DOCUMENT RESUME

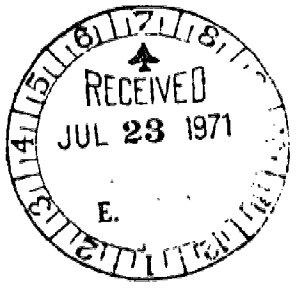
ED 057 930 RC 004 831

AUTHOR Stodola, Janet
TITLE Mathematical Contributions of the Mayas, Aztecs & Incas: A Native American Curriculum Unit for Middle and High School. NATAM XIX.
INSTITUTION Minnesota Univ., Minneapolis. Training Center for Community Programs.
PUB DATE May 71
NOTE 19p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS American Indian Languages; *American Indians; Curriculum Development; *Mathematics; Mayan Languages; *Number Systems; *Resource Units; *Secondary Grades; Uto Aztecan Languages
IDENTIFIERS Aztecs; Incas; Mayas

ABSTRACT

Written to fulfill the requirements for a University of Minnesota College of Education off-campus Indian education course for public school teachers, this Native American curriculum unit for middle and high school reflects the mathematical achievements of the Maya, Aztec, and Inca Indians. The number systems, notation, and calendar techniques of the 3 groups are discussed. Included is an 8-item bibliography. (MJB)



MATHEMATICAL CONTRIBUTIONS OF THE MAYAS, AZTECS & INCAS: A NATIVE AMERICAN CURRICULUM UNIT FOR MIDDLE AND HIGH SCHOOL

NATAM XIX

by

Janet Stodola

Series Coordinators:

Arthur M. Harkins, College of Education

I. Karon Sherarts, CURA

G. William Craig, General College

Richard G. Woods, CURA

Charles R. Bruning, College of Education

Training Center for Community Programs

in coordination with

Office of Community Programs
Center for Urban and Regional Affairs

Training of Teacher Trainers Program
College of Education

Minnesota Federation of Teachers

University of Minnesota
Minneapolis, Minnesota

May, 1971

This series of Native American Curriculum Units was authored by Minnesota public school teachers while they were enrolled in a University of Minnesota College of Education off-campus Indian education course. The course was taught in the suburbs largely through the initiation of the staff of the Indian Upward Bound Program, an Office of Education funded Minneapolis junior high community school program staffed by Indians and under Indian board control.

The production and distribution of these curriculum units to teachers across the State of Minnesota was made possible by the cooperation and contributions of several agencies.

The Minnesota Federation of Teachers is a teacher union movement affiliated with the AFL-CIO which seeks to promote collective bargaining relationships with school boards and other educational employers. Its activities at the national, state and local levels are directed to all the concerns of teachers about developing a better educational climate for children.

The Training of Teacher Trainers Program, College of Education, University of Minnesota, attempts to help Minnesota colleges and the Minneapolis and St. Paul school systems do a better job of training teachers for inner-city jobs.

The Training Center for Community Programs and the Office of Community Programs are operating divisions of the Center for Urban and Regional Affairs (CURA), University of Minnesota. CURA was established by the Regents to help make the University more responsive to the needs of the larger community, and to increase the constructive interaction between faculty and students, on the one hand, and those dealing directly with major public problems, on the other hand.

These curriculum units are an outgrowth of the participation of the University of Minnesota in the National Study of American Indian Education, USOE Number OEC-0 - 8 - 08 - 147 - 2805.

