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ABSTRACT

The concept of structure has become a central concern in the study of human perception, learning, memory, and recall. The purpose of this paper is to present a method for analyzing structure--the theory of directed graphs--and to review research on teaching that has employed this method to investigate input structure, memory structure, or both. To this end, the paper is divided into two parts. The first part presents fundamental concepts of the theory of directed graphs--digraphs: the axiom system, types of digraphs, adjacency, reachability, connectedness, distance, and vulnerability. In the second part, research is reviewed in which content structure, cognitive structure, and the correspondence between these structures are examined with implications for the teaching-learning process. Throughout this review, implications for further research on teaching using digraphs are set forth. (Author/RT)

STANFORD CENTER
FOR RESEARCH AND DEVELOPMENT
IN TEACHING

Research and Development Memorandum No. 71

THE THEORY OF DIRECTED GRAPHS: SOME APPLICATIONS TO RESEARCH ON TEACHING

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Introductory Statement

The Center is concerned with the shortcomings of teaching in American schools: the ineffectiveness of many American teachers in promoting achievement of higher cognitive objectives, in engaging their students in the tasks of school learning, and, especially, in serving the needs of students from low-income areas. Of equal concern is the inadequacy of American schools as environments fostering the teachers' own motivations, skills, and professionalism.

The Center employs the resources of the behavioral sciences--theoretical and methodological--in seeking and applying knowledge basic to achievement of its objectives. Analysis of the Center's problem area has resulted in three programs: Heuristic Teaching, Teaching Students from Low-Income Areas, and the Environment for Teaching. Drawing primarily upon psychology and sociology, and also upon economics, political science, and anthropology, the Center has formulated integrated programs of research, development, demonstration, and dissemination in these three areas. In the Heuristic Teaching area, the strategy is to develop a model teacher training system integrating components that dependably enhance teaching skill. In the program on Teaching Students from Low-Income Areas, the strategy is to develop materials and procedures for engaging and motivating such students and their teachers. In the program on Environment for Teaching, the strategy is to develop patterns of school organization and teacher evaluation that will help teachers function more professionally, at higher levels of morale and commitment.

Research and Development Memorandum No. 71, which follows, presents the theory of directed graphs and reviews research on teaching which has used this method to investigate input structure, memory structure, or both. Since teaching involves the communication of the structure of a discipline to the learner, methods for investigating structure in teaching and learning are critical to research on teaching. They are thus of integral concern to the program on Heuristic Teaching, for which this paper was prepared.

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Abstract

The human teacher and learner may be characterized as processors of information. It is of prime importance in the investigation of human information processing to describe the structure of the input, the various structures through which the information is processed, the structure of storage, and the retrieval mechanisms for searching memory for information. The concept of structure has become a central concern in the study of human perception, learning, memory, and recall. The purpose of this paper is to present a method for analyzing structure (the theory of directed graphs) and to review research on teaching that has employed this method to investigate input structure, memory structure, or both. To this end, the paper is divided into two parts. The first part presents fundamental concepts of the theory of directed graphs (digraphs): the axiom system, types of digraphs, adjacency, reachability, connectedness, distance, and vulnerability. In the second part, research is reviewed in which content structure, cognitive structure, and the correspondence between these structures are examined with implications for the teaching-learning process. Throughout this review, implications for further research on teaching using digraphs are set forth.

THE THEORY OF DIRECTED GRAPHS: SOME APPLICATIONS TO RESEARCH ON TEACHING

Richard J. Shavelson
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Research by Miller (1956) and Broadbent (1958) along with computer science's interest in artificial intelligence (e.g., Feigenbaum & Simon, 1962) has greatly influenced the current view of man as a processor of information. Norman (1969) expressed this view:

> In particular, we are concerned primarily with verbal, meaningful information in acoustical and visual form. The aim is to follow what happens to information as it enters the human and is processed by the nervous system. The sense organs provide us with a picture of the physical world. Our problem is to interpret the sensory information and extract its psychological content. To do this we need to process the incoming signals and interpret them on the basis of our past experiences. Memory plays an active role in this process. It provides the information about the past necessary for proper understanding of the present. There must be temporary storage facilities to maintain the incoming information while it is being interpreted and it must be possible to add information about presently occurring events into permanent memory. We then make decisions and take actions on the information we receive (pp.3-4).

It is implicit in this point of view that psychological processes such as learning and/or memory are highly organized and complex. Of prime importance in investigating human information processing is the description of the structure[1] of the input, the various structures through which the information is processed, the structure of the storage, and the retrieval mechanism for searching memory structure for information. In short, in studying human behavior it has become increasingly important to operationalize the term structure; both stimulus structure (the structure of the input) and cognitive structure (e.g., the organization of facts and concepts as they are interrelated in long-term memory). This view of man has been extended to the teaching-learning process by Gage (1963):

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[1]Structure is defined as an assemblage of identifiable elements and the relationships between those elements (Shavelson, 1970).

> If the sets of facts, ideas, concepts, principles, and so on, that we want to teach were themselves mere unrelated congeries of items, such an approach to teaching [cognitive processes] would face severe difficulties. The teacher would have trouble in finding sets of ideas, or cognitive structures, that he could use to apply cognitive force, to compel understanding or acceptance or belief, and produce learning. Fortunately, of course, many subjects taught in schools have highly organized structures. Arithmetic, chemistry, and German are examples at the high extreme of structure. Subjects like literature and history are far from devoid of structure in the views of scholars in these fields. The implication of the cognitive approach to learning and teaching is that maximum advantage should be taken of the cognitive properties of learners and subjects. . . . Properly organized subject matter presented to learners whose cognitive development and processes are correctly understood will produce learning--learning of the best kind, according to the value systems of many educators (p. 138).

With increasing emphasis being placed on the analysis of behavior according to different structures, the behavioral sciences require a method for formally analyzing structure. In their book, Structural Models: An Introduction of the Theory of Directed Graphs, Harary, Norman, and Cartwright (1965) provide such a method. These authors have taken the mathematical theory of graphs and developed a lucid explanation of the theory with special application to social scientific variables.

The purpose of this paper is to present the fundamental concepts of the theory of directed graphs (called digraph theory) and to indicate the ways in which this theory may be applied to research on teaching. The presentation of digraph theory will be drawn directly from Harary et al. (1965) and is intended to introduce the reader to this theory and to provide sufficient information to allow him to comprehend research employing this technique. Any serious application of this theory should be preceded by a thorough study of Harary et al. (1965, Chapters 1-5, especially). Once the fundamental concepts of digraph theory have been presented, research involving the application of digraph theory to the teaching-learning process will be described. Suggestions for future applications of this theory to research on teaching are made throughout the last section.

