Included are papers presented at the University of Illinois Colloquium on the Preparation of Teacher Educators in Mathematics. The first paper, "On the Preparation of the Teacher of Teachers" by B. Othanel Smith, examines the prominence of research in the teacher training programs. The second, "Source and Supply of Leadership in Mathematics Education" by Howard F. Fehr, discusses the characteristics of leaders and the tasks of future leaders in mathematics education. The last, "Preparation for Effective Mathematics Teaching in the Inner City" by Irvin K. Feinstein, outlines what should be the mathematical and professional background of those teachers working with inner city children. Following each of these three papers are two response papers written by other mathematics educators. (RS)
The TTT (Training of Teachers of Teachers) Program at the University of Illinois has been made possible through a grant from the United States Office of Education under the Education Professions Development Act.
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"EDUCATION FOR EFFECTIVE MATHEMATICS TEACHING IN THE INNER CITY"

Irwin K. Feinstein, Professor of Mathematics, University of Illinois at Chicago Circle

Discussants:
Mrs. Lucile B. Jautl, Mathematics Consultant, Chicago City Schools
M. Dennis Maher, Consultant, Department of Curriculum, Chicago City Schools
FOREWORD

The current realigning of priorities in education has been a welcome one indeed. Teacher education for too long has been low man on higher education's totem pole. And the Training of Teachers of Teachers (TTT) section of the Education Professions Development Act has provided a welcome impetus in directions in which some teacher educators have been eager to move. Peculiar to the Act is an insistence on the generation of unique and imaginative projects which commit unprecedented amounts of energy and talent from the school, the college and the community to the task of upgrading the quality of professional input into the classrooms of educational systems for the 1970's.

The 1969 University of Illinois Colloquium on the Preparation of Teacher Educators in Mathematics, an outgrowth of our TTT mathematics program, was addressed to three broad areas: provision of training in professional (as opposed to academic) competencies for teachers; leadership needs in mathematics education for the coming decade; particular requirements for the preparation of teachers for effective service in the inner city. We were indeed fortunate to be able to obtain papers on the three major problem areas by these distinguished spokesmen, respectively: B. Othanel Smith, Professor of History and Philosophy of Education, University of Illinois, Urbana, Illinois; Howard F. Fehr, Professor of the Teaching of Mathematics, Teachers College, Columbia University; Irwin K. Feinstein, Professor of Mathematics, University of Illinois at Chicago Circle.

The colloquium was held with two primary purposes in mind; (1) to formulate some well defined statements which would provide appropriate guidelines for mathematics teacher education for this decade and (2) to gather together representatives of the teacher education community so that deliberation and comment on the points of view presented might be possible. The attendance of nearly one hundred mathematics educators from the midwest and the resulting open and enthusiastic
dialogue attested to the attainment of the latter goal. The degree to which the former goal was realized is left for the reader's judgment, and will in the last analysis be known only when this decade is itself history.

The production of a written record of such a conference can only result, of course, from the hard work and cooperation of many people. To this end, debts of gratitude are due to James R. Lockwood and Garth E. Runion, doctoral students in mathematics education at the University of Illinois and to Mrs. Pat Roseberry and Mrs. Pat Dunkeld of the TTT secretarial staff.

January 1970
Urbana, Illinois

Kenneth J. Travers
Editor and Colloquium Coordinator
INTRODUCTORY REMARKS
J. Myron Atkin, Dean
College of Education

I think you are convened, as I understand from your program, for a purpose that is very close to the heart of a good many concerns on campus these days. We have not had a proud history, I think, in finding connections between the world of research, the world of scholarship, and the world of practice. I do not know that people who teach often feel that what we do under the label of research in education bears much relationship with what they see themselves needing to do in school classrooms. And there is a considerable sentiment on this campus at least that we need somehow to find a way to bridge the gap.

Presumably if we have some talents in a university community it is in the area of generating new knowledge. To what extent does this new knowledge relate to the kinds of jobs that need to be done around the state, around the nation? I myself hold a rather pessimistic view that if we do not find ways of making more clear how our activities relate to the work of professionals in the field, then other ways will be found to train professionals than by going to universities. Either that, or the work of the university that supposedly relates to the kind of practice they engage in will be reduced to a bare minimum.