A Note on the First NATAM Curriculum Series

During the Spring of 1970, a special University of Minnesota course in Indian education was offered through the College of Education and the General Extension Division to public school teachers in the school system of Columbia Heights, a Minneapolis suburb. This course--which was taught in Columbia Heights--was arranged and specially designed as a result of a request from Columbia Heights school officials and teachers to Mr. Gene Eckstein, Director of Indian Upward Bound. (Indian Upward Bound is a special Indian education program funded by the U.S. Office of Education, the University of Minnesota, and the Minneapolis Public Schools. It operates at two inner-city Minneapolis junior high schools, and functions under the control of an all-Indian board of directors.) In addition to the usual on-campus course requirements, such as reading, rollees were given special lectures by invited Indians in addition to the person responsible for accreditation, Dr. Arthur Harkins. Lecturers were compensated for their contributions by a special fee paid by the course enrollees. A complete listing of the lecture sessions follows:

April 1, 1970	Mr. Charles Buckanaga (Chippewa) "Indian Americans and United States History" Mr. Buckanaga presented a brief resume of the relationship of the American Indian and the in-coming European Cultures. He also discussed a three-dimensional view of historical data, emphasizing the development of gradual feelings toward and the eventual end result of the native Americans.
April 8, 1970	Mr. Roger Buffalohead (Ponca) "Urban Indian" Mr. Buffalohead discussed the conflicts and problems confronting the Indian in the migration to the Urban setting.
April 15, 1970	Lecture on Urban Indians Dr. Arthur Harkins - University of Minnesota.

April 15, 1970	Gene Eckstein (Chippewa) "Cultural Conflict and Change." Mr. Eckstein discussed the changing cultures of the Indian American and the problems encountered.
April 22, 1970	G. William Craig (Mohawk) "Treaties and Reservations." Treaties by the United States and American Indian Nations. The out growth of reservations and their influences on the American Indian.
April 29, 1970	Lecture H Ed. III Dr. Arthur Harkins
May 6, 1970	Gene Eckstein (Chippewa) The psychological and sociological challenges of the Indian American citizen in the transition from the Indian reservation to an urban area.
May 13, 1970	Lecture H Ed. III Dr. Arthur Harkins
May 20, 1970	Mr. Will Antell (Chippewa) "Indian Educational Conflicts" Director of Indian Education in Minnesota, Mr. Antell presented the challenges of the teacher in Indian Education, together with their relationship to the Indian student, Indian family and Indian community.
May 29, 1970	Lecture H Ed. III Dr. Arthur Harkins Comments from the class - final examination.

As a course requirement, each teacher taking the course for credit authored a curriculum unit for the grade level or subject area which he or she was actively teaching. The best of these units - a total of nineteen - were selected, and the over-all quality was judged to be good enough to warrant wider distribution. It was felt that the units were a good example of what professional teachers can do--after minimal preparation, that the units filled an immediate need for the enrolled teachers for curriculum material about Indian Americans, and that they served as an opportunity to test a staff development model. The units were endorsed by a special motion of the Indian Upward Bound Board of Directors.

From Indian Upward Bound Board meeting--Thursday, January 7, 1971.

Certain people are asking that the curriculum guide of the NATAM series be taken from school teachings. There was discussion on this and it was suggested instead of criticizing the writing make suggestions on how to better them. Gert Buckanaga made a motion that we support the experimental curriculum guides. Seconded by Winifred Jourdain. Motion carried.

To accomplish distribution, the units were typed on stencils, mimeographed, assembled and covered. Costs were shared by the University's Training Center for Community Programs and the Training of Teacher Trainers Program of the College of Education. The units were then distributed throughout the state by shop stewards of the Minnesota Federation of Teachers, and AFT affiliate. The entirety of these distribution costs were borne by MFT.

A new NATAM series is currently being prepared. It will focus upon contemporary reservation and migrated Native Americans.

The Coordinators,
May, 1971

TABLE OF CONTENTS

Introduction

Upon completion of research concerning the mathematical contributions of the Mayas, Aztecs, and Incas, I have chosen to write the paper in two parts. The mathematical systems established by the Mayas and the Aztecs are similar in numerous ways, and therefore will be treated as one topic. Both of these systems originated primarily for religious reasons, and were fundamentally concerned with the establishment of a calendar. The Incas, on the other hand, used their mathematical system basically for computational purposes, and devised a rather sophisticated method for recording data.