Fundamental Concepts of Digraph Theory

The rationale for and the limitations of applying the abstract mathematical theory of directed graphs (or, more succinctly, digraphs) to empirical structures is aptly stated by Harary et al.:

> [The] theory [of directed graphs] is concerned with patterns of relationships among pairs of abstract elements. As such, digraph theory makes no reference to the empirical world. Nevertheless, it has potential usefulness to the empirical scientist, for it can serve as a mathematical model for the structural properties of any empirical system consisting of relationships among pairs of elements (p. 9).
>
> If an appropriate coordination is made so that each entity of an empirical system is identified with a point and each relationship is identified with a line, then for all true statements about structural properties of the obtained digraph there are corresponding true statements about structural properties of the empirical system (p. 22).

"The theory of digraphs is based on an axiom system consisting of four primitives (undefined terms), together with four axioms (or postulates) which give us an understanding of the primitives and of their relations to one another" (Harary et al., p. 4). The primitives are (p. 9):

P_1: A set V of elements called "points."

P_2: A set X of elements called [directed] "lines."

P_3: A function f whose domain is X and whose range is contained in V.

P_4: A function s whose domain is X and whose range is contained in V.

If the appropriate coordination is made between the points and lines set forth above and the empirical world, the primitives may be tied to the empirical world. For example, if communication structures are studied, the "points" might represent people in the structure and the "lines" might represent the direction of communication within the structure. Or, the points might represent concepts in a textbook and the lines might represent the links between the concepts as specified by the text. "The second two [primitives] relate the lines to the points by means of two functions f and s which serve to identify the 'first' and the 'second' point of each line, respectively" (Harary et al., p. 5).

The axioms are (Harary et al., p. 9):

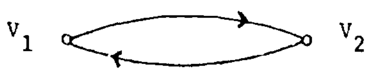
A_1: The set V is finite and not empty.

A_2: The set X is finite.

A_3: No two distinct lines are parallel.

A_4: There are no loops.[2]

The first two of these axioms are self-explanatory. Further, A_2 is implied from A_1 and A_3. Since digraph theory has grown out of more general theories about nets and relations, the last two axioms refer to a limitation of digraph theory not inherent in general graph theory. Given two points and two lines, A_3 restricts us to case 1 (below) and A_4 indicates that digraph theory does not permit the situation shown



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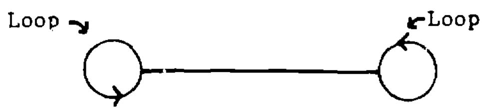

II

in case II. Using the communication examples, A_3 requires that plural communications from person 1 to person 2 be considered as essentially a single relation. Since there is no formal provision for loops, A_4 implies that an individual communicates with himself. When distance[3] is discussed, these last two restrictions will become clearer.

Up to this point the axiomatic system of digraphs has been presented and described using a number of concepts. These concepts now will be explained. To facilitate explanation, an example of classroom communication is described. In the class there are a teacher and, say, three students. This teacher is known to start talking at the bell beginning the class period and to stop talking only at the second bell closing the period. Further, his students have been observed passing notes to each other while he drones on. (There is a middleman and all notes pass through

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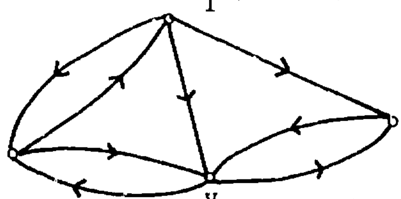
[2] "A line X of a net is called a loop if fx=sx . . . [i.e.] if it has the same first and second point" (p.6).

[3] Distance is the smallest number of lines between two points on a digraph.

him because of his location.) One of the three students is a cute coed who has been seen talking to the teacher after the second bell. The problem now is to digraph this situation. Let the teacher and students be represented by points in our digraph and the communication between teacher and students by lines. The following digraph is obtained:



In this example, the teacher and the students are joined in communication. Following the arrows, any student can reach the teacher while the teacher can reach all of the students. The lines in the digraphs can be thought of as paths between individuals in the classroom. "A (directed) path from v_1 to v_n is a collection of distinct points, v_1, $v_2, \ldots, v_n$, together with the lines v_1v_2, $v_2v_3, \ldots, v_{n-1}v_n$, considered in the following order: v_1, v_1v_2, v_2, v_2v_3, $v_3, \ldots, v_{n-1}v_n$, v_n" (Harary et al.,[4] p. 32). Thus if there is a path from V_1 to V_2, then V_2 is reachable from V_1. In our example, there is a path between the teacher and the students and the students and the teacher. However, if students V_3 and V_4 decided not to pass notes to student V_2, then there would not be a path from students V_3 and V_4 to V_2 or V_1 by this definition.

This problem brings us to the next concept, semipath: "A semipath joining v_1 and v_n is a collection of distinct points, v_1, $v_2, \ldots, v_n$, together with n-1 lines, one from each pair of lines, v_1v_2 or v_2v_1, v_2v_3 or $v_3v_2, \ldots, v_{n-1}v_n$ or v_nv_{n-1}" (Harary et al., p. 31). From this definition it becomes clear that if there is a semipath between two points, these points are joined but not reachable. We can generalize the definitions

[4]Small "v's" in quoted material follow the authors, Harary et al. For better readability, however, capital "V's" are used in the body of the text.

of path and semipath to include cases in which a repetition of points and lines is permitted. A sequence can be defined as a path in which points and lines are repeated[5] and a semisequence can be defined as a semipath in which the points and lines may be repeated. "The difference between reachability and joining is that in the latter instance we ignore the direction of lines" (Harary et al., p. 45).

Intimately related to the concepts of reaching and joining is the concept of distance:

> The number of lines in a path is called its length. A geodesic from u to v is a path from u to v of minimum length. If there is a path from u to v in a digraph, then the distance from u to v, denoted d(u, v) is the length of a geodesic from u to v. . . . If there is no path from u to v, the distance from u to v is called infinite (Harary et al., p. 32).

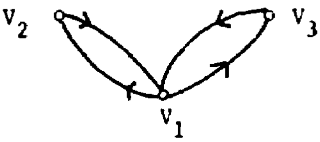
In the classroom example, the distance from the teacher to the students is equal to one. The distance from student V_2 to the teacher (V_1) is one; the distance from student V_3 to the teacher is two; the distance from student V_4 to the teacher is three.

Description of Digraphs

Digraphs can be characterized by: (a) relations, (b) connectedness, (c) point bases, (d) vulnerability, and (e) matrices. From these concepts research evolves.