I debated with myself about whether or not to try to launch into this serious note and expose my concerns, or just stay on the more formal note of welcoming you officially on the part of the university. So I decided to do both. You are most sincerely welcome on the campus. I think that you are convened for a most serious purpose and I think that the program in which you are engaged, the Triple T effort, represents one of the few activities that I know of that is trying to meet this rather serious problem of university connections with social problems that are broadly seen by many people in the nation. I am glad you are here.
We regret that the presentation made by Professor Smith at the colloquium was not available to us in time for inclusion in the present version of this report. The following paper is instead the original one given earlier by Professor Smith and contains essentially the main thrust of his talk last October.
ON THE PREPARATION OF THE TEACHER OF TEACHERS

B. Othanel Smith
University of Illinois

The expression "teacher of teachers" is ambiguous. Since we are interested in the preparation of those who teach the teachers, it is well that we make an effort to clarify the meaning of this expression. We shall use it to refer to (a) the teachers in the common school who supervise and instruct student teachers, (b) the supervisors and directors of instruction who give in-service training, and (c) the college instructors, irrespective of departments, who provide initial preparation for the teacher. We may refer to these persons individually or collectively as teacher trainers, teacher educators, or teacher of teachers, depending upon our semantic taste. It is this group whose preparation we are to discuss.

The college instructors who educate and train the teacher of teachers, especially the supervisors and directors of instruction and the college teachers, are the graduate faculties in education and in nonpedagogical departments such as English, history, fine arts, and so on. It is the work of these faculties that we are primarily concerned about when we discuss the preparation of the teacher of teachers.

Of course, these groups are not as clear cut as I have made them appear, for there is some overlapping of personnel. But functionally they are fairly distinct.

1 I have used the term 'train' to refer either to cognitive learning or to repetitive learning or skill and sometimes to both. This will irritate some of you, but I hope not too many. I think the referent is evident in most cases. To have done otherwise would have made the discourse more burdensome than it is.

2 Throughout this paper I have used the term "nonpedagogical" instead of "academic" because the latter term is associated with the disciplines of the liberal arts and sciences. We then tend to overlook the disciplines of music, art, physical education, agriculture, etc., when we use it. The term "nonpedagogical," it is hoped, will not prove to be more confusing.

This paper was prepared for the Tripli-T Project, Midwest Area, Michigan State University, East Lansing, Michigan, February 26-29, 1968.
Until recently the task of preparing those persons who train teachers received very little attention. In his book entitled The Teacher of Teachers, Harold Rugg tried to focus attention upon this task almost two decades ago. But his voice went unheeded if not actually unheard. This lack of sensitivity to the problem can be attributed in large measure to the fact that a great deal of the energy and resources of the universities and colleges has been spent in the task of training teachers at the preservice level. It can be attributed even more to preoccupation of graduate faculties with research and to the belief that the teacher's deficiencies are remedial and can be adequately corrected through institutes, extension work, and other forms of in-service training. As a result, we have spent much of our time and energy in trying to clean up the water down stream while it was being polluted at its source faster than we could purify it. In this Triple-T Project we now have the opportunity to direct some of our time and energy to the task of cleaning up the source.

Preparation Centered in Graduate College

The preparation of the teacher of teachers came, by circumstances we are not at the moment interested in, to be located in the graduate college. Inconsequence, the graduate frame of mind has shaped the program for the preparation of those who were to prepare teachers. The tenacity with which the graduate mentality has held on to the program is clearly seen in the title given to the few departments and schools of education that have acquired a measure of autonomy. They chose to call themselves graduate schools of education. And even the program leading to the professional degree of doctor of education was never quite pulled off. This so-called professional degree turned out to be a research degree in the same sense as the doctor of philosophy with the same trappings—preliminary examinations, theses, and other appropriate rituals. And, of course, the nonpedagogical departments, being even more insensitive to the task, made little or no effort to gear their programs to the preparation of teacher-training personnel.
The time when the training of the teacher of teachers might have been placed in a professional school has passed. Perhaps there never was a time when it would have been wise to do so anyway. But be that as it may, the fact is that the job of preparing the teacher of teachers is now located in the graduate college where it will remain. The problem of improving the program for the preparation of the teacher of teachers therefore centers in the graduate colleges of the universities.

To clarify still further who the teacher of teachers is and who is accountable for his education and training, we note that the undergraduate teacher-training program is made up of two components: pedagogical and nonpedagogical. Just as the preservice program consists of these two components so does the program for the preparation of the teacher of teachers. The question naturally arises as to where one component ends and the other begins. We thus face the perennial controversy over what should be the role of the nonpedagogical faculties, on the one hand, and the pedagogical faculties, on the other.

