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ABSTRACT

The objective of the investigation was to develop and test procedures for training children in coordination of projective space. (Projective concepts involve apparent distance, relative position, shape of figures, and other topological factors. A person with a command of projective space sees objects as a coordinated system of figures in space.) A total of 36 8-year-old middle class children who could not coordinate projective space were trained in a carefully sequenced instructional program embodying heirarchies based on Piagetian theory as one dimension, Brunerian modes of representation as a second, and complexity of task as the third. Gains in ability to coordinate projective space, as measured by the Perspective Ability Test developed by the senior author, exceeded those of a control group. There were no significant differences between performance gains of males and females. It is recommended that previous studies that suggested Piaget's normative process is immutable--and that basic cognitive development cannot be enhanced significantly through instruction--be reevaluated in terms of theoretical deficiencies, inappropriate methodology, measurement problems, and other factors. The possibility of successful training on other spatial tasks and with younger children should be considered. Appendixes provide statistical data, training lessons, and explanations of Piagetian theory. (Authors/DR)

A SUCCESSFUL ATTEMPT TO TRAIN CHILDREN IN COORDINATION OF PROJECTIVE SPACE

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A SUCCESSFUL ATTEMPT TO TRAIN CHILDREN IN COORDINATION OF PROJECTIVE SPACE[1]

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BACKGROUND

Ability to operate effectively in a spatial environment is an imperative for man, whatever his cultural milieu. In non-industrial societies men must master tasks such as judging relationships between size of an image and the probable distance of an object, compensate for distortion caused by light refraction in water, and develop a spatial guidance system sufficient to permit locating and returning to a camp or village site. Individuals in an industrial society face similar and more complex demands. Proficiency in judging the speed of an approaching vehicle by the rate of its apparent size increase, visualization of three-dimensional objects and object relationships presented in two-dimensions, and representation of relationships to an audience far removed in space and time through drawings and maps have become cultural imperatives for twentieth century man. There is evidence that spatial capabilities develop sequentially in any culture, are influenced to an unknown degree by the environment, are quite variable among individuals within a society, and are related to later success in a number of areas, including reading, mathematics, engineering, geography, and graphic arts.

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Piagetan Theory

Only Piaget offers a comprehensive theory and supporting body of research concerning development of spatial schemas. According to him and his associates, adaptations in spatial cognition are analogous to those in time. Both begin with the local and immediate and are applied separately to each situation or movement. For the very young child there seem to be as many spaces as there are objects or patterns. Intervals between distant elements are perceived as belonging either to the elements themselves or as not being spatial at all. Only later are all of these separate factors fused together into an interlocking and coherent system.

The Genevans organize their theory on a mathematical model, with strong interrelationships and considerable overlapping. Thus the child is seen as entering the first stages of projective space operations at the same time that he is completing mastery of the preceeding topological operations. As he approaches maturity in the latter, he is already dealing successfully with some aspects of Euclidean space.

Topological concepts involve the relationship between parts of a single object or between an object and its immediate environment, continuously and independent of distance. Elements arrayed in topological space can be expanded or contracted at will without conservation of line, distance, or angle. While a child in the pre-operational or early period of concrete operations (ages 4-9 years) may be able to solve some spatial problems, he demonstrates no stable system for conservation of elements and the spaces that separate them. Analyses tend to show that initial concepts of space are influenced by notions of proximity, separation, order, enclosure, and continuity--operations limited to spatio-temporal, non-metric perception.

Projective concepts involve apparent distance, relative position, shape of figures, and other topological factors. However, objects are not viewed in isolation, but are seen as part of a coordinated system of objects in space. The individual with a command of projective space understands and can apply the understanding that appearance of elements and relationships in space depends heavily on viewing point. While the projective system does not conserve dimensions and distances, it does conserve relative positions of figures in relation to one another, parts of figures, and the whole in relation to the observer or a plane corresponding to his visual field--the essential factor being entry of the observer into the system. Mastery or projective space is signified by ability to coordinate all possible perspectives.

Euclidean concepts incorporate all elements of topological and projective space and, in addition, involve conservation of length, angularity, parallism, distance, area, and interior volume. Thus, the individual with mature competence in Euclidean space is able to simultaneously conserve over three dimensions. For most humans this is hypothesized to occur in late adolescence; for many it may never appear in mature form without special training.

Antecedent Studies

Training human subjects in spatial operations based on the Piagetan model is a comparatively recent concern of educational researchers. Five reports serve as direct antecedents to the present investigation.

