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ABSTRACT

Some general philosophy about maps is presented in the introduction. It is noted that the main purpose of a map is to generate an intellectual response through visual symbols. Quantitative distribution maps present a number of design problems that have led to continuing investigations of graduated circles as point symbols. A summary of the original graduated circle study, by the author, is presented. Six advantages to using graduated circles are summarized, however, map users consistently underestimate size differences when relying on the circle. One thousand forty college students from five colleges were tested and there was 70.5 percent underestimation. Additional tests were given to provide data for the construction of an appartent of visual size curve. Logarithmic transformation and the method of least squares were used to derive an estimating equation for circle tests to convert data into symbolic form. Recent test results, including R.V. Crawford's dissertation on the validation of apparent size scale for graduated circles, are included and generally support the author's research. It is noted that wedges and bars as symbols have some advantages, but that circles are generally preferred. (SLD)

THE RELATIVE EFFECTIVENESS OF SOME COMMON GRADUATED POINT SYMBOLS IN THE PRESENTATION OF QUANTITATIVE DATA

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A paper for presentation during a Symposium on the Influence of the Map User on Map Design to be held at Queens University, Ontario, Canada, from September 8 to September 10, 1970.

Introduction - Some General Philosophy

It is the fundamental purpose of a map to generate through visual means desired intellectual responses on the part of the map user. The map as a whole with its points, lines, areas, tones, patterns, colors and lettering functions as a kind of Gestalt stimulus, but order can be made out of this seeming chaos through the application of the principles of good map design and a knowledge of how effective specific map symbols are in eliciting the desired intellectual response.

In the case of the general purpose or reference map, no single intellectual response is usually intended. Nor, in general, is there any reason to intellectually and therefore visually order the various kinds of data on such maps. There may be order within classes of data, large cities vs. small cities, major streams vs. minor streams, etc., and landforms frequently are visually emphasized at the expense of other elements on reference maps. But in the main, reference maps are designed to serve a variety of purposes in sometimes rather precise ways. Many are much like nomograms in their use. The visual design of such maps consequently poses problems which can usually be resolved through the application of the principles of clarity and legibility in graphic design. There is no intention here to imply that general purpose cartography is free of design problems. Instead it should be noted that the problems are not the same kind as those in thematic cartography.

It is not quite true that if a reference map is analagous to a newspaper, a thematic map is a billboard. To be sure, the thematic map has as its primary, perhaps only function, the visual projection of a message which is in most cases of limited subject variety. At first glance this kind of cartography would seem to present few graphic design problems. On the other hand, the message usually involves complexities in precision and accuracy of statement which lead to, if not more involved, more critical design considerations than are found in general purpose cartography.

Quantitative distribution maps in particular, present a host of critical graphic design problems, and thus, for more than two decades thematic cartographers have been investigating various of these symbols. My interest has been in quantitative point symbols, particularly in areally varying interval point symbols and specifically graduated circles.

The original research done on the graduate circle was completed almost fifteen years ago. Some research done recently by myself on the graduated circle and related symbols is reported herein, as is an investigation on the circle recently completed by Paul V. Crawford.

Before moving to this subject, however, I should like to make a plea for continued and increased testing of specific map symbols at all map user levels. Objectively designed tests of individual map symbols are the most expeditious approach to the acquisition of empirically derived data so essential to the development of systems for symbol utilization.

There are numerous symbol problems awaiting investigators willing to use the testing approach. The testing of map symbols should be a continuing activity. We should not rest on our laurels. Those of us who in the past have used tests in our investigations should check on our earlier work. Tests of a particular map symbol should be repeated after a period of time to assure that those universals derived at a given time and in a given place are still applicable.

We should also check on one another. None of us is so expert that those investigations we have made could not stand the scrutiny of tests conducted by others at different times with different subjects. Of course, it is not an inspiring thing to redo a job already done. But studies which expand or complement work done at an earlier time lend themselves to a reevaluation of the original research.

Hopefully such reexaminations would fortify the results of earlier investigations, but whatever the results, contradiction or confirmation, this kind of corroboration is of major importance to the field in this early period in the development of objectively designed, user-oriented thematic cartography.

