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ABSTRACT

The University of California considers the faculty/student ratio a principal determinant of quality. Requests made by the University for new faculty positions are based on the student/faculty ratios proposed by the University in its 5-year program for each campus. Given the forecasts of student enrollments, these proposed ratios determine the required increase in faculty positions. What the ideal ratio is depends on the Regent's policy guidelines and these may change over the years. The difference between the University's request for new positions and the number approved by the state poses a difficult problem for the administration, and this paper addresses itself to that problem. It formulates and solves a mathematical model used to calculate lower and upper bounds on the number of new faculty positions allocated, over a finite planning horizon, to a multicampus educational institution. In this model the student/faculty ratios must meet certain growth rate restrictions imposed by the faculty and the administration. The initial student/faculty ratios, forecasts of student enrollments, and certain critical ratios are assumed known and given. (AF)

BOUNDS FOR NEW FACULTY POSITIONS IN A BUDGET PLAN

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UNIVERSITY OF CALIFORNIA

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PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

This paper formulates and solves a mathematical model used to calculate lower and upper bounds on the number of new faculty positions allocated, over a finite planning horizon, to a multi-campus educational institution. In this model the student/faculty ratios must meet certain growth rate restrictions imposed by the faculty and the administration. The initial student/faculty ratios, forecasts of student enrollments and certain critical ratios are assumed known and given.

The assistance of Professors Robert M. Oliver, Charles R. Glassey, and Richard C. Grinold in formulating this paper is greatly appreciated.

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I. INTRODUCTION

1. A Budgetary Planning Problem

In this paper we address a specific budgetary planning problem which exists at the University of California. In this University, as in many other organizations, administrators try to measure the achievements of the institution by various performance criteria other than total cost or profit. These measures may have the form of ratios such as: students per faculty member, library acquisitions per student, expenditures per hour of student's instruction, and many others. In certain cases accomplishments of the institution are then evaluated by trends or rates of change in these ratios.

One of these measures, namely the student/faculty ratio, has received a great deal of attention in the development of short and long range budgetary plans for institutions of higher education. For example, in his budget message[1] the President of the University of California recalled three expressions of policy which were made by the Regents of the University. The third, dated February 17, 1968, reads, "The Regents reaffirm that there should be no lessening of quality in teaching and research at the University of California, particularly as it might apply to the student/faculty ratio."

Following these guidelines we find the President's own statement that "the budget for Current Operations, 1970-71, has been prepared in accordance with these policy guidelines." Later in his message we find that "the number of faculty engaged in University service in relation to the numbers of students at the various levels of instruction is a principal determinant of qual-

[1] Office of the President, University of California, September 19, 1969.

ity. In the 1970-71 Budget we have projected additional faculty positions to preserve the numerical relationship between teachers and students which prevailed in 1967-68."[2]

The budgetary planning problem that we address in this study arose from the procedure which is followed in determining the allocation of funds for new budgeted faculty positions.

At the beginning of each year (usually during March) the Chancellors of the eight campuses of the University of California submit to the President's Office their tentative budgets for the next five years. Budgets are reviewed by the President's Office and the Program Review Board. They are combined into a five year university-wide proposed budget which is then submitted to and reviewed by the Regents. In September of the same year the proposed budgets are submitted to the Department of Finance of the State of California. The proposed budget represents the University's request for funds from the California State government. The Department of Finance reviews the proposed budget for the first year of the five year plan and negotiates various items in it with the University. These negotiations result in the Governor's budget message to the Legislature in January of the following year. The Legislature, which has to vote on the State's budget by the end of June because the state fiscal year begins July 1, may also impose changes in the total amount of funds allocated to the University. The amount approved by the Legislature is transferred to the University and becomes a part of its budget. Although the Legislature approves only the total allocation of funds to the University, implicitly it determines

[2] The use and discussion of student to faculty ratios is found in numerous places. See for example: Barzum [1968], Bowen [1968], Correa [1967], Harris [1962], Judy [1969], Keeney [1967], Knight, et al [1969], Oliver [1970], Radner [1968], Shepard [1965], Williams [1966], Zemach [1968].

the major items of the budget, including the number of budgeted faculty positions.

The process is repeated annually. Hence, any given year appears in five budgets, first as the last year of a five year fiscal program. Eventually it becomes the first one on which the state acts.[3]

Requests made by the University for new faculty positions are based on the student/faculty ratios proposed by the University in its five year program for each campus. Given the forecasts of student enrollments, these proposed ratios determine the required increase in faculty positions. The change in the value of these proposed ratios over time should be, for each campus, in accordance with the Regents' policy guidelines. The difference, however, between the request made by the University for new faculty positions and the number approved by the state poses a difficult problem for the University-wide administration: what is the minimal number of new faculty positions that the University needs, during the next five years, in order to meet the increase in student enrollment without violating the Regents' guidelines? How is this lower bound, for the number of new faculty positions, distributed over the five year period? The purpose of this study is to answer these questions.

As a first step, we describe mathematically the budgeting procedure outlined above. Then we introduce a set of restrictions on the annual variations and rates of change of the student/faculty ratios. These restrictions reflect Regential guidelines and faculty pressure for lower ratios on one hand, and the State's desire for more "productive" faculty, i.e., more students per faculty member, on the other hand.

[3]Starting with the budget submitted in September, 1969, a six-year program is now requested by the Department of Finance. It is now called, for example, 1970-71 Budget and Five Year Fiscal Program 1971-72 to 1975-76.

Given these restrictions, we find the smallest feasible increase in the number of faculty positions over the planning horizon. This will give a lower bound on the number of new faculty positions. If the allocations approved by the State are smaller than this lower bound, the restrictions imposed on the variations of student/faculty ratio will have to be violated.

Currently the actual allocations are close to these lower bounds. Yet it is also interesting to know what the upper bounds are on the increase in faculty positions subject to the restrictions mentioned above. While the lower bound is the least desirable allocation from the point of view of the University administration and faculty, the upper bound is the least desirable (i.e., most expensive) allocation from the State's point of view.

The restrictions on the variations of the student/faculty ratios, under which we compute the lower and upper bounds for new faculty positions, have a special structure. In the University of California the ratio of 28 weighted FTE students per FTE faculty position is used as a critical value in the following sense: If the ratio in a given year is below 28 then the State will not provide faculty positions to reduce this ratio in the next year. If the ratio is above 28, then the University Administration and faculty will prevent any additional increase in the ratio.[4] As stated, these constraints permit a sharp increase in the ratio if initially it is below 28, and a drastic decrease if it is above 28. This is undesirable, particularly in the case of a young campus with low initial ratio. To avoid this unrealistic property, we introduce in the model bounds on the magnitude of the changes in the ratios.

