Twenty-seven kindergarten children were trained on two different double classification matrix tasks in an attempt to determine whether the tasks were hierarchically related. Prior behavior analyses of the tasks suggested that the two tasks shared many components, but that the more complex task had in addition components not included in the simpler task. For this reason it was predicted that learning the simpler task first, then the complex, was the "optimal," learning sequence. As predicted, children who learned the tasks in the optimal order learned the more complex task in fewer trials than those who learned the tasks in the reverse order. In addition, the reverse order group showed evidence of having acquired the simpler task in the process. Both of these findings are in accord with the hypothesis that the two tasks are hierarchically related. It is suggested that acquisition of complex cognitive skills may be a matter of learning specific relevant prerequisites. (Author)
TRANSFER AND SEQUENCE IN LEARNING DOUBLE CLASSIFICATION SKILLS

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DOUBLE CLASSIFICATION SKILLS

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The presence of multiplicative classification skills has generally been considered one index of the child's having reached the stage of concrete logical operations (Flavell, 1963). One of the most interesting manifestations of multiplicative classification skill is the child's ability to deal with two aspects of a situation at a time. A reasonable approach to studying this ability is to examine it in the context of a logically complex classification task, the matrix, which involves the simultaneous ordering of two dimensions. A child who completes or who can construct a double classification matrix is showing some evidence of multiplicative classification ability (Inhelder & Piaget, 1964).

Most studies of matrix behavior have been developmental in nature, seeking to investigate the relationship between the ability to perform various kinds of matrix tasks and age (e.g., Bruner & Kenney, 1966; Lovell, Mitchell, & Everett, 1962; Overton & Brodzinsky, 1969; Shantz, 1967; Siegel & Kresh, 1970; Smedslund, 1964). As a group, these studies demonstrate an increase in matrix classification skills up to about eight years of age, at which point an asymptotic level of performance is reached. Several of these studies (Overton & Brodzinsky, 1969; Siegel & Kresh, 1970) have found relatively similar levels of performance between ages four and seven, with a relatively sharp increase between ages seven and eight. Other studies (Smedslund, 1964) have found a more linear increase in performance between these same ages.
Several different types of matrix tasks have been used in these studies. Lovell, Mitchell & Everett (1962) had children sort sets of pictures that were susceptible of arrangement in a matrix format. Smedslund (1964) and Shantz (1967) used an incomplete matrix task in which the child had to select the object that belonged in the empty cell. Inhelder and Piaget (1964) used both of these kinds of tasks in their studies of multiplicative classification. Bruner and Kenney (1966) had children reconstruct and transpose a matrix after observing it in completed form. Siegel and Kresh (1970) used two tasks in which the child was presented an empty matrix, but the relevant attribute was displayed in a special cell at the head of each row and column. The child described the object belonging in each cell, or placed objects in the appropriate cells (as defined by the attribute cells).

Although variations in the nature of the task might be expected to materially affect performance, few studies have attempted to systematically analyze the behaviors required by the task. In two studies, Smedslund (1967a, 1967b) was concerned with the effects of perceptual and labelling variables in children's performance in matrix tasks. In the first study (Smedslund, 1967a) he found that covering the objects in the filled matrix cells before allowing the child to indicate what belonged in the empty cell had no effect on performance. However, requiring S to verbally describe both the objects in the matrix and the missing object increased the number of correct responses, suggesting that labelling the attribute mediates successful solution of matrix tasks. In the second study, Smedslund (1967b) found that visually presenting the objects in the filled cells facilitated performance relative to providing verbal description alone, suggesting that, to some extent, performance depends on the presence of perceptual cues.

Smedslund's hypotheses concerning the relative effects of perceptual and verbal factors on performance in matrix tasks derived largely from a general theory of cognitive and
Resnick perceptual development, supported by peripheral data from an earlier study (Smedslund, 1964). Neither he nor other investigators of multiplicative classification skills have used systematic behavior analysis procedures to determine the precise behaviors required for solution of specific matrix tasks. Furthermore, although different types of matrix tasks have been compared for general level of difficulty, there have apparently been no studies of transfer effects among different types of tasks.

