Two computer programs are described, with the
development and implementation of the first program described in some
detail. This is a student-computer dialogue for beginning or
intermediate physics classes entitled "A Computer-Based Dialogue for
Deriving Energy Conservation for Motion in One-Dimension." A portion
of the flowchart is included, along with a general discussion of the
analysis of the program using student response information. The
second program simulates a pulse in a rope. The student is provided
with a "measurement" facility; if he enters time and position, he is
provided the rope displacement. Checks are made as to the reasonableness
of the student strategy, and suggestions are given based on these
checks. Auxiliary facilities such as plotting and listing are
provided. (Author/TS)
A COMPUTER-BASED DIALOGUE FOR DERIVING ENERGY CONSERVATION FOR MOTION IN ONE-DIMENSION*

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ABSTRACT
This paper describes a student-computer dialogue for beginning or intermediate physics classes. The dialogue enables the student to take some initiative in showing that energy conservation in one-dimension is a consequence of the laws of motion.

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Physics, mathematics, and other science courses, use the mathematical derivation or proof of a result, starting from some other theorem or physical principle, as a staple of such courses at the beginning, intermediate, and advanced levels. Such derivations often constitute the main portions of lectures and textbooks; in a mathematics course they may constitute the entire course.

A derivation can serve several purposes. First, a particular result is needed, often an important result useful to the student in future work. Secondly, as teachers, some of us are eager to show that classical physics can be developed as a well-constructed logical net, and that austere beginnings can yield powerful results. A third and perhaps more important reason for derivations in physics courses is that we hope to teach students the "art" of deriving physical results. A complicated derivation often involves much trial and error. We want to help students become sophisticated at deriving results. Teaching the techniques of proof is one of the most important goals of physics courses, and it is one of the hardest goals to accomplish. (George Polya's How to Solve It is one attempt to teach this art.)

Many teachers have heard the archetypal student's comment on a complicated proof presented in lecture. The student announces that he could follow the proof, but he does not feel he could find the derivation himself! This hardly surprises the teacher, who may not, on first encountering the problem, have produced the derivation as facilely as he duplicated it in lecture. But not realizing that everyone gropes initially, the student feels insecure because he cannot generate quickly such a smooth and elegant proof. (It could
be argued that the less polished lecturer might provide better insight as to how proofs are developed than the person who carefully prepares and rehearses an elegant derivation.

The Energy Conservation Dialogue
The computer dialogue described here is designed to make the student an active participant in the development of the proof, to let them take at least some of the steps along the way on their own. Some of these steps can be large, while others will be relatively small. At worst, the dialogue corresponds to something like the lecture situation, where the student is told the proof; however, he probably receives more detail than in lecture, through the remedial sequences in the dialogue.

The dialogue develops a proof of the conservation of mechanical energy for a mass moving in one-dimension and subject to a force that depends only on the position. The proof starts with the law of motion; we multiply by the velocity and write the resulting equation in the form

\[ \frac{d}{dt} \text{(something)} = 0. \]

In the process of the proof we introduce the concept of potential energy as the quantity whose negative spatial derivative is the force, and the student is asked to enter the potential energy and total energy for several different forces.

The first flow chart shows the general form of the dialogue, the second shows greater detail in one section, and the third is a page from the full flow chart.
Overview of the Tutorial Program on Energy Conservation

Sign on and problem statement

Student starts derivation by choosing to modify the equation of motion. He multiplies both sides of the equation by $v$.

Other alternatives, such as working from kinetic energy, integration, etc. (Presently unimplemented)

Rewrite $mv \frac{dv}{dt}$ as:

$$\frac{d}{dt} \left( \frac{1}{2} mv^2 \right)$$

Rewrite $Fv$ as:

$$\frac{dP}{dt}$$

(Introduce Potential Energy $P$)

Write result as:

$$\frac{d}{dt} \left( \frac{1}{2} mv^2 + P \right) = 0$$

Conservation Law

Practice Example

END
Can student suggest some physical variables that forces depend on?

Yes

No

He is told

He is given a force that depends only on position. The problem is to find the "something" whose time derivative is the righthand side of the modified equation:

\[ m \frac{dv}{dt} = Fv \]

Can the student enter the relationship between \( F \) and \( W \) which will make the following equation an identity.

\[ \frac{dW}{dt} = F(x) \cdot v = F(x) \frac{dx}{dt} \]

Yes

No

Find \( \frac{dh}{dt} \) for the equation \( h(g) = g^3 \)

Hint: Does he know the chain rule?

Yes

No

Chain rule is briefly explained.