It is apparent that nonpedagogical faculties sometimes think that they have something to contribute to the pedagogical knowledge and skill of the teacher trainer as well as the teacher. But just what this contribution consists in has never been clearly identified and described. By the same token, the pedagogical faculties have sometimes claimed that they should deal with nonpedagogical content relevant to the work of the teacher as well as the teacher of teachers. But again just what this claim consists in and just what the evidence is in support of it has never been made clear. If there is to be conjoint development of the program for educating the teacher of teachers, the responsibilities of the various faculties of the graduate college must be worked out to the mutual satisfaction of the various faculties. To let this matter go unattended, or to settle it on the basis of institutional politics, is an inadequate response and the program will itself be weakened thereby.
Disciplines and Their Functions

What is ultimately needed is a criterion by which to decide who is to do what. It is hazardous to suggest a criterion for this purpose, but that is what I am about to do. I do so because it seems necessary at this point for further consideration of the problem. It goes without saying that the nonpedagogical graduate faculties are basically responsible for the education of the teachers of teachers in their disciplines when their disciplines are outside of pedagogy. But what pedagogical training are these graduate faculties supposed to give? That is the issue. It is here suggested that they provide instruction in those aspects of pedagogical knowledge and skill that are rooted in, or derived from, their own disciplines per se. What these are I shall attempt to indicate in a few moments. It seems equally evident that the pedagogical faculties should be held accountable for the training of the teachers of teachers in their particular discipline; that is, in pedagogical skills and knowledge that are derived from research in teaching, in learning, in school operations, and in the school as an institution.

Since the role of the graduate faculties goes back to their disciplines, it is appropriate that we ask what is meant by the term "discipline" and how a discipline is related to schooling. Like most terms, this one is defined in several ways. Perhaps the most inclusive and useful definition for our purpose is this: a discipline is an area of inquiry the boundaries of which can be somewhat specified and for which there is a body of knowledge and techniques for exploring the area and for correcting and expanding the body of knowledge. This definition holds for such diverse areas as history, music, physics, engineering, psychology, sociology, pedagogy, and so on.

I am indebted to S. L. Smith for this definition and some of the ideas that follow it. See Identification and Analysis of Problems Concerning the Study of Education in Australian Universities, by S. L. Smith, University of Sydney, 1967 (unpublished doctor's dissertation).
A discipline serves at least three instructional functions. The first of these is the preparation of research workers in the discipline itself. Most of the graduate programs leading to advanced degrees are designed to prepare students to till their respective areas and to explore new territory. It is not surprising that such programs as these dominate the interests of graduate faculties, for it is through research that their disciplines grow in power and prestige. The second instructional function served by the disciplines is to impart information to those who wish simply to gain knowledge of the disciplines as a matter of general education. Undergraduate programs, especially in the disciplines of the liberal arts and sciences, serve this purpose even though they may not be designed to do so in some cases. The third is fulfilled by programs of instruction that use the disciplines to prepare persons who are to perform a social function such as the production of food, maintenance of communication, reduction of ignorance, and healing of the sick. The occupations of persons who perform these social functions are typically referred to as professions.

We are concerned in the Triple-T Project with the third of these instructional purposes as it relates to the production of professional workers in the field of education. But our interest runs up against the plain fact that graduate faculties have shown little, if any, interest in the use of their disciplines for this purpose. It should be emphasized that this lack of interest is not confined to the nonpedagogical faculties. For perhaps there is no group more oblivious to the problem of using their discipline to train teachers than the graduate faculties of Education. Like the faculties of universities in the Commonwealth of Nations, our graduate faculties of pedagogy have been, and continue to be, preoccupied with the preparation of research workers in their discipline. Almost every individual who is awarded either a doctor of education or a doctor of philosophy degree is prepared in the techniques and procedures of investigating his field, and he is led to think
that this activity is the most important one for him to pursue. The fact that comparatively few individuals so schooled engage in productive research is no evidence to the contrary. These individuals, while they are trained to do research and to prepare papers for publication, typically find themselves employed in institutions where they are responsible for training teachers at the preservice level. While thus engaged they try to carry on research or to write articles and books that will help them to escape from this assignment as early as possible and to climb to the level of the graduate faculty where they themselves can engage in the process of preparing individuals to do research who will in turn be employed to train teachers.

