Game theory may be a fruitful basis for educational theory. After describing games and related concepts like "utility," the author sets up examples of how games may be used as a basis for selecting teaching strategies. For instance, a teacher may decide to change many aspects of his teaching style, rather than just one at a time, if there is a greater probability that the changes will affect his pupil's performance. (JK)
PRELIMINARY WORK:
AN EDUCATIONAL THEORY BASED ON GAME THEORY

Educational Theory Center
Occasional Paper 64-170
As a requirement for Education 800V, a seminar in theory construction offered by E. S. Maccia in the spring of 1964, L. H. Chapman* presented her preliminary work. This work is of outstanding merit, and so was made a part of the occasional paper series.

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I. INTRODUCTION

Game theory is an applied mathematical theory which can be utilized with respect to interest conflict and its resolution. Interest conflict is implicated in situations where one of several possible outcomes will obtain and a person has certain personal preferences among them. The person confronting the situation may or may not have full control over all of the variables influencing the outcome, but he is motivated to resolve the situation according to an intended outcome, which could be viewed as maximizing benefits to himself in any event.

It is obvious that interest conflict is present in teaching; if not between teacher and pupils then between the alternative ways of structuring the game of learning which are open to the teacher. Since the teacher can be assumed to intend a certain outcome, teaching seems to involve resolution of conflict through decision. It seems reasonable to consider game theory a model for educational theory.¹

The strength of formal game theory rests with a group of assumptions summarized in the concept of rational behavior. Players are assumed to act on the assessed consequences of a choice, to be fully informed about contingencies, and to know the probabilities of outcomes associated with each decision. In decision making under certainty, a choice between two actions is known to lead invariably to a specific outcome. In decision making under risk, a choice between two actions is known to lead invariably to one of a set of possible outcomes.
Many conflict situations are not amenable to strict game theoretic interpretations because one cannot reasonably assume the degree of certainty and consistency in player behavior that is required by the central axioms of game theory. Theoreticians who have been willing to compromise the mathematical rigor of game theory have found it a fruitful model for theorizing about international conflict, psychological processes, and legal and bargaining maneuvers.

II. THE EDUCATIONAL THEORY MODEL

Utility Theory

To begin setting forth game theory as a model for educational theory, utility theory must be presented. While utility theory is independent of game theory, the concept of utility is central to game theory. Utility theory sets forth an idealized model of human preferences. It is idealized in the sense of being fruitful for mathematics, i.e., the theory assures consistency in the ordering of preferences. These consistency demands are described by Luce and Raiffa and are presented here without modification:

1. Any two alternatives shall be comparable, i.e., given any two, the subject will prefer one to the other or he will be indifferent between them.
ii. Both the preference and indifference relations for lotteries are transitive, i.e., given any three lotteries $A$, $B$, and $C$, if he prefers $A$ to $B$ and $B$ to $C$, then he prefers $A$ to $C$; and if he is indifferent between $A$ and $B$ and between $B$ and $C$, then he is indifferent between $A$ and $C$.

iii. In case a lottery has as one of its alternatives (prizes) another lottery, then the first lottery is decomposable into the more basic alternatives through the use of the probability calculus.

iv. If two lotteries are indifferent to the subject, then they are interchangeable as alternatives in any compound lottery.

v. If two lotteries involve the same two alternatives, then the one in which the more preferred alternative has a higher probability of occurring is itself preferred.

vi. If $A$ is preferred to $B$ and $B$ to $C$, then there exists a lottery involving $A$ and $C$ (with appropriate probabilities) which is indifferent to $B$.$^2$

These assumptions: continuity (i.), ordering of alternatives (ii.), reduction of compound lotteries (iii.), substitutability (iv.), monotonacity (v.), and transitivity (vi.), permit the following definition of a utility function:

If a person imposes a transitive preference relation, $\succeq$, over a set of lotteries and if to each lottery, $L$, there is assigned a number $u(L)$ such that the magnitudes of the numbers reflect the preferences, i.e., $u(L) \geq u(L')$ if and only if $L \succeq L'$, then we say there exists a utility function, $u$, over the lotteries.$^3$

Having presented a definition of a utility function, it remains to offer evidence that a utility function can be empirically determined.

