DOCUMENT RESUME

ED 062 194 SE 013 686

AUTHOR Deichmann, John; Beattie, Ian
TITLE Spatial and Modality Effects in Simple Mathematical Computation.
PUB DATE Apr 72
NOTE 10p.; Paper presented at the meeting of the American Educational Research Association, April 1972

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Addition; *Algorithms; Arithmetic; Basic Skills; *Elementary School Mathematics; Multiplication; *Oral Communication; *Research; Subtraction; *Visual Stimuli

ABSTRACT

This study explored the effects of visual (vertical and horizontal) and oral presentation modes upon simple mathematical computations (addition, subtraction, and multiplication). Seventy-two undergraduate education majors were employed as subjects. The placement of the process sign (left, middle, right) and whether a one or two digit number appeared first in the mathematical sentence was manipulated. The results demonstrated significant differences for modality, type of computation, sign and two-digit placement. Further, it appears that for the oral presentation, the process sign placed last is superior to the first position. (Author/MM)

3.8

Paper Presented to a Meeting of the
American Educational Research Association

April 1972

SPATIAL AND MODALITY EFFECTS IN SIMPLE MATHEMATICAL COMPUTATION

John Deichmann
Southern Illinois University
at Carbondale

Ian Beattie
Southern Illinois University
at Carbondale

SPATIAL AND MODLITY EFFECTS IN SIMPLE MATHEMATICAL COMPUTATION

John Deichmann, Southern Illinois University
Ian Beattie, Southern Illinois University

ABSTRACT

This study explored the effects of visual (vertical and horizontal) and oral presentation modes upon simple mathematical computations (addition, subtraction, and multiplication). Seventy-two undergraduate education majors were employed as subjects. The placement of the process sign (left, middle, right) and whether a one or two digit number appeared first in the mathematical sentence was manipulated. The results demonstrated significant differences for modality, type of computation, sign and two-digit placement. Further, it appears that for the oral presentation, the process sign placed last is superior to the first position.

SPATIAL AND MODALITY EFFECTS IN SIMPLE MATHEMATICAL COMPUTATION

The objective of the following study was to obtain data on the perceptual processes and habits which may enter into simple mathematical computations. It is an admittedly normative approach. The research questions stem less from any theoritical position than they do from a puzzlement over on-going practice. Research attempts have thus far ignored the effects of problem presentation configuration upon acquisition and performance of simple mathematical computations. It appears that no empirically based rationale has been advance for: 1) the visual presentation (vertical or horizontal) of the problems, 2) the placement of the operation sign within the problem, and 3) the alternate placement of one and two digit numbers, in addition, the possible modality (visual or aural) effects have not been explored. Whether any of the above manipulations differentially effect initial acquisition, long term retention, or actual computational ease is not known. For example, if reading habits produce a left to right eye scan habit, computation sign placement might be more efficient if placed first in the mathematical sentence, at least for the horizontal presentation method. This initial placement would allow the student to encode the required operation and develop the appropriate set in order to act correctly upon the following numbers. The possibility of age or grade differences interacting with the perceptual displays is also unknown. The specific purpose of the present research was therefore, to demonstrate possible performance differences related to the method of presentation of simple mathematical computations.

METHOD

The subjects (Ss) consisted of seventy-two undergraduate education majors who were randomly assigned to one of three presentation groups: Group I: problems were presented with digits arranged vertically (V); Group II: presentation was horizontal (H); Group III was presented the material aurally (A). All groups were presented the same 180 problems: sixty of the problems contained the operation (+, -, x) sign on the left for H (top for V and first for A); sixty in the middle;

and the remaining sixty on the right (bottom for V or last for A). For each group of sixty problems, twenty were addition, twenty were subtraction, and twenty were multiplication. For each of these twenty, half were presented with a two-digit number (2) first; with the remaining problems a one-digit number (1) first.

All manipulations with the problems were randomized. Each presentation group received the same order (once randomized) of the problems. The visual presentations (Hand V) were via a carousel projector, with the A presentation via a tape recorder. The words plus, minus and times were used in the A group; the respective visual presentation was +, -, and x.

Problem exposure time for V and H was equated with the A presentation time, which resulted in approximately a three-second presentation rate with a one-second inter-problem interval. Subjects recorded their responses on a numbered form provided.

