Although many schools claim to make use of individualized instruction, no common definition of this term has been agreed on. The author reviewed definitions of "individualized instruction" in five studies and then surveyed 30 community and junior colleges who claimed to be using this method of instruction to learn what their programs consisted of. It was learned that most programs prescribed objectives, partially set the time of classes, and partially set the location of the media used. The programs did not agree on the location of evaluation of student progress, limits of the test time, or the rate of accomplishment. Tables show characteristics of the various programs. Suggestions for incorporating these elements into the program at Moraine Valley Community College conclude the document. (JK)
ELEMENTS OF
INDIVIDUALIZED INSTRUCTION
ELEMENTS OF INDIVIDUALIZED INSTRUCTION

Ronald Svara

Presented to
Dr. Berlin,
Loyola University of Chicago

April 10, 1972
I would like to thank Dr. Walter, Dean of Institutional Services at Moraine Valley Community College for helping me start this project in the right direction.

Thanks is also due Dr. Berlin for giving me my first choice in subject matter.

Ronald Svara
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INTRODUCTION

I began working in formal individualized instruction three years ago. I worked on a project involving one hundred mathematics students. As an outgrowth of this project, my Associate Dean asked me if I would like to visit Purdue and examine Dr. Postelwait's project. I happily ran home, packed my bag, and was off to see "Dr. Sam."

Dr. Postelwait directs an individualized instruction project in biology. I examined his project and it was very much different in approach and format from mine. Well, this didn't bother me too much. After all, Sam was the expert and I was a young upstart just beginning to teach using an individualized approach. The people at Purdue used the terminology audio-tutorial and individualized instruction interchangeably. Thus, I ran into the situation of people doing different things but calling them by the same name.

My next experience was traumatic. I attended the National Convention of the Association for Educational Communications and Technology held in Philadelphia during March of 1971. I attended a number of presentations on individualized instruction and found that the approaches and formats were all different. Some approaches were very successful. Some failed. It was then that I began to wonder what individualized instruction was.

When I returned to work, I requested that I be put on the agenda of the Dean's Council Meeting. At the meeting, I contended that we claim we are providing individualized instruction, yet I have not seen a definition of individualized instruction. So, how do we know that this is what we are doing? Our Vice President agreed and commissioned our Assistant Dean of Instruction to find "the" definition, or should I say "a" definition?
A week or two passed and the Assistant Dean notified me that he could not find "the" definition, but he had analyzed a few projects and each had its own definition. Eureka! Projects in "individualized instruction" were being written up, yet no one could agree on the definition of the term. If we could determine characteristics of curriculum that make an individual approach successful, then let us identify guidelines for successful projects.

I must first start by defining what "successful" means. If an instructor can maintain the same teaching load, expend the same effort, and increase the quality of instruction, then the teaching is successful. He will also be successful if he can exert the same effort, maintain the same quality of instruction, but handle at least thirty percent more students.
Let's look at some definitions of individualized instruction.

(1) Alexander Frazier lists seven elements. He defines "individualized instruction as the answer to the problem of how to teach everybody what everybody needs to know." His seven elements are:

1. "Goals" involving "continuous progress" and "failure free learning."
2. "Nature of the learner."
3. "Content analysis," a scientific analysis to help us identify what is learnable.
4. "Materials. We have for the first time the kind of study materials needed to individualize instruction toward mastery."
5. "Methodology. We must provide a one-to-one correspondence between teacher and learner."
6. "Evaluation. We must be able to check the progress of many learners progressing at various rates."
7. "Organization." We can use learning centers for the dissemination of information.

(2) William Hedges lists eight "factors" as characteristics of individualized instructional programs.

1. "Students do not leave one unit and begin a new one until they have attained a predetermined level of proficiency in the former unit."
2. "Students must be allowed varying amounts of time (and practice) to achieve mastery of specific instructional goals."
3. "Permitting students to proceed at varying rates necessitates provision for frequent and diagnostically oriented evaluations of each student's progress."

4. "The teacher's role changes to learning manager."

5. "Students become more actively involved in the learning process than before by assuming more responsibility for their own development."

6. "With individualized instruction, almost every child becomes a teacher part of the time."

7. "Our classrooms must be arranged differently in a physical sense."

8. "We must begin to apply systems analysis approach to schools as learning centers."

(3) O'Donnel and Lavaroni list five elements:

1. "Purposeful pacing."
2. "Alternative means of learning."
3. "Self evaluation process."
4. "Student decision making."
5. "Grouping by needs of learner, not instructor."

(4) The S.R.A.-Research report, Study 4, lists eight elements, seven of which are parallel to seven of the ten I will propose. My guess is that the reason we have so many parallel items is that we both have experience and use the scientific method. These elements are now being used at the Lincoln School at Staples, Minnesota. The student moves along at his own rate and the curriculum is designed to meet
his needs. He also has some voice in choosing activities in his academic subjects. The objectives are prescribed and the level of accomplishment is mastery. There is more than one mode per unit. I found a contradiction in the report. At one point, they say "He has his own folder that tells him when to study, how much time to put in," and at another point, they say study time varies.

(5) In the Shanberg report on Individualized Instruction Systems at Hillsborough Junior College in Florida, the student's "learning style" is identified. Their students perform at an "agreed level of proficiency" at their own rate. Students can enter and complete courses "at any time." Multiple modes are available. Most of the materials they use are "canned" materials. The objectives are prescribed.

Looking at Table I, we can see that Studies 1, 2, 3, and 5 have four items each that are parallel to my elements. It is interesting to note that only one item is commented upon in all five studies. That is Item 8; rate of student accomplishment. Four of five have open rates and one "purposeful pacing."

If all students are progressing at their own rates, how is the testing handled? How does one keep track of where the students are, with reasonable time expenditures?

It is also interesting to note that no one commented on the location of the evaluation of students' progress. Where is the student tested? In the classroom, in a central testing center?

