and aid determination has not been taken into account. According to Pogue and Sgontz, biased estimates of the aid effect will be obtained if either (a) aid payments are a function of expenditures, or (b) aid payments are a function of some of the exogenous determinants of expenditures. It does not seem likely that case (a) is of great importance, although some feedback effects from expenditures to aid undoubtedly exist. Case (b), however, is almost certain to be relevant, since income, urbanization, and possibly other variables omitted from the expenditure equation appear frequently in federal grant-in-aid formulas.

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* E.g., the assumption that neither education aid, other state spending nor private spending is an inferior good is sufficient to preclude a net stimulative effect of lump-sum grants-in-aid.


**** For a feedback effect to exist, there must be federal matching at the margin. Such is not the case, however, with the major federal aid programs. Aid to education generally does not carry matching requirements. Highway aid takes the form of a fully-utilized matching grant. Welfare (AFDC) payments in most states are above the level at which there is federal matching. However, welfare is an open-ended program in that the federal government will provide additional funds as more welfare recipients (but not higher payment levels) are added. The overall extent of federal matching at the margin has not been determined, but the likelihood that the overall marginal matching ratio (if such a concept can be defined) is sufficiently great to produce the observed stimulative effect seems negligible.
To allow for the second kind of simultaneity, we reestimated the state equations by two-stage least squares, incorporating such additional variables as population density and state size in the first stage equation for $f$. The results, however, did not constitute an improvement. In the case of the state aid equation, several coefficients changed in a direction contrary to theory and the federal aid coefficient increased to values of 5.0 or more. These results are clearly not acceptable either from a theoretical or common sense point of view. In the case of the state tax and other state spending equations, the federal aid coefficient changed only slightly, decreasing in some cases and increasing in others. In no instance did the value become less than one as required by the theory. Therefore, although it is certainly proper to acknowledge econometrically that some determinants of state spending also act as determinants of federal aid, that refinement alone does not resolve the basic problem.

Another possible explanation of the disturbing empirical results rests on the categorical nature of most federal aid to states. As a general proposition, the expenditure effect of a categorical grant will be greater than the effect of an unrestricted grant if the categorical grant is accompanied by binding constraints, (i.e., conditions that force the aid recipient to spend more for the aided function than he would have in their absence). If, in addition, the aid recipient is required to match categorical grants, then even where matching is not open-ended, it becomes possible for the aid effect to be greater than one.

The practical problem that makes it difficult to include categorical aid in the model is that one cannot do so using aggregative data. It is necessary to observe the magnitudes of both federal categorical grants and required state matching funds. We are attempting to determine whether the necessary data can be obtained or approximated from published statistics. Until such a test is carried out, it remains possible that the interrelated phenomena of categorical aid and state matching requirements (plus the relatively few open-ended matching formulas that exist) may account for the otherwise unacceptable empirical results.
GENERAL AID TO STATES VS. DIRECT AID TO
SCHOOL DISTRICTS: SOME ILLUSTRATIVE COMPARISONS

It is convenient to dichotomize the analysis of impacts of federal aid into two parts, the first concerned with the effects of a given type of aid on an "average" state or on the nation as a whole, the second with differential effects among states and their distributional implications. The reason for this separation is that different kinds of information, or analytical results, are needed to address the two questions. To estimate the average impact of a grant-in-aid proposal, one needs to know only (a) the form of aid (e.g., whether a lump-sum or matching formula is used and whether there are restrictions on the use of aid funds); (b) the method of financing the aid (e.g., out of taxes, by reducing other programs, or by borrowing); and (c) the response characteristics (coefficients) of the "average" state. Analysis of distributional aspects requires much more, including (d) the formula for allocating aid among the states and (e) information on the response characteristics of each state and interactions between the response characteristics and the form of aid.

Given the results of the state-level empirical analysis, there is little enough information on which to base even the "average state" projections. Essentially arbitrary assumptions must be made about the magnitudes of state fiscal responses to federal aid and other variables. There is even less information bearing on distributional effects, especially since the regional parameters in the state equations, which provide the main evidence for differential effects among states, do not appear to be reliable. Consequently, in developing illustrative projections of grant-in-aid impacts we have concentrated on the average, or national, effects, focusing on simple aid alternatives with clear-cut differences in effects.

ALTERNATIVES

Specifically, we consider two types of aid and two contrasting
assumptions about financing. The aid types are (1) unrestricted aid to states (revenue sharing) and (2) direct federal aid to local school districts. The two assumptions about financing are (1) that aid is financed out of reductions in other federal programs (i.e., the total federal budget and federal taxes remain unchanged), and (2) that aid is fully financed out of an increase in per capita taxes equal to the increase in per capita federal aid.

