This booklet is a sample activity from an individualized instruction unit in mathematics. Agreement between the performance specified in the units' objectives, the performance taught in the instruction activity, and performance required on the posttest was a key criterion during the development of this material. The student is told what he is expected to be able to do at the end of the activity, and how the particular activity relates to the entire instructional unit. The material presented deals with the solution of linear equations and the graphing of linear inequalities. (Author/RS)
AN INDIVIDUALIZED INSTRUCTION MODULE

for

Specific

Performance

Objectives

in

SETS, NON-METRIC GEOMETRY and RELATIONS

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Introduction

This booklet contains a sample activity of an INDIVIDUALIZED INSTRUCTION MODULE based upon a specific performance objective in mathematics. The student is told at the beginning of the activity precisely what he is expected to be able to do at the end of the activity. He is also told at the beginning of the activity where the activity fits into the instruction module's learning hierarchy.

Agreement between the performance specified in the objective, the performance taught in the instruction activity, and performance required on the posttest was a key criteria during the development of this material. This booklet is a "first-generation" edition of the material.

Information concerning the complete INDIVIDUALIZED INSTRUCTION MODULE may be obtained by writing:

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5401 Wilkens Avenue
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The Objectives of Lesson VII Continued

When you have completed Lesson VII-b, you should be able to perform the following three tasks:

**Task 1:** GIVEN an equation from Set B and a value of x, COMPUTE the corresponding value of y.

The Set B referred to in the above statement of Task 1 is:

\[ y = |x + a| \]
\[ y = x + b \]
\[ y = c \cdot x \]

Note: a, b, and c are integers

**Example of Task 1:**

GIVEN the equation
\[ y = |x - 5| \]
with \( y = -4 \)

COMPUTE the corresponding value of y.

**Solution:** \( y = |-4 - 5| = |-9| = 9 \)
Task 2: GIVEN a relation from the set of relations shown below, CONSTRUCT a table of ordered pairs from which the graph of the relation could be constructed.

The set of relations referred to in the above statement of Task 2 is:

\[
\begin{align*}
\{(x, y) \mid y \geq x + a, x \text{ real}\} \\
\{(x, y) \mid y \geq x + b, x \text{ real}\} \\
\{(x, y) \mid y \geq c - x, x \text{ real}\}
\end{align*}
\]

Note: \(\geq\) may be replaced by \(=\), \(>\), \(<\), or \(\leq\); \(a, b, c, d,\) and \(e\) are integers.

Example of Task 2:

GIVEN the relation

\[
\{(x, y) \mid y \leq 3 - x, x \text{ real}\}
\]

CONSTRUCT a table of ordered pairs from which the graph of the relation could be constructed.

Solution:

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
Task 3: GIVEN a relation from Set A, CONSTRUCT a graph of its solution set.

The Set A referred to in the above statement of Task 3 is:

\[
\begin{align*}
\{ (x, y) \mid y &\geq |x + a|, x \text{ real} \} \\
\{ (x, y) \mid y &\geq x + b, x \text{ real} \} \\
\{ (x, y) \mid y &\geq c - x, x \text{ real} \} \\
\{ (x, y) \mid y &\geq d, x \text{ real} \} \\
\{ (x, y) \mid x &\geq e, x \text{ real} \}
\end{align*}
\]

Note: \( \geq \) may be replaced by \( = \), \( > \), \( < \), or \( \leq \);
a, b, c, d, and e are integers.

Example of Task 3:

GIVEN the relation

\[
\{ (x, y) \mid y \geq |x + 2|, x \text{ real} \}
\]

CONSTRUCT a graph of its solution set.

Solution:

\[
\begin{array}{c|c|c|c|c|c}
 x & -6 & -4 & -2 & 0 & 2 \\
 y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

\[
\begin{array}{c}
\end{array}
\]
LEARNING SEQUENCE

A
GIVEN a system of two relations of the type in Set A; CONSTRUCT a graph of its solution set and NAME the solution set in terms of the U or N of points, line segments, rays, half-lines, angles, or triangles.

4
GIVEN the graph of a system of two relations of the type in Set APCM4STRUCT; NAME the solution set in terms of the U or N of points, line segments, rays, half-lines, angles, or triangles.