Can the student enter the value for \( F \) which makes an identity of

\[ \frac{dW}{dt} = F \cdot \frac{dx}{dt} \]

He is told the answer

F = - \( \frac{dW}{dt} \) or

F = \( \frac{dW}{dt} \)

Yes

No

No
1. Assume that you are familiar with the laws of motion. As with many laws in science, these laws have powerful consequences. Let's explore one of these consequences, a conservation principle. First, recall the meaning of a conserved quantity. Please enter the numbers identifying the lines which you consider to be correct statements:

1. A conserved quantity does not change when the coordinate system is changed.
2. A conserved quantity does not change in time.
3. A conserved quantity has zero spatial derivative.
4. A conserved quantity has zero time derivative.

Statement 1 is the usual simple statement of the property of invariance under coordinate transformation. This property is different from that of a conserved quantity.

Statement 2 is a simple statement of the property of a conserved quantity.

Statement 3 describes a quantity which is the same everywhere in space, at a given instant, but may change from instant to instant. A quantity that changes with time is not conserved.

Statement 4 is a property of a conserved quantity, since the time derivative of a quantity not changing in time vanishes.

To be sure that we mean the same thing by "laws of motion" enter the equation of motion of a one-dimensional simple harmonic oscillator. Use M for mass, X for position, A for acceleration, and K for spring constant.

The equation of motion should contain the mass M. If equal, yes. If not, no.

I can't identify your response as an equation. Use an = sign in your equation.

Don't forget the acceleration at

Input

Set TSC = 0

Try again to enter the equation of motion for the oscillator.

The equation of motion should contain the mass M.

I can't identify your response as an equation.

Use an = sign in your equation.

Don't forget the acceleration at

Input

Increment TSC

Not M

M = A + Kx OR EQUIV.

Not a

F = -Kx

Not a

I can't read you...
Development

The development of computer dialogues as a self-instructional resource is still relatively new, so a description of the process we followed may be of some interest. First we discussed which areas and approaches in physics might lend themselves to an effective computer-student conversation. Then we decided to pursue two dialogues, the one discussed here and another involving simulation in the study of plane electromagnetic radiation (a dialogue still under development).

The energy conservation proof was developed first as a flow chart showing what is typed to students, the expected responses, and the actions in each case. The two of us spent approximately three days working together on the flow chart, with occasional assistance from a student and a secretary. We did not use standard flow chart conventions.

The flow chart approach was appealing for a number of reasons. We were working at the University of California, Irvine, where a change was under way in computer facilities and no local computer was available. We were very concerned with the question of spreadability of such material. Computer dialogues have often existed only as computer programs in specialized languages, not usable outside the environment in which they were developed! The simplified flow chart seems a reasonable approach to developing computer conversations in a language-independent form. Furthermore we felt that pedagogical details should come first: we decided what we wanted to do, knowing something about the potentialities of the computer, before putting ourselves into the straight-jacket of a particular set of computer
languages and facilities. We feel that successful use of the computer in education demands that learning details have priority over computer software details. In addition the flow chart furnishes a view of the dialogue to a teacher who is considering its use in his classroom.

We sent the flow chart to friends for comments and suggestions. Particularly useful criticisms came from Edward Lambe, of the State University of New York at Stony Brook, and Kenneth Ford, of the University of California, Irvine. Students within the project also suggested improvements.

Implementation

After a brief time together working on the dialogue we returned to our respective institutions and proceeded to implement and use the dialogue on local timesharing facilities.

At the University of Michigan the dialogue was implemented in an existing FORTRAN-based conversational computer language, FOIL, developed by Karl Zinn and others at Michigan and running under the Michigan Timesharing System for the IBM 360/67. This language has since been superseded and the program will be rewritten. The original FOIL version is still in use.

The development of the dialogue as a computer program at the University of California, Irvine proceeded differently. The change in computer facilities at Irvine provided an XDS SIGMA 7, with little directly applicable software. Hence, development had to proceed in two parallel directions, generating facilities for student dialogues
and developing the dialogue itself. The language facilities were
developed as quickly as possible for the energy conservation program
so as to allow easy extension to the electromagnetic simulation dialogue
mentioned above. The dialogue facilities were developed as SIGMA 7
assembly language macros, "procedures," making it easy to extend and
modify the language for new uses. The macros were oriented toward
professors and secretaries who are not experienced programmers. A
current description of this system, with examples of usage, is
available; it has proved to be flexible to changing needs, and is now
being used by others also.