In the mode of learning, four of five articles inferred more than one mode per unit but did not comment directly to that effect.
<table>
<thead>
<tr>
<th>Item</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Location - where media is used</td>
<td>Greatly contributes to identifying instruction. Each student a learner, not an instructor. Needs of basis of learner. More student responsibility. Grouping by grouped on learning capacity.</td>
</tr>
</tbody>
</table>

**TABLE I**

<table>
<thead>
<tr>
<th>Prescribed</th>
<th>Prescribed</th>
<th>Prescribed</th>
<th>Stated Indirectly, Prescribed</th>
<th>Prescribed</th>
<th>Prescribed</th>
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<td>Shanghera</td>
<td>Sample, Min.</td>
<td>Lavenston &amp; Lincoln Sch.</td>
<td>Sarra &amp; Study 3</td>
<td>Lodge &amp; Study 2</td>
<td>Hedgees &amp; Study 1</td>
</tr>
</tbody>
</table>
Three reports that I read have another interesting element. Groups are arranged for short periods of time, grouped by needs of learner, not instructor.

The best way to identify how people are handling these elements is to make up a questionnaire, which I did. I sent the questionnaire to 73 colleges throughout the country and, as of this writing, I have 30 responses. The list of colleges outside the state came from a list that the Community College Affiliate of the Association for Educational Communication and Technology developed. This list is comprised of colleges and universities that are working in individualized instruction. The list of colleges within the state came from the same report, plus some colleges about which I knew personally, or colleges I learned of at one convention or another. The questionnaire is attached to the end of this report.
COMPILATION AND INTERPRETATION OF DATA

Table II shows a compilation of the survey. In Item 5, mode of learning, twenty-five of twenty-eight responding to this question had more than one mode per unit. These people generally agree that the objectives should be prescribed. The schedule, or time of classes, should be partially set, and location where "media" is used is partially set. At least, audio tapes can be checked out for home use.

Three items that people do not seem to agree upon are: the location of evaluation of student progress, chronological evaluation and test time limits, and the most controversial item - rate of accomplishment.

Florissant Valley Community College (3400 Pershall Road, St. Louis, Missouri 63135. David Underwood, Dean of Instruction), in my opinion, uses individualized instruction heavily (although on the questionnaire, he replied "medium" to this item) and they have all three rates now being used. The most common rate is set, week by week progress. Second, comes minimum rate set, and, in some classes, it's open.

Table III is an effort to evaluate and identify characteristics of colleges using open, minimum, and set rates of accomplishment. Only one thing is clear: the set rate is the most conservative. Three of five use individualization "lightly." All five have centers with "moderate" hours and location of evaluation is not "open."

It is interesting to note under "minimum rates," only three schools claim "heavy" use but five schools claim the media is available over one hundred hours a week, which indicates to me a center open many hours per day. I can observe no obvious patterns between the "open" and "minimum" groups. The reason I selected use of individualization,
| TABLE II |

<p>| 1. Objectives | 19 prescribed | 9 partially prescribed | 0 not prescribed |
| 2. Time of Study | 9 open. Available at least 100 hrs. per wk. | 15 semi-open. Available 30 to 99 hrs. per wk. | 3 set. Less than 30 hrs. per wk. |
| 3. Location of evaluation of students' progress | 5 open. Anywhere upon agreement between student &amp; instructor | 13 semi-open. At least two locations, i.e.: the classroom &amp; a testing or learning center | 9 set. One place, such as: 1. in the class 2. in a learning center |
| 4. Degree of accomplishment | 9 set by agreement between student &amp; instructor | 15 &quot;mastery level&quot; | 8 &quot;A,B,C&quot; levels |
| 5. Mode of learning | 3 open. Set by agreement between instructor | 25 varies. Could be more than one mode per unit | 0 audio only |
| 6. Schedule or time of classes | 4 open. Individual meetings only | 19 partially set. Students have some choice during entire smstr. | 5 set |
| 7. Location - where media is used | 2 open. All forms of media can be checked out for home use including such items as T.V. tapes | 19 partially set. Audio tapes can be checked out for home use. | 1 other |
| 8. Rate of student accomplishment | 11 open. Completely at their own rate | 11 minimum rate set | 6 set. Week by week progress |
| 9. Chronological evaluation - when do students take their tests? | 10 open. Whenever they are ready. In order to circle this, you must have circled item immediately above this one. | 13 varies. When they are ready, within the prescribed minimum rate or week by week progress | 6 at more or less &quot;set&quot; times |
| 10. Subject matter covered | 1 open. Student has very wide selection | 16 varies. Student could select, say 7 of 10 units, or 13 of 20, etc. | 11 &quot;pretty much&quot; set |</p>
<table>
<thead>
<tr>
<th></th>
<th>Open</th>
<th>Minimum</th>
<th>Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy use of individualization</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Light</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>When students use media over 100 hours</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>30 to 99 hrs.</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Under 30 hrs.</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Location of evaluation, open</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Two places</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>One place</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Location of evaluation</td>
<td>No answer circled</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE III**
"heavy," "medium," or "light" is I thought if I could identify "heavy" users, this would indicate to me previous experience in individualized instruction and, maybe, knowhow. Although the "open" group rated themselves as "heavy" users six times compared to the "minimum" raters three times, the rest of the chart left me with inconclusive results.

I selected the time when students use media as another criteria because if a number of colleges selected over one hundred hours, this would indicate a strong commitment to individualized instruction, e.g. perhaps some important knowledge. There is no obvious difference between "open" and "minimum" in this grouping.

Finally, I analyzed the location of evaluation, the significant factor indicating the sum of "one place," "two places." Any large scale individualized instruction evaluation, in my opinion, cannot take place "anywhere upon agreement between student and instructor." Again, you see that there is no trend comparing "open" against "minimum" rate. My conclusion is that I cannot identify why some people use "open" rates and some use "minimum" rates from the statistics I have compiled here.

My own evaluation of my elements is that I omitted one element and one important suggestion. The element is - how much traditional class time is replaced by the automated method? I would classify as follows: (1) independent study, at least 67%; (2) individualized instruction, from 33% to 67%; and (3) audio-tutorial, from 0% to 33%. This element was in my head, but was a plain, old-fashioned oversight.

The important suggestion is - if possible, attempt to determine under which media or which instructor a student learns better. Develop a student profile in the beginning
and advise the student of which courses he should take.