*The term 'lottery' refers to a single entity conducted once and only once (a spin of a roulette wheel). A lottery ticket is a chance mechanism yielding certain prizes or alternatives. Choices (preferences) are thus between tickets or chance mechanisms, i.e., lotteries.
Unfortunately there is not such evidence. It does not follow that the concept is therefore useless.

Decision Making Under Uncertainty

We turn to a decision problem under uncertainty (hereafter denoted by 'd.p.u.u.') as an example of a one-person game against nature. This part of game theory appears most likely to provide a point of view for educational theorizing about teacher behavior in the setting of the art classroom. The choice is based upon the poor likelihood that objective probabilities will enter into teacher behavior. This estimate of the situation does not preclude considering teacher behavior as rational behavior nor does it mean that probabilities are meaningless with respect to teacher behavior.

Perhaps the most succinct illustration of the problem of decision making under uncertainty is that given by Savage:\footnote{5}

You are to finish preparations for making an omelet another has already started to make. There is one egg remaining which may be either (1) broken into the bowl already containing five eggs, (2) broken into a bowl for inspection, or (3) thrown away without inspection. Depending upon the state of 'nature' (the egg) each of these acts has a differing consequence for you. The situation may be represented thus:

<table>
<thead>
<tr>
<th>Act</th>
<th>Good</th>
<th>Rotten</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break into bowl</td>
<td>Six-egg omelet</td>
<td>No omelet, and five good eggs destroyed</td>
</tr>
<tr>
<td>Break into saucer</td>
<td>Six-egg omelet, and a saucer to wash</td>
<td>Five-egg omelet and a saucer to wash</td>
</tr>
<tr>
<td>Throw away</td>
<td>Five-egg omelet, and one good egg destroyed</td>
<td>Five-egg omelet</td>
</tr>
</tbody>
</table>

TABLE 1
For the purpose of representing the d.p.u.u., assume a person's preferences will be consistent in the sense that they may be described by a utility function. Also assume that the states of nature form a mutually exclusive and exhaustive set of aspects of a situation which circumscribe a decision making problem. The problem is summarized in the following matrix:

**TABLE 2**

<table>
<thead>
<tr>
<th>ACTS</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_j$</th>
<th>$s_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$u_{11}$</td>
<td>$u_{12}$</td>
<td>$u_{1j}$</td>
<td>$u_{1n}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$u_{21}$</td>
<td>$u_{22}$</td>
<td>$u_{2j}$</td>
<td>$u_{2n}$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$u_{11}$</td>
<td>$u_{12}$</td>
<td>$u_{1j}$</td>
<td>$u_{1n}$</td>
</tr>
<tr>
<td>$A_m$</td>
<td>$u_{m1}$</td>
<td>$u_{m2}$</td>
<td>$u_{mj}$</td>
<td>$u_{mn}$</td>
</tr>
</tbody>
</table>

In terms of game theory, the problem for the decision maker is to choose an act (to select a row) or, more generally, to rank the rows according to some optimality criterion.

Optimal choices are generally represented in one of two ways. Either a numerical index is attached to each act and all acts in a set are rank ordered, or a criterion isolates subsets of acts which are optimal or non-optimal. In either case the criterion, if it is to be precise, must prescribe a procedure which, for any given d.p.u.u., leads one to select acts which are tautologically termed "optimal according to the criterion."
Informal Characterization of Games

Components of a Game

Certain terms representing components of the game along with their conventional meanings are presented below.

1. A **play** is a series of choices in sequence until the game is terminated.
2. A **choice** is an alternative selected by a player at a given point in the game.
3. A **decision point** is a specification of a set of alternatives and the designation by some rule, that a choice is to be made among them.
4. A **pure strategy** is a complete statement of all the choices a player will make in every situation in the game, which statement could be given to a referee before the start of a game.
5. An **information set** is a set of alternatives assigned to each player which may or may not give him information about prior moves in the game.

Types of Games

1. Games against nature. A typical case has been presented in the discussion of decision making under uncertainty.
2. Strictly competitive games. Such games are usually termed two-person zero-sum games. Players have precisely opposite preferences which give rise to a pair of equilibrium strategies such that any improvement in the status of the game for one person necessitates
A deviation from an equilibrium strategy, therefore, carries with it a risk of loss greater than with the equilibrium strategy. A player might also profit from such a deviation.