In 1963 Eliot reported an attempt to teach spatial relationships to third grade pupils in Boston elementary schools. Concepts of direction, scale, and distance were introduced through an emphasis on hand work,

rather than on verbalization. Subjects began with tactual activities involving texture and density and progressed to construction of topographic sketches of wooden models. The principal criterion measure was a three mountain map test similar to the one described by Piaget and Inhelder in The Child's Conception of Space. While Eliot's results were not statistically significant, he did conclude that a change in behavior had been produced and that some lessons were considerably more successful than others. Later, Eliot undertook a projective spatial training program based on the Piagetan principle of reversibility (1966). Although a rather large N was involved (257 Ss enrolled in kindergarten, first grade, and third grade) and training extended over a four-week period, results were mixed and did not generally support the principle of reversibility as a key to successful projective space coordination.

The senior author of the present paper has reported three earlier investigations related to projective space coordinations (1967, 1968, 1969). The first two dealt with development of a satisfactory measure of ability to coordinate perspectives, following the pattern employed by Piaget and Inhelder, Eliot, and others. This instrument has been used with nearly a thousand subjects between five and sixteen years of age, and is described in detail elsewhere (1968).

In 1969 the senior author and two associates reported a successful training study involving four groups of seven-year-old children. One group was provided a teacher-directed program in the enactive-iconic modes, utilizing spatial projections and sighting. A second group underwent an automated program that involved aiming and line segment extension, organized to some extent on the logic of Euclidean space. The third group experienced

a combination of the two programs, beginning with automated aiming and line segment lessons and then converting to teacher-directed activities. A control group received no special training over the two weeks duration of the study. Posttesting revealed an overall tendency toward improvement of ability to coordinate perspectives. As anticipated, however, the gain was differential, showing an interaction between method of instruction and sex of the subject. Among the boys the teacher-directed activity program produced gains over the controls significant at the .05 level. Gains of males in the combination program approached significance; the automated program proved to be totally ineffective. Girls in the sample were found to be extremely variable. While significant differences were not found, their mean gains under the three experimental procedures approximated that of the boys, so that improvement under the "activity" program was highest, combination next, and automated aiming-sighting least.

THE PRESENT STUDY

The present investigation had as its basic objective development and trial of procedures for successful training of children in coordination of projective space. The primary hypothesis was:

> Children who are provided a series of lessons related to spatial cognition will make significantly greater gains between pre- and posttest sessions than will children who are not exposed to the special lessons.

Because of sex differences frequently found in spatial visualization studies, recently summarized by Sherman in an issue of Psychological Review (1967) and apparently supported by the earlier training study conducted by the senior author and his associates (1969), a secondary hypothesis stated:

> Within the experimental group, boys will demonstrate significantly greater improvement than will girls.

Frankly, we hoped that our training procedures would be so powerful and universal that this hypothesis would not be sustained.

Rationale for Training Procedures

The primary dimension on which training modules were organized was founded on the work of Piaget and Inhelder. Their summary in The Child's Conception of Space (1948) lists eight factors as essential to cognition of projective space. These are addition and subtraction of projective elements, rectilinear order, complementary perspective relations, symmetrical interval relations, one-one multiplication of elements, one-one multiplication of relations, one-many multiplication of elements, and one-many multiplication of relations. The list is generally hierarchical, with subfactors useful for training within several of the elements. A brief description of each element is provided in Appendix B.

A second dimension of the training protocol was based on the enactive, iconic, and symbolic modes of representation discussed in Toward A Theory of Instruction (1966) and other writings of Jerome Bruner. The modes also are hierarchical, ranging from minimal to maximal abstraction and internalization of thought.

The third dimension in structuring training modules was level of complexity. Within a cross-classified cell of spatial element and Brunerian mode of representation, subjects were called upon to deal first with very simple tasks, perhaps involving one or two objects or perspective positions, and later with increasingly difficult ones involving multiple objects, relations, and perspectives.

This three-dimensional design of training modules is primarily the result of analysis and pilot work by Haroldine C. Miller, my co-author.

To bring the descriptions to a more managable level, Appendix C summarizes the teaching strategies along the dimensions of Piagetan hierarchy and level of complexity; Appendix D expands the Piagetan scheme to eleven elements and relates these to Bruner's modes of representation; and Appendix E contains one of the actual lesson plans followed in the study.

Each child in the experimental group was provided a total of eight lessons over a three-week period. Average length of the lessons was 30 minutes, with some variation to accommodate individual learning patterns. In anticipation of subject differences, lessons were structured in modules with an estimated length of 5-15 minutes. Subjects sometimes completed only one or two modules of a lesson, and at other times moved rapidly through three, four, or five modules. The final module of each lesson was diagnostic, providing feedback to both subject and experimenter on the child's true grasp of the spatial operation just covered. If the results were negative an appropriate review or repetition was undertaken before moving into the next group of lesson modules.