This is the state of affairs in the discipline of Education and it is to be found with equal emphasis in almost every discipline offered in the universities. The purpose here is not to depreciate in the least the importance of research in any field whatever. It is through research activities that our knowledge increases and that the performance of social functions is ultimately improved. No one can deny the fact that it is through research that the practice of medicine, agriculture, engineering, and teaching have all been significantly improved in this century. But it is important to note that the knowledge of a discipline can be used for some purpose other than the production of further knowledge. And it is this point that we wish to make here in connection with the training of the teacher of teachers.

**Nonpedagogical Disciplines and the Preparation of the Teacher of Teachers**

Those of us assembled in this room occupy different positions in the educational scheme. Some of us come from the public schools. Others of us are primarily concerned with the training of teachers at the preservice level. And still others are members of graduate faculties in either pedagogical or nonpedagogical departments and schools. The question naturally arises as to the role which each of us should play in the improvement of programs for the preparation of the teacher
of teachers. What I shall attempt to do therefore is to identify and discuss some of the basic tasks in the preparation of the teacher of teachers, and, in the course of doing so, I shall suggest roles which each of us may appropriately play in the performance of these tasks.

As we move to identify these tasks an important question naturally arises. What difference, if any, is there between an instructional program for the preparation of research workers and one for the preparation of teacher trainers. This is a crucial question, for if there is no difference, the distinctions which have been made thus far fall flat. The answer to this question is not far to seek. The first difference is the fact that the knowledge to be taught to the person who is to prepare teachers does not depend for its significance upon its contribution to investigations in the discipline, but rather upon its use in teaching. This is true of the nonpedagogical disciplines no less than the discipline of pedagogy itself.

To see this point, we have only to begin with what the teacher in the common school tells us and then feel our way back to the disciplines. The teacher is constantly faced by the question, repeatedly put to him by his pupils, of the usefulness of the subject matter of history, mathematics, biology, or what have you. It makes a great deal of difference to a pupil whether or not a teacher can give satisfactory answers to questions of this order. For it is the nature of motivations in our time that the individual tends to reduce his load of knowledge to that which he believes is most useful to him as a citizen and a worker. We may decry this state of affairs, but we are talking about the fact, not our desires.

The teacher who has been taught the usefulness of knowledge in his particular discipline has an advantage in reducing the sense of futility in his pupils. To illustrate, we shall refer to the discipline of history. The history teacher in the common schools is often asked: Why do we have to study that? What good is it? The teacher, no less than the subject of history, is at a decided disadvantage in
the teaching game unless he is able to show with clarity and with a ring of truth that what is being taught helps in some significant way to understand contemporary events. He must be able to come up with something better than the shopworn answer: "We study history to understand the present." The day when the pupil will accept this sort of vacuous answer has passed, if indeed it was ever so. Today the pupil may demand that he be given specific cases in which the understanding of an event is markedly enhanced by having knowledge of its historical antecedents, and he may also want to know just how one determines the antecedents that are relevant to a particular event. If he is keen, the pupil may want to know further whether or not what is said about the applicability of historical knowledge rests upon questions of sampling and the validity of inductions and deductions. These observations about the uses of history can be made, in a different fashion, of course, about any and every subject offered in the common schools. The problem boils down to the humanization of knowledge not only in the sense in which James Harvey Robinson spoke of it but also in a more thoroughgoing fashion.

Persons who are to train teachers must themselves have this humanistic command of their own disciplines. And they are not apt to have it if the graduate faculties who educate the teacher trainers are not themselves able to handle the content of their disciplines humanistically. It is a sad commentary on the state of the disciplines that no concerted effort has been made on the part of the masters of these disciplines to humanize the content. When we have properly related the knowledge of the nonpedagogical disciplines, especially those which make up the liberal arts and sciences, to human interests, values, and activities, we will have taken a long step toward the creation of a more humanistic school system and perhaps a more humane society.

The task of showing how the nonpedagogical disciplines are related to human interest is a major responsibility of the graduate faculties in those disciplines in cooperation with elementary and high school teachers and others who are directly
engaged in teacher training. What is required is a pragmatics of knowledge that takes full account of the experience and interests of individuals in different age groups and with varying experiential backgrounds. There are generic uses of knowledge, but their specific application to the disciplines has not been worked out for pedagogical purposes. It would not be too difficult to do so.

