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WORKING PAPER NO. 36
COGNITIVE OPERATIONS
IN CONCEPT LEARNING

REPORT FROM THE PROJECT ON
SITUATIONAL VARIABLES AND
EFFICIENCY OF CONCEPT LEARNING

Wisconsin Research and Development
CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
Madison, Wisconsin

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COGNITIVE OPERATIONS IN CONCEPT LEARNING

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Report from the Project on Situational Variables and Efficiency of Concept Learning
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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Working Paper is from the Situational Variables and Efficiency of Concept Learning Project in Program I. General objectives of the Program are to generate new knowledge about concept learning and cognitive skills, to synthesize existing knowledge, and to develop educational materials suggested by the prior activities. Contributing to these Program objectives, the Concept Learning Project has the following five objectives: to identify the conditions that facilitate concept learning in the school setting and to describe their management, to develop and validate a schema for evaluating the student's level of concept understanding, to develop and validate a model of cognitive processes in concept learning, to generate knowledge concerning the semantic components of concept learning, and to identify conditions associated with motivation for school learning and to describe their management.
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ABSTRACT

A tri-level structure of cognitive operations in concept learning is proposed, based on results of controlled experimentation and factor analytic studies. Global strategies in concept attainment are hypothesized to consist of three phases: attending to the situation, searching for information, and processing and using the information. Further, specific cognitive operations entailed in each of the three phases are identified. Research related to the structure and to each of the operations is summarized. Clarification of the cognitive operations in concept learning may lead to a better understanding of individual differences in learning, provide a theoretical basis for a process approach to the curriculum, and suggest variables which may be effective in facilitating concept learning.
I

Introduction

In this report cognitive, or mental, operations in concept learning are identified tentatively and described in terms of a tri-level structure of cognitive operations. Concept learning is conceptualized as involving a global strategy that controls three sets of information securing and processing activities. Each of these is further analyzed into more discrete operations. Other operations that precede the actual learning of a specific concept and still others that follow after concept learning are also indicated.

The present status of the structure is considered tentative. Some of the controlled experiments from which the structure was derived were not designed solely to identify operations. The interests of the graduate students, their preferences for methodology, and practical concerns involving sources of funding and availability of experimental subjects posed some constraints. More systematically controlled experimentation is projected during 1969-1972 to validate the structure and operations.

The emphasis is on the cognitive operations, rather than on the content, of concept learning. Therefore, only minor consideration is given to instances of concepts, attributes, and rules for joining attributes. The differentiation between operations and contents as applied
to concepts is well stated by Kagan (1966): "Cognitive units are the hardware of mental work; the things that get manipulated in mentation. Three basic classes of cognitive units include perceptual schema, language units, and rules of transformation or principles. Cognitive processes refer to the more dynamic events that act on the cognitive units, much like catalysts act on basic compounds in chemical solution. The processes of labeling, evaluation, hypothesis production, and transformation are fundamental (p. 98)."

Three main lines of investigations during the past two decades lay the substantive basis for the structure of cognitive operations. Bruner, Goodnow, and Austin (1955) dramatized the movement away from narrowly conceived S-R methods in the study of human learning. Their methods for studying operations and their accompanying approach to concept learning as searching for, achieving, retaining, and transforming information, rather than as associating stimulus and response events or as differential reinforcement of operants, generated much research and knowledge about cognitive operations. Miller, Galanter, and Pribram (1960) further confirmed the feasibility of studying internal structures and operations as represented by such labels as images, plans, intentions, and decision-making, or TOTE, units. Finally, Guilford and his associates early differentiated between operations and contents, and carried out a large number of experimental and factor analytic studies from which the structure-of-intellect model emerged. This model has recently been put in fairly final form by Guilford (1967). Common to all of these approaches is the attempt to specify more precisely the
mental operations, or processes, that result in relatively permanent changes in behavior.

Controlled experimentation and factor analyses are used to identify cognitive operations. Brune et al. (1956) developed a methodology for controlled experimentation applied to concept learning, while Fleishman and Bartlett (1969) and Guilford (1967) have used factor analytic methods for identifying abilities. The operations in the factor analytic studies are defined in terms of performances on tests.

The present hypothesized structure of cognitive operations is based on the results of controlled experiments and factor analytic studies carried out by the senior author and his students over a period of about 7 years. The operations identified or clarified in these studies were inferred from performances on tests, from responses during a sequence of learning trials, or from other response measures. Findings from other sources are also noted and incorporated into the structure.

Clarification of the specific operations related to outcomes of learning, such as concepts, should aid in developing a taxonomy of cognitive operations. This taxonomy may, in turn, lead to a better understanding of individual differences in learning (Melton, 1967). Further, curriculum designers and test developers concerned with a process or abilities approach to the curriculum need a systematic basis for developing internally consistent instructional objectives, materials, and tests. The structure of operations when more fully validated should provide a theoretical basis for these activities.
In the next pages, ideas are reported in approximately the same order as the research was carried out. The first work was done on global strategies and the delineation of a strategy into a three-phase informational sequence. Subsequent research was concerned with identifying the more specific operations involved in each of the three phases.
Figure 1 outlines a tri-level structure of cognitive operations. A brief overview of it is now given. Klausmeier, Harris, and Wiersma (1964) identified variants of two strategies by analyzing the consecutive responses of subjects in a series of concept attainment experiments. These strategies appeared to function in the experimental situations in the same manner as the Plan described by Miller, Galanter, and Pribram (1960): "A Plan is any hierarchical process in the organism that can control the order in which a sequence of operations is to be performed [p. 16]." The sequence of operations in concept learning inferred by Klausmeier, et al. (1964), were attending to the situation, securing information, and processing the information. If one attains a concept on the first attempt, there is a straightforward sequential engagement in the three sets of activities. When several trials are required, there is a moving back and forth. These three sets of activities are considered to be applicable to attaining concepts under various conditions, such as by reading material that presents information about a concept, by hearing information presented about a concept, or by attending to actual examples or instances of the concept and subsequently inferring the concept.
Figure 1. A Tri-Level Structure of Cognitive Operations