Criterion Measure

The Perspective Ability Test, developed earlier (Miller, 1967. 1968) served as the criterion measure. Essentially it consists of presenting the subject with a circular map table and 20 slides taken from positions around its circumference. The subject is asked to indicate the location of the camera for each slide, a procedure providing a quick and reliable estimate of his ability to coordinate projective space. Since the procedure is unusual, brief, and provides little feedback, using it for successive testings has not presented difficulties to experimenter or subject.

Other Details

Subjects were middle class white pupils enrolled in third grade at Hill Elementary School in Nashville, Tennessee. Of 83 potential subjects in the school, 31 were eliminated because they already demonstrated ability to coordinate some perspectives through error scores of 50 or less; 9 others were eliminated because of error scores above the chance level (100 points). The remaining 43 third graders were dichotomized into two numbered lists based on sex. Nine males and nine females were assigned to each treatment, for a total N of 36 in a 2x2x2 repeated measures mixed ANOVA design.

Results

Appendix A presents mean error scores for all groups and the results of data analyses. As these data show, the experimental group made extraordinary gains in ability to coordinate perspectives, while the gains made by the control subjects in the period were nonsignificant. (A retention test seven months after the close of the study revealed that the subjects in the experimental group had experienced only a slight regression in ability to coordinate projective space. The controls had continued a rather steady developmental gain in ability to coordinate perspectives, achieving significant improvement over their pretest and posttest scores.) Sex differences were nonsignificant in either experimental or control groups, although the control girls showed the usual pattern of lagging behind the boys in spatial performance. Mean error scores for the experimental and control groups at pretest, posttest, and retention test points are shown graphically on page 10. (See Figure 1.)

Discussion

The children involved in this study, averaging about nine years of age,

were assumed to be transitional so far as projective spatial operations were concerned. Normal expectations were that a majority of them would make steady progress in conceptualization of space and would become able to solve most projective situations within the next three or four years. This assumption seems borne out by the performance of children in the control group, who made gradual but significant progress from the first to the third testing.

On the other hand, we were able to assist children assigned to the experimental group to improve dramatically ($P < .000000$) in a very short period of time. Those gains were sustained over many months when we had absolutely no contact with the children.

In view of this success, previous investigations which failed to produce significant results and which contributed to judgments by some that the normative process described by Piaget is immutable--and that basic cognitive development cannot be enhanced significantly through instruction--might well be reevaluated in terms of theoretical deficiencies, inappropriateness of methodology, measurement problems, and other factors. The possibility of successful training on other spatial tasks and with younger children than those involved in this study becomes increasingly attractive.