A second difference between the instructional program provided for those who are going to prepare the teacher of teachers and those who are going to pursue research activities is to be found in the fact that the subject matter of the disciplines has a pedagogical aspect. Sometime ago I recorded and transcribed the classroom discourse of some twenty high school teachers representing the conventional content subjects. About three pages of their classroom discourse was given back to them and each teacher was asked to identify the places in the discourse where he was dealing with definitions, values, facts, and the like. The ability of these teachers to identify the places at which they were dealing with these types of information was not any greater than that of individuals of comparable education who had not been trained to teach. This bit of information tells something significant about the content preparation of the teacher. Illustration after illustration could be given to show that the teacher's ignorance of these elements of content leads him into pitfall after pitfall as he teaches.

From the pedagogical standpoint each discipline is a mixture of different forms of knowledge. There is in every discipline a set of concepts, and most of the disciplines contain rules and laws or law-like statements comprised of combinations of concepts. In addition, a considerable number of the disciplines contain value statements. It is very important pedagogically for the teacher to be aware of these different knowledge forms, because studies of learning and teaching have shown that the way in which each of these forms of content is taught and learned is different one from another. It is equally clear that preparation in the non-pedagogical disciplines does not now enable a teacher to identify these elements
of content and to relate teaching behavior appropriately to their requirements.
What is here being said is that the graduate faculties in the nonpedagogical fields, and I might add in pedagogy as well, have simply failed to analyze the content of instruction into its pedagogically significant elements. My observations of instruction in the common schools lead me to believe that teachers who are ignorant of these different elements do not know how to handle their content in ways that surpass common sense performance. How can we expect a teacher to handle deliberately and with skill an element of knowledge which he cannot identify and the structure of which he does not know and which is as foreign to his experience as the stars?

The graduate faculties in the nonpedagogical disciplines are responsible for the analysis of their content into pedagogically significant elements of knowledge. They are responsible also for the structural analysis of these elements.

There is still another pedagogical dimension of these disciplines. Each discipline not only contains elements of knowledge such as definitions and cause-effect relationships but also they involve such operations as defining, explaining, and justifying. It would seem highly desirable that the teacher of teachers be thoroughly trained in the performance of these operations as well as in the structural analysis of them. For these operations are performed in all classrooms, though poorly. Instruction of teachers in the structure and performance of these is of primary importance if they are to understand their content more deeply, to manipulate it so as to bring out its various aspects, and to treat it with clarity and rigor. The teacher of teachers in the discipline of history, for example, who treats the law-like statements, typically bootlegged into instruction, as if they were to be taken at face value, is simply laying the groundwork for loose thinking by teachers in the common schools. It is through an understanding of these operations and skill in analyzing them that the teacher of teachers comes to understand the structure of his discipline and thereby able to prepare teachers who understand their field of instruction in a more adequate way.
We turn now to the pedagogical faculties and ask what their responsibilities are in the preparation of the teacher of teachers. We recall that their role is fixed by the discipline of pedagogy. They are to instruct the teacher of teachers in the knowledge and techniques derived from research in pedagogy itself. The content of the discipline of pedagogy, however, is to be treated not in its relationship to research but to the social function the school is designed to serve.

It goes without saying that the program of preparation for the teacher of teachers must include systematic work in various aspects of pedagogy such as human development, learning, deviant behavior, curriculum development, sociology of education, philosophy of education, and other aspects. He need not, and indeed cannot, be a specialist in any of these. But he must be familiar with their basic concepts and principles because these are the tools that the teacher trainer uses as he interprets and modifies the behavior of his students, who are prospective teachers, and thereby trains them to do likewise with their own pupils.

But if we stop at this point, his pedagogical training would not be qualitatively different from that of the research worker. In addition to his mastery of concepts and principles, the teacher of teachers must be prepared to carry on the job of training teachers. His area of specialization is teacher training. To perform this job is to know the classroom work of the teacher—what he does from day to day, from moment to moment; it is to know the structure and dynamics of teaching behavior; to know how to direct the prospective teacher as he improves his teaching behavior; it is to know what the teacher does outside the classroom in the school and community; and to be able to shape the prospective teacher's behavior to the requirements of these extraclassroom duties.

