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ABSTRACT

This document is a report on a series of statewide conferences organized by the Committee on the Undergraduate Program in Mathematics (CUPM) in 1962-65 to enable representatives of mathematics departments, schools of education, elementary schools and state departments of education to discuss the content and implementation of the 1961 report, "Recommendations for the Training of Teachers of Mathematics". The major part of the report consists of addresses by Folson, Hlavaty, Roush, and Davis. There is also a report of four studies conducted by the National Association of State Directors of Teacher Education and Certification and the American Association for the Advancement of Science. (MM)

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COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

REPORT

NUMBER 15

JUNE, 1966

FORTY-ONE CONFERENCES
ON THE
TRAINING OF TEACHERS
OF
ELEMENTARY SCHOOL MATHEMATICS

A SUMMARY

SE 013 605

MATHEMATICAL ASSOCIATION OF AMERICA

FORTY ONE CONFERENCES ON THE TRAINING OF TEACHERS OF ELEMENTARY SCHOOL MATHEMATICS

A SUMMARY

Edited By
Harold T. Slaby

Committee on the Undergraduate Program in Mathematics
Mathematical Association of America

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

Additional copies of this report may be obtained without charge from CUPM, Post Office Box 1024, Berkeley, California 94701.

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS
June 1966

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Indiana University

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TABLE OF CONTENTS

THE NATURE OF THE CONFERENCES

Introduction

The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America has as one of its basic concerns the improvement of college and university mathematics education of future teachers of mathematics. To this end, the Committee established the Panel on Teacher Training as an action group to recommend revisions of mathematics teacher education curricula and to work toward the realization of such revisions.

In 1961 the Panel on Teacher Training published its report Recommendations for the Training of Teachers of Mathematics.[1] These recommendations suggest the type and amount of mathematical training which should be required of teachers of mathematics at each of five levels. Specific courses were suggested for teachers in elementary schools (Level I), junior high schools (Level II), and high schools (Level III). Subsequently the Panel produced two documents which outlined these courses in more detail: Course Guides for the Training of Teachers of Elementary School Mathematics and Course Guides for the Training of Teachers of Junior High and High School Mathematics.

In the fall of 1962 the Panel began a series of statewide conferences in order to give mathematicians, educators, administrators, classroom teachers, and representatives of state departments of education an opportunity to discuss the content and implementation of its recommendations for elementary teachers. Subsequently these "Level I" conferences were held each fall at the rate of approximately ten a year. This series of conferences was completed with the eleven held in the fall and winter of 1965. Each of the fifty states, as well as the District of Columbia, participated in one of these meetings.

This document is a report on the forty-one conferences of this series.

The activities of CUPM

The Committee on the Undergraduate Program in Mathematics was established in January, 1959, as an action committee of the Mathematical Association of America. The latter is a professional organization of mathematicians, concerned with mathematics at the undergraduate level. CUPM was formed for the purpose of bridging the gap between present practices in business, industry, graduate schools, and even some secondary schools on the one hand, and the outmoded, stagnant undergraduate programs in mathematics at many (if not most) colleges, on the other hand. Its general aim is the improvement of the undergraduate mathematics curricula in colleges and universities. The Committee operates under a grant from the National Science Foundation.

1. Copies of most of the CUPM publications mentioned in this report may be obtained at no charge by writing to CUPM, P.O. Box 1024, Berkeley, California 94701. A list of the material available is given in Appendix B.

Upon its formation CUPM distributed most of its work among four subcommittees, referred to as "panels." They were the Panel on Teacher Training, the Panel on Mathematics for the Physical Sciences and Engineering, the Panel on Mathematics for the Biological, Management, and Social Sciences, and the Panel on Pregraduate Training (the last being concerned with the undergraduate education of professional mathematicians). Each of these panels devoted much time and energy toward preparing sets of recommendations on the college mathematics curriculum. These recommendations were subsequently published by CUPM in the form of individual pamphlets.

A number of other subcommittees have also played important roles in the work of CUPM. One of these is the Subcommittee on the General College Curriculum. The report of this committee is contained in CUPM's pamphlet, A General Curriculum in Mathematics for Colleges. The report describes an undergraduate mathematics program designed to meet the diverse needs of today's students while remaining within the capabilities of a small mathematics staff.

The Advisory Group on Communications is concerned with all CUPM publication activities. An outstanding project of this subcommittee is the assembling and maintenance of the Basic Library List, a list of books and periodicals recommended for the shelves of every college mathematics library.

The ad hoc Subcommittee on Applied Mathematics studied the entire complex question of the place to be occupied by applications of mathematics in the undergraduate curriculum. The curriculum report of this subcommittee was approved for publication by CUPM in January, 1966.

One of CUPM's most important activities is its Consultants Bureau. The Consultants Bureau is a group of forty qualified people distributed throughout the country who visit any interested college to help the college plan improvements in its mathematics curriculum, as well as to make any other suggestions which will help to improve its mathematics department. These visits are made at the request of the college seeking help.

Recently CUPM has opened attacks on several new fronts. It has formed a Panel on College Teacher Preparation, concerned with training and retraining programs for college teachers, a Panel on Mathematics in Two Year Colleges, and three new panels devoted to applications of mathematics: Computing, Statistics, and Life Sciences.

The administrative work of the Committee is performed by its Central Office.

The recommendations of the Panel on Teacher Training

The concern of the Panel on Teacher Training is the mathematical preparation of teachers of mathematics. The Panel has worked on the premise that there will be no substantial improvement of mathematics education at the elementary and secondary levels until or unless a concerted effort is made to provide appropriate mathematical programs for prospective teachers.

The Panel divided its work into five so-called "levels," each level determined by the ultimate goal of the prospective teacher. In this report we are concerned with Level I—prospective teachers in grades K through 6.

The original recommendations of the Panel were published in December, 1960, in the American Mathematical Monthly. A month later CUPM published a reprint of this article in pamphlet form, called Recommendations for the Training of Teachers of Mathematics. Here are the Panel's recommendations for Level I:

> As a prerequisite for the college training of elementary school teachers, we recommend at least two years of college preparatory mathematics, consisting of a year of algebra and a year of geometry, or the same material in integrated courses. It must also be assured that these teachers are competent in the basic techniques of arithmetic. The exact length of the training program will depend on the strength of their preparation. For their college training, we recommend the equivalent of the following courses:
>
> (A) A two-course sequence devoted to the structure of the real number system and its subsystems.
>
> (B) A course devoted to the basic concepts of algebra.
>
> (C) A course in informal geometry.
>
> The material in these courses might, in a sense, duplicate material studied in high school by the prospective teachers, but we urge that this material be covered again, this time from a more sophisticated, college-level point of view.
>
> Whether the material suggested in (A) above can be covered in one or two courses will clearly depend upon the previous preparation of the student.
>
> We strongly recommend that at least 20 per cent of the Level I teachers in each school have stronger preparation in mathematics, comparable to Level II preparation but not necessarily including calculus. Such teachers would clearly strengthen the elementary program by their very presence within the school faculty. This additional preparation is certainly required for elementary teachers who are called upon to teach an introduction to algebra or geometry.

The initial responses included many requests that CUPM provide detailed outlines of the content of the proposed courses. Therefore, in the spring of 1962, CUPM gathered together 12 mathematicians, expert in the area of elementary education, to prepare course guides. The guides were discussed at ten Level I conferences in the fall of 1962, and during the summer of 1963 they were substantially rewritten. In July, 1964, CUPM published the fourth draft of these guides under the title Course Guides for the Training of Teachers of Elementary School Mathematics.

It might be appropriate to emphasize, in connection with the production of the course guides, one of the points made in the original recommendations:

> The recommendations are not motivated by a desire to meet the demands of any special program of mathematics education; nor do the descriptions or outlines of courses to be taken by prospective teachers represent an attempt on the part of this committee to further the goals of any particular school curriculum planning organization. The recommendations are meant to be the minimum which should be required of teachers in any reasonable educational program, and the course descriptions are presented only to illustrate what is meant by the course titles.

In the fall of 1962, the Panel instituted its series of Level I conferences in order to determine how well it had done its job, and to acquaint people with these recommendations and course guides.

The members of the Panel

PANEL ON TEACHER TRAINING

E. G. Begle, Stanford University, 1960-1967
Roy Dubisch, University of Washington, 1962-1965
Mary Folsom, University of Miami, 1963-1967
W. T. Guy, Jr., University of Texas, 1960-1962
Clarence E. Hardgrove, Northern Illinois University, 1963-1968
P. S. Jones, University of Michigan, 1960-1962
John L. Kelley, University of California at Berkeley, 1960-1961
John G. Kemeny, Dartmouth College, 1960-1962 (Chairman 1960-1962)
E. R. Kolchin, Columbia University, 1966-1968
Bruce E. Meserve, University of Vermont, 1960-1963
Edwin E. Moise, Harvard University, 1960-1966 (Chairman 1963)
George Springer, Indiana University, 1966-1968
Rothwell Stephens, Knox College, 1960-1965
Henry Van Engen, University of Wisconsin, 1960-1961
Stephen S. Willoughby, New York University, 1966-1968
Gail S. Young, Tulane University, 1963-1967 (Chairman 1964-1967)

A typical Level I conference program

The format of the conferences varied little throughout the four year series. All were two day meetings, beginning at noon of one day and terminating at noon of the next. Here is the program of a typical Level I conference:

First Day of Conference

11:30 a.m.	Registration
12:30 p.m.	Luncheon
	Welcoming address
2:00 p.m.	General session
4:00 p.m.	Group discussions
6:00 p.m.	Dinner
7:30 p.m.	Plenary session

Second Day of Conference

9:00 a.m.	Group discussions
11:00 a.m.	Plenary session
12:00 noon	Adjournment

The welcoming address, given at the conclusion of the opening luncheon, was always delivered by a member of the Panel. A representative of the CUPM Central Office opened the general session by discussing CUPM and the Panel on Teacher Training and explaining the nature of the conference. These representatives were:

Robert J. Wisner, New Mexico State University
Bernard Jacobson, Franklin and Marshall College
B. E. Rhoades, Indiana University
Robert H. McDowell, Washington University
Harold T. Slaby, Wayne State University

Following this talk a local speaker discussed problems of raising standards of teacher preparation in his state, and a national speaker stated his views on trends in mathematical training at the elementary level.

After the general session the participants separated into small groups in order to discuss the recommendations and the course guides. At the evening plenary session the group leader or recorder for each group gave a report of the discussion which took place in his group. This was followed by general discussion.

The morning group discussions centered around problems related to implementing the recommendations. At the closing plenary session the feelings of the conference regarding the recommendations were expressed in the form of resolutions.

The conferences and local speakers

The following list contains the dates and sites of the Level I conferences, together with the states represented and the local speakers:

October 1-2, 1962	Oklahoma City	Oklahoma	Dorothea Meagher
October 5-6, 1962	Miami Beach	Florida	J. T. Kelley
October 19-20, 1962	Lansing	Michigan	John Wagner
October 22-23, 1962	Concord	New Hampshire, Maine, and Vermont	George S. Cunningham
October 26-27, 1962	Chicago	Illinois	Joseph J. Stipanovich
October 29-30, 1962	Minneapolis	Minnesota	Donovan A. Johnson
Nov. 30-Dec. 1, 1962	Albany	New York	Robert W. Sloan
December 3-4, 1962	Nashville	Tennessee	James A. Cooley
December 7-8, 1962	Dallas	Texas	John Wagner
December 10-11, 1962	San Francisco	California	Marguerite Brydegaard
Sept. 30-Oct. 1, 1963	Baltimore	Maryland, Delaware and District of Columbia	Mildred L. Sowers
October 4-5, 1963	Harrisburg	Pennsylvania	Joy Mahachek
October 14-15, 1963	Seattle	Washington	Joseph Hashisaki
October 18-19, 1963	Madison	Wisconsin	Arnold M. Chandler
October 28-29, 1963	Wichita	Kansas	Marjorie L. French
November 1-2, 1963	Albuquerque	New Mexico	George L. Baldwin
November 11-12, 1963	Indianapolis	Indiana	Philip Peak
November 15-16, 1963	Cleveland	Ohio	Clarence H. Heinke
December 6-7, 1963	Roanoke	Virginia	Isabelle P. Rucker
December 9-10, 1963	Louisville	Kentucky	Louise Combs
September 28-29, 1964	Salt Lake City	Utah	N. Blaine Winters
		Idaho	Richard Kay
October 2-3, 1964	Denver	Colorado, Wyoming	William E. Briggs
October 12-13, 1964	Portland	Oregon, Alaska, and Nevada	J. Richard Byrne
October 19-20, 1964	East Brunswick	New Jersey	Max A. Sobel
October 26-27, 1964	Hartford	Connecticut	Elizabeth Glass
		Rhode Island	William P. Robinson, Jr.
October 30-31, 1964	Boston	Massachusetts	J. Fred Weaver
November 9-10, 1964	Des Moines	Iowa	E. Glenadine Gibb
November 13-14, 1964	Omaha	Nebraska	Milton W. Beckman
Nov. 30-Dec. 1, 1964	St. Louis	Missouri	Lois Knowles
December 4-5, 1964	Atlanta	Georgia	Gladys M. Thomason
September 27-28, 1965	Phoenix	Arizona	Arthur H. Steinbrenner
October 4-5, 1965	Charleston	West Virginia	Genevieve Starcher
October 25-26, 1965	Great Falls	Montana	Donald A. Kamp

October 29-30, 1965	Bismarck	North Dakota	Kiaran L. Dooley
		South Dakota	Lloyd T. Uecker
November 8-9, 1965	Durham	North Carolina	Thomas D. Reynolds
November 12-13, 1965	Columbia	South Carolina	Daniel H. Sandel
November 19-20, 1965	New Orleans	Louisiana	Z. L. Loflin
November 22-23, 1965	Little Rock	Arkansas	Helen Graham
December 6-7, 1965	Biloxi	Mississippi	John O. Ethridge
December 10-11, 1965	Birmingham	Alabama	Charlotte H. Jones
January 7-8, 1966	Honolulu	Hawaii	Ruth E. M. Wong

WELCOMING ADDRESS
Indianapolis, Indiana, November 11, 1963

Gail S. Young
Tulane University

I want to talk for a few minutes on why I believe this conference, and the others like it, are of great importance.