The secretary helped greatly in preparing the dialogue. Those
acquainted with computer dialogues will realize that even an elemen-
tary dialogue entails a vast amount of typing. Experienced typists
should do this typing. Hence we taught the secretary to type at the
terminal, directly from the flow chart, the macros which constitute
the final program. Only a short amount of instruction was necessary.
The secretary cannot handle all details, and she was instructed to
enter a row of asterisks when she was uncertain about what to type.
Several secretaries at Irvine have been successfully trained in this
procedure! The students who worked on the program after could easily
recognize points marked by the secretary for further editing. The
secretary worked at the terminal typing directly into a disk file; no
cards were punched. The normal editing facilities of the computer were
used to modify the program, by both the secretary and the student
programmer.
The first running version of the dialogue was available in January. This program was informally tested within the project and by colleagues in the physics department, whose useful suggestions we were able to incorporate. It was used with 150 science and engineering students at University of California, Irvine, in the first quarter of a five quarter introductory course. It was also used by a group of University of California, Berkeley students in a similar class taught by Richard Ballard, through the University of California tie-line facilities. A section of student use follows; student inputs follow the question marks at the start of the line.
TO BE SURE THAT WE MEAN THE SAME THING BY "LAWS OF MOTION" ENTER THE EQUATION OF MOTION OF A ONE-DIMENSIONAL SIMPLE HARMONIC OSCILLATOR. (USE M FOR MASS; X FOR POSITION; A FOR ACCELERATION; AND K FOR SPRING CONSTANT.)

\[ \text{F} = ma \]

PLEASE ENTER THE EQUATION WITH THE FORCE FOR THE HARMONIC OSCILLATOR:

\[ \text{F} = -kx \]

THE EQUATION OF MOTION SHOULD CONTAIN THE MASS M
TRY AGAIN TO ENTER THE EQUATION OF MOTION FOR THE OSCILLATOR

\[ \text{ma} = -kx \]

WATCH THE SIGN ON THE FORCE
TRY AGAIN.

\[ \text{ma} = -kx \]

YOUR RESULTS SEEM RIGHT.
A USUAL FORM FOR HARMONIC OSCILLATOR EQUATION OF MOTION IS \( \text{ma} = -kx \).

NOW CONSIDER ANY ONE-DIMENSIONAL SYSTEM, NOT NECESSARILY AN OSCILLATOR.

LET'S SEE IF WE CAN FIND A CONSERVED QUANTITY BY "PLAYING" WITH THE EQUATIONS OF MOTION. IF WE MANIPULATE THE EQUATION, \( \text{ma} + \text{dv} / \text{dt} = \text{f} \), UNTIL WE HAVE TRANSFORMED IT INTO THE FORM

\[ \text{d} / \text{dt} (\text{something}) = 0, \]

THEN "SOMETHING" IS THE CONSERVED QUANTITY WE ARE SEEKING....!
DO YOU WANT TO BEGIN BY MODIFYING THE EQUATION OF MOTION, OR BEGIN WITH SOME OTHER QUANTITY?

LETS MODIFY THE EQUATIONS OF MOTION

WHAT OPERATIONS DO YOU WISH TO PERFORM?
MULTIPLY BOTH SIDES BY THE SAME QUANTITY?
ADD THE SAME QUANTITY TO BOTH SIDES?
INTEGRATE? SOMETHING ELSE? IT MIGHT BE FUN TO EXPLORE THESE POSSIBILITIES WITH PENCIL AND PAPER. YOU MIGHT EVEN BE ABLE TO ANTICIPATE WHERE WE ARE GOING....
Feedback

Two types of feedback were obtained at Irvine, using questionnaires and selective storage of student responses on the disk. The questionnaire showed that the average time at the terminal was 58 minutes; about 15 or 20 minutes is required by a knowledgeable student. Most students completed the material in one section (the dialogue offered a restart facility if the student did not complete the program). The students could use either Model 33 teletypes or Datapoint 3300 alphanumeric CRT. Students preferred the Model 33 over the Datapoint, because the previous responses were often useful to them, and they were only available in the hardcopy printing of the Model 33. (Neither terminal is ideal for student use.) We also queried students on a stylistic aspect of the program. We chose the grammatical first person in addressing for the computer to use students. Some of our consultants objected, but student response was overwhelmingly favorable toward the first person style. Perhaps it alleviates the feeling that computers are at best impersonal; such a style may tend to humanize the computer. In spite of the problems to be mentioned next, two-thirds of the students who used the program claimed to enjoy it.

Some difficulties quickly developed with our new programming system, and it was not surprising that they showed up in the student survey. Our testing had proceeded with only one user; when many students were simultaneously using the system, conflicts not provided for arose in use of the files. Some users were bounced out of the program, or received unintelligible error messages. Some students complained that the questions were vague or hard, and some also complained, sometimes justifiably, that the computer did not accept correct answers. The
fast speed on some Datapoints (run at 1200 baud) which presented information faster than the student could read caused another problem.

Another very useful form of feedback was obtained internally in the program. When the student types in a reply, the program attempts to analyze the answer, looking for both right and identifiable wrong responses. In some cases it can find none of these expected responses. In about 40 places we inserted instructions for saving the student response in a special disk file, if we failed to analyze the response. Several thousand such responses were saved and we examined them daily. They indicated where we were missing correct responses, wrong responses we should have responded to, the weak places in the program, and ways of using the system that we had not contemplated.