Contributions of the Mayas and Aztecs

The relationship between the Mayan and the Aztec empires has been likened to that which existed between the Greek and the Roman empires.[1] The Mayan civilization was founded first, and like the Greek was more theoretical in its establishment of its number system. When the Aztecs became the major power they assimilated most of the Mayan work in the same manner as the Romans had done to the Greeks. Few new advances were made by the Aztecs.

The primary contributions of the Mayan empire were the rigorous mathematical systems they devised. Little is known of the advanced mathematical studies made by the Mayans, but the remains of their buildings and temples attest to the fact that they had a deep understanding of many of the sciences. The Mayans appear to have spent most of their time in the study of astrology and the prediction of various solar phenomena. They devised a calendar as early as 3372 B. C.[2] which is still being used by many of the ancestors, including the Jacaltins of Northeast Guatemala.[3] This calendar is so accurate that it has not lost a day in over 5000 years of existence. It is the oldest calendar in use today.

Archaeologists have found Mayan temples which date back as far as 900 B. C. The artifacts found in these temples show that the Mayans had a fully developed numeration system at that time. The Mayan number system is unique from any that history records. All of the ancient number systems (including ours today) use ten as their base. In other words, the present number 4,873 equals $(4 \times 1000) + (8 \times 100) = (7 \times 10) + (3 \times 1)$ or $(4 \times 10^3) + (8 \times 10^2) + (7 \times 10) + (3 \times 1)$. The symbols used by the Egyptians[4] or the Romans[5] may have been different from the ones presently used, but they were still based on the number ten. The Mayans, in contrast, used a system based on the number twenty.[6] This we will refer to as the vigesimal system. In their system the number 4,873 would equal $(4 \times 8000) + (8 \times 400) + (7 \times 20) + (3 \times 1)$ or $(4 \times 20^3) + (8 \times 20^2) + (7 \times 20) + (3 \times 1)$ which is equivalent to 35,413 in a base ten system. They used this system primarily for their business transactions.

When writing numbers in vigesimal they used two different forms. One was a system based on dots and bars. In comparison with the ancient systems of the Romans and Egyptians, the simplicity of the Mayan ststem makes it superior. The Mayans are known as one of the first civilizations to devise a symbol for zero. It is often called the "Shell Design" and is believed to represent the frontal view of a closed fist.[7] The dot (or pebble) was the symbol for one, and the horizontal bar (stick) represented five. This system is the most easily understood, and the more convenient to use. The additions were made vertically so that seventeen would be represented as $\overline{\overline{\cdot\cdot}}$. In order to write numbers greater than twenty, they added the groupings vertically with the top grouping having the highest positional value. To illustrate: the number 2,431 would be written in vigesimal as --

$\underline{\cdot}$ -- $(6 \times 20^2) = 6 \times 400 = 2400$

$\cdot$ -- $(1 \times 20) = 20 = 20$

$\overline{\cdot}$ -- $(11 \times 1) = 11 = 11$

2431

and four hundred one would be represented by -

$\cdot$ -- $(1 \times 20^2) = 400$

-- $(0 \times 20) = 0$

$\cdot$ -- $(1 \times 1) = 1$

401 [8]

The second form of numbers the Mayans used roughly corresponds to the Arabic numerals we presently use. They employed head-variant symbols which represented thirteen of their gods for the numbers zero through nine and eleven through thirteen. Ten was represented by the death head, and fourteen through nineteen by affixing the lower jaw of the death head to the symbols for three through nine.[9] These are the symbols found on many of their artifacts. Often these symbols were used to represent dates. From top to bottom the first represented the year, the next the month and the last the day.

The Mayans modified the vigesimal system when devising their calendar. The modification was made in the third (or hundreds) represent 8 x 400 or 8 x 20^2 as it did in the vigesimal system. They changed 20^2 to 18 x 20 which made it easier to devise a calendar with exactly 360 days. The 360 days constituted an official year which was called a Tun.[10] In their calendrics system each higher power of twenty was modified to be 18 x 20^{n-1} instead of 20^n. For example in the calendar number 3789541, the seven would represent 7 x 18 x 20^4 instead of 7 x 20^5 as it did in the vigesimal system. Because of the difference in these two systems, one number could have two different values depending upon whether you were talking in terms of calendar days, or you were talking in terms of numerical commodities.

