Part of the program of the Center for Social Organization of Schools at Johns Hopkins University is one which examines how a student's education affects his actual occupational attainment, and how education results in different vocational outcomes for blacks and whites. This report comes out of such a context. Properties of measures and models of social mobility are analyzed in relation to the conceptualization of mobility. Two main objectives of mobility research are identified: one is the study of determinants of occupational achievement; the other is the study of mobility as a characteristic of social systems. It is shown that the realization of both objectives is hindered by a failure of commonly used models and measures of mobility to separate out the various individual and structural factors responsible for mobility. (Author/JW)
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Models of Social Mobility

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INTRODUCTORY STATEMENT

The Center for Social Organization of Schools has two primary objectives: to develop a scientific knowledge of how schools affect their students, and to use this knowledge to develop better school practices and organization.

The Center works through five programs to achieve its objectives. The Academic Games program has developed simulation games for use in the classroom, and is studying the processes through which games teach and evaluating the effects of games on student learning. The Social Accounts program is examining how a student's education affects his actual occupational attainment, and how education results in different vocational outcomes for blacks and whites. The Talents and Competencies program is studying the effects of educational experience on a wide range of human talents, competencies and personal dispositions, in order to formulate -- and research -- important educational goals other than traditional academic achievement. The School Organization program is currently concerned with the effects of student participation in social and educational decision making, the structure of competition and cooperation, formal reward systems, ability-grouping in schools, effects of school quality, and applications of expectation theory in the schools. The Careers and Curricula program bases its work upon a theory of career development. It has developed a self-administered vocational guidance device to promote vocational development and to foster satisfying curricular decisions for high school, college, and adult populations.

This report, prepared as part of the Social Accounts program, is a review of models and measures of social mobility. The concept of social mobility, for the Social Accounts program, is important in predicting and evaluating the effects of educational achievement on occupational attainment and vocational outcomes.
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Properties of measures and models of social mobility are analyzed in relation to the conceptualization of mobility. Two main objectives of mobility research are identified. One is the study of determinants of occupational achievement, the other is the study of mobility as a characteristic of social systems. It is shown that the realization of both objectives is hindered by a failure of commonly used models and measures of mobility to separate out the various individual and structural factors responsible for mobility.
1. **INTRODUCTION**

One of the essential tasks in research is to make sure that there is a close correspondence between the concepts of a theory and the empirical measures of these concepts. When the focus is on mathematical models of some phenomenon, this demand is formulated as one of asking that the mathematical model be isomorphic with the conceptualization of the phenomenon (Coleman, 1964).

It often can be difficult to tell to what extent there is a close correspondence between measures and concepts. The conceptualization of social phenomena studied in empirical research is often imprecise, and many measures and models have rather obscure properties. It is easy to lament this situation, and too much lamenting may very well inhibit original and creative research. Few will disagree, however, with the ultimate objective, a high degree of validity. An analysis of the extent to which this goal was achieved in a specific research area, pointing out where things went wrong, should therefore have some value.

It is a precondation for such a task that the area of research focused on must have sufficient coherence, so that a confrontation of concepts with measures is possible. The field of social mobility seems to have the necessary coherence. A strong consensus prevails regarding the conceptualization of mobility, and this consensus may be formulated in rather precise and unequivocal terms. In addition, a large body of empirical research exists which has often placed emphasis on the development of adequate measures and models.
This paper will analyze the adequacy of some measures and models used in mobility research in relation to the conceptualization of mobility. The analysis will be made in the context of the goals of mobility research. The realization of these goals, rather than correspondence with some epistemological principles, should be used as the main criterion of evaluation.

The paper will not deal with problems caused by the quality of data, since this is not a problem of internal validity. The type of data used in mobility research, especially the heavy reliance on intergenerational data, does create some validity problems. These problems are analyzed at length elsewhere (Duncan, 1966), and only scant attention will be given this aspect of the validity question here.

2. THE CONCEPTUALIZATION OF MOBILITY

The prevailing conceptualization of social mobility may be largely traced back to Sorokin's pioneering work in the 1920's. Sorokin (1927) gave a definition of the phenomenon, an account of the main factors responsible for it, and a classification of various forms of mobility. His general outline has been followed in subsequent empirical research. No controversy is known to have existed around this conceptualization, although there has been a tendency here, as elsewhere in sociological research, to develop private vocabularies. Our presentation of the conceptualization of mobility will not follow any single author, but will summarize the most important notions.

Basic in the conceptualization of mobility is the distinction between the positions in a social structure and the individuals who occupy these positions. Most research has used occupational groups as a criterion of
classification of positions. Often the motivation for this procedure has been an interest in the reward the occupation conveys to its holder in the form of prestige and income. In some mobility studies the criterion of classification has been solely the prestige of the occupation (Glass, 1954; Svalastoga, 1959).

Social mobility is now defined as movements of individuals between social positions. The unit of analysis has varied; it may be the individual proper, in which case mobility is usually referred to as intragenerational or career mobility. The unit may be taken as the family, as indexed by its male head, and we then speak of intergenerational mobility. Finally, the unit of analysis can be larger social groups; however, this unit is seldom used explicitly in empirical research.