The purpose of this study was to test several hypotheses concerning the nature of hierarchical transfer relationships among a set of interrelated matrix classification tasks. According to Gagne (1962), two tasks are hierarchically related when a) one of the tasks is easier to learn than the other, and b) learning the simpler task produces positive transfer in learning the more complex task. Hierarchical transfer relationships are thus asymmetrical: one order of task acquisition is more favorable than the other.

Gagne's research has shown that when instruction in complex intellectual tasks proceeds upward through a hierarchy of increasingly complex tasks, each one prerequisite to the next, nearly uniform positive transfer from one task to the next occurs (Gagne, 1962; Gagne & Paradise, 1961; Gagne et. al., 1962). Furthermore if the subject has learned the prerequisites in order of increasing complexity, the terminal task itself can often be "learned" without explicit instruction. None of Gagne's studies of learning hierarchies, however, has directly tested the asymmetry of the transfer effect. That is, the studies were not designed so that the effects of learning the tasks in the hypothesized optimal order could be compared with learning them in non-optimal orders.

The present study is specifically designed to make such a comparison for two different matrix classification tasks. The hypotheses concerning optimal learning order (hierarchical relationships) for these two tasks were derived from a systematic behavior
analysis, using a method developed by Resnick (1968; Resnick, Wang, & Kaplan, 1970). The behavior analysis specifies both component behaviors of the task and likely pre-requisites for learning each component. By performing behavior analyses of two or more related tasks, it is possible to identify component behaviors common to all tasks as well as components unique to a particular task. This extended behavior analysis provides a basis for predicting a) which tasks will be more easily learned or performed, and b) which "simpler" tasks, learned first in an instructional sequence, will facilitate learning the more complex tasks (Resnick, Wang, & Kaplan, 1970).

For the present study, two matrix tasks were analyzed. These tasks were chosen as representative of two distinctive types of matrix tasks: a) tasks in which attribute cells explicitly define the objects for each matrix cell (see Figure 1) and b) tasks in which the subject must infer the common row and column attributes on the basis of the arrangement of objects in a partially completed matrix (see Figure 2). Behavior analyses of the two tasks appear in Figures 3 and 4. In each analysis, box Ia behaviorally defines the child’s task. The entry above the line describes the stimulus situation, the entry below the

Insert Figures 1 - 4 about here.

line the appropriate response. This convention is followed in all boxes. Line II in each analysis describes the chain of component behaviors; these behaviors are performed in the temporal sequence indicated by the arrows.

For the task with filled attribute cells (Figure 3), three steps must be followed in order to place an object in the proper cell. The correct row must be found (Box IIa); the correct column must be found (Box IIb); and the intersect of the row and column must
be found (IIc). The process of identifying the appropriate row or column is actually a form of matching-to-sample task in which the object to be placed is the "sample" stimulus and the attribute cells constitute the "choice" stimuli. Box IIIa therefore describes matching-to-sample behavior as a prerequisite to both IIa and IIb. Only a relatively simple form of matching-to-sample is required, as the choice stimuli vary in only a single dimension (e.g. color or shape); there is no intruding irrelevant dimension which S must learn to ignore. This restriction is indicated in Box IIIa. A still simpler form of matching-to-sample, in which an identical match is possible, is shown as a lower-level prerequisite (Box IVa). No linguistic encoding appears necessary to the solution of this task.

Once the proper row and column have been identified, finding the intersect is a fairly mechanical matter. However, it does involve certain spatial organization behaviors which permit one to "keep one's place" in a relatively complex visual field. A hypothesized sequence of such spatial organization skills, cumulatively prerequisite to locating the intersect of a row and column, is shown in Boxes VIA, Va, IVb, and IIIb.