3
GIVEN the graphs of two relations of the type in Set A; IDENTIFY by shading the N of the two graphs.

2
GIVEN the graphs of combinations of two of the following sets of points on a line: points, lines, line segments, half-lines, or rays; IDENTIFY their N by shading on the line.

1
GIVEN the graphs of combinations of two of the following sets of points: points, lines, line segments, half-lines, or rays; IDENTIFY their N by shading on the line.

B
GIVEN two relations of the type in Set A; CONSTRUCT a graph of the solution set of each relation.

C
GIVEN a relation from Set A; CONSTRUCT a graph of its solution set.

D
GIVEN a relation from Part 1 of Set A, CONSTRUCT a table of ordered pairs from which the graph of the relation could be constructed.

C
GIVEN a set of at least four ordered pairs, IDENTIFY the points on a Cartesian plane which correspond to the ordered pairs by plotting the points and naming them by the corresponding ordered pairs.

B
GIVEN an equation from Set B and a value of x, COMPUTE the corresponding value of y.

<table>
<thead>
<tr>
<th>Set A:</th>
<th>Set B:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1: {(x, y)</td>
<td>y \leq x + 4, x \text{ real}} {x, y)</td>
</tr>
<tr>
<td>Part 2: {(x, y)</td>
<td>y \geq 4, x \text{ real}} {(x, y)</td>
</tr>
</tbody>
</table>
The place of Lesson VII-b is shown on the OPPOSITE PAGE by the three red blocks. The lesson begins with the task of step 1-D and progresses to step 2-B and then to step 3-C. Having already acquired the skill to perform the task of step 2-A (shown in blue), acquiring the skill to perform the task of step 2-B will equip you to learn to perform the task of step 3-C.

Why does the author want you to acquire the skill for step 3-C? He considers the skill of step 3-C will enable you to learn to perform the task of step 4-B. Lesson VII-a will be concerned with step 4-B. Refer to the example of step 4-B in the green folder to obtain an idea of what type of task Lesson VII-b leads into.

Turn the page.
VII-b. CONSTRUCTING GRAPHS

We have discussed ordered pairs \((x, y)\) in previous lessons. For each value of \(x\) in an ordered pair, there is a corresponding value of \(y\). In the ordered pair \((-1, 4)\) the value of \(x\) is -1 and the corresponding value of \(y\) is 4.

Ordered pairs are the coordinates of points on a Cartesian plane:

\((-1, 4)\)

Equations like \(y = x + 5\) have sets of ordered pairs which satisfy the conditions of the equation. For the equation \(y = x + 5\), there is a specific set of ordered pairs which satisfy the equation. That is, there is a solution set of ordered pairs for \(y = x + 5\). The relation \(\{(x, y) \mid y = x + 2, \ x \text{ real}\}\) describes the solution set of \(y = x + 2\). The solution set is determined by computing corresponding values of \(y\) for different selected values of \(x\).

For the equation \(y = x + 5\), let \(x = 0\). The corresponding value of \(y\) can be computed by substituting 0 for \(x\) in the equation

\[
\begin{align*}
y &= x + 5 \\
y &= 0 + 5 \\
y &= 5
\end{align*}
\]
Thus, when \( x = 0 \), \( y = 5 \). That is, \((0, 5)\) is a member of the solution set described by the relation
\[
\{(x, y) \mid y = x + 5, x \text{ real}\}.
\]

If \( x = -2 \), let's compute the corresponding value of \( y \) for \( y = x + 5 \):

\[
\begin{align*}
y &= x + 5 \\
y &= -2 + 5 \\
y &= 3
\end{align*}
\]

Thus, \((-2, 3)\) is also a member of the solution set of the relation \( \{(x, y) \mid y = x + 5, x \text{ real}\} \).

