Projections of the size and composition of future student populations are crucial to effective planning in an institution of higher education. Using the mathematical model presented here, future enrollments and degree production can be predicted from admission, transfer, and attrition rates, as well as from administrative policies and current birth rates. Mathematical formulas and examples are included. (RA)
MODELS FOR UNIVERSITY SYSTEMS PLANNING

by

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ABSTRACT

The Pennsylvania State University system, with an enrollment of 50,000 students, is located on 22 campuses throughout the state with half the students on the main campus at University Park. The basic planning model developed for the system is a growth model of the logistic form. Status quo projections using current practices and procedures are used to provide the base for charting the future course, and deriving desired changes in objectives and policies. The paper also describes some of the key models used, including one that defines the flow of students into, within, and through the system, with degree production as a measure of output.
INTRODUCTION

Measures of the size and composition of future student populations are a basic requirement for planning in an institution of higher education. Both tactical and strategic assessments of resources require quantitative appraisals of enrollments over time. However, during the last several decades the demands for higher education have consistently outstripped available resources. Consequently, no matter what happened, enrollments could be structured in some way in order to make efficient use of resources. As a result the need for careful appraisal of future student populations was limited if not totally lacking.

However, the characteristic feature of the future which mankind knows with certainty is that the future is uncertain! Already straws are in the wind which are suggestive of shifts in student demand. For some years the absolute number of recorded births has been dropping and recently the impact of this trend has been felt at the primary school level. In higher education, the proportion of enrollment represented by women is rising. The choices exercised by students in the selection of their fields of study are shifting. Collectively, these and other symptoms suggest that significant changes may be anticipated in the number and characteristics of future populations of students. Such an environment will require a greater understanding of the variables and relationships among student progression than has been needed in the past.
THE EDUCATIONAL SYSTEM AS A FLOW PROCESS

The progression of students into, within, and from a system of higher education is essentially a flow process. Some of the variables which affect both sizes and characteristics are controlled by the institution itself, although others are completely external to it; many are influenced by events occurring during the actual period of enrollment while others take place before this period of time even begins. For example, the number of births affects the size of enrollments in later years in primary and secondary schools. Secondary school enrollment controls the output of high school graduates. The proportion of these graduates electing to continue their formal education influences admissions to colleges and universities. Admissions over sequential years, coupled with retention rates and the discipline area selected for study, control the levels of enrollments and the number of degrees awarded in subsequent years.

As a flow process the educational system is composed of a number of sequential steps through which students must pass. These steps may be defined in terms of the similar characteristics — such as age, grade-level, discipline of study, etc. — of the students temporarily residing in them. A unique set of common characteristics may be referred to as a "state". The interval of time to progress from one state to another is normally defined by the system and the progression of a group of students can usually be measured from historical observation. Thus if the size of a student population in a particular state is known, it is usually possible to estimate the size of the group reaching some other feasible state at a later date.
In order to estimate the size of a student population in a given state, some relative measure between its size and the magnitude of the population in some previous state is required. Various approaches have been used and reported upon for computing these measures. (1,2,3,4,5,6,7) However, in their simplest form, all may be classified within one of two general types and the choice between the two is usually dictated by the scope of historical observations that are available. The total scope of possible measurements which may be available on the progression of students between two feasible states is illustrated in Figure 1.
FIGURE 1

POSSIBLE MEASUREMENTS OF STUDENT FLOW

$S_{i,n-1}$ is the size of the student population in state $i$ measured at time $n-1$.

$S_{j,n}$ is the size of the student population in state $j$ measured at time $n$.

$S_{j(i),n}$ is the number of students from state $i$ at time $n-1$ moving to state $j$ at time $n$.

$N_{j,n}$ is the number of new students migrating into the system into state $j$ at time $n$.
If historical data are limited to the sizes of total student population, then a measure of progression between two successive states may be computed as a ratio of the size of the population in the later state to that of the earlier state as follows:

$$P_{ij} = \frac{S_{j,n}}{S_{i,n-1}}$$  \hspace{1cm} (i)

In this case projections of the sizes of populations in successive states over the normal interval of time required by the system for advancement may be computed as follows:

$$S_{j,t} = S_{i,t-1} \cdot P_{ij}$$  \hspace{1cm} (2)

where:
- $S_{j,t}$ is the size of the student population in state $j$ at time $t$
- $S_{i,t-1}$ is the size of the student population in state $i$ at time $t-1$
- $P_{ij}$ is the ratio of student populations in state $j$ to $i$