The analysis of the incomplete matrix task is shown in Figure 4. There are four component behaviors (Boxes IIa-IIId). Instead of matching-to-sample, the S must determine what attributes a set of objects has in common (Boxes IIa and b). Hypothesized prerequisites for this behavior are both spatial (IIIa and its prerequisites) and conceptual (IIIb and its prerequisites). An important set of prerequisites involve naming attributes of objects (IVc, Vb, VIa). Thus, some form of linguistic encoding behavior seems necessary to solution, although it should be noted that an S might use "private" rather than standard language labels for the attributes and still solve the matrix task.

Having identified the row and column attributes, the S must next combine the attribute names into a description of an object (Box IIc) and then select the object that
meets the description as the appropriate one for the cell (IIId). Hypothesized pre-
requisites for composing the description involve grammatical behavior (Box IIIc),
while selecting the appropriate object shares with earlier components in the chain the
prerequisites of responding to a verbal label (Boxes Vb and VIb). Thus, these components,
too, are heavily linguistic in nature.

These analyses suggest that the incomplete matrix task should be considerably
more difficult to learn than the task in which attribute cells are given. However, since the two
tasks are similar in stimulus format and logical structure, and since they share the same
spatial organization prerequisites, it seems reasonable to assume that learning the
easier task first would significantly facilitate learning the incomplete matrix task. The
two tasks were therefore hypothesized to be hierarchically related, with the attribute
cell task prerequisite to the incomplete matrix task. From this general hierarchical
hypothesis, three specific hypotheses were derived:

1) The incomplete matrix task will be learned in fewer trials when the
attribute cell task has been learned first.

2) Trials to criterion for the two tasks combined will be lower if the
tasks are learned in the optimal order (attribute cell, then incomplete matrix) than
if they are learned in the reverse order.

3) If the incomplete matrix task is taught first (i.e., non-optimal order),
Ss who succeed in learning it will show nearly immediate mastery of the attribute cell
task, since Ss who had learned the more complex task would have acquired the elements
of the less complex one in the process.

**Method**

**Subjects**

Ss were 53 kindergarten children in a predominantly white, middle class
Resnick

school. All children in each of two classes were pretested on three matrix tasks; only Ss who failed all three tasks were included in the experimental sample. The final sample consisted of 11 boys and 16 girls, ranging in age from 5 years-3 months to 6 years-5 months. Ss were matched as closely as possible for total number of errors on the pretests. One member of each pair was randomly assigned to each of two treatment groups, with the restriction that the number of boys and girls in each group be as equal as possible.

Description of Tasks

Three different matrix classification tasks were studied. In addition to the two tasks analyzed above, a form of the attribute-cell task with high feedback was used as a warmup task.

1) Describing (warmup task). S was presented with a 3 x 3 matrix in which the attribute cells were filled and open and the interior cells were filled and covered (see Figure 1). E pointed to the attribute at the beginning of row one and said, "This object is (blue). That tells you that everything in this row (runs finger across row) is (blue)." E continued in a like manner for the rest of the rows and columns. Then E said, "In each box there is an object that has a color and a shape. You guess what color and shape it is; then you may lift up the cover to see if you are right." After stating his answer for each cell, S was permitted to lift a flap covering the cell. A drawing of the correct object appeared underneath.

The task was used both as a pretest and as a training task for the experimental Ss. In the pretest, two matrices were presented. In each matrix, E pointed to all 9 interior cells in a random order. S was scored as passing the test if he responded correctly for six cells consecutively on each matrix.

In the training phase, only six of the nine cells in each matrix were pointed
to by E. If the child responded correctly, E simply pointed to the next randomly selected cell. If S responded incorrectly, one or both of two correction procedures was followed by E.

If the S failed to name both attributes, E said, "You must tell me a color and a shape. Always tell me two things, a color and a shape." If the S named two attributes but an error was made in either color or shape, E said, "The object is (red) because this (E pointed to attribute cell) tells you that everything in this row (column) is (red)." This procedure was repeated until S reached a criterion of no more than one error on two consecutive matrices.