We are interested in planning for an entire five year program (or T year program in general); therefore, we have to compare the value of a new

[4] See for example: Office of the President [1969], Knight, et al [1969], and Office of the Chancellor [1969].

faculty position in various time periods. This is done in the traditional way by introducing a discount factor, α , where α is between zero and one. From the State's perspective, a new budgeted faculty position approved this year is more expensive than a position filled in the next year. From the University's point of view, a new position allocated for the current year is certainly better than a position promised to be opened in the future. Thus we are interested in the present value of the total number of new faculty positions available in the entire planning period. Analytically, we may write this $\sum_{T} \alpha^{t-1} y(t)$, where $y(t)$ is the number of new faculty positions opened in year t . The lower and upper bounds on the number of new faculty positions are the solutions for a corresponding minimization and maximization problem respectively, where the sum given above is the objective function. It should be emphasized that these bounds cannot be viewed as an optimal policy for the University or for the state because these bounds may have features which are not acceptable for other reasons. The actual allocation of new faculty positions will probably be chosen within the range determined by these bounds. How this choice is made is beyond the scope of this study.

The bounds for the number of new faculty positiosn depend on the changes in student enrollment. We assume that the forecasts of student enrollments are given for every campus in the University. Under this assumption, a decision about the number of new faculty positions is equivalent to a decision about the student/faculty ratios. In this case, the bounds on the increase in the number of faculty positions for the whole University are equal to the sum of the bounds for the individual campuses. We refer to this single campus faculty allocation problem as problem Q_1 . If, however, we assume that only the University-wide increase in enrollment is given, then the allocation of this increase among the campuses is an important factor in determining the level of the bounds on the number of new faculty positions. We refer to this multi-campus student and faculty allocation problem as problem Q .

As a conclusion for this introductory section we present an example of the requests for new faculty positions made by the University, and the allocations approved by the State.

Table 1 shows the enrollments of students and the number of faculty positions for the year 1968-69, the five year program proposed by the University for the years 1969-70 through 1973-74, and the budget approved by the State for 1969-70.

The State approved 178.40 new faculty positions for 1969-70, which is less than one-half of the 393.52 requested by the University. These differences are reflected in the individual campus allocations. For example, Berkeley requested 32 new positions in 1967-68, 52 in 1968-69, and 67 in 1969-70. The actual allocations were 0, 10, and 0, respectively.[5]

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[5] Office of the Chancellor, 1969, page 8.

TABLE 1: Students' Enrollment and Faculty Positions

1968-69 to 1973-74

	Weighted FTE Enrollment*	FTE Faculty Positions*	Student to Faculty Ratio
1. 1968-69 Budget	146,407	5,455.75	26.84
2. 1969-70 Proposed	155,099	5,849.27	26.52
3. 1970-71 Proposed	166,749	6,224.17	26.79
4. 1971-72 Proposed	176,743	6,532.28	27.06
5. 1972-73 Proposed	186,538	6,821.61	27.35
6. 1973-74 Proposed	196,909	7,157.76	27.51
7. 1969-70 Budget	155,107	5,634.15	27.53

* The transformation from headcount to full time equivalent (FTE) for students and faculty is described, for example, in Knight [1969] pp. 116-118. Total weighted students are the sum of the FTE enrollments at four levels multiplied by the following weights: 1.00 lower division, 1.50 upper division, 3.50 advanced doctoral, and 2.50 other graduate students.

Source: lines (1) - (6): Office of the President, [1968], p. 220
line (7): Office of the President, [1969], p.86.

2. A Mathematical Formulation of the Problem

In this section we present a mathematical formulation of our budgetary planning problem.

We divide the time interval under consideration, or the planning horizon, into equal periods (e.g., years) by a set of points (dates) $t = 0, 1, \ldots, T$. Furthermore, period t is the time interval $(t - 1, t]$. The campuses of the University are numerated by $i = 1, \ldots, n$.

Let

$w_i(t)$ = the number of students at campus i in period t

$x_i(t)$ = the number of faculty positions at campus i in period t

then

$$r_i(t) = \frac{w_i(t)}{r_i(t)} = \text{the student/faculty ratio at campus } i \text{ in period } t .$$

In particular

r^* = a given critical value of student/faculty ratio (e.g., $r^* = 28$ in the case of the University of California).

In the previous section we described how the Regents' policy guidelines and a pressure from the faculty, will prevent an increase in the student/faculty ratio if the current ratio is above r^*. The State, however, will not accept a reduction in the ratio which it views as being too small. The acceptable reduction may depend on the initial value of the ratio. Let

$G\left(r_i(t)\right)$ = the largest decrease in the student/faculty ratio of campus i that is allowed to occur in period $t + 1$, given $r_i(t) \geq r^*$.

These remarks are formulated as the following restriction:

$$\text{If} \quad r_i(t) > r^* \quad \text{then} \quad r_i(t) - G\{r_i(t)\} \leq r_i(t+1) \leq r_i(t) \quad .$$

Similarly if $r_i(t) < r^*$, then pressure from the state will prevent a further reduction of the student/faculty ratio at campus i in period $t + 1$. To avoid a sharp increase in the ratio, which will be objectionable to the University, we impose a limit to the increase of the ratio; this limit may be dependent upon $r_i(t)$. We define

$F\{r_i(t)\}$ = the largest increase in the ratio of campus i that is allowed to occur in period $t + 1$, given $r_i(t) \leq r^*$.

Thus, the second restriction on the variation of the ratio is

$$\text{If} \quad r_i(t) < r^* \quad \text{then} \quad r_i(t) \leq r_i(t+1) \leq r_i(t) + F\{r_i(t)\} \quad .$$

Finally, if the ratio at any given period is equal to r^*, then it is possible to change it either by increasing or decreasing it within the bounds mentioned above. In symbols,

$$\text{If} \quad r_i(t) = r^* \quad \text{then} \quad r^* - G(r^*) \leq r_i(t+1) \leq r^* + F(r^*) \quad .$$

The bounds G and F described above are assumed to be non-negative for all values of $r_i(t)$. Various types of bounds on the variation of the ratio may be considered, and one specific bound is used in the numerical examples.

We turn now to the problem of allocating the increase in student enrollment among n campuses. We begin by defining

$v_i(t) = w_i(t) - w_i(t - 1)$ = the change in the number of students at campus i in period t.