- Preconceptual Operations
- Concept Learning Operations
- Postconceptual Operations

- Processing and using information
- Setting the stage
- Post hoc operations

- Searching for the defining attributes and rules when responding or when presented with the instances, or when responding to presented instances
- Differentiating the stimulus characteristics
- Constructing the concept--the conceptual population
- Cognizing the structure of the concept--the conceptual population's defining basis
- Cognizing the concept--the defining attributes

Global Strategy
Each of the three sets of activities is further analyzed into specific cognitive operations, as shown in Figure 2, 3, and 4. Three of the five labels for operations shown in Figures 2, 3, and 4—cognizing, remembering, and evaluating—have been used systematically by Guilford (1967). The other two operations that complete Guilford's structure are divergent production and convergent production. Other terminology used in the concept attainment literature was considered more descriptive than divergent and convergent production, although these two terms might incorporate some of the more specific operations.

The specific operations enumerated in Figures 2 and 3 under the two main headings clarify more precisely what is involved in attending to the situation and securing information. The bottom section of Figure 4 indicates the operations that precede the actual learning of a concept through the lowest level of concept formation as defined by Bruner, et al. (1956); that is, simply categorizing two instances as belonging to the same concept without being able to indicate the defining attributes of the concept. In the middle section of the figure are the operations that were identified in the present series of experiments as being directly involved in attaining more complex concepts. In the upper part of Figure 4 the specific operations involved in using a concept are indicated. The latter are based primarily on the work of Bruner, et al. (1956), and Gagné (1965, 1966).
Attending to the situation--the whole and its components

Cognizing the definitional basis of the concept population in terms of perceptible defining attributes, semantic meanings, operations, relations, or use.

Cognizing the structure of the concept population--instances, attributes, and their relations.

Differentiating the characteristics of the stimulus situation.

Figure 2. Specific Operations Involved in Attending to the Situation

Securing information--searching the instances for common elements

Searching for the defining attributes and roles by responding to instances presented simultaneously or consecutively.

Searching for the defining attributes and roles by selecting instances from a total array.

Figure 3. Specific Operations Involved in Securing Information
Processing and using information

Postconceptual Operations
Transferring the concept to problem-solving activities.
Generalizing to other conceptualizing situations.
Cognizing other concepts as coordinate, superordinate, and subordinate.
Generalizing to other instances of the newly acquired concept when encountered for the first time.

Operations During Concept Learning
Inferring the concept by inductively arriving at the common defining properties and rules; or by cognizing logical relations among attributes and rules.
Evaluating relations among attributes and rules.
Remembering information concerning instances, attributes, rules, and hypotheses.
Hypothesizing the concept.
Cognizing a criterial attribute or rule by comparing the information presented in positive and negative instances.
Cognizing that instances do or do not belong to the same set.

Preconceptual Operations
Cognizing attributes common to at least two instances of the same concept and responding with the appropriate concept label (corresponding to late preoperational and early concrete operations of Piaget).
Acquiring labels for single-instance, or identity, concepts and associating the label with the instance (corresponding to the preoperational stage of Piaget).
Acquiring labels for potential instances and attributes and associating the label with the instance (corresponding to the preoperational stage of Piaget).
Discriminating among potential instances and attributes and responding with appropriate nonverbal responses (corresponding to sensorimotor stage of Piaget).

Figure 4. Specific Operations Involved in Processing and Using Information
It is these operations in Figure 4 from bottom to top that are proposed as involving both a vertical and a developmental sequence. The bottom set of operations are characteristic of young children who are incapable of the middle set of operations. Children at the sensorimotor (ages 0-1½) and preoperational stages (ages 1½-6) in Piaget's system (Ripple & Rockcastle, 1964) are thought to engage in the bottom set of operations. The middle set of operations is involved in all concept learning except the lowest level of concept formation, which in Figure 4 is included in the preconceptual set. Children in Piaget's stage of concrete operations (ages 6-12) supposedly engage in these operations. A concept must be acquired before the operations in the upper part of Figure 4 can be manifested. Children late in Piaget's stage of concrete operations probably manifest all the operations in the top part of Figure 4 with some concepts. More complex concepts with highly abstract defining attributes and other concepts whose instances are not readily available to the senses or cannot be readily represented visually are attained only by older children who are in Piaget's stage of logical operations.

We now turn to a more detailed treatment of the various components of the proposed structure of cognitive operations in concept learning, beginning with strategies.
Strategies

In their experiments to identify strategies, Bruner, et al. (1956), constructed an array of 81 concept instances which represented all possible combinations of four trivalued attributes. The attributes were type of figure--cross, circle, or square; number of figures--one, two, or three; color of figure--red, black, or green; and number of border lines--one, two, or three. This stimulus array was designed as a model population of the real-world population of concepts that are defined in terms of attributes and values joined by conjunctive, disjunctive, or relational rules.

At the start of the experiment the nature of the concept to be attained was described to the subject. A positive instance of the particular concept was pointed out. The subject then selected additional instances for testing to identify the attributes and rule that defined the concept. Following each choice, the subject was told whether it was or was not an instance of the concept to be attained. Also, after each choice the subject could offer an hypothesis, or his best guess, concerning the concept that the experimenter had in mind. When the hypothesis offered was correct, the subject had identified the concept. This type of task represented
a selection paradigm; that is, the subject selected instances from the total array.

Bruner, et al. (1956), also employed a somewhat different arrangement in which the experimenter presented the instances to the subject and told him whether they were or were not instances of the concept; this represented a reception paradigm. Here also the subject's task was to infer the concept the experimenter had in mind.