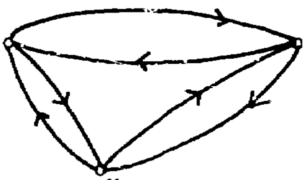
Relations. Digraphs are characterized by relationships between points. All digraphs are irreflexive; i.e., no point in a digraph has a loop (A_4). Given that digraphs are irreflexive, some of the major types of digraphs classified according to relationships between points are:

1. Symmetric digraph - for every line uv there is also a line vu:

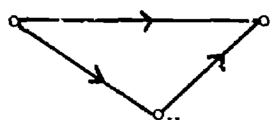


[5]The difference between a sequence and a path is that the latter requires all points to be distinct whereas the former permits repetition of the same point.

2. Complete symmetric digraph - every pair of points of the digraph is joined by two lines, one in each direction (complete and symmetric):



3. Complete asymmetric digraph - at least one of the ordered pairs (uv or vu) exists and uv precludes vu:

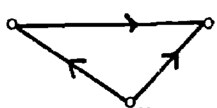


4. Totally disconnected digraph - a digraph without any lines:

V_2 o o V_3

o V_1

5. Transitive digraph - "Contains a line uw whenever lines uv and vw are in D (the digraph), for any distinct points u, v, w" (Harary et al., p. 12).

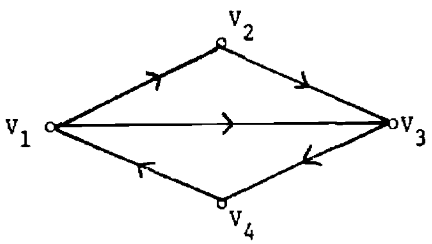


Connectedness. One can attend to the degree of linkage or connectedness between any two points of a digraph, or of all pairs. Five terms are used to classify the degree of connectedness: strong, unilateral, weak, disconnected, and trivial.

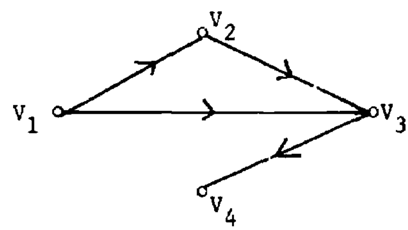
> A digraph D is strongly connected, or strong, if every two points are mutually reachable; D is unilaterally connected, or unilateral, if for any two points at least one is reachable from the other. We say that D is weakly connected, or weak, if every two points are joined by a semipath. A digraph is disconnected if it is not even weak. . . . A digraph with just one point is called trivial (Harary et al., p. 51).

> The digraph D is strong if and only if every pair of its points is 3-joined; D is unilateral if and only if every pair of its points is 2-joined; and D is weak if and only if every pair of its points is 1-joined (Harary et al., p. 51).

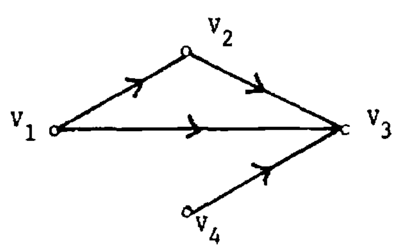
Harary et al. give the following examples of the different levels of connectedness:



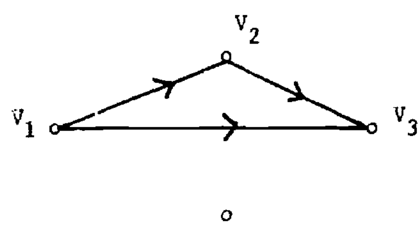
Strong: Each pair of points is 3-joined (i.e., there is a sequence from each point to the other).



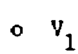
Unilateral: Each pair of points is 2-joined (there is a sequence from one point to the other).



Weak: Each pair of points is 1-joined (there is a semisequence joining them).



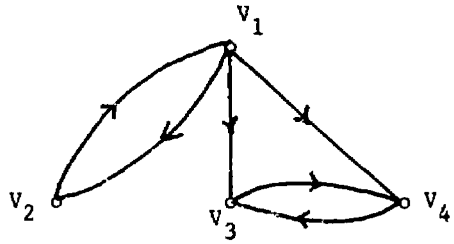
Disconnected: Each pair of points is 0-joined (every pair of points of a digraph is 0-joined by definition).



Trivial digraph

The digraph from the example of classroom communication can be characterized as strong. What would happen to the type of connectedness of that digraph

if students V_3 and V_4 decided not to talk to student V_2? The following would result:



The entire nature of the classroom communication system now has changed; the connectedness is weak.

But, although the connectedness of the digraph of classroom communication has changed from 3-joined (strong) to 1-joined (weak), strong components still remain. A more complete characterization of a digraph can be achieved by breaking it down into subgraphs. Then, a simpler digraph can be obtained by replacing certain subgraphs by points and joining those points by lines. This procedure is referred to as condensation, and a full discussion may be found in Chapter 3 of Harary et al.

Vulnerability. The third method for characterizing digraphs will be presented only briefly to facilitate a discussion of research in the next section. The interested reader is referred to Harary et al. (Chapter 7). As has been said above, a digraph may be characterized by its connectedness: strong, unilateral, weak, and disconnected. If lines of the digraph are removed (or added), the strength of the resulting digraph is decreased (or increased). Thus, the greater the redundancy or number of lines in a digraph of a given strength the more invulnerable it is to losing strength if some of those lines are removed. Vulnerability gives an index of the number of lines which must be removed in order to reduce the strength of a digraph. Stated more formally:

> The line vulnerability of a digraph D . . . is the minimum number of lines in any strengthening set Y of D. In other words, this index gives the smallest number of lines whose removal reduces the category of a digraph. Thus, any property of D that depends upon the category of D may be destroyed if the number of lines removed equals or exceeds the line vulnerability of D. Clearly, however, the lines that are removed must all be contained in the same strengthening set of D (Harary et al., p. 213).

This concept can readily be seen in the example above where students V_3 and V_4 stopped communicating with student V_2. By removing lines V_2V_3 and V_3V_2 the strength of the digraph was reduced from 3 to 1.

Point bases. A fourth method for characterizing a digraph is point bases. "A point basis of a digraph is a minimal collection of points of D from which all its points are reachable" (Harary et al., p. 86). A point bases analysis is used to find the minimum number of points in D from which all other points within D are reachable. Conversely, given all of the points in D, what are the minimum number of points from which all points in D are reachable? "Stated in terms of communications networks, these two problems are (1) how to find a minimal collection of people required for a message to reach everyone (any one person in our example), and (2) how to find a minimal collection of people who, together, will learn of any message originating in the network" (Harary et al., p. 85; all persons in our communications example).