Two restrictions are imposed on the changes in student enrollments to reflect both continuing student demand and administrative policies. The first is that the enrollment changes will be non-negative for each campus in every period, namely

$$v_i(t) \geq 0 \ .$$

In other words, no campus will experience a decrease in total enrollment over time. The second restriction requires that the total enrollment in all n campuses be increased in period t by a predetermined non-negative amount, $b(t)$, which recognizes the increasing demand for admission generated by increasing population in the state. Analytically, this constraint is

$$\sum_{i=1}^{n} v_i(t) = b(t) \qquad .$$

Furthermore, for each campus we restrict the changes in the number of faculty positions to be non-negative. Therefore, if

$$y_i(t) = x_i(t) - x_i(t-1) = \text{the change in the number of faculty positions at campus } i \text{ in period } t$$

we require that $y_i(t) \geq 0$. In other words, faculty positions are not eliminated over time.

Subject to these constraints, we want to find lower and upper bounds for the discounted sum of new faculty positions over the planning horizon. To find these lower and upper bounds, we either minimize or maximize, respectively, the sum

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \alpha^{t-1} y_i(t)$$

where $0 \leq \alpha \leq 1$ is a discounting factor.

Mathematically, the multi-campus problem, denoted by Q, is

$$Q: \quad \text{Minimize or Maximize} \quad \sum_{i=1}^{n} \sum_{t=1}^{T} \alpha^{t-1} y_i(t) \tag{1}$$

subject to

$$\text{If} \quad r_i(t) < r^* \quad \text{then} \quad r_i(t) \leq r_i(t+1) \leq r_i(t) + F\big(r_i(t)\big) \tag{2}$$

$$\text{If} \quad r_i(t) = r^* \quad \text{then} \quad r^* - G(r^*) \leq r_i(t+1) \leq r^* + F(r^*) \tag{3}$$

$$\text{If} \quad r_i(t) > r^* \quad \text{then} \quad r_i(t) - G\big(r_i(t)\big) \leq r_i(t+1) \leq r_i(t) \tag{4}$$

$$\sum_{i=1}^{n} v_i(t) = b(t) \tag{5}$$

$$v_i(t) \geq 0 \tag{6}$$

$$y_i(t) \geq 0 \tag{7}$$

for all campuses $i = 1, \ldots, n$ and time period $t = 0, \ldots, T$; where $w_i(0)$ and $x_i(0)$ for $i = 1, \ldots, n$ and $b(t)$ for $t = 1, \ldots, T$ are given.

When the problem is solved for a single campus, i.e., $n = 1$, the subscript i may be dropped. Restriction (5) is now the identity $v(t) = b(t)$ and since by assumption $b(t) \geq 0$, restriction (6) may also be dropped. The single campus problem, denoted by Q_1 , is therefore

$$Q_1: \quad \text{Minimize or Maximize} \quad \sum_{t=1}^{T} \alpha^{t-1} y(t)$$

subject to

$$\text{If} \quad r(t) < r^* \quad \text{then} \quad r(t) \leq r(t+1) \leq r(t) + F\big(r(t)\big)$$

$$\text{If} \quad r(t) = r^* \quad \text{then} \quad r^* - G(r^*) \leq r(t+1) \leq r^* + F(r^*)$$

$$\text{If}\quad r(t) > r^* \quad \text{then} \quad r(t) - G\{r(t)\} \leq r(t+1) \leq r(t)$$

$$y(t) \geq 0$$

for all $t = 0, \ldots, T$.

In Chapter II we present the solution to the single campus problem, Q_1 , and in Chapter III the solution to the multi campus problem Q . Finally, Chapter IV discusses extensions and other applications of the model.

II. LOWER AND UPPER BOUNDS FOR THE NUMBER OF NEW FACULTY POSITIONS - SINGLE CAMPUS

In this chapter we present the numerical results for the lower and upper bounds in six campuses of the University of California. (The two new campuses of Irvine and Santa Cruz were not included.)

To solve problem Q_1 numerically, we have to specify several parameters and the functions $F(r)$ and $G(r)$. We choose the constraint on the increase of a ratio, F, to be

$$F\big(r(t)\big) = c + d\big(r^* - r(t)\big) \quad .$$

Thus, whenever the student/faculty ratio in period t is not greater than r^*, i.e., $r(t) \leq r^*$, the allowed increase in the ratio in the next period is the sum of two numbers. The first number is a constant, c, which is the same for all ratios less than r^*. The second number is a decreasing function of $r(t)$. For example, if $r^* = 28$, $c = 100$, and $d = 0.10$, then for $r(t) = 23$ and $r(t) = 27$ we have $F(23) = 1.00 + 0.50 = 1.50$ and $F(27) = 1.00 + 0.10 = 1.10$, respectively.

Similarly, the function chosen for G which applies when $r(t) \geq r^*$ is

$$G\big(r(t)\big) = c + d\big(r(t) - r^*\big) \quad .$$

In this section we describe in detail the solution for one campus, the University of California at San Diego. For the other campuses we give the lower and upper bounds on the number of new faculty positions. These bounds are compared with the requests for new positions made by the University in its five year fiscal program. The necessary data of forecasts of students' enrollment and initial number of faculty positions is given in Table 2.

The other parameters which have to be specified are the constraints r^* , α , c , and d for which the values chosen were:

Parameter	Value
r^*	28.0
α	0.9
c	1.0
d	0.1

The choice $r^* = 28.0$ reflects the current standards assumed by the University. The values of the other three parameters were chosen more arbitrarily, although they represent some limits accepted by both the University and the State's Department of Finance (see at the end of this chapter for a comment on the sensitivity of the bounds to changes in the values of these parameters).

Our first step in the construction of the network for the minimization problem (see Figure 1) is as follows:

$$r(0) = 27.17 < 28 = r^* \tag{1}$$

$$s(1) = \frac{w(1)}{x(0)} = \frac{7611}{237.35} = 32.07$$

$$\begin{aligned} r(0) + F\big(r(0)\big) &= r(0) + c + d\big(r^* - r(0)\big) \\ &= 27.17 + 1.00 + 0.10\ (28.00 - 27.17) = 28.25\ . \end{aligned}$$

Hence,

$$\begin{aligned} S(1) &= \mathrm{Min}\{s(1),\ r(0) + F\big(r(0)\big)\} \\ &= \mathrm{Min}\{32.07,\ 28{,}25\} = 28.25 > 28 = r^*\ . \end{aligned} \tag{2}$$

Table 2: Students' Enrollment and Faculty Positions - San Diego Campus

a.

Year	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74
t	0	1	2	3	4	5
Weighted FTE Students' Enrollment $w(t)$	6449	7611	9389	11124	12717	14585

b. FTE Faculty Positions in 1968-69: $x(0) = 237.35$

Student/Faculty Ratio in 1968-69: $r(0) = w(0)/x(0) = 27.17$

Source: Office of the President [1968], p. 220.

As shown in Halpern [1970], the structure of this problem can be transformed into the network shown in Figure 1. The number associated with each node is the student/faculty ratio for this node. The number along each arc is the length of the arc and represents the discounted increase in faculty positions. The shortest path from the source node at $t = 0$ to a sink node at $T = 5$ is the crossed path. The minimal (discounted) increase in the number of faculty positions over the five year planning horizon is equal to the length of the shortest path, 211.98.