Consider the equation \( y = -3 - x \). Let \( x = 0 \):

\[
\begin{align*}
y &= -3 - x \\
y &= -3 - 0 \\
y &= -3
\end{align*}
\]

\((0, -3)\) is therefore a member of the solution set of the relation \( \{(x, y) \mid y = -3 - x, x \text{ real}\} \). For \( x = -4 \):

\[
\begin{align*}
y &= -3 - x \\
y &= -3 - (-4) \\
y &= -3 + 4 \\
y &= 1
\end{align*}
\]

\((-4, 1)\) is a member of the solution set of
\[
\{(x, y) \mid y = -3 - x, x \text{ real}\}, \text{ also.}
\]

Some equations may involve absolute values; for example, \( y = |x + 6| \). We mentioned in Lesson VII-a that the absolute value of a number like \(-5\) was \(+5\). That is, \(|-5| = 5\). Also, \( |+7| = +7\). The absolute value of a number refers to the distance of that number from 0.
Consider the location of the point representing -5 on the number line:

```
  -5     0
```

How far is -5 from 0? Did you write "5" or "5 units"? You are correct. We certainly don't say, "Joe is standing -5 feet to the left of the table and Jack is standing 7 feet to the right of the table." Distance is always given in terms of a positive number. When we are working with distance we are interested in the absolute number of units from one point to another point. Direction is of no consequence. The distance from -5 to 0 is the same as the distance from 0 to -5. The absolute value symbol is used to denote distance. Thus, |−5| = 5 and |5| = 5.

Consider the equation \( y = |x - 4| \) and let \( x = -3 \). What is the corresponding value of \( y \)? Computing for \( y \):

\[
y = |−3 − 4| = |−7| = 7
\]

\((-3, 7)\) is a member of the set of ordered pairs which satisfy the relation \( \{(x, y) | y = |x - 4|, x \text{ real}\} \).
\(- 4 -\)

Let \(x = 6\) for \(y = |x - 4|\):

\[
\begin{align*}
y &= |x - 4| \\
y &= |6 - 4| \\
y &= |2| \\
y &= 2
\end{align*}
\]

For each of the following equations **compute** the corresponding values of \(y\) for the values of \(x\) given (show your calculations):

1. **Compute \(y\) from \(y = x - 7\),**
   a. when \(x = -4\)
   b. when \(x = 0\)
   c. when \(x = 3\)
2. Compute $y$ from $y = 5 - x$,
   
   a. when $x = -3$

   b. when $x = 0$

   c. when $x = 3$

3. Compute $y$ from $y = |x - 2|$, 
   
   a. when $x = -2$

   b. when $x = 0$
Solutions to these problems are found on page 29.

In Lesson II in our discussion on the line, we noted that any two points define the location of a line. Suppose we denote two points on a Cartesian plane by their ordered pairs:

\[ (-2, 4) \quad \text{and} \quad (3, -1) \]
Next, let's draw a line through the two points:

Hence, knowing two ordered pairs we have been able to draw a straight line on a Cartesian plane. Of course, there are also other points on the line. Notice that the point (0, 2) is on the line:

The point \((a, b)\) is noted here to illustrate that there can be points on a line in a plane whose coordinates are real numbers but not integers. A line on a plane includes all real values of \(x\). However, we have noted that only two points are needed to establish the location of a line.

In Lesson VII-a, we stated that a relation or its graph describes the solution set of an equation or an inequality. Let's consider the solution set of \(y = 2 - x\).
with the domain of \( x \) being the real numbers. The solution set is described by the relation \( \{ (x, y) \mid y = 2 - x, \ x \text{ real} \} \).

In order to describe the solution set of \( y = 2 - x \) with a graph, we must plot the points of ordered pairs which satisfy the relation \( \{ (x, y) \mid y = 2 - x, \ x \text{ real} \} \).

Since the domain of \( x \) is the set of real numbers, we can choose \( x \) to be \(-3\). What is the corresponding value of \( y \)? Substituting \(-3\) for \( x \), we have:

\[
\begin{align*}
y &= 2 - x \\
y &= 2 - (-3) \\
y &= 2 + 3 \\
y &= 5
\end{align*}
\]

That is, \((-3, 5)\) is a member of the solution set.

Letting \( x = -2 \), we have:

\[
\begin{align*}
y &= 2 - x \\
y &= 2 - (-2) \\
y &= 2 + 2 \\
y &= 4
\end{align*}
\]

\((-2, 4)\) is also a member of the solution set.