3. Non-strictly competitive games. Such games are usually termed non-zero-sum games. They are not strictly competitive games because there is at least one lottery which neither player prefers over the other, i.e., there is an element in the game over which there is no disagreement or preferential treatment by both players. Temporary gains can be effected by one player, and reversed by a change in strategy on the part of the other player.

4. Cooperative games. In a cooperative game the players are permitted to make binding agreements, to use either correlated or randomized strategies, to make side payments, and to communicate prior to the game. When complete communication is possible the game is said to be trivial.

5. N-person games. These are games which involve more than two players. Their form may be as indicated in 1, 2, 3, and 4 above.

Psychological Functions of a Strategy

A given selection strategy may be said to benefit the person using it in any or all of the following ways:

1. A strategy may maximize or increase the likelihood that a person gains information (feedback) from each test of the appropriateness of a choice.
2. A strategy may regulate the amount of risk associated with a given test of the appropriateness of a choice, and thus regulate the risk of failing to solve a problem within a specified period of time, or within a given energy limit.

3. A strategy may reduce the cognitive strain of assimilating and keeping track of information obtained from prior choices. Preferences for certain selection strategies will be influenced by the nature of the goal one seeks to maximize.

In summary, game theory is mathematical theory about interest conflict. It is prescriptive theory in the sense that logic is prescriptive theory. Game theory tells what ought to be the case if a person is to maximize his utility, i.e., to maximize benefits for himself in a given situation. Certain aspects of game theory have been abstracted to serve as a point of view for theorizing about teacher behavior in the setting of the classroom. It remains now to demonstrate the relevance of this framework for educational theory.

III. THE EDUCATIONAL THEORY

Teaching as a Game

We may begin by describing teaching as formulating problems or sets of alternatives which could be a basis for the pupil to engage in a selection process. The problems to be formulated may be viewed as discrete units patterned in time. We will assume that the teacher is motivated to maximize benefits to himself, i.e., he maximizes pupil learning. The point of the game of teaching is to formulate problems so the student can solve them and
will solve them. 'Can' refers to the readiness of the pupil to solve the problems because he has sufficient background in the content to do so. 'Will' refers to motivation.

A rational teacher would possess knowledge of subject matter, learner capacity, and environmental variables which would enable him invariably to present solvable problems to the student. Such perfect consistency would permit precise formulation of rules for teaching. In the absence of perfect knowledge and thus perfect control, discrepancies between wanted and observed pupil behaviors are noticed. The teacher may be competent and need more information to solve a teaching problem.

Specification of Some Variates

In order to reduce the burden on the reader for the remainder of this paper, certain variates and informal definitions of them will be presented.

1. A teaching consists in something taught—content—and the way it is taught—method. A teaching will sometimes be referred to as a trial or test. This usage treats a teaching as a discrete unit.

2. The content of teaching is that component of a teaching consisting in a curricular problem or set of alternatives which serves as a basis for the pupil to engage in a selection process.

3. A method of teaching is that component of a teaching consisting in a set of teacher behaviors and a pupil behavior such that certain teacher behaviors in the set induce a pupil behavior.

4. Pupil readiness refers to the ability of the pupil to respond to the content of a teaching because he has residues of past experience or cognitive schemas which are in some sense congruent with the content to which he will be introduced.

5. Pupil motivation refers to the tendency of the pupil to respond to an influence because he has residues of past experience or
motive schemas which are in some sense complemented by the context in which teaching occurs.¹³

6. An influence base consists in a relationship between persons, or between persons and things, or between persons and events which can bring about a change in behavior. Since content is presented to a pupil through a teacher behavior, and method consists in a set of teacher behaviors which induce pupil behaviors, content (a thing) may appear to influence or bring about a change in pupil behavior.¹⁴

7. Problem solution refers to any degree of congruence between a pupil response and the problem or content alternatives presented to the pupil by the teacher.¹⁵

Characteristics of Teaching Which Are Implicated in a Concept of Utility

Utility refers to an ordered array of preferred outcomes of a game. A theoretically fruitful approximation of a utility function is not inconceivable. Its relevance to the solution of educational problems would need to be demonstrated and will not be undertaken in this paper. The purpose of the following discussion is to indicate some characteristics of teaching that not only would be implicated in establishing utility functions but also could be so interpreted. A secondary purpose is to indicate further the extent to which the present theory of teaching as a game will depend upon concepts presented in Construction of Educational Theory Models by E. Maccia, G. Maccia, and R. Jewett.