The use of this type of relationship for projecting future student populations has one deficiency which under certain circumstances may introduce serious discrepancies. Essentially when progression is simply measured as the ratio of two inventories of student populations, there is no assurance that all of the students in the later state were in reality in the previous state in the same system, since the effect of migration of students from other systems has been ignored. Thus measures of progression based upon populations carry the implicit assumption that any migration trends existing in the past will be continued in the future.
This deficiency can be overcome if the scope of historical measurements on the progression of students between two feasible states is such that the composition of the student populations may be identified with respect to origin. Again referring to Figure 1, the progression of students between states i and j may be measured as follows:

\[ f_{ij} = \frac{S_{j(i),n}}{S_{i,n-1}} \]  

(3)

In this case, projections, which take into account the effect of migration, may be computed as follows:

\[ S_{j,t} = S_{i,t-1} f_{ij} + N_{j,t} \]  

(4)

where: 
- \( f_{ij} \) is the transition ratio of students from state i to state j  
- \( N_{j,t} \) is the new student population migrating into the system in state j
MODEL DEVELOPMENT

In the previous section, it was shown how the size of a student population in a terminal state may be projected from the size of a known or previously estimated population in an origin state and a progression rate between the two states. In addition, it was shown that a progression rate may be developed from historical data by computing the

1. ratio of populations in successive states — the ratio of population in the terminal state to that in the origin state, or
2. transition rates of students between successive states — the fraction of those students in the origin state who move to the terminal state.

The choice between the two methods is usually dictated by the scope of historical data available for computing measures of progression. Application frequently necessitates the use of both. An example will be described to illustrate how the two may be employed in conjunction with one another. The example concerns the flow of students into, within, and from baccalaureate programs in The Pennsylvania State University. It should be noted, however, that applications of this type exhibit a tendency to problem specific and what may prove to be suitable for use in one institution may not be in another.
Entering Flow

The flow of students into the University may be divided into two parts. The first consists of high school graduates seeking admission as first-time freshmen; the second is comprised of students who have already attended an institution of higher education and seek admission with advanced-standing status.

The first of these two flows is of greater numerical importance. Since, on the basis of policy, the number of admissions granted to out-of-state students is limited, the level of freshman admissions in any one year may be related primarily to the number of students graduating from high schools within the Commonwealth of Pennsylvania during the same year. Projections of high school graduates are developed from the most recent census of primary and secondary enrollments which summarize for a given year the number of students in each grade level and the number of high school graduates. Projections of the flow from each grade level through graduation are developed from the census data for the most recent year.

\[ S_{j,t} = S_{i,n} P_{ij} \quad \text{for} \quad t = n+1, n+2, \ldots, n+12 \]  

(5)

where: 
- \( S_{j,t} \) is the population in the terminal state — high school graduates in year \( t \)
- \( S_{i,n} \) is the population in the origin state — enrollment by grade-level in primary and secondary school system recorded in year \( n \)
- \( P_{ij} \) is the progression rate — a particular grade-level to graduation from high school
In those cases where the census of school enrollment is not stratified by grade-level, or is incomplete, the number of high school graduates may be estimated from actual records of births or projected values based upon appropriate fertility rates in the actual female population between ages 14 and 44.

\[ S_{j,t} = S_{i,n} \cdot p_{ij} \quad \text{for } t = n+18 \]  \hspace{1cm} (6)

where:

- \( S_{j,t} \) is the population in the terminal state — high school graduates in year \( t \)
- \( S_{i,n} \) is the population in the origin state — births recorded in year \( n \)
- \( p_{ij} \) is the progression rate — birth to graduation from high school

Only a fraction of these high school graduates will elect to continue their formal education in a college or university. However, unlike the progression rates applicable to the primary and secondary school system, the proportion of high school graduates entering colleges has been increasing over time. Thus the total number of high school graduates electing to pursue a higher education may be projected from the following:

\[ S_{j,t} = S_{i,t} \cdot p_{ij,t} \]  \hspace{1cm} (7)

where:

- \( S_{j,t} \) is the population in the terminal state — high school graduates to college in year \( t \)
\( p_{ij,t} \) is the population in the origin state — high school graduates in year \( t \)

