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This report describes the planning, writing, evaluation, and results and recommendations of the Secondary School Mathematics Curriculum Improvement Study. This study was begun "to formulate and test a unified secondary school mathematics program (7-12) that will take capable students well into current collegiate mathematics" and to "determine the education required by teachers who will implement such a program." Also included in the report is a scheme for a taxonomy of objectives and course contents for each of the three courses developed (grades 7-9). Recommendations are that the program be reexamined for possible adoption as a curriculum for all junior high school students, that the preservice mathematics education of prospective junior high school teachers be reexamined, and that the unified approach developed for junior high school be extended throughout the senior high school. (FL)
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SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY

August, 1970

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Office of Education

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HEALTH, EDUCATION, AND WELFARE
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SUMMARY

The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) has two main objectives:

1) To formulate and test a unified secondary school mathematics program (7 - 12) that will take capable students well into current collegiate mathematics;

2) To determine the education required by teachers who will implement such a program.

To inaugurate the study, leading United States and European mathematicians and educators met in June 1966 to formulate a position paper stating the aims and procedures of the study, to construct a flow charted analysis of the proposed 7 - 12 mathematics courses, and to make detailed recommendations for the mathematical content of Course I. Using this detailed syllabus as a guide, a team of eight mathematics educators wrote a textbook for Course I during that summer. Each chapter was written by one writer, reviewed by the other writers and a consulting mathematician, and then revised for printing. Teachers' guides and solutions to exercises were written and distributed to the teachers.

In each subsequent year (1967, 1968) a two-week June working conference was held to review and revise the previous year's experimental text and to make specific recommendations for the content and teaching of a new course—Course II and Course III, respectively. As in the first year the writing team used the chapter guides developed by the June working conference to write the new texts for the following year.

Six junior high schools in the metropolitan New York area have participated in the experimental teaching of Courses I, II, and III from the initiation of the experiment. Each of these schools designated a team of two capable and interested teachers who taught all the pilot classes using the experimental textbooks. Each summer, while new materials were being written, six weeks of instruction was given to these teachers in preparation for teaching the new SSMCIS course. This instruction included 50 hours in the fundamental concepts underlying the unified mathematics program and 50 hours in contemporary methods of teaching those concepts.
The experimental teaching was evaluated in three ways. The director and project staff members made frequent visits to the classes for direct observation. The students were tested by examinations - prepared by the project staff - designed specifically to measure learning of important new concepts introduced in the courses. Teachers, staff, and consultants met at full day conferences to discuss progress and problems in the experimental teaching.

Results of the experimental teaching have shown that the new mathematics courses, based on fundamental concepts and structures, give promise of meeting the expectations of the proposed six year program.

Introduction

During the past decade the United States has been engaged in reforming the elementary and secondary school mathematics curriculum - primarily by updating the existing traditional curriculum. Modest recommendations of the Commission on Mathematics have been largely accepted by curriculum and syllabus bodies and by writers of commercially produced textbooks. Implementation of this program by the SMSG has led to wide acceptance and massive experimental use throughout the country.

Throughout all of our reform movements the traditional division of mathematics instruction into separate years of arithmetic, algebra, and geometry has been maintained. Beyond introduction of new concepts, little has been gained in bringing more advanced study into the high school through more efficient methods of organizing the subject matter. Holders and more radical recommendations for the improvement of secondary school education in mathematics have been made both in this country, notably by the UICSM, and in Europe, notably in Belgium, Switzerland, and Denmark.

What has been called for is reconstruction of the entire curriculum from a global point of view - one which eliminates the barriers separating the several branches of mathematics and unifies the subject through its general concepts (sets, operations, mappings, and relations) and builds the fundamental structures of the number systems, algebra, and geometry (groups, rings, fields, and vector spaces). The efficiency gained by such organization should permit introduction into the high school program of much that was previously considered undergraduate mathematics.
In September 1965, the Commissioner of Education, Department of Health, Education, and Welfare, Office of Education, approved for support for a period of 18 months the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), an experimental study whose objective would be the construction of a unified school mathematics curriculum for grades seven through twelve.

In June, 1967, continuation of this support was granted for an additional 36-month period ending June 30, 1970. This is a report of the activities and findings of the SSMCIS during this latter period, covering the writing and teaching of experimental Courses II and III, the revision and further, teaching of Courses I and II, and the final revision of Courses I and II.

Planning the 7 - 12 Program

In June 1966, a group of eighteen leading United States and European mathematicians and educators met for 20 days to outline the scope and sequence of a six year unified secondary school mathematics program. The first half of the conference was devoted to producing a complete flow charted analysis of the proposed course. Then topics planned for the seventh grade were expanded in working papers which outlined the mathematical content of each textbook chapter and made specific suggestions for writing and teaching these ideas.

Writing of Courses I, II, and III

During July and August 1966, a team of eight mathematical educators wrote the textbook for Course I, using the syllabus produced in June as a guide. Each textbook chapter was written by one writer, reproduced for review by the other writers and consulting mathematicians, and then rewritten, incorporating the reviewers' suggestions. Teachers' guides and solutions to exercises were written for each chapter. These notes, mimeographed and distributed to teachers of experimental classes, included discussions of fundamental mathematical ideas underlying each chapter, hints for possible class activity to accompany reading of the text, and suggested time allotment to the various topics. The Course I textbook was then published in three volumes.

To initiate the detailed planning for Course II and for the revisions of Course I, a pre-planning session was held on March 11-12, 1967. The recommendations of this pre-planning group were considered in detail at a working conference held in June, 1967, at which the full group of
writers and consultants was present. This group produced a
detailed set of writing guidelines, which were then used
by the summer writing teams to produce the text materials
for experimental Course II and to revise Course I.

Beginning with a pre-planning meeting in December, 1967,
and followed by a full working conference in June, 1968,
a similar procedure was used to plan and write experi-
mental Course III, to rewrite Course II, and to finalize
Course I. In addition, in order to make the fullest
possible use of the experience of two years of teaching
Course I, ten of the experimental teachers re-wrote and
expanded the original teachers' commentaries for Course I,
which had been originally written by the authors of the
various chapters. These chapter-by-chapter commentaries
were then bound into a single volume and made available
for public use along with the texts.

In 1969, the writing activities of the SSMCTS, supported
by the Office of Education, consisted of rewriting Course
III and making final revisions in Course II. Again, as
in the previous year, a group of teachers of experimental
classes worked during the summer, to rewrite the teachers'
commentaries for Course II, which were then printed in a
single volume for use in conjunction with the texts.

The texts and Teachers Commentaries for Courses I and II
(1 each in two volumes) are now available for widespread
classroom use, and are published and distributed by the
Teachers College Press. When revision of Course III, now
under way, is completed, the two volumes of the Course III
text and the accompanying Teachers Commentary will be
similarly available.

Education of Teachers

Each summer, beginning in 1966, the teachers of experimental
classes have participated in a six-week program of special
study at Teachers College, designed to prepare them to
teach an experimental course in the following school year.
This program of study was in two parts. The first was a
course in the mathematical subject areas underlying the
experimental course content, such as: abstract algebra,
linear algebra, transformation geometry and probability.
These subjects were taught from a modern and unified point
of view. The second part of this program was a course in
pedagogical methods of teaching secondary mathematics as a
unified branch of knowledge, with special emphasis on the
specific structures and principles to be covered in the
following year of experimental teaching.
The following is a list of the teachers and the schools in which they taught the experimental classes (Course III) during the 1968-69 school year:

Elmont, New York  
Schools: Alva T. Stanforth Junior High School  
       Sewanhaka High School  
Teachers: Samuel Backer  
          Alexander Imre  
          Edward Keenan  
          Mary Murray

Leonia, New Jersey  
School: Leonia High School  
Teachers: Christine McGoe  
          David Swaim

New York, New York  
School: Hunter College High School  
Teachers: Douglas Bumby  
          Ruth Cohen  
          Richard Klutch

Teaneck, New Jersey  
Schools: Benjamin Franklin Junior High School  
         Thomas Jefferson Junior High School  
         Teaneck High School  
Teachers: Franklin Armour  
          Annabelle Cohen  
          Otto Krupp  
          Mary Renda

Westport, Connecticut  
Schools: Bedford Junior High School  
       Coleytown Junior High School  
       Long Lots Junior High School  
       Staples High School  
Teachers: David Fays  
          Robert Keller  
          John Pepe  
          Daniel Sullivan

All teachers showed intense interest and cooperated splendidly in acquiring the spirit and content of the proposed new curriculum, and in teaching it. As a result of this training we now have a core of demonstration teachers and also a body of subject matter that must constitute teacher preparation in the future.
Teaching Courses I, II, and III

Six junior high schools in the New York Metropolitan area have taken experimental classes through Course I, II, and III. Five of these classes covered the text material in its first experimental revision. Another fifteen experimental classes have completed these three courses in the revised versions and another fifteen non-experimental classes in these schools have completed both Course I and II in their final revisions. Since the SSMCIS program is at present designed for those students in roughly the top 15% of their class with respect to mathematical ability, the original selection of students for the twenty experimental classes was made by the participating schools with prior mathematics achievement and scores on aptitude tests as main criteria.

Because the teachers of pilot classes were working as a team in the experimental class, they were often able to help each other with difficulties that arose in understanding or teaching the new material. Having had this year of team teaching experience, the teachers are now prepared to teach Courses I, II, and III on their own.

During the school year, the director and project staff members made frequent personal visits to observe the experimental teaching. Each class was observed at least four times. Visits to these schools included discussions with the teachers and administrators concerning progress and problems with the experimental course.

The teachers were further assisted by several full-day meetings at Teachers College where teaching problems were reviewed with selected consultants and the project director. At these meetings many teaching difficulties were resolved and valuable criticisms of the textbook were gathered.

Evaluation of Courses I, II, and III

The six year mathematics program introduces many new concepts into the secondary school mathematics curriculum and integrates both standard and new topics in a global organization not characteristic of existing programs. Student achievement in such a program cannot adequately be measured using conventional standardized tests. For this reason, student learning was tested by extramural examinations constructed by the project staff.

To guide construction of these and future measurement instruments, the textbooks were analyzed to produce a taxonomy of cognitive objectives. This taxonomy aided in
delineating goals of instruction in terms of subject matter and related behaviors. The categories of behavior appear in Table I.

Each year, two examinations were prepared; one for a mid-year evaluation, and one for an evaluation at the end of the year. These instruments were used as a measure of the teachability as well as of the learning of the prepared content. They also guided the revisions that were subsequently made.

Although achievement on standardized traditional mathematics tests was not accepted as a measure of the success of the experimental program, it was of interest to determine at the start whether or not study in the experimental Course I affected learning of traditional topics. To accomplish this objective all students were administered the Sequential Test of Educational Progress - Mathematics, Form 3A - in September 1966 and again in September 1967. The test results clearly show that students in the project classes suffered no decline in mathematical skills when compared with students studying more traditional programs. Moreover, the achievement of these students on the project tests shows that they were learning to work with many new and powerful mathematical tools not a part of the traditional mathematics fare of seventh graders.

Future Activity

The Secondary School Mathematics Curriculum Improvement Study received support from the National Science Foundation in June 1969 to continue designing and experimenting with a unified secondary school program for college capable students in the senior high school. This support has enabled the project to produce Course IV in the six year sequence and to begin planning for Course V and VI.

Course IV, which had been written in the summer of 1969, was pilot-tested in five experimental schools during the 1969-1970 school year. As a result of this experimentation, Course IV will be revised during the summer of 1970 and will be available by the early fall (1970). After further testing, the final version will be available by September 1971.

A tentative outline for Course V was developed during a two day conference of the advisory council in January 1970. This outline was expanded during a nine day conference of writers and consultants into detailed guides for the writers of the chapters in Course V. This course will be written during the summer of 1970 and experimented with in five schools during 1970.
The planning for and writing of Course VI will follow a similar procedure to that outlined for Course V with classroom testing during the 1971-72 school year.

By the end of the academic year 1973, the SSMCIS will have completed its task - to make a reconstruction of the secondary mathematics curriculum by presenting the subject as an integrated body of knowledge reflecting the spirit of contemporary mathematics.

Conclusions and Recommendations

After 5 years of classroom experimentations with students selected in the upper 15 to 20% of academic ability and taught by interested classroom teachers with special training in subject matter and pedagogy, a new curriculum in mathematics has been designed for the junior high school study, grades seven, eight, and nine. This curriculum breaks down the traditional barriers separating arithmetic, algebra, and geometry, and unifies the study through those fundamental concepts underlying all the branches, namely sets, relations, functions, and operations. The resulting curriculum is like a double helix in which the important structures - group, ring, field and vector space - form one strand, while the other strand consists of the important realizations: the number systems and the several geometries; synthetic, coordinate, vector, and transformation. Interwoven with both these strands are the activities and applications including the study of function, conditional sentences, statistics and probability.

Students who complete this three year program are advanced in knowledge more than one year beyond the present college preparatory program. This is accomplished by eliminating a great deal of traditional content that today is of little or no value in further study or application of mathematics. The increase in learning is also brought about by the unification of all the study under the more general concepts and structures of contemporary mathematics.

The teachability has been tested and verified, each course undergoing three years of thorough examinations and revision into its present form. Concomitantly, five doctoral studies researching the learning and teaching aspects of the program were completed. These studies were carried out by the research assistants associated with the project. They were:

A study in one school has shown that the same material can be learned by students of average ability, if pursued at a slower rate over a longer period of time.

It is strongly recommended that the program developed by SSMCIS be reexamined for adaptation as a curriculum for all students in the junior high school. It is also recommended that the SSMCIS program serve as a basis for re-examining the pre-service mathematics education of prospective junior high school teachers of mathematics. For these teachers, far more attention must be given to abstract and linear algebra, and to geometry of a modern variety, with less stress on advanced analysis. Lastly it is recommended that the unified approach be extended throughout the senior high school study to give college preparatory students an advanced knowledge of all aspects of mathematics on entering college and not only that of the calculus.
### TABLE I
**TAXONOMY OF OBJECTIVES**

**Mathematical Objectives**

- **Structures:** Arithmetic, Geometry, Probability, Analysis, and Algebra
- **Fundamental Concepts:** Sets, Operations, Relations, Mappings, Logic

**Behavioral Objectives**

1. Ability to recall definitions, notations, operations, concepts.
2. Ability to manipulate and calculate efficiently.
3. Ability to interpret symbolic data or processes.
4. Ability to communicate mathematical ideas.
5. Ability to apply concept to a purely mathematical situation—solve problems.
6. Ability to apply concept to problems in other situations—solve word problems.
7. Ability to transfer learning to a new situation in mathematics.
8. Ability to construct or follow a mathematical argument.

Of course not all these categories apply to each subject matter topic, but the goals were checked against subject matter.
APPENDIX A

COURSE I CONTENT

Chapter

1  FINITE NUMBER SYSTEMS
   Jane Anderson's Arithmetic
   Clock Arithmetic
   \((Z_{12},+)\) and \((W,+}\)
   Calendar Arithmetic
   Open Sentences
   New Clocks
   Rotations
   Subtraction in Clock Arithmetic
   Multiplication in Clock Arithmetic
   Comparison of \((W,\cdot)\) and Clock Multiplication
   Division in Clock Arithmetic
   Inverses in Clock Arithmetic
   The Associative and Distributive Properties
   Summary

2  SETS AND OPERATIONS
   Ordered Pairs of Numbers and Assignments
   What is an Operation?
   Computations with Operations
   Open Sentences
   Properties of Operations
   Cancellation Laws
   Two Operational Systems
   What is a Group?
   Summary

3  MATHEMATICAL MAPPINGS
   Assignments and Mappings
   Mappings of Sets of Whole Numbers
   Mappings of Clock Numbers
   Sequences
   Composition of Mappings
   Inverse and Identity Mapping
   Special Mappings of \(W\) to \(W\)
   Summary

4  THE INTEGERS AND ADDITION
   Introduction
   Some New Numbers
   The Integers and Opposites
   Properties of \((Z,+}\)
   The Integers and Translations on a Line
   Subtraction in \((Z,+}\)
   Subtraction as Addition of Opposites
   Equations in \((Z,+}\)
   Cancellation Law
Chapter

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Absolute Value
Summary

5
PROBABILITY AND STATISTICS
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Discussion of an Experiment
Experiments to be Performed by Students
The Probability of an Event
A Game of Chance
Equally Probable Outcomes
Another Kind of Mapping
Counting with Trees
Preview
Research Problems
Statistical Data
Presenting Data in Tables
The Frequency Histogram and the Cumulative Frequency Histogram
Summary

6
MULTIPLICATION OF INTEGERS
Operational Systems (\(W, \cdot\)) and (\(Z, \cdot\))
Multiplication for \(Z\)
Multiplication of a Positive Integer and a Negative Integer
The Product of Two Negative Integers
Multiplication of Integers through Distributivity
Dilations and Multiplication of Integers
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7
LATTICE POINTS IN THE PLANE
Lattice Points and Ordered Pairs
Conditions on \(Z \times Z\) and their Graphs
Intersection and Unions of Solution Sets
Absolute Value Conditions
Lattice Point Games
Sets of Lattice Points and Mappings of \(Z\) into \(Z\)
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Summary

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COURSE II CONTENT

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   Mathematical Statements
   Connectives: And, Or
   Conditional and Bi-conditional Statements
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   Substitution Principle for Equality (SPE)
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   Indirect Mathematical Proof
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   A Non-Geometric Model of the Axioms
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   Getting Some Field Theorems Painlessly
   Trouble with 0
   Subtraction and Division in Fields
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Axiom 5, Relating Two Coordinate Systems on a Line
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   Transition Matrices
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   Systems of Linear Equations and Matrices
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