A concept of utility in educational theory would need to take into account the structure of a given curriculum. Two types of structure will be considered. If a curriculum is hierarchically ordered, then goals are interrelated. Achievement of one goal is contingent upon achievement.
of another. By means of a ranking criterion, curricular goals that are
treated in a taxonomic structure could be interpreted as utility functions. 16
Conversely, if curricular goals are not hierarchically ordered, not all
goals are interrelated. If subsets of outcomes which are optimal or not
optimal can be identified, these subsets could be interpreted as utility
functions. 17 It is, therefore, conceivable that curricular goals could be
formulated in a way that permits an informal use of utility in game theory.

A concept of utility in educational theory would need to take into
account the range of influence of the teacher. E. Maccia has achieved some
formalization of a theory of interpersonal influence ingroups. 18 From this
effort one may project not only a typology of influence relationships but
also a scale indicating range and strength of teacher influence. Since
scaling of some form is a minimal criterion for establishing a utility
function, this aspect of teaching could also be formalized to meet the
requirements of game theory.

A concept of utility in educational theory would need to take into
account the probabilistic character of outcomes, given a well-trained and
competent teacher. The uncertainty of teaching rests not only with the dif-
ficulty of measuring changes in behavior 19 but also from the interaction
between amounts and kinds of method and content in a given teaching. A
typology of effects produced by interaction of content and method has been
presented by E. Maccia. 20 This typology—together with more adequate speci-
fication of contents and methods themselves—would provide a clearly defined
set of variables whose systematic manipulation could assure certain
probability levels for pupil learning. Furthermore, specification of systems for manipulation of variables, the very heart of game theory, could permit formalization of likelihoods of outcomes of teachings in specific contexts.

Identification of Teaching Problems

The purpose of the following discussion is to indicate that a given teaching may appear to have little or no effect in a limited context, but may nevertheless be strategically important in a larger context.

The tentative educational theory based upon an information theory model offers a means for measuring the exhaustiveness and dependability of a teaching through measurement of differences between "problems as presented" to the pupil, and "problems as solved" by the pupil. This criterion presupposes that a pupil has acquired a motive or influence base of sufficient strength to have engaged him with the problem. There is also an implicit suggestion that a teaching can be effective when it matches and extends pupil's schemas (or residues of past experience) with respect to content.

Both game theory and the tentative educational theory based on pharmacology suggest to me that consideration be given to the possibility that teaching may occur even when problem solution does not. If, as the pharmacology model suggests, a given teaching represents an interaction between method and content, then the impact of method upon pupil behavior would seem relevant to measuring the effect of a given teaching. The fact that method is fully described when both teacher and pupil behaviors are specified suggests that method be viewed as the component in teaching which gives rise to influence.

The influence may be interpersonal, as between pupil and
teacher. But it may also be primarily between pupil and content. Thus the pupil may engage himself with the content, and be influenced to learn, even if he is indifferent to the way in which content is presented to him.

From these considerations the following suggestion is made. The content component of a teaching is generally selected to match and extend pupil schemas with respect to content (cognitive schemas). Similarly the method component may be selected to match and extend pupil motive schemas in order to induce content learning. If motivation is taken to be a prerequisite for learning, then certain teaching problems and tentative prescriptions for their solution can be projected. Consider the state of the pupil with respect to problem solution:

<table>
<thead>
<tr>
<th>State of Pupil</th>
<th>Solved</th>
<th>Unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ready and Motivated</td>
<td>If motivation is strong and readiness is maintained, learning will persist.</td>
<td>Logically impossible unless teacher has made a mistake.</td>
</tr>
<tr>
<td>Ready, Not Motivated</td>
<td>Compliant learning, weak influence base, learning not likely to persist.</td>
<td>Stronger Influence base appears to be required.</td>
</tr>
<tr>
<td>Not Ready, Motivated</td>
<td>Logically impossible unless pupil has been lucky or has given a substitute response.</td>
<td>Maintainence of motivation accompanied by remedial help appears to be required.</td>
</tr>
<tr>
<td>Neither Ready nor Motivated</td>
<td>Logically impossible unless pupil has given substitute response.*</td>
<td>If motivation is established, readiness could be improved.</td>
</tr>
</tbody>
</table>