Mobility can be classified, furthermore, according to the direction of move. A much used distinction, introduced by Sorokin (1927), is one in which a movement that involves a change in a stratification criterion (such as prestige) is denoted vertical mobility; and all other moves, horizontal mobility. The latter is thus movement between jobs at the same prestige or income level.

Mobility is commonly seen as a function of two main sets of factors: characteristics of individuals and structural characteristics of society. In the case of occupational mobility the latter consists of characteristics of the occupational structure. Most treatments of mobility stop short of identifying, explicitly, the relevant individual and structural characteristics. The following account, therefore, is partly an attempt to make explicit what has been treated only sporadically and implicitly in the literature.
The individual characteristics are of two kinds. First, individuals vary with respect to their propensity to move, no matter where they are placed in the occupational structure. The propensity to move, in turn, may be related to such characteristics as an individual's age and marital status. Another set of relevant individual characteristics are those that determine his direction of movement. This may be referred to as constituting his "occupational ability." Prominently used variables here are an individual's family background and education.

The structural characteristics influencing the probability of an individual's moving between two occupations are characteristics of the origin occupation, the destination occupation and the degree of affinity or distance between the two occupations. With respect to both the origin and destination occupations, the characteristics directly affecting the probability of move will be the number of positions (jobs) in the occupations relative to the number of individuals demanding those positions.

In the origin occupation the relative supply of positions will determine the pressure to leave that occupation. In the destination occupation the relative supply will determine the availability of vacant positions, or the "openness" of that occupation.

Sociologists usually mention only in passing the major sources of variation in the supply of positions in different occupations, their sources being technological, economic and demographic. In contrast, much attention has been paid to the notion of affinity or distance between occupational groups, and several measures of this variable exist.

The factors discussed above as influences on social mobility -- the individual characteristics and the structural characteristics --
can be summarized in the following heuristic expression:

$$p_{ij}(t) = f(a_v, b_i(t), c_j(t), d_{ij})$$  \quad (2.1)$$

where $p_{ij}(t)$ is the probability that individual $v$ moves from occupation $i$ to occupation $j$ in a given unit of time. This probability then is a function of individual characteristics $a_v$, characteristics of the origin and the destination occupation $b_i(t)$ and $c_j(t)$, and the affinity between the two occupational groups, $d_{ij}$. With respect to the last parameter we will ordinarily assume that $d_{ij} = d_{ij}$.

In general it must be assumed that $p_{ij}(t)$ is dependent on time. Even if we assumed that $a_v$ and $d_{ij}$ remain relatively stable over time, $b_i(t)$ and $c_j(t)$ would be a function of mobility at earlier points in time, since the net mobility in and out of occupations $i$ and $j$ would influence the pressure to leave occupation $i$ and the openness of occupation $j$ respectively.

In the remainder of the paper, the conceptualization of mobility expressed by eq. (2.1) shall be compared with currently used measures and models of mobility. With the formulation in eq. (2.1), the demand for internal validity can be formulated as one of being able to measure the parameters in eq. (2.1) without confounding the measures with the remaining parameters or with exogenous factors. This rather innocuous sounding demand will be shown to have some rather far-reaching implications.

The analysis will be organized around what seem to be the main objectives of mobility research. An outline of what these objectives are is given in the next section.
3. THE OBJECTIVES OF MOBILITY RESEARCH

In the heuristic expression, eq. (2.1), it is of course possible to derive a variety of measures from the dependent variables, the probability \( p_i^V(t) \). A simple measure would be:

\[
m(t) = 1 - \sum_{i,j} p_{ij}^V(t), \text{ if } i \neq j, \tag{3.1}
\]

which represents the total amount of mobility that takes place within a given society in a certain period of time, \( t \).

A large, if not the major part of mobility studies have focused primarily on the measurement of \( m \) or similar expressions for the amount of mobility within a society. This is a worthwhile beginning step, but in itself of limited interest. The next step is the analysis of such measures. Carlsson (1958) lists three objectives of such an analysis. Paraphrased, these objectives are:

1) The comparison of measures of mobility over time.
2) The comparison of measures of mobility between different societies.
3) The analysis of measures of mobility for different groups and classes within a society.

The first two objectives clearly refer to the use of mobility as a characteristic of social systems. The rationale for comparing measures of mobility over time and places is to relate variations in mobility to variations in other characteristics of society. Such an attempt may be made with the goal of learning more about the causes of mobility, or from the perspective of using mobility as an independent variable in the study of other characteristics of social system (e.g., its political structure).
The third objective of mobility research is less clearly classified. Carlsson seems to refer to both the study of mobility for categories of individuals with certain characteristics (education, family background, etc.) and to the study of mobility in subsets of the larger social system (e.g., in and out of farm occupations). The latter objective is again clearly an example of studying mobility as a system characteristic; the former, however, is an instance of studying mobility with the goal of accounting for individual variations in mobility. The primary interest is not in the person's propensity to move, but in the distance moved, or the occupational achievement. Hence, this objective may be formulated as one of wanting to study individual variations in occupational ability.