To begin with, education—the aims of education—change to meet changes in society. I do not have to explain that remark to this audience. I believe our society is now in one of those periods of transformation that mankind has been through so many times. Let me go back twenty years. At the end of World War II, there were many predictions as to the changes in our lives that would be brought about by atomic energy. By 1965, coal would be obsolete, every home would have its own little reactor, cars would be atom-powered, etc. These predictions do not seem to have materialized yet. Few people talked then of what was the really revolutionary development of the war, the electronic computer, with its power of extending man's brain in the same sort of way the machines of the Industrial Revolution did man's muscle. One consequence of this new tool is automation, which is already putting a strain on our society. It is this new power that I believe is causing the revolution we are living in, and in this revolution mathematics plays a principal part.

The revolution is also a fast-moving one. It took a thousand years for the Bronze Age to come into being, and at any given time in that thousand years most people's lives were not affected by it. Even the Industrial Revolution moved slowly. In this country, from 1800 to 1900 most people could plan a livelihood and count on being able to spend their working life at it. That had, of course, become more difficult before automation, but now, how many jobs can you urge a child to prepare for if you must say those jobs will still be there forty years from now?

This puts a tremendous responsibility on the educators, one no other generation of educators has had to face. We must prepare our children for a world that can only be dimly seen. All we can say surely is that mathematics and science will be more important for everyone, and that the ability to learn will be needed all our students' lives.

In the schools of my childhood in the '20's, the emphasis in arithmetic—no one spoke then of the mathematics of the elementary school—was on speed and accuracy in arithmetic computation. Much of the content was directly useable. For example, I remember making out grocery bills—2 dozen eggs at 25¢ a dozen, 3 cans of beans at 10¢ a can, that sort of thing. This was exactly what a store clerk did then. Now no one in a supermarket needs any arithmetical skill more advanced than counting money. The cash register even tells the checker what the change will be. What our elementary students need now is a beginning of insight into mathematics.

I find it is hard to be concrete as to why this is the need. It requires that I be able to describe jobs that do not even exist yet. But let me talk about skilled labor for a moment. In the industry of 1930, perhaps the highest position of skilled labor was the tool-and-die maker. He needed some mathematical skills: how to read a vernier, how to handle right triangles, various formulas from geometry, the ability to use tables. The traditional high school program more than met his needs. The successor to the skilled workman is now the technician who will more and more be trained in technical institutes. These institutes are still developing their programs, but typically they seem to take their students up through some sort of a course in calculus. Partly they do this because the technician will need some mathematics as a tool. More important is that the things the technician is learning how to do can best be understood in a mathematical formulation.

It is also going to be essential for most people to maintain their ability to learn for all their working life. We have already seen this in engineering. A specialist in propellor design has had to learn nose-cone design, or retire. This capacity for relearning will have to be possessed by most people.

Let me turn for a moment to the college-bound student. We have not worried very much in designing the second grade curriculum about the needs of the college student. I believe we have to start doing this, in our thinking about mathematics. After all, about half the students now in a second grade classroom will go to college. Of that half, a very rough extrapolation would be that half will take two years of mathematics for their various fields. In my own university we discovered this year that our freshman-sophomore mathematics enrollment amounted to one third of the entire undergraduate enrollment. This means that, right now, 1963, at my school most students are taking two years of mathematics. Just last week I received from Dr. Lindquist of the Office of Education, who is the authority on such things, the latest estimates on bachelor's degrees in science fields. By 1970, one student out of 19 will graduate as a mathematics major, a much higher figure than for all the physical sciences combined, but still below engineering. Mathematics, the physical sciences and engineering will account for 16.3% of the total bachelor's degrees, or about one out of six. By 1975, more people will graduate in mathematics than in all the physical sciences or in engineering. If we assume that half the present elementary school population will go on to college, this means that in a 6th grade class of thirty-five, three will get degrees in mathematics, the physical sciences, and engineering, and at least three more will take more than two years of college mathematics, and three, four, or five will take two years.

Figures like these make me feel a great urgency about our particular task today. After all, if a change in teacher preparation is started next year at the freshman level, it is five years from now before any teachers trained by it come into the schools. Five years now becomes a long time. First graders taught then will finish high school in 1980.

I would like to give some idea of the changes in the elementary school programs that our teachers will have to cope with. We have supplied you with a summary of the report of last summer's Cambridge Conference on School Mathematics. This was a group of 25 mathematicians and mathematics users, from industry or the colleges, with others brought in later to comment on their work. The conference included or involved many of the leading figures from the mathematics side in the movement to change the mathematics curriculum. I would like to read to you the recommendations for the elementary school, leaving out motivations or reservations:

> The conference found itself essentially in complete agreement on the mathematical aims of the elementary school.
>
> Through the introduction of the number line, the child would be started immediately on the whole real number system, including negatives...By this wedding of arithmetic and geometry at the pre-mathematical level, the intuition of the child would be developed and exploited, and the significance of the arithmetical operations enriched. Moreover, the child provided with these complementary viewpoints, would have a very good chance to understand the essential nature of mathematics and its relationship to the "real" world.
>
> The order properties of the real number system would be studied from the beginning, and would be used in inequalities, approximation, and order of magnitude estimates.
>
> The use of Cartesian coordinates ("crossed" number lines) would begin almost as soon as the number line itself. Moreover, we agree with Freudenthal and other pioneers, that an early development of the child's spatial intuition is essential. Study of the standard shapes in two and three dimensions would continue concurrently, and would include discussion of their symmetries.
>
> The notions of function and set are to be used throughout; of course, set theory and formal logic should not be emphasized as such, but the child should be able to build his early mathematical experience into his habitual language. Informal algebra should be taken up along with the arithmetic operations.
>
> The Conference agreed that reasonable proficiency in arithmetic computation and algebraic manipulation is essential to the student of mathematics. But this is not an argument in favor of a curriculum devoted primarily to computation with

contrived numbers through the whole of grammar school...

Because of both its intuitive appeal and its basic importance, there should be an introduction to the elementary ideas of probability and statistical judgment, accompanied by concrete experimentation with random processes...

Having studied arithmetic and geometry, mostly informally, in the elementary school, the student will be prepared for a sound treatment of geometry and the algebra of polynomials, beginning in the seventh grade...

This audience, certainly, knows how very far such a program is from what we do now, and how few teachers now in the schools are prepared to teach such a program.

It is clear that if this is to be the curriculum of the future, our Level I recommendations are only makeshift, and represent only the best we can do with the students now entering the schools and with the present staff engaged in teacher training. In the future, we will have to do a good deal better.

Excerpt from WELCOMING ADDRESS
Hartford, Connecticut, October 26, 1964

E. E. Moise
Harvard University

I shall have very little to say, this afternoon, about the substance of the problems that we have come here to discuss. I shall speak only of the urgency of these problems. It seems to me that in the training of elementary teachers in mathematics, we are in even worse trouble than is commonly supposed.

I should begin, however, with a disclaimer. It very often happens that eager reformers, trying to improve the quality of a certain job, are unfair to many of the people who were doing the job before the reformers came along. Thus even the warmest admirers of the Illinois and SMSG programs should bear in mind that good teaching of high school mathematics was not invented in Urbana in 1954 or in New Haven in 1958. Similarly, I have no doubt that some elementary teachers have taught mathematics very well, at some times and in some places. The problem before us today, however, is the statewide fraction of an immense national enterprise. We are concerned not with isolated successes of this enterprise, but with its overall quality. In these terms, it is an understatement to say that the teaching of elementary mathematics has been a failure. In general, the teaching of elementary mathematics deserves to be described as a disaster.

From time to time we hear scandalous test data cited to prove this point. I am not concerned, however, about this sort of evidence. I suspect that you can prove just about anything you want to prove with test data, including even the truth, if the truth happens to be what you had in mind at the outset. I am concerned, rather, about evidence that we all know about, if we stop to think. The most distressing evidence is the image of the mathematician in the popular culture. In popular fiction, mathematicians hardly occur at all. A few weeks ago I had the pleasure of reading a detective story entitled <u>A Murder of Quality</u>, by John Le Carré, the author of <u>The Spy Who Came in from the Cold</u>. One of the characters in this story is a young high school mathematics teacher who intends to go back to Cambridge for his doctorate. This young man is a quite normal and sympathetic character, and his desire to go back to graduate school and do mathematical research is described as a worthy enterprise, leading to a bright future. The reason I am telling you about this story of Le Carré is that it is the only such case that I know of. In popular fiction, it almost never happens that any character whatever is a mathematician; plainly we don't exist. When mathematicians do occur in fiction, they are nearly always villains or helpless eccentrics.

Now this conception of mathematicians is not the work of unfriendly propagandists. As far as I know, we have never attracted the attention of extremists of either the Left or the Right. In the last analysis, all attitudes are based on experience; and the attitude that most people have to us, and to our work, is based on the experience that most of them had studying mathematics in school.

In fact, mathematics is the most obscure of the sciences, and this is a paradox, because it is by far the most widely studied. It is the only science which is universally taught in elementary schools, and is thus the only science whose rudiments are known to everybody. But the paradox is easy to resolve: the point is that very few people are aware that there is a science of which elementary arithmetic is the beginning. This idea can only be imparted by experience, and few people have had the experience. Indeed, arithmetic as commonly taught does not appear to be knowledge at all. It appears to be a set of procedures, which get more and more complicated as we go along, but which do not appear to be based on ideas. Rather simple calculations depend on rather complicated mathematical principles, but on the basis of the traditional study of arithmetic, most pupils would be at a loss to state to you any mathematical principle, however simple. Thus, at worst, arithmetic is the subject in which you obey complicated orders which you do not understand. The unpopularity of such pseudo-study does credit to the popular taste: it deserves to be disliked; it is revolting.

The reasons why the teachers of arithmetic convey to students this conception of the subject is that this is the conception that they have. To put it bluntly, most of them are trying to teach something they do not understand. In some cases, their methods may be poor, but for most of them the question of methods hardly arises. You cannot convey knowledge which you do not have.

A couple of years ago in the New Yorker there was a cartoon which has a moral for us today. The scene was in the parking lot of an elementary school. One of the teachers was looking at her car. Seeing that the fender was smashed, she said "Oh! Oh! Oh! Look! Look! Look! Damn! Damn! Damn!" Ordinarily there is not much point in explaining why a joke is funny. But the reasons for the humor of this one have a great deal to do with the problem before us. It is taken for granted that a person who teaches reading in the first grade is a literate person. In fact, it is this background of literacy which gives a sense of conviction to first grade teachers. The mechanics of learning to read are rather dull in themselves, but every teacher knows that these mechanics lead to a skill which will be a source of pleasure forever after. In fact, the teacher is so sure of this that it hardly occurs to her that the idea needs to be expressed. This is not entirely an intellectual matter. It is also a matter of attitude. Students are very likely to sense and to share the attitudes that their teachers really have. The teaching of reading is greatly aided, psychologically, by the solid conviction of the teacher that the process is worthwhile, and the teaching of arithmetic is badly handicapped by the teacher's frequent conviction that it is not. No doubt all specialists are convinced that teachers are badly trained in their specialties, but it is hard to believe that any teacher is as poorly prepared in the language arts as many of them are in mathematics. A person as weak as this in English would speak in the language of a primer, like the teacher in the New Yorker cartoon. In English this doesn't happen, and so the idea strikes us as funny. In mathematics the same level of understanding occurs all the time, and so it isn't funny even slightly. What teachers need to have is mathematical literacy. If we manage to give it to them, then we can break the vicious circle in which mathematical illiteracy has reproduced its own kind in the classroom.

