The arithmetic portion of the Developing Mathematical Processes (DMP) program, as it applies to children of ages 5 to 8, is described in some detail. The terminal objective of the number program of the primary segment of DMP is the ability of the child to correctly write, read and validate mathematical sentences of the form $A = B + X$ or $A = B - X$. For a more complete review of the DMP program, write the author at the Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, Madison, Wisconsin 53706. (DB)
NUMBER PROGRAM FOR PRIMARY SCHOOLERS

by

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The purpose of this paper is to describe in some detail the arithmetic portion of the Developing Mathematical Processes program as it applies to children of ages 5 to 8. Developing Mathematical Processes (DMP) is a research-based, elementary mathematics program. It is currently under development by the Analysis of Mathematics Instruction project of the Wisconsin Research and Development Center for Cognitive Learning. The developmental process includes classroom testing and validation in an increasingly large number of schools; this will culminate in large-scale field tests involving several hundred schools. It is expected that the complete program will undergo large-scale testing during the academic years 1972-73, 1973-74, and 1974-75. Following these field tests, the instructional materials will be available for use in elementary classrooms throughout the country. With the approval of the U. S. Office of Education and the National Science Foundation, the agencies that have supported the project, the R and D Center has entered into a contract with Rand-McNally and Company for the dissemination and distribution of materials over a period of five years. When completed, DMP will be a carefully sequenced, complete mathematics instructional program for grades K-6 that will include subject matter from the arithmetic of the rational numbers, the geometry of time-space, and fundamentals of probability and statistics.

The most notable feature of DMP's approach to the understanding of number concepts, relations, and operations at the primary level is its use
of activity learning. In other words, any symbolic work is treated as a symbolic representation of some prior experience and action of the child that he has undergone as the result of physical and pictorial stimuli.

For purposes of delimiting the scope of this paper, the terminal objective of the number program of the primary segment of DMP is the ability of the child to correctly write, read and validate mathematical sentences of the form \( A = B \pm X \). Conceptually, these sentences require the student to

1. compare two objects, sets, or events on some particular attribute;
2. if the two are non-equivalent, order the two by stating which is greater (or smaller) on the particular attribute; and
3. equalize the two by adding onto the smaller or taking away from the larger.

These compare-order-equalize processes have been found through classroom experimentation and validation to be well within the intellectual capacity of the child for which the program is designed.

The pedagogical approach of DMP is to elicit the compare-order-equalize processes initially with perceptually and intuitively meaningful materials and then gradually establish the meaning of arbitrary mathematical symbols by constantly referring to the same familiar processes. A child cannot, for example, be expected to discriminate between 17 and 19 marbles (without prior meaningless rote training on counting or one-to-one correspondence). Yet, the same child can immediately discern the difference between a stick that is 17 inches long and one that is 19 inches.
long. The non-equivalence of length is perceptually obvious. For this reason, the initial DMP experiences for the kindergarten child deal with length and direct comparisons and orderings of objects on the attribute of length. It should be noted that no reference to number is made at this time.

Soon, the child is confronted with the problem of making indirect comparisons and orderings. He is then introduced to the process of representing. This is the beginning of the development of one of the most important problem-solving skills - translation from the real world into the symbolic world of mathematics, and vice versa. The child is led through the various stages of representation - physical, pictorial, and finally symbolic. Naturally, these do not come all at once. As he is led gradually to the symbolic representation of an attribute of an object, the child learns the process of measuring, which is the assignment of a number.

DMP has been characterized by its developers as a measurement approach to the study of mathematics. The need for number and numeral is presented as a way of efficiently carrying out the compare-order-equalize process with objects, sets, or events, and their relevant attributes. In this regard, counting is considered as a special case of measurement; namely, measuring the numerousness attribute of a set. While one may argue that counting is conceptually easier than measuring because measuring involves an origin and units of measure, classroom experimentation with DMP materials indicate that young children in first and second grade are quite capable of handling not only the assignment of a measurement, but also the inverse relationship between size of unit and the size of the measure.
assigned.

Returning to the sequence of ideas presented to the children at kindergarten, numerousness is introduced as an attribute of sets. Sets are subjected to the compare-order-equalize process. At this time, the need for number and numeral is introduced and the kindergarten program has as its final instructional topic, the introduction of the numerals and their recognition. The child is not required to write numerals, though he may do so if he is capable.

Beginning the first grade portion of the number program of DMP is the attribute of weight. Using arbitrary units of measure, the children assign measurements to objects on the basis of their length or weight, or to sets on the basis of numerousness. Having learned, in the meantime, how to write numerals the child begins to write simple mathematical sentences representing the static conditions observed in the comparison process. Sentences such as $5 = 5$ and $7 \neq 3$ are written. This is followed by sentences representing the ordering process; sentences such as $5 > 2$ and $1 < 8$ are examples of these sentences. Finally, the child learns the meaning of the $+$ and $-$ symbols as they are used in sentences representing the action involved in the equalizing process. The important feature of the equalization process and its companion sentences is that the action of either adding on or taking away results in two sets or objects that are equal. Thus, the meaning of the equality relation and its symbol $=$ is graphically illustrated to the young child. Equality is a state that can be visually checked by equality of two lengths being placed side by side, by two objects or sets of units of weight causing the two pans of a balance beam to balance, or by two sets being in a one-to-one corre-
spondence, or by counting. Regardless of the type of sentence or the relation or operation it represents, there has been an experiential base to produce the sentence. This experiential referent is used in the validation of sentences that are written. By validating, the child becomes an independent problem solver.

In summary, the DMP early number program is one in which the numbers are tied to reality and come about as a result of a process by attribute measurement approach. This paper is but a brief summary of a portion of a complete instructional management program that also includes a great deal of geometry and probability and statistics. For a more complete review of the DMP program, the reader should write to the author at the Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin-Madison, Madison, Wisconsin 53706.