In terms of eq. (2.1), the objective of research on mobility as a system characteristic is to study the parameters $b_1$, $c_j$ and $d_{ij}$, whereas the objective when studying variations in occupational ability is to account for variations in $a_v$. We shall treat these two objectives separately in the following, since they present rather different, although not independent, problems.

4. THE STUDY OF OCCUPATIONAL ABILITY

The approach to the study of individual occupational ability has greatly over time. Early research tended to focus on the social recruitment to elite groups, such as professionals, business leaders or political leaders. It is an obvious advantage here that data often are readily available in biographical lexica, "Who's Who," etc. In historical studies of mobility, such data usually are the only ones available.
Studies of the recruitment to elites are appropriate if the objective is to relate the social composition of the elite and its behavior. Such studies are, however, usually inadequate for assessing the probability of gaining entry to the elite, since the magnitude of the population groups from which the elite is recruited rarely is established precisely, if at all. 1

The gathering of mobility data showing the intergenerational mobility between father and son in a community or a nation clearly solves some of the problems that plague recruitment studies. Such data not only give the necessary information to analyze the chances of entering certain occupational levels, but also make possible an analysis of the effect of various individual characteristics, such as education. This is an alternative not permitted by recruitment studies.

Although it is possible to analyze the effect of individual characteristics on occupational ability with the traditional intergenerational mobility data, the problem was not posed as such in most studies before the appearance of Blau and Duncan's work in 1967. Rather, the dependent variable was the position of son relative to the position of father. The effect of education, for example, is formulated in terms of the son's being upwardly mobile, stable or downwardly mobile. More precisely, the purpose is to determine the correlation

\[ r(y(x_1, x_2)) \]

(4.1)

where \( y \) is, say education, and \( x_1 \) and \( x_2 \) are father's and son's occupational level, respectively.

1 If information is available, it is possible to estimate the probabilities of getting entry, given the social origin, by using Bayesian probabilities (Sørensen, 1969).
It is a rather obvious drawback of this approach, as shown with nearly sadistic enjoyment by Blau and Duncan (1967), that the possible values of \((x_1 - x_2)\) are determined by \(x_1\). Although at face value this is a reasonable approach, some rather unreasonable results are apt to come of it.

The problem seems avoidable if one controls for father's social status, and then analyzes the effect of the individual characteristic within each of the occupational levels. Great care must be exercised, however, not to confound substantive results with the regression effect (i.e., the tendency toward regression toward the mean, produced by measurement error).

Tumin (1957) constructed a mobility measure, called Goms, where son's occupational level is measured in relation to the average occupational level of sons from the original stratum. This measure avoids the problem connected with taking the distance between father and son as the dependent variable. The natural next step is to use regression analysis explicitly, a procedure associated especially with Duncan (for example, Duncan and Hodges, 1963; Duncan, 1966; and Blau and Duncan, 1967), and generally considered the most important recent innovation in mobility research.

Regression analysis on mobility data, and the use of such analysis in the creation of causal models (path-analysis) represents not only a technical innovation, but also a conceptual change. The dependent variable is not the distance between father and son's occupational level, but the actual occupational achievement of the person. The occupational level of father becomes an indicator of the person's social origin, rather than a criterion of reference in measuring the son's occupational level.
Father's occupational level, then, is only one of a set of independent variables alongside such characteristics as father's education and son's own education.

Since the dependent variable in the approach used by Duncan and others is actual occupational achievement, this characteristic becomes an indicator of the occupational ability of the individual. It is important to note that occupational achievement is not only a function of the characteristic it is supposed to measure ---occupational ability--- but is also a function of the characteristics of the occupational structure. In particular, since the occupational achievement is determined by the job an individual holds at a particular point in time, the achievement depends on the availability of jobs in different occupational groups.

The dependence of occupational achievement on structural characteristics means that it is not possible to compare the effect of an independent variable on occupational ability in different occupational structures when using the actual achievement as the dependent variable. Using the approach of Duncan and others, it is not possible, for example, to test a hypothesis stating that the direct influence of the family of origin on occupational ability decreases, whereas the indirect influence, via education, increases, as the level of modernization of society goes up.

The problem can be illustrated by a simple example. Assume a conditional distribution of occupational ability for a given value of an independent variable, say respondents' education, \( f(z; \cdot) \), where \( i \) denotes the given educational level and \( z \) the occupational ability. The dependency of occupational achievement on structural characteristics can now

\[ \text{Comparison of results such as those obtained by Blau and Duncan (1967) is furthermore hindered by a technical problem -- their use of the standardized regression coefficients as measures of effect. Since standardized regression coefficients give the effect with the standard deviations of the independent variables as units, the measures will be population specific.} \]
be formulated as one of the availability of positions of a given occupational level at a point in time, \( z_i \), determining a minimum level of occupational ability, or a price of the occupational level \( C_T \).