Bruner pointed out that the subject made a series of decisions in attaining a concept under the selection paradigm; that is, he decided which attributes to test, which hypothesis to offer, when to offer an hypothesis, and what changes to make in his selections and hypotheses when various contingencies were encountered. Regularities in these decision-making process were called strategies and served to control the sequence of activities carried out by the subject.

Selection Strategies

Bruner, et al., formulated a set of ideal strategies which met certain objectives with "maximum rationality." The actual performance of the subject was then compared with these ideal strategies, and a best fit was determined. Four ideal strategies were identified under the selection paradigm:

1. **Simultaneous Scanning.** The subject initially entertains all possible hypotheses and subsequently selects instances for testing on the basis of securing a maximum amount of information. After testing each instance, the subject deduces which hypotheses are still tenable and which have been eliminated.
2. Successive Scanning. The subject initially entertains a single hypothesis and he chooses instances which will provide a direct test of that hypothesis.

3. Conservative Focusing. The subject holds no hypotheses initially. A positive instance is chosen as a focus and each attribute of this focus is directly tested for relevance to the concept. This is accomplished by testing a sequence of instances, each differing from the focus in only one attribute value. When instances are tested in this manner, a "yes" indicates that the changed attribute is irrelevant to the concept and a "no" that it is relevant.

4. Focus Gambling. In this strategy, a variant of conservative focusing, the subject chooses a positive instance as a focus but then varies more than one attribute at a time. When instances are tested in this manner, a "yes" indicates that all changed attributes are irrelevant to the concept. A "no," on the other hand, provides no information concerning which of the changed attributes is relevant.

Reception Strategies

In addition to the four ideal strategies outlined under the selection paradigm, Bruner characterized two strategies which might occur in the reception paradigm:

1. Wholist. The subject initially adopts a hypothesis which consists in toto of the first positive instance encountered. Subsequently, this hypothesis is revised by taking the intersect of the initial hypothesis and all other positive instances.

2. Partist. The subject entertains an initial hypothesis consisting of only part of the positive focus. If this hypothesis is disconfirmed by a subsequent instance, the subject formulates a new hypothesis consistent with all instances encountered.
Comparing actual sequences of responses, or decisions, of experimental subjects with these ideal strategies, Bruner found a high degree of similarity which permitted him to characterize each sequence of decisions as representing a particular strategy. Bruner's techniques, then, attempted to make decision-making operations open to direct observation.

In a subsequent study of selection strategies using similar stimulus material and experimental procedures, Byers (1961) demonstrated that the simultaneous scanning strategy produced a sequence of choices indistinguishable from that resulting from the conservative focusing strategy. Furthermore, Byers pointed out that Bruner was not wholly objective in his method of characterizing a particular sequence of responses as a particular strategy. Byers also reported that practice modified the probability with which subjects used various strategies. In general, the probability of the conservative focusing strategy increased with practice, while the probabilities of other strategies decreased. The type of strategy employed had an influence on efficiency of performance, the most efficient performance being associated with the conservative focusing strategies.

Klausmeier, Harris, and Wiersma (1964), employing the Bruner-type selection paradigm, were unable to identify systematic differences in the responses of subjects using simultaneous scanning and conservative focusing strategies. Instead, the strategies were identified and designated as one of three variants of a conservative focusing strategy or as one of two variants of a focus gambling strategy. Table 1 on Page 15 summarizes the criterion behaviors for
<table>
<thead>
<tr>
<th>Designation</th>
<th>Observable Behavior</th>
<th>Inferences Based on Knowledge of Observable Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservative Ca</td>
<td>Selected instances from a total array that contained the essential information, i.e., criterion attributes. Tested all attributes. Offered 1st hypothesis which was correct. Made below-median number of redundant choices.</td>
<td>Cognized the essential information. Remembered the essential information. Correctly inferred the attributes and rule.</td>
</tr>
<tr>
<td>Conservative Cb</td>
<td>Selected instances from a total array that contained the essential information, i.e., criterion attributes. Tested all attributes. Offered 1st hypothesis which was correct. Made above-median number of redundant choices.</td>
<td>Same as Ca, except in relation to above-median number of redundant choices. It is inferred from the above-median number of choices that the subject intentionally retested more instances, or did not cognize potentially available information, or remembered information less well.</td>
</tr>
<tr>
<td>Conservative Cc</td>
<td>Selected instances from a total array that contained the essential information, i.e., criterion attributes. Tested all attributes. Offered 1st hypothesis which was incorrect.</td>
<td>Did not cognize potential information, or did not remember it, or made incorrect inferences.</td>
</tr>
<tr>
<td>Gambling Gh</td>
<td>Selected instances from a total array that contained the essential information, i.e., criterion attributes. Did not test all attributes. Offered 1st hypothesis which was correct.</td>
<td>Gambled successfully about nature of concept. Cognized the essential information. Remembered the essential information. Inferred correctly.</td>
</tr>
<tr>
<td>Gambling Gi</td>
<td>Selected instances from a total array that contained the essential information, i.e., criterion attributes. Did not test all attributes. Offered 1st hypothesis which was incorrect.</td>
<td>Gambled unsuccessfully about the nature of concept, or gambled successfully on nature of concept but did not cognize, or remember information, or did not draw correct inferences.</td>
</tr>
</tbody>
</table>
classifying the five strategies and also the inferences drawn from the behavior.

1. **Conservative Strategy (Ca)**. When using Ca, the subject checked each attribute to ascertain whether it defined the concept. From this we infer that he potentially cognized all the information essential to attain the concepts. He offered a first hypothesis which was the correct concept. From this we infer that he actually cognized all the essential information, and correctly combined the information about attributes and rule to arrive at the concept. He made redundant choices below the median number made by all subjects using Ca andCb strategies. From this we infer that he used or remembered information better than did subjects making above the median number of redundant choices.