For these reasons, I believe very strongly in the urgency of the task that we are here to discuss. It deserves, and it needs, the best efforts of all of us.

THE NATIONAL SPEAKERS

At each of the Level I conferences some aspect of the problem of the mathematical training of elementary teachers was discussed by someone well acquainted with the situation at the national level. These speakers were:

Truman A. Botts, Associate Professor of Mathematics at the University of Virginia and Executive Director of the National Academy of Science's Committee on Support of Research in the Mathematical Sciences

Robert B. Davis, Associate Professor of Mathematics at Syracuse University and Director of the Madison Project

Mary Folsom, Professor of Education at the University of Miami

Helen L. Garstens, Assistant Professor of Mathematics and Education at the University of Maryland and Associate Director of the Maryland Project

Leonard Gillman, Professor and Chairman of the Department of Mathematics at the University of Rochester

Clarence E. Hardgrove, Professor of Mathematics at Northern Illinois University

Shirley Hill, Professor of Mathematics at the University of Missouri

Julius H. Hlavaty, New Rochelle, New York

H. Stewart Moredock, Chairman of the Division of Science and Mathematics at Sacramento State College

Donald C. Roush, Dean of the College of Teacher Education at New Mexico State University

Merrill E. Shanks, Professor of Mathematics at Purdue University

This section of this report contains four of these talks by "national speakers."

1962

NATIONAL PROBLEMS AND TRENDS IN THE MATHEMATICAL TRAINING OF ELEMENTARY SCHOOL TEACHERS

Mary Folsom
University of Miami

Since I was asked to speak to you today, I assume that you know I have a great interest in the work you will do during this conference. This interest is strong because it stems from two different sources. Every day I must face the fact that the mathematics background of my students, on both the preservice and inservice levels, is inadequate. These students expect me to teach them HOW to teach mathematics to children, yet they have not mastered the mathematics they expect to teach. In my opinion, they do not even know very much ABOUT mathematics. I also must face the devastating knowledge that I must be concerned with this problem for the sake of my grandchildren, since it is already too late to improve the situation as far as my own children are concerned.

In 1951, Grossnickle (3:205) in a survey of state certification requirements adapted from the nineteenth edition of Woellner and Wood (8), reported that thirty-five states had no requirements in mathematics for elementary teachers, three had blanket requirements, and ten had specific requirements ranging from a minimum of two to a maximum of six semester hours. Now, thirteen years later, the twenty-sixth edition of Woellner and Wood (9) supports the statement that no improvement has been made from the certification aspect of the situation. Hawaii and Alaska may have joined the Union, but they also have joined the ranks of those states which require no mathematics of their elementary school teachers.

If we have been complacent for this period of time, why is there now a feeling of urgency about this matter? That this feeling of urgency has grown in the last five years cannot be denied.

Although I have taught mathematics to children and to teachers for several years, I am not making the progress I would like. If I am doing a better job of teaching today, and I certainly believe that I am, why is there not a feeling of satisfaction in a job well done? Even though I may accomplish more, the growth in mathematics content and changes in approach to content simply leave more to be accomplished.

The typical program in teacher education—and my university is typical—does not allot sufficient time to the study of mathematics. What we are doing now may have been satisfactory in 1745 when Yale included arithmetic in the undergraduate program (2:13), but it will not do at all today when the role of mathematics in contemporary society is not only unique but quite literally overpowering us. The problems we face are widespread and national in scope.

We all realize that everyone should not only KNOW more mathematics but also, as I have implied, more ABOUT mathematics. The phrase "about mathematics" has some interesting connotations. We are being forced to use this phrase. There is no living research mathematician who knows all the mathematics man has created. It has been said many times that Gauss was the last mathematician who could make this claim. Today mathematicians are generalists in mathematics and specialists on one topic or a small cluster of related topics within the field of mathematics. We might well ask: "What has caused this change; why have mathematicians been forced to give Gauss this particular claim to fame?"

It is not the fact that our society is changing. Times always change. History is the record of these changes. It is the accelerated pace of these changes. The middle aged person of today, and I will allow you to define "middle aged" for yourself, has witnessed more change than anyone living in Western civilization before him. For example, it took one hundred years to perfect the steam engine, fifty years to perfect the electric motor, but only fifteen years to get the computer from the drawing board to use in the computer center of the university. What may have seemed pure fantasy in early science fiction may actually be within the realm of conservatism today. The growth of knowledge is not linear but exponential.

While the man on the street is not consciously aware of this accelerated change, particularly in the field of mathematics, it certainly should be apparent to us. Forcing awareness is a large part of our problem.

One half of the mathematics we have today was created by men who are still alive. The mathematics of Pythagoras is not even considered mathematics by today's standards. It was only during the nineteenth century that man was able to break away from Euclid to a world in which the criterion for judging good mathematics is the model of consistency rather than the model of reality. This change began to register in the twentieth century, and we are now feeling the backlash.

One of the chief tools for obtaining knowledge and arriving at conclusions is the deductive method; and, although much of mathematics is discovered or invented inductively, mathematics is still, par excellence, the science of deductive reasoning. It is clear, therefore, that man needs to understand mathematical methods and the language of mathematics in order to apply them to the physical, biological, and social sciences.

If one were to ask: "What are the distinguishing characteristics of mathematics?" the answer would have to include: (1) its abstract nature, (2) its generality, and (3) its concern for structure and pattern. This answer no doubt led to the popular opinion that the ivory towered mathematician sat alone absorbed only in the creation of abstract systems which had no practical application in the sense of physical or tangible interpretations. If the well educated man has "any doubts about the 'utility' of such abstract systems, he may be reminded of the central role played by complex numbers of the form a + bi in the theory of alternating currents; or of Boolean algebra in relation to the theory of electronic computers; or perhaps even more dramatic, the role of noneuclidean geometry in the development of relativity theory. Surely A. N. Whitehead was not exaggerating when he said: 'It is no paradox that in our most theoretical moods we may be nearest to our most practical applications' " (5:28)

One cannot predict with assurance either the mathematical needs of today's learner, or the mathematical ideas required by tomorrow's society, but we do know that the learner must be mathematically literate. He must have some feeling for mathematics and not be afraid of it. The businessman does not have to know how to program, but he has to know about linear programming. Nearly every field has been touched by this accelerated growth in mathematics: medicine, psychology, biology, oceanography, geography, and business are just a few. We have applications we never dreamed of: linear programming and queueing theory are just two. There is no dimension that cannot be touched by mathematics.

As a consequence of these forces and pressures, the mathematics of the elementary school curriculum takes on added importance. Children cannot be denied the opportunity to learn the mathematics they need as citizens or the mathematics that is a necessary steppingstone to many careers.

A small government publication entitled "Mathematics and Your Career" makes this statement: "Even if you don't become an engineer or statistician, knowing what mathematics can do will be a great help in business or any other career." (7:3) I must admit, however, that the problem of awareness is even greater than I had thought. Among the careers mentioned which require some training in mathematics, that of the elementary school teacher is conspicuous by its absence. Since more children are bored rather than scared out of the study of mathematics, the ability of the elementary school teacher to develop in children a good attitude, a better background, and an interest in mathematics is crucial.

To meet these new demands, curricular changes have been made and are being made at every level in our educational system. It is evident that there is a tendency to push topics downward in the curriculum. During the nineteenth century calculus was taught in a few colleges at the senior level. By the turn of the century it was a sophomore level course. Today it is considered a freshman course and even taught in some high schools. Certainly by 1970, we can expect calculus to be taught in all good college preparatory programs.

Are people more intelligent today than they were in 1830? No, the answer for this speedup lies in mathematics education.

1. Today we place a greater value on education, and we have better teaching techniques. During the days of faculty psychology, the mind was considered a muscle. The techniques of developing a muscle were applied to the mind. The more difficult the task, the better the development. The teacher deliberately tried to stump the class. If the teacher made the learning difficult, it was considered excellent training for the student. Today we have approaches which make it difficult for the student not to get over the hump.

2. There is emphasis on the discovery method. The high school people are just getting this message. The elementary school people knew about it in the forties. I quote from Spitzer's book The Teaching of Arithmetic: "The emphasis upon 'find out for yourself,' whether it be in the development of a new fact, in the consideration of ways of study, or in other situations, gives to the method a decided advantage over all others. Through this emphasis the arithmetic classroom assumes some of the naturalness, creative drive, and problem solving attitude characteristic of elementary science and social studies classrooms..." Later, he says, "Learning in such situations is characterized by the spirit of inquiry, the method of the scholar. In contrast, most learning in arithmetic classes is a case (maybe somewhat sugar coated by play settings and make believe) of learning facts and processes that are explained and presented by the text or teacher." (6:387)

3. Since the textbook is the common denominator of curriculum, the fact that we now have available new and better textbooks was bound to affect the curriculum. Today we have mathematicians who are willing to devote part of their time to such endeavors. If we want good mathematics at the lower levels, the mathematician must, since he has the knowledge, join forces with those who know students, and write textbooks which contain good mathematics and are teachable.

4. Better means of communication and transportation make it possible to bring people together to discuss mutual problems and disseminate knowledge. Such a conference as this would not have been possible fifty years ago.

5. A special application of the increased ability to disseminate knowledge is found in the use of television. Has any other generation had the opportunity to take a course in abstract algebra or nuclear physics while still unshaven and in pajamas, lounging in an easy chair with a cup of coffee?

Since instruction in college mathematics rests firmly on the foundation laid in high school and instruction in secondary mathematics rests squarely on the foundation laid in elementary school, let us examine the changes and the reasons for the changes being made at the elementary school level.

First, there was a need to include ideas of "modern mathematics." Since the term "modern mathematics" means different things to different people, I define the term as referring essentially to mathematical ideas which were either unknown or not widely accepted as recently as one hundred years ago. The language of sets is an example of this need. This language provides the tools needed to speak succinctly and precisely about concepts of non metric geometry.

Second, since arithmetic is a system of thought, a structure, which man has devised in order to deal with the quantitative situations he faces in the world in which he lives, it must be approached as the system of thought which it is. To look upon it as a hodgepodge of accepted facts, which somehow the student must swallow whole while in school so that it may be regurgitated later on when practical circumstances require, is largely wasted effort. A knowledge of the properties of the set of whole numbers enables the child to master the basic facts of the operations of addition, subtraction, multiplication and division with greater ease. It certainly adds to his understanding of these operations. It makes it easier for him to deal with other sets of numbers of similar structure even though they possess additional properties.

Third, the need for interesting and challenging material has resulted in the introduction of some new topics. One such topic is inequalities. Not only is the fundamental idea of order and its representation as inequalities important, but it permits the introduction of the appropriate symbols, as well as providing a new and possibly exciting setting for needed review.

Fourth, change has resulted from the search for more effective means of dealing with individual differences. The introduction of simple equations, aimed first at the bright child, resulted in experimentation with the topic with children of lesser abilities. Now the topic is considered appropriate for practically all children at all levels in the elementary school. Since all problem solving is basically algebraic in nature, the introduction of the equation has been effective in improving the ability of children to solve word problems.

Fifth, changes have taken place as a consequence of the search for more effective methods of instruction. The introduction to the topic of multiplication of fractions as given in one of the new programs is an example of this. It does not help any of us to be told that I found this same approach to the topic in a seventeenth century Italian arithmetic book!

The reasons I have given for some of the changes in the curriculum are reflected nicely in the kinds of changes being made. Perhaps the most startling change is the appearance of such topics as sets, other systems of numeration, metric and nonmetric geometry, the integers and coordinates. The push from the top moved such ideas as prime and composite numbers, rules of divisibility, factoring, greatest common factor and least common multiple within the sphere of the elementary school mathematics program. The emphasis on structure and precise language has had a tremendous impact on teachers. They find themselves bogged down in terminology which is completely foreign to them—commutative, associative, distributive, sets of points, array, closure, rational number are just a few. It is impressive to see the ease with which children handle this language, while the teacher tends to find it confusing and much more difficult than the "old" way. It is even more impressive to watch the spirit of discovery grab the class as they struggle to find the pattern—the Euler Formula, Fibonacci's Sequence, the formula which will yield the number of line segments it is possible to draw when given 'n' number of points. The beginning of mathematical thinking of the "if...,then..." type is fun for youngsters when David Page's hidden number idea is the vehicle of instruction. Suddenly there are many exciting, challenging things to learn in mathematics. As one youngster put it, "Yes, she teaches arithmetic but it isn't the kind of arithmetic we have every day. You have to think."

In light of these changes and other changes that will undoubtedly be made, what are the implications for the elementary school teacher? I believe that we all know the answer. It is a truism that one cannot teach a subject effectively unless his knowledge and understanding go well beyond the scope of that which he is expected to teach. If we continue at this pace, within ten or perhaps five years the elementary school teacher will not even be able to read the sixth grade pupil text in arithmetic. The changes in content and approaches to content, to say nothing of the emphasis on structure and precise language, will leave him so frustrated and insecure that he will not be able to do as well as he is doing today.