In each case it is assumed that aid takes the form of a lump-sum grant, either to the state or the school district, as the case may be. There are assumed to be no binding restrictions on uses of aid funds. Incremental state and federal taxes are treated as simple subtractions from personal income (i.e., no price effects are included).

The projections are for the school year 1970-71. Extrapolated values of the various national price indexes for that year are as follows: $P_e = 1.26$, $P_g = 1.21$, $P_o = 1.16$ (base year = 1967). The only other required datum when personal income and prices are assumed constant is the pupil/population ratio, $a$, for which we assume a value of 0.25.

ASSUMPTIONS ABOUT STATE RESPONSES

Since the empirical analysis did not confirm the hypotheses about state responses to federal grants and did not yield usable values of federal aid coefficients, our only recourse is to examine the consequences of a series of assumptions about the magnitudes of state responses. The key parameters can be defined in several different ways. Our choice is to examine alternative assumptions about the following three response coefficients:

a. The fraction of an increment in unrestricted federal

*The aid is considered "direct" even if funds are channeled through the state, provided that a full pass-through is required.
aid that the average state will allocate to state tax reduction.

b. The fraction of aid not allocated to tax reduction (i.e., the fraction spent) that will be used to increase state aid to education.

c. The fraction of direct federal aid to local districts that will be offset by an induced reduction in state aid to local districts.

These are designated $\sigma_1$, $\sigma_2$, and $\sigma_3$, respectively.

We have almost no valid information on the probable magnitude of the first parameter. According to the estimated equations (e.g., the second equation on p. 39) the fraction is negative; i.e., the state allocates none of the aid receipts to tax reduction, but rather increases state taxes to a higher level than would have obtained in the absence of aid. For reasons already stated, and regardless of the factors that led to those empirical results, it remains unreasonable to assume a net stimulative effect of lump-sum, unrestricted aid. Therefore, we test assumptions spanning the range from zero to one hundred percent allocation of aid receipts to tax reduction, using as intermediate values fractional reductions of $1/3$ and $2/3$.

Our "base case" assumption about the second parameter, the fraction of incremental state expenditure allocated to public school aid, is that the fraction will be the same as the average ratio of public school aid to total state expenditures in the U.S. in 1970. That ratio is calculated from Census data as 0.22. The sensitivity of fiscal outcomes to that ratio is tested by using values of one-half and twice the base value, 0.11 and 0.44.

As an initial estimate of the state aid offset to direct federal aid to districts, we use the value obtained in the equation on p. 37, a coefficient of -0.4. The values used to test sensitivity are -0.2 and -0.8. To complete the specification of state behavior in offsetting federal aid, it is necessary to postulate some application of the released funds. We will assume that the "savings" in state aid funds are distributed proportionately between state tax reduction and other state expenditures.
It is also necessary to make assumptions about two other parameters, the responses of state aid and state taxes, respectively, to changes in the level of federal taxes. Since it has been assumed that the state response to a change in taxes is the same as the response to any equivalent change in disposable personal income, we set these parameters at the values of the income effects estimated in the state aid and state tax equations (pp. 37 and 39), i.e., .07 and .024 respectively.

Finally, we add one behavioral element that was not included in the econometric model. This is an assumption that some fraction of "other state expenditures," $e_S$, is used for aid to local governments other than school districts and ultimately translates, in part, into reductions in non-school local taxes. Arbitrarily, we assume that 0.25 of $e_S$ goes into tax reduction, and that the reduction affects state school aid and state taxes in the same way as an equivalent reduction in federal taxes.

**PROJECTION MODEL**

The above assumptions are embodied in the following set of equations for projecting changes in local school spending and taxes, $\Delta e$ and $\Delta t_L$, in response to changes in federal aid to states, $\Delta f_S$, direct federal aid to districts, $\Delta f_D$, and federal taxes, $\Delta t_F$.

**State-Level Equations**

\[
\Delta t_S = \alpha_1 p e \Delta f_S - \alpha_4 \Delta t_F - \alpha_7 \alpha_3 p e \Delta f_L \\
\Delta e = (1/p e) [(c_2 (1 - c_1) \Delta f_S - c_6 \Delta t_F) + c_3 \Delta f_L] \\
\Delta t_o = \alpha_3 [(1 - c_2) (1 - c_1) p e \Delta f_S - c_8 \alpha_3 p e \Delta f_L].
\]

**Local Equations** (from extrapolated equation, p. 30)

\[
\Delta e = 1.35 p e a (\Delta e + \Delta f_L) - .197 (\Delta t_S + \Delta t_o + \Delta t_F) \\
\Delta t_L = p e a (\Delta e - \Delta t_S - \Delta f_L).
\]
The only variable not previously defined is $\Delta t_{01}$, the change in taxes by local governments other than school districts, which was introduced immediately above.