2. **Conservative Strategy (Cb)**. The criteria for identifying Cb are the same as Ca, except that the number of redundant choices is above the median. The inferences drawn about Cb are the same as for Ca, except for those based on the above-median number of redundant choices. On that latter basis we infer that the subject intentionally retested more of the same instances or instances containing equivalent information or remembered information less well and therefore retested the same attributes, or did not cognize the potentially available information from earlier card choices.

3. **Conservative Strategy (Cc)**. When using Cc, the subject tested each attribute, thus having all the necessary information potentially, but after doing so, offered an incorrect hypothesis. Apparently, this resulted from one of a combination of the following:
the subject made the card choices essential for securing the necessary information but did not cognize part of the information, or he forgot, or he drew an incorrect inference relative to the criterial attributes or rule comprising the concept.

4. Gambling Strategy (Gh). When using Gh, the subject offered a first hypothesis which was the correct concept, without having checked all the attributes. In order to accomplish this, the subject gambled correctly that the concept was defined by a certain number of relevant attributes and a certain rule. He was then able to identify the concept. In addition, one may infer that the subject actually cognized the potential information from his card choices, remembered it, and drew the correct inferences from it.

5. Gambling Strategy (Gi). When using Gi, the subject offered a first hypothesis which was incorrect without having checked all the attributes. In contrast to a subject using the Gh strategy, he did not gamble correctly that the concept had a certain number of relevant attributes joined conjunctively; or, if he did so, he did not cognize the potential information, or did not remember it, or did not draw the correct inferences.

The preceding strategies were based on analysis of the responses of hundreds of subjects. Subsequently, an attempt was made to instruct some subjects to use a conservative focusing strategy, and others, a focus gambling strategy (Klausmeier & Heinke, 1968). Subjects were readily taught to use the conservative focusing strategy and performed significantly better than those not instructed. Subjects could not
be taught to use the focus gambling strategy consistently. Like the uninstructed subjects of Byers (1961), they tended to use a conservative focusing strategy.

Subjects, as noted in the prior studies, use various strategies in attaining concepts. While it is possible to identify strategies on the basis of the patterns and accuracy of responses as shown in Table 1, the carrying out of a strategy involves a number of cognitive operations which may be inferred from the patterns of responses. Major attention was given to operationally defining, based on controlled experiments and factor analytic studies, the specific cognitive operations associated with the three sets of activities: (1) attending to the situation, (2) searching the instances to identify the attributes and rules comprising the specific concept to be attained, and (3) processing information to identify the attributes and rule that define the concept.
III
Attending to the Situation

Three experiments were carried out in which instructions were manipulated to permit inferences concerning the cognitive operations involved in attending to the concept population. The optimal instructions in these experiments were formulated to enable subjects to cognize the attributes of the concept population, to cognize the rule joining the attributes of specific concepts to be attained in the experiments, and to draw correct inferences from positive and negative instances. The stimulus materials in all three experiments were geometric forms varying on five bivalued dimensions. The subjects were college students enrolled in educational psychology classes.

In the first experiment (Fredrick & Klausmeier, 1968), the task consisted of identifying a two-attribute conjunctive concept from a series of six slides presented sequentially in a reception paradigm. The optimally instructed subjects read instructions concerning the nature of the concepts to be identified. The optimal instructions were as follows:

In this experiment you are going to identify concepts that I have in mind. A concept in this experiment is used to classify sets of cards into two groups, one set belongs to the concept and the other set does not. Let's clarify
further how we are using the term concept in this experiment. Here is a card with one large textured green square. (Slide.) Suppose that I told you "yes," meaning the card belongs to the concept I have in mind. This would tell you that the concept I have in mind might be large square, or one large, or one textured, or green textured, or any other combination of features of the card. You would need more cards, however, to tell exactly what the concept is. Suppose I presented a second card that was identical to the first one except that it had one small textured green square, instead of one large textured green square. If I told you "no," meaning this card does not belong to the concept, you could infer that all cards that are small do not belong to the concept. The third card I present might be identical to the first one except that it contained a circle instead of a square. I might tell you "yes," meaning it does belong to the concept. You would know that both circles and squares belong to the concept. Still other cards would be needed to tell exactly what the concept is. Thus, concepts in this experiment are combinations of the features of the cards and are used to classify sets of cards. After seeing a series of cards you can decide what the concept is; you can tell which cards do and do not belong to the concept. The label below each card will tell you which are in the concept.

All subjects in both the optimal and minimal instruction groups were told the following before beginning the task.

You are going to see slides which have geometric figures on them. Some of these figures will be circles and some will be squares. The figures can be large or small, red or green, solid or textured. There can either be one circle or two circles, or one square or two squares on a slide. For examples, look at this slide. (Slide.) We could describe it as two, large, plain, green, square, figures. Now will you please describe the next figure?

Subjects who received the optimal instructions made significantly fewer errors in classifying new instances than subjects receiving only minimal instructions.

In a second experiment (Kalish, 1966), the same instructions and task were employed. The mean error scores for the optimal and minimal conditions were 1.99 and 3.21, respectively. The difference between these means was significant. A third experiment (Lynch, 1966), combined
the instructional conditions of the previous experiments with high- and low-frequency labels placed on the instances. In addition, both conjunctive and disjunctive concepts were attained by each subject. Again, optimal instructions significantly facilitated concept attainment for both types of concepts.

Thus, cognizing the definitional basis of the concept population and cognizing the structure of the concept population at the outset of concept attainment tasks were inferred as facilitating the acquisition of concepts. The extent to which these two operations may be univocal or overlapping was not tested.

The third operation in this first set is labeled as differentiating the characteristics of the stimulus situation. This is perhaps more a perceptual than a cognitive operation, as judged from the early research on cognitive style reported by Witkin, Dyk, Paterson, Goodenough, and Karp (1962) and Kagan, Moss, and Sigel (1963). They found a marked difference in how subjects selectively attended to a stimulus array. Some subjects tended to perceive a stimulus array globally, while others attended to the cues or attributes within the whole and organized the parts according to the common or relatable attributes.