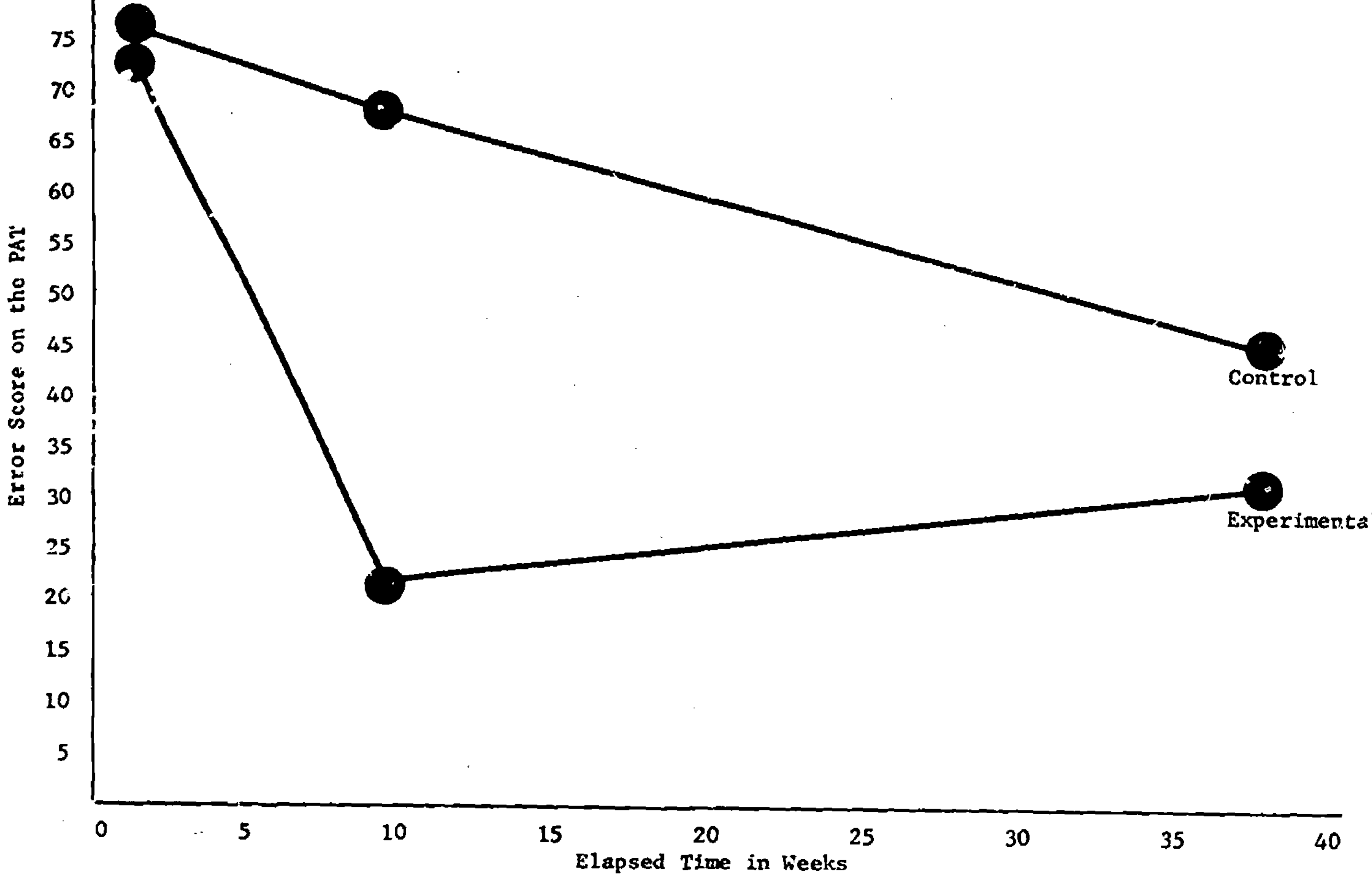

Figure 1. Mean error scores on the Perspective Ability Test (PAT): Pretest, Posttest, and Retention.

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APPENDIX A

MEANS AND ANOVA TABLES

TABLE 1: MEAN ERROR SCORES ON THE PERSPECTIVE ABILITY TEST (PAT): PRETEST, POSTTEST, AND RETENTION TEST

Group	N	Pretest	Posttest	Retention Test
Experimental				
Males	9	74.11	24.78	33.44
Females	9	72.78	18.44	29.56
Combined	18	73.44	21.61	31.50
Controls				
Male	9	73.78	60.78	34.89
Females	9	78.67	74.67	56.11
Combined	18	76.22	67.72	45.50

TABLE 2: ANALYSIS OF VARIANCE, PRETEST PAT ERROR SCORES OF EXPERIMENTAL AND CONTROL GROUPS

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Groups	1	69.4444	69.4444	.23	.64
Within Groups	34	10,091.5556	296.8104		
Totals	35	10,161.000			

TABLE 3: ANALYSIS OF VARIANCE, PRE- AND POST-TEST PAT ERROR SCORES FOR EXPERIMENTAL AND CONTROL GROUPS

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Treatments (B)	1	10,755.5555	10,755.5555		
Between Girls and Boys (C)	1	138.8888	138.8888	.28	.61
Treatment and Sex Interaction (B X C)	1	786.7223	786.7223	1.56	.22
Error Between Subjects	32	16,154.3334	504.8229		
Total Between Subjects	35	27,835.5000			
Change in Scores, Pre- to Post-tests (A)	1	16,380.5000	16,380.5000		
Test and Treatment Interaction (A X B)	1	8,450.0000	8,450.0000	24.73	.0001
Test and Sex Interaction (A X C)	1	18.0000	18.0000	.05	.82
Test, Treatment, Sex Interaction (A X B X C)	1	220.5000	220.5000	.65	.57
Error Within Subjects	32	10,935.0000	341.7187		
Total Within Subjects	36	36,004.0000			
Grand Total	71	63,839.5000			

TABLE 4: ANALYSIS OF VARIANCE, PRE- TO POST-TEST PAT ERROR SCORE CHANGE FOR THE EXPERIMENTAL GROUP

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Tests	1	24,180.2500	24,180.2500	117.87	.0000
Within Tests	34	6,974.7233	205.1389		
Total	35	31,154.9733			

TABLE 5: ANALYSIS OF VARIANCE, PRE- AND POST-TEST PAT ERROR SCORE CHANGE FOR THE CONTROL GROUP

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Tests	1	650.2500	650.2500	1.04	.32
Within Tests	34	21,278.7223	625.8447		
Total	35	21,928.9723			