Perhaps a chart or table might help us to organize our findings:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

The table of ordered pairs shows that when \( x = -3 \), \( y = 5 \) and when \( x = -2 \), \( y = 4 \).
Let's determine more ordered pairs in the solution set and place them in the table of ordered pairs. For

\[ x = -1 : \]

\[
\begin{align*}
y &= 2 - x \\
y &= 2 - (-1) \\
y &= 2 + 1 \\
y &= 3
\end{align*}
\]

Hence,

\[
\begin{array}{c|ccc}
x & -3 & -2 & -1 \\
\hline
y & 5 & 4 & 3 \\
\end{array}
\]

For \( x = 0 \):

\[
\begin{align*}
y &= 2 - x \\
y &= 2 - 0 \\
y &= 2
\end{align*}
\]

Hence,

\[
\begin{array}{c|cccc}
x & -3 & -2 & -1 & 0 \\
\hline
y & 5 & 4 & 3 & 2 \\
\end{array}
\]

For \( x = 1 \):

\[
\begin{align*}
y &= 2 - x \\
y &= 2 - 1 \\
y &= 1
\end{align*}
\]

Hence,

\[
\begin{array}{c|cccc}
x & -3 & -2 & -1 & 0 & 1 \\
\hline
y & 5 & 4 & 3 & 2 & 1 \\
\end{array}
\]
For \( x = 2 \):

\[
\begin{align*}
  y &= 2 - x \\
  y &= 2 - 2 \\
  y &= 0
\end{align*}
\]

Hence,

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

For \( x = 3 \):

\[
\begin{align*}
  y &= 2 - x \\
  y &= 2 - 3 \\
  y &= 1
\end{align*}
\]

Hence,

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Plotting the ordered pairs in the table, we have:

![Graph](image)

The line is drawn solid because the domain of \( x \) is the set of real numbers. Since the graph of the relation

\[
\{(x, y) \mid y = 2 - x, \ x \text{ real}\}
\]

is a line, only two ordered pairs would have been required. Perhaps a third ordered pair would have been helpful to check the accuracy of the
other two ordered pairs. That is, if all three points computed do not lie on a straight line, we know that at least one of the ordered pairs was incorrectly computed.

Selecting any three values of \( x \) we could have computed the corresponding values of \( y \) and obtained a table of ordered pairs. If we had selected \( x = -2 \), \( x = 1 \), and \( x = 3 \), we would have had the table:

\[
\begin{array}{c|ccc}
  x & -2 & 1 & 3 \\
  y & 4 & 1 & -1 \\
\end{array}
\]

and the graph:

Since the graph of equations of these two types

\[
y = x + b \\
\text{and } y = c - x
\]

(Note: \( b \) and \( c \) are integers)

are always straight lines, we can plot their graphs by computing the ordered pairs for only three points for each equation.
Let's construct the table of ordered pairs from which the graph of the relation \( \{(x, y) \mid y = x + 3, \ x \ \text{real}\} \) could be constructed. Since any three points are sufficient to plot the straight line, we can select any three values of \( x \). Let's choose \( x = -4 \), \( x = -2 \), \( x = 0 \) and compute the corresponding \( y \)'s to complete the table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( y = x + 3 )</td>
<td>( y = x + 3 )</td>
<td>( y = x + 3 )</td>
</tr>
<tr>
<td>( y = -4 + 3 )</td>
<td>( y = -2 + 3 )</td>
<td>( y = 0 + 3 )</td>
<td></td>
</tr>
<tr>
<td>( y = -1 )</td>
<td>( y = 1 )</td>
<td>( y = 3 )</td>
<td></td>
</tr>
</tbody>
</table>

Hence, we have the table of ordered pairs:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( 3 )</td>
</tr>
</tbody>
</table>

Notice, that for convenience and ease in plotting the values of \( x \) start from some low value and increase in value by the same amount; i.e., \(-4\) to \(-2\) increase of 2 units
\(-2\) to \(0\) increase of 2 units

Here are several relations for you to practice constructing tables of ordered pairs from which the graph of each relation could be constructed:

4. \( \{(x, y) \mid y = x - 3, \ x \ \text{real}\} \)
5. \( \{(x, y) \mid y = x - 5, \ x \text{ real} \} \)

6. \( \{(x, y) \mid y = 3 - x, \ x \text{ real} \} \)

7. \( \{(x, y) \mid y = -2 - x, \ x \text{ real} \} \)

Solutions for these problems are found on page 29.