The role of the graduate faculty in pedagogy is to prepare the teacher of teachers to give this sort of training. This role can be clarified still further by identifying some of the tasks which the teacher of teachers must train the
prospective teacher to do. The tasks that the classroom teacher performs and for which the teacher of teachers must prepare him, and for which the graduate faculty must in turn prepare the teacher of teachers to give training in, may be classified in the following way:

1. Classroom tasks
   1.1 Instructional tasks
      1.11 Input operations
      1.12 Diagnostic operations
      1.13 Reinforcement operations
      1.14 Assessment of pupil learning
   1.2 Tasks of classroom management and control
      1.21 Physical aggression
      1.22 Peer affinity
      1.23 Attention seeking
      1.24 Challenge of authority
      1.25 Critical dissension

2. Extraclassroom tasks
   2.1 Collective tasks
      2.11 Salary committees
      2.12 Grievance committees
      2.13 Academic freedom committees
   2.2 Staffing tasks
      2.21 Staffing deviant cases
      2.22 Staffing learning problem cases

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2.3 Interviewing tasks
   2.31 Parents
   2.32 Pupils
   2.33 Principals and supervisors

2.4 Programming tasks
   2.41 Curriculum planning
   2.42 Instructional planning (team work)

2.5 Community tasks
   2.51 Special interest groups
   2.52 Other community groups—parent-teacher, open board meetings, etc.

These categories are shaped in keeping with the work of the classroom teacher. Were we considering the training of administrators, counselors, school psychologists, supervisors, and so on, we would come up with other categories, although they would in all probability be somewhat similar. These tasks provide a basis for selecting the content of a teacher-training program and for deciding in turn what the teacher of teachers is to be taught as well as what he is to be trained to do.

The logic of a task-oriented program in pedagogy leads to the conclusion that the training of teachers, and inconsequence the training of the teacher of teachers, is to be carried on by examining tasks and by practicing appropriate ways of dealing with them. To follow this mode of training is to meet certain conditions as to the materials of instruction and their use. The first condition is an adequate supply of protocol materials such as tape and video tape recordings of classroom behavior, committee meetings, small conferences, interviews, and the like, supplemented with pen sketches of situations not available in other forms. Furthermore, simulations of various types of situations must be introduced as well as actual teaching situations ranging in complexity from those that involve four or five pupils to a full size classroom group.
The second condition is that these materials be analyzed and classified for instructional purposes. Protocol materials are now being used in teacher training, but with less effectiveness than is potentially possible, because they are used in the rough without proper analysis and without grouping them in relation to the objective of instruction. Taken as a running picture of what goes on in a classroom, committee meeting, and so forth, a tape recording or video tape contains all sorts of tasks. Where one task ends and another begins is not easily detected. To use protocol materials effectively requires that they be analyzed into segments or units corresponding to input tasks, diagnostic tasks, reinforcement tasks, and management and control tasks. Input tasks in turn would be analyzed into subsets. Corresponding to each of these subsets would be ample units of protocol materials selected to emphasize different aspects of the input process. Similarly protocol units would be selected for subsets of other task categories—diagnostic, reinforcement, and management and control. Furthermore, the fields of instruction at different grade levels would be represented in the protocol materials, and these materials would also be representative of classroom work in schools of culturally disadvantaged areas, middle-class rural communities, rural slums, suburban areas, and the like. The protocols would also represent work in classrooms that give heuristic as well as didactic instruction.

The third condition is that each set of materials be analyzed for instructional purposes. That which is to be taught about the behavior exemplified in these materials as well as the ends to be achieved through the use of them must be decided prior to the beginning of instruction. This point is so obvious that it need not be made were it not that such materials are used even today in program after program with only a superficial notion of what they exemplify or the lesson they may be used to teach. It should go without saying that no instruction should begin until the instructors have analyzed and studied the materials with far more care than they would require of their students.
Suppose that the protocol materials have been collected, classified, and analyzed. How are they to be used? The answer is not far to seek, for they are the objects, not the means of study. In themselves they afford little, if any, content. They are simply samples of behavior to be examined, reflected upon and assessed. But with what cognitive resources are they to be studied? This brings us up short to the question: What is the content of pedagogical training and where does it come from? Suffice it to say here that it obviously comes from the discipline of pedagogy and from its theoretical underpinnings taken from the disciplines of psychology, sociology, logic, and epistemology. The time does not permit me to explore the question of what this content is or how it should be organized and taught. What I have said about protocol materials makes it clear, however, that only that content which is demonstrably useful in interpreting and performing such tasks as I have suggested be included in the preservice curriculum.