TABLE 6: ANALYSIS OF VARIANCE, PRE-, POST- AND RETENTION TEST PAT ERROR SCORES FOR EXPERIMENTAL AND CONTROL GROUPS

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Treatments (B)	1	11,865.0369	11,865.0369		
Between Males and Females (C)	1	606.8147	606.8147	.79	.62
Treatment and Sex Interaction (B X C)	1	1,993.4817	1,9[illegible]3.4817	2.59	.11
Error Between Subjects	32	24,584.6667	768.2708		
Total Between Subjects	35	39,050.0000			
Change in Scores over Three Tests (A)	2	27,218.0000	13,609.0000		
Treatment and Test Interaction (A X B)	3	9,104.5186	3,034.8395	9.27	.0001
Test and Sex Interaction (A X C)	3	226.0741	75.3580	.23	.88
Test, Treatment, Sex Interaction (A X B X C)	3	432.5184	144.1728	.44	.73
Error Within Subjects	61	19,964.8889	327.2932		
Total Within Subjects	72	56,946.0000	790.9166		
Grand Total	107	95,996.0000			

TABLE 7: ANALYSIS OF VARIANCE, PRETEST, POSTTEST, AND RETENTION TEST PAT ERROR SCORE CHANGE FOR EXPERIMENTAL GROUP

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Tests	2	26,417.3703	13,208.6851	39.70	.0000
Within Tests	51	16,969.2223			
Total	53	46,386.5926			

*Since cell sizes were equal, samples were originally from the same population, and homogeneity of variance across all cells was confirmed by the Hartley F_{max} test, the Newman-Keuls procedure was followed in comparing differences between tests. These confirmed that error scores dropped significantly (.01) from the pretest to the training posttest (determined also in the earlier analysis) and that the slight increase from posttest to retention test 28 weeks later was nonsignificant.

TABLE 8: ANALYSIS OF VARIANCE, PRETEST, POSTTEST, AND RETENTION TEST PAT ERROR SCORE CHANGE FOR CONTROL GROUP

Source of Variability	df	Sum of Squares	Mean Squares	F	P(F)
Between Tests	2	9,059.5926	4,529.7963	7.49	.0018
Within Tests	51	30,839.2223	604.6906		
Total	53	39,898.8149			

*Newman-Keuls subanalyses confirmed that PAT error scores at pretest and posttest were not significantly different. However, the reduction in PAT error score over the 28 weeks between posttest and retention test was significant at the .01 level.

TABLE 9: PAT ERROR SCORES OF ALL SUBJECTS OVER THREE TESTS

Males				Females			
S	Pre	Post	Retention	S	Pre	Post	Retention
			Experimental Groups				
1	87	27	28	1	61	21	14
2	60	17	13	2	78	7	14
3	85	19	70	3	88	20	29
4	51	55	29	4	64	19	55
5	100	19	[illegible]	5	64	26	35
6	86	29	22	6	62	9	9
7	53	16	11	7	100	34	70
8	53	4	15	8	78	19	17
9	92	27	93	9	60	11	23
			Control Groups				
1	52	41	27	1	87	45	66
2	52	27	13	2	100	96	72
3	96	96	25	3	60	72	25
4	76	65	34	4	80	51	37
5	62	18	15	5	55	74	38
6	54	113	63	6	94	83	53
7	97	8	11	7	94	108	51
8	90	94	60	8	82	76	64
9	85	85	66	9	56	67	94

APPENDIX B

PIAGETAN ELEMENTS OF PROJECTIVE SPACE COGNITION

1. Addition and Subtraction of Projective Elements. Changes in projective figures result from changes in point of view, implying a coordination system in which conservation of the whole is maintained throughout a series of transformations. In some perspectives, spaces are blocked from view, causing objects to be joined together (addition). From other vantage points, objects are completely hidden (subtraction). These additions and subtractions systematically change as the viewpoint is changed.

2. Rectilinear Order. Since the straight line segment is the only shape that remains unaltered throughout perspective transformations, rectilinear order constitutes a basic operation acquired through practice in taking aim. This involves reduction of a line segment length to a single point and facilitates recognition of a straight line segment as such. The ability to perceive and conceptualize a straight line represents an important step in cognition of projective space. Piaget and Inhelder regard (the power to imagine . . . straight lines facing in any direction as the essential requirement for forming perspectives (1948, p. 171).

3. Complementary Perspective Relations. When an observer assumes a reciprocal position in regard to elements, the sequence of left-right and before-behind are reversed. Since the viewpoints determining these perspectives are complimentary to each other, the operation expresses the conservation of relative position of elements with viewpoint reversals.