A Brief Summary of the Original Graduated Circle Study

My original study considered all aspects of the graduated circle as a quantitative map symbol. For example, when compared to other quantitative point type symbols, it was clearly demonstrated that there are distinct advantages in favor of the graduated circle:

1) It is relatively easy to convert basic quantitative data into symbolic form.
2) Circles can be placed on maps more rapidly than other types of point symbols.
3) Circles use map space efficiently, at least when compared to bars.
4) Aesthetically, users prefer circles (62%) over bars (38%), and 60% ranked circles first over triangles, squares, and

rectangles.

5) Circles represent patterns of distribution reasonably well.

6) Circles, when used as pie charts, more effectively communicate parts of the whole than do segmented bars.

The last advantage was discovered by Croxton, Stryker, and Eells in studies done on bars and circles in a non-map context in 1926 and 1927.

But in one, in fact, the very most important trait, it was discovered that the circle malfunctioned. When graduated in the traditional manner with areas in precise proportion to the quantities represented, map users consistently underestimated size differences and consequently perceived smaller quantitative differences than intended by the cartographer. The primary objective of the cartographer, therefore, that is, the expression of "graphically accurate" quantitative differences was shown to be persistently in error when circles are graduated on areal size basis.

Tests to demonstrate persistent underestimation were given to a sample of 1040 students from five colleges and universities around the country. They made 46 individual judgments of the size differences of black circles on a white background in both a map and non-map context and with areal size differences ranging from 2 to 1 to 32 to 1, and diameter sizes ranging from 3 to 24 millimeters. These absolute sizes are in general characteristic of circles on page sized maps. Comparisons were made with both larger and smaller circles serving as a base of comparisons and estimates recorded as size differences.

The results were: underestimates, 70.5%, correct estimates, 16%, and overestimates, 13.5% (see Figure 1). In no single set of comparisons were there ever more overestimates than underestimates. In addition to the percentage distribution, the significance of the underestimates was checked statistically by comparing the mean, median, and mode of the estimates against the actual size differences and by two more rigorous tests of statistical significance. One was a test of the significance of the proportion of underestimates and the other a 't' test on the weakest underestimate. All checks overwhelmingly supported the results of these tests. Underestimation was a fact. A recommendation could have been made that the circle be abandoned as a quantitative map symbol or some attempt made to correct the deficiency of an otherwise valuable map symbol. The latter decision prevailed.

Additional tests were then given to provide sufficient data for the construction of an apparent or visual size curve. The second group of tests were primarily in a map context with black circles on a white background and with a new sample of approximately 1000 students from six colleges. They made 97 comparisons of circle size differences ranging from 1 to 1.50 to 1 to 160, but ranged primarily from 1 to 50. Medians of the estimates were used to derive an estimating equation. An estimating or regression equation is a measurement of correlation which describes the functional relationship between two variables. The equation was structured on median estimates because quantitative graphic devices of this type can be expected to indicate only approximate values. Therefore, in any series of test of estimates of the value or relative size of the symbol, there should be approximately as many overestimates as underestimates but with a concentration of estimates near the correct value. Since the distribution of estimated circle size or value differences is usually moderately skewed, the means of estimates, even though they are underestimates, do not subdivide estimates equally. The median of estimates is thus the better measure.

The specific procedure used to derive the estimating equation for circle tests was to use a logarithmic transformation and the method of least squares, permitting the derivation of an equation which best describes the results of the tests. The procedure was selected after graphic examination of the data showed that when areal sizes were plotted against median estimates on a log-log scale, the points fell so as to suggest a straight line. This is in contrast to a plot on an arithmetic scale where a second degree curve was suggested.

Two curves for two sets of test data were derived. They eventually were combined to give the following equation which is the more convenient expression in terms of the original data:

$Y_c = .98365X^{(.8747)}$, in which

Y_c = a computed or estimated apparent relative size value,

.98365 = the value of Y_c when $X = 1$,

X = the logarithm of the actual relative size, and

.8747 = a constant.

The curve for the equation is shown in <u>Figure 2</u>.