Now to train the teachers of teachers is partly to require that they themselves participate in the preservice training program. In other words, the laboratory as well as the source of protocol material for the training of the teacher of teachers is the program of teacher training. In the same sense the laboratory and source of protocols for the preservice training of the teacher is the program of the common school.

A Cooperative Venture

It may appear from what I have said thus far that the graduate faculties are the whole cheese. But this is not the case. A program for the training of the teacher of teachers must be designed and carried out conjointly by the graduate and undergraduate faculties and the personnel of the common school. The graduate faculty can neither design the program nor carry it on alone. The graduate faculty in all the disciplines, including pedagogy, is isolated. Its members know little about teaching in the common school, and the faculty of the common school is equally removed from the work of the graduate faculty. The undergraduate faculty which
gives the bulk of preservice training is suspended between the two with little support from either side. Inconsequence the current teacher-training program typically consists of nonpedagogical courses in the prospective teacher's field of specialization plus a few courses in pedagogy, capped with student teaching. The courses are shaped by the requirements of the research orientation of the graduate faculties and are designed and taught with little or no reference to the usefulness of their content in teaching. The program for training the teacher of teachers, if indeed it exists at all, suffers from the same compartmentalization and is influenced even less by the practical necessities of either the common school or the training program. The only way to break this system of compartmentalization and isolation is to develop a mechanism for bringing these three faculties together to consider their common problem, to work out a plan for dealing with it and for sharing in the development and execution of the plan.

Some Issues

As we move toward such a conjoint effort some issues are certain to arise. One of these, and perhaps the most difficult to resolve, is the question of who shall be permitted to give graduate work. The gates to most graduate colleges are guarded by intellectual angels with flaming swords that turn in every direction 'to keep the way of the tree of life.' This is the way it is, and I do not wish to see the angels removed from the gate. But as the graduate college recognizes the relationship of its disciplines to such basic social functions as education and revamps its programs and degrees, will it also make appropriate modifications in the criteria for deciding who shall be permitted to give professional courses at the graduate level. Would it be letting down the bars were outstanding competence in the performance of professional work be allowed as a criterion? Would that criterion, rigorously enforced, be as adequate for professional programs as outstanding competence in research and scholarly production is for research programs?
Another issue arises from the fact that not all the college personnel involved in the training of the teacher of teachers is in either pedagogical or nonpedagogical departments. They are distributed throughout many departments of the university. This distribution of personnel means that each teacher of teachers must be prepared to fit the requirements of his respective department. He must master his own discipline in addition to preparing himself for his role as a teacher trainer. If he is to be a member of a department of English, for example, and share in teacher education, he will be able to take only minor work in pedagogy. The same is true of students in all other nonpedagogical disciplines. By the same token, if the student is majoring in pedagogy to become a teacher of teachers, he will be able to take only minor work in the nonpedagogical fields. The problem of deciding what work and how much of it should constitute minors in each of these cases is of great importance. Should the prevailing conception of the minor, shaped by the research mentality, be altered? Is there justification for a more balanced program as between the major and the minor for the teacher of teachers?

Still another issue is that of how programs for the training of the teacher of teachers are to be administered? If the design and execution of these programs is to be the business of each discipline or department taken separately, it seems likely that little, if anything, will ever be accomplished. It would seem that the dimensions of the administrative machinery should be consonant with the dimensions of the program. This principle suggests that perhaps something like a graduate college council on the education of college teachers including the teacher of teachers might serve as the machinery through which departments and schools could coordinate their programs.

The final issue I wish to open up is that of where the emphasis in the preparation of the teacher of teachers is to be placed. The answer to this question, like all the others, will, of course, be given by each team assembled here. A team
can decide to spend its time upon the development of a program to prepare directors of instruction to give on-the-job training and to prepare the classroom teachers who are to direct student teachers and interns. Or a team may choose to place its bets upon the preparation of the college teacher of teachers. Or again a team may decide to formulate a program that includes both of these alternatives or elements of them. I believe that in any case the general questions I have raised, and the analysis I have made, will be applicable. The point of decision is that of choosing the target group at which your program is to be aimed.