\( p_{ij,t} \) is the progression rate — high school graduates in year \( t \) to high school graduates electing college

Since the value of \( p_{ij} \) is dependent upon time \( t \)

\[
p_{ij,t} = p_{ij,n} + (t-n) \Delta p_{ij}
\]

where: \( p_{ij,n} \) is the progression rate recorded for year \( n \)

\( \Delta p_{ij} \) is the annual change in progression rate

The number of first-time freshmen admitted to the University from Pennsylvania is a fixed proportion of the number of high school graduates electing to go to college. In turn, the number of out-of-state applicants who are granted admission is limited to a specified fraction of the in-state freshmen. Thus total freshmen admissions may be determined from the following:

\[
f_{ij,t} = f_{ij} + N_{ij,t}
\]

where: \( f_{ij,t} \) is the population in the terminal state — first-time freshmen admissions to the University in year \( t \)

\( f_{ij} \) is the progression rate — Pennsylvania high school graduates electing college to Pennsylvania first-time freshmen admissions to University
\( N_{j,t} \) is the population migrating into the system in the terminal state — first-time freshmen admissions from out-of-state applicants to the University.

In turn the number of out-of-state freshmen admissions is computed as:

\[
N_{j,t} = q \ S_{i,t} \ f_{ij} \tag{10}
\]

where: \( q \) is the ratio of out-of-state to Pennsylvania admissions.

The number of students admitted with advanced standing status is related to the total number of freshmen admissions by a constant factor. Thus total admissions — freshmen and advanced-standing — may be computed as follows:

\[
\sum_{j=1}^{m} S_{j,t} = S_{i,t} \ f_{ij} + \sum_{j=1}^{m} N_{j,t} \tag{11}
\]

where: \( S_{j,t} \) is the population in the terminal state — total admissions for all academic achievement levels in year \( t \).

\( S_{i,t} \) is the population in the origin state — first-time admissions to the University in year \( t \).

\( f_{ij} \) is the progression rate, which in this case is simply 1.0.

\( N_{j,t} \) is the population migrating into the system in the terminal state — advanced standing admissions to the University.

In turn the advanced standing admissions may be computed as:

\[
\sum_{j=1}^{m} N_{j,t} = a \ S_{i,t} \ f_{ij} \tag{12}
\]

where: \( a \) is the ratio of advanced standing to freshmen admissions.
Internal Flow

The flow of students within the University is treated as a system separate and distinct from that concerned with the entering flow. The entering flow — admissions — is treated as a flow of students migrating from the "outside world" into the University. Projections of admissions by discipline and academic achievement level are then cycled through a Markov type model, together with existing enrollments, in order to generate distributions of students for sequential years. For this purpose the following type of relationship is employed:

\[ S_{j,t} = \sum_{i=1}^{m} S_{j(i), t-1} + N_{j,t} \quad \text{for } j = 1, 2, \ldots, m \]  

(13)

where:

- \( S_{j,t} \) is the population in the terminal state at time \( t \) — enrollment by discipline and academic achievement level
- \( S_{j(i), t-1} \) is the population in the origin state who move to the terminal state at time \( t \) — students enrolled in the University at time \( t-1 \) who move to a particular discipline and academic achievement level in time \( t \)
$N_{j,t}$ is the migration of population into the system in the terminal state at time $t$ — admissions of students by discipline and academic achievement level.

Since $S_{j(i)}$ represents the movement of students who are already enrolled in the University from an origin state ($i$) to a terminal state ($j$), transition ratios expressing the distribution of the students in state $i$ among state $j$ may be computed as follows:

$$f_{ij} = \frac{S_{j(i)}_{j, n}}{S_{i, n-1}}$$

where: $f_{ij}$ is the progression rate between states — fraction of students enrolled in state $i$ in time $n-1$ moving to state $j$ in time $n$.