![Figure 1](image)

The proportion of persons, \( P_j \), that obtains the occupational level is now indicated by the shaded area to the right of \( C_T \). If \( C_T \) varies from one time point to another or from one place to another, so will \( P_j \), and so will the occupational achievement of individuals with the given value of the independent variable. Such a change will be observed even if the distribution of \( z_i \) does not change at all; that is, even if the effect of the independent variables on the occupational ability remains unchanged. Hence changes in the distribution of occupational achievement may be produced solely by changes in the occupational structure, without any changes in the distribution of occupational ability.

A completely satisfactory solution to the problem does not seem to be available. Inferences about the effect of independent variables on occupational ability may be drawn, however, using an approach previously applied to comparisons of inequality of education (Sørensen, 1969). This approach relies on a stochastic process model developed by Coleman (1964).

Suppose that at any point in time, \( t \), an individual can be characterized by a probability, \( P_j \), of attaining at least the occupational level \( j \). This probability changes over time according to the process:

\[
\frac{dp_j}{dt} = q^j
\]
with the initial condition that $P_j = 1$ for $t = 0$. The parameter $q$ may be conceptualized as the transition rate of going from the state "does have the necessary resources to obtain occupational level $j$" to the state "does not have the necessary resources." The transition rate $q$ is then a function of the occupational ability $z$, and $C_T$ the minimum level of occupational ability for obtaining occupational level $j$. Hence,

$$q = a_1 y + a_0 \quad (4.3)$$

where

$$y = z - C_T \quad (4.4)$$

The effect of independent variable on the occupational ability can now be measured, assuming that the independent variable is linearly related to occupational ability, by $b_1$ in the equation:

$$z = b_1 x + b_0 \quad (4.5)$$

where $x$ is the independent variable. Inserting and collecting the constant terms, of which $C_T$ is one, we get

$$q = b_1 x + b_0 \quad (4.6)$$

Integrating equation (4.2) and inserting, we get

$$\log P_j = (b_1 x + b_0) t \quad (4.7)$$

Since $t$ may be assumed constant we expect a linear relationship between the logarithm to the proportion obtaining a given occupational level and the
independent variable -- the slope giving the effect of the independent variable on occupational ability.

As an illustration, Figure 2 shows the model applied to British and Danish mobility data. The independent variable is father's social status. The occupational level chosen as reference is stratum 4 in Denmark, stratum 3 in Britain. It appears that the model fits, and the slopes are rather similar. A slightly lower effect of father’s social status on occupational ability in Britain than in Denmark may, however, seem to be indicated.

The model does not make very good use of the data, and models that enable individual regressions and introduction of several independent variables need be developed. However, the separation of structural and individual characteristics is possible with this model, and it thus seems that the approach is one of higher validity than the direct study of occupational achievement.

5. THE STUDY OF MOBILITY AS A SYSTEM CHARACTERISTIC

The interest here is focused on the three last parameters in eq. (2.1). As before, this does not mean that the remaining parameter (αv) is without importance. However, in the measures and models to be reviewed below this parameter has been ignored, which means that the measures of structural characteristics will be population specific. They will, for

---

Other strata could have been used as reference and should give identical results. When the cumulative percentage is close to 0 or 100 the corresponding points have to be deleted, and a loss of information occurs. The strata shown were chosen to safeguard the most information.
FIGURE 2

The Effect of Father's Social Status on Occupational Ability

Father's Social Status

BRITAIN

DENMARK

Log $\log j$

0

-1.0

-2.0

-3.0

-4.0

-5.0

1 2 3 4 5 6 7

1 2 3 4 5 6
example, differ in general according to the age of the population. No solution has been given to this problem of measures being population specific although it obviously hinders the comparison of the structural parameters.

This section will first discuss some simple measures of the affinity or distance parameter in eq. (2.1), i.e., $d_{ij}$. Then follows a discussion of the application of Markov models to mobility data. Finally, some recently proposed models for the mobility process will be reviewed.

The point of departure for most attempts to use mobility as a system characteristic is a matrix where the marginals $(n_{i1}, n_{j1})$ represent the occupational distribution of fathers and sons and the cell entries represent the number of individuals going from one father occupational category to a son occupational category. The difficulties encountered when using such data to draw inferences on structural characteristics have been thoroughly analyzed by Duncan (1966). A main problem is that even if the generation of sons is a well defined cohort, the generation of fathers will not be so. The mobility between father and son cannot be specified in time with the usual intergenerational data. This is unfortunate since the structural parameters $b_{ij}$ and $c_{ij}$ in eq. (2.1) are assumed time dependent.

### Measures of Structural Characteristics

Considerable attention has been paid to the question of how to measure $d_{ij}$ or some derivative of this parameter such as the overall openness of a society. A variety of indices of this parameter exist, many of them reviewed by Svalastoga (1959). The best-known attempt to use mobility as a system characteristic (Lipset and Bendix, 1959) does not rely on such an index, however, but uses the percentage moving between blue-collar and white-collar occupations -- a measure that has no clear
interpretation in terms of eq. (2.1).