To learn more about this phenomenon in concept attainment, Fredrick (1968) ran an experiment involving 6th, 8th, and 10th Graders. They were administered the Hidden Figures Test, which is a measure of cognitive style; an information processing test; and a series of concept learning problems. Total scores and subscores from the three tests were correlated and various ANOVA's were run. An increase in analytic ability, treated here as the ability to differentiate attributes, was
observed in students from age 12 to 16 years. At all ages the subjects higher on analytic ability—those who scored higher on the Hidden Figures Test—also processed information and attained concepts more efficiently. In comparison with other subjects, the analytic Ss made fewer inclusion and exclusion errors in categorizing instances of the concepts they were to attain. Two other experiments involving 12th Grade students (Davis, 1967) similarly showed analytic students to attain concepts more efficiently than the global or non-analytic. Thus, differentiating the attributes embedded in an array of concept instances is considered an important operation in this initial set.
Securing Information

No experiment was undertaken in the present series specifically to clarify this configuration of activities systematically. However, four main stimulus presentation methods were used in the experiments: (1) A total stimulus array was presented simultaneously and the subject selected successive instances according to his own choice. (2) All instances necessary to attain the concept were presented to the subject simultaneously. (3) Instances were presented successively. In some experiments the instance remained in view for inspection, while in others the instance was withdrawn shortly after presentation. (4) Verbal propositions combined with instances were presented successively.

Observation of the subjects during concept acquisition and analysis of their final performances led to the conclusion that cognizing the criterial attributes and rules of the various positive and negative instances is the key operation here. The search for instances is not the critical matter. Rather, it is the searching for and cognizing what is embodied in the instance and the relations among instances. Whether the individual searches for instances himself or has them presented to him, his main task is to cognize what the defining attributes and rules may be.
VI
Processing Information

As noted earlier in Figure 4, three sets of cognitive operations are outlined in a vertical sequence: preconceptual operations, operations during concept learning, and postconceptual operations. In the experiments on which this report is based, primary attention was given to the set of operations directly involved in attaining fairly complex concepts. In the next pages these are reported in more detail than are the preconceptual and postconceptual operations.

Preconceptual Operations

A quotation from one of Piaget’s addresses at a conference at Cornell University is helpful in understanding an operation as conceived by Piaget and also in clarifying the operations set forth in Figure 4:

To understand the development of knowledge, we must start with an idea which seems central to me—the idea of an operation. Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy, or image, of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge. For instance, an operation would consist of joining objects in a class, to construct a classification. Or an operation would consist of counting, or of measuring. In other words,
it is a set of actions modifying the object, and enabling
the knower to get at the structures of the transformation.

An operation is an interiorized action. But in
addition, it is a reversible action; that is, it can
take place in both directions, for instance, adding or
subtracting, joining or separating. So it is a par-
ticular type of action which makes up logical structures.

Above all, an operation is never isolated. It is
always linked to other operations, and as a result it
is always a part of a total structure. For instance,
a logical class does not exist in isolation; what exists
is the total structure of classification. An asym-
metrical relation does not exist in isolation. Seriation
is the natural, basic operational structure. A number
does not exist in isolation. What exists is the series
of numbers, which constitute a structure, an exceedingly
rich structure whose various properties have been revealed
by mathematicians.

These operational structures are what seem to me to
constitute the basis of knowledge, the natural psychological
reality, in terms of which we must understand the develop-
ment of knowledge. And the central problem of development
is to understand the formation, elaboration, organization,
and functioning of these structures.

I should like to review the stages of development
of these structures, not in any detail, but simply as
a reminder. I shall distinguish four main stages. The
first is a sensory-motor, pre-verbal stage, lasting
approximately the first 18 months of life. During
this stage is developed the practical knowledge which
constitutes the substructure of later representational
knowledge. An example is the construction of the schema
of the permanent object. For an infant, during the first
months, an object has no permanence. When it disappears
from the perceptual field it no longer exists. No attempt
is made to find it again. Later, the infant will try to
find it, and he will find it by localizing it spatially.
Consequently, along with the construction of the permanent
object there comes the construction of practical, or
sensory-motor, space. There is similarly the construction
of temporal succession, and of elementary sensory-motor
causality. In other words, there is a series of structures
which are indispensable for the structures of later repre-
sentational thought.

25
In a second stage, we have pre-operational representation—the beginnings of language, of the symbolic function, and therefore of thought, or representation. But at the level of representational thought, there must now be a reconstruction of all that was developed on the sensory-motor level. That is, the sensory-motor actions are not immediately translated into operations. In fact, during all this second period of pre-operational representations, there are as yet no operations as I defined this term a moment ago. Specifically, there is as yet no conservation which is the psychological criterion of the presence of reversible operations. For example, if we pour liquid from one glass to another of a different shape, the pre-operational child will think there is more in one than in the other. In the absence of operational reversibility, there is no conservation of quantity.

In a third stage the first operations appear, but I call these concrete operations because they operate on objects, and not yet on verbally expressed hypotheses. For example, there are the operations of classification, ordering, the construction of the idea of number, spatial and temporal operations, and all the fundamental operations of elementary logic of classes and relations, of elementary mathematics, of elementary geometry and even of elementary physics.

Finally, in the fourth stage, these operations are surpassed as the child reaches the level of what I call formal or hypothetic-deductive operations; that is, he can now reason on hypotheses, and not only on objects. He constructs new operations, operations of propositional logic, and not simply the operations of classes, relations, and numbers. He attains new structures which are on the one hand combinatorial, corresponding to what mathematicians call lattices; on the other hand, more complicated group structures. At the level of concrete operations, the operations apply within an immediate neighborhood; for instance, classification by successive inclusions. At the level of the combinatorial, however, the groups are much more mobile. These, then, are the four stages which we identify, whose formation we shall now attempt to explain [Piaget, 1964, pp. 8-10].