Student Response Information
Even in this first Irvine version we did a respectable job in matching student responses; the number of places where we failed to analyze a reasonable student input—either a correct or incorrect response—is smaller than we would have predicted. Certainly there were such places, but for many questions we anticipated most of the responses.

It was comforting to note some "convergence" in the unanalyzed responses stored on the disk. As the week progressed we found fewer and fewer new corrections needed. The difference was sufficient as the week progressed to suggest that the program will soon reach the stage where we will be able to analyze almost any reasonable student response from students at this level, although our relatively crude matching techniques cannot analyze all possible responses. However, additional experiences are needed with students of diverse background.
Student responses indicated a number of weak points in the program. Some of these were simply programming errors on our part. In one place, for example, we look for a "no" response but unfortunately the number 0 (zero) had been typed in our program instead of the letter 0. We received vast numbers of no's listed as unanalyzed! (In the new version we always look for both.) This is a trivial error that would be difficult to spot without student feedback.

Probably the weakest section was where we introduced and used potential energy. Many students noted that we went too rapidly there.

Nor did we give students enough assistance in calculating potential energy for particular forces. A number of people arrived at this point not knowing how to make the calculation, perhaps because calculus was still a new tool for them.

Calculus notation was another problem. This may be a particular problem at Irvine, but it may be more widespread. The calculus course uses two notations for derivatives which we almost never use in physics! They indicate derivatives by a prime, or by writing a big "D," avoiding the "(d/dt)" and the dot notation common in physics. Although the "d/dt" notation was employed in the course, a number of people used the alternate notations, particularly when asked to differentiate \( F \times G \).

The responses show that a few students do not use the program as a dialogue at all, but simply use it as an information source, much the same way that students would use a book. These students, 5% of the
users, either enter no response at all for question after question, or enter garbage. Should we worry about such a student? He is not using the program to maximum educational advantage, but he is probably no worse off and perhaps better off than if he were reading the same material in a text; at least he is sent into various remedial branches which he would not have seen in a standard text, and he is "paced" through the proof.

Using the feedback mentioned above, particularly the selective disk storage of responses, we have prepared a second version of the conservation dialogue for the SIGMA 7. The dialogue is available in flow chart and program form for those who wish to implement it elsewhere. While we would not claim any degree of perfection in its present form, the program was considerably improved by the sizable student feedback.

Potential users should recognize the limitations in the present program. Only one proof is possible, a proof which starts with the laws of motion, multiplies both sides by $v$, and writes everything as $\frac{d}{dt} (\text{something}) = 0$. A flexible program should follow the students' whims, at least to some extent. We have not followed all the branches we can contemplate in the program; some we hope to add in later versions. No computer program could allow all the possibilities, with present day technologies and know-how. But we hope that the conversation would encourage most students to take some steps themselves, and thus to develop the analytical abilities necessary for future physics progress.

Another limitation in a computer dialogue is our inability to recognize all correct responses. Recognition is particularly difficult
if, as here, we restrict the student as little as possible with regard to possible input. Most inputs are free-form, with no directions about typing; even with formulas, we adapt the program internally to accept the various notations the students may use. Since we cannot recognize all correct responses, modest comments to students are in order when we have not recognized the response; it is dangerous in this environment to tell the student he is wrong. Hence we use comments which emphasize our limitations within the program as well as the fact that he has not put in what we expected. It should be emphasized that every implementation, in a different language facility, is bound to differ in its capabilities, and even possibly tactics, for recognizing student responses. Thus although the initial versions at Ann Arbor and Irvine were very similar, since they were both based on the flow chart, the student would not necessarily receive identical responses for identical inputs, because of different tactics of string matching to identify the critical components of the input.

We are eager to talk with people who want to implement this dialogue on other systems or use it with other groups of students besides those we have worked with. The detailed flow-chart is available on request.
A Computer Simulation for the Study of Waves

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Abstract
The computer program described here is designed for use in the second quarter of the beginning course for science and engineering majors at the University of California, Irvine, simulates a pulse in a rope. The student is provided with a "measurement" facility; if he enters time and position he is told the rope displacement. His problem is to understand the disturbance sufficiently to answer. Auxiliary facilities such as plotting and listing are provided. Checks are made as to the reasonableness of the student strategy, and suggestions are given based on these checks. It is hoped that through this simulation students will be able in many cases to "discover" the preservation of "shape," the x - vt dependence of the pulse.
Newly developed beginning physics courses often make strong demands on the students' mathematical ability. Thus the Feynman course or the Berkeley Physics course are mathematical techniques that previously had been confined to the junior/senior level in physics courses. Hence, a major problem associated with the teaching of newly developed high-level beginning courses is that of overcoming the mathematical barriers in students' background. Students do not come into the physics course with noticeably more mathematics background so the burden of dealing with this new mathematical complexity must fall to the individual physics instructor.