In more recent comparative investigations of mobility (Fox and Miller, 1965, Svalastoga, 1959) the product-moment correlation coefficient has been used as a measure of the openness of a society. This measure, of course, does demand a scoring of occupational categories in terms of prestige. The measure will be identical to Duncan's measure of the effect of father's status on son's occupational achievement. Here the interpretation is structural, however. Rather than giving the correlation coefficient an interpretation in terms of any of the parameters in eq. (2.1), it more appropriately may be seen as a formulation of the dependent variable in eq. (2.1), i.e., \( p_{ij} \).

The most commonly used measure of mobility is probably the mobility ratio simultaneously but independently constructed by Glass and associates (1954), Rogoff (1954) and Bressard (1950). This measure is:

\[
e_{ij} = \frac{\hat{n}_{ij}}{n_{i}^{*} n_{j}^{*}}
\] (5.1)

The measure is commonly conceived of as the ratio of actual mobility over the amount of mobility expected under the hypothesis of statistical independence between occupation of origin and occupation of destination. The measure can be derived in a way that lends itself more closely to an interpretation in terms of eq. (2.1), if it is taken as a measure of distance (Svalastoga, 1959; Carlsson, 1958). Using the so-called gravitational model, we get

\[
n_{ij} = k \frac{n_{i}^{*} n_{j}^{*}}{d_{ij}}
\] (5.2)
where \( y_{ij} \) is a measure of the distance between occupation \( i \) and \( j \). It is easily seen that \( d_{ij} \) and \( c_{ij} \) are related as

\[
c_{ij} = \frac{k}{d_{ij}} \tag{5.3}
\]

The distance measure in eq. (5.3) has a definite interpretation in terms of the conceptualization of mobility presented in eq. (2.1) if it is assumed that the availability of positions in occupations \( i \) and \( j \), and thus the parameters \( b_i \) and \( c_j \), are proportional to the number of individuals there, \( n_i \), and \( n_j \).

The expression in eq. (5.2) with \( n_i \) and \( n_j \) taken as measures of \( b_i \) and \( c_j \) (cf. eq. (2.1)) may be seen as a testable model of mobility. Svalastoga (1959) gives a test of this model. He delineated his social strata roughly equidistant in social status (measured by occupational prestige). He could therefore determine the distances a priori and test the model by calculating the expected rates of mobility and comparing them to the actual. The resulting deviations were too large to be accounted for by chance. Svalastoga attributed the deviations to coding unreliability. The substitution of the marginal entries \( a_{i1} \) and \( a_{1j} \) for the parameters \( b_i \) and \( c_j \) also may be responsible for the lack of fit.

The measure \( c_{ij} \) has been criticized extensively (Killevitz, 1955; Yassie, 1964; Carlsson, 1958) for having a range of variation that depends on the marginals. An even more serious drawback is that the measure is not well-suited for comparative purposes (Duncan and Blau, 1967). From the definition of \( c_{ij} \) it follows that

\[
c_{ij} = n_{i1}/n_{j} \tag{5.6}
\]
and therefore that

\[ x_{ij} \cdot n_{ij} = n/n \cdot y_{ij} \cdot n_{ij} = n.. \]  \hspace{1cm} (5.5)

If we know \( c_{ij} \), eq. (5.5) may be seen as an equation in \( k \) unknowns (\( k \) equal to the dimension of the mobility matrix), from which we can determine the \( n_{ij} \)'s and the \( n_{..j} \)'s. This means that we cannot have identical matrices of \( c_{ij} \) (e.g., for two time periods), without the marginals being identical. Hence, it is not possible to observe unchanged mobility ratios when the marginals differ. The \( c_{ij} \)'s are not well-suited for comparative purposes, since it obviously should be possible to observe unchanged ratios or distances even if the occupational distribution has changed.

Duncan's solution to the problem is to suggest what he calls "simultaneous adjustment" of the cell entries \( n_{ij} \) according to the differences in marginal distribution between two mobility matrices (Duncan, 1966). He suggests the following model:

\[ m_{ij} = n_{ij} + n_{i..} s_i + n_{..j} v_j + n_{ij} e_{ij} \]  \hspace{1cm} (5.6)

where \( m_{ij} \) are the entries in one matrix and \( n_{ij} \) the corresponding entries in another; \( s_i \) and \( v_j \) are parameters for characterizing the changes in the marginal distribution, and \( e_{ij} \) an error term. A test of this model then provides a test for whether all the difference between \( m_{ij} \) and \( n_{ij} \) is caused solely by changes in the marginal distribution. The model is not justified by Duncan in terms of mechanisms of mobility, and a negative outcome of the test is, therefore, not very instructive.

The model presented in eq. (5.6) can be given a formulation in terms of the parameters in eq. (2.1), however. If we assume that
\[ s_i = b_i^1 \cdot b_i^2 \\
\nu_j = c_j^1 \cdot c_j^2 \\
e_{ij} = d_{ij}^1 \cdot d_{ij}^2 \]  

(5.7)

where the superscripts characterize the two matrices \( n_{ij} \) and \( m_{ij} \), then eq. (5.6) can be rewritten as

\[ m_{ij} = n_{ij} + n_{ij} (b_i^1 - b_i^2) + n_{ij} (c_j^1 - c_j^2) + n_{ij} (d_{ij}^1 - d_{ij}^2) \]  

(5.8)

This is a definite interpretation in terms of eq. (2.1), which gives the matrix \( m_{ij} \) as a function of the matrix \( n_{ij} \) and the structural parameters.