You recall in Lesson VII-a that we worked with graphs which had the shape of an angle:

The graph of an equation involving an absolute value always has this angle shape. Notice that there are no values of \( y \) which are negative.
Let's check that statement by seeing what the graph of the relation \( \{ (x, y) \mid y = \vert x - 2 \vert , \text{ x real} \} \) looks like. Selecting values for \( x \) we have the incomplete table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Computing the corresponding values of each \( y \) like you did in your practice work on page 5, we have

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Plotting the points from the table, we have the graph:

Notice that in the above graph the three points on each ray, including a common point \((2, 0)\) at the vertex of the angle, were a sufficient number of points (total of five) to draw the graph. The graph of the equation \( y = \vert x - 2 \vert \) is solid rather than just the points corresponding to the table of ordered pairs because the domain of \( x \) is the set of real numbers. The value of \( y \) at the vertex of the angle will always be \( 0 \) for such an equation as \( y = \vert x - 2 \vert \).
Let's construct the table for the relation 
\[ \{(x, y) \mid y = |x + 3|, \ x \text{ real}\} \]. Only five points are necessary, so we will select only five values of \(x\). However, since we've stated that the value of \(y\) at the vertex of the graph will always be 0, then let's first choose a value of \(x\) whose corresponding value of \(y\) is 0. Letting \(x = -3\) will assure that \(y = 0\):

\[
\begin{align*}
y &= |x + 3| \\
y &= |-3 + 3| \\
y &= |0| \\
y &= 0
\end{align*}
\]

Therefore, we can begin the construction of the table of ordered pairs at the point \((-3, 0)\):

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-3)</th>
<th>(y)</th>
<th>(0)</th>
</tr>
</thead>
</table>

Notice that since the vertex of the angle is the center point of the graph, we place the ordered pair \((-3, 0)\) in the center position in the table. Moving in equal distance steps to the left and to the right of \(x = -3\), we have

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-7)</th>
<th>(-5)</th>
<th>(-3)</th>
<th>(-1)</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>(0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Computing the corresponding values of $y$:

$y = |x + 3|$
$y = |-7 + 3|$
$y = |-4|$
$y = 4$

$y = |x + 3|$
$y = |-5 + 3|$
$y = |-2|$
$y = 2$

$y = |x + 3|$
$y = |-1 + 3|$
$y = |2|$
$y = 2$

The completed table is:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-7</th>
<th>-5</th>
<th>-3</th>
<th>-1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

and the graph looks like

![Graph](image)

Construct the table of values for the relation

$\{(x, y) \mid y = |x - 4|, \, x \text{ real} \}$:
Remember that ALL VALUES OF y WILL BE EITHER 0 OR A POSITIVE NUMBER. If your computations for y yield a negative value for y, you have made an error. Except for the center point, you may use any value of x in the table. One correct table using equal steps of two from the value of x at the vertex is

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Here are several other relations for your practice:

8. Construct the table of ordered pairs for the relation \( \{(x, y) \mid y = |x - 1|, x \text{ real}\} \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
</tr>
</tbody>
</table>

9. Construct the table of ordered pairs for \( \{(x, y) \mid y = |x + 2|, x \text{ real}\} \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
</tr>
</tbody>
</table>

10. Construct the table of ordered pairs for \( \{(x, y) \mid y = |x - 3|, x \text{ real}\} \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
</tr>
</tbody>
</table>

Solutions to these problems are shown on page 30.
When a relation involves an inequality, we use the table of ordered pairs to graph the edge of the region which is to be shaded. Consider the relation \( \{(x, y) \mid y \geq |x + 3|, \ x \text{ real}\} \). In order to construct the graph of this relation we perform the following three steps:

i. Construct the table of ordered pairs of the equation \( y = |x + 3| \).

ii. Plot the ordered pairs from the table to locate the edge of the shaded region of the graph of the inequality.

iii. Determine as we did in Lesson VII-a whether or not the origin is in the shaded region; shade the appropriate region.

Let's go through each of these steps for the relation \( \{(x, y) \mid y \geq |x + 3|, \ x \text{ real}\} \):

i.  