One feature of these newer courses is a more sophisticated approach to the study of waves which assumes that even a beginning student, in the freshman year, can see the wave equation and explore some of its simple consequences. The wave equation, and the associated mathematics necessary for the student to understand what the equation means and how to generate solutions, is typical of the mathematical problems and demands of the newer courses. Students are likely to be relatively unfamiliar with both the notion of partial derivative and the idea of differential equations, either ordinary or partial. They are not likely to be equipped with a great facility for solving such equations, so even if one has a rational way of arriving at the equation, the solution must be developed and motivated entirely within the physics class.

Experience shows that the computer can often be useful in a physics course, in overcoming mathematical handicaps. For example computational use of the computer allows the beginning course to get
directly to the equations of motion as differential equations, rather than the usual algebraic treatment. (Add reference) Hence, it seems reasonable to ask if there are effective ways of using the computer to overcome the limitations associated with the wave equation. One might use computational approaches here. But one aim, to have students able to see that the wave equation has certain types of solutions which are very characteristic of waves, the traveling solutions which depend on \( x - vt \) or \( x + vt \), is difficult to satisfy with directly nomical work. One can simply produce these solutions "out of the blue," but one hopes to motivate students to understand that one might expect such solutions forwards; this would offer a sounder basis for introducing these travelling patterns into the class.

The simulation described attempts to have students discover, through interaction with a computer simulation, the \( x - vt \) dependents of a wave in a rope. It does not explicitly use this terminology; success in the program is measured by performance criteriun. Students must use this relation or something equivalent to it to calculate values of the disturbance. Hence it would be followed immediately by another program or lecture material showing that the \( x - vt \) disturbance is indeed a solution of the wave equation. (In the case of the Physics 5A-5B sequence at Irvine the students would have seen the wave equation just before seeing this dialog.)

It is also possible that this dialog might be used in a phenonomo-logically oriented course which does not introduce the wave equation.
at this level, but where it is deemed important to have students learn about the x - vt dependence.

A Trial Run
To give some of the flavor of what it is like for a student to go through the particular program at hand, we will examine here a sample of a complete (but abbreviated) student use of the simulation dialog. It should be realized that the situation would be different for different students, and that any one trip through the program does not show all the aspects of the dialog. Thus some of the help messages are tailored to the particular situation, the type of input (requests for measurement) the student has been putting in up to that point. But talking our way through an example should give a useful view of what is happening.

We start assuming that the student has signed on the computer, and he knows that the name of the dialog is ROPEGAME. He requests the dialog.

\[ \text{LET DIALOG} \]
\[ \text{PROGRAM NAME? ROPEGAME} \]

The items entered by the student are underlined; everything else is typed by the computer.
First he starts with an introduction.

THE PHYSICAL SYSTEM WE WILL EXPLORE IS
AN EXTREMELY LONG ROPE WITH A DISTURBANCE
IN IT. IF YOU TELL ME A POSITION ALONG THE ROPE
AND A TIME, I WILL GIVE YOU THE DISTURBANCE,
THE DISPLACEMENT FROM EQUILIBRIUM. YOUR JOB
IS TO LEARN WHAT IS HAPPENING IN THE ROPE.

I WILL EVENTUALLY TURN THE TABLES, GIVING YOU
INFORMATION AND ASKING YOU TO PREDICT VALUES.

'POSITION IS IN METERS AND TIME IN SECONDS;
DON'T ENTER UNITS.'

After the introduction the dialog expects the students to enter
values for the position along the rope, and the time in which he
wants disturbances along that position. The computer calculates
a disturbance at that point and tells him the result. He starts
with no initial information about the disturbance, but he has a
measurement-like facility for gathering information. In the
following case it is noted that the student tries more or less
random values of position and time and does not find the disturbance.
As we will see, it is at any one time almost zero for most of the
rope. Here are the initial measurements.

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISTURBANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>9.9</td>
<td>0</td>
</tr>
<tr>
<td>1.8E17</td>
<td>6.4</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>0</td>
</tr>
</tbody>
</table>

Actually many students will find a disturbance in these first few
measurements, because if the student makes the most likely choices,
x = 0 and + = 0, the origins, he will encounter a non-zero
disturbance. But we don't want students to miss the action forever
so if a student has gotten only zero disturbances in the rope up in
his first five measurements the program offers him some guidance as
to where to look for non-zero values.

JUST TO CONVINCE YOU THAT THE DISPLACEMENT
IS NOT ALWAYS ZERO, HERE ARE SOME POSITION
AND TIMES AT WHICH THE DISPLACEMENT IS
DISTINCTLY NON-ZERO.