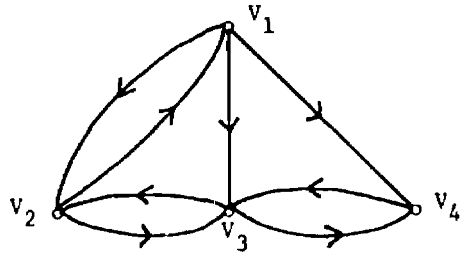
Matrices. The last method for describing digraphs is matrix analysis. Matrices permit quantification of the digraph. The value of matrix analysis is aptly stated by Kiss (1968, p. 708):

> There is an intimate connection between graphs and matrices. Indeed, one of the attractive features of this field is that one can shift between the graphical, intuitively more meaningful, and the matrix, sometimes analytically more powerful, methods of dealing with problems.

A matrix is defined as follows (Harary et al., p. 14):

> An r x s matrix is a rectangular array of rs numbers called the entries of the Matrix, arranged in r rows and s columns. We denote the entry in the i^{th} row and j^{th} column of matrix M by m_{ij}.

From the original classroom example, the digraph was:



From this digraph, an adjacency matrix A(D) can be formed (Harary et al., p. 15):

> Given a digraph D, its <u>adjacency matrix</u>, $A(D) = (a_{ij})$, is a square matrix with one row and one column for each point of D, in which the entry $a_{ij} = 1$ if line V_iV_j is in D, while $a_{ij} = 0$ if V_iV_j is not in D.

Referring to the digraph above, the following adjacency matrix is obtained.

A(D) =		V_1	V_2	V_3	V_4	Row Sum	
	V_1	0	1	1	1	3	
	V_2	1	0	1	0	2	Outdegree (od)
	V_3	0	1	0	1	2	
	V_4	0	0	1	0	1	
	Col. Sum	1	2	3	2		
			Indegree (id)				

Severa' features of the matrix need to be noted. First, any change in the orderir.g of points of the digraph will change A(D); the adjacency matrix is determined by the particular ordering of points in D. Next, if a digraph is asymmetric, then the existence of V_1V_2 precludes the existence of V_2V_1 and $a_{12} = 1$ while $a_{21} = 0$ in the A(D) matrix. In a symmetric digraph, the entry $a_{12} = 1$ implies the entry $a_{21} = 1$ in the adjacency matrix. The sum of the rows indicates the number of lines originating from V (<u>outdegree</u>) and the sum of the columns indicates the number of lines terminating at V (<u>indegree</u>). For example, the outdegree of V_1 equals 3 [we write od $V_1 = 3$] while the indegree of V_1 equals 1 [we write id $V_1 = 1$]. In other words, the teacher can communicate directly with each of the students, but only one student can communicate directly with the teacher. The total degree of V_i [we write td (V_i)] is the sum of the outdegree and the indegree (Td = od + id). Furthermore, the sum of the outdegrees across V equals the sum of the indegrees across V which equals the total number of lines in the digraph. In our example,

$3 + 2 + 2 + 1 = 1 + 2 + 3 + 2 = 8$; count the lines in the digraph. Finally, a transmitter is a point whose indegree equals zero and whose outdegree is greater than zero. A receiver is a point whose indegree is greater than zero and whose outdegree equals zero. A carrier is a point whose indegree and outdegree are both greater than zero. An isolate has an indegree of zero and an outdegree of zero.

"We now consider the reachability matrix R(D) whose entries are denoted r_{ij} and defined as follows: $r_{ij} = 1$ if v_j is reachable from v_i; otherwise $r_{ij} = 0$" (Harary et al., p. 117). In short, if D contains a sequence from v_i to v_j, then $r_{ij} = 1$. We consider all points reachable from themselves and thus: r_{ij} where $i = j$ equals 1. With respect to the adjacency matrix, if $a_{ij} = 1$, then $r_{ij} = 1$, but not conversely. The following reachability matrix is obtained for the digraph of classroom communication:

$$R(D_1) = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 1 & 1 & 1 \\ V_2 & 1 & 1 & 1 & 1 \\ V_3 & 1 & 1 & 1 & 1 \\ V_4 & 1 & 1 & 1 & 1 \end{array}$$

The obtained reachability matrix indicates that every person in the class can communicate (directly or indirectly) with everyone else in the class. Furthermore, this matrix confirms that the digraph demonstrates strong connectedness. In the example on page 9 where students V_3 and V_4 decided not to talk to the student V_2, the following reachability matrix is obtained for the digraph:

$$R(D_2) = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 1 & 1 & 1 \\ V_2 & 1 & 1 & 1 & 1 \\ V_3 & 0 & 0 & 1 & 1 \\ V_4 & 0 & 0 & 1 & 1 \end{array}$$

Since row 1 contains all ones, we know that the teacher is able to communicate with all the students. Of the students, only V_2 can communicate with everyone else (directly or indirectly).

"The connectedness matrix C(D) has the number n=0, 1, 2, 3 in its i, j location whenever the points v_i and v_j are n-connected in the digraph D. Clearly C(D) is a symmetric matrix" (Harary et al., p. 132). To obtain a connectedness matrix, an analysis by inspection can be performed, or the C matrix can be obtained from the R matrix. If the R matrix is used, then $C_{ij}=r_{ij}+r_{ji}+1$. The following matrices are obtained for the two digraph examples given in reachability matrices above.[6]

$$C(D_1) = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 3 & 3 & 3 & 3 \\ V_2 & 3 & 3 & 3 & 3 \\ V_3 & 3 & 3 & 3 & 3 \\ V_4 & 3 & 3 & 3 & 3 \end{array}$$

$$C(D_2) = \begin{array}{c|cccc} V_1 & 3 & 3 & 2 & 2 \\ V_2 & 3 & 3 & 2 & 2 \\ V_3 & 2 & 2 & 3 & 3 \\ V_4 & 2 & 2 & 3 & 3 \end{array}$$

That is:

$$\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} + \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array} + \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

---

[6] "If D is a weak digraph with connectedness matrix C and reachability matrix R, then C = R + R' + J" (Harary et al., p. 133. Where R' is R transpose and J is the universal matrix (all entries = 1).

Several properties of the connectedness matrix deserve mention. First, the lowest entry in C(D) gives the strength category of the entire digraph. Secondly, the strong components are given by 3s and the weak entries by nonzero entries.