Table IV correlates my definition with the survey I took. Notice that Items 3 and 9 are the weakest in my list of elements and both pertain to testing; "when" and "where." This is probably because the testing I advocate necessitates the use of a computer. Moraine Valley's use of the computer as a test-scoring and record-keeping device is unique as far as I know. I designed the system from its inception, trying to incorporate printouts in the format needed by the instructors and the students. At the end of this paper, you will find examples of daily printouts for students' information in social security number order and weekly printouts for instructors' use in course and section order.
<table>
<thead>
<tr>
<th>Element</th>
<th>My Definition</th>
<th>Agreed Responses</th>
<th>Out of</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Objectives</td>
<td>Prescribed</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>2. Time of study</td>
<td>Over 30 hours per week in a center</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>3. Location of student evaluation</td>
<td>Two locations - with the instructor or in a learning center</td>
<td>13</td>
<td>27</td>
</tr>
<tr>
<td>4. Degree of accomplishment</td>
<td>Mastery or &quot;A, B, C&quot; levels</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td>5. Mode of learning</td>
<td>Varies; could be more than one mode per unit</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>6. Schedule or time of classes</td>
<td>Partially set</td>
<td>19</td>
<td>29</td>
</tr>
<tr>
<td>7. Location - where media is used</td>
<td>Partially set. Audio tapes can be checked out for home use</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>8. Rate of student accomplishment</td>
<td>Minimum rate set; could be week by week</td>
<td>17</td>
<td>28</td>
</tr>
<tr>
<td>9. When do students take their tests?</td>
<td>Varies; but must be within minimum rate set by instructor</td>
<td>13</td>
<td>29</td>
</tr>
<tr>
<td>10. Subject matter covered</td>
<td>Varies. Student has choice, such as 7 of 10 units or 13 of 20, etc.</td>
<td>16</td>
<td>28</td>
</tr>
</tbody>
</table>

TABLE IV
What are the implications of individualized instruction for Moraine Valley? Dr. Turner, our President, is very much interested in trying to develop good teaching and improved learning with this mode of instruction. He has been the driving force behind our successes in individualized instruction.

How do we implement these elements? Most of our instructors develop their "units" first, then their objectives. They have a resource man to go to for help; Dr. Al Hecht, Director of Institutional Research.

On the subject of time of study, or when our students use the media; our Individualized Learning Center is open sixty-seven hours per week. We have three video tape recorders, five 3M Sound on Slide machines, twenty cassette playbacks, and four other types of lesser used machines available all sixty-seven hours per week.

The location of the evaluation of the progress the students are making in individualized courses is generally the Individualized Learning Center. The student may take a test anytime the Center is open. If the test is an objective test, it is computer scored. If the student takes the test before 3:00 P.M. one day, he will get his results by 10:00 A.M. the next school day. The results are posted in the Center by social security number. No names are posted. During the fall of 1971, we administered over 7,000 tests for 32 instructors.

Tests can be administered by the instructor if he so wishes. It is up to the instructor as to how and where the evaluation takes place. (See attached examples of test sheet, daily printout, and weekly printout.)
The degree of accomplishment is decided by the instructor, but generally it is on an "A, B, C, D" grading system.

In our setup, 98% of our units have one mode of instruction. We have four units which now have two modes. Our experience has been that developing one mode of instruction for each unit in a three-credit-hour course takes from a year to a year-and-a-half. Most of our people are still at this stage. Two of our math instructors are past this stage; they have been "tinkering" over two years now.

Our schedule of classes is generally partially set. The students are assigned to come to class one or two days a week and have an option on other days.

Most of our media is used on campus, but we do permit students to check out cassettes and cassette players for home use.

I strongly suggest that a minimum rate of accomplishment be prescribed for all students. This could mean week by week progress. I do not recommend completely open pacing. One of our instructors tried that with 140 students and 70 did not complete the course within the time prescribed. Normally, she would have 7 or 8 students who would not complete the course within the time prescribed. Our students need goals set for them and a minimum rate still allows students to work ahead if they wish.

The student should be able to take the test whenever he is ready within the minimum rate prescribed. This leads to two problems. First, Charley Brown takes the test on Tuesday, tells Benedict Arnold what's on the test, and Ben takes the test on Thursday. The way to combat this problem is to generate multiple forms of the test, preferably a random generation of tests from stored test items.
The second problem is "substitute test takers." The way I suggest combating this problem is to have student I.D. cards with pictures on them and that these be checked as the student leaves the testing area.

I suggest a "core curriculum" be outlined. These units are designated by an Advisory Committee as necessary for successful functioning in the career in which the Advisory Committee are expert. Other units designed to broaden the student should be selected by the student, with a choice of, say 7 out of 10, 13 out of 20, etc.
HOW TO DO IT

For my summary, I will suggest "how to do it."

2. Read Gronlund's *Stating Behavioral Objectives for Classroom Instruction.*
3. Read Bloom's *Taxonomy.*
4. Write objectives for a three-credit hour course.
5. Group the objectives into "natural" teaching units.
6. Prepare a media for each unit.
   (a) Use 3M's Sound on Slide for "easy" cognitive domain objectives.
   (b) Use an audio tape with a handout for "medium" difficult cognitive domain objectives.
   (c) Use T.V. tape for the more difficult units.
   (d) Don't forget programmed instruction books or booklets.
   (e) Be imaginative. Keep the students active and thinking.
   (f) Make no presentation longer than 30 minutes.
7. Develop a student handout for each unit, including a self-administered and self-scored pretest and practice problems. (See accompanying examples)
8. Group your units for testing purposes. In a three-credit hour intermediate algebra course, we have 26 units and 9 tests.
9. Design two forms of the test. (See attached examples)
10. Open an Individualized Learning Center, which should be open at least 30 hours per week and should include a testing center. Stock necessary hardware, depending on types of media generated and number of students using center during a fixed time interval.
11. Outline minimum pace for your students.
12. Give "traditional lecture" as homework assignment and do "traditional homework assignment" in class.
13. Group your students into teams of two or three so that they may help one another.
14. The instructor will do no traditional lecturing, but instead will table hop and help people who cannot help one another individually.

The above is certainly not detailed, but I'm at 2,609 words now, excluding my examples (not necessary to read these, but glance over them), and excluding my tables. If one picture is worth 10,000 words, I'm in trouble.
FOOTNOTES


March 7, 1972

Dear Sir:

My name is Ron Svara, Director of the Individualized Learning Center at Moraine Valley Community College in Palos Hills, Illinois. I have written tentative criteria for defining individualized instruction. I am trying to get a compilation from colleges that are doing what they call "individualized instruction."