4. Symmetrical Interval Relations. In a series, the middle element position is conserved throughout transformations. The left-right relationships of elements and spaces always remain symmetrical in relation to the unchanged middle element.

5. One-one Multiplication of Elements. With the introduction of the perspective viewpoint, the system takes into account projective dimensions. These include relationships among a number of figures corresponding to left-right, above-below, and before-behind. Together with perspective viewpoints, relationships in these three dimensions result in the concept of a plane or a set of planes. It is worthy to note that a plane seen from a given viewpoint is reduced to a straight line--all of which run either left-right or before-behind.

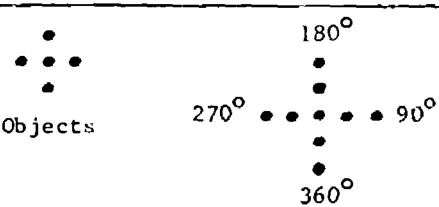
6. One-one Multiplication of Relations. Not only objects, but spaces between them are related simultaneously in terms of left-right, before-behind, and above-below dimensions. These relationships can be observed in perspective drawing, where the height and width of the background are related to the dimensions of the foreground and shown through a projective decrease in length. Such an operation represents an intensive multiplication of relationships that would be demonstrated by a child if he drew a slanting stick as foreshortened.

7-8. <u>One-many Multiplication of Elements and Relations</u>. The one-many relationship schemata bring the child closest to extensive quantification (mathematical treatment of perspective), which would be a part of the hierarchally superior Euclidean space. One-many multiplication of elements and relations expresses change in size with change in distance, therefore, the enlargement is continuous and uniform. For example, in a drawing of a railroad track the rails will meet at the horizon or zero point. On another dimension, the longest crosstie would be in the foreground and the shortest at the zero point, with all others in ratio between. One-many correspondence may be expressed either in terms of elements or of relationships.

APPENDIX C

MATRIX OF TEACHING STRATEGIES BASED ON LEVELS OF COMPLEXITY AND THE PIAGETAN HIERARCHY OF SPATIAL OPERATIONS

Level of Complexity	I Addition and Subtraction of Projective Elements	II Rectilinear Order	III Complementary Relations
Low	Experimenter and subject stand in specified overlapping positions in front of mirror.	Experimenter and subject stand in front of mirror in rectilinear order, showing four rotations of view. 90° - 180° - 270° - 360°	Experimenter, subject and Snoopy stand in front of mirror on circular rotating disc in irregular positions. Child predicts reciprocal position. Disc is then rotated 180° to check prediction.
	Manipulate two objects in specified overlapping positions.	Sight across circular surface to specific object using black rod to represent sighting line. Sight at 0° elevation with camera as sighting device.	Manipulates two, three and four objects in irregular positions or circular surface.
	Manipulate three and four objects in specified overlapping positions.	Sight across two objects on circular surface. Represent sighting line with black rod. Use camera with string attached to form sighting line to first object at 0° elevation.	Child arranges duplicate set of objects as they appear to him when he moves 180° around circular surface (reciprocal position).
	Manipulate the rectilinear order of three and four objects, with specified objects or parts of objects visible from defined positions.	Sight across two, three and four objects on round surface. Use string to represent sighting lines at from 0° at designated positions around table.	Child arranges duplicate set of objects as they appear to him when he moves 90° around circular surface, then 270° (reciprocal positions).
High	Manipulates relations of three and four objects, with specified objects or parts of objects visible from defined positions	Sights across three and four objects on round surface from 0° at designated positions around table. No props (imaginary line)	Child arranges duplicate set of objects as they would look to Snoopy when Snoopy moves to 180° position. 90° and 270° positions.

Level of Complexity	IV Symmetrical Relations	V Left-Right Relations	VI Before-Behind Relations
Low	Experimenter, subject and Snoopy stand in rectilinear order in front of mirror on rotating disc. Disc is rotated 90°, 180°, 270° to see if middle person (Snoopy) maintains middle identify	Use handshaking, hopping, turn about, and mirror games to establish left-right relations of child and experimenter. (Rotate disc.)	Use turn about game with child, experimenter and Snoopy to establish before-behind relations. (Rotate disc in front of mirror.)
	Arrange three objects in rectilinear order on circular surface. Rotate surface 180° to see if middle object maintains middle identify	Use Linnus with blanket on right arm to establish his life & right side. Reverse his position. Remove blanket and reserve his positions.	Use two, three and four objects child moves to different positions around table and describes before-behind.
	Arrange five objects--four equidistance from the fifth object in middle. Child predicts pattern of objects during rotations. Will 5th maintain middle identify.	Left-right relations using doll & object with doll facing in same direction as child--next with doll facing the child.	A doll and five objects are positioned on circular surface. As doll is maneuvered the child describes before-behind relationships to the doll.
	Arrange nine objects in form of cross with ninth object in middle. Will relative position be maintained. Will 9th object maintain middle identify in rotations.	Left-right relation of doll and three objects--doll facing same direction as child--then opposite.	Child stays in one position and predicts view when disc is rotated 90°, 180°, and 270°.
High	 5 Objects 9 Objects	Array of 5 objects on rotating disc, child describes left-right relations as they related to different position of view.	Child predicts view Snoopy will see as Snoopy is positioned at different points around circular surface.