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISPLACEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.24</td>
<td>-8.24</td>
<td>4.66E+1</td>
</tr>
<tr>
<td>-0.06</td>
<td>0.34</td>
<td>0.06</td>
</tr>
<tr>
<td>-1.64</td>
<td>-6.38</td>
<td>0.28</td>
</tr>
<tr>
<td>-0.66</td>
<td>-2.63</td>
<td>0.32</td>
</tr>
</tbody>
</table>

These values are generated partially random within the program,
but in such a way that there is a non-zero disturbance. Incidentally
the pattern of the disturbance is partially the result of random
choices; each student receives a slightly different disturbance.
However, the form of the disturbance has been chosen to make the
dialog as profitable as possible and so the general form stays the
same for all students. We also cue the same wave-velocity for all
students.

Our hypothetical student now continues to make more measurements.

GRAPH OR SKETCHES MIGHT BE USEFUL.

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISTURBANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>0.0</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

THIS PUZZLE HAS A 'PAYOFF'. IF YOU
CAN DETERMINE HOW THIS DISTURBANCE BEHAVES,
YOU WILL UNDERSTAND AN IMPORTANT PRINCIPLE
INVOLVED IN MANY PHYSICAL SYSTEMS.
AFTER A FEW MORE MEASUREMENTS YOU CAN
TURN-THE-TABLE AND TRY TO PREDICT THE
BEHAVIOR OF THE ROPE.

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISTURBANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.5</td>
<td>0.0</td>
</tr>
<tr>
<td>0</td>
<td>1.5</td>
<td>0.0</td>
</tr>
<tr>
<td>0</td>
<td>1.1</td>
<td>0.09</td>
</tr>
<tr>
<td>0</td>
<td>1.2</td>
<td>0.11</td>
</tr>
<tr>
<td>0</td>
<td>1.3</td>
<td>0.11</td>
</tr>
</tbody>
</table>
After each group of five measurements he is given some additional advice about what is happening. He is confronted with the problem of dealing with a situation with two independent variables. The complexity of the situation is such that if one simply picks unorganized values of these two variables, success or understanding is unlikely. We expect that some students, but not all, will develop in these early measurements what we consider to be a reasonable strategy. The student at this point is still in the dark as to the full details of what we expect of him. He has been told that he needs to study the disturbance to understand what is happening in the rope, but he does not know what kind of information about the rope we are going to request of him. So he may not go into what we think of as a reasonable strategy.

We define reasonable strategy as meaning one of two things. Either the student clusters his measurements around certain particular times, the snapshot point of view in which he looks at the detailed behavior of the rope at a number of different times, or he clusters his measurements around relatively few values of position—the point of view in which he stands at particular positions and watches what happens as the disturbance in the rope passes that position.

This particular student has decided to find out what is happening in the rope at time 0, following the snapshot standby.
After the student has made fifteen measurements we now turn on a new set of facilities for him.

You may have some idea of how the rope is behaving. At this point I will change the rules of the game. For:

- Measurement Type M
- Turn-the-tables Type T
- List of Measurements Type L
- Graph Type G

Don't be disturbed if you can't turn the tables at first—I will give you other chances.

You can see that he can get a list of his values, useful particularly if the student is running the program on a CRT terminal, where he has no hard copy. He can also receive a graph of the data in selected ways. Here is such a request, and the results.

Furthermore, he can for the first time now determine what it is that we are going to ask him to do in the program, the measure of success, by asking to "turn the tables." Since he can do this over and over again we expect many students to try this at the earliest possible moment.
Our student first asks for a list.

Table:

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISTURBANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>26.9</td>
<td>0</td>
</tr>
<tr>
<td>1.80E17</td>
<td>6.40</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>-2.24</td>
<td>-8.24</td>
<td>4.66E-2</td>
</tr>
<tr>
<td>-0.06</td>
<td>0.34</td>
<td>0.06</td>
</tr>
<tr>
<td>-1.64</td>
<td>-6.36</td>
<td>0.28</td>
</tr>
<tr>
<td>-0.66</td>
<td>-2.63</td>
<td>0.32</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.07</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.10</td>
</tr>
<tr>
<td>0</td>
<td>-0.5</td>
<td>0.09</td>
</tr>
<tr>
<td>0</td>
<td>1.5</td>
<td>0.09</td>
</tr>
<tr>
<td>0</td>
<td>1.10</td>
<td>0.09</td>
</tr>
<tr>
<td>0</td>
<td>1.20</td>
<td>0.10</td>
</tr>
<tr>
<td>0</td>
<td>1.30</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Trying everything, he now investigates what graphic facilities are available to him.


WHAT INDEPENDENT VARIABLE DO YOU WANT FOR YOUR GRAPH?  TIME

FOR WHAT VALUE OF POSITION?  1

NOT ENOUGH MEASUREMENTS AT THAT VALUE TO BOTHER PLOTTING.