I would appreciate it if you would fill out the enclosed form and return it in the enclosed envelope. The form should take no longer than four or five minutes to fill out. I will present the compilation at the National Convention of the Association for Educational Communications and Technology to be held in Minneapolis, Minnesota in April. Results will be sent back to all those responding.

Thank you.

Sincerely,

Ron Svara
Director, Individualized Learning Center

RS:pn
Enc.
Circle one: You may/may not print our name.
(You may remain anonymous on Items No. 1 and 2.)

1. (name of college)

2. (location)

3. (name and title of person completing this form)

3. Extent of your use of individualized instruction (in your opinion).
   Circle one: heavy medium light none

Answer the following, using as a frame of reference, only those courses that you would classify as in the "individualized instruction" mode. Use your own good judgment.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>CIRCLE ONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Objectives</td>
<td>prescribed partially not prescribed</td>
</tr>
<tr>
<td>2. Time of Study</td>
<td>Open. Available at least 100 hrs. per wk. Semi-open. Available 30 to 99 hrs. per wk. Set. Less than 30 hrs. per wk.</td>
</tr>
<tr>
<td>(when your students use the &quot;media&quot;)</td>
<td></td>
</tr>
<tr>
<td>3. Location of evaluation of students' progress</td>
<td>Open. Anywhere upon agreement between student &amp; instructor Semi-open. At least two locations, i.e.: the classroom &amp; a testing or learning center Set. One place, such as: 1. in the class 2. in a learning center</td>
</tr>
<tr>
<td>4. Degree of accomplishment</td>
<td>Set by agreement between student &amp; instructor &quot;Mastery level&quot; &quot;A, B, C,&quot; levels</td>
</tr>
<tr>
<td>5. Mode of learning</td>
<td>Open. Set by agreement between instructor Varies. Could be more than one mode per unit Audio only</td>
</tr>
<tr>
<td>6. Schedule or time of classes</td>
<td>Open. Individual meetings only Partially set. Students have some choice during entire semester Set.</td>
</tr>
<tr>
<td>ITEM</td>
<td>CIRCLE ONE</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>7. Location - (where &quot;media&quot; is used)</td>
<td>Open. All forms of media can be checked out for home use including such items as T.V. tapes</td>
</tr>
<tr>
<td>8. Rate of Student Accomplishment</td>
<td>Open. Completely at their own rate</td>
</tr>
<tr>
<td>9. Chronological evaluation - when do students take their tests?</td>
<td>Open. Whenever they are ready. (In order to circle this, you must have circled item immediately above this one.)</td>
</tr>
<tr>
<td>10. Subject matter covered</td>
<td>Open. Student has very wide selection</td>
</tr>
</tbody>
</table>

**OTHER COMMENTS**

__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
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THIS IS YOUR RECEIPT NO.

4408

COPY IT DOWN ON YOUR OWN PAPER.

IF YOU HAVE ANY QUESTIONS ABOUT THIS TEST, BE SURE TO TALK TO YOUR INSTRUCTOR.
QUADRATIC EQUATIONS
COMPLEX ROOTS UNIT XI

Give yourself this pre test. If you get 7 correct you do not need to go through this unit, you may go on to the next unit. If you have less than 7 correct, listen to audio tape numbers 418 and 419 in the Individualized Learning Center. The test answers are given at the end of this test. Grade yourself.

1. What is the real part of this complex number: $5 + 2i$
2. $i^{23} = i^?$
3. What is the conjugate of $3 - 2i$?
4. What is the discriminant in the quadratic formula?
5. Simplify: $\frac{6i}{2-3i}$
6. Simplify: $\sqrt{-9} + \sqrt{-36} - \sqrt{-48} + \sqrt{12}$
7. Use the discriminant test to determine the nature of the equation $2x^2 + 3x + 4 = 0$
8. Solve: $6x^2 - 3x + 3/4 = 0$
ANSWERS

1. 5
2. $i^{23} = i^3 = -i$
3. $3 + 2i$
4. $b^2 - 4ac$
5. $\frac{12i - 18}{13}$
6. $9i - 2i \sqrt{3}$
7. two complex roots
8. $x = \frac{1^+ i}{4}$
FRAME ONE
1. Define complex number
2. Convert $i^n$ where $n \geq 2$ to $i$, $-1$ or $+1$
3. Add, subtract, multiply and divide complex numbers
4. Simplify polynomials with negative radicals

FRAME TWO
1. $a + bi$
2. $a$ is the ______
3. $bi$ is the ______

FRAME THREE
1. The complex part ______ has two parts. the ______ part of $bi$ is $b$. The complex part is _____
FRAME FOUR

1. \(5 + 2i\)

   The real part is ________.
   The complex part is ________.

2. \(-3 + i\)

   Real part ________
   Complex part ________

FRAME FIVE

\[
i = \text{______}
\]

\(i^2 = i \cdot i\)

\(i^3 = i^2 \cdot i\)

\(i^4 = i^2 \cdot i^2\)

\(i^5 = i^4 \cdot i\)

\(i^6 = i^4 \cdot i^2\)

\(i^{17} = \text{______}\)

\(i^{18} = \text{______}\)

\(i^{23} = \text{______}\)

\(i^{104} = \text{______}\)
FRAME SIX

Add complex numbers

\[(3 + 2i) + (2 + 4i) = 5 + 6i\]

\[(2 - 3i) + (4 + i) = \]

FRAME SEVEN

\[(2 - 3i) + (4 + i) = (6 - 2i)\]

try this subtraction

\[(4 + 5i) - (2 - 3i) = \]

FRAME EIGHT

\[(4 + 5i) - (2 - 3i) = \]

\[4 + 5i - 2 + 3i = 2 + 8i\]

try this multiplication

\[(2 + 3i)(2 + i) = \]
FRAME NINE

\[
\begin{align*}
2 + 3i \\
2 + i \\
+ 2i + 3i^2 \\
4 + 6i \\
4 + 8i + 3i^2 \\
\downarrow \\
3 (-1) = -3
\end{align*}
\]

so

\[
4 + 8i - 3 = 1 + 8i
\]

FRAME TEN

Division

(1) \[
\frac{6}{2 + 3i}
\]

(2) \[
\frac{6}{(2 + 3i)} \cdot \frac{(2 - 3i)}{(2 - 3i)}
\]