Level of Complexity	VII Above-Below Relations	VIII One-One Multiplication of Elements	IX One-Many Multiplication of Elements
Low	Experimenter, subject & Snoopy stand in front of mirror and use yarn as sight line from eye level to eye level as one looks at the others eyes.	Child lies on rotating disc in front on mirror and sees when he looks the longest and shortest. He predicts other rotations.	Child stands close to mirror and his size is noted. Experimenter then moves back until he looks the same height or shorter than the child.
	Manipulate two, three and four objects in rectilinear order. Describe view from 45° elevation when circular surface is rotated 90°, 180°, and 270°.	Child rotates a 12" colored rod in different positions and notes apparent lengths relations in different positions.	Child maneuvers three equal size dolls in front of mirror until all three look a different size. Discuss how they would look to Snoopy from another vantage point.
	Sight from 30°, 15°, and 0° elevation. Have Snoopy sight from these positions.	Child rotates two, three, and four 12" colored rods to different positions and compares apparent lengths from a specified veiwpoint	Child extends line of blocks until size of farthest block appears smaller than nearest block. Discuss how formation would look to Snoopy from another vantage point.
	Manipulate objects in irregular pattern and sight from 15°, 30°, 45° and 0° elevation. Rotate circular surface 90°, 180° and 270°.	Child rotates four 12" colored rods to different positions and predicts how these apparent distances compare from Snoopy's viewpoint as Snoopy is moved to different positions.	Child links together a straight railroad track with farthest section appear narrower and shorter than nearest position. Discuss appearance from another vantage point.
High	Child predicts view he would see from specified elevations.	Child rotates four, five, and six colored toothpicks to different positions and compares apparent lengths from different vantage points.	Child extends two parallel pieces of black yarn with equidistant cross pieces placed between until farthest portions appears narrower and shorter. Discuss vantage point.

Level of Complexity	X Horizontal and Vertical Axis	XI Coordination of Viewpoints
Low	Child would drink ½ bottle of coke in front of mirror noting position of liquid in relation to bottle as he tips bottle at different angles.	Child is rotated on disc in front of 3-way mirror and sees how he looks from different views. Next use experimenter and Snoopy.
	Child will then predict levels if turned 45°, 90°, and 180°. He can draw in water levels.	Experimenter, subject, Snoopy, Lucy, and Linus stand at specified points on rotating disc in front of mirror and view scene in middle of disc. Predict views of each person--turn disc and check with mirror.
	With circular cardboard surface on the wall the child will mark points and snap a chalk line in a vertical position to cut circular surface in half.	Place four dolls on circular surface within 3-way mirror--child can see 3 views other than his own. Remove mirrors and child walks around and verifies views.
	Section circular cardboard surface in half at 15° intervals but always with chal line at vertical position--snap line. Watch vertical line become horizontal line when turned 90°.	Round surface is then rotated and child predicts views. Will predict views from ten different points around the table.
High	Child will label one and two points used in sighting across the sectioned rotating round sur-	Use red and green blocks of different heights and size to form four similar figures. Use 3-way mirror and rotating table. Predict views from 20 viewpoints.

APPENDIX D

APPLICATIONS OF ENACTIVE, ICONIC, AND SYMBOLIC REPRESENTATIONAL MODES TO TEACHING STRATEGIES

Addition & Subtraction of Elements	Rectilinear Order	Complementary Relations	Symmetrical Relations	Left-Right Relations	Before-Behind Relations	Above-Below Relations	One-One Mult. of Elements & Relations	One-Many Mult. of Elements & Relations	Horizontal & Vertical Axis	Coordination Viewpoints

Enactive Mode	=	The experimenter, subject, and numbers of objects are placed in the positional relationships indicated above. They are viewed from specified vantage points as transformations are made by experimenter and subject.
Iconic Mode	=	Child represents positions of self, experimenter, and objects in above positional arrangements on flannel board using cut-out representations and yarn. Transformations will be predicted on flannel board.
Iconic Mode	=	Child identifies photographs and diagrams that represent positional relationships of objects from specific viewpoints. Child will make some representative diagrams of transformations.
Symbolic Mode	=	With screen in front of all objects and operations, the child verbalizes positional arrangements and perspective views when specific transformations are made. Screen is removed to verify his solution.