The network generated to solve the maximization problem is presented in Figure 2. The longest path from the source to a sink node is the crossed path. Correspondingly, the maximal (discounted) increase in the number of faculty positions over the five year planning horizon is the length of this path, 243.25.

The results for all six campuses are presented in Table 3. For each campus the table shows the annual increases in the number of faculty positions associated with the lower and upper bounds. These increases are compared with the annual number of new faculty positions proposed by the University in its five year fiscal program.

To compare the University's proposal with the lower and upper bounds, we use the difference between the proposal and the lower bound as a percentage of the difference between the upper and lower bounds. Thus, we compute

$$P = 100 \cdot \frac{\text{Proposal} - \text{Lower Bound}}{\text{Upper Bound} - \text{Lower Bound}} .$$

These percentages are presented in Table 4.

The reason for the low percentages in Riverside and Santa Barbara is the initially low student/faculty ratios in these two campuses (23.07 and 25.09, respectively). Since we do not require that a low ratio will be increased

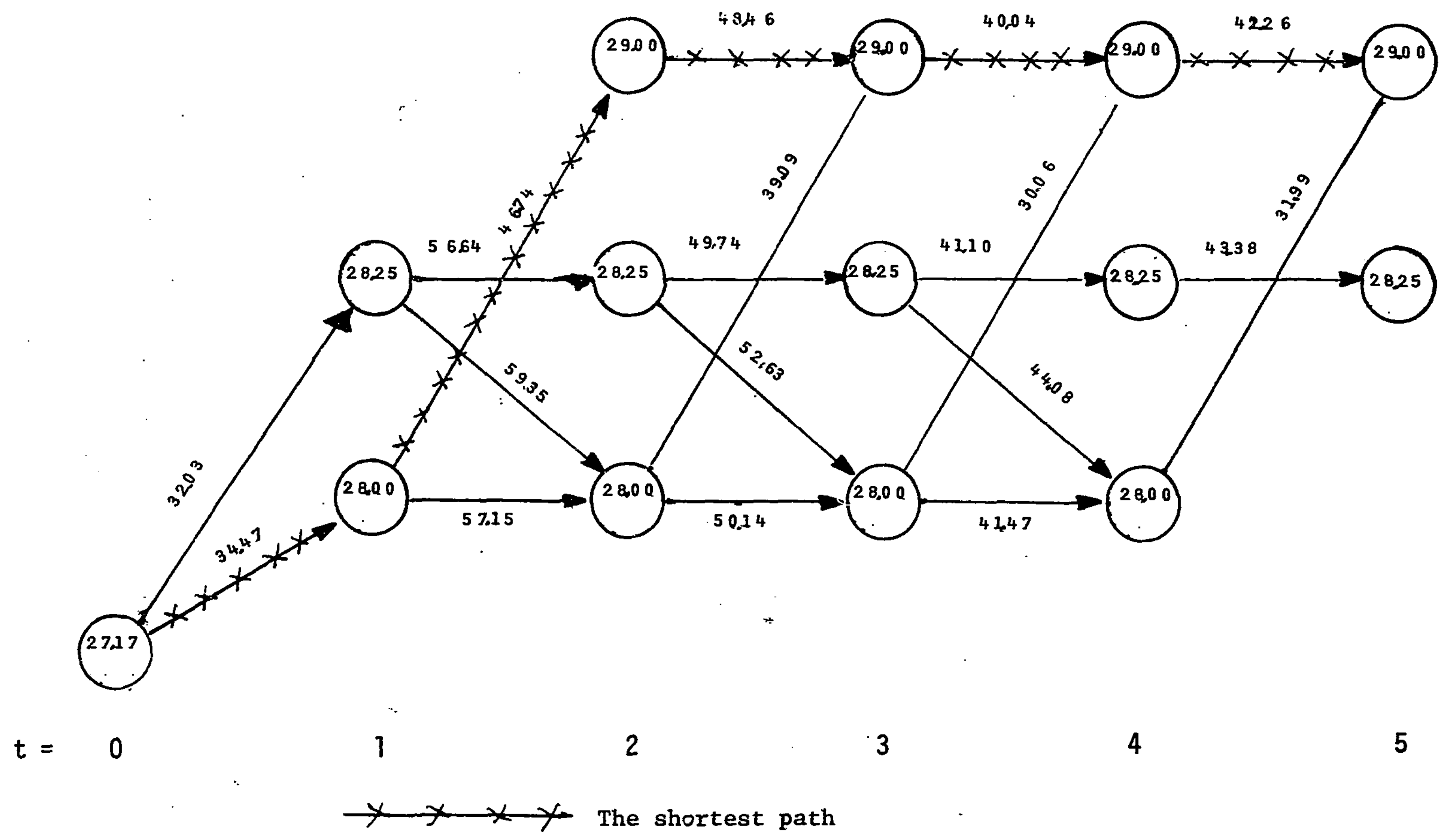

Figure 1: Network Presentation for the Minimization Problem Q_1 - San Diego Campus