There are several things that we might do to insure that future generations of elementary school children have well trained teachers of arithmetic. We can improve the undergraduate program in mathematics; we can improve certification requirements; we can provide extensive inservice programs such as we have undertaken in my home county in Florida. Since our first responsibility is to children, we will have to do all of these things. But it does seem to me that the improvement of the undergraduate program offers more substantial reward.

Universities and colleges should be leaders in the improvement of teacher education programs and not depend on changes in certification requirements to force them to improve their programs. The state usually sets up minimum requirements, since these requirements are, in effect, law. Inservice programs are designed to upgrade existing instruction and provide better learning experiences for children. It seems a bit incongruous to work on inservice programs while still continuing to graduate elementary teachers whose mathematics education is inadequate even before they leave our institutions. We are in effect telling these teachers that the day they graduate they should enroll in an inservice course! These preservice teachers are our responsibility. We cannot afford to shirk this responsibility by continuing to perpetuate the obsolete programs that exist.

If we had an adequate program in mathematics for preservice teachers, only 5 per cent of the 900,000 elementary school teachers would be directly affected each year (1). While this is a small percentage, members of the group would be spread throughout the schools of the nation. These teachers who are mathematically aware could give help and encouragement to other teachers. If we start NOW, it will take at least twenty years to achieve our goal—a well trained arithmetic teacher in every elementary school classroom. If we procrastinate, as we have in the past, this goal will not be achieved within our lifetime.

Since you are here to discuss the CUPM course outlines and recommendation that every elementary school teacher have twelve semester hours of mathematics, I will not attempt to outline a course of study for you. Nor will I debate number of courses or number of semester hours of work. On the basis of my own work with preservice teachers, I would like, however, to bring you some of my own feelings and convictions.

1. The content of these courses should capture the spirit of contemporary mathematics and include, among other things, the nature of number and of systems of numeration, the logical structure of arithmetic, the number system of arithmetic and algebra, informal and formal geometry, measurement, trigonometry, functional relations, and certain concepts of statistics and probability. That this need is great has been pointed out repeatedly. The effective teacher of any discipline should live intimately with that discipline. He should himself have far greater insight than he strives for in his students.

2. These mathematics courses should be specifically designed for elementary teachers. These teachers need more than a simple review or refresher course in seventh or eighth grade arithmetic, or a traditional course in algebra, geometry, trigonometry, and analytics. Nor should it be an experience designed to achieve desirable computational efficiency. On the contrary, these courses should strive to give some insight into the nature and structure of mathematics, including not only arithmetic but algebra and geometry as well.

3. These courses should be taught by teachers who are competent in two fields—mathematics and education. This is not a job for a graduate assistant. We need the very best. We need people who know mathematics and the elementary school program. It is not easy to find teachers who meet these requirements. The quality of these courses will depend on the instructor.

4. Since the elementary school teacher is, generally speaking, a victim of poor mathematics instruction and has, therefore, a built in negative reaction to mathematics, the instructor must be prepared to be patient, to make haste slowly, to overcome bad attitudes and to instill in these preservice teachers the confidence they need to do a good job with youngsters. Above all else, elementary teachers have to be convinced that they can learn mathematics, that they can teach mathematics effectively, and that the subject is interesting and stimulating.

A study made in 1957-1958 (4:304-306) revealed that the typical student completing a four year program for certification to teach in the elementary grades had completed two years of high school mathematics and had either a three semester hour course in general mathematics or a two semester hour course in the teaching of arithmetic.

The situation has improved slightly in the last four years. CUPM is now in the process of making a study of the status of the undergraduate mathematics training of preservice teachers in 1962. The data received to date show that in the United States today, 23 per cent of the programs for the preservice education of elementary teachers require no mathematics, 77 per cent require a three semester hour course and this course has a 50 per cent chance of not being specifically planned for elementary teachers.

Individuals and groups in the United States who have planned or are now planning up-to-date mathematics courses for elementary teachers have concluded that a three to four hour course is not adequate to teach the mathematics that is necessary or desirable. CUPM has recommended twelve semester hours. That is only 10 per cent of the average teacher preparation program. Since the elementary school teacher spends approximately 16 per cent of his time teaching mathematics, the recommendation seems reasonable. Michigan State University at Oakland, and the University of Southwestern Louisiana are following these recommendations. Many others are seriously considering their adoption. Leadership is coming from the universities and colleges of the nation.

The national trend is toward improvement in teacher education programs with more time allotted to the study of mathematics. To those of you who have wondered if emphasis would ever be placed on mathematics, and to those of you who have watched the other disciplines steal the limelight, I say, "Enjoy! Enjoy!"

I sincerely hope that the day is not too far distant when the essence of mathematical thinking, the significance of mathematical systems and models, and the relation of the rational numbers to the natural numbers, for example, will be as familiar to future elementary teachers as "borrowing in subtraction" and the "partition idea of division" have been to an earlier generation of teachers.

Although the experimental programs have shown us that we can teach more mathematics earlier than we had thought, and we have had improvements in content and techniques, we still need a major breakthrough. The problem in mathematics education, as I see it, is a human problem—teachers. In business, no matter how good the product, distribution is the problem. The analogy applies here. No matter how good the research, distribution is the problem of the teacher.

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1964

SOME TRENDS IN THE MATHEMATICAL TRAINING OF ELEMENTARY SCHOOL TEACHERS

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Mark Twain in his "Roughing It," you will remember, gives a wonderful account of his trip to our great West about a hundred years ago and of his adventures there. Among the many things that impressed Mark was the remarkable speed with which the trip from St. Joseph, Missouri, to Sacramento, California (about two thousand miles) was accomplished by the overland coach.

Mark Twain reports that it was the special genius of a man named Ben Holliday which made this phenomenal speed possible. He says "...Ben Holliday, a man of prodigious energy, used to send mails and passengers flying across the continent in his overland stagecoaches like a very whirlwind; 2,000 long miles in fifteen days and a half by the watch!"

But the story Mark tells is not really about Ben Holliday. It is rather about a pilgrimage to the Holy Land that Mark Twain made sometime later. Among the members of that pilgrimage was a nineteen year old New York boy named Jack who had once made a trip to California on one of Ben Holliday's coaches but who, alas, was very ignorant about scriptural history. Another member was a very learned elderly pilgrim who was very knowledgeable and enthusiastic about scriptural history.

At one point the pilgrimage was camped near the ruins of Jericho and the elderly pilgrim was moved to speak. Let me read from Mark Twain's account:

> "Jack, do you see that range of mountains over yonder that bounds the Jordan Valley? The mountains of Moab, Jack! Think of it...we are actually standing face to face with those illustrious crags and peaks; and for all we know, our eyes may be resting at this very moment upon the spot where lies the mysterious grave of Moses! Think of it, Jack! 'Moses who?' Jack, you ought to be ashamed of yourself. Why, Moses, the great guide, soldier, poet, lawgiver of ancient Israel! Jack, from this spot where we stand, to Egypt, stretches a fearful desert 300 miles in extent, and across that desert that wonderful man brought the children of Israel! Guiding them with unfailing capacity for forty years! 'Forty years? Humph! Ben Holliday would have fetched them through in 36 hours'."

Now, no story of Mark Twain's, I believe, ever needs an apology. But I believe this story does have a relevance to our discussion today.

Why did it take forty years for the children of Israel to reach the Promised Land? Was there no Ben Holliday to speed up the trip? In that most perspicuous of the commentaries and exegeses on the Bible, the Talmud, the learned rabbis who compiled it, addressed themselves to the question. And they came to the conclusion that it was special divine providence that detained the children in the desert. Only after the generation that had known the slavery and fleshpots of Egypt had died out would there be a generation that could appreciate and enjoy the Promised Land. Even Moses himself, you will remember, was not judged worthy to enter the Promised Land.

In the last ten years we have witnessed the emergence of many prescriptions for a new world in mathematics and science education—from UICSM and the Commission on Mathematics to the Cambridge Report. In all of these attempts, the questions that have come up most sharply and most insistently have been: Can the present generation of teachers realize the promises of the new curricula? How can the generation of teachers to come be equipped for their tasks?

As you know we have chosen not to follow the Biblical example; we are not waiting for the present generation of teachers to die out before we start instituting the promised land. We all know the heroic job that has been done and is being done by individual teachers, by schools, and by school systems (sometimes reaching the proportions of statewide efforts) to achieve a better program of mathematical education. While it hasn't been all plain sailing, and while the advances sometimes are modest, there has been significant progress.

At the high school level, a great deal has been achieved by means of the various NSF and state sponsored institutes for teachers—whether summer, in-service or academic year institutes. A significant fraction of the high school teachers of the country has been influenced by these institutes.

But our topic for today is not how to help the current generation of teachers of elementary or secondary mathematics to cope with the problems that confront them. Not that we may not get very valuable help in the problem that we are discussing by examining carefully the work of the dedicated teachers, curriculum counselors, and, yes, school superintendents and trainers of teachers, for one of the major considerations in our discussion of the training of future teachers must be the real situation into which they will be entering. Let us then turn our attention to today's topic, to the training of future teachers of mathematics, especially at the elementary level.

There are several aspects to this problem. Who are the people we are talking about? What will they be like when they teach? What do they need—today as college students, and tomorrow as teachers? Specifically, what mathematics should they know when they teach?

I am sorry to have to say that CUPM—especially in its early deliberations on the problem—ignored the first two questions almost entirely, and gave a correct but far from helpful answer to others. The CUPM recommended, and recommends, and properly so, the glittering, austere beauty of the real number system. This certainly is the fundamental and minimum requirement that teachers of elementary mathematics should satisfy. But how do we achieve this? I am happy to cite as a counterweight to this criticism the statement of Professor Moise in May, 1964, where he does discuss the first two questions. I commend the statement to your attention. While his appraisal of the undergraduates we are discussing tends to be negative on the whole:

> "...they stopped studying mathematics at the end of the tenth grade, because they wanted to stop...
>
> "The majority of them suffer not only from a lack of knowledge, but also from indifference, complicated in many cases by antagonism..."

he nevertheless senses and enunciates the problem and he states strongly:

> "This means that an orderly exposition is not enough. Motivation and stimulation are essential."

He closes modestly by saying:

> "We have no simple remedy to propose for these difficulties. We have described them...merely to avoid suggesting by indirection that they can safely be ignored."

Let us add some positive considerations to the picture. The undergraduates who are considering (more or less firmly) going into elementary teaching are young. This may not be an unmixed blessing, but, hopefully, they have enthusiasm and energy, both of which they will need as teachers! Secondly, these undergraduates have a feeling that they like children and that they would like to work with children. In the third place, we are getting more of the capable undergraduates interested in teaching as a career. For some years past it has been taken for granted, and I will not cite Shaw's dictum about teaching, that the future teachers came from the lowest quartile of the graduating class. I think this is no longer true; I have the testimony of the Dean of the School of Education of the University of Illinois (and others) to support this.

These are three solid foundations on which we can build the education of the next generation of teachers.

If we can motivate and stimulate our students sufficiently, we can give them the preparation that they will need for their future work. Let me take one more suggestion from Professor Moise's note:

"It seems an unpromising proceeding to try to teach to such an audience a straight forward course in the foundations of mathematics...the students may not see the point of building careful foundations for a subject whose superstructure is neither known nor highly valued...a heavy emphasis on logical foundations may make it hard to find stimulating problem materials...(and this) is crucial...people learn mathematics by doing it."

What do future teachers need? Of course they need to understand the structure of the real number system, at least. But they must also know and appreciate that they will teach a broad spectrum of skills and understandings in a variety of disciplines and that they will be teaching children.

The renaissance that is taking place in education has affected all the disciplines—elementary science, social studies, language arts, even art and music. We are being told by the educators and research men in all these areas that the teacher of elementary school has to attain understandings in depth in every area. A modest demand by each field is that the elementary school teacher have the equivalent of at least a major in the field at the undergraduate level—but preferably the equivalent of a master's degree. How can the poor prospective teacher meet all these demands?—remember she also has to be able to help the children put on their rubbers!

Clearly, more is needed than merely subjecting the prospective teacher to the individual pressures and propaganda of competing fields. An interdisciplinary approach—from which we must not exclude either the psychologist or the professional educator—is indicated as a start. In the meantime, if instructors in mathematics of undergraduates heading for teaching kept in the foreground of their consciousness this situation—they might help a great deal. If they (the instructors) acquainted themselves with the programs and demands of the other fields, they might be on the lookout for sensible and helpful contacts that could be made with mathematics. It should be possible, at a minimum, to prepare problem materials that might provide these contacts.

Finally, let us say a word about the children who are expected to be the beneficiaries of this better education in mathematics. It may be unusual, if not heretical, to mention children in a conference on education, but in what way are the children supposed to be better off as a result of a better program in mathematics? Why do I raise the question? Because what teachers do in the classroom must be guided by what effects they are supposed to have on children.