*A substitute response is only an apparent solution to the problem.
These possible states of the pupil with respect to problem solving presuppose that the teacher can know that a pupil is ready to learn even if he does not solve the problem. Such knowledge may be available, if only approximate, either from readiness tests or a reasonable expectation based on a pupil's earlier problem solutions. Similarly, the entry in the cell denoted by 'problem solved, pupil ready, but not motivated' presupposes a criterion for motivation other than mere problem solution. The criterion is implied through the choice of the terms, 'compliant learning.' It suggests that an influence base exists but that it is not strong. Persistence in learning or persistence of a learning is unlikely. Just as the physician cannot treat a condition that is assumed not to be problematic, so the teacher cannot effect a change in behavior if teaching problems are not identified. Given diagnostic cues about the state of the pupil, one would expect to be able to measure teaching by amount or degree of content learning. One might also measure how frequently the teacher succeeds in getting the pupil to the point of content learning. (Frequency and degree of problem solution are not necessarily separate measures. If problem solution cannot occur in degrees, problem solution reduces to a measure of frequency. The emphasis upon frequency here is intended to underscore the fact that problem solution can be either more or less or all or nothing.)

A Conflict of Interest Teaching Game

A situation analogous to a zero-sum two person game occurs in the classroom when teacher and a pupil have opposite values with respect to some curricular outcome. Typically, the range of teacher influence is limited to
pupil behaviors he can observe directly and which can be reinforced through rewards and punishments. Typically, the pupil adopts a pattern of behavior or strategy which regulates and minimizes the risk of being deterred from his commitment. Several forms of pupil behavior associated with the teaching problem will dramatize the difficulties likely to be encountered by the teacher:

1. The pupil may appear to be physically weak, unintelligent, or otherwise powerless to control or modify his behavior. This strategy gains its strength from the pupil's belief that the teacher will not threaten or use punitive measures on a person who is not in full control of his own behavior.

2. The pupil may systematically close off information from the teacher which would require the pupil to abandon his position. If the pupil is unavailable for the receipt of messages, through physical or mental absence, he cannot be deterred from a commitment.

3. The pupil may threaten the teacher with a behavior that would embarrass the teacher or place the reputation of the teacher in jeopardy if the teacher attempts to influence the pupil to change his commitment.

4. The pupil may give substitute responses which disguise his real commitment.

5. The pupil may adopt a sophisticated behavior pattern so that a real commitment is not revealed.

The effectiveness of such behaviors depends upon the pupil assuming that the teacher is naive and a failure of the teacher to introduce measures which deter the pupil from initiating such behaviors. An efficient strategy for the teacher will depend upon the structure of the curriculum to which he is committed and in terms of which he will seek to maximize pupil learnings.
Case 1

Assume that curricular materials are interrelated—that content is hierarchically structured. Loss of pupil learning on one level will prevent him from learning subsequent materials. Ideally the teacher would seek information that would enable him to assess the full range of consequences of each teaching that may be available to him. Unless the teacher is a tutor, his curricular commitment probably has a time schedule associated with it. Other pupils, too, demand his attention. Postponement of decision in this situation is likely to reduce the teacher's chance of recovering losses in pupil learning as time passes. Thus there is a point of no return: a point at which measurement of pupil learning virtually reduces to all or none. In short, the teacher cannot afford to make many wrong moves if he is to maximize learning. The efficient delineation of and selection among attributes of teaching (i.e., content and method components) which will induce the pupil to learn is crucial to the outcome of the game.

The fewer number of teaching attributes the teacher can afford to test because of time restrictions, then the greater value of any information that helps the teacher to solve his problem. The teacher's task is to maximize his utility—to gain information or to get a pupil response. If he is cautious in selecting attributes of teaching to test for effectiveness, he can be assured of some information at every trial. If he changes several attributes of his teaching at each trial, he gains much information only if the pupil responds.
The expected utility of the situation can be represented in approximate form if a payoff matrix can be formulated. Consider the following description in matrix form of the problem confronting the teacher.