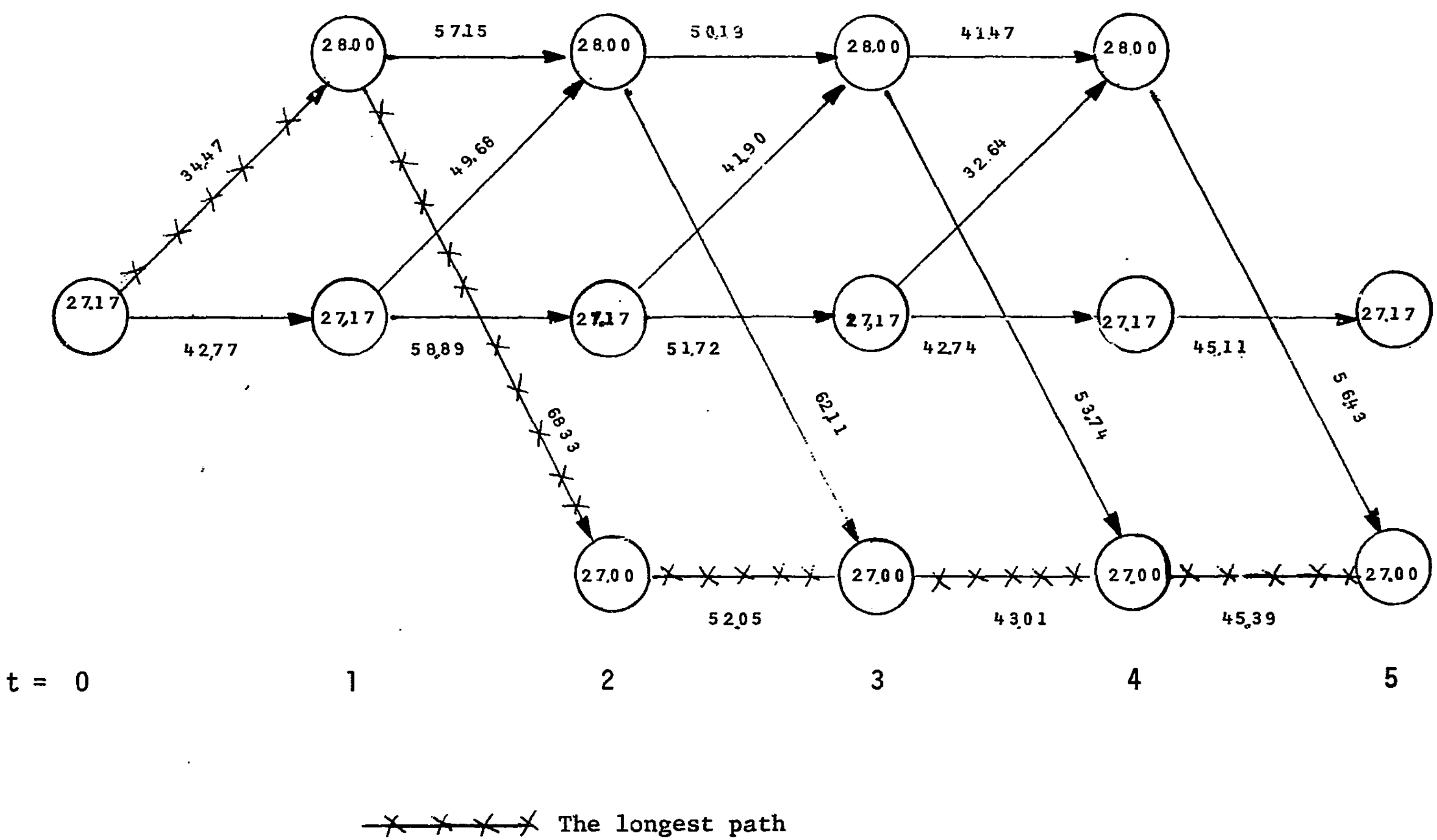

Figure 2: Network Presentation for the Maximization Problem Q_1 - San Diego Campus

Table 3: Annual New Faculty Positions for Lower and Upper Bounds and the University Proposal - Six Campuses

For 1969-70 to 1973-74

	1969-70	1970-71	1971-72	1972-73	1973-74	Direct Sum	Dis-Counted Sum
Berkeley							
Lower Bound	42.01	3.90	20.45	14.00	13.41	93.77	81.09
Upper Bound	42.01	136.11	21.96	15.04	14.41	229.53	202.72
Proposed*	42.01	67.65	21.18	14.50	13.89	159.23	139.73
Davis							
Lower Bound	44.14	26.16	50.34	47.79	55.79	224.22	179.91
Upper Bound	44.14	73.01	54.07	51.33	59.93	282.48	230.39
Proposed*	44.14	48.75	52.14	49.50	57.79	252.32	204.25
Los Angeles							
Lower Bound	96.30	-	50.50	-	20.21	167.01	150.47
Upper Bound	150.38	31.96	20.29	38.35	21.39	262.37	237.57
Proposed*	96.30	30.90	19.60	37.07	20.60	204.47	180.53
Riverside							
Lower Bound	9.93	20.82	22.48	25.24	35.00	113.47	88.24
Upper Bound	30.34	41.74	44.04	44.39	60.64	221.15	175.74
Proposed*	17.31	25.62	25.68	24.07	35.75	128.43	102.17
Santa Barbara							
Lower Bound	-	55.28	27.55	19.51	39.93	142.27	112.49
Upper Bound	21.20	93.74	59.19	53.93	46.15	274.21	223.11
Proposed*	16.60	50.40	39.80	24.35	41.36	172.51	139.08
San Diego							
Lower Bound	34.47	51.94	59.83	54.93	64.41	265.58	211.98
Upper Bound	34.47	75.92	64.26	59.00	9.19	302.84	243.25
Proposed*	67.09	56.68	50.88	42.18	66.71	283.54	233.83
SIX CAMPUS TOTAL							
Lower Bound	226.85	158.10	231.15	161.47	228.75	1006.32	824.18
Upper Bound	322.54	452.48	263.81	262.04	271.71	1572.58	1312.78
Proposed*	283.45	280.00	209.28	191.67	236.10	1200.50	999.60

*Source: Office of the President [1968], p. 220.

Table 4: The Difference Between the University Proposal and the Lower Bound as a Percentage of the Difference Between Upper and Lower Bounds

Campus	P
Berkeley	48.2
Davis	48.2
Los Angeles	34.5
Riverside	15.9
Santa Barbara	24.0
San Diego	69.9
SIX CAMPUS TOTAL	35.9

toward $r^* = 28$, the upper bound for these campuses is relatively high. It is reached by keeping the low ratio unchanged over the whole planning horizon. It is simple to add a constraint which will eliminate this possibility.

The values of the bounds depend on the choice of paramets, α , c , and d . At most campuses, the annual allocations of new positions, $y(t)$, which yields the lower and upper bounds are not very sensitive to changes in the value of α for values between 0.7 and 1.0. However, the present value of the new faculty positions, $\Sigma\alpha^{t-1}y(t)$, is changing with α even when $y(t)$ are not. In general the lower bound is a decreasing function of c and d , and the upper bound is an increasing function of c and d . Furthermore, if the initial ratio, $r(0)$, is within the range $(\underline{r}, \bar{r})$, where $\underline{r} = r^* - G(r^*) = r^* - c$ and $\bar{r} = r^* + F(r^*) = r^* + c$, then both bounds are sensitive to changes in c more than they are to changes in d . If $r(0)$ is significantly lower than r^* , then usually the upper bound will not be sensitive to c and d while the lower bound will be quite sensitive to changes in any of them. The situation is reversed when $r(0)$ is significantly higher than r^* . In Figure 3 the lower and upper bounds for San Diego are plotted as a function of c for given $d = 0.1$. Both bounds are not very sensitive to changes in d for a given c .

A further study is needed in order to present general results for the sensitivity of the bounds to changes in c and d for various mixtures of $r(0)$ and $w(t)$.