The answer to the question about children is too often either assumed to be obvious, or the question is dismissed glibly by saying that they will learn better mathematics or that they are having a good time. As to the first, I don't think the answer is that obvious. As to the second, while there is nothing wrong with better mathematics or with children enjoying themselves, neither is a sufficient answer to the question raised.

1964

IMPLEMENTATION OF CUPM RECOMMENDATIONS

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For many human beings, change is a most painful experience. Moreover, in institutions of higher learning change is further complicated by the forces of traditionalism, institutionalization, misunderstanding, departmentalization, etc.

The implementation of four required mathematics courses recommended by CUPM for elementary teachers will necessitate institutional changes. Last year only five institutions in America had adopted the entire CUPM program, although several had adopted part of the recommendations. Since I represent one of these five institutions and since institutional change is such a tremendously complicated process, it seemed appropriate for me to share with you what we believed to be the significant forces in the change process at New Mexico State University. I would make it crystal clear that this description is not intended as a prescription for any other institution. Rather, it is hoped that in the identification of significant change processes and the forces affecting these processes, there might be a page or two of some value to another institution.

The first and probably most important process in the eventual adoption of the CUPM program at NMSU was the establishment of mutual trust and respect between the Mathematics Department and the College of Teacher Education. I doubt that many institutions will be adopting the CUPM recommendations if members of the mathematics area and members of the education area consider themselves to be "on different teams" within a university and "on different sides of the fence." This kind of immature behavior is a luxury (if indeed it is a luxury) which no institution can afford. Certainly the public schools cannot afford to have the personal likes and dislikes of university faculties prevent the achievement of excellence in the public school program. These campus differences have retarded public school progress for at least a century.

NMSU was no exception during the first century. It made its "contribution" to the prevention of public school excellence, but this is no longer the case. Listen to what one of the sixth grade teachers said:

> "Many teachers dislike mathematics because they do not understand it. This dislike is unconsciously transmitted to the children. Under confident, understanding teachers, children, instead of developing a dislike for the subject, develop an affection for it. I feel competent to teach mathematics and therefore enjoy teaching math."

What were the forces which contributed to the development of mutual trust and respect between the two groups?

I believe the first force was the insistence of public school personnel to improve the subject matter preparation of the elementary teacher. The second force was the involvement of the Head of the Mathematics Department in the improvement of the public school mathematics curriculum. The time and interest of Dr. Ralph Crouch, Head of the Mathematics Department, was a source of encouragement to public school personnel and to teacher educators.

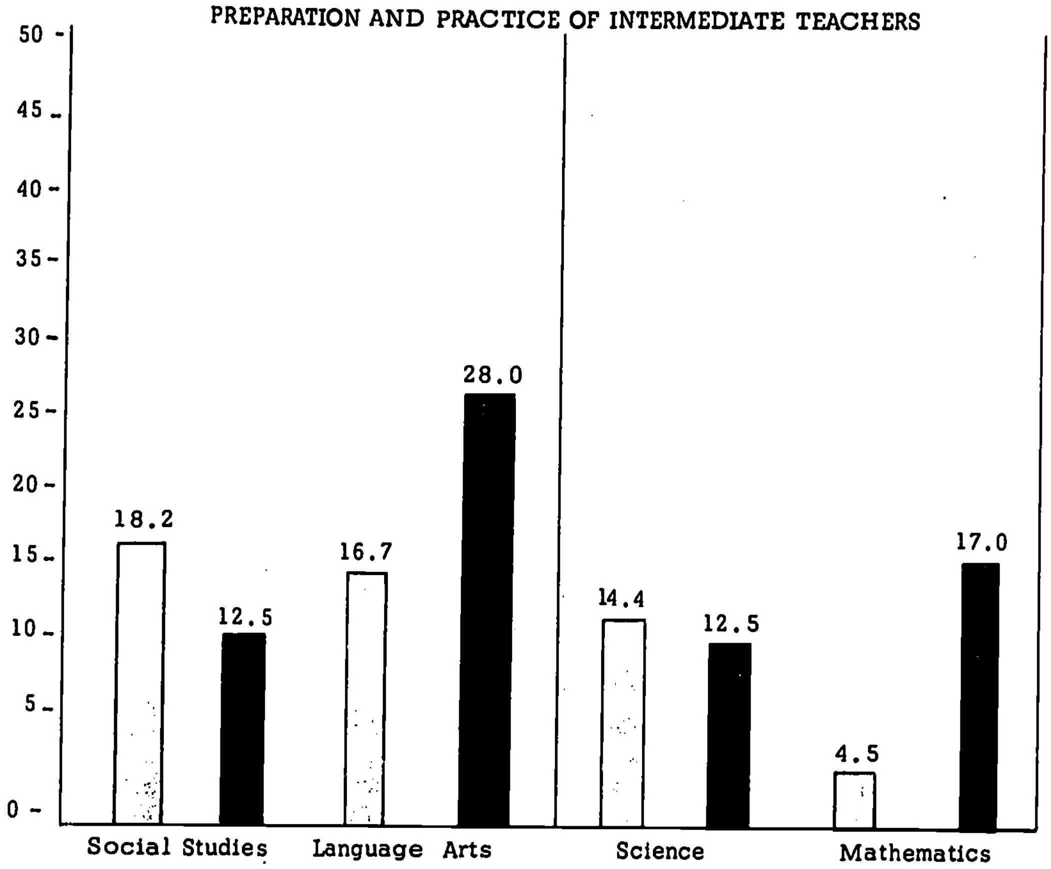
As a result of these two forces, i.e., the public school request and Dr. Crouch's involvement, a course was developed and taught by Dr. Crouch for the inservice education of teachers. The course was approved for either senior or graduate credit. Several members of the Graduate Council were reluctant to give graduate approval and their reluctance was quite understandable. However, in a professional school, there is perhaps more than one way to view graduate credit. If one holds to the usual viewpoint that graduate credit is something that follows a long sequence of undergraduate courses, then the mathematics course was not a graduate course. If, on the other hand, one considers that a professional person with years of experience in a profession might have considerable background as an elementary teacher for a course in the subject matter of the elementary school, then perhaps some justification is found for offering such a course. Also, when university personnel want a profession to adopt new rules for "playing the game," they must consider the possibility of "adopting new rules for the game" across the board. The fact is that no profession requires persons who have completed a professional program of preparation to return to a university for the purpose of completing additional undergraduate courses. In short, if the CUPM recommendations are to be adopted and available to teachers in service, some flexibility is mandatory. This flexibility, motivated by an urgency to improve the preparation program for elementary teachers, was another significant force in developing mutual trust and respect. It is also significant that success with a mathematics course for teachers in service became a force for adopting the CUPM recommendations in the preservice program. It was clear that if teachers in the field needed additional education in mathematics, then certainly students preparing for teaching needed additional education.

The commitment of the faculty of the College of Teacher Education to achieve excellence in the teacher preparation programs was a second significant change process. Changes in the curriculum were not limited to mathematics. Elementary teachers were required to complete all general education requirements of the College of Arts and Sciences. Moreover, three other Arts and Sciences Departments developed and taught a subject matter course for the elementary teacher. When we had "overhauled" the curriculum, we found that the minimum requirement in mathematics was indefensible. The requirements in the four main areas of subjects taught in elementary school were as follows: (See Chart I and Chart II on the following pages). The desire for a rigorous program and the obvious need to increase mathematics requirements became forces for adopting the CUPM program.

One of the first hurdles to be overcome was "finding room" in the curriculum for an additional six hours of mathematics. We, like any institution which adds credit hours to the required curriculum, had three alternatives: (1) reduce the number of elective credit hours; (2) reduce the number of required credit hours; or (3) increase the number of credit hours required for a degree. Reducing the number of required credit hours in our case would have meant removing courses in general education. This route was not chosen because of the commitment to a rigorous, superior program and because the politics of removing a requirement from one Arts and Sciences Department in order to add a

CHART I

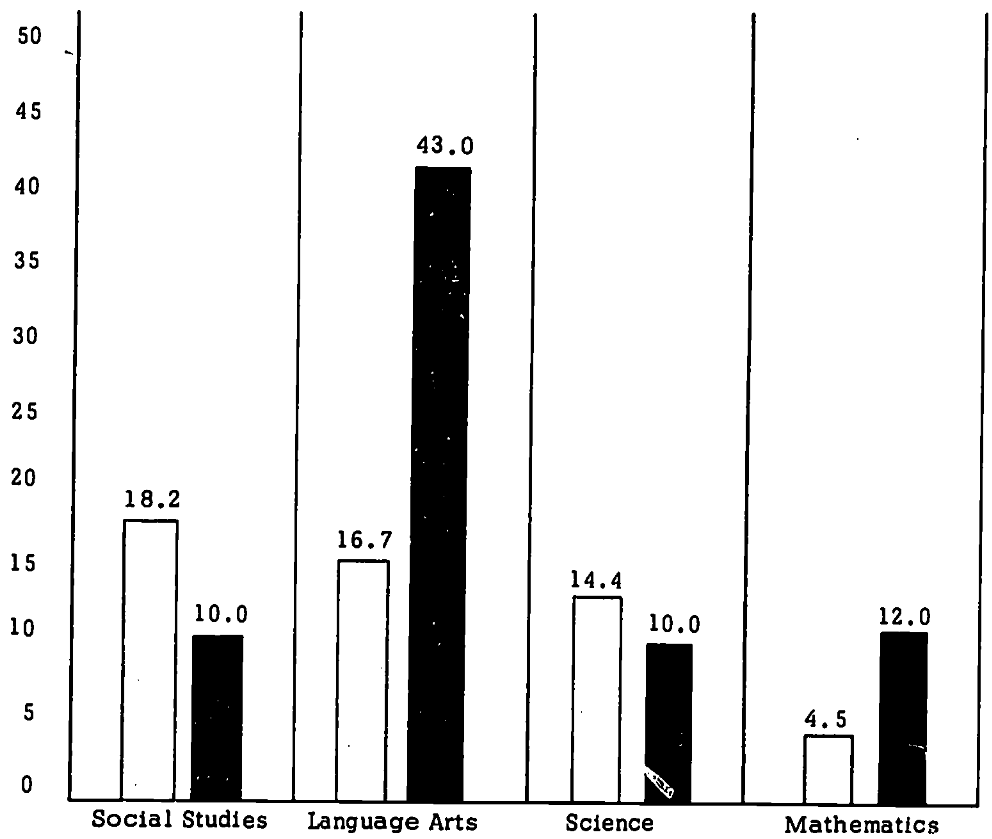
PREPARATION AND PRACTICE OF INTERMEDIATE TEACHERS



% of 132 semester hours. % of time devoted to subject by teachers.

CHART II

PREPARATION AND PRACTICE OF PRIMARY TEACHERS



% of 132 semester hours % of time devoted to subject by teachers requirement in another Arts and Sciences Department are incomprehensible.

The route chosen was the cowardly one--elective credit hours were reduced. Perhaps the route is cowardly, but it is, at least, immediate. It is conceivable that the CUPM recommendations would never be implemented on a campus if one were to choose the route of reduction in required credit hours--especially in general education. Maybe this, in fact, is really the cowardly route because it avoids the fact that elementary teachers are not receiving adequate preparation in mathematics.

I do not believe that "finding room" in the curriculum is the real problem because most curricula have ample electives, some of which could be used for implementing the mathematics program. Some of my colleagues admit this fact, but are leery of including more mathematics because of the demands being made on them from other disciplines as well. There are at least two answers to these demands: (1) each request to increase requirements should be considered on its own merit, and (2) each request should be evaluated according to the amount of time given to the particular subject in the elementary school. (See Chart I and Chart II.)

A third change process was experimentation and research. The Mathematics Department received a research grant to develop and test materials for the first two courses recommended by CUPM. In order to conduct the experiment, it was necessary to require a year of mathematics of freshman elementary education majors. During the second semester (or second course) it was reported that these students were getting "downright cocky" about mathematics. At the end of the semester, Dr. Crouch and I discovered a correlation between the failures in mathematics and the failures in Introduction to Education. The fact that the same people who failed mathematics also failed education, added to the other factors stated above, convinced the faculties of both areas that what had begun as an experiment should become a requirement. Moreover, the correlation between the two courses became another contributing force in the establishment and reinforcement of mutual trust and respect between mathematics and education.

A fourth change process was the development of enthusiasm among students who had completed the mathematics program. They were proud of their achievement and appreciative of their competency. Here is what some of them had to say:

> A Seventh Grade Teacher: "I would recommend the mathematics program to all seventh grade mathematics teachers because we do teach a lot of elementary mathematics."

> A Third Grade Teacher: "The concepts inherent in the 'new' method of mathematics were difficult for me to learn, as they were for many. A number of teachers agree with me now that this method, if learned in the initial stages of arithmetic training makes it easier for a child to grasp higher math. I believe the processes of this new math are utilized (and thereby retained better by the teacher) to much greater extent, in a level higher than 3rd grade, but even there the child is better able to understand why a math problem is set up the way it is, and can think through the logic involved in what is represented. This method of learning gives him

greater reasoning power, and he will eventually see more reason for studying math."