As a formula used to draw circles on maps with the appropriate relative sizes, the equation is transposed so as to solve for 'X' and is as follows:

$$X_c = 1.0166Y^{(1.1432)}$$

In order to simplify the application of the equation for converting the data into symbolic form, the value 1.0166 can be dropped because it is so close to one. In addition, since $Y^{(\frac{1}{2})} = \sqrt{Y}$, simply by dividing the exponent 1.1432 by 2, an exponent (.5716) is obtained, and the final recommended formula for transposing data into a form suitable for drawing circles on a map is therefore $X = Y^{(.5716)}$. From this formula, values can be directly derived for the radii of a series of circles, which can then be drawn on the apparent size scale.

This formula substitutes logarithmic tables for the square root tables used in the traditional method of graduating circles and is therefore somewhat more clumsy. A short cut is provided in tables which give radii values for circles on the apparent size scale. One such set of tables appears in the Appendix F of Robinson and Sale's Third Edition of the Elements of Cartography. It gives radius index values ranging from apparent size 1 to 999, a range, to say the least, adequate for most purposes.

Some Recent Test Results

In spite of the publication of the essential results of this investigation in the Second Edition of the Elements of Cartography ten years ago, there has apparently been no widespread acceptance of the system. Perhaps the system is suspect. Nevertheless, other researchers such as Robert L. Williams in "Statistical Symbols for Maps" cite evidence of underestimation for a number of areal size varying point symbols and suggest an apparent size scale equally applicable to circles, squares, and triangles. Gösta Ekman and others at the Psychological Laboratory at the University of Stockholm conducted tests in the late 1950's which produced an exponent of .85 (remarkably close to .8747), but again, as with Williams, for a group of areally varying point symbols, not a specific geometric form. In spite of this support, it was decided to retest circles to determine if the apparent size scale was still valid and also to compare the effectiveness of the circle in conveying relative quantity with two other point symbols. One of these was the areally graduated circle sector or wedge which, in the period since the completion of the original study, has achieved some popularity, particularly among European cartographers. The other was the bar graph which was also examined in the first study and has somewhat the status of a control in testing graduated symbols.

Individual circle sectors or wedges have certain desirable attributes as quantitative map symbols. Locations can be pinpointed with the tip of the wedge centered on a given location. This pleases the locational perfectionist. The shape of the wedge also lends itself to more effective space utilization than the graduated circle, which is particularly true if the data being presented have a linear arrangement. Consequently, wedges are favored in the mapping of port data.

Tests were made of black circles, bars, and wedges on white backgrounds in a map context. A sample of 200 college students each made a total of 44 different comparisons. Circle size differences range areally 2 to 1 to 44 to 1, and absolute sizes were similar to those used in the original tests. Wedge sizes differences ranged from 1 to 52 with approximately the same absolute size variations used in the circle tests. The tests are shown in Figure 3, reduced considerably from their original size.

Figure 4 shows the percent distribution of circle size estimates for the new test data. 73.5% were underestimates, 19.5% overestimates, and 7.0% correct estimates, substantially the results of the original tests. Tests on wedges indicate that they are also underestimated as illustrated in Figure 5. There were 70% underestimates, 18% overestimates, and 12% correct estimates. Bars tests on the other hand show no evidence of underestimation as can be seen in Figure 6, thus substantiating tests made on bars in the original study. It is recommended that bars be drawn on maps in the traditional linear graduated scale.

Applying the same statistical methods used in the original study, that is, a logarithmic transformation of median estimates and actual area size differences plus the method of least squares resulted in the two following estimating equations:

For the new circle test data the equation is

$$Y_c = .99013X^{(.8583)}$$

For the wedge data it is

$$Y_c = 1.08X^{(.82051)}$$

Figure 7 compares the original curve of the estimating equation with those derived from the more recent tests on circles and wedges. Note that over the range of area size differences up to 50 times the curves are remarkably similar, particularly the two circle curves.

Figure 8 shows that when the median estimates of the new circle tests are graphically compared to the regression line of the original data, the trend of new median estimates coincides closely. A standard error of the estimate computed linearly shows, when plotted, all new median estimates within two standard errors and 75% within one standard error of the estimate. If the original regression line is to be applied universally, 95% of all estimates should fall within two standard errors, which they do. More correctly a logarithmic computation should have been employed in the deviation of the standard error or estimate. When plotted graphically, such lines limiting one and two standard errors would diverge at the higher end of the scale and would show even a closer coincidence of the new median estimates to the original curve. The linear computation provides a more stringent test and strongly supports the universality of the original estimating equation.