<table>
<thead>
<tr>
<th>x</th>
<th>-7</th>
<th>-5</th>
<th>-3</th>
<th>-1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

ii. 

[Diagram of the graph of the inequality]
iii. 

\[ y \geq |x + 3| \]

substituting \((0, 0)\), 

\[ 0 \geq |0 + 3| \]

\[ 0 \geq |+3| \]

\[ 0 \geq 3 \] is not true.

Therefore, we have

If you were asked to construct the table of ordered pairs from which the graph of the relation 

\[ \{(x, y) \mid y < |x - 1|, \ x \text{ real}\} \] could be constructed, you would perform step (i.) as follows:

Set: \( y = |x - 1| \)

Choose the values of \(x\): 

\[
\begin{array}{c|ccccc}
 x & -3 & -1 & 1 & 3 & 5 \\
\hline
 y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

Complete the construction of the table:

Notice step (i.) is the same whether the inequality is of the form \(\leq, \geq, <, \text{ or } >\).
Later in this lesson you will be asked to construct a graph for a relation similar to \( \{(x, y) \mid y < \left| x - 1 \right| \} \). Steps (i.), (ii.), and (iii.) should be followed:

i. We have already constructed the table.

\[
\begin{array}{ccccc}
 x & -3 & -1 & 1 & 3 & 5 \\
 y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

ii. Since we have <, the edge of the shaded region will be dashed:

iii. Checking the origin (0, 0)

\[
y < \left| x - 1 \right|
\]

Substituting (0, 0),

\[
0 < \left| 0 - 1 \right| \quad \text{is true}
\]

Hence, we have
Construct the table of ordered pairs from which the graph of each of the two following relations could be constructed:

11. \[\{ (x, y) \mid y \geq |x + 2|, x \text{ real} \}\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
</table>

12. \[\{ (x, y) \mid y > |x - 4|, x \text{ real} \}\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
</table>

Solutions for these problems are shown on page 30.

Now, let's try bring what we have learned about constructing graphs of relations into a clear focus. We know that:

i. If we are working with an inequality with \(\leq\) or \(\geq\), the edge of the shaded region on the plane will be solid;

ii. If we are working with an inequality with \(<\) or \(>\), the edge of the shaded region on the plane will be dashed;

iii. If we are working with a relation involving an inequality, we construct the table of ordered pairs using an equation;

iv. If we are working with a relation involving an inequality, the graph will have a shaded region, while if the relation involves only an equation, the graph will not have a shaded region;
v. If we are working with a relation involving an inequality, the location of the shaded region is determined by whether or not the origin \((0, 0)\) satisfies the conditions of the inequality.

There are some relations for which it is not necessary to construct tables of ordered pairs in order to construct their graphs. Consider the line shown on the Cartesian plane shown below:

Now, let's notice the coordinates of some points on the line.

![Graph](image)

The value of \(x\) for every point is \(x = 2\). It doesn't seem to matter what the value of \(y\) is for points on the line, the \(x = 2\). Hence, the line is the graph of the relation

\[ \{(x, y) \mid x = 2, x \text{ real}\} \]
Given the relation \( \{(x, y) \mid x = 2, x \text{ real} \} \),
the graph is

The graphs of the following two relations are constructed
in the same manner:

\[ \{(x, y) \mid x = -5, x \text{ real}\} \]

\[ \{(x, y) \mid x = 3, x \text{ real}\} \]
Consider the graph of the line shown below and notice the coordinate points on the line:

\((-4,4)\)
\((-2,4)\)
\((0,4)\)
\((3,4)\)

For every point on the line, \(y = 4\). Hence, the line is the graph of the relation \(\{(x, y) \mid y = 4, x \text{ real}\}\).

The graphs of the following two relations are constructed in the same way:

\(\{(x, y) \mid y = -1, x \text{ real}\}\)

\(\{(x, y) \mid y = 3, x \text{ real}\}\)
Construct the graph for the relation
\[ \{(x, y) \mid x = -1, \ x \text{ real}\} : \]

The solution is shown on page 30.