Values of the parameters are as follows (multiple entries indicate that alternative parameter values are to be tested):

$$
\begin{align*}
\sigma_1 & = 0, -1/3, -2/3, 1 \\
\sigma_2 & = .11, .22, .44 \\
\sigma_3 & = -.2, -.4, -.8 \\
\sigma_4 & = .024 \\
\sigma_5 & = .25 \\
\sigma_6 & = .07 \\
\sigma_7 & = \sigma_1 / (\sigma_1 + (1 - \sigma_2)(1 - \sigma_1)) \\
\sigma_8 & = 1 - \sigma_7
\end{align*}
$$

RESULTS

Table 3 displays the effects of each federal aid alternative on the major state and local fiscal variables under various sets of assumptions about parameter values. The four alternatives, as previously identified, are revenue sharing and direct aid to school districts, each with and without increases in federal taxes. The effects shown in the table are (a) changes in outside aid per pupil to local districts (state, federal, and total); (b) changes in all non-school taxes (state, federal, other local, and total); and (c) changes in school spending and local school taxes, each stated in per pupil and per capita terms. All values in the tables correspond to a federal aid increment of one dollar per capita.

The sensitivity of outcomes to alternative assumptions about state responses to federal aid is explored most fully in connection with alternative $A_1$, revenue sharing with no change in federal taxes. Cases $A_1 - A_4$ show what happens as the fraction of aid allocated to state tax reduction rises from 0 to one hundred percent. The surprising result is the relative insensitivity of school spending. Increments in per pupil expenditure fall only from $0.36 to $0.21 as $\sigma_1$ varies from zero to one. The amount of local property tax relief does depend strongly, however, on the extent to which federal funds are used to relieve state taxes. A distinct trade-off, or substitution, effect between state and local taxes is evident. The degree of total (state + local) tax relief rises
Table 3. Effects of a One Dollar Increase in Federal Aid per Capita to an "Average" State Under Various Assumptions About State Behavior

<table>
<thead>
<tr>
<th>Form of aid and financing</th>
<th>Assumed values of State response parameters</th>
<th>Change in aid to local districts ($/pupil)</th>
<th>Change in state, federal, and &quot;other local&quot; taxes ($/capita)</th>
<th>Change in school spending</th>
<th>Change in local school taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>State</td>
<td>Fed.</td>
<td>Total</td>
<td>State</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td></td>
</tr>
<tr>
<td>A. Revenue sharing, no increase in federal taxes</td>
<td>A1</td>
<td>0</td>
<td>.22</td>
<td>NA</td>
<td>.88</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>-1/3</td>
<td>.22</td>
<td>NA</td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>-2/3</td>
<td>.22</td>
<td>NA</td>
<td>.29</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>-1</td>
<td>.22</td>
<td>NA</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A5</td>
<td>-1/3</td>
<td>.44</td>
<td>NA</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>A6</td>
<td>-1/3</td>
<td>.11</td>
<td>NA</td>
<td>.29</td>
</tr>
<tr>
<td>B. Revenue sharing fully financed out of federal taxes</td>
<td>B1</td>
<td>-1/3</td>
<td>.22</td>
<td>NA</td>
<td>.51</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>-2/3</td>
<td>.22</td>
<td>NA</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>-1/3</td>
<td>.44</td>
<td>NA</td>
<td>1.10</td>
</tr>
<tr>
<td>C. Direct aid to districts, no increase in federal taxes</td>
<td>C1</td>
<td>-1/3</td>
<td>.22</td>
<td>-.4</td>
<td>-1.60</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>-1/3</td>
<td>.22</td>
<td>-.8</td>
<td>-3.20</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>-1/3</td>
<td>.22</td>
<td>-.2</td>
<td>-.80</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>-2/3</td>
<td>.22</td>
<td>-.4</td>
<td>-1.60</td>
</tr>
<tr>
<td></td>
<td>C5</td>
<td>-1/3</td>
<td>.44</td>
<td>-.4</td>
<td>-1.60</td>
</tr>
<tr>
<td>D. Direct aid to districts, fully financed out of federal taxes</td>
<td>D1</td>
<td>-1/3</td>
<td>.22</td>
<td>-.4</td>
<td>-2.03</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>-1/3</td>
<td>.22</td>
<td>-.8</td>
<td>-3.63</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>-1/3</td>
<td>.22</td>
<td>-.2</td>
<td>-1.23</td>
</tr>
</tbody>
</table>
and the expenditure increment diminishes as the state diverts increasing fractions of aid funds to state tax reduction.