The last type of matrix to be considered is the <u>distance matrix</u> [N(D)]. <u>Distance</u> was defined as the smallest number of lines between u and v; if one point was not reachable from another, the distance between these two points was considered infinite. A distance matrix may be characterized in the following manner (Harary et al., p. 135):

1. Every diagonal entry d_{ii} is 0 (the distance from every point to itself is zero),
2. $d_{ij} = \infty$ if $r_{ij} = 0$, and
3. Otherwise d_{ij} is the smallest power n to which A must be raised so that $a^{(n)}_{ij} = 0$, that is, so that the i, j entry of A^n # is 1.

Since this digraph is small, the distance matrices for the two examples can be derived by inspection. The procedural steps for determing N(D) from A(D) are given by Harary et al. (pp. 135-136).

$$N(D_1) = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 1 & 2 \\ v_3 & 2 & 1 & 0 & 1 \\ v_4 & 3 & 2 & 1 & 0 \end{array}$$

$$N(D_2) = \begin{array}{c|cccc} v_1 & 0 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 2 & 2 \\ v_3 & \infty & \infty & 0 & 1 \\ v_4 & \infty & \infty & 1 & 0 \end{array}$$

In the classroom communication structure example, only one student communicated directly with the teacher while the teacher communicated with

all students. The adjacency matrix partially reflected this but did not characterize the indirect communication in the classroom. Both the reachability and the connectedness matrices took into consideration the indirect communication as well as the direct, but made no discrimination between them. However, the value of the distance matrix is that it permits characterization and encoding of direct and indirect influences of the communication from the students to the teacher.

Applications of Digraph Theory to Research on Teaching

This section reviews research which: (a) has employed digraph theory and (b) has implications for research on teaching. The information processing view of man provides the frame of reference for this section. Research is reviewed wher stimulus structure, cognitive structure, or both have been analyzed with digraph (or graph) theory. Social psychological studies have been omitted because Harary et al. (1965) use many of these studies as examples and include a bibliography.

Research, Teaching, and Research on Teaching

Gage's (1963) definitions are adopted in this paper. "By research, we mean activity aimed at increasing our power to understand, predict, and control events of a given kind" (p. 96).

> By teaching, we mean . . . any interpersonal influence aimed at changing the ways in which other persons can or will behave. The restriction to "interpersonal" influence is intended to rule out physical (e.g., mechanical), physiological, or economic ways of influencing another's behavior, such as pushing him, drugging him, or depriving him of a job. Rather the influence has to impinge on the other person through his perceptual and cognitive processes, i.e., through his ways of getting meaning out of the objects and events that his senses make him aware of.
>
> The behavior producing the influence on another person may be "frozen" (so to speak) in the form of printed material, film, or the program of a teaching machine, but it is considered behavior nevertheless. . . .
>
> We define research on teaching . . . as research in which at least one variable consists of a behavior or characteristic of teachers" (pp. 96-97).

Structural Analysis of Instructional Material

Frase (1969, p. 2) examined "how we might quantify the depth and structure of text information, and how this structure can be related to

the processing activities of readers which result in learning." He used digraph theory to quantify the depth and structure of text information, and he used <u>orienting directions</u> ("a general class of goal-inducting stimuli including questions, verbal commands, typographical cues, etc., which might be used to alter the effective stimulation from text," p. 2), to control the processing activities of the readers.

Frase (p. 2) constructed a passage "from four primary sentences, which assert relationships among five classes. The classes are (A) farmers, (B) peace loving people, (C) hill people, (D) outcasts, and (E) Fundulas [a mythical people]. . . The primary assertions of this passage are: 'Farmers are peace loving'; 'Hill people are farmers'; 'Outcasts are hill people'; and 'Fundulas are outcasts.'"

From these four primary sentences, the following digraph was constructed:

$$V_B \longleftarrow V_A \longleftarrow V_C \longleftarrow V_D \longleftarrow V_E$$

An adjacency matrix was formed from the digraph.

A(D) =		V_B	V_A	V_C	V_D	V_E	od
	V_B	0	0	0	0	0	0
	V_A	1	0	0	0	0	1
	V_C	0	1	0	0	0	1
	V_D	0	0	1	0	0	1
	V_E	0	0	0	1	0	1
	id	1	1	1	1	0	

From the adjacency matrix, the following information is obtained. V_B is a <u>receiver</u> and V_E is a <u>transmitter</u>; all other points are <u>carriers</u>. Also, we can assume that the total degree value of a point (td = id + od) indicates the "number of encounters with that text point which are induced by Ss' reasoning behaviors. Retention of the text points should be a

function of the number of predicted encounters" (Frase, pp. 4-5). These assumed encounters are open to confirmation from empirical data. "The average outdegree value (which equals the average indegree value) must then represent the overall levels of encounters induced by the inferential behaviors" (Frase, p. 5). Furthermore, indegree and outdegree values represent class inclusion in this study.

The adjacency matrix represents each primary assertion in the text; i.e., A—>B, C—>A, etc. This matrix does not indicate whether S, reasoning from C, will arrive at A. The latter case Frase terms "deeper inferences." This information can be obtained by constructing a distance matrix. If it is possible to reason from C to B, the distance from C to B is finite and equals the number of lines in the shortest patch between C and B. If it is not possible to reason from C to B, the distance is infinite. The greater the distance between two points on the digraph, the greater the cognitive processing required to verify an assertion about two points on the digraph. The distance matrix, formed by inspection, is shown below.

$$N(D) = \begin{array}{c|ccccc} & V_B & V_A & V_C & V_D & V_E \\ \hline V_A & 0 & \infty & \infty & \infty & \infty \\ V_B & 1 & 0 & \infty & \infty & \infty \\ V_C & 2 & 1 & 0 & \infty & \infty \\ V_D & 3 & 2 & 1 & 0 & \infty \\ V_E & 4 & 3 & 2 & 1 & 0 \end{array}$$

Data in this matrix indicates that E—>B represents the highest level inference ($d_{EB} = 4$). The distance matrix "indicates all permissable universal inference in the passage, as well as the structural distance among the subject and predicates of those inferences" (Frase, p. 4). The distance matrix analysis assumes that the learner will give extraneous points only minimal processing; their information should not reach long-term memory.

> The view presented here, [sic] is that S can function as a logical processor, and the flow of text through this processing system is filtered in terms of the conceptual relations expressed in the text.

> If the conceptual distance, represented by that matrix, has cognitive reality for Ss exposed to the text, we would expect a significant difference in Ss' ability to recall and produce assertions of various distances. Figure 2, Section e [our distance matrix above], also predicts that these differences would not be found for backward associations (Frase, p. 5).