Therefore, Equation 13 may be expressed as:

$$S_{j,t} = \sum_{i=1}^{m} f_{ij} S_{i, t-1} + N_{j,t}$$

The values assignable to $N_j$ — admissions — are influenced as far as disciplines are concerned by the trends in choices in the "outside world" and the proportion of male to female students admitted. In order to assess the impact of both time and sex upon initial choices of study, sets of historical time-series of admissions by discipline are prepared for male and female students separately. Each set is then normalized by total male and female admissions on a combined basis for each year in the series to provide two sets of "psuedo" historical time series. These two sets reflect the composition by discipline of newly admitted students if the student body were either 100% male or 100% female. Projections are then made using regression equations for the disciplines.
within each set and these are normalized by using the total admission levels projected from Equation 11. Corresponding disciplines within each of the two sets are combined on the basis of the proportion of male and female students to be admitted.

\[ N_{j,t} = N_{j,t}^* w + N_{j,t}^{**} (1-w) \]  

where:  
- \( N_{j,t}^* \) is the number of projected admissions in state \( j \) at time \( t \) if all students were male  
- \( N_{j,t}^{**} \) is the number of projected admissions in state \( j \) at time \( t \) if all students were female  
- \( w \) is the fraction of admissions represented by male students

**External Flow**

Basically the flow of students from the University consists of those who are awarded degrees and those who simply leave without completing the degree requirements. It is the first of these two which is of primary concern.

At this point some explanation is in order. Obviously for projecting the number of degrees awarded by discipline, a relationship similar to that employed for enrollments — Equation 15 — would seem to be in order. However, from a practical standpoint such an approach proved to be inadequate due to data limitations which could not provide relationships to take into account degrees awarded in dual majors, students awarded degrees after completing their studies elsewhere, and degree requirements extending beyond the normal fourth year of baccalaureate study. As a result the number of
degrees awarded are computed from the combined fourth and fifth year enrollment for each discipline in the following manner:

\[ S_{j,t} = \sum_{i=1}^{5} S_{i,t} P_{ij} \]  

(17)

where:
- \( S_{j,t} \) is the population in the terminal state in time \( t \) — degrees awarded in one discipline area
- \( S_{i,t} \) is the population in the origin state in time \( t \) — enrollments in fourth and fifth academic achievement level in the same discipline
- \( p_{ij} \) is the progression rate between origin and terminal states

The progression rate, \( p_{ij} \), is determined on an empirical basis from historical data.

**APPLICATION**

Utilizing relationships of the types described, it is possible to develop projections of student flow which, by introducing the currently applicable values for variables under institutional control, reflect continuance of existing policies and practices. On the other hand, by changing any one or more of these variables, projections can be generated which take into account the impact of the changes.

In Table 1 through 3 are shown sample projections for each of the three sequential phases of the application.
<table>
<thead>
<tr>
<th>Year</th>
<th>Pa. High School Grads.</th>
<th>Grads. To College</th>
<th>Total Freshmen Admissions</th>
<th>Total Advanced Standing Admissions</th>
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<td>189,000</td>
<td>94,000</td>
<td>10,593</td>
<td>1,377</td>
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<tr>
<td>1973</td>
<td>187,000</td>
<td>95,000</td>
<td>10,706</td>
<td>1,391</td>
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<tr>
<td>1974</td>
<td>194,000</td>
<td>102,000</td>
<td>11,495</td>
<td>1,494</td>
</tr>
<tr>
<td>Year</td>
<td>Academic Level</td>
<td>Enrollment by Discipline</td>
<td>Total Enrollment</td>
<td></td>
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<td>--------------------------</td>
<td>------------------</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>#1</td>
<td>#2</td>
<td>#12</td>
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<td></td>
<td>4</td>
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<td>921</td>
<td>459</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>38</td>
<td>75</td>
<td>18</td>
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TABLE 3
EXTERNAL FLOW

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<tr>
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<th>Degrees by Discipline</th>
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<td>930</td>
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<td>1,107</td>
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<tr>
<td>1974</td>
<td>287</td>
<td>1,112</td>
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</table>
FIGURE 2
SCHEMATIC DIAGRAM OF STUDENT FLOW

OUT-OF-STATE PRIMARY AND SECONDARY SCHOOLS

ADVANCED STANDING APPLICANTS

IN-STATE PRIMARY AND SECONDARY SCHOOLS

Grade Level
7 8 9 10 11 12 G

Academic Level

Discipline
1 2 3 4

UNIVERSITY ADMISSIONS

Academic Level

Discipline
1 2 3 4

UNIVERSITY ENROLLMENT
GRADUATES
REFERENCES


2: Connell, J.L., "A Proposed Model for Predicting Student Flows through a University", Project G.U.M. Phase III, University of Texas, Austin, November 1969