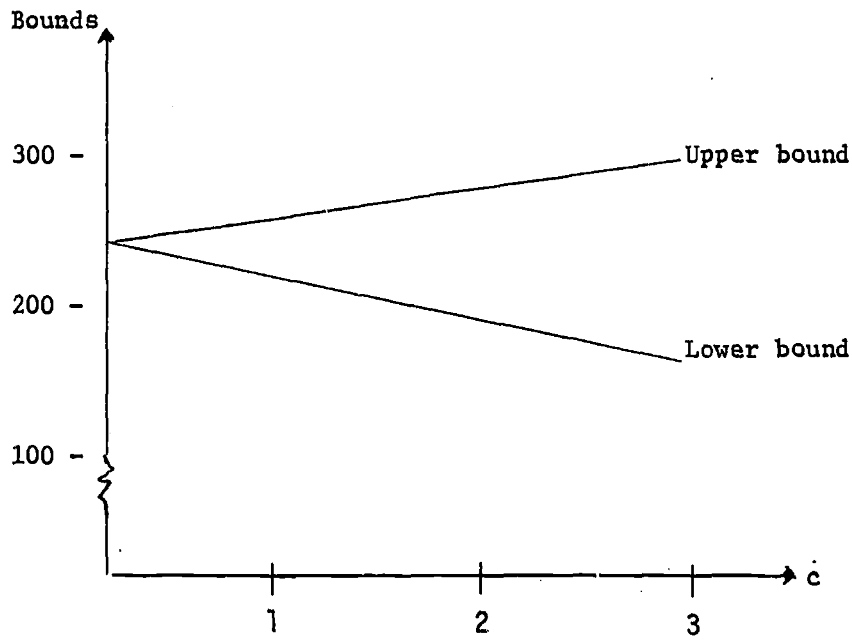

Figure 3: Lower and Upper Bounds as a Function of C - San Diego Campus

III. LOWER AND UPPER BOUNDS FOR THE NUMBER OF NEW FACULTY - A MULTI CAMPUS UNIVERSITY

In this chapter we present the numerical solution for the multi campus problem Q, where six campuses of the University of California are considered. The campuses are those for which the single campus problem, Q_1, was solved in Chapter II, namely, Berkeley, Davis, Los Angeles, Riverside, Santa Barbara, and San Diego. They are denoted by $i = 1, \ldots, 6$ respectively. The planning horizon, also the same as in Chapter II, covers the years 1968-69 through 1973-74 denoted by $t = 0, \ldots, 5$ respectively. The values of the parameters r^* and α and the bounds F and G are the same as those which were assumed in Chapter II for the single campus problem.

The necessary data are presented in Tables 5 and 6. Table 5 gives the FTE student enrollments, $w_i(0)$, and the FTE budgeted faculty positions, $x_i(0)$, at campus i in period $t = 0$. The increases in the total enrollment for the six campuses in period t, $b(t)$, are given in Table 6.

The network presentation of the solution of this problem is illustrated in Figure 4 (see Halpern [1970]). The shortest path from the source to the set of sinks, on the network, corresponds to an optimal solution to the minimization problem Q. This optimal solution is presented in Table 7. The length of the shortest path equals the lower bound for the number of new faculty positions.

In a similar way we solve the minimization problem Q in order to get an upper bound for the number of new faculty positions.

Table 5: Number of Students and Faculty, and Student/Faculty Ratios at Year $t = 0$

Campus	i	FTE Student Enrollments $w_i(0)$	FTE Budgeted Faculty Positions $x_i(0)$	Weighted Student/Faculty Ratio $r_i(0)$
Berkeley	1	49,994	1,738.95	28.75
Davis	2	16,162	562.18	28.75
Los Angeles	3	41,161	1,466.34	28.07
Riverside	4	7,045	305.39	23.07
Santa Barbara	5	17,672	704.35	25.09
San Diego	6	6,449	237.35	27.17