**Anticipated Pupil Responses and Outcome Values**

<table>
<thead>
<tr>
<th>Teacher Decision Alternatives</th>
<th>Response</th>
<th>No Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change one attribute of a teaching at a time.</td>
<td>Gain moderate amount of information and small gain in solving problem of inducing pupil to learn.</td>
<td>Gain moderate amount of information and small gain in solving problem of inducing pupil to learn.</td>
</tr>
<tr>
<td>Change more than one attribute of a teaching.</td>
<td>Gain much information and gain much in getting problem solved.</td>
<td>Gain little or no information and make little progress in getting problem solved.</td>
</tr>
</tbody>
</table>

With only a rough estimate of the likelihood of outcomes, the expected utility can be represented more formally. To accomplish this we may assign values which suggest the desirability of each outcome. Let gain in information and much gain in solving problem have the value 1.0; gain little or no information have the value 0.0; and gain moderate amount of information have the value 0.5. Now suppose the chance of getting a response from the pupil is greater than getting no response. Let these probabilities be represented by .80 and .20, respectively. We solve for
expected utility by multiplying estimated probability by value assigned to outcomes and sum the products across a row.

<table>
<thead>
<tr>
<th>Teacher Decision Alternatives</th>
<th>Anticipated Events</th>
<th>Pupil Response</th>
<th>No Pupil Response</th>
<th>Expected Utility of Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated probabilities</td>
<td>0.80</td>
<td>0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change one attribute</td>
<td>0.5 x 0.80</td>
<td>0.5 x 0.20</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Change many attributes</td>
<td>1.0 x 0.80</td>
<td>0 x 0.20</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

Under these conditions the teacher would maximize his utility by changing many attributes of a teaching at one time.

Now suppose that even with his best preliminary selection among teachings, the likelihood of a pupil response is small. Let the chance of a pupil response be assigned the probability of .20 and the chance that he won't be .80.

<table>
<thead>
<tr>
<th>Teacher Decision Alternatives</th>
<th>Anticipated Events</th>
<th>Pupil Response</th>
<th>No Pupil Response</th>
<th>Expected Utility of Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated probabilities</td>
<td>0.20</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change one attribute</td>
<td>0.5 x 0.20</td>
<td>0.5 x 0.80</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Change many attributes</td>
<td>1.0 x 0.20</td>
<td>0 x 0.80</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

Given these probabilities, the effective strategy or decision procedure is reversed. It is important to note that when the number of attributes in teaching which may be relevant to determining an outcome...
exceeds the number of occasions for testing their relevance, the teacher cannot succeed in solving the problem, except by chance, if he tests one attribute at a time. Only if he changes many attributes at each test can he succeed. He may also fail completely.

Several specific ways of inducing a pupil response are suggested in other theoretical work.

1. The teacher could take advantage of interactions among content and methods so that so-called non-effective teachings, together with effective teachings, increase the effectiveness of the latter. The gain would be especially evident if the non-effective teaching is not usually associated with learning, and can be used as a focus to engage the student with a problem without the student anticipating and thus avoiding the problem.28

2. Another suggestion may be termed the ‘Maccia effect.’29 If a teaching influence is maintained at a level beyond the necessary steps required in the possible learning and a disturbing influence is introduced at one of the intermediate steps, then learning at that step will be intensified.29

This suggests that the incrementalism assumed to be required in learning could be produced by a disturbing influence being introduced at some level of content. A disturbing influence for a given value of content could be associated with an extreme method. An extreme method might be one which is complementary to pupil motive schemas. A symmetry between method and motives at this extreme could be termed ‘psychological comfort.’ An extreme method might also be non-complementary to pupil motive schemas. Asymmetry between method and motives at this extreme could be termed ‘anxiety.’ Contrast in teaching influences may intensify pupil response.