A Fourth Grade Teacher: "The math courses have been of inestimable value to me in teaching the 'new' arithmetic. Without these courses I would have been as bewildered as the parents to whom I've been able to explain the reasons behind the change in approach to arithmetic."

A Second Grade Teacher: "I took the courses to help me understand what concepts the primary child would need and why; in order for him to make continuous growth at each level. The courses did that and added to my own growth."

A Sixth Grade Teacher: "The courses in modern mathematics were especially helpful to me. I have a clearer understanding of the true purpose of the program as well as a better grasp of fundamental concepts that I have been attempting to teach."

A First Grade Teacher: "The math courses which I took gave me a better basic understanding of mathematics and would certainly have been a help in my teaching if I were a teacher in the upper grades. Actually the one-to-one correspondence and position of the numbers in relation to the story problem is about all I use in the first grade."

There are many other factors worth mentioning if time permitted. However, in summary, the four change processes described above brought about institutional changes which led to the adoption of the CUPM program. These were: (1) establishment of mutual trust and respect between mathematics and education; (2) the commitment to a preparation program of excellence; (3) experimentation and research; and (4) the development of enthusiasm and pride of students completing the mathematics program. I believe that the development of any one of these changes on a campus could result in eventual adoption of the CUPM program.

If you return to your state or to your university and find that a stalemate exists and changes in the mathematics program for elementary teachers seem hopeless, take heart, here are some "sure fire" suggestions:

(1) If public school personnel make their requests heard loud and long enough, eventually a university will meet these demands. Public school personnel can be very effective if they believe that teachers in tomorrow's schools need the CUPM program.

(2) Concentrate on one university in the state. If one school of the state adopts the CUPM program, others will soon do likewise. (A second institution in New Mexico adopted the 12-hour CUPM program last Spring.)

Finally, during the five-year development of the program for elementary teachers at NMSU, the following facts have been gathered:

(1) The mean percentile on the Culture I part of the Teacher Education Examination Program (TEEP) in 1960-61 was 49.0. In 1963-64, the mean percentile was 69.5. The mean percentile on the Culture II part (mathematics and science) in 1960-61 was 55.5. In 1963-64, the mean percentile was 71.0. The only change made in the curriculum during this period was in mathematics.

(2) Enrollment in the College of Teacher Education doubled.

I hope the story I have told will be of some value to you. Regardless of the problems and differences, I trust that each person charged with the responsibility of educating American youth will keep one central fact ever before him as improvements for educating teachers are considered: what happens to American education will most certainly happen to America.

1965

THE NEEDS OF THE CURRICULUM AND THE NEEDS IN TEACHER EDUCATION

Robert B. Davis
The Madison Project

My remarks are based upon one view of the future needs of the mathematics curriculum in the elementary school. I want to indicate briefly what this view is and why it implies a need to bring a higher level of mathematical expertise inside the walls of the elementary school house.

First, what are the needs of the elementary curriculum in the next few years? I would argue that they are primarily these:

1. An early introduction of the "big ideas" of mathematics.
2. Replacing "arithmetic" with a combination of arithmetic, algebra, geometry, and science.
3. Developing a more student centered approach to classroom instruction in mathematics.
4. A greater emphasis on student creativity and originality.
5. A more extensive use of physical materials in the mathematics classroom.
6. More emphasis on individualized instruction in mathematics.
7. Freeing the curriculum from its present bondage to grade levels and textbook series.
8. Teaching a great deal more mathematics in the elementary school than is ordinarily done at present.

These eight directions for change may require a word or two of explanation:

Early Introduction of "Big" Ideas. Nearly all modern curriculum studies, in any subject matter field whatsoever, usually advocate identifying the central "structural" ideas of a subject and introducing them to quite young children. The argument in support of such a course of action is simple: over several thousand years man has developed effective "basic" ideas that organize a large body of knowledge in a powerful and efficient way. If we teach these "organizers" to young children, they will make use of them in all of their subsequent learning. If we do not teach these "organizing" ideas, then either the child must discover them for himself—which requires the genius of centuries of Euclids and Newtons and Cauchys and Hilberts—or else the child must get along without them and fight his way through a dense forest of "facts" with nothing by way of a conceptual road map to help him maintain his bearings.

The "big" ideas of mathematics include such concepts as variable, open sentence, truth value of a statement, function, graph, the arithmetic of signed numbers, the relation of abstract models to physical reality, implication, contradiction, axioms and theorems, matrices, inference schemes, and so on. We know from various experimentation in recent years that all of the ideas mentioned above can be understood by elementary school children.[1]

Replacing "Arithmetic" with a Combination of Arithmetic, Algebra, Geometry, and Science. This is not entirely different, of course, from the preceding point. The argument in support of such a "unified" or "combined" program of study is primarily this: arithmetic, by itself, has little capacity for continuing growth. When, however, we combine it with algebra, geometry, and science, we have a curriculum that contains within itself the seeds for its own future growth and development. We expect the child to grow with each successive year; the teacher should also, and so should the school program. The school program at any grade level for 1967 should be a natural outgrowth of, and an improvement upon, the program at that same grade level that was taught in 1966. A unified program will tend to grow in this way, because it embodies a potential for growth. A straight "arithmetic" program does not embody such a potential for growth and will tend to stagnate. In a straight arithmetic program we would find our schools teaching the same things in 1970 that they taught in 1960, 1950, or even 1940.

Developing a More Student Centered Approach to Classroom Instruction. Much has been written lately about "discovery" teaching, about flexible sequencing of topics of study, and so on. At the core of all of this is probably the idea of interactive teaching, in which tomorrow's classroom discussion and study grow organically out of today's discussion and study. An example may help to show what this means. In a fifth and sixth grade "mixed" class in the Chicago Public Schools recently, some students made up the function indicated by the table

$\square$	$\triangle$
0	$^{-}3$
1	$^{-}2$
2	0
3	1
4	3
5	4
6	6
7	7
⋮	⋮

1. The best evidence I can cite in support of this claim is the collection of 16mm. sound motion picture films made by the Madison Project that show actual classroom lessons with children, K-9. These films are available from: The Madison Project, 8356 Big Bend Boulevard, Webster Groves, Missouri 63119.

Other students in the class had the task of finding an algebraic formula to represent this function.

Now, in fact, the table above came about from an unintentional systematic arithmetical error that involved neglecting remainders after dividing by two. What kind of class discussion should now ensue?

The teacher was familiar with the mathematical function, "square brackets,"

$$[X] = \text{the largest integer N such that } N \leq X,$$

and was also familiar with the creative work David Page has done in using this idea with young children. Hence, the class discussion was allowed to move in this direction, and the function in question was identified as either

$$\left[\frac{3 \times \square}{2}\right] - 3 = \triangle$$

or (if you prefer)

$$\left[\frac{\square}{2}\right] + \square - 3 = \triangle$$

This led to considerable valuable work in estimation, "rounding off," and so on, as well as an example of a new type of function.

If the teacher is to allow class discussion to proceed organically as a continuing inquiry, the teacher must have a broad knowledge of the mathematical terrain that is being explored.

Greater Emphasis on Student Creativity and Originality. Studying tape recordings of many classroom lessons makes it clear that teachers are confronted almost daily with student suggestions that are very clear and deserve consideration. Most of these student suggestions are ignored or rejected by most teachers or else accepted uncritically in a fashion that leads to homogenized chaos. An actual example may help (from a Weston, Connecticut, third grade class):

The third grade teacher was explaining, "I can't take 8 from 4...

$$\begin{array}{r} 64 \\ -\underline{28} \end{array}$$

...so I regroup the 60 as 50 + 10..."

Kye, a third grade boy (who had learned the arithmetic of signed numbers in the second grade), interrupted:

"Oh, yes, you can! Four minus eight is negative four...

$$\begin{array}{r} 64 \\ -\underline{28} \\ -4 \end{array}$$

...and 20 from 60 is 40...

$$\begin{array}{r} 64 \\ -\underline{28} \\ -4 \\ 40 \end{array}$$

...so the answer is 36...

$$\begin{array}{r} 64 \\ -\underline{28} \\ -4 \\ \underline{40} \\ 36 \end{array} \quad ."$$

This particular teacher recognized the merit of Kye's method, and some of the children in the class went on to study it.

Kye was lucky. In many classes the teacher would have said, "No, Kye, that's not the way you do it. Now watch carefully, and I'll go through it again."

In such a "traditional" class, Kye would then have been left with an unresolved conflict between his own mental picture of how mathematics works out versus the authoritarian pronouncements of the teacher. For some children this might be a challenge, but for others it would be more than they could cope with.

There is no need here to continue this catalog of what is versus what might be. All of the proposed changes in the curriculum will tend to require more mathematical knowledge from elementary school teachers—in some cases, very much more. Rather than present an argument couched in the unconvincing rhetoric of educational English, I want to plead with the interested to view a few Madison Project films, showing actual classroom lessons. Is it not true that these children are enjoying themselves more than most children do in most schools? Is it not true that they are learning much more mathematics than children their age usually do? And is it not true that one indispensable step toward bringing this sort of thing to many more children is to educate more teachers who are flexible and creative, who conduct a student centered classroom, and who are at home in the world of mathematics?

If so, our course is clear.

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THE NASDTEC-AAAS STUDIES

The following is a report on four studies, conducted by the National Association of State Directors of Teacher Education and Certification and the American Association for the Advancement of Science, which constitute a part of the current programs in improving the quality of our educational system. This report is a composite of speeches given by the following persons at one or more of the ten CUPM Level I conferences held in 1963:

Louise Combs, Kentucky State Board of Education

J. T. Kelley, Florida State Department of Education

Wayland W. Osborn, Iowa State Department of Public Instruction

William P. Robinson, Jr., Rhode Island State Department of Education

William P. Viall, Western Michigan University

N. Blaine Winters, Utah State Department of Education

The joint studies of NASDTEC and AAAS began in December, 1959, under a grant from the Carnegie Corporation of New York and were completed in August of this year. There have been four studies, the first of which produced Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics published in September, 1961.[1] This document is concerned mainly with the subject matter preparation of secondary teachers of science and mathematics, because too little attention has been given to content by regional and national accrediting bodies. Sections of the publication are assigned to biology, chemistry, physics, physical sciences, junior high school science, earth and space science, and mathematics.

It should be noted that the Guidelines are recommendations—not an outline for a national curriculum. The Guidelines say:

1. Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics, 1961. National Association of State Directors of Teacher Education and Certification in cooperation with the American Association for the Advancement of Science, Washington, D. C.

It is not necessary that all colleges adopt a uniform pattern of organization for providing the subject matter preparation for the science teacher. Some institutions with a divisional organization may offer a curriculum for the preparation of science teachers through a single division. Others, with a departmental pattern of organization, may offer individual teaching majors through separate science departments, such as physics, chemistry, biology, etc. Whatever the type of organization, the subject matter portion of the teacher's preparation should constitute a pattern carefully planned in accordance with these Guidelines.

In response to a questionnaire sent to state directors of teacher education and certification throughout the country during the spring of 1963, it was reported that the Guidelines had been adopted totally or in part by a Teacher Education Council, State Board of Education, State TEPS Commission, State Higher Education Board, or a similar body having control of or influence on the state teacher education program in the following 14 states and Puerto Rico: Arizona, Arkansas, Georgia, Indiana, Kansas, Louisiana, Mississippi, Nebraska, New Mexico. North Carolina, Oregon, Pennsylvania, Utah, and West Virginia. "Adoption" does not mean word for word compliance with the Guidelines, but rather a reasonable reflection of the recommendations expressed in it.

In addition, 12 other states: Alabama, Colorado, Connecticut, Illinois, Maryland, Massachusetts, Montana, Nevada, New York, Oklahoma, Rhode Island, and Vermont signify "approval" by these bodies, which, although somewhat less effective will undoubtedly have influence on states and institutional programs. In states where adoption or approval has not yet taken place, several directors of teacher education and certification held statewide meetings or meetings with faculties to study the Guidelines with a view toward subsequent adoption. Still others reported that although no progress had been made to date, progress was expected during the next year. Lack of progress was often due to changes in administration in a state education department. In at least one state the autonomy of its several teacher education institutions prevented the director from working closely enough with the institutions to effect any change. However, even in this state two of the institutions are known to be studying the Guidelines.

Another count revealed that at least 170 institutions throughout the country had adopted the Guidelines. It is believed that there are additional institutions who have adopted them but had not reported the action to their state education departments by the time the questionnaire was returned. Nebraska and North Carolina made considerable use of the Guidelines in preparing their own guidelines for a number of subjects, including science and mathematics.