RESULTS AND DISCUSSION

The analysis of correct scores is presented first. Subtraction answers were scored as correct regardless of the plus or minus answer if the number was correct. The modality employed, sign placement, type of sign, and digit placement produced a 3x3x3x2 factorial design: 3 modalities (V,H,A) x 3 signs (+, -, x) x 3 placements (L, M, R) x 2 digit placement (two digit first, one digit first). The score of each cell reflected the number of corrected responses for the 24 Ss on each of the 10 problems for that cell. Table 1 contains the summary statistics for correct answers. The 3x3x3x2 ANOVA for correct scores produced significant results for all four main effects: modality $F(2,486) = 138.148$, $p < .01$; sign $F(2,486)$ 138.470, $p < .01$; placement $F(2,486) = 6.935$, $p < .01$; digit placement $F(1,486) = 5.401$, $p < .01$. Furthermore the mode x sign interaction was also significant $F(4,486) = 6.024$, p .01. The latter would appear to have been caused by the extremely low x cell of the H group. Although not explicitly stated in the in-

troduction, some results were expected by the experimenters based solely on the very overlearned habit of reading from left to right, combined with the equally overlearned habit of horizontal visual presentation with the computational sign in the middle. (It is much to soon to predict results from information processing theory, analysis by synthesis, etc.) The vertical condition had no "real life" similar condition of practice, for the usual presentation mode is with the computation sign to the visual left of the bottom digit.

Based on the above, it was predicted that the horizontal condition would produce the most correct, with the vertical condition next and with the oral (least prior practice) the poorest. Further, it was predicted that the M sign position would be the most affective (due to prior practive, at least in the H and O conditions) with L position next (due to L to R reading habit which would allow a set to develop in S to operate correctly upon the following presented numbers), R position would therefore be the poorest. Predictions regarding sign were based on classroom observation with ease of computations from high to low in the order of +, -, x. Two digit number first was expected to produce a greater number of correct than one digit first again due to the usual method of placing the 2-digit number on top or to the left in text and workbooks. The obtained results quite nicely demonstrated the inadequacy of our common sense prediction. The modality results

TABLE 1

Summary Statistics For Correct Answers

	A	V	H	L	M	R	+	-	x
$\bar{X}$	18.183	16.839	11.639	14.844	16.506	15.337	18.117	17.650	10.839
SD	5.019	4.645	6.091	6.011	5.439	6.337	4.405	4.620	5.836

were the reverse of the initial prediction. The means for A, V, and H were 18.18, 16. 84, and 11.64 respectively. (A vs. V, t (180) = -2.638 p < .01; A vs. H, t(180)= 11.125, p < .001; t vs. H, t(180) = 9.107, p < .001)

Regarding sign placement, the M position did produce the most correct responses. The R position however was consistantly better than the L position although the differences did not reach significance. The last result certainly deserves further exploration. Regarding the type of sign and digit placement, the results matched the prediction. (two-tailed test; + vs. -, NS; + vs. x, $\underline{t}(180) = 13.354$, $p < .001$; - vs. x, $\underline{t}(180) = 12.227$, $p < .001$).

The A group would appear (in retrospect) to possess two advantages over the two visual presentations modes. During presentation, the Ss in the A group could be looking at the answer sheet, this was not true in the V and H groups. Further, A presentation could quite possibly have allowed the Ss to rearrange the aural input (possibly in some ikonic form) to match the form most suited for them. The possibility of modality differences beyond those mentioned above are certainly possible, the authors believe however, that further speculation at this point is unwarranted.

Both of the above mentioned factors could have combined to produce the superior results for tha A group. The V and H differences, however, can be explained by neither, but may be explained by employing the concept of overlearned habit. Although not done by the authors, if during presentations of the A condition the Ss were allowed to write the problem down, it would be predicted (at this point in time at any rate) that Ss would do so in a vertical manner. Previous vertical computation practice would therefore appear to over-ride any horizontal reading habit. It would appear that Ss are capable of adapting their scanning habit to the appropriate material, i.e. words or numbers. How the vertical process habit affects the introduction of algebraic formulations is unknown. The complete reversal of the predicted L/R superiority is fascinating. Replication is obviously in order to verify this finding.

Error Analysis

As with the number of correct items, analysis of the error scores do not include the errors with the negative sign missing. For analysis, errors were broken down into

three categories: Incorrect (W), no answer written (B), and operation (C). An operation error was considered to have occurred if the result could have been obtained if S has in fact employed a different operation than directed by the sign presented. For example, if the actual problem presented was 12 + 3, a C error was considered to have occurred if the answer presented was either 9 or 36. Table 2 contains the summary statistics for errors and Table 3 contains the **t test results** appear in the appendix. ANOVA at this point became too cumbersome and therefore multiple T's were employed in the analysis. An inspection of Table 3 reveals some trends. It would appear that multiplication for the H group (regardless of sign position) was extremely difficult as demonstrated by the significantly more blanks than answers (two tailed: (L) W vs B t (20) = 11.296 $p < .001$; (M) W vs B t (20) = 9.094 $p < .001$; (R) W vs B t (20) = 13.115 $p < .001$). This was also true for addition when the sign was anywhere other than the middle position ((L) W vs B t (20) = 3.602 $p < .01$; (R) W vs B t (20) = 2.864 $p < .01$). Little differences between presentation modes were demonstrated for incorrect answers. Operation errors within the visual mode (V vs H) were significant only in +L and xR problems ((+L) V vs H t (20) = 3.051 $p < .01$; (xR) V vs H t (20) = 3.090 $p < .01$). The A condition produced significantly less operation errors relative to the V condition for +L and -L((+L) A vs V, t (20) = 4.114 $p < .01$; (-L) A vs V, t (20) = 2.845 $p < .01$); and relative to the H condition for +R, xR, xM, xR ((+R) A vs H, t (20) = 2.864 $p < .01$; (xL) A vs H t (20) = 5.724 $p < .01$; (xM) A vs H, t (20) = 4.706 $p < .01$; (xR) A vs H t (20) = 4.423 $p < .01$). It would appear that operation errors have a greater probability of occurance with visual presentation than with aural.