(3) \[
\frac{6(2 - 3i)}{13} \cdot \frac{2 + 3i}{2 - 3i}
\]

(4) \[
\frac{12 - 18i}{13} \cdot \frac{4 + 6i}{4 - 9i^2}
\]

TRY THIS

\[
\frac{6i}{2 - 3i}
\]

\[
- 9 (-1) = + 9
\]

\[
4 + 9 = 13
\]
FRAME ELEVEN

\[
\frac{6i}{(2 - 3i)} \cdot \frac{(2 + 3i)}{(2 + 3i)} = \frac{6i(2 + 3i)}{4 - 9i^2} = \frac{12i + 18i^2}{4 + 9} = \frac{12i - 18}{13}
\]

FRAME TWELVE

TRY THIS

\[
\frac{(2 + 3i)}{(1 - 2i)}
\]
FRAME THIRTEEN

1. \( \frac{(2 + 3i)}{(1 - 2i)} \cdot \frac{(1 + 2i)}{(1 + 2i)} \)

2. \( \frac{3 + 7i + 6i^2}{1 - 4i^2} \rightarrow \frac{6}{-4} \cdot (-1) = -6 \)

3. \( \frac{3 + 7i}{1 + 4} \rightarrow -6 \cdot (-1) = +4 \)

4. \( \frac{-3 + 7i}{5} \)

FRAME FOURTEEN

1. \( \sqrt{-32} + \sqrt{-18} \)

2. \( \sqrt{32} \cdot (-1) + \sqrt{18} \cdot (-1) \)

3. \( \sqrt{32} \cdot \sqrt{-1} + \sqrt{18} \cdot \sqrt{-1} \)

4. \( \sqrt{16.2} \cdot i + \sqrt{9.2} \cdot i \)

5. \( \sqrt{16} \cdot \sqrt{2} \cdot i + \sqrt{9} \cdot \sqrt{2} \cdot i \)

6. \( 4 \cdot i \sqrt{2} + 3 \cdot i \sqrt{2} \)

7. \( 7 \cdot i \sqrt{2} \)

TRY \( \sqrt{-27} + \sqrt{-12} \)
FRAME FIFTEEN

1. \(\sqrt{-27} + \sqrt{-12}\)
2. \(\sqrt{9 \cdot 3 \cdot (-1)} + \sqrt{4 \cdot 3 \cdot (-1)}\)
3. \(\sqrt{9} \sqrt{3} \sqrt{-1} + \sqrt{4} \sqrt{3} \sqrt{-1}\)
4. \(3 \sqrt{3} \ i + 2 \sqrt{3} \ i\)
5. \(5 \ i \sqrt{3}\)
PRACTICE PROBLEMS

1. In $5 + 3i$ 5 is the _____ part and 3i is the _____ _____.

2. $i^{40} =$

3. $i^{13} =$

4. $(2 - 3i) + (-3 + 5i) =$

5. $(6 + 3i) - (2 - 3i) =$

6. $(2 + 5i) (2 - 3i) =$

7. $(3 + 2i) (3 - 2i) =$

8. \[
\frac{3}{4 - i} = \]

9. \[
\frac{1 + i}{1 - i} = \]

10. $\sqrt{-20} - \sqrt{-80}$

11. $\sqrt{-8} + \sqrt{-18} - \sqrt{-50}$

12. $\sqrt{-12} - \sqrt{-48}$

THE ANSWERS ARE ON THE NEXT PAGE!
ANSWERS

1. Real
   Complex part
2. 1
3. i
4. (-1 - 2i)
5. (4 + 6i)
6. (19 - 4i)
7. 13
8. \[
   \frac{12 \div 3i}{17}
\]
9. i
10. -2i \sqrt{5}
11. zero
12. -2i \sqrt{3}
**DIRECTIONS:** The problems in this test correspond with the three units in quadratic equations. There is no penalty for guessing. There is only one correct answer per problem.

1. If you solve this equation by factoring: \(2X^2 + X = 6\), one of the factors will be:
   a. \(X (2X + 1)\)
   b. \((2X + 2)\)
   c. \(2 (X + 1)\)
   d. \((2X - 3)\)
   e. None of the above.

2. Solve this equation by factoring. Pick out the line that shows up in your problem.
   \[3X^2 - X - 4 = 0\]
   a. \(3X = 4\)
   b. \((X - 4)\)
   c. \(X - 1 = 0\)
   d. None of the above.

3. Solve this equation by factoring: \(X^2 + 6X + 5 = 0\)
   a. \(X = -5\)
   b. \(X = -3\)
   c. \(X = -5, X = -1\)
   d. None of the above.

4. The answers to this problem \((X - 2) (X + 3) (X -4) = 0\) are
   a. \(X = -2, X = 3, X = -4\)
   b. \(X = 2, X = 3, X = 4\)
   c. \(X = 2, X = 4\)
   d. None of the above.

5. The answers to this problem \(X (X + 2) (X -1) = 0\) are
   a. \(X = 2, X = -1\)
   b. \(X = -2, X = 1\)
   c. \(X = 0, X = -2, X = 1\)
   d. \(X = +\sqrt{2}, X = -\sqrt{2}, X = -1\)
   e. None of the above.
6. Solve by factoring \( X^3 + 3X^2 + 2X = 0 \)
   a. \( X = -2 \) \( X = -1 \)
   b. \( X = 2 \) \( X = 1 \)
   c. \( X = 0 \) \( X = -2 \) \( X = 2 \)
   d. \( X = 0 \) \( X = -1 \) \( X = 3 \)
   e. None of the above.

7. Solve by factoring \( X^3 + 2X^2 - 16X - 32 = 0 \)
   a. \( X = 4 \) \( X = -4 \) \( X = -2 \)
   b. \( X = 4 \) \( X = -4 \) \( X = 2 \)
   c. \( X = 16 \) \( X = -4 \)
   d. None of the above.

8. What do I add to both sides of this equation to make the left side a perfect square?
   \( X^2 + 4X - 2 = 0 \)
   a. 4
   b. 2
   c. -2
   d. 6
   e. None of the above.

9. Given this equation \( 3X^2 + 8X - 3 = 0 \) and the instructions "solve by completing the square", the first thing you would do is:
   a. divide by 3
   b. take half of 8
   c. square four
   d. add 19
   e. None of the above.