Cases A5 and A6 show effects of variations in the school aid share of state expenditures. An increase in that ratio channels a greater share of federal aid to local districts to be allocated between increased spending and local property tax relief. Note, however, that even in the most favorable case from the local point of view (case A5), only $0.13 out of each dollar of federal aid translates into increased school spending and only $0.16 into local property tax relief.

Cases B1, B2, and B3 illustrate the effects of paying for aid out of increased federal taxes. Comparing B1 and A2, which are identical except for the federal tax, it can be seen that a dollar of aid produces only 25 percent as great an increase in per pupil spending when financed out of taxes as when financed out of resources drawn from other programs. The amount of local tax relief is not very much affected. In fact, more local tax relief is forthcoming in cases B2 and B3 than in the corresponding cases with no federal taxes. Again, the phenomenon of tax substitution is shown to be important. As federal taxes increase, the local response is to generate a partially compensating local tax reduction primarily at the expense of public school outlays.

When aid is channeled directly to school districts rather than to the state, the direct effect, of course, is to make a considerably larger fraction of aid available for school spending and local property tax relief. But the extent of the local gain—or whether there is any gain at all—depends critically on how the state adjusts its own aid apportionments. As can be seen by comparing cases C1, C2, and C3 (or D1, D2, and D3), the results are highly sensitive to the value of the state aid offset parameter, $c_3$.

Assuming $c_1 = -1/3$, $c_2 = .22$, and $c_3 = .4$, the effect of shifting from revenue sharing to direct aid to districts (cases C1 vs. A2) is to multiply the school spending increment by more than two and one-half and the amount of local property tax relief by five. However, if the state offsets twice as much federal aid (case C2),
then direct aid gives local districts only a very slight advantage.

Finally, in comparing direct aid to districts with and without corresponding tax increases, we see that most of the impact of tax financing falls on school spending, as was the case when we compared alternatives A and B. The amount of local tax relief falls only very slightly, but the increment in school spending falls by a considerable percentage. If we compare alternatives B and D, both of which include federal tax financing, it can be seen that direct aid generates a much greater boost in spending and much more property tax relief when only 40 percent of federal aid is offset by the state, but no increase in spending and only slightly greater property tax relief when federal aid acts primarily as a substitute (case D2) for state assistance.

**IMPLICATIONS**

Although this rather artificial exercise adds little to our substantive knowledge of fiscal impacts of federal aid, it does have implications for both empirical research and aid formula design.

With respect to the former, the main contribution of sensitivity testing is to identify the critical parameters on which future empirical work must focus. Clearly, the two most important are (a) the marginal ratio of state school aid to total state expenditure and (b) the coefficient of state aid response to a federal aid increase. Additional empirical work should be targeted specifically on estimates of those critical ratios. Our reformulation of the state model for the purpose of these projections suggests an alternative estimation procedure. Instead of estimating an aid equation directly, the alternative would be to develop the estimate in two stages. Stage one would be a total state expenditure equation, fitted to time-series, cross-section data with appropriate allowances for structural factors, categorical aid, and simultaneity of expenditure and aid determination. Stage two would estimate state school aid as a function of total state expenditure and other relevant variables. The point of the two
stage separation is that the second step could be conducted on a state-by-state basis, using time series data for individual states, if that appeared necessary to avoid the problems caused by structural differences across states. At the same time, the interplay between state aid and direct federal aid to school districts could be examined as part of the individual state, time-series studies. A further advantage is that it would be possible to construct different price indexes for the individual states without having to be concerned about the cross-state validity of price level comparisons. Except for the latter improvement, the data are available to carry out the two-stage analysis. That will be our first order of business in attempting to upgrade the empirical results.

The relevance of the projections for aid formula design is that they point to facets of state-local behavior that must be modified or controlled if federal aid is to have its intended effects. This statement is true independent of what the effects are intended to be. Whether the goal is higher per pupil spending, local property tax relief, or reduced state taxes, so long as there is a preferred set of outcomes, state responses are too uncertain to make general, unrestricted aid an effective fiscal instrument.

A way of using the projection model, therefore, is to turn the analytical question around, asking not what state responses are likely to be or what outcomes they imply, but what values or ranges of values are necessary if desired results are to be achieved. This leads naturally to the study of appropriate aid conditions: incentive provisions; constraints; effort maintenance, matching, and tax reduction requirements; and other devices for controlling the state response parameters. Many possible conditions can be built into the projection models and tested under alternative sets of assumptions about the form of federal aid and the underlying pattern of state behavior. Adaptation of the models to this use will be a major objective of our future work.