The adoption, approval, and use of the Guidelines to this degree so soon after their publication in September, 1961, reflects not only the urgency and importance with which institutions and the states regard the raising of educational standards but also reflects the considerable impetus given to the study by the use of the technique of asking large numbers of persons to review the Guidelines prior to publication. It also demonstrates the support and publicity given to them by the American Association for the Advancement of Science and by many of its members. In addition, the American Association of Colleges of Teacher Education requested several of its institutions to review the Guidelines while the study was in progress. The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America invited a NASDTEC member to speak on the Guidelines at each of its nine regional meetings prior to publication. Several staff members from the National Science Foundation and the United States Office of Education helped in constructing, publicizing, and distributing copies of the Guidelines.

In September, 1961, the NASDTEC-AAAS Studies undertook a study of the mathematical and science preparation in elementary school education programs. Involved in this study, supported by the Carnegie Corporation of New York (like the one for secondary teachers), were elementary, high school, college, and university teachers and administrators; scientists and mathematicians from higher education, government, and industry, and a number of representatives from various agencies, associations, and groups. They participated in regional conferences in Chicago, Salt Lake City, and Washington, D. C. in the spring of 1962.

In the year that followed, the compiled reports of these regional conferences were reviewed by thousands on college campuses and in conferences organized by state education departments and professional and scientific societies. The study was completed this past summer with the publication of Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers.[2]

The Elementary Guidelines read:

GUIDELINE I: The faculty of each institution should design its program for the preparation of the elementary school teacher after careful analysis of the role of (1) the elementary school in American society, (2) the elementary school teacher, and (3) the institutions preparing teachers.

GUIDELINE II: The program of preparation for the elementary school teacher should include a broad general education with attention to human growth, learning, and behavior.

2. Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers, 1963. NASDTEC-AAAS Studies, 1515 Massachusetts Avenue, N. W., Washington, D. C. 20005.

GUIDELINE III: Instruction in science and mathematics should be conducted in ways that will develop in teachers an understanding of processes and in scientific inquiry and mathematical thinking.

GUIDELINE IV: The program of preparation for the elementary school teacher should include breadth of preparation in the sciences and in mathematics most appropriate as background for the elementary school program, with emphasis on concept development and interdisciplinary treatment.

GUIDELINE V: The program of preparation for the elementary school teacher should include study of the aims and methods of teaching science and mathematics in the elementary school.

GUIDELINE VI: Professional laboratory experiences, including observation and student teaching, should provide opportunities for the prospective teacher to work with experienced elementary school teachers who are competent in the subject area, skilled in nurturing the spirit of inquiry, and effective in helping children benefit from the study of science and mathematics.

GUIDELINE VII: The program for the preparation of the elementary school teacher should provide opportunities for pursuit of additional undergraduate study in a carefully planned program in science and mathematics.

GUIDELINE VIII: Fifth year and sixth year programs for the elementary school teacher should offer appropriate science courses and mathematics courses which might be applied toward an advanced degree.

GUIDELINE IX: Inservice education should provide opportunities for the elementary school teacher continually to improve and extend the competencies required for effective teaching of science and mathematics.

Following each Guideline, the recommendation is developed in some detail. For example, the section on mathematics in Guideline IV reads in part as follows:

The Recommendations of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America for the preparation in mathematics of prospective elementary teachers are strongly endorsed in principle. The amount of time needed to satisfy the CUPM Recommendations is dependent upon the ability and previous preparation of the teacher candidate and will vary from student to student. Programs should be based on at least two years of college preparatory mathematics and more if feasible."

Then there are brief descriptions of essential mathematical preparation of the elementary teacher.

The key to the improvement of any program in the school curriculum is surely the teacher. This is so commonly recognized that we can be sure of unanimous agreement. Inservice education is the best way of improving or changing teachers. Inservice education is a continuing need if we are to do away with obsolescence and replace it with more significant, currently proven, methods.

Now what can we do about it?

1. We must encourage school districts and higher institutions to provide methods of "retreading" teachers.

2. We must encourage school districts to revise curricula.

3. We must encourage teachers to visit classes in school districts where contemporary materials and methods are being used. Districts must be asked to furnish substitute teachers so that regular teachers may be released to make such visits.

4. We must encourage teachers to join their professional associations in mathematics or science and actively participate in programs and projects sponsored by their professional associations.

5. We must encourage teachers to ask for assistance from consultants in their field.

6. Higher institutions must be encouraged to offer extension courses on a level in which newer methods can be learned and newer materials can be used.

7. We must encourage administrators in local school districts to use more discretion in the employment and assignment of teachers.

8. We must encourage teachers to refuse to accept assignments in areas where they have inadequate training.

9. We must work cooperatively with directors of teacher education and certification to the end that a revised pattern of preparation for teachers on the elementary and secondary levels may be attained.

Some of these items will require additional expenditures of funds, but it must be remembered if we are to secure the type of teaching needed, we must be willing to pay for it.

Without adequate numbers of well prepared teachers who can interest and inspire young people, we cannot have quality education in the area of mathematics, science or any other field. Without good education, we cannot hope to meet the challenges of a rapidly changing world in which we are witnessing an explosion of knowledge.

We must also be constantly aware that neither the amount of time spent on a subject nor the size of the home assignments given to pupils can be used as a measure of quality of a program. Many teachers because of the caustic criticism directed at education have sought to improve the quality of their courses by "pouring it on" or "getting tough." This is not the kind of change we want for providing improvement.

A third project of NASDTEC-AAAS Studies,[3] a survey, the report of which was published by the National Science Foundation, concerns itself with qualification and service loads of teachers of secondary science and mathematics in the United States.

The sample was chosen by stratified random sampling from the U. S. Registry of Junior and Senior High School Science and Mathematics Teaching Personnel, 1960-61, compiled by the National Science Teachers Association and containing names, school addresses, and other information for some 142,000 school teachers in public and private schools, grades 7-12, across the country. The stratification covered proportional representation by geographical region, size of school, grade levels within the school and number of classes in science and mathematics taught by the individual teacher. The sampling ratio of 1 in 37 produced a total sample of 3,957 teachers. Seventy-six per cent returned usable questionnaires.

3. Secondary School Science and Mathematics Teachers: Characteristics and Service Loads, NASDTEC-AAAS Studies. Order from Superintendent of Documents, Government Printing Office, Washington, D.C. (Catalog No. NS 1.2: T22/2) $.35, 1963.

Half the teachers in the study were less than 35 years old—only a quarter 45 years old or older. Over half received their bachelor's degrees in 1950 or later. All but one per cent have earned bachelor's degrees.

Most have courses beyond the baccalaureate. Twenty-nine per cent hold the master's degree; a fraction of one per cent have the doctorate; over seventy-five per cent have at least ten hours graduate credit; and many have substantial credit above the master's. However, only forty per cent have credit beyond the baccalaureate in the subjects they teach.

Salaries ranged from below $3,000 to over $10,000, with the national median in the range of $5,000-$5,499. The regions vary considerably in salary—the median for the South is below $4,500, and for the West it is above $5,500.

Teachers have considerable opportunity to be full-time teachers of mathematics and of general science—but not biology, chemistry, or physics. Only twelve per cent of the science teachers teach physics and four out of five of them have only one or two classes in physics.

When all five subjects—biology, chemistry, physics, general science, and mathematics—are combined under the heading of science teaching, much over half of the science and mathematics teachers teach in these fields exclusively. Thirty per cent of all science and mathematics classes are being taught by teachers who give some or even most of their teaching to nonscientific and nonmathematics subjects.

If 18 semester hours credit is minimally adequate preparation to teach a subject, twenty-three per cent of the mathematics classes in grades 9-12 and fifty-three per cent of the mathematics classes in grades 7 and 8 are taught by inadequately prepared teachers. Teachers of mathematics in grades 7 and 8 usually have less than 9 semester hours in mathematics.

How close our survey can be related to the qualifications of teachers of such other areas as English and the social sciences I cannot guess, but, until I see some research to the contrary, I shall regard it as dangerous to assume that teachers in general in our secondary schools have more preparation in subject matter for the classes they teach than do secondary science and mathematics teachers. We have a massive job to do.

The fourth NASDTEC-AAAS project was a series of nine regional conferences for school administrators around the country last year. A report of these conferences was published recently under the title, The New School Science.[4]

4. The New School Science: A Report to School Administrators on Regional Orientation Conferences in Science, 1963. NASDTEC-AAAS Studies, 1515 Massachusetts Avenue, N. W., Washington, D. C. 20005.

These conferences, financed by the National Science Foundation, brought information about the new science curricula for high schools to administrators so that they would be better informed in making judgments about the introduction of new courses into their schools. About one hundred and fifty administrators were invited to each conference where they were addressed by scientists, science educators, and other administrators. Teams of college and high school teachers discussed and demonstrated the new courses and materials. Fifty thousand copies of the report of these conferences have been mailed to administrators and those who work with them through lists furnished by the American Association of School Administrators, the National Association of Secondary School Principals, the American Association of Colleges for Teacher Education, and the National Commission on Teacher Education and Professional Standards.

The Mathematical Association of America can be proud of its Committee on the Undergraduate Program in Mathematics, which has been a strong force in the forward movement of teacher education. That it has been able to conduct itself with such strength and yet such discretion during a time when tact is the essence of progress is a credit to all its members. I thank you for NASDTEC and for myself for the opportunities we have had to work with CUPM in achieving, within such a short period of time, a certain prospect of better education for the youth of America in the field of mathematics. Although the NASDTEC-AAAS Studies have been formally closed, you may be sure that NASDTEC as an organization and its members, as individuals with heavy responsibilities in education, will look to you for help in the years ahead.

THE RESULTS

The participants

Each conference included representatives from the mathematics and education departments of those colleges and universities in the state which were involved in elementary teacher training. Officials from the state department of education, school administrators, and classroom teachers also participated. Invitations were extended to individuals on the basis of interest or involvement in the mathematical training of elementary teachers. A total of 2,376 people attended the 41 conferences. A breakdown of this total gives the following distribution of participants:

	Number	Per cent
Representatives of mathematics departments	949	39.9
Representatives of schools of education	512	21.5
State department of education officials	226	9.5
Teachers and school administrators	689	29.0

The discussions

Reviewing all forty-one of the conferences one finds some patterns in the kinds of questions discussed. Different aspects of the problem were stressed in different states, but there were certain fundamental questions for which concern was displayed in all of the states. A gradual change in emphasis over the four years is also apparent: discussions of ways of improving the recommendations, which occupied a good share of the time in earlier conferences, gave way to attention to problems related to the implementation of the recommendations.

The overall reception given the Level I recommendations, generally favorable from the beginning, became even more enthusiastic as the awareness of the need for adequate training became more widespread. All were agreed that the elementary teacher must be given the kind of mathematical training which will prepare him to meet the demands of the new mathematics programs, and that doing this will require considerably more time than has been allotted to mathematics in the past. Some participants expressed concern over the possibility that the CUPM recommendations are too modest and that the amount of mathematics recommended is not enough to meet the needs of the near future.

Everyone agreed that it was desirable for each student going into elementary education to have had at least two years of college preparatory mathematics in high school. Most felt that it was not unreasonable to require this much mathematics for entrance into the elementary education program. The exceptionally well prepared students, it was felt, should be encouraged to take a program of the sort recommended by CUPM for Level II.

Although there was general agreement that the Course Guides provide a reasonable program for achieving the goal of the Level I recommendations, some of the details were questioned at first. By the time the Guides had undergone their third revision, however, the participants found little to criticize in them. The most serious fault remaining in the Guides, apparently, is a lack of clarity as to the level of rigor of the proposed courses. (The Panel intends to rectify this deficiency by revising the Guides in order to emphasize the fact that elementary school teachers need a very careful but highly intuitive approach to mathematics.)

The problem of fitting 12 hours of mathematics into the already overcrowded elementary education curriculum was the subject of a great deal of discussion at each conference. Although there was a strong feeling that mathematics warrants a larger share of the time in the elementary education curriculum, the immediate installation of a 12 hour requirement was considered unfeasible at many colleges: for the present, 6 to 9 hours were considered all that could be fitted into the required program. Two methods of working the full 12 hour requirement into the program were discussed extensively. One was borrowing from the student's elective time, and the other was the adoption of a five year program. In any event it was generally agreed that all of the recommended courses should be available to the student at least as electives.

At most conferences in the Level I series the use of mathematics specialists was proposed as a means of improving the quality of mathematics instruction in the elementary schools. The suggestion often gave rise to heated and lengthy debates, and usually no clearcut concensus was established on the question.

A major impediment to the implementation of the Level I recommendations is the shortage of competent people with whom to staff the proposed courses. This problem seems to be a basic difficulty for all mathematics departments. It is an especially serious one because the success of such a program depends so much on the person teaching it. In the past the mathematics courses for elementary teachers evoked little interest on the part of mathematicians and as a consequence were often poorly planned and poorly taught. Fortunately, the mathematical community has taken increased interest in these courses during recent years. There was general agreement that these courses should be taught only by able teachers who are mathematically capable and who at the same time have an interest in, and sympathy with, the problems of teaching mathematics in the elementary school.