Wedges are another matter, however. Only 56% of the median estimates of wedge size differences fall within two standard errors of the estimating equations for circles (see Figure 9). On the other hand, the standard error of the estimate for the wedges is more than twice that of circles when computed linearly and, of course, even larger when computed logarithmically. Thus, even if a separate apparent size scale is used for wedges there is still ample evidence from these tests that the wedge is not as consistent in expressing quantity variation as are circles drawn on an apparent size scale. An examination of the raw test data substantiates this conclusion.

Crawford's Validation of the Apparent Size Scale for Graduated Circles

Paul V. Crawford, in a Ph.D. dissertation entitled "Visual Properties of Uniform Grey-Toned Point and Line Symbols Used in Graduated Series on Maps," University of Kansas, 1969, conducted a series of tests of black and grey-toned circles. It is Mr. Crawford's prerogative to report the detailed results of his study to the profession. On the other hand, since his investigations substantiate the discoveries of fifteen years ago, I am pleased to mention two aspects of his research.

In one series of tests of black circles over a size range difference of from 1 to 2 to 1 to 14 with a sample of 100 students, Crawford found 70% underestimates, 28% overestimates, and 2% correct estimates of circle size differences.

A logarithmic transformation of the test data and the method of similar squares were used by Crawford to statistically derive an estimating equation. Given in the form most convenient for use with original data, it is

$$Y_c = .96044X^{(.90421)}$$

Figure 10 illustrates the similarity between the Crawford and Flannery black circle curves. Although not shown, all fourteen median estimates fall within one standard error of the estimate based on the original apparent size curve.

Crawford completed a number of tests with grey-toned circles. Only the results of the 30% grey circle tests, the one deviating the most from the original curve are included here. Figure 11 shows that the two regression lines are quite similar and only one of the fourteen median estimates fell outside one standard error of estimate.

Conclusion

In summary, the original apparent or visual size scale for graduated circles works well. It has stood the test of time and the objective evaluation of an investigation free from any parental or vested interest. It works equally well for black circles as for grey, and probably will eventually be found to function as effectively for circles of different hues. Bars function effectively when graduated in a linear series and require no apparent size scale but there are some disadvantages to using it as a quantitative map symbol when compared to the circle. It uses map space less efficiently, takes more time to draw and has less aesthetic appeal. It seems that areally varying individual circle sectors or wedges should also be drawn on a visual size scale. As a substitute for the graduated circle, however, the wedge is inferior in representing quantitative variation, since median estimates deviate considerably from the predictions of either the apparent size circle scale or a specifically derived wedge scale.

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3. Frederick E. Croxton and Roy E. Stryker, "Bar Charts Versus Circle Diagrams," Jr. Am. Statistical Assoc., XXII, 1927, pp. 119-132.
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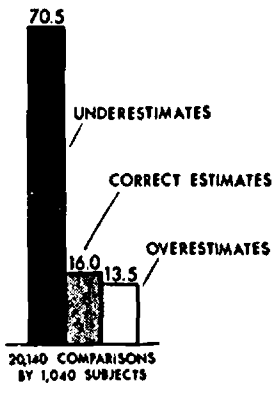
5. Robert L. Williams, Statistical Symbols for Maps: Their Design and Relative Values. Map Laboratory, Yale University, New Haven, Conn., 1956.
6. Gösta Ekman and Kenneth Junge, "Psychophysical Relations in Perception of Visual Length, Area, and Volume" Report #83 from The Psychological Laboratory, University of Stockholm, Stockholm, Sweden, 1960.
7. Paul V. Crawford, Visual Properties of Uniform Grey-Toned Point and Line Symbols Used in Graduated Series on Maps. Unpublished Ph.D. Dissertation, University of Kansas, 1969.