3. If the teacher cannot influence the pupil directly, he may be able to influence the pupil by influencing another person who influences the pupil.30
4. If the teacher fails in a series of attempts to induce the pupil to learn, and if the discrepancy between wanted and observed behavior increases, there is a maximum point at which the teacher can sustain efforts to induce the pupil to learn without producing excessive strain on teaching capacity. Reduction of strain can be accomplished through reduction of information intake and increasing the risk of failure, compromising the curricular constraints, or delegation of teaching to machines, other persons, and agencies.31

Case 2

Assume that curricular materials are not interrelated. Loss of pupil learning in one part of the curriculum does not prevent learning of other parts of the content. The teacher may be committed to a minimum proportion of pupil gains against pupil losses. At the point where pupil failures are in danger of being outweighed by pupil gains, the teaching strategy might take the form already presented. The relative weight attached to a given learning would suggest differing strategies too.

Consider two subcases: (1) a series of losses could be wiped out by a single success with a problem more heavily weighted than the sum of all the previous losses; (2) if learnings are equally weighted and numerous, the teacher could afford to risk some failures if gain in information is likely to assure systematic pupil learning on later occasions.

A Coordinated Teaching Game

A situation analogous to a coordinated or cooperative game occurs in the classroom when the teacher has an influence base with the pupil. If the basis for influence is the pupil's perception of the teacher's behavior as legitimate or expert, the range of teacher influence is less than if the pupil identifies with the teacher.32 The problem for the teacher is to
devise problems which will (1) maximize pupil learning, (2) maintain a favor-able influence base so learning continues to occur, and (3) prevent behavior patterns to generate which are likely to prevent the utilization of one stage of pupil learning as a stage for another learning.33

As in the case of interest conflict situations, the curricular structure suggests teaching strategies which are likely to maximize learnings. When curricular materials are interrelated, and the pupil identifies with the teacher, then the teacher is not restricted to those trials in which a direct and positive confirmation of pupil change in behavior is present. The pupil is motivated to please the teacher. He may undertake or seek out remedial measures if he falls behind in his work, since he perceives that the teacher values his achievement. The range of teacher influence is thus moderately extended.

When there is no problem in communication between pupil and teacher, the pure coordination of interests ceases to be a game or problematic with respect to motives. The focus then shifts to content and the efficient organization of content to maximize learning.

III. SIGNIFICANCE OF GAME THEORY

"Formalize as much and as little as possible, so that your theorizing can be evaluated and so that your inquiry is not constricted,"34 Much of the apparent significance of game theory is due to concepts which are not indigenous to the theory itself. Many have been abstracted from the work.
presented in *Construction of Educational Theory Models* and simply placed in a game theoretic framework.

Common sense illustrations which tend to indicate relevance of theory to practice have been omitted at this stage of theorizing in order to get on with the task of setting forth the theory. Such illustrations could be projected in several ways. The emphasis upon problem identification, efficient selection strategies, rationality, curricular constraints, and interaction of content and method—all have direct bearing on problems in teacher education. The distinction among teaching strategies which are tenable in the face of certain curricular commitments offers a help in understanding both how controversies in education could develop when commitments cannot be met—and why a truly fantastic array of teaching devices and media of instruction tend to be introduced when the going gets rough. The rationality of the pupil, evident in behaviors he may adopt to avoid being deterred from a commitment, illustrates the diagnostic sophistication required of the teacher if he is to succeed. The theory suggests that teaching may appear to be less frantic, where curricular constraints permit the teacher to average gains and losses or to introduce fell swoop techniques to cancel out pupil losses.

The insistence that method in teaching is an influence relationship does not preclude describing the preliminary theory based on game theory—a theory of teaching method. If behavioral descriptions of teaching methods are an innovation, perhaps more work on teaching as a game is warranted.
FOOTNOTES


3. Ibid., p. 29.


7. Ibid., p. 278.


10. Ibid., pp. 325-26.


13. Ibid. The term "schema" added to "motive" does not appear in the work cited.

14. E. Maccia et al., *op. cit.*, "An Educational Theory Model: Graph Theory," p. 110. Influence base is not taken to be limited to inter-personal relationships *per se*, even if valuing of content or things can ultimately be traced backward to such relationships.


17. Luce and Raiffa, op. cit., p. 286.


23. Ibid., p. 208.

24. Persistence of learning is related both to strength and length of influence. See pages 126 and 251 respectively of the work cited above.


27. The matrix and following illustration have been adopted from Bruner et al., op. cit., pp. 115-16.

28. Interpreted from the Pharmacology Model, E. Maccia et al., op. cit.