The Quinche tribe at Monostenango, Mexico still uses the ancient Mayan calendar. They now use it chiefly for religious purposes and as a means of divination. The Mayan calendar names the days after various gods. Each day is given a number plus a name. The numbers and names rotate as follows:

	Day-sign	Presiding God
1.	Kan (maize)	God E
2.	Chicchan (serpent)	God H
3.	Cimi (death)	God A
4.	Manik (deer)	God F.
5.	Lamat (planting)	Grain-god
6.	Muluc (cloud)	God K
7.	Oc (dog)	Lightning-god
8.	Chuen (monkey)	God C
9.	Eb (grass)	A goddess
10.	Ben (reed)	Chahalhuc
11.	Ix (jaguar)	Jaguar-god
12.	Men (moan bird)	God G
13.	Cib (vulture)	The vulture-being

1.	Caban (earth)	The Four Bacabs, or deities of the four quarters
2.	Eznab (knife)	God I
3.	Cauac (rain)	Tortoise-god
4.	Ahau (god Ahau)	God, or Ahau
5.	Imix (honey)	Honey-god
6.	Ik (air)	God B (Kukulkan)
7.	Akbal (darkness)	God L
8.	Kan (maize)	God E -- repeat, continued[11]

Since the numbers are also named after gods, the days achieve different significance depending upon the combination of gods represented. The calendar is devised in such a way that each pair of numbers and day names occur only once each religious year. For example, the day 8 Batz is considered to be the most fortunate day of the year, and 5 Toh is considered unlucky for women.[12]

In the Mayan calendar, each month has twenty days and is given a name and number like the days. They set up two systems of calendars using this twenty day month. The first was the religious calendar which consisted of thirteen months or 260 solar days. The second calendar was used for official purposes and consisted of eighteen months or 360 days plus five days which they titled "unlucky days." Using these two calendars concurrently enabled them to predict eclipses to a high degree of accuracy. These predicted eclipses were an important part of their religious lives.[13]

Since each month and each day was given a name and number, the exact combination of numbered month and numbered day occurred only once every fifty-two official years. This time period of fifty-two official (or seventy-three religious) years was called a "calendar round" and was celebrated in various ways.[14] Upon completion of a temple a stela or stone monolith was erected each succeeding "calendar round" or each fifty-two civil years. This custom has been useful to archeologists when studying Mayan ruins.[15]

Aztecs

Upon conquering the Mayas, the Aztecs assimilated many of their customs and ideas. They did, however, discard the two types of Mayan number systems and replace them with a base ten number system which in certain aspects resembled the Egyptian system. The symbols used in their number system were the following:

units 10 20 100 200 300 400 8000

The Aztecs modified the Mayan calendar by changing the names of the months and days. The basic idea behind the Mayan religious calendar, nevertheless, remained the same. The Aztecs used the twenty day month, and the year of thirteen months of 260 days. The days and months also maintained the same numbering system with certain days having different significance depending upon the god after which it was named. The Aztec calendar was called the Tonalmanatl and the days were named as follows:

	Day-Sign	Presiding God
1.	Cipactli	Crocodile (good)
2.	Eecatl	Wind (uncertain)
3.	Calli	House (uncertain)
4.	Cuetzallin	Lizard (good)
5.	Coatl	Serpent (bad)
6.	Miquiztli	Death's - head (unlucky)
7.	Mazatl	Deer (unlucky)
8.	Tochtli	Rabbit (good)
9.	Atl	Water (bad)
10.	Itzcuintli	Dog (unlucky)
11.	Malinalli	Monkey (uncertain)
13.	Acatl	Grass (unlucky)
1.	Ocelotl	Ocelot (bad)
2.	Quauhtli	Eagle (lucky)
3.	Cozcaquauhtli	Vulture (bad)