During the training phase, the six responses for each matrix in the Describing task constituted one "trial." The trial was counted correct if S had no more than one incorrect response in the six. The procedure was continued to a criterion of two consecutive successful trials (i.e., matrices) or to a maximum of twelve trials without reaching criterion.

2) Placing (attribute cell task). S was presented with a 3 x 3 matrix in which the attribute cells were filled but the interior cells were empty. E held a set of the nine objects defined by the attribute cells, explained the meaning of the attributes as in the Describing task, and then said: "I'm going to give you an object. You put it in the right box." E handed S an object and S was required to place it in the appropriate cell. E presented all nine objects in random order. After S placed each object, E recorded its placement and then removed it from the matrix; thus there were nine cells from which S could choose for each response.

In the pretest, there was no feedback to the child as to the correctness of his choice. S was scored as passing the test if he consecutively placed six objects correctly.
on each of two matrices.

In the training phase only six (randomly selected) objects of the nine were presented for each matrix. After S responded correctly, E removed the object and handed him the next object. If S responded incorrectly, E pointed to the correct row and column and said, "Everything in this row is (green) and everything in this column is (square)." E visually emphasized the intersection of the row and column by bringing his fingers together at the point of intersection (i.e., the appropriate cell). As in Describing, this procedure was repeated until S reached a criterion of no more than one incorrect placement on two consecutive matrices, or until a maximum of twelve matrices had been presented. For purposes of analysis, six responses on one matrix constituted one trial.

3) **Inferring** (incomplete matrix task). S was presented with a partially filled 3 x 3 matrix without attribute cells. One, two, or three cells of the matrix were empty. S's task was to infer the attributes of the object belonging in the empty cells, given the arrangement of the objects already in the matrix. A nonsystematic array of the nine possible objects for the matrix was shown to S. E said, "The object(s) for one, (two, three) of the boxes is (are) missing. Find the object(s) that is (are) missing and put it (them) in the right place(s)."

The pretest consisted of four matrices: one with one empty cell; one with two empty cells; and two with three empty cells. An S was scored as passing the pretest if he missed no more than the first two responses.

In the training phase each series of three matrices constituted a "trial." If S made a correct response, he proceeded to the next item. After each incorrect response, E pointed to the appropriate row and column for each choice and said, "Everything in this
row is (blue) and everything in this column is (a circle). So here (E pointed to the empty cell) you need something that is (blue) and (a circle). " This procedure was continued until S reached a criterion of no more than one incorrect response on two consecutive trials (i.e. series of three matrices). The maximum number of trials given to any S was twelve.

Procedure

Two Es, one male and one female, conducted the pretest and experimental sessions. Children were assigned randomly to the Es with the restriction that each E tested approximately the same number of boys and girls in each treatment group. All testing was done individually in spaces provided in a corner of the classroom. At the beginning of the first pretest session, Ss were tested to determine whether they could readily identify the various colors and shapes being used in the tasks. All experimental Ss had ready labels for the colors and shapes. The order of tasks in pretesting was: Placing, Describing, Inferring.

The experimental sessions began ten days after the end of the pretest sessions. Each S was given one or two training sessions per week. Only one task was taught in each session. The session was terminated when S reached criterion, after he had completed six trials, or after approximately 20 minutes had elapsed. If a child did not reach criterion on a task in one session, he was given up to six trials on that task in the next session. After twelve trials or a maximum of four sessions on a task, training was begun on the next task. This schedule permitted a minimum of three and a maximum of twelve training sessions for each S.

Design

There were two treatment conditions, defined by the order in which the matrix
tasks were taught. Both groups learned Describing first. This functioned as a "warmup" task, and assured that all Ss entered the experimental training phase with equal skill in performing the simplest task under study. Group A learned the other two tasks in the hypothesized optimal order: Placing then Inferring. Group B learned the tasks in reverse order. Dependent measures were trials to criterion on Describing on Placing and on Inferring and trials to criterion for Placing and Inferring combined.