10. Solve by using the quadratic formula: \( X^2 + 3X + 1 = 0 \)
    a. \( X = \frac{-3 \pm \sqrt{13}}{2} \)
    b. \( X = \frac{-3 \pm \sqrt{5}}{2} \)
    c. \( X = \frac{-1 \pm \sqrt{5}}{2} \)
    d. None of the above.
11. \(2x^2 - 3x - \frac{3}{8} = 0\)
   a. \(x = \frac{3 \pm 2\sqrt{3}}{4}\)
   b. \(x = \frac{3 \pm \sqrt{12}}{4}\)
   c. \(x = \frac{3 \pm \sqrt{3}}{2}\)
   d. \(x = \frac{-3 \pm \sqrt{3}}{2}\)
   e. None of the above.

12. \(\frac{3}{2} x^2 + 3x + 21/2 = 0\)
   a. \(x = \frac{-3 \pm 36\sqrt{2}}{3}\)
   b. \(x = \frac{3 \pm 6\sqrt{2}}{3}\)
   c. \(x = \frac{-3 \pm 6\sqrt{2}}{3/2}\)
   d. \(x = \frac{3 \pm 6\sqrt{2}}{3/2}\)
   e. None of the above.

13. \(x^2 + 3x + 21/4 = 0\)
   a. \(3 \pm 2i\sqrt{3}\)
   b. \(3 \pm 2i\sqrt{3}\)
   c. \(-3 \pm i\sqrt{3}\)
   d. \(x = \frac{-3 \pm 2i\sqrt{3}}{2}\)
   e. None of the above.
14. What is the real part of this complex number $-2 + 3i$
   a. $3i$
   b. $i$
   c. 3
   d. -2
   e. None of the above.

15. $\frac{33}{i} = ?$
   a. $i$
   b. $i^8$
   c. 1
   d. $i^4$
   e. None of the above.

16. The word used to describe the relation between $2 + 4i$ and $2 - 4i$ is:
   a. discriminant
   b. conjugates
   c. complex
   d. elephants
   e. None of the above.

17. $(3 + 2i) - (-2 + 4i) =$
   a. 7
   b. 5 $-2i$
   c. 5 $-2i$
   d. None of the above.

18. $\frac{1 + i}{1 - i} = ?$
   a. $\frac{2 + 2i}{2}$
   b. $1 + i$
   c. $1 + 2i + i^2$
   d. $i$
   e. None of the above.
19. \( \sqrt{-36} - \sqrt{-2} + \sqrt{-12} + \sqrt{-3} = \)
   a. \( -\sqrt{53} \)
   b. \( -7 \)
   c. \( 5i\sqrt{6} \)
   d. \( 2i\sqrt{2} + 3i\sqrt{3} \)
   e. None of the above.

20. Use the discriminate test to determine the nature of the roots of \( X^2 + 3X + 3 = 0 \)
   a. two equal real roots
   b. two different real roots
   c. two different complex roots
   d. None of the above.
   e. Who cares!

MATCH

21. Discriminant
   a. \( \sqrt{-1} \)
   b. Coefficient of the complex part of a complex number.
   c. Answer not given
   d. \( b^2 - 4ac \)

22. i

23. Real number

24. Complete the square
DIRECTIONS: The problems in this test correspond with the three units in quadratic equations. There is no penalty for guessing. There is only one correct answer per problem.

1. If you solve this equation by factoring; \(3X^2 + 7X = 6\), one of the factors will be:
   a. \((3X + 3)\)
   b. \((X + 3)\)
   c. \((3X + 7)\)
   d. \(3 (X + 1)\)
   e. None of the above.

2. Solve this equation by factoring. Pick out the line that shows up in your problem.
   \(2X^2 + X - 6 = 0\)
   a. \(2X = -2\)
   b. \((X - 3)\)
   c. \(2X = 3\)
   d. \(X - 3 = 0\)
   e. None of the above.

3. Solve this equation by factoring: \(X^2 + 4X + 3 = 0\)
   a. \(X = -3\)
   b. \(X = -2\)
   c. \(X = +2 X = -2\)
   d. \(X = -3 X = -1\)
   e. None of the above.

4. The answers to this problem \((X - 3) (X + 2) (X + 5) = 0\) are:
   a. \(X = -3 \quad X = +2 \quad X = +5\)
   b. \(X = 2 \quad X = 3 \quad X = 5\)
   c. \(X = 3 \quad X = -2 \quad X = -5\)
   d. \(X = 3\)
   e. None of the above.

5. The answers to this problem \(X (X - 3) (X + 4) = 0\) are:
   a. \(X = 3 \quad X = -4\)
   b. \(X = 0 \quad X = 3 \quad X = -4\)
   c. \(X = -3 \quad X = 4\)
   d. \(X = +\sqrt{3} \quad X = -\sqrt{3} \quad X = -4\)
   e. None of the above.
6. Solve by factoring $x^3 + x^2 - 2x = 0$
   a. $x = 0$  $x = 1$  $x = -2$
   b. $x = 1$  $x = -2$
   c. $x = -1$  $x = 2$
   d. $x = 1$  $x = -1$  $x = -2$
   e. None of the above.

7. Solve by factoring: $x^3 + x^2 - 4x - 4 = 0$
   a. $x = 4$  $x = -1$
   b. $x = 0$  $x = -1$
   c. $x = -2$  $x = -1$
   d. $x = 1$  $x = 2$  $x = -2$
   e. None of the above.

8. What do I add to both sides of this equation to make the left side a perfect square?
   $x^2 + 6x - 3 = 0$
   a. 9
   b. 6
   c. 12
   d. -6
   e. None of the above.

9. Given this equation, $2x^2 + 4x - 1 = 0$ and the instructions "solve by completing the square", the first thing you would do is:
   a. take half of 4
   b. add 4 to each side
   c. divide by 2
   d. square 2
   e. None of the above.

10. Solve by using the quadratic formula: $x^2 + 3x + 1 = 0$
    a. $x = \frac{-3 + \sqrt{5}}{2}$
    b. $x = \frac{-3 + \sqrt{13}}{2}$
    c. $x = \frac{-1 + \sqrt{5}}{2}$
    d. None of the above.
11. \( \frac{3}{2} x^2 - 2x - \frac{3}{8} = 0 \)
   
a. \( \frac{-2 + 3\sqrt{2}}{3} \)
   
b. \( \frac{2 + 9\sqrt{2}}{3} \)
   
c. \( -2 + \sqrt{2} \)
   
d. \( \frac{2 + 3\sqrt{2}}{3/2} \)
   
e. None of the above.