4.	Ollin	Motion (uncertain)
5.	Tecpatl	Flint knife (bad)
6.	Quiauitl	Rain (unlucky)
7.	Xochitl	Flower (good)

The months were also named as follows:

Gods of the "weeks"

1. Tonacatecutli
2. Quetzalcoatl
3. Tepeyollotl
4. Ueuecoyotl
5. Chalchihuitlicue
6. Tecciztecatl
7. Tlaloc
8. Mayauel
9. Xiuhtecutli
10. Mictlantecutli
11. Patecatl
12. Itzlacoliohqui
13. Tlazolteotl
1. Xipe Totec
2. Itzpapalotl
3. Xolotl
4. Chalchuihtotollin
5. Chantico[16]

The Aztecs made a further contribution to this system of recording time, by dividing the day into thirteen hours and the night into nine hours. The hours were each named for "lords" or patron gods with each of these having a designated significance which was derived from its numerical standing. To the Aztecs the numbers three and four were thought to be lucky, five and six ominous or threatening, seven good, eight and nine bad, and ten through thirteen good. The hours were named as follows:

Day Hours	Night Hours
1. Xiuhtecutli	1. Xiutecutli
2. Tlaltecutli	2. Itztli
3. Chalchihuitlicue	3. Piltzintecutli
4. Tonatuih	4. Centeotl
5. Tlazolteotl	5. Mectlantecutli
6. Teoyaomiqui	6. Chalchihuitlicue
7. Xochipilli	7. Tlazolteotl
8. Tlaloc	8. Tepeyollotl
9. Quetzalcoatl	9. Tlaloc[17]
10. Tezcatlipoca	
11. Mictlantecutli	
12. Tlauizcalpantecutli	
13. Ilamatecutli	

The Aztec recorded their writings on paper made of agave plant or painted on the skins of animals. Their early writing is often described as being "pictographic," but as they continued to communicate from generation to generation it became more phonetic. Reading and writing were indulged in only by the elite of the Aztec society. The Aztecs have the distinction of leaving the most complete history of the American continent from which historians were able to derive most of their knowledge of the civilization of that time.[18]

Contributions of the Incas

At the height of the Inca Empire they controlled nearly all the peoples of South America. Their culture and society differs greatly from that of the Mayan and Aztecs which were located in the northern hemisphere. Both the Mayan and Aztecs at an early age developed systems of alphabet and numbers, but this is not true of the Incas. When Pizarro conquered the Incas in the first half of the sixteenth century, he discovered no written form of communication. In place of written communication the Incas developed a system called quipou which consisted of little more than knotted cords. This they used for innumerable purposes including recording decisions, checking arms and soldiers, and drawing up statistics about population and land.[19]

Chosen people in the community were designated as "Rune Quipoc Inca" or superintendent of the Quipou. At first appearance the quipou seems to be nothing more than an aid to memory, but it had more significance than this. For example, when Huascar, the twelfth emperor of the Inca, died, his quipou was buried with him to help him in his after life. The Incas also did this in order to prepare for the new era under the next emperor Alahualpa.[20]

> The Incan quipou was principally made of a greyish-white rope which was twisted between two thinner cords. From this rope hung 48 secondary cords, divided into five groups. To some of these cords were affixed extra threads. There were in all 87 cords...Knots were made on each cord, starting from the lower end...The first series of knots represented units, the second tens and the third hundreds. The small cords were colored. In collating the accounts given by the chroniclers it may be supposed that black meant time, red the army or king, green meant enemies, yellow meant gold, and white represented silver. There were cords of several colors whose position indicated certain meanings. Each quipou was arranged to register one of the activities of the administration; the whole collection recorded the entire life of the Empire.[21]

Even the courts used quipous to determine which taxes or tariffs to levy or what the punishment should be for various crimes.[22] The superintendents of the quipou instructed each new generation of superintendents so that they were able to account for everything that had happened in the country since the Inca government begun. Huaman Poma, an Inca historian, stated that "the whole realm was ruled by the cords!"[23]