If it is assumed that

\[ \Sigma s_i = \Sigma v_j = \Sigma e_{ij} = \Sigma e_{ij} = 0 \]  

(5.9)

it is possible to estimate all parameters in the model. The model now not only can be used to test whether all changes can be attributed to changes in the marginal distribution, but is also provides the necessary information to evaluate how much the three sets of parameters contributed to the differences in mobility over time or between places.

It is unfortunately necessary to assume that the \( \Sigma d_{ij} = 0 \). Thus an overall change in the \( d_{ij} \)'s is, by definition, excluded. It is not possible to solve the system of eq. (5.8) otherwise.

Simple Markov Models

Mobility is conceptualized as being a probabilistic phenomenon, and a process in time. Hence it is natural to attempt to represent mobility

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This conclusion is based upon work with an identical model in another context, the Tukey model to be discussed below.
as a stochastic process. And the Markov chain model is a natural choice, since it has nice mathematical properties, is relatively simple and demands that data be presented in a turn-over table. There exist quite a few applications of Markov chain models to mobility data. Work on intergenerational data has been done by Prais (1955) and Matras (1960). On intragenerational data the best-known application is by Blumen and Associates (1955).

Because of an obvious lack of data, no test of the Markov model on intergenerational data has been performed. A test is possible with intragenerational data -- a test that shows that the simple Markov model cannot account for the process of mobility (Blumen, Kogan and McCarthy, 1955).

The rationale for using the Markov model is not made explicit in most applications, except for the observation that the entries in the mobility matrix are readily conceived of as bases for the estimates of conditional probabilities, the governing parameter in the Markov chain model. A somewhat more explicit derivation of the model is provided by Blumen and Associates. They assume that at any point in time a person is exposed to a constant probability, λ, of leaving his job in a time interval, dt. The probability of leaving within a time interval t, \( p_t \), is then exponentially distributed:

\[
p_t = \lambda e^{-\lambda t}
\]  

(5.10)

If it is assumed that job moves are independent of each other, and that everybody has the same probability of leaving, then the proportion having \( i = 0, 1, 2, \ldots, n \) moves in the time interval, t, will be poisson distributed.
Once a person leaves a job, we assume that his movement is determined by a set of probabilities which are conditional on the occupation left. If there are \( n \) occupations, then these probabilities can be set up in a matrix.

\[
P = \begin{bmatrix}
P_{11} & \cdots & P_{1n} \\
P_{i1} & \cdots & P_{ij} \\
P_{n1} & \cdots & P_{nn}
\end{bmatrix}
\]  

(5.12)

The turnover table for a period \( t \) can now be written as

\[
M(t) = r_0 \mathbf{1} + r_1 \mathbf{P} + r_2 \mathbf{P}^2 + r_3 \mathbf{P}^3 + \ldots + r_s \mathbf{P}^s
\]

(5.13)

where \( \mathbf{1} \) is the identity matrix. Inserting eq. (5.11), eq. (5.13) can be written as

\[
M(t) = e^{-\lambda t} \mathbf{1} + e^{-\lambda t} \frac{\lambda t \mathbf{1}}{1!} + e^{-\lambda t} \frac{(\lambda t)^2 \mathbf{1}}{2!} + \ldots
\]

(5.14)

Blumen and Associates show, through a rather elaborate argument, that the expression 5.14 gives \( M(t) \) all the properties of a Markov chain. That \( M(t) \) is indeed a Markov chain can be seen easily by noting that the right hand side of eq. (5.14) is the matrix equivalent to an exponential series. Hence eq. (5.14) can be written as

\[
M(t) = e^{Qt}
\]

(5.15)

which is the definition of a continuous time Markov process (Coleman, 1964).
This derivation of the model makes clear some of the crucial features of the Markov model. These are: (1) all individuals have the same probability of leaving a job in a time interval, \( t \); (2) moves are independent of each other; (3) the transition probabilities are a function of the occupation of origin only. Of those assumptions, the first has been held as the most important, and it is the one which gets all the blame for the lack of fit of the Markov model. It is easily shown (Bartholomew, 1967) that if this assumption is relaxed and a heterogeneity of individuals is introduced, then the model will provide a better fit to the data.

Blumen and Associates (1955) introduced this notion by assuming that individuals can be divided into two groups, "movers" and "stayers." Recently McGinnis (1968) has introduced a model where \( \lambda \) is a function of the length of stay in an occupation.

The other two assumptions are at least as crucial in view of the conceptualization of mobility. These assumptions imply that the movement of an individual is in no way constrained by the movement of other individuals, and related to this, that the transition probabilities are in no way a function of characteristics of the destination occupation. That the movement of individuals and exogenous factors changes the supply of positions in different occupational groups, and thus changes the pressure to leave an occupation and the availability of positions in the destination occupation, is a notion completely foreign to the Markov model. Hence, the simple Markov model is blatantly in disagreement with the conceptualization of mobility.