Table 6: Increases in Total Enrollment for the Six Campuses

t	1	2	3	4	5
$b(t)$	5,675	9,217	6,838	6,800	7,011

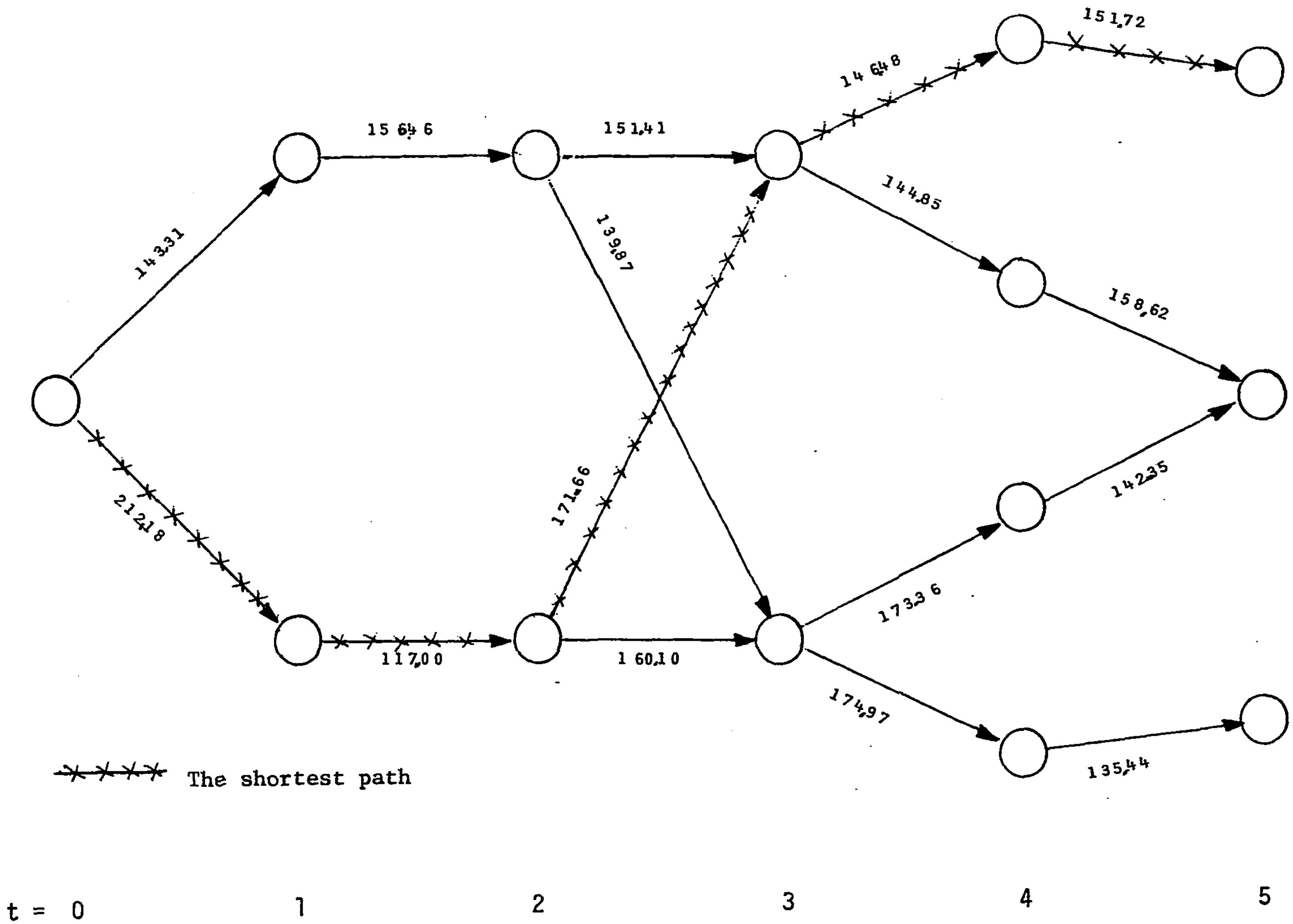

Figure 4: A Network Presentation of the Minimization Problem Q

Table 7: An Optimal Solution for the Minimization Problem Q

a. Optimal Allocation of the Increase in Enrollment

Campus	i	$v_i(1)$	$v_i(2)$	$v_i(3)$	$v_i(4)$	$v_i(5)$	$\sum_i v_i(t)$
Berkeley	1	---	5,556	---	---	---	5,556
Davis	2	---	557	6,146	---	---	6,723
Los Angeles	3	---	1,470	---	---	3,800	5,270
Riverside	4	456	411	370	269	305	1,811
Santa Barbara	5	909	819	322	704	2,906	5,660
San Diego	6	4,310	384	---	5,827	---	10,521
Six Campus Totals	$b(t)$	5,675	9,217	6,838	6,800	7,011	35,541

$v_i(t)$ = the increase in enrollment in campus i at period t.

Table 7 - Con't.

b. Optimal Allocation of New Faculty Positions

Campus	i	$y_i(1)$	$y_i(2)$	$y_i(3)$	$y_i(4)$	$y_i(5)$	$\sum_t y_i(t)$	$\sum_t \alpha^{t-1} y_i(t)$
Berkeley	1	46.55	130.00	----	----	----	176.55	163.55
Davis	2	15.03	----	211.93	----	----	226.96	186.69
Los Angeles	3	3.70	----	----	----	131.03	134.73	89.67
Riverside	4	----	----	----	----	----	----	----
Santa Barbara	5	----	----	----	----	----	----	----
San Diego	6	146.90	----	----	200.93	100.21	448.04	359.13
SIX CAMPUS TOTALS	Direct	212.18	130.00	211.93	200.93	231.24	986.28	
	Discounted	212.18	117.00	171.66	146.48	151.72		799.04

$y_i(t)$ = the increase in faculty in campus i at period t .

To conclude the numerical example we compare the lower bounds, upper bounds and the university budget proposal for the allocation of new budgeted faculty positions. The numbers which we collate are the discounted total increases in faculty over the five years 1969-70 through 1973-74. Two types of lower and upper bounds are presented. We call multi-min the lower bound for new faculty positions which may be reached when the redirection of the increase in student enrollment among the campuses is allowed (i.e., the solution for the minimization problem Q). This lower bound is denoted by $\underline{h}(Q)$. The solution to the minimization problem Q also gives us the discounted increase in faculty in each campus, $\underline{h}_i(Q)$, where $\underline{h}(Q) = \sum_i \underline{h}_i(Q)$. (The last column of Table 7.b gives the values of $\underline{h}_i(Q)$.)

Similarly, we use the term multi-max when we refer to the upper bound for new faculty positions when the redirection of students is allowed. This bound is the solution for the maximization problem Q and is denoted $\bar{h}(Q)$. The solution also provides us the distribution of the increase in faculty among the campuses, i.e., $\bar{h}(Q) = \sum_i \bar{h}_i(Q)$.

The second pair of lower and upper bounds is the solution for the six independent Q_1 type problems presented in Chapter II. In this case we assumed that no redirection of new students is allowed. The enrollment forecasts are given for each campus and are equal to those published in the 1969-70 Budget of the University. Here we use the term single-min for the lower bounds for new faculty, and denote it by $\underline{h}_i(Q_1)$. We define $\underline{h}(Q_1) = \sum_i \underline{h}_i(Q_1)$. Similarly, the upper bounds are called single-max and are denoted by $\bar{h}_i(Q_1)$. We define $\bar{h}(Q_1) = \sum_i \bar{h}_i(Q_1)$.

Finally, we call "proposed" the increase in the number of faculty positions which is the proposal presented by the University in its 1969-70 Budget. The proposed increase for campus i is denoted by P_i and we let $P = \sum_i P_i$.

Assuming that a proposal, P_i , should always be feasible, it is simple to establish the following inequalities:

$$\underline{h}_i(Q_1) \leq P_i \leq \bar{h}_i(Q_1) \qquad \text{for all } i = 1, \ldots, n$$

$$\underline{h}(Q) \leq \underline{h}(Q_1) \leq P \leq \bar{h}(Q_1) \leq \bar{h}(Q) \qquad .$$

Note that the inequalities $\underline{h}_i(Q) \leq \underline{h}_i(Q_1)$ and $\bar{h}_i(Q) \geq \bar{h}_i(Q_1)$ are not true in general.

The numerical results are given in Table 8.

Table 8: Lower and Upper Bounds for the Number of New Faculty Positions

CAMPUS	i	Multi-Min $\underline{h}_i(Q)$	Single-Min $\underline{h}_i(Q_i)$	Proposed P_i	Single-Max $\bar{h}_i(Q_i)$	Multi-Max $\bar{h}_i(Q)$
Berkeley	1	163.55	81.09	139.73	202.72	106.07
Davis	2	186.69	179.91	204.25	230.39	34.27
Los Angeles	3	89.67	150.47	180.53	237.57	54.76
Riverside	4	-----	88.24	102.17	175.74	1259.90
Santa Barbara	5	-----	112.49	139.08	223.11	-----
San Diego	6	359.13	211.98	233.83	243.25	-----
Six Campus Totals		799.04	824.18	999.60	1312.78	1455.00

IV. EXTENSIONS AND OTHER APPLICATIONS OF THE MODEL

In this chapter we describe some possible variations and extensions of the model. In a few cases, some minor modifications in our method of solution might be necessary. We do not give here the details of these required changes, if any. In other extensions a further study is necessary in order to modify our method to these cases. In the last section of the chapter we briefly remark about other possible uses of the model.