Results

Pretest. The percentages of all Ss (N = 53) passing each pretest were: Describing, 36%; Inferring, 21%; and Placing, 19%. Apparently, these tasks presented real challenges for most of the Ss—only slightly more than a third of the children passed even the easiest task. Only Ss who failed all three pretests were included in the experimental sample (N = 27, 14 in Group A, and 13 in Group B).

Training. Table 1 presents mean trials to criterion on the three training tasks for both groups of experimental Ss. The difference between the experimental groups in the warmup task, Describing was not significant (.< 1.00), indicating that the groups were equivalent in ability to learn tasks of this type. All but one S learned this task; the number of trials to criterion for the learners ranged from two to twelve.

Hypothesis 1 stated that the group learning Placing first (Group A) would learn Inferring more quickly than the group that began with Inferring (Group B). This hypothesis was supported, but not strongly when the data from all 27 S is considered. Group B took more trials to learn Inferring than did Group A (. = 1.36, df = 25,.05< p
This effect is shown graphically by the solid curves in Figure 5.

The study was specifically concerned with transfer effects of mastering (as opposed to simply being exposed to) one task on the learning of the next task. Therefore, a rigorous test of Hypothesis 1 would require looking at, for each successive task, only those Ss who had succeeded in learning the preceding task. This method of analysis would treat the data as if any S who failed to learn a task had been dropped from the study and not allowed to proceed to the next task. The mean number of trials to criterion for Placing and Inferring considering only Ss who reached criterion on the preceding task appears in Table 2. One can see that the difference between Groups A and B for Inferring was more clearcut ($t = 1.92, df = 22, p < .05$, one-tailed) when Ss who had failed to reach criterion on Placing were dropped from Group A. The dotted curve in Figure 2 shows this heightened effect.

Hypothesis 2 stated that Ss who learned the tasks in the optimal order (Group A) would learn the two tasks combined (Columns 7 and 8 of Table 1) more quickly than Ss who learned the tasks in the reverse order (Group B). Since the difference between the groups on this measure was not even marginally significant ($t = 1.04, p > .10$), there was no support for this hypothesis. However, the lack of significant difference might have been produced by a ceiling effect, especially for Ss in Group B. Had more than twelve trials been allowed, Ss who failed to learn a task would have had scores ranging upward from twelve. Since there were more failures to learn in Group B than in Group A
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a greater maximum of training trials would have differentially raised the mean for Group B, thus increasing the difference in trials to criterion for the two groups. This would have affected differences both on Inferring alone and on the combination of Placing and Inferring.

Hypothesis 3 stated that Ss who learned Inferring first would demonstrate immediate "learning" of Placing. A test of this hypothesis requires examination of the data only for Group B Ss who learned Inferring. Figure 6 shows a plot of trials-to-

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Insert Figure 6 about here.
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criterion scores for Ss in Group B, with Placing on the vertical axis and Inferring on the horizontal axis. Ss who failed to reach criterion on Inferring are designated by "FI". Ss who failed to reach criterion on Placing are designated by "FP." Five Ss failed to learn Inferring, one of whom also failed to learn Placing. Of the eight Ss who did learn Inferring, all but two took the minimum possible number of trials on Placing—i.e., they "learned" Placing immediately. One S took five trials to learn and one S, who had had considerable difficulty with Inferring, reaching criterion only on the final trial, failed to learn Placing at all. The mean number of trials to criterion on Placing for Group B Ss who reached criterion on Inferring first was 3.63; if the single extreme case is not considered the mean drops to 2.43. Thus, the data for most Ss is in support of Hypothesis 3. In the absence of further information on the single extreme S it is difficult to interpret this exception.