In closing I am moved to observe that the universities of the middle west award approximately half of the doctor’s degrees in this country. These schools have tremendous resources. I firmly believe that with a little determination and cooperation with the common schools these universities, in a very short time, could have a profound impact upon pedagogical practices at all levels of the educational system.
COMMENTS ON B. O. SMITH'S PAPER
"ON THE PREPARATION OF THE TEACHER OF TEACHERS"

Max Beberman, University of Illinois, Urbana

I find myself in the rather embarrassing position of being asked to give a rebuttal to a talk that I discover that I agree with. I read very carefully the printed talk and struggled to find something wrong with it. Then when I saw that Bunnie was not going to read it, I thought perhaps there was another talk coming that I could disagree with. It was in a sense another talk and I can't disagree with that either. I think Bunnie often does this—takes hold of some problem and really lays it out. And although he doesn't offer us too much in the way of solutions, except gigantic projects at the end, the problem is there for us to look at.

I'm faced with one or two problems. I keep thinking of the people in this room and what their role is in the training of teachers. Some of us teach mathematics courses in mathematics departments and that means probably we can do some of the things that Bunnie is calling for with respect to analyzing content of the subject and even trying to humanize the subject. Then there are other people here who teach methods courses. And one wonders what that would consist of. Is this the course where you take a look at protocols of classroom episodes and analyze the behavior of teachers? Or is this the course where you try to humanize mathematics? And so on. Perhaps if we could build a university from scratch, we might be in a much better position to assign these various functions to various parts of the university. That is, in our math department, we will take care of humanizing mathematics, and we will take care of analyzing the knowledge and the reasoning processes of the subject. In the department of education, we will take care of how to write a lesson plan and how to hold a conference with parents. But unfortunately it's not that neat. And even if you set up the course properly, it's the person who teaches that course who makes the difference. There are
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courses that are labeled, "Mathematics for Prospective Elementary School Teachers," and this course varies from how to solve systems of equations to what to do with Catherine Stern blocks. It's very hard to single out the various responsibilities that have to be met in the training of teachers.

I'm also wondering about the kind of end product that Bunnie has in mind. Is he talking about a program for training secondary school teachers who do nothing but teach mathematics all day? Or is he perhaps referring to a program for training teachers who will teach in elementary schools and who will do lots of things different than teaching mathematics? I think at the moment there are different needs. Until very recently I gave all my attention to secondary school teaching and I think that had I heard this talk three years ago, I would have said, "That's what's needed," for I would then have been thinking about the training of teachers of secondary school mathematics.

For the last year or so I've been working in an elementary school with children and with teachers all day long and the issues are not so clear at that level.

Let's consider for a moment the whole question of humanizing mathematics to make it more acceptable for children. Well, it's hard to get this point across merely by telling children that mathematics is useful. You know, they just don't really care about being told that. Rather what is important in the elementary school is to have children discover that mathematics is useful because they are using it to solve problems that are important to them. Now, I have a hunch that five years from now I'll also be saying that's the way they ought to do it in secondary school also. In elementary school one cannot teach mathematics as such but has to work with children who are playing with problems vital to them, and which cut across subjects. I suspect that it really will make no sense to look at mathematics as mathematics and worry about the concepts, and definitions and values in that subject as opposed to those in physics or social studies or English. I have a feeling that the elementary school
tradition, at least the best kind of tradition, is probably closer to being more effective in getting children to learn things than has been the tradition in the secondary school and has been the tradition in the college, as you all know.

A further problem which I would like to hit at has to do with distinguishing between the training of educational researchers and the training of people who will train teachers. Bunnie has pointed out that as far as he can tell there doesn't seem to be any difference between the two types of programs. That is, the doctor of education degree which was once proposed as a way of giving academic respectability to people who train teachers or work in schools, has now become in the case of many institutions just a different version of the Ph.D. which was originally designed for research. A way out of that dilemma is to broaden somewhat the concept of research. In the field of education, we have taken as our model for research a kind of model that works well in some of the social sciences, particularly psychology. And we have tried to force research on educational problems to fit that model. Well, there is a lot more to it than that.

There are problems that need solving in the field of education, and participating in the solution of these problems is a wholesome activity for people who are going to train teachers. For example, take the whole field of curriculum development. Now in Bunnie's paper that was listed along with educational psychology, and sociology and social foundations as one of the basic things that you do in getting an advanced degree in education. It turns out, however, that one who does research in curriculum development will find himself worrying about all the issues that Bunnie has said are important. For example, you will worry about how one teaches a concept, how one teaches a definition, how one handles value judgments. All of these things have to be considered carefully in carrying out curriculum development. You will also worry about humanizing the