Frase investigated these hypotheses in a series of studies. In Study 1, the effect of different orienting directions on Ss recall of assertions was examined. Twenty-four Ss were assigned at random to one of two groups. Ss in group 1 were asked to verify the inference C—>B (from the digraph, C—>A—>B). Ss in group 2 were asked to verify the inference E—>C (from the digraph, E—>D—>C). In either condition, the distance from one assertion to the second was 2 (i.e., $d_{CB} = d_{EC} = 2$). Frase found no difference between groups in total recall. However, Ss in group 1 recalled assertions represented by A and B more frequently than Ss in group 2; Ss in group 2 recalled assertions represented by E and D more frequently than Ss in group 1. Both groups recalled assertions represented by C equally well.

In Study 2, Frase explored the effect of orienting directions of different path length (distance) on recall of text. This study is similar to the first except that inferences of the orienting direction ranged from A—>B ($d_{AB} = 1$) to E—>B ($d_{EB} = 4$). Recall of text points across inference levels was analyzed in a 5 x 4 repeated measures design; recall for primary (text) assertions versus inferential assertions across inference levels was analyzed in a 2 x 4 design. The results indicated that the points receiving direct analysis under the orienting direction were recalled more frequently than points not receiving analysis under the orienting direction. As path length increased, recall of text points was significantly greater for path length 4 than for path length 1. For primary (text) assertions such as "Hill people are farmers," recall of Ss in groups of path length 2-4 differed significantly from Ss in the path length 1 group. "There were no significant differences among the path lengths in recall of inferences. . . . The benefit of a deeper analysis of the text was primarily for the recall of text information, and not for the deeper knowledge which Ss might have derived" (p. 10).

In Study 3, sentence order was varied to determine whether order of passages interacts with inference. In addition, Ss were required to do the following: (a) determine if the orienting instruction is a valid deduction, (b) take a free recall test for each passage (three passages including the one about Fundulas, were used), and (c) take a multiple choice test which presented conclusions in the logical form; ten valid conclusions (forward associations) and ten invalid conclusions (backward associations). The study, then, was a 3 (sentence order) x 3 (path lengths of 1, 2, and 4) analysis of variance with repeated measures on the last factor. For the repeated measures factor, the same Ss read three different passages of five text points each. The findings of this study indicated that correct judgments of orienting direction validity for path lengths of 1, 2, and 4 were 95%, 78%, and 42%, respectively. The frequency of recall findings confirmed those of Study 2. The correct recall of primary statements was significantly higher than for inferences. Further, the deeper the orienting direction, the greater the recall of correct assertions. No difference was found between orienting direction groups and recall of primary and inferential information; recall of primary assertions was significantly greater than recall of inferential assertions across groups. Analysis of the multiple choice test demonstrated that Ss were good at identifying the invalidity of backward associations (70% correct). However, they did not do as well with forward associations (correct 50%).

In summary, orienting directions were influential in selective recall. Study 1 demonstrated that those points required by the orienting direction in analyzing text were recalled more frequently than those not required. Studies 2 and 3 extended this finding to demonstrate that higher-level processing (as defined by digraph <u>distance</u>) required by orienting directions added additional items to memory. "Surprisingly, however," says Frase (p. 15), "the consequence of deeper processing was to influence the retention of text information, and only slightly the deeper inferences which could be drawn from the text." He concluded (pp. 15-16):

> We might thus distinguish between two separate functions of orienting directions; one to control the learning of text by producing appropriate cognitive encounters, and another

> to control the processing activities of Ss in order to achieve some higher level learning outcomes. It was indicated in the introduction that orienting directions function to specify the range of stimuli and the behaviors which are to operate upon those stimuli. In a sense, they specify the content, product, and operation of an intellectual task. . . . If the required operations or behaviors are weak, these directions apparently can still function adequately as a method of controlling the reproduction of text. They are not necessarily adequate, however, in terms of the higher productive functions defined by the operation demanded.

Frase's monograph may have relevance to research at the Stanford Center for Research and Development in Teaching. Claus (1969), for example, investigated the effect of training programs designed to increase a teacher's use of "higher-order questions." Higher-order questions are assumed to require the respondent to use "higher-order cognitive processes" such as "analysis," "synthesis," and "evaluation," as defined by Bloom (1956). Higher-order questions may be contrasted with "lower-order questions" which require the respondent to recall facts from memory.

If Frase's "level of inference" is, in some fashion, analogous to "level of question," his findings suggest that the assumption that higher-order questions lead to higher-order cognitive processing is tenuous. Data bearing on this assertion are being analyzed at SCRDT, but the results are not yet available.

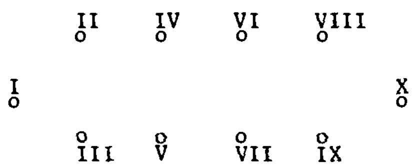
Frase's work also suggests that digraph theory can be used to determine the level of inference required to answer a teacher's question. The levels of inference, then, determines the level of question. The transcript of student-teacher interaction would be digraphed (Shavelson, 1970, provides rules for digraphing text); the distance (path length) between concepts in the teacher's question could be determined. Digraph theory, then, provides a more objective approach to determining level of question than, say, judges. The effects of "level of question" could be studied as Frase (1969) did. These ideas will be discussed again after other studies are reviewed.

Whereas Frase (p. 2) reasoned that "it seems more natural, and of greater practical importance, to induce these [Ss'] encounters [with

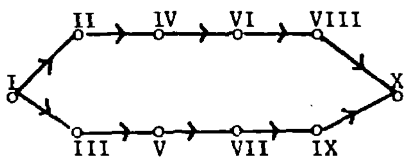
stimulus structure] by using different verbal orienting directions, rather than modifying the stimulus materials," Kopstein and Hanrieder (1966) chose to manipulate digraph-defined text structures. More specifically, they used digraph theory to determine five levels of content structure.

Their study was influenced by the concept of vulnerability. Digraph theory suggests that the more invulnerable (redundant and hence reliable) the digraph, the smaller the probability of its losing its degree of "connectedness"[7] when one line is removed. Applying the concept of vulnerability to subject matter, content structures can be more or less vulnerable. The less vulnerable the structure, the greater the probability the learner will inspect all of the elements in the content. Kopstein and Hanrieder hypothesized, therefore, that the more invulnerable the content structure, the more accurate the students would be in spontaneously reproducing what they had read.