WHAT INDEPENDENT VARIABLE DO YOU WANT FOR YOUR GRAPH?  POSITION

FOR WHAT VALUE OF TIME?  0

-1  MIN  HORIZONTAL  MAX  2
0  MIN  VERTICAL  MAX  0.3207

The dialog will only graph his data when a reasonable amount is available.
Our student decides to try a few more measurements:

How many measurements in this block? 7

| TIME = 0 | POSITION = -9 | DISTURBANCE = 0 |
| TIME = 0 | POSITION = -8 | DISTURBANCE = 1.01E-2 |
| TIME = 0 | POSITION = -7 | DISTURBANCE = 2.77E-3 |
| TIME = 0 | POSITION = -6 | DISTURBANCE = 5.30E-4 |
| TIME = 0 | POSITION = -5 | DISTURBANCE = 0.16 |
| TIME = 0 | POSITION = -4 | DISTURBANCE = 0.20 |
| TIME = 0 | POSITION = -3 | DISTURBANCE = 0.26 |

How many measurements in this block? 7

| TIME = 0 | POSITION = -1 | DISTURBANCE = 0.31 |
| TIME = 0 | POSITION = 1 | DISTURBANCE = 0.31 |

He is clearly filling in what appear to be tides in the data, as shown on the graph. We allow him to specify a block of measurements, so that he is not queried each time as to what he wants to do. Note that he has not yet completed the second block of seven measurements; at this point we are prepared to assist him.

After the student has reached the "more than 15 measurements" situation, we periodically give advice and assistance. The type of advice depends on whether the student has developed a reasonable strategy in the sense already suggested. If he has not developed a reasonable strategy, taking either the snapshot point of view or the stand-at-one-point point of view, then we try through a series of successively strong hints to suggest a strategy. In the end we practically tell him how to proceed, because we don't want him to sit forever taking random measurements. After he has a reasonable
strategy, a second set of hints takes over. These hints are designed to slowly suggest to him the moving pattern idea, the x - vt dependence that we hope will be his eventual conclusion.

In some of the later hints we actually give him additional values, showing him what would happen if he followed a consistent strategy. For example in one place we plot a picture of the curve for t = 0, giving values that he may not have asked for. Thus we show him the disturbance at one time, and hope that will be enough to get him going in a reasonable direction.
Our prototype student has been making meaningful measurements, so he gets a hint from the second set, and a suggestion that he "turn the tables." Then measurements continue, and he asks for a more complete graph.

BREAK AT LOC #:SAVETS+.18

BREAK AT LOC #:SAVETS+.25

YOUR STRATEGY SEEMS REASONABLE. YOU SHOULD HAVE SOME IDEA AS TO HOW THE ROPE IS BEHAVING.

YOU HAVEN'T TRIED TO BE ON THE OTHER END YET, WHERE YOU TELL ME THINGS! YOU SHOULD HAVE ENOUGH INFORMATION.

| TIME = 0 | POSITION = .2 | DISTURBANCE = 0.26 |
| TIME = 0 | POSITION = .3 | DISTURBANCE = 0.21 |
| TIME = 0 | POSITION = .4 | DISTURBANCE = 0.15 |
| TIME = 0 | POSITION = .6 | DISTURBANCE = 0.06 |
| TIME = 1 | POSITION = 1  | DISTURBANCE = 0    |

WHAT INDEPENDENT VARIABLE DO YOU WANT FOR YOUR GRAPH? POSITION

FOR WHAT VALUE OF TIME? 0

<table>
<thead>
<tr>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0.3207</td>
</tr>
</tbody>
</table>
The disturbance at \( t = 0 \) clearly has two humps. Each student using the dialog receives a disturbance of this type, but with parameters chosen randomly within limits.

The student next follows the suggestion to try turn the tables.

**MEASURE? LIST? TURN? GRAPH?**

YOU KNOW ALREADY THAT AT \( T = -1.64 \) AND AT \( X = -6.38 \)

\[ \text{THE DISTURBANCE} = \theta.28 \]

AT \( T = 4.28 \) THE DISPLACEMENT IS TO BE THE SAME.
WHAT VALUE OF POSITION MAKES THIS THE CASE?

\( \theta \)

TRY ONCE MORE. ACCURACY .1.

\( \theta \)

I CAN'T IDENTIFY YOUR RESPONSE AS CORRECT.
THE POSITION SHOULD BE 16.97
TRY MORE MEASUREMENTS, AND TYPE TURN WHEN YOU THINK YOU CAN ANSWER QUESTIONS LIKE THIS.

This student clearly realizes that he doesn't have enough information to make the prediction, so he types only zeros. Note that he works partially with data already obtained, and partially with randomly generated new data.