In view of the fundamental conceptual troubles with the Markov model it may seem futile to repair the model only as far as the hetero-
geneity of individuals is concerned. This heterogeneity is seen as a function of variations in an individual's propensity to move. But the pressure to leave will change as a function of the movement of other individuals, so the actual probability to leave will change over time even if the individual heterogeneity in propensity to move is accounted for.

It should be noted that some have claimed that the statistics of the model are useful in investigations of mobility (Duncan, 1966; Carlsson, 1958). The model then is a "descriptive device" (Carlsson, 1958) for presentation of data. It is, however, difficult to see the usefulness of this application of Markov chains, unless the parameters have a clear interpretation in terms of the conceptualization of mobility. The parameters of the simple Markov model do not lend themselves to such an interpretation.

Some Alternatives to the Simple Markov Model

White (1968) has attempted to overcome some of the difficulties of the simple Markov model by turning it on its head. As mentioned, independence of individuals cannot, in general, be assumed. An individual's probability of finding a vacant position will depend on the movement of other individuals, unless, of course, there is a great number of vacant positions relative to the number of individuals. White assumes that in general the opposite will be the case, that is, there will be a great number of individuals relative to the number of vacancies. But then the movement of vacancies may be assumed independent of each other, where the movement of vacancies is the opposite of the movement of individuals. If an
individual moves from occupation i to j, then a vacancy may be seen as moving from j to i. Hence a Markov model that describes the movement of vacancies may be assumed a more valid model of the mobility process.

The validity of the vacancy-chain model, as White calls it, rests on the assumption of a much larger number of individuals than vacancies. The validity of the model, therefore, depends on the technological, demographic and economic factors which determine the supply of vacancies relative to the number of individuals. This is somewhat bothersome. It is also a serious problem, since it rarely is possible to ascertain the movement of vacancies. Only in very special cases (of which White's own application is one) do we know the number of vacancies in different social strata.

A parametrization of the transition rates in the continuous time Markov model, suggested by Tukey, is a partial solution to the problem of giving the Markov model an interpretation in accordance with eq. (2.1). Coleman (personal communication) has suggested applying the model to mobility data and it has been applied to intragenerational data with some degree of success (Sørensen, 1968). The parametrization is actually taken from a model used in chemistry. The transition rates are written as

$$q_{ij} = e^{s_i - s_{ij}}$$ (5.16)

where $s_i$ is a characteristic of the state left, and $s_{ij}$ one of the boundaries between states. In mobility data $s_i$ can be interpreted as the pressure to leave an occupation i, and $s_{ij}$ interpreted as the affinity between i and j. Hence the transition rates are functions of $b_i$ and $d_{ij}$. 

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in eq. (2.1). The same parametrization may be applied to the vacancy chain model in which we will get measures of $c_j$ and again $d_{ij}$, provided we do have data on vacancies.

It is tempting to introduce all structural parameters in eq. (5.16), and thus make the model a full interpretation of eq. (2.1). We will get

$$q_{ij} = b_i + c_j - d_{ij}$$

(5.17)

in the notation of eq. (2.1). Taking logarithms we get

$$\log q_{ij} = b_i + c_j - c_{ij}$$

(5.18)

The correspondence between equations 5.8 and 5.18 is obvious. Unfortunately it has not been possible to solve the system of equations for $b_i$, $c_j$, and $d_{ij}$ without some rather unreasonable restrictions on the $d_{ij}$'s. The parametrization 5.18 imposes certain constraints on the system of equations that make the solution matrix singular, at least for the size matrix that corresponds to mobility matrices of "normal" magnitude.

If should be emphasized that the model presented in eq. (5.17) is an attempt to utilize the Markov model as a descriptive device. Since $b_i$ and $c_j$ are function of time, so are the $q_{ij}$'s, and the Markov property is lost.

On the basis of considerations similar to the ones the led to the rejection of the simple Markov model above, Coleman (1968) has proposed $^5$ other than the one of $d_{ij} = d_{ji}$
a model for mobility that does see mobility as a function of the availability of positions. He assumes that the rate of movement from occupation $i$ to occupation $j$ is proportional to the number of individuals in $i$ and the number of vacant positions or jobs in $j$. Hence the rate of movement from $i$ and $j$ is $n_i \cdot m_j \cdot d_{ij}$, where $n_i$ is the number of filled positions in occupation $i$, (that is, the number of individuals there) and $m_j$ is the number of vacant positions in $j$.