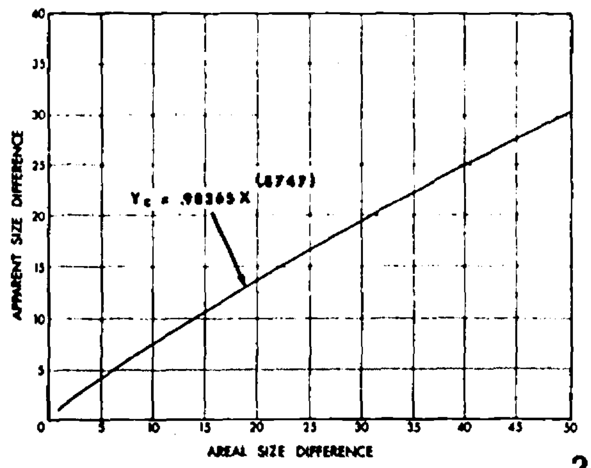
FIGURES

PERCENT DISTRIBUTION OF CIRCLE SIZE ESTIMATES
ORIGINAL TEST DATA

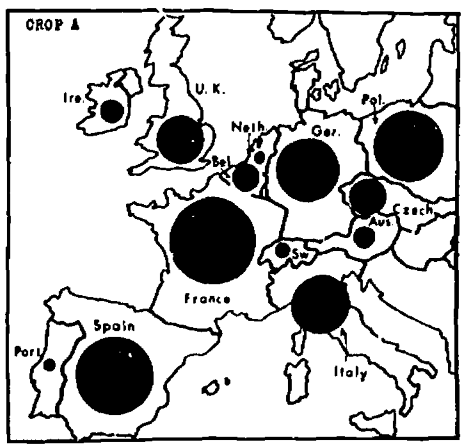


1

ORIGINAL APPARENT SIZE CURVE



2



Directions:

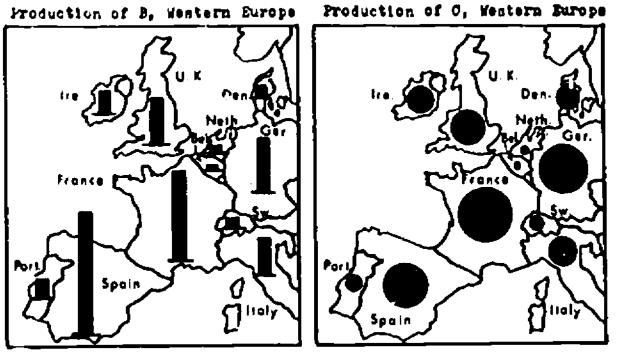
The map above is designed to test reactions to map symbols. The student is requested to make the comparisons listed below. For example, in comparing how much more "Crop A" is grown in one country than in another, make a quick estimate of the size difference of the circles and record your answer. Simply estimate how much bigger one circle <u>looks to be</u>. Do not complicate your judgment by applying geometric theorems. <u>Your first reaction is wanted</u>. In other words, how much difference in size does there <u>seem to be</u> or how many of the smaller circles would fit into the larger. Make judgments by eye only from standard reading distance. Please be conscientious.

How much more of Crop A is grown in

1.	Austria	than	Switzerland	________
2.	Italy	"	Austria	________
3.	United Kingdom	"	Switzerland	________
4.	Czechoslovakia	"	Switzerland	________
5.	Poland	"	Switzerland	________
6.	France	"	Austria	________
7.	France	"	Switzerland	________
8.	France	"	Germany	________
9.	France	"	Belgium	________
10.	Spain	"	Portugal	________
11.	Germany	"	Belgium	________
12.	France	"	Netherlands	________
13.	France	"	Ireland	________
14.	Spain	"	Switzerland	________
15.	Germany	"	Austria	________
16.	Germany	"	Netherlands	________

FIGURES

Production of B, Western Europe   Production of C, Western Europe



Using the maps B & C above, the student is requested to make the estimates requested below. Make all judgments by eye only.

I. Map B. Production of "B" in Western Europe.
By estimating the differences in the length of the bars--How much more B is produced in

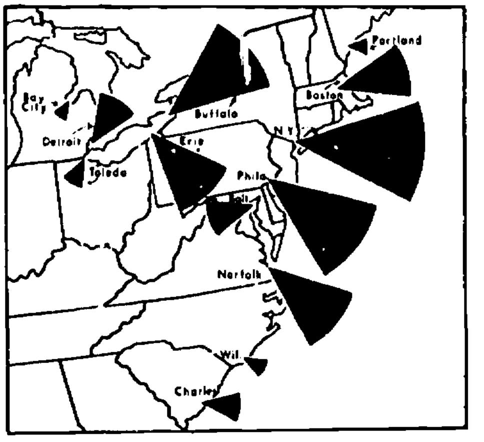
1. France than Belgium ______
2. Portugal " Belgium ______
3. Italy " Belgium ______
4. Spain " Belgium ______
5. Netherlands " Belgium ______
6. Germany " Belgium ______
7. U. Kingdom " Ireland ______
8. Spain " Portugal ______