12. \( x^2 + 2x - 11 = 0 \)
   
a. \( x = \frac{2 + 4\sqrt{3}}{2} \)
   
b. \( x = -2 + 4\sqrt{3} \)
   
c. \( x = -1 + 2\sqrt{3} \)
   
d. \( x = -1 + 4\sqrt{3} \)
   
e. None of the above.

13. \( 2x^2 + 2x + \frac{7}{2} = 0 \)
   
a. \( x = \frac{-1 + 3i\sqrt{2}}{2} \)
   
b. \( x = \frac{-2 + 3i\sqrt{2}}{4} \)
   
c. \( x = \frac{-2 + 9i\sqrt{2}}{4} \)
   
d. \( x = \frac{-2 + 3i\sqrt{2}}{2} \)
   
e. None of the above.
14. What is the real part of this complex number $-3 + 2i$?
   a. $-3$
   b. $2$
   c. $i$
   d. $2i$
   e. None of the above.

15. $|i| = ?$
   a. $1$
   b. $i$
   c. $1$
   d. $i$
   e. None of the above.

16. The word used to describe the relation between $3 + 2i$ and $3 - 2i$ is:
   a. complex
   b. discriminants
   c. conjugates
   d. elephants
   e. None of the above.

17. $(2 - 2i) - (-3 + i) =$
   a. $5 - 3i$
   b. $5 - 3i$
   c. $8$
   d. None of the above.

18. $\frac{1-i}{1+i} = ?$
   a. $\frac{2 - 2i}{2}$
   b. $1 - i$
   c. $1 - 2i + i^2$
   d. $-i$
   e. None of the above.
19. \( \sqrt{-12} - i\sqrt{-3} + \sqrt{-18} + \sqrt{-8} = \)

a. \( \sqrt{-41} \)

b. \( \sqrt{-35} \)

c. \( 6i\sqrt{6} \)

d. \( i\sqrt{3} + 5i\sqrt{2} \)

e. None of the above.

20. Use the discriminants test to determine the nature of the roots of \( X^2 + 4X + 3 = 0 \)

a. Two equal real roots

b. Two different real roots

c. Two different complex roots

d. None of the above.

e. Who cares!

MATCH

21. Real number

22. Complete the square

23. Discriminant

24. i

a. \( \sqrt{-1} \)

b. Coefficient of the complex part of a complex number.

c. Answer not given.

d. \( b^2 - 4ac \)
MTH 110 - Fundamentals of Mathematics
Units of Instruction Defined by Behavioral Objectives

Unit I - Sets

The student:

A. Understands the concept of set and set operations as evidenced by his ability to diagram and solve simple problems of these types:

2. Picturing sets using Venn diagrams and number lines.
3. Union and intersection of sets.
4. Compound sentences (word problems) involving "and" and "or".

B. Correctly uses these related words and symbols in problem solving:

1. Set
2. Subset, \( \subseteq \)
3. Element or member, \( \in \)
4. Null or empty set, \( \emptyset \), \( \{ \} \)
5. Identical or equal sets, \( \equiv \)
6. Finite sets
7. Infinite sets
8. Venn diagram
9. Number line
10. Descriptive (rule) and roster method of specification of sets
11. Union, \( \cup \), or
12. Intersection, \( \cap \), and

Unit II - Subsets of the Real Numbers

The student:

A. Differentiates between the sets of natural numbers, integers, rational numbers, irrational numbers, and real numbers.

B. Correctly uses these related words in problem solving:

1. Counting or natural number
2. Whole number
3. Integer
4. Rational number
5. Irrational number
6. Real number
7. Positive
8. Negative
9. Non-positive
10. Non-negative
Unit III - Signed Number Arithmetic

The student:

A. Adds, subtracts, multiplies and divides integers.

B. Understands the order in which the four arithmetic operations are performed and uses this knowledge to solve problems involving several operations.

C. Uses parentheses to indicate order of operation.

D. Evaluates expressions containing parentheses.

Unit IV - Properties of the Real numbers

The student:

A. Explains "binary operation".

B. Identifies examples of the following:
   1. Commutative property
   2. Associative property
   3. Identity element
   4. Inverse element
   5. Closure
   6. Distributive property

C. Uses the above properties in solving these types of problems:
   1. Addition, subtraction, multiplication and division of real numbers.
   2. Simplification of expressions containing linear terms and involving addition, subtraction, and multiplication.

Unit V - Linear Equations

The student:

A. Solves linear equations in one variable algebraically.

B. Sets up and solves ratio and proportion type word problems.

C. Manipulates given formulas to solve for a specified variable.
D. Correctly uses these related words in problem solving:

1. Open or conditional sentence
2. Variable
3. Domain, replacement set, universal set
4. Solution set or truth set
5. Equation
6. Algebraic expression
7. Term
8. Members or sides of an equation
9. Coefficient
10. Equivalent equations
11. Linear
12. Constant
13. Ratio
14. Proportion

Unit VI - Order on the Real Numbers and Linear Inequalities

The student:

A. Understands the concept of order on the real numbers as evidenced by his ability to:

1. Graph sets of numbers on the real number line
2. Correctly use these related words and symbols in problem solving:

   a. Comparison property (axiom of Trichotomy)
   b. Transitive property of inequality and equality
   c. Absolute value as distance on a number line
   d. \( \leq \)
   e. \( \leq \)
   f. \( \geq \)
   g. \( \geq \)

B. Demonstrates proficiency in dealing with simple one dimensional linear inequalities as evidenced by his ability to:

1. Solve them algebraically
2. Solve their solution sets on a real number line
3. Correctly use these related words in problem solving:

   a. Inequality
   b. Open interval
   c. Closed interval
   d. Compound inequality
Unit VII - Exponents and Simple Polynomial Operations

The student:

A. Defines positive integral exponent.

B. Demonstrates proficiency in applying the distributive, associative, and commutative laws to polynomials with only positive integral exponents as evidenced by his ability to:

1. Multiply monomials and raise monomials to positive powers.

2. Add and subtract polynomials.

3. Multiply a monomial times a polynomial.

4. Correctly use these related words in problem solving:
   a. Monomial
   b. Binomial
   c. Trinomial
   d. Polynomial
   e. Exponent
   f. Base
   g. Degree
   h. Quadratic
   i. Cubic
   j. Quartic

Unit VIII - The Cartesian Coordinate System

The student:

A. Plots a point in the plane given its ordered pair.

B. States the approximate coordinates for a given point in the plane.

C. Draws the graph of a two dimensional linear equation.

D. Correctly uses the following words in problem solving:

1. Ordered pair
2. Cartesian coordinate system
3. Quadrant
4. Axes
5. Origin
6. Abscissa
7. Ordinate
8. Graph
9. Coordinates
Unit IX - Slope and Intercepts

The student:

A. Finds the slope of a line given two points it passes through.
B. Finds the slope of a line given its equation.
C. Finds the X and Y-intercepts of a line given its equation.
D. Writes the equation of a line given a point and the slope by using the point-slope form for the equation of a line.
E. Writes the equation of a line given two points by using the point-slope form for the equation of a line.
F. Draws the graphs of lines given:
   1. Their equations
   2. Two points on each line
   3. The slope of each line and a point on each
G. Correctly uses the following words in problem solving:
   1. Slope
   2. Intercept
   3. Point-slope form

Unit X - Simultaneous Linear Equations

The student:

A. Determines whether a pair of equations represent the same line, parallel lines, or intersecting lines.
B. Solves systems of two linear equations by graphing them and reading their approximate point of intersection.
C. Solves systems of two linear equations algebraically by using either the substitution or the addition-subtraction method.
D. Sets up and solves worded problems that result in two linear equations in two variables. Types of problems to be solved include:
   1. Geometric
   2. \( D = rt \)
   3. Simple interest

Contrived problems are to be avoided.
Units of Instruction Defined in Terms of Behavioral Objectives

Unit I - Multiplication of Polynomials
The student:

A. Multiplies polynomials and simplifies the results by combining similar terms.
B. Utilizes the special products \((a+b)^2 = a^2 + 2ab+b^2\) and \((a + b)(a - b) = a^2 - b^2\) to write products of this type on inspection.

Unit II - Factoring - Part 1
The student:

A. Factors algebraic expressions using the following techniques:
   1. Removing a common factor
   2. Difference of two squares
   3. Sum and difference of two cubes
B. Correctly uses these related words in problem solving
   1. Greatest common factor
   2. Factor completely

Unit III - Factoring - Part 2
The student:

A. Factors algebraic expressions using the following techniques
   1. Quadratic trinomial
   2. Grouping
B. Factors algebraic expressions using combinations of the techniques present in Units II and III

Unit IV - Algebraic Fractions - Part 1
The student:

A. Simplifies algebraic fractions to lowest terms
B. Multiplies and divides algebraic fractions

Unit V - Algebraic Fractions - Part 2
The student:

A. Finds the least common multiple (common denominator) of a set of algebraic expressions.
B. Adds and subtracts algebraic fractions
C. Simplifies elementary types of complex fractions
Unit VI - Exponents - Part 1; Zero and Negative Integral Exponents

The student:

A. Defines the zero exponent and negative integral exponent.
B. Manipulates and simplifies expressions containing integral exponents.

Unit VII - Exponents - Part 2; Radicals

The student:

A. Simplifies, rationalizes and/or evaluates expressions involving radicals.
B. Correctly uses these related words in problem solving:
   1. Principal root and square root
   2. Radical
   3. Index
   4. Radicand
   5. Rationalize

Unit VIII - Exponents - Part 3; Rational Exponents

The student:

A. Expresses rational exponents in radical form and conversely.
B. Simplifies and/or evaluates expressions involving rational exponents.

Unit IX - Quadratic Equations - Part 1; Factoring

The student:

A. Solves quadratic equations with rational coefficients by factoring
B. Solves simple higher degree polynomial equations by factoring.

Unit X - Quadratic Equations - Part 2; Real Roots

The student:

A. Solves quadratic equations with real coefficients and real roots by completing the square.
B. Solves quadratic equations with real coefficients and real roots by using the formula.

Unit XI - Quadratic Equations - Part 3; Complex Roots

The student:

A. Simplifies expressions containing complex numbers.
B. Determines the nature of the roots of a quadratic equation by using the discriminant test.
C. Solves quadratic equations with real coefficients and complex roots by using the formula.

D. Correctly uses the following words and symbols in problem solving:

1. Complex number
2. Real part
3. Imaginary part
4. $j = \sqrt{-1}$
5. Conjugate
6. Discriminant

**Unit XII - Radical and Fractional Equations**

The student:

A. Solves equations containing radicals by using the squaring technique.
B. Solves equations involving rational expressions.
C. Identifies extraneous roots.

**Unit XIII - Quadratic Applications**

The student sets up and solves the following types of quadratic word problems:

A. Geometric
B. $D = rt$
C. Rate of work
D. Displacement and other appropriate physics types

Contrived word problems are to be avoided.

**Unit XIV - Functions - Part 1**

The student:

A. Defines function, domain, and range in his own words.
B. Determines the domain of the following types of functions:

1. Linear, $f(x) = mx + b$
2. Quadratic, $f(x) = ax^2 + bx + c$
3. Radical, $f(x) = \sqrt{x + d}$
4. Fractional, $f(x) = \frac{1}{x + e}$

C. Determines the value of a function $f(x)$, given the value of the independent variable $x$. 

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Unit XV - Functions - Part 2

The student:

A. Differentiates between graphs that represent functions and graphs that do not.

B. Relates f(x) notation to the height of a function as evidenced by his ability to graph functions of the form f(x) = mx + b and f(x) = ax^2 + bx + c by plotting points (x, f(x)) in the rectangular coordinate plane.

C. Uses X and Y intercepts as helpful graphing techniques.

Unit XVI - Inequalities

The student:

A. Solves absolute value inequalities of the form |x + a| ≤ b and |x + a| > b as evidenced by his ability to graph their solution sets on the number line.

B. Solves one dimensional quadratic inequalities and graphs their solution sets on the number line.

C. Explains the relationships between the solution sets of quadratic inequalities and the heights of their corresponding quadratic functions.

D. Graphs the solution sets of two dimensional linear inequalities.