The total rate of change in $n_i$, that is, the loss from $i$ to all other occupations and the gain to $i$ from other occupations, can now be written as

$$\frac{dn_i}{dt} = - \sum_{j \neq i} n_i \cdot m_j \cdot d_{ij} + \sum_{j \neq i} n_j \cdot m_i \cdot d_{ji}$$

(5.19)

The quantities $n_i$ and $m_j$ are assumed a function of time. The model implies, as it stands, that everybody in occupation $i$ is an active job seeker, and that every vacancy is an equally active individual seeker. Coleman suggests adding exponents to the quantities $n_i$ and $m_j$ and assumes that these exponents are a function of the level of employment. If there is a very high level of employment and this many vacant positions, the exponent to $n_i$ would be close to zero, and the exponent to $m_j$ close to 1, i.e., there will be few active job seekers and many active individual-seeking vacancies.

The same idea may be introduced by assuming that only a fraction $y_i$ of the individuals in occupation $i$ are active job seekers and a fraction $z_j$ of the positions in $j$ are active vacancies. The magnitude of these
fractions may be seen as governed by the parameters $b_i$ and $c_j$ in eq. (2.1).

If the pressure to leave an occupation is high, then the proportion of active job seekers will be high. If the availability of positions is high, this is equivalent to stating that the fraction of jobs that are active "individual-seeking" vacancies is high. Hence the model (5.19) can be given a clear interpretation in terms of eq. (2.1), if the terms $p_i$ and $p_j$ are replaced by parameters $s_i$ and $v_j$ that are simple functions of $b_i$ and $c_j$.

One possible way of realizing this is to let the movement between $i$ and $j$ be expressed as

$$n_i \cdot n_j \cdot d_{ij} = v_i \cdot s_j \cdot d_{ij} = n_i \cdot b_i \cdot c_j \cdot d_{ij} \quad (5.20)$$

But the last quantity can be expressed as

$$n_i \cdot (b_i \cdot c_j \cdot d_{ij}) = n_i \cdot q_{ij} \quad (5.21)$$

following the parametrization in eq. (5.18). Hence, (5.19) may be written as

$$\frac{dn_i}{dt} = -\sum_{j \neq i} q_{ij} \cdot n_i + \sum_{j \neq i} q_{ji} \cdot n_j \quad (5.22)$$

This equation has the same form as the defining equation for a continuous time Markov model, although the Markov property is lost since the $q_{ij}$'s are functions of time. Hence, the two models (5.18) and (5.19) may be given the same form.

With continuous time data it may be possible to gain insight into the behavior of the $q_{ij}$'s and possible to obtain estimates of the different
parameters. Empirical work with the last-mentioned models as a framework thus does seem to be a fruitful next step toward the derivation of testable models that are in accordance with the conceptualization of mobility.

CONCLUSION

Our review of models and measures of social mobility has indicated two main unresolved problems. One is the contamination of structural and individual factors in the study of occupational achievement and ability; another the apparent failure to obtain satisfactory models that will enable measurement of structural factors determining mobility.

The first problem is important if an attempt is made to study determinants of occupational ability in different occupational structures, separated, for example, by time. The relation between structural characteristics and occupational achievement should, on the other hand, not hinder analysis of the relative importance of different variables for occupational ability, which has been the primary objective in recent mobility studies. If results from these studies are to be used for predictive purposes (for example, to evaluate the effect of a change in level of education on the occupational achievement of blacks), the inability to evaluate the importance of structural factors for occupational achievement is a hindrance. The established relation between occupational achievement and education is partly a function of the availability of jobs at different occupational levels, at a particular point in time. To predict the occupational achievement of a group if its educational level is
changed a certain amount is only meaningful if the availability of jobs in different occupational groups can be assumed unchanged. But this assumption leads to a contradiction since the availability of jobs depends, among other things, on the number of people with the necessary occupational ability to enter a given occupational level -- a function of the level of education.

A somewhat unsatisfactory solution is proposed in this paper to the problem of the contamination of structural and individual factors in the study of occupational achievement. The method suggested enable multivariate analysis of individual data. However, the does fulfill our main demand by enabling analysis of the effect of independent variable on occupational ability independent of structural variations.

The second major problem, the difficulties encountered in using structural models and measures of mobility, is partly caused by lack of concern for the development of models isomorphic with the conceptualization of mobility. The various attempts to measure the difference in occupational groups thus ignores variations in the available jobs. The prominent use of Markov models in mobility research the fact that the assumptions of the simple Markov model are in disagreement with the conceptualization of mobility.

Mathematical problems are responsible for some undesirable features of various models and measures. The most commonly used are affinity or distance between occupational groups thus has a role when used for comparative purposes. An alternative method is
mobility differences between occupational structures can be explained by structural differences only has an interpretation in terms of the conceptualization of mobility if it is assumed that the overall affinity between occupational groups remains the same. This assumption is necessary to solve the system of equations developed. The same problem, caused by the structure of a system of equations, hinders a satisfactory solution of a parametrization of a continuous time Markov model.

Empirical analysis of transition rates between occupational groups has been suggested as a fruitful next step in the development of models of mobility. Such work might indicate what will be reasonable assumptions for structural models of mobility.

Another problem, not touched upon in the structural models discussed, is the separation of individual and structural factors. The parameters in the presented models will vary with population characteristics, since they are dependent on individual characteristics. When analyzing differences between occupational structures in mobility, assumptions therefore have to be made concerning concurrent population differences.
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