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It will be noted that Piaget's definition of operations is restrictive and does not include discriminating, acquiring labels or vocabulary, and cognizing the common properties of instances as operations. American psychologists tend to be more inclusive. The operations as outlined in the lower part of Figure 4 which are fundamental to later concept learning are now considered in some detail.

Bruner, Olver, and Greenfield (1966) outline the course of cognitive growth and give special attention to enactive representation. Early in life and prior to the development of speech, the child has many sensory experiences with a variety of objects and events. He also manipulates many objects. He eventually discriminates among these various objects and events and responds differently to them, although now with comprehensible words. Thus, many objects in the environment, e.g., ball, bottle, and mother; and physical dimensions; e.g., hot, rough, and sweet, which later will be the basis for classifying or conceptualizing, are discriminated and responded to differentially. At this young age, the discriminations are represented enactively rather than in words or in images.

As soon as the child develops comprehensible speech, labels or words are acquired and are associated with objects and events that have been discriminated, and further discriminations are made. This initial learning probably follows along the line of acquiring chunks of behavior through observation and imitation as proposed by Bandura and Walters (1963). That is, the young
child observes a parent or some other person call an object by a name and then imitates the person by calling the object the same name.

Later, the child acquires identity concepts. For example, a single instance of a ball is treated as a ball, regardless of the different viewpoints from which it is experienced. Thus, a many-colored ball that a child may play with is treated as a ball, regardless of whether the red or white side is viewed and regardless of whether it is seen only or seen and touched. Many other single objects in the child's environment are treated as identity, or concrete, concepts. (Bruner, et al. 1966, pp. 186-192, present an excellent treatment of this phenomenon.)

Still later, two balls that differ in color or size are responded to as if they belonged to the same class and are given the proper label. This is the lowest level of concept formation, corresponding to the S-R behavioristic definition of a concept as making a common response to dissimilar stimuli. At this level of concept mastery, the individual merely responds with the same label to two instances of the same class; he does not specify the basis of the classification.

It may be argued that labels are not essential to the learning of concepts and that when lower-form animals such as pigeons and rats, make the same response to two different stimuli, they have acquired concepts. No attempt is made here to refute either of the preceding propositions; however, G. Miller (1966) and Harré (1966) make a strong case for including the
label in the definition of the concept and both treat labeling as an important operation in conceptualizing. The point of view here is that most human beings learn to talk and one can readily determine whether a concept has been acquired and the level at which it has been acquired through communicating orally or in written form with human beings.

**Operations During Concept Learning**

At this point it may be well to recall the operations already discussed under attending to the situation and securing information. Cognizing the definitional basis of the concept population, discriminating the characteristics of the stimulus situation, and searching for the criterial attributes are the antecedents of the more direct operations that lead to attainment of the concept. The more direct operations as shown in Figure 4 are cognizing a relevant attribute or rule by comparing the information presented in two or more positive and negative instances; hypothesizing the concept; remembering information concerning instances, attributes, rules, and hypotheses; evaluating relations among attributes and rules; and inferring the concept. Each of these is now considered in more detail.

**Cognizing that instances do or do not belong to the same set.** Students may be informed in advance that instances do or do not belong to the same set, or concept; they may observe the instances and independently make this decision; or they may attend to the instances, make the decision, and then check against some criterion to verify the
decision. Cognizing, by any of these means, that instances which vary from one another do or do not belong to the same set was shown to facilitate the attainment of concepts (Fredrick, 1968).

In this experiment, subjects were given a test comprised of a series of short items. In each item an instance was presented and the subject was told that it was an instance of an undefined concept. This was followed with two positive instances, two negative instances, or a positive and a negative instance. In regard to the last or third instance, the subject was asked to respond in one of three ways: the last instance belonged to the same concept as the first, it did not belong to the same concept as the first, or its membership could not be determined due to lack of sufficient information. Subjects in the experiment who had high scores on this test attained concepts more efficiently than those who had low scores.

Cognizing a criterial attribute or rule by comparing the information presented in positive and negative instances. The same three experiments noted early in connection with the cognizing of the definitional basis and the structure of the concept population also provided evidence that cognizing a criterial attribute or a criterial rule (conjunctive or disjunctive) facilitated concept attainment.

Optimal instructions were formulated to enable subjects to cognize the attributes and the rules for joining the attributes and also to enable them to draw correct inferences from "yes" and
"no" instances which varied on only one attribute from the focus instance. The stimulus materials in all three experiments were geometric forms varying on five bivalued dimensions. Optimal instructions, in comparison with minimal instructions that clarified only task requirements, significantly facilitated concept attainment. From these experimental arrangements and results it was inferred that cognizing the criterial attributes and the disjunctive or conjunctive rules enabled the subjects to make more correct responses in classifying instances as belonging to the same conjunctive or disjunctive concepts.

**Hypothesizing the concept.** Adult subjects participated in three consecutive experiments designed to clarify hypothesizing behavior in concept learning (Klausmeier, Harris, Davis, Schwenn, & Frayer, 1968; Lynch, 1968). A hypothesis is the prediction of what the concept is and includes both the internal cognitive operation and the observable response manifested as a result of that hypothesizing.

In the first experiment the effects of two learning set orders were compared. Subjects received 24 four-trial concept learning problems, 18 outcome and 6 nonoutcome problems. Nonoutcome problems were systematically interspersed with outcome problems. The task consisted of identifying a two-attribute conjunctive concept from a series of four slides presented successively. Based on analysis of the responses to the nonoutcome problems, the major conclusions were:

1. Adult subjects offered hypotheses in a systematic predictable manner, apparently searching for the attribute that was the cue for correct responding.
(2) Certain attributes were initially hypothesized more frequently than others, apparently because of response sets, or preference for selecting certain attributes over others.

(3) Greater proficiency was attained on the first learning set than on the second; having tested and rejected a hypothesis during the first learning set, the probability was decreased that it would be retested.