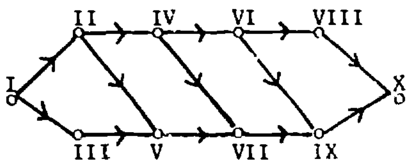
The following five digraphs were identified, and prose passages were constructed to fit each of the structures:



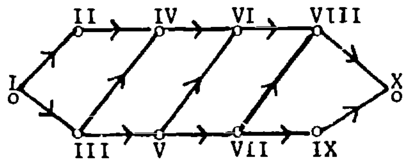
A



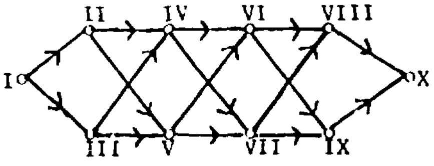
B



C



D



E

[7] In digraph theory, "connectedness" is defined as the degree of linkage between any two points of a digraph.

The prose passages were constructed to match each digraph by manipulating the meaning imposed on the concepts and the syntax of the instructional material. Each of the five passages contained approximately the same number of words, their only difference being the number and direction of cross-references between concepts.

The data did not confirm the hypothesis that as invulnerability increased, the accuracy of Ss' recall of the prose passages from long-term memory would increase.

> Although a number of dependent measures can be examined, results of the first administration were not analyzed extensively, because of the Ss' uniformly high response production. The mean numbers of responses produced for the five structures (i.e., 0, 2, 5-even route, 5-odd route, and 12 equipaths) were 39.12, 33.25, 41.00, 38.25, and 39.00.
>
> [In the second study, the authors report that] while the absolute magnitudes are substantially less than for the first administration, the differences between levels form no consistent pattern (Kopstein & Hanrieder, p. 14).

To explain these finds, the authors suggest that their analysis was too general and a more molecular analysis of structure would be required before learner differences could be observed. A number of alternative explanations can be advanced to account for these findings. For example, since the passages were relatively short (approximately 15-30 minutes of exposure time required), invulnerability may not have been critical. S may have reread the passage more than once or he may have organized the material in long-term memory by connecting concepts himself. If the stimulus material had been longer, redundancy (invulnerability) might have been critical.

Another hypothesis is suggested by a connectedness matrix analysis of Kopstein and Hanrieder's digraphs. The "strength" of four digraphs, B, C, D, and E, is "weak" (1-joined) while Digraph A is "disconnected" (0-joined). The most important feature of vulnerability is the specification of a subset of lines which, when removed, would reduce the strength of the digraph. The hypothesis is that invulnerability is an important factor in S's recall of text when, by not processing a text point, the strength of the digraph is changed. In the structures represented by

Kopstein and Hanrieder, the major comparison is between Digraph A and Digraph B since they represent different strengths. However, S's recall of Digraph A might equal his recall of Digraph B due to the loss of lines (e.g., S did not process several relationships); the consequence would be disconnected subgraphs in Digraph B. Perhaps, then, no difference in S's recall of Digraphs A and B would be expected. Given this condition, the concept of invulnerability (redundancy) becomes important. The greater redundancy in Digraphs C through E compared to Digraph B would prevent the structure from becoming "disconnected."

The Kopstein and Hanrieder study should be replicated with prose passages of greater length, and with digraph structures representing different levels of strength. Under these conditions, the structure variable might prove important.

This study shows digraph theory to be useful in generating alternative instructional treatments. Much of the research on teaching compares the effect of different instructional treatments. These facts suggest a natural bond between digraph theory and research on teaching. Digraph theory enables the researcher to generate alternative instructional treatments; the researcher can state precisely the ways in which his treatments differ.

Structural Analysis of Long-term Memory

Previous studies have dealt with the effect of stimulus structure on recall of that structure from long-term memory. In this paper the storage of stimulus structure in long-term memory is termed cognitive structure. The studies reported below use graph theory to investigate cognitive structure.

Before discussing these studies, certain features of graph theory, excluded from digraph theory, must be presented. By permitting "loops" in the digraph and numbers to be associated with lines in the digraph, the digraph becomes a "network." For studying cognitive structure, the most attractive feature of the network is that it allows the relationships (directed lines) between words, concepts, facts (points) to be quantified.

Kiss (1967, 1969) employed networks to study associative (cognitive) structure. He designated the points of his networks as words and the lines between points as associative relationships.

> Word-association norms usually specify the "strength" of an association by giving the relative frequency of one word as a response to another. It seems natural, therefore, to use these relative frequencies (estimates of probabilities) as the value of the network. The network of associative connections among a set of words can be ascertained by using each word in the set as a stimulus with a group of Ss, or with the same S repeatedly. The data from such an experiment can then be used for the construction of a network which is descriptive for the particular set of words and set of Ss on that particular occasion (Kiss, 1967, p. 709).

Kiss (1967) reported that the norms generated by the network mapping of the responses of 50 Ss to stimulus words showed a significant correlation between predicted (network) and observed (Ss) values with the average correlation being .61. Kiss (1969) demonstrated that a computer simulation using associative networks of cognitive structure is capable of generating responses which correlate with human Ss' responses from .58 to .73. Quillian (1967) and Giuliano (1963) have had success with similar types of networks.

Rapoport and Fillenbaum (1967) used "undirected graphs" to investigate cognitive structure. An undirected graph contains points and lines. The lines do not indicate direction of relations; they indicate, simply, that a relation exists. Rapoport and Fillenbaum instructed their students to build a graph where the points represent words and the lines represent the relationship among the words. The students were required to rank order these relationships by placing ranks on each line; the ranks represented similarity among words (the most similar pair of words received a rank of 1). From the students' graphs, a symmetric distance matrix was obtained. (The distance matrix for undirected graphs is symmetric since the direction of the relation is not specified.) The distance matrix was considered a proximity matrix and scaled using a multidimensional scaling procedure. The scaling solution identified the ways in which Ss clustered the words in long-term memory. In other studies (Rapoport, 1967; Rapoport, Rapoport, Levant, & Boyd, 1966) Rapoport has extended his procedures for using graphs to examine cognitive structure.

The contribution of these studies to research on teaching is in providing methodology for investigating cognitive structure. By using the method of word associations and networks (Kiss, 1967) or undirected graphs (e.g., Rapoport, 1967), the structure of a student's memory, at least in part, can be investigated. For example, digraph theory was suggested as a method for analyzing "higher-order questions." The effect of a teacher's higher-order question on a student's cognitive processing could be studied by having the student construct a graph in response to the question. The points on the student's graph and the interpoint distances should correspond to the digraph of the question in some specified way.

Digraph (or graph) analysis, then, provides a method of focusing on teacher (human, textbook)-student interaction. The teacher's cognitive structure, when communicated to the student, becomes the stimulus structure for the student, and vice versa. The teaching-learning process may be considered to be the transfer of cognitive structures. At certain times, the student is learning; at other times, he is teaching. The same applies to the teacher. Digraph theory provides an objective and reproducible method for analyzing structure.