APPENDIX E

Lesson VI

OBJECTIVE: When child views four Peanuts characters on a round disc, the child can reconstruct each of the eight views with cutouts of characters. Using photos he can identify by number the camera position where the photo was taken.

ELEMENTS: Addition and Subtraction of Elements, Complementary Relations, Left-Right Relations, Before-Behind Relations, Above-Below Relations, Coordination of Viewpoints (decentering).

MATERIALS: 2 round discs that turn, round disc for reconstruction of viewpoint, cutouts of four Peanuts characters, large Snoopy, Charlie Brown, Lucy, Linus and Snoopy, photos, rods, numbers 1-8, 3-way mirror, camera with red yarn attached for Snoopy.

PROCEDURE:

I. Characters and Mirros
Position: Child in front of 3-way mirror

A. Name the characters. Place each character on rotating disc within view of 3-way mirror and discuss how each character looks from all four sides.

B. Put Snoopy behind mirror and get reverse views.

Questions:
1. What does Snoopy see?
2. What is to Snoopy's left? His right?

II. Characters on circular disc
Position: Child behind black sighting line

A. Order of positions to use

1. Lucy at sighting line (No. 1)
2. Linus at sighting line (No. 3)
3. Charlie Brown at sighting line (No. 5)
4. Snoopy at sighting line (No. 7)
5. No. 2 at sighting line
6. No. 4 at sighting line
7. No. 6 at sighting line
8. No. 8 at sighting line

B. Turn the crank and have child use cutouts to represent the 8 views. Lay the red rod on disc to show sighting line for each of the views. Show the child the cutouts--front and back, left and right sides.

C. Throughout the exercise ask the following helping questions if needed.

Questions:
1. Which character is closest to you?
2. Which view of him do you see?
3. Where should the cutout be placed on the disc? (at bottom)
4. Which character is facing you?
5. Which character is farthest from you?
6. Where should the cutout be placed on the disc?
7. Which character is to your left?
8. Which view of him do you see?
9. Which character is to your right?
10. Which view of him do you see?
11. Which two characters are closest to you?
12. Which two characters are farthest away?
13. Where should the cutouts be placed?
14. Which are to your right? Which to your left?
15. Does Charlie Brown have his glove on his right or left hand?
16. Does Linus have his blanket on his right or left arm?

If necessary, use a signal dot to help child stay aware of the right and left sides of Lucy and Snoopy.

III. Characters and 3-way mirror
Position: In front of mirror

A. Put all four characters on rotating disc inside 3-way mirror. Put Snoopy behind mirror at 3 points. As disc is rotated to different views, ask what child sees and what Snoopy sees.

IV. Move characters back to original disc--place numbers outside of cutout circle.
Position: Place Lucy at No. 1 and child at the sighting line (No. 1)

A. Have the child use cutouts to represent views of Snoopy.

B. With Snoopy and camera behind Charlie Brown, place rod across to show Snoopy's sighting line. Also extend red yarn to show sighting line of camera. Have child reconstruct views as seen by Snoopy as he moves from number to number in this order
(5, 7, 3, 2, 4, 6, 8)

C. Throughout the exercise, ask the following helping questions.

Questions:
1. Which character is closest to Snoopy's camera?
2. Which view of him does Snoopy's camera see?
3. Where should the cutout be placed on the disc?
4. Which character is facing Snoopy's camera?
5. Which character is farthest from Snoopy's camera?
6. Where should the cutout be placed?
7. Which character is to Snoopy's right?
8. Which character is to Snoopy's left?
9. Which two characters are closest to Snoopy's camera?
10. Which are farthest away?
11. Which are to his left? To his right?
12. Where should the cutout be placed?

Use Charlie's glove and Linus' blanket as cues

V. Photos and Characters
Position: Child at sighting line

A. Have the child turn the crank to match the photos. Divide the photo in half. Call attention to character or characters closest to the bottom.

VI. Photos and Characters

A. Place child at No. 1 and have him identify 8 camera positions.

B. Place child at No. 3 and have him identify 8 camera positions.

C. Place child at No. 4 and have him identify 8 camera positions.

D. Put screen in front of the Peanuts characters so child cannot see characters. Move the circular disc until Snoopy's back is toward child and have the child identify the character to Snoopy's left and Snoopy's right.