The lack of textbooks suitable for the Level I courses is no longer as great a problem as it was when the Level I series of conferences began. Since that time a number of books having content resembling the Level I recommendations have been published, but the choice of textbooks having the content and spirit of the algebra and geometry recommendations is severely limited. (In order to assist colleges in evaluating Level I textbooks, the Panel made a survey of commercial textbooks intended for the undergraduate education of elementary school teachers. The results were published in November, 1965, in the CUPM pamphlet Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers.)

At many of the conferences the question was raised whether mathematics courses especially designed for elementary teachers might serve to satisfy a general education requirement as well. Opinion was divided on this. There was some agreement, however, that the number systems course would be suitable for general education.

During the last half of the series of conferences, methods courses were frequently discussed, although this subject was outside of the conference agenda. Serious concern was expressed about the adequacy of the methods courses being given in the elementary curriculum. There was no general agreement as to what the nature of these courses should be. Many of the participants expressed the hope that the Panel on Teacher Training would come up with recommendations on this question.

At most conferences the participants considered the inservice training of elementary teachers sufficiently important to warrant discussion even though this, also, was not on the agenda. Participants generally agreed that while necessary and valuable, such training is at best only a holding operation. They felt that our major efforts should be toward graduating new teachers who are not in immediate need of help.

It was considered important that a continual effort be made to achieve the goals set by the participants for their states. At a great many of the conferences, therefore, machinery was set up for forming committees to pursue follow up activities. Among these activities was disseminating information about the conference throughout the state's educational community. Most follow up committees made special efforts to ensure that high school principals and counselors learn of the CUPM recommendations and the conference resolutions.

The resolutions

At each conference, a set of resolutions was passed expressing the sentiments of the participants. There was no particular pattern for these resolutions, but a number of expressions of feeling were common to many of the sets.

Forty-two of the states gave their unqualified endorsements of the Level I recommendations. The following is an example of such a resolution:

> BE IT RESOLVED that this Conference does hereby declare its endorsement of the CUPM recommendation for twelve semester units of instruction in the number systems, algebra, and geometry for elementary school teachers, and its belief that the implementation of this recommendation involving only

eight per cent of the college course of study is essential as a minimum to ensure adequate instruction in elementary mathematics.

(California, December 11, 1962)

Many of the resolutions were directed toward the colleges and universities in the state, some suggesting timetables for the implementation of the proposed Level I program. Here is one of these:

It is recommended that each college preparing elementary teachers should scrutinize its program for elementary teachers looking forward to the eventual adoption of a required program of twelve semester hours of mathematics reflecting the approach and essential content of the CUPM Level I Recommendations and Course Guides. To do this, the following timetable is suggested:

By September 1964—The requirement of six semester hours of mathematics which will give the prospective elementary school teacher the background incorporated in Course (A) of the above mentioned Course Guides, the structure of the real number system, as a minimum.

By September 1965—The requirement of nine semester hours. The first six hours to be the above mentioned Course (A), and three hours to follow in either Course (B), the basic concepts of algebra, or Course (C), informal geometry.

By September 1966—The requirement of twelve semester hours which incorporates Courses (A), (B), and (C). Courses (B) and (C) may be taken in either order, after the completion of (A).

(Pennsylvania, October 5, 1963)

About half of the sets of resolutions contained specific instructions to the state boards of education regarding the action the conference felt it should take. Typical among these was this one:

The State Departments of Education should anticipate making the Level I courses a certification requirement for the elementary school teacher as soon as is administratively feasible. The Department of Education should encourage a step by step approach via elective courses in the interim.

(Delaware, Maryland, and District of Columbia, October 1, 1963)

Some of the resolutions indicated concern about the high schools' contributions to the mathematical training of elementary teachers. One such is the following:

> We recommend that potential college students in North Carolina be advised to take strong mathematics courses in high school, these courses to include at least one year of algebra and one year of geometry. The quality of college preparation in mathematics for future elementary teachers clearly depends on the suitability and strength of their precollege mathematics.
>
> (North Carolina, November 9, 1965)

In a few cases the conferees registered their belief that some of the elementary teachers should have more than the minimum training recommended for Level I. Here is a recommendation of this sort:

> We further recommend that those students who have unusually strong high school preparation be encouraged to take work along the lines of the CUPM Level II courses. Such persons would strengthen the elementary program by their presence within the school faculty.
>
> (Oregon, October 13, 1964)

Follow up activities of various sorts were recommended at the majority of the conferences. Most often committees were planned to carry out these activities, which often included follow up conferences. Here is an example:

> We recommend that the Directors of Certification of North and South Dakota form a steering committee consisting of themselves plus six other people, three from each state, each three to consist of an elementary teacher, a member of a department or college of education, and a professional mathematician from a college or university in the given state. The purpose of this committee shall be to attempt to coordinate the implementation of the recommendations and resolutions passed by the states of North and South Dakota at this conference, and to continue to study the problems of mathematics education in the two states.
>
> (North and South Dakota, October 30, 1965)

At a few of the conferences resolutions were passed concerning the teaching of the Level I courses. Typical of these is this one:

It is especially important that such courses be taught by persons who are mathematically competent, who are acquainted with the problems of the elementary teacher, and who, by example, can demonstrate a high level of expository skill.

(Washington, October 15, 1963)

A number of other aspects of the mathematical training of elementary teachers were topics of resolutions, including selection of textbooks, retraining of inservice teachers, methodology, and mathematics for general education.

The effects of the conferences

From time to time studies have been made to determine what sort of mathematical training our undergraduate elementary education majors were receiving. Until recently the results have revealed that the situation was indeed a sorry one. There are indications, however, that the conditions are changing for the better.

A study made in 1957-58 showed that of 96 colleges and universities selected at random which educated elementary teachers, about 39 per cent required no mathematics, 5 per cent required a combined content and methods course, and 56 per cent required a mathematics course of 2 or 3 semester hours. This course was described as one concerned mostly with the uses of mathematics.*

By the time the Level I series began, and two years after the appearance of the CUPM Level I recommendations, the situation in the colleges had improved somewhat. In the fall of 1962 CUPM made a study of the state of the undergraduate mathematics programs for elementary teachers. The data obtained show that of the 762 colleges which reported, 173 or 22.7 per cent of them required no mathematics, while 31.8 per cent required 5 or more semester hours. Only about half of the colleges requiring some mathematics designed their courses especially for preservice elementary teachers.

In January, 1966, upon the completion of the Level I conference series, the Teacher Training Panel initiated another study of the Level I situation. Each college in the country was asked how many semester hours of mathematics (exclusive of mathematics methods courses) it required of its elementary education majors for graduation as of January 1, 1966. Replies were received from

* Arden K. Ruddell, Wilbur Dutton, and John Reckzeh, "Background Mathematics for Elementary Teachers," Instruction in Arithmetic. Twenty-fifth Yearbook. Washington, D.C.: National Council of Teachers of Mathematics.

901 of the 909 colleges known to be involved in elementary teacher training. A summary of the results of the 1962 and 1966 CUPM studies may be found in Appendix A.*

A comparison of the results of the 1962 and 1966 studies shows continued improvement in the Level I situation. The number of colleges requiring no mathematics of elementary education students dropped to 75, or 8.3 per cent of the total, in this four year period. At the same time the percentage of colleges requiring 5 or more semester hours rose to 50.2 percent. In January, 1966, colleges required 4.6 semester hours of mathematics, on the average, compared to an average of 3.4 semester hours in 1962.

There are other encouraging facts not revealed by these figures. Many colleges revealed that they are planning to increase their requirements soon, while others pointed out that in addition to their required courses they offer elective courses patterned after the Level I recommendations. Even the fact that some of the correspondents indicated acute embarrassment in reporting how little mathematics their colleges require is a hopeful sign.

Meanwhile, the follow up committees formed at the Level I conferences have been active. Many of the states have held follow up conferences in order to continue their discussions of the problems connected with implementing the Level I recommendations. In several instances, improvements in the state certification requirements or in the curricula of individual colleges can be attributed directly to the efforts of these committees and conferences.

It appears, then, that progress has been made. But these efforts at reform have been directed at an area which has long been neglected. This neglect is ending, but there are still very difficult and very challenging problems to overcome.

* The results of the 1962 and 1966 studies are given in more detail in CUPM's report "A Study of Mathematics Requirements for the Preparation of Elementary School Teachers."

APPENDIX A

In the fall of 1962 and again in January of 1966 CUPM conducted studies of the amounts of mathematics colleges are requiring of their elementary education majors. Tables 1, 2, and 3, below, summarize the results of these studies.

Table 1

Number of semester hours of mathematics required of elementary education majors for graduation by American colleges:

	Number of Colleges		Percentage of Colleges	
Hours	1962	1966	1962	1966
0	173	75	22.7	8.3
1-2	39	29	5.1	3.2
3-4	308	345	40.4	38.3
5-6	209	339	27.4	37.6
7-8	17	63	2.2	7.0
9-10	11	40	1.4	4.4
11-12	5	10	0.7	1.1
Total	762	901	99.9	99.9

Table 2

Changes in the number of semester hours of mathematics required by colleges of elementary education majors for graduation over the period 1962-1966:

Change in hours	+12	+10	+8	+7	+6	+5	+4	+3	+2	+1	0	-1	-2	-3	-4	-5	-6
Number of colleges	2	1	3	2	40	19	27	152	41	34	343	9	9	34	2	3	4

Table 3

Number of semester hours of mathematics required by colleges of their elementary education majors for graduation—average, by states.

	1962	1966		1962	1966		1962	1966
Ala.	3.5	3.8	Ky.	2.0	5.9	N.D.	3.7	5.5
Alas.	6.0	7.0	La.	6.3	7.8	Ohio	2.9	3.5
Ariz.	3.0	2.0	Me.	2.7	7.0	Okla.	3.2	4.8
Ark.	3.2	5.3	Md.	6.1	6.3	Ore.	3.0	4.8
Cal.	2.1	4.0	Mass.	2.0	3.9	Pa.	2.9	3.6
Colo.	1.1	1.5	Mich.	2.5	3.2	R.I.	4.0	3.8
Conn.	4.4	6.0	Minn.	2.4	2.8	S.C.	3.4	5.0
Del.	4.5	4.5	Miss.	4.2	3.6	S.D.	1.7	3.5
D.C.	-	5.2	Mo.	3.7	3.5	Tenn.	4.2	5.4
Fla.	4.1	5.4	Mont.	3.4	4.4	Tex.	3.3	4.2
Ga.	4.2	5.9	Neb.	1.8	3.0	Utah	1.5	1.8
Ha.	4.5	4.0	Nev.	2.0	6.0	Vt.	5.2	6.7
Ida.	3.0	4.5	N.H.	6.0	9.0	Va.	5.0	6.6
Ill.	4.2	4.8	N.J.	4.6	4.9	Wash.	1.6	2.8
Ind.	4.4	8.0	N.M.	5.3	5.7	W.Va.	3.3	3.4
Ia.	2.0	3.4	N.Y.	4.4	4.8	Wisc.	3.9	4.8
Kan.	1.5	2.5	N.C.	3.5	5.8	Wyo.	0.0	6.0

APPENDIX B

Listed below are the CUPM publications which are currently available. The starred items are ones which are relevant to the mathematical training of elementary teachers. Copies of any of this material may be had at no charge by writing to CUPM, Post Office Box 1024, Berkeley, California 94701.

- *Report No. 1--Five Conferences on the Training of Mathematics Teachers (1961)
- Report No. 8--Annual Report, August 1962-August 1963
- *Report No. 9--Ten Conferences on the Training of Teachers of Elementary School Mathematics (1963)
- Report No. 10--Annual Report, August 1963-August 1964
- Report No. 12--Annual Report, August 1964-August 1965
- *Report No. 13--Eleven Conferences on the Training of Teachers of Elementary School Mathematics (1966)
- Report No. 14--Annual Report, August 1965-August 1966
- *Report No. 15--Forty-one Conferences on the Training of Teachers of Elementary School Mathematics
- *Recommendations for the Training of Teachers of Mathematics (Revised 1966)
- *Course Guides for the Training of Teachers of Junior High and High School Mathematics (1961)
- *Course Guides for the Training of Elementary School Mathematics (Fourth Draft, 1964)
- *Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers (Revised 1965)
- A General Curriculum in Mathematics for Colleges (1965)
- A Curriculum in Applied Mathematics (1966)
- Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists (Revised 1967)
- Recommendations on the Undergraduate Mathematics Program for Work in Computing (1964)
- Mathematical Engineering: A Five Year Program (1966)
- Pregraduate Preparation of Research Mathematicians (Revised 1965)
- Preparation for Graduate Study in Mathematics (1965)
- Tentative Recommendations for the Undergraduate Mathematics Program for Students in the Biological, Management, and Social Sciences (1964)
- Qualifications for a College Faculty in Mathematics (1967)
- CUPM Basic Library List (1965)
- *Teacher Training Supplement to the Basic Library List (1965)
- R. W. Hamming: Calculus and the Computer Revolution (1966)
- T. E. Hull: The Numerical Integration of Ordinary Differential Equations (1966)