Correspondence between Stimulus Structure and Cognitive Structure

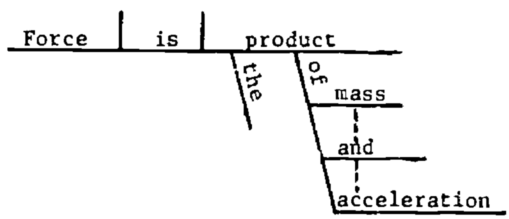
Studies reviewed above used graph theory (e.g., digraph, networks) to describe stimulus structure or cognitive structure. A study by Shavelson (1970, 1971, in press) investigated the correspondence between content structure (stimulus structure) and cognitive structure with digraphs and word associations.

Structure was defined as an assemblage of identifiable elements and the relationships between those elements. Content structure was defined as the web of facts and their relationships in the instructional material. Cognitive structure was defined as a hypothetical construct referring to the organization (relationships) of concepts in long-term memory.

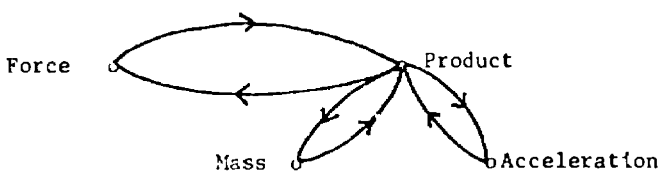
Instructional material used involved Newtonian mechanics; 14 concepts received special study. Content structure was represented using digraph theory. Each point on the digraph corresponded to one of the 14 concepts. The lines on the digraph corresponded to the relationships between those

concepts as specified by the syntax of sentences. What follows is an enumeration of the steps in the analysis (for a more detailed description, see Shavelson, 1970).

The first step was to identify the key concepts. Next, every sentence and equation containing two or more key concepts in the text (Dull et al., 1960) was diagrammed using the procedure suggested by Warriner and Griffith (1957). Then, each diagram was converted into a digraph using rules as reported by Shavelson (1970). For example, the sentence, "Force is the product of mass and acceleration," was diagrammed as:



Using the conversion rules, the following digraph was obtained:



The symmetric relation between force and product is specified by the rule for linking verbs; a linking verb does not specify action and is to be digraphed as a symmetric relation between two points. The symmetric relation between product and acceleration is specified by the rule for prepositions; if the preposition does not specify direction, the relation is digraphed symmetrically. The absence of a line symmetrically connecting mass and acceleration follows from the rule that whenever two words or groups of words are joined by a coordinating conjunction such as and, those words are digraphed independently of each other.

The distance between two points on a digraph is the number of lines in the shortest path connecting the two points. Only those digraphs representing the shortest distance between pairs of concepts received further

analysis. In this manner, 170 digraphs were reduced to 52. These digraphs contained key concepts and concepts lying in a path between them. In the example above, the key concepts of force, mass, and acceleration and the concept of product were contained in the digraph. To combine all 52 sentence digraphs into one digraph representing content structure, an adjacency matrix was formed. Once the adjacency matrix was formed for all 52 digraphs, this matrix was converted to a distance matrix using procedures given by Harary et al. (1965, pp. 135-136).

Cognitive structure was inferred from the organization of responses on a word association (WA) test. The number of responses to a concept and the overlap between responses to pairs of concepts were converted into a relatedness coefficient (RC) (Garskoff & Houston, 1963). This coefficient was used to determine relationships between pairs of concepts retrieved from long-term memory.

Forty high school students were pretested and placed at random into one of two groups: instruction (N-28) or control (N-12) group. At pretesting, all took aptitude tests, a WA test, and one of two forms of an achievement test. For the next five days, the instruction group studied the instructional material. At the end of each period of instruction, a WA test was administered. After the last period, Ss received the alternate form achievement test. The control group served as a methodological check; these Ss received all tests in a condensed period of time, three days, but received no instruction.

All Ss performed above chance on the achievement pretest. Instruction Ss gained significantly from pre- to posttest Control Ss showed no gain.

Data from the WA test confirmed these findings. The number of responses to concepts increased from day to day for the instruction Ss. Control Ss' responses increased in a similar manner for the first three tests, then leveled off well below the instruction group average on the last three. Late in instruction, responses of instruction Ss were qualitatively different from responses of control Ss.

RCs were calculated for the WA data. The median RC for the six control-group WA tests never exceeded .06. Median RCs for instruction Ss for tests 1 through 6 were: .00, .09, .15, .22, .27, and .32, respectively.

Instruction and control RC matrices became increasingly dissimilar as instruction progressed. The RC matrices were scaled using Kruskal's (1964) procedure. Four concept clusters emerged at pretesting and remained across RC matrices for both groups.

Comparison of the measure of content structure (digraph distance matrix) with the measure of cognitive structure (RC matrices) indicated that instruction Ss' memory structure corresponded more closely to content structure as instruction progressed. For control Ss, cognitive structures were as unlike the content structure on the sixth WA test as on the first. When the digraph matrix was scaled, three clusters emerged which were very similar to three of the four clusters for cognitive structure.

Verbal communication (spoken, written) is the most frequently used medium in teaching. Yet methods for analyzing the content of this medium are of the most rudimentary types (e.g., Berelson, 1954). Usually categories are set up, a unit of analysis is determined, and judges count the frequency with which the content units are classified into a category. The Shavelson study provides a method for translating prose into digraphs. Digraph analysis offers a myriad of ways for analyzing the content ranging from frequency counts (adjacency matrix) to measures of linkages between concepts (e.g., distance matrix).

One of the most important applications of digraph theory to research on teaching is interaction analysis. Teacher-student interaction can be mapped and the correspondence between the two structures ascertained. Rather than applying a label such as "Teacher summarizes student's statement," the structures can be compared directly. Labels are not necessary to describe the communication between the teacher and student.

Gage and his students (personal communication), using Shavelson's method for mapping prose, have applied digraph analysis to essay tests. They have been working on methods for training teachers to use adequate explanations. An explanation contains concepts; part of the adequacy of an explanation rests on how the concepts are interrelated. Digraph theory is used to explore the correspondence between the teacher trainee's explanation (essay) and the "ideal" explanation.

Shavelson's study, then, points toward research on teaching which compares stimulus structure with cognitive structure, a direct comparison of the effects of teacher-student interaction. Teaching is viewed as the communication of the structure of a discipline to the learner through an _inter_personal process. By focusing on the structure in the input and the structure in the learner's memory, research on teaching also focuses on learning itself, an _intra_personal even (cf. Snow, 1969).

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