Discussion

The results of the experiment serve to partially confirm the hypothesized hierarchical relationship between the two matrix tasks, and thus lend support to the
technique of detailed behavioral analysis used in generating the hierarchy. Placing and Inferring superficially seem to be very similar tasks. However, prior behavior analyses of the tasks had suggested that Inferring required all the critical components of Placing, plus the additional one of discovering the common attribute value for each row and column. Inferring was therefore put above Placing in a hierarchy, implying that prior learning of Placing would facilitate learning Inferring. The advantage of Group A over Group B in learning Inferring confirms this hypothesis. The fact that a majority of Ss who first learned Inferring made no errors on Placing lends further support to the behavior analyses.

This experiment also demonstrates that children well below the age normally associated with concrete operations can learn to perform a complex task involving multiplicative classification when they are given the opportunity to learn component and prerequisite behaviors through corrected practice on a series of simpler, related tasks. This is in accord with studies of "programming" and successive approximation in children's learning of discriminations (e.g. Hively, 1963; Jeffrey, 1958). It suggests that acquisition of more complex cognitive skills as well may be a matter of learning specific relevant prerequisites rather than of entering a general level or "stage" of development.
References


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Siegel, A. W. & Kresh, E. Children's ability to operate within a matrix: A developmental study. Developmental Psychology, in press.


Table 1
Mean Trials to Criterion on the Three Training Tasks

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Order of Testing</th>
<th>Describing</th>
<th>Placing</th>
<th>Inferring</th>
<th>Placing + Inferring</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14</td>
<td>Describing-Placing-Inferring</td>
<td>3.43 3.13</td>
<td>4.36 3.58</td>
<td>6.86 3.66</td>
<td>11.21 6.68</td>
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</table>
Table 2
Mean Trials to Criterion for Placing and Inferring Considering only Ss who Reached Criterion on the Preceding Tasks

<table>
<thead>
<tr>
<th>Group</th>
<th>Placing</th>
<th>Inferring</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>X</td>
</tr>
<tr>
<td>A</td>
<td>13</td>
<td>4.23</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>3.63</td>
</tr>
</tbody>
</table>
Figure 1. Stimulus Layout for Tasks in Which Attribute Cells are Given
(Describing and Placing)
Figure 2. Stimulus Layout for Incomplete Matrix Task (Inferring)
Figure 3. Behavioral Analysis for Task in which Attribute Cells are Given
Figure 4. Behavioral Analysis for Incomplete Matrix Task
Figure 5. Cumulative Percent Ss at Criterion on Inferring
Figure 6. Trials to Criterion on Placing and Inferring for Group B
Figure 1. Stimulus Layout for Tasks in Which Attribute Cells are Given (Describing and Placing)
Figure 2. Stimulus Layout for Incomplete Matrix Task (Inferring)
Figure 3. Behavioral Analysis for Task in which Attribute Cells are Given
Two dimensional matrix with some cells empty. Identify object belonging in empty cell.

Ia
Empty cell
State what all objects in a row have in common.

Iib
Empty cell
State what all objects in column have in common.

Iic
Common attributes for row and column
Describe an object having both attributes.

IId
Description of an object in terms of two attributes and an assortment of objects.
Select object described.

IIia
Several parallel pathways
Follow one of them.

IIib
Several sets of objects, each set alike in one dimension but differing in another dimension
State how the objects in a given set are alike.

IIic
Two attribute names
Describe object as adj. + noun (e.g. "brown dog"); noun + noun (e.g. "a boy + a house"); adj. + adj. (e.g. "large green"); or noun + verb (e.g. "cat running").

IVa
A line with several points marked
Visually scan line from a given point to another given point.

IVb
Several sets of objects, each set identical within itself but differing in one dimension from other set
State how the objects in a set are alike.

IVc
An object
Name its attributes.

Va
A line with several points marked
Physically trace line from one point to another.

Vb
Array of objects varying in several dimensions
Select an object with a named attribute.

Via
Array of objects varying in a single dimension
Select an object with named attributes.

Figure 4. Behavioral Analysis for Incomplete Matrix Task

23
Figure 5. Cumulative Percent Ss at Criterion on Inferring
Figure 6. Trials to Criterion on Placing and Inferring for Group B