Summary

Although dealing with colleges and therefore trememdous over-learning, significant differences were demonstrated for all manipulations. The causal factors operating to produce the differences are unknown. If the aural "minus" was replaced with the term "subtract" differential results might be obtained. That is, a statement of the operation rather than the sign might produce greater correct responses. Further, this initial probe did not attempt to break the presentation into the stages of recognition and computation. During H presentation for example, a minus left problem might produce difficulties in recognizing the sign requirements of the answer. Once this was resolved, computation might easily follow.

Whether any of the differences in this research can be demonstrated with younger children and if various presentation methods might facilitate or hinder acquisition is unknown. Future research in mathematics education should direct some attention toward these possibilities.

Table 2

Summary Statistics For Error Answers

Addition Problems

	L						M						R					
	A		V		H		A		V		H		A		V		H	
	X	SD	X	SD	X	SD	X	SD	X	SD	X	SD	X	SD	X	SD	X	SD
W	1.6	1.4	2.1	1.3	2.5	1.9	1.5	1.1	2.3	3.5	2.9	1.9	1.3	1.7	1.9	1.5	2.4	2.2
B	2.4	4.5	3.0	2.8	5.9	3.7	2.3	2.5	3.1	2.5	3.9	2.7	1.8	2.6	2.2	2.2	5.2	3.6
C	.2	.4	.9	.6	.4	.5	.1	.2	.3	.6	.2	.4	0	0	.6	.9	1.4	2.2

Subtraction Problems

W	2.5	1.9	2.3	2.3	3.5	2.5	1.2	1.2	2.0	1.6	2.9	2.3	1.6	1.2	2.7	1.8	3.3	3.4
B	1.8	2.6	2.3	1.6	7.3	4.4	1.5	1.9	2.5	1.9	4.8	2.8	2.1	2.3	2.3	1.7	6.5	4.2
C	.1	.2	.4	.5	.2	.4	.1	.2	.5	.8	.3	.6	.1	.2	.4	.5	.5	.9

Multiplication Problems

W	3.7	1.8	4.2	2.5	3.8	2.5	4.2	1.6	5.4	2.5	3.6	1.7	3.8	1.8	5.3	2.6	2.8	1.6
B	6.7	5.1	5.5	2.4	13.5	2.9	3.2	1.7	3.7	2.2	13.1	4.4	5.2	4.4	6.2	3.5	13.9	3.4
C	.1	.2	1.0	1.4	1.3	.9	.1	.2	.6	1.0	1.0	.8	.1	.2	.3	.6	1.2	1.1

Table 3
Summary of Significant t's For Error Computation
(* = p <.01 O=p <.001, Two Tailed Test)

Comp.	L V +	L V -	L V x	L A +	L A -	L A x	L H +	L H -	L H x
N/B	NS	NS	NS	NS	NS	NS	*	*	O
W/C	*	*	O	O	O	O	O	O	O
B/C	*	O	O	NS	*	O	O	O	O

Comp.	M V +	M V -	M V x	M A +	M A -	M A x	M H +	M H -	M H x
N/B	NS	NS	NS	NS	NS	NS	NS	NS	O
W/C	NS	*	O	O	O	O	O	O	O
B/C	O	O	O	O	*	O	O	O	O

Comp.	R V +	R V -	R V x	R A +	R A -	R A x	R H +	R H -	R H x
N/B	NS	NS	NS	NS	NS	NS	*	NS	O
W/C	*	O	O	*	O	O	NS	*	*
B/C	*	O	O	*	O	O	O	O	O

Comp.	Item	L +	L -	L x	M +	M -	M x	R +	R -	R x
	W	NS	NS	NS	NS	NS	NS	NS	NS	NS
V/A	B	NS	NS	NS	NS	NS	NS	NS	NS	NS
	C	O	*	NS	NS	NS	NS	NS	NS	NS
	W	NS	NS	NS	NS	NS	NS	NS	NS	*
V/H	B	NS	O	O	NS	O	O	*	O	O
	C	*	NS	NS	NS	NS	NS	NS	NS	*
	W	NS	NS	NS	*	*	NS	NS	NS	NS
A/H	B	NS	O	O	NS	O	O	*	O	O
	C	NS	NS	O	NS	NS	O	*	NS	O