Summary

Nearly all of the mathematical knowledge we currently have was derived from the systems set up by the Egyptians and Greeks, yet the mathematical systems established by the early Western culture have numerous similarities, and even areas where they are superior. The simplicity of the Mayan system to dots and bars is in most respects superior to the early Greek and Roman systems and in some respects even to our own digital system. The Mayans, for example, founded a symbol to represent zero centuries before it was derived in the East. Most of our current mathematical knowledge would be severely decreased had not the concept of zero been established.

Upon reviewing the contributions of the Mayans, Aztecs and Incas, one must remember that these systems were being formulated at a time when most civilizations were in a relatively primitive state. The fact that the Western cultures did not deal as much with geometry as the Egyptians and Greeks did, is not that surprising. Both the Greeks and Egyptians began their study of geometry for utilitarian purposes; they needed to devise irrigation systems. This need probably never arose in the West, because the climate and land structure is so different. Both the Aztecs and Mayans, nevertheless, built pyramids or temples which, by their structure, showed a deep understanding of various geometric concepts and properties.

When the Spanish overtook the Western civilizations of the Aztecs and Incas, most of their documents were destroyed. When one studies the ruins of the temples and buildings they built, one realizes that their knowledge of mathematics and the sciences must have been far greater than that reported here. So much of what they accomplished has been lost to civilization, that we are left with mere speculations of what might have been.

Footnotes

[1]Herbert Joseph Spinden, Mayan Art and Civilization (The Falcon's Wing Press, 1957), p. 346.

[2]Donald R. Byrkit, Early Mayan Mathematics (The Arithmetic Teacher, Vol. 17, May, 1970), p. 389.

[3]F. Eric S. Thompson, Maya Archaeologist (Norman: University of Oklahoma Press, 1963), p. 188.

[4]Lancelot Hogben, Mathematics in the Making (New York: Doubleday and Co., Inc., 1960), p. 30.

[5]Hogben, op. cit., p. 31.

[6]Byrkit, op. cit., p. 387.

[7]Ibid., p. 388.

[8]Ibid., p. 388.

[9]Ibid., p. 387.

[10]Ibid., p. 389.

[11]Lewis Spence, The Magic and Mysteries of Mexico (David McKay Co.), p. 223.

[12]Thomas Gann, Glories of the Maya (New York: Charles Scribner's Sons, 1939), p. 147.

[13]Brykit, op. cit., p. 387.

[14]Ibid., p. 389.

[15]Thompson, op. cit., p. 190.

[16]Spence, op. cit., pp. 146-47.

[17]Ibid., pp. 149-50.

[18]Thompson, op. cit., p. 184.

[19]Bertrand Flornoy, The World of the Inca (The Vanguard Press, 1956), p. 113.

[20]Burr Cartwright Brundage, Empire of the Inca (Norman: University of Oklahoma Press, 1963), p. 295.

[21]Flornoy, op. cit., p. 115.

[22]Brundage, op. cit., p. 223.

[23]Flornoy, op. cit., p. 115.

Bibliography

Brundage, Burr Cartwright, Empire of the Inca. Norman: University of Oklahoma Press, 1963.

Brykit, Donald R., Early Mayan Mathematics. The Arithmetic Teacher, Vol. 17, May, 1970.

Flornoy, Bertrand, The World of the Inca. The Vanguard Press, 1956.

Gann, Thomas, Glories of the Maya. New York: Charles Scribner's Sons, 1939.

Hogben, Lancelot, Mathematics in the Making. New York: Doubleday and Co., Inc., 1960.

Spence, Lewis, The Magic and Mysteries of Mexico. David McKay Co.

Spinden, Herbert Joseph, Mayan Art and Civilization. The Falcon's Wing Press, 1957.

Thompson, F. Eric S., Maya Archaeologist. Norman: University of Oklahoma Press, 1963.