II. Map C. Production of "C" in Western Europe.
By estimating the differences in the sizes of circles (how many smaller ones could fit into or be made from a larger one)--How much more C is produced in

1. Ireland than The Netherlands ______
2. Denmark " The Netherlands ______
3. Germany " The Netherlands ______
4. France " The Netherlands ______
5. U. Kingdom " The Netherlands ______
6. Denmark " Belgium ______
7. Netherlands " Belgium ______
8. Germany " Belgium ______

3b

CARGO D



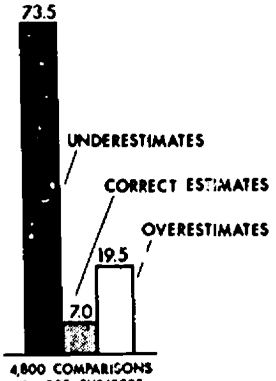
Directions: The map above is designed to test reactions to map symbols. The student is requested to make the comparisons listed below. For example, in comparing how much more "Cargo D" is handled in one city than in another, make a quick estimate of the size difference of the wedges and record your answer. Simply estimate how much bigger one wedge looks to be. Do not complicate your judgment by applying geometric theorems. Your first reaction is wanted. In other words, how much difference in size does there seem to be or how many of the smaller wedges would fit into the larger. Make judgments by eye only from standard reading distance. Please be conscientious.

How much more of Cargo D is handled in

1. Toledo than Wilmington ______
2. Erie " Toledo ______
3. Boston " Wilmington ______
4. Baltimore " Wilmington ______
5. Buffalo " Wilmington ______
6. New York " Toledo ______
7. New York " Wilmington ______
8. New York " Norfolk ______
9. New York " Detroit ______
10. Philadelphia " Portland ______
11. Norfolk " Detroit ______
12. New York " Bay City ______
13. New York " Charleston ______
14. Philadelphia " Wilmington ______
15. Norfolk " Toledo ______
16. Norfolk " Bay City ______

3c

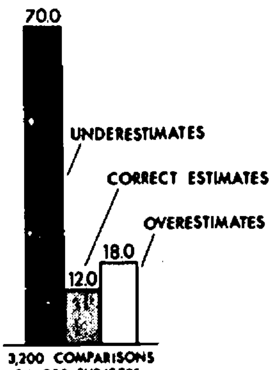
PERCENT DISTRIBUTION OF
CIRCLE SIZE ESTIMATES
NEW TEST DATA



4,800 COMPARISONS
BY 200 SUBJECTS

4

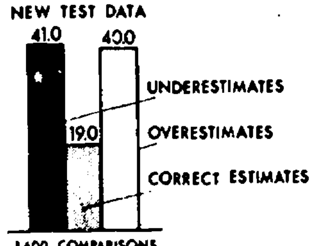
PERCENT DISTRIBUTION OF
WEDGE SIZE ESTIMATES