In the second experiment an attempt was made, using the same experimental materials and procedure, to ascertain the effects of preexperimental training on learning set. Based on the analysis of nonoutcome problems, the major conclusions were: (1) Adult subjects offered hypotheses in a systematic, predictable manner. (2) Pretraining on a certain attribute increased the probability of that attribute being offered as the hypothesis on the first experimental nonoutcome problem; however, an already established response set for the attribute, color, outweighed the effect of pretraining on the attribute, form.

In the third experiment, verbal stimuli were used. The design was formulated to further clarify the effects noted in the first two experiments. Results indicated:

(1) The pattern of hypothesizing behavior did not vary significantly as a result of the specific dimension which was relevant.

(2) A series of more than eight problems was required to establish a learning set (i.e., significant increase in the probability of hypothesizing a particular dimension).
Development of a learning set increased the probability that the hypothesis relevant to it would be retested on subsequent problems.

Pretraining effects were strong but transitory; that is, the small number of reinforcements on the pretraining problem increased the probability that the hypothesis would be subsequently offered. However, nonreinforcement of the pretraining hypothesis and reinforcement of another hypothesis was associated with rapid extinction of the pretraining hypothesis.

In these studies, the subjects offered hypotheses in a systematic predictable pattern, this being related to informative feedback provided by the experimenter. When a subject was told that a hypothesized value was correct, he maintained the hypothesis on subsequent trials. When told it was incorrect, he offered a different value. In addition, the subject tended not to offer the incorrect hypothesis until all other hypotheses had been offered or tested. This conclusion is probably the most significant of the entire set in this project, confirming many informal observations that human beings of various ages actively search in a systematic manner for cues that enable them to categorize instances as belonging or not belonging to a set. It is the defining attributes or values searched for, not the instances, that are critical. The instances only carry the essential information.

Remembering information concerning instances, attributes, rules, and hypotheses. Students from introductory educational psychology classes participated in four consecutive experiments (Miller & Davis, 1968) that were designed to ascertain whether stimulus variables and other conditions that supposedly impeded concept attainment
by increasing the memory load actually resulted in lower retention. Variables manipulated in the experiments were concept complexity (one or three relevant attributes comprising the concept), method of presenting instances (simultaneous or successive presentation), stimulus exposure time (5, 10, or 15 seconds), and method of recall (unrestricted recall, in which the subjects were to recall the instances and categories—whether the instance was or was not a member of the concept—in the order presented in the experiment, and random recall, in which the subjects were to recall the instances and categories in a nonsequential random order fixed in advance by the experimenter). In the first three experiments the stimulus material consisted of four bivalued dimensions: shape (triangle or rectangle), number (one or two), color (red or blue), and size (large or small). In the fourth experiment two other dimensions were included: position (right or left) and orientation of figures (upright or tilted). The dependent variables were number of concepts identified, number of instance values recalled, and number of categories recalled.

The simultaneous method of presentation resulted in significantly better recall of instances in two of three experiments and better recall of categories in three of three experiments, as hypothesized. The unrestricted recall of instances and categories was significantly better in one of two experiments and in the same hypothesized direction in the other. Stimulus exposure time of 5 seconds produced significantly poorer retention of instances and categories in two of two experiments as hypothesized.
Complexity of the concept yielded mixed results in that in only one case was the three-relevant-attribute concept associated with significantly poorer recall. The other small differences generally were in the same hypothesized direction. The major contribution of these studies was to demonstrate that variables assumed to increase memory load were, in fact, associated with poorer retention scores. Moreover, the absolute level of recall under most conditions was sufficiently high to render questionable the limited memory assumption of various models of concept identification.

Tests of memory were administered to subjects and related to concept attainment in two factor-analytic studies (Lemke, Klausmeier, & Harris, 1967; Jones, 1968). In the first study, rote memory and span memory did not load on concept attainment factors. In the second study memory did load on concept attainment factors for low achievers but not for high achievers. Apparently memory is not a critical process in concept attainment when the concept population is simple (e.g., geometric forms of four bivalued dimensions) nor when the subject correctly categorizes instances on the basis of the relevant values. When the subject does not categorize the instances in more complex concept populations, the dimensions and values successively tested must be remembered as relatively discrete elements in order to eventually identify the values comprising the concept.

Evaluating relations among attributes and rules. In a factor-analytic study (Jones, 1968), six consecutive propositions were presented in written form and each was followed with a written statement
of a positive instance of the concept to be attained, a negative instance, or both. The material was designed as a model of difficult concept formation tasks appropriate for university students.

After studying the proposition and instances, the subject sorted test instances as belonging or not belonging to the concept. At the end of each of the six consecutive trials, a dependent measure was taken. Scores from 16 tests, two for each of eight abilities (Memory for Semantic Classes, Memory for Semantic Relations, Memory for Semantic Transformations, Induction, Syllogistic Reasoning, Cognition of Semantic Systems, Evaluation of Semantic Relations, and Cognition of Semantic Units), and six scores from different stages (trials) of the concept learning task were obtained from each of 102 female Ss enrolled in educational psychology. This total group was subsequently divided into two groups of higher achievers and lower achievers with Ns of 50 and 52. The division was based on the median number of errors on the sixth and last trial.

The derived orthogonal solution was obtained by Kaiser's normal varimax rotation procedure (1958). Six interpreted and one uninterpretable factor were identified. The six interpreted factors were Meaningful Memory, Within-Task "Practice," Verbal Comprehension, Early-Task "Practice," Reasoning, and Logical Reasoning. Of particular interest were the abilities associated with the task factors and the differences between higher achievers and lower achievers. The Within-Task factor showed significant loadings on all trials for higher achievers, for Trials 3-6 for
lower achievers. Memory test scores loaded on the Within-Task factor for the lower achievers but not for the higher achievers; whereas both inductive reasoning and cognizing semantic relations loaded on this factor for the higher, but not for the lower, achievers. Thus, after the first trial, the higher achievers were already cognizing the relationships among the propositions, instances, and the concept. This did not occur systematically in the loadings until the third trial for the lower achievers who apparently had to memorize instances and propositions rather than cognizing relationships and drawing correct inferences concerning class membership. In the Early-Task Factor, which was the best indicator of efficient learning, Evaluating Semantic Relations loaded heavily for both groups, suggesting that of importance was not only cognizing the relations among propositions and instances, but also evaluating them on the basis of the defining attributes of the concept.