1. Alternative Types of Bounds on the Variation of the Ratios

In the numerical examples we used one possible bound on the variation of ratios, namely

$$\text{(i)} \quad F\big(r(t)\big) = c + d\big(r^* - r(t)\big)$$
$$G\big(r(t)\big) = c + d\big(r(t) - r^*\big) \quad .$$

We give here only three other possible bounds, and note that many more may be used.

$$\text{(ii)} \quad F\big(r(t)\big) = c/r(t)$$
$$G\big(r(t)\big) = r(t)/c$$

$$\text{(iii)} \quad F\big(r(t)\big) = G\big(r(t)\big) = cr(t)$$

$$\text{(iv)} \quad F\big(r(t)\big) = G\big(r(t)\big) = c$$

where c is a positive real number.

2. Ratios Converging Toward the Critical Ratio, r^*

One of the features of the optimal solution is that in the minimization problem, the trend of the optimal ratios is toward $\bar{r} = r^* + F(r^*)$, and in the maximization problem, toward $\underline{r} = r^* - G(r^*)$. Furthermore, if a ratio is equal to or greater than $\bar{r}$ it remains unchanged in the minimization problem and similarly, if it is equal to or less than $\underline{r}$ in the maximization problem. It is interesting to know how much it would cost, (i.e., how the lower bound is increased and the upper bound is decreased) if we try to force the ratios outside the interval $[\underline{r}, \bar{r}]$ to converge toward this interval, or ratios different from r^* to converge toward r^* . This inquiry may be answered by various methods. One, for example, may be to add restrictions of the type

$$\text{If } r(t) > \bar{r} \text{ then } r(t+1) \leq r(t) - \bar{H}(t)$$

and

$$\text{If } r(t) < \underline{r} \text{ then } r(t+1) \geq r(t) + \underline{H}(t)$$

where $\bar{H}(t)$ and $\underline{H}(t)$ may be of various forms.

Another possible way may be the following:

Let $c(t)$ be a penalty on having a ratio outside the interval $[\underline{r}. \bar{r}]$ at time period t . Let

$$z(t) = 0 \text{ if } r(t) \varepsilon [\underline{r}. \bar{r}]$$
$$= 1 \text{ otherwise.}$$

The objective function will now have the form

$$\text{Minimize } \sum_t \{\alpha^{t-1} y(t) + c(t) z(t)\} .$$

It is simple to show that the method of solution presented in Chapter II is useful in solving this type of problem.

Finally, we mention the possibility of imposing target ratios to be reached at the end of the planning horizon, i.e., we add the constraint $r(T) = r_0$.

3. Different Bounds and Critical Ratios for Different Campuses

In the model investigated in the previous chapters, we have assumed that the critical ratio r^* and the bounds on the variation of $r_i(t)$, i.e., $F(r_i(t))$ and $G(r_i(t))$, are the same for all campuses. We may relax this assumption and assume r_i^* , $F_i(r_i(t))$ and $G_i(r_i(t))$ for the i^{th} campus.

4. Different Bounds and Critical Ratios for Different Time Periods

Similarly to comment (3) above, we may choose to have changing critical ratios and bounds on variation of ratios over the planning horizon. For example, if $r(t)$ is the expenditures of instructional use of computer per student, we may choose to have the sequence of critical ratios, $r^*(t)$, with an upward trend over our planning horizon. Similarly, $F_t(r(t))$, $G_t(r(t))$, etc., may be time dependent.

5. Alternative Forms of Enrollment's Changes

When we formulated the problem in Chapter I, we have assumed that $b(t)$ are non-negative. We may relax this assumption and also include the possibility of $b(t) < 0$ for one or more time periods. This, however, requires us to give another set of constraints for $r(t)$ whenever $b(t) < 0$. For example, if $r(t) < r^*$ and $b(t + 1) < 0$, then if $y(t + 1) = 0$, we shall have $r(t + 1) < r(t)$. Is this possibility feasible - or not?

Another extension is to let $b(t)$ be a random variable whose real value is realized only after the decision about $y(t)$ was made. In this case we want to minimize the expected increase in faculty positions.

The last variation to be mentioned for $b(t)$ is to let $b(t) = \sum_i b_i(t)$. In the context of the student-faculty example, $b_i(t)$ is the number of new students in time period t who prefer to be enrolled at campus i . If p is the probability that a student will decline to be enrolled in a campus different than his preference, we may want to minimize the following objective function:

$$\lambda \sum_t \sum_i \alpha_1^{t-1} p\left(b_i(t) - v_i(t)\right)^+ + (1 - \lambda) \sum_t \sum_i \alpha_2^{t-1} y_i(t)$$

where

$$0 \leq \lambda \leq 1$$

and

$$(x)^+ = x \qquad \text{if } x \geq 0$$
$$= 0 \qquad \text{otherwise.}$$

Hence, $p\left(b_i(t) - v_i(t)\right)^+$ is the loss of students due to redirecting them to a campus which is not their choice. For $\lambda = 0$ or close to zero, the problem is the one which was solved in previous chapters. For $\lambda = 1$ or close to one, we shall allocate $b_i(t)$ to i^{th} campus, and then solve the problems by means shown in Chapter II. More investigation is needed to formulate a method for solving the problem for an arbitrary λ .

6. Different Types of Faculty, e.g., Faculty by Ranks

The last variation that we shall mention is that of considering $x(t)$ and $y(t)$ as vectors. We may consider faculty by ranks and let $x(t) = \left(x_1(t), \ldots, x_m(t)\right)$, $y(t) = \left(y_1(t), \ldots, y_m(t)\right)$ where

$j = 1, \ldots, m$ are ranks of faculty. If P is an m by m promotion matrix, then we have for the faculty flows the equation:

$$x(t) = x(t-1)P + y(t) \quad .$$

Our student/faculty ratio is defined by $r(t) = \frac{w(t)}{x(t)e}$ where $e^T = (1, \ldots, 1)$. Our objective function may be the same as before, namely

$$\text{Minimize} \sum_t \alpha^{t-1} y(t)e$$

or we may include a salaries vector $Q^T = (Q_1, \ldots, Q_m)$ and solve for the objective function

$$\text{Minimize} \sum_t \alpha^{t-1} y(t)Q \quad .$$

7. Other Applications of the Model

We have already noted that a situation such as that of student/faculty ratio, which led to the formulation of the model, may be found elsewhere in the educational system. Other ratios which are currently in use as performance criteria in the University of California are: undergraduate students to teaching assistants ratio; instructional support per student; total library volumes per student or per faculty member; students per library employee; instructional use of computer per student; and so forth.

In other social organizations, other ratios are widely used. For example, hospital administrators use the ratio of patients per doctor or per nurse, and the number of beds per room. In short, in many non-profit organizations whose output is service, we may find a similar conflict of interests with respect to the trends of certain performance criteria. In some of these cases, an estimate of the magnitude of the gap between the opposing points of view may be of use.

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