This student fails for a variety of reasons. He does not understand how to make the calculation and furthermore he is not in a position to make it even if he does understand, because his measurement has not been in enough detail to tell him the velocity of the disturbance. You can see, although the student will not see it yet, that his ability to answer the questions is based on an understanding that the displacement function, giving this disturbance in the rope as a function of position and time, depends on position and time always...
in the combination of $x - vt$. The student has given several tries. We know from observing students, that initially he may make a cruder calculation, so we advise him with regard to accuracy if he gives us a wrong answer.
Our hypothetical student, quicker than most, now goes after the velocity of the disturbance.


HOW MANY MEASUREMENTS IN THIS BLOCK? 10

<table>
<thead>
<tr>
<th>TIME</th>
<th>POSITION</th>
<th>DISTURBANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.32</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0.32</td>
</tr>
</tbody>
</table>

ARE YOU MAKING GRAPHS OF THE ROPE?
HAVE YOU THOUGHT ABOUT THE BEHAVIOR OF THE STRING AT DIFFERENT TIMES. WHAT DOES IT LOOK LIKE AT ANY ONE TIME? THEN WHAT HAPPENS TO IT?
I SEE THAT YOU ARE HOLDING THE TIME CONSTANT IN MANY MEASUREMENTS.

TIME = 1
POSITION = 5.21
DISTURBANCE = 0.10

TIME = 1
POSITION = 5.3

JUST A NUMBER, PLEASE.

TIME = 1
POSITION = 5.3
DISTURBANCE = 0.11

TIME = 1
POSITION = 5.4
DISTURBANCE = 0.10

TIME = 1
POSITION = 6
DISTURBANCE = 0

TIME = 1
POSITION = 3.5
DISTURBANCE = 0.09

MEASURE? LIST? TURN? GRAPH? 0

WHAT INDEPENDENT VARIABLE DO YOU WANT FOR YOUR GRAPH?
POSITION

FOR WHAT VALUE OF TIME? 1

<table>
<thead>
<tr>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>HORIZONTAL</td>
<td>6</td>
</tr>
<tr>
<td>VERTICAL</td>
<td>0.3207</td>
</tr>
</tbody>
</table>
Here the choice of pattern we have made, a double baussio with unequal peaks, assist him in identifying the form that he is dealing with. In particular if he has found the maximum, he can try to find how the maximum moves in order to determine the wave velocity. His graph suggests that he is dealing with a moving pattern, keeping the same shape in time, and moving with a velocity of 4 meters/second. So now he returns, with confidence, to turn the tables! Most students will not succeed this quickly, so this chap must be bright.


YOU KNOW ALREADY THAT AT T = 0 AND AT X = 0 THE DISTURBANCE = 0.32

AT T = 5.06 THE DISPLACEMENT IS TO BE THE SAME. WHAT VALUE OF POSITION MAKES THIS THE CASE?

?21.5

SEEMS GOOD. LET’S TRY ANOTHER OF THE SAME TYPE.

YOU KNOW ALREADY THAT AT T = 0 AND AT X = 1.20 THE DISTURBANCE = 0.10

AT T = 3.60 THE DISPLACEMENT IS TO BE THE SAME. WHAT VALUE OF POSITION MAKES THIS THE CASE?

?14.4

TRY ONCE MORE. ACCURACY .1.

?15.6

FINE.... NOW WE'LL PLAY THE GAME A SLIGHTLY DIFFERENT WAY.
Here are two measurements for time $T = 1$
for $x = 3.5$ the rope displacement is $0.09$

and for $x = 6$ displacement is $0$

At time $T = 0.15$ and at the position $x = 0.11$
the displacement is the same as in the first measurement above: $0.09$

At the same time what will be the value of $x$ that gives the displacement the same as
that of the second measurement above?

$22.6$

Give it another try...think about what happens with the string in time.

$22.6$

As far as I can tell with my limited ability you have successfully learned how the
rope is behaving. The behavior that we have been examining is typical of a wave, an
important physical concept.

Please type a verbal description of what is happening with our rope. Use the line feed
for multiple lines, only using carriage return when finished. If you wish to have your
description evaluated by the instructor to see if you understand this aspect of wave
behavior, type your name also.

Any comments about the program are also welcome.

A pattern with two humps is moving down the string. The pattern appears to stay to same shape. Its velocity
is four meters per second.

Congratulations and goodbye!
If the student is successful we ask him to comment on the dialog which might help us to improve it, and we congratulate him on his understanding what is happening in the rope.

Clearly we do not want the students to spend forever at this game. It may be that it will be too difficult and that the student needs other ways of learning the $x - vt$ dependence. At the moment we have an arbitrary cutoff of 100 measurements. If the student has not succeeded, we first check to see if he has tried to "turn the tables." If he has not we send him in to try that. But if he has already turned the tables, we ask him for comments, express our sorrow that we have not succeeded in accomplishing our objective, and suggest that he might want to talk directly with the teacher. As in similar situations with dialogs the comments are stored in a file for future evaluation.

[Add comments about usage]