Inferring the concept inductively or deductively. Individuals may cognize attributes that are common to instances, evaluate the information, and arrive at a concept inductively. Situations may also be arranged whereby individuals arrive at a concept by logical reasoning. At an oversimplified level this may be illustrated by the student knowing that a rhombus is an instance of the concept but that a parallelogram is not. On the basis of this information the individual properly infers that equal length of sides is a criterial attribute. No inference can be made as to whether number of sides is a criterial attribute. With only this information he further deduces that right triangles might not be a member of the concept but that all equilateral triangles might be. In general, logical reasoning to arrive at the concept occurs when
negative instances are compared with positive instances. Attributes which differ in positive and negative instances may be inferred to be defining attributes.

In another factor analytic study (Lemke, Klausmeier, & Harris, 1967), geometric stimuli were employed and the concepts to be attained were conjunctive of two or three values. In one condition the subject selected instances from an entire array and in the other condition only the instances needed to attain the concept were presented simultaneously. Scores from 16 tests, two for each of eight abilities (General Reasoning, Verbal Comprehension, Induction, Deduction, Spatial Scanning, Perceptual Speed, Rote Memory, and Span Memory) and 18 scores from concept-attainment and limited information-processing tasks were obtained from each of 94 female subjects enrolled in educational psychology. The 34 task and ability variables were intercorrelated, then factored using Alpha factor analysis. The 12 Alpha factors were rotated to an oblique solution according to the Harris-Kaiser (1964) criterion. Seven of the eight hypothesized ability factors were identified, the only exception being Perceptual Speed. Five factors associated with the tasks were identified: three concept-attainment and two limited information-processing factors. The intercorrelations of the factors showed that General Reasoning, Induction, and Verbal Comprehension (to a lesser extent) correlated positively with the three concept attainment factors. Thus, both induction and deduction were related to learning of the concepts.
Postconceptual Operations

Bruner, et al. (1956), indicated that acquisition of concepts leads to five other achievements. First, categorizing reduces the complexity of the individual's environment by making equivalent discriminably different objects and events. Second, categorizing enables one also to identify other similar things. Third, categorizing reduces the need for instance learning or relearning. This is closely related to the first. Having acquired a concept of baby, the individual does not need to learn that each new baby encountered may be put into the baby category. Fourth, categorizing provides direction for instrumental activity. For example, knowing that cooked beef is edible is to know in advance that other cooked beef, when encountered, can be eaten. Further, if it can be eaten, it probably has certain nutrients such as proteins, fats, vitamins, and minerals. In this way, knowing one criterial attribute that objects have in common leads to inference of additional attributes and directs appropriate behavior toward the object. Fifth, categorizing enables the individual to relate and order objects or events with regard to one another.

The postconceptual operations outlined in the upper part of Figure 4 refer essentially to use or transfer of the learned concept to new situations.

Generalizing to other instances of the newly acquired concept when encountered for the first time. When as a result of concept learning novel instances are correctly categorized, the first four effects cited by Bruner, et al., are achieved. For example, assume that a child has
first learned to call correctly a number of figures squares and that he also cognizes, but not necessarily states, that a square is a closed, plane, simple figure that has four sides of equal length and four angles of equal size. Having acquired a concept of square at this level should enable him to properly categorize other square figures that might be encountered and also to treat them as squares rather than as rectangles or other four-sided figures.

Cognizing other concepts as coordinate, superordinate, and subordinate. This operation is essential to the fifth achievement noted by Bruner, et al. (1956). Guilford (1967) has indicated that classes, or concepts, may be related to one another and in turn that the concepts and their relationships may be organized into systems. A child may have a fairly well established concept of square and rectangle and a beginning concept of quadrilateral and pentagon. Having these concepts should enable him to cognize more readily that pentagon is coordinate with quadrilateral, and square and rectangle are subordinate to quadrilateral.

Generalizing to other conceptualizing situations. Human beings have the ability to learn how to learn concepts. This phenomenon was observed routinely in a series of laboratory and school experiments (Klausmeier, Harris, Davis, Schwenn, & Frayer, 1968). In these experiments the students learned consecutive concepts of the same type but each one was different from the other. Both the amount of time required and the number of errors decreased as successive concepts were attained. Related to the geometric figures, we have
not done the experiments but would predict that the students would require less time to acquire each of these coordinate concepts consecutively—quadrilateral, triangle, and pentagon.

Transferring the concept to problem-solving activities. A principle is defined by Gagné (1966) as a relationship between two or more concepts. [In this discussion, principle and generalization are used synonymously.] A problem is encountered when one does not have a solution, a method, or both to deal with a situation that one must resolve. According to Gagné (1966) concepts are requisite for the cognizing of principles and the solution of problems. Again to illustrate with the geometric figures, a student may have learned the concept quadrilateral. This would enable him to more readily acquire the principle that the perimeter is the distance around a quadrilateral and to solve problems requiring determination of the perimeter of a given quadrilateral.

Thus, knowledge of a specific concept may transfer to a wide range of situations. Further research is required to specify in greater detail the operations entailed in this transfer.
Summary

This paper has proposed a model of cognitive processes in concept learning. In this model, global strategies in concept attainment are hypothesized to consist of three phases: attending to the situation, searching for information, and processing and using the information. Specific cognitive operations presumed to be entailed in each of these phases are identified.

Future research will be directed to testing and refining this model. It is anticipated, in turn, that the model will prove to be a powerful tool in guiding research on concept instruction.
References


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