3,200 COMPARISONS
BY 200 SUBJECTS

5

PERCENT DISTRIBUTION OF
BAR SIZE ESTIMATES
NEW TEST DATA

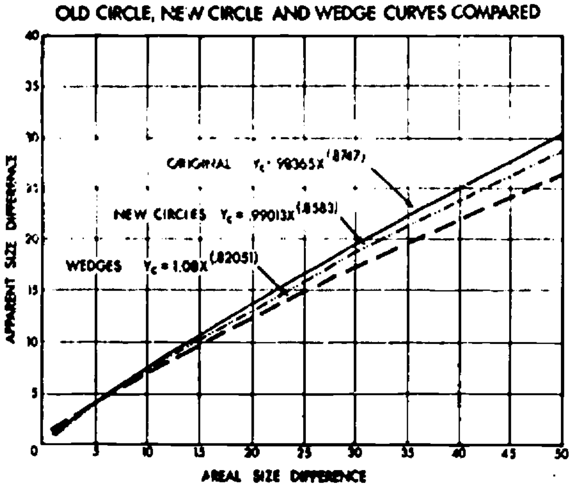


1,600 COMPARISONS
BY 200 SUBJECTS

6

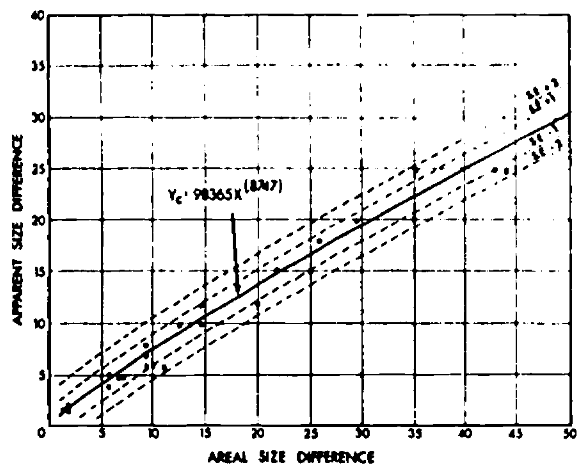
FIGURES

OLD CIRCLE, NEW CIRCLE AND WEDGE CURVES COMPARED



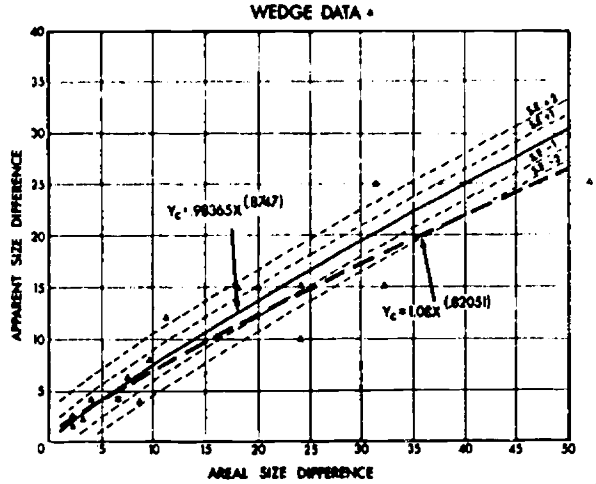
7

NEW TEST DATA - CIRCLES •



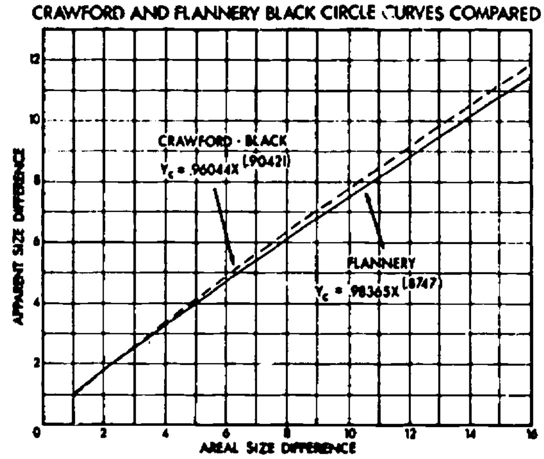
8

WEDGE DATA •



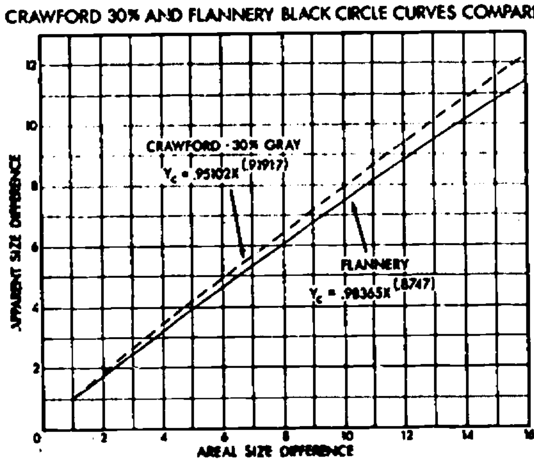
9

CRAWFORD AND FLANNERY BLACK CIRCLE CURVES COMPARED



10

CRAWFORD 30% AND FLANNERY BLACK CIRCLE CURVES COMPARED



1