If the relation involves an inequality then the graph includes a shaded region. Several relations and their respective graphs are shown below:

i. \[ \{(x, y) \mid y \geq 2, \ x \text{ real}\} \]

In (i.) notice that the ordered pairs in the solution set consist of all ordered pairs for which the y coordinate is \( \geq 2 \). Select any point in the shaded region or on the line to convince yourself that its y coordinate is \( \geq 2 \).
ii. \[ \{ (x, y) \mid y > 3, x \text{ real} \} \]

In (ii.), notice that \( > \) and the dashed line "go together."

iii. \[ \{ (x, y) \mid x \leq -2, x \text{ real} \} \]

iv. \[ \{ (x, y) \mid x < -2, x \text{ real} \} \]

To assure yourself that you can construct graphs when given relations, practice on these problems:
14. Given the relation
\[ \{ (x, y) \mid y < -2, \ x \text{ real} \} \]
Construct a graph of its solution set.

15. Given the relation
\[ \{ (x, y) \mid y = |x - 3|, \ x \text{ real} \} \]
Construct the graph of its solution set.

16. Given the relation
\[ \{ (x, y) \mid y > |x - 2|, \ x \text{ real} \} \]
Construct the graph of its solution set.

Note: Remember that \( > \) and a dashed edge "go together."
17. Given the relation
\[ \{(x, y) \mid y \leq 1 - x, \ x \text{ real}\} \]
Construct the graph of its solution set.

Note: Remember that \( \leq \) and a solid edge "go together."

Solutions of these problems are shown on page 31.
Answers to Performance Tasks

Pages 4, 5, and 6.

1a. \( y = -11 \)
1b. \( y = -7 \)
1c. \( y = -4 \)
2a. \( y = 8 \)
2b. \( y = 5 \)
2c. \( y = 2 \)
3a. \( y = 4 \)
3b. \( y = 2 \)
3c. \( y = 0 \)
3d. \( y = 2 \)
3e. \( y = 4 \)

Pages 12 and 13.

(The tables are correct solutions. However, your table may be different if you chose different values of \( x \).)

4. \[
\begin{array}{c|ccc}
 x & -3 & -1 & 1 \\
--- & --- & --- & --- \\
y & -6 & -4 & -2 \\
\end{array}
\]

5. \[
\begin{array}{c|ccc}
 x & -2 & 0 & 2 \\
--- & --- & --- & --- \\
y & 3 & 5 & 7 \\
\end{array}
\]

6. \[
\begin{array}{c|ccc}
 x & -1 & 1 & 3 \\
--- & --- & --- & --- \\
y & 4 & 2 & 0 \\
\end{array}
\]

7. \[
\begin{array}{c|ccc}
 x & -3 & -1 & 1 \\
--- & --- & --- & --- \\
y & 1 & -1 & -3 \\
\end{array}
\]


(These tables are correct solutions. However, your table may be different except the center position depending upon what values of x you selected. There must be five ordered pairs.)

8. \[
\begin{array}{c|ccccc}
  x & -3 & -1 & 1 & 3 & 5 \\
  \hline
  y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

9. \[
\begin{array}{c|ccccc}
  x & -6 & -4 & -2 & 0 & 2 \\
  \hline
  y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

10. \[
\begin{array}{c|ccccc}
  x & -1 & 1 & 3 & 5 & 7 \\
  \hline
  y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

(If you chose different values of x, your table may be different, except for the center position.)

11. \[
\begin{array}{c|ccccc}
  x & -6 & -4 & -2 & 0 & 2 \\
  \hline
  y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

12. \[
\begin{array}{c|ccccc}
  x & 0 & 2 & 4 & 6 & 8 \\
  \hline
  y & 4 & 2 & 0 & 2 & 4 \\
\end{array}
\]

13. 

[Diagram of a coordinate plane with arrows indicating positive x and y directions]
Page 27.

14.

15. \[ \begin{array}{cccccc}
    x & -1 & 1 & 3 & 5 & 7 \\
    y & 4 & 2 & 0 & 2 & 4 
\end{array} \]

16. \[ \begin{array}{cccccc}
    x & -2 & 0 & 2 & 4 & 6 \\
    y & 4 & 2 & 0 & 2 & 4 
\end{array} \]

Page 28.

17. \[ \begin{array}{cccc}
    x & -\infty & 0 & 2 \\
    y & 3 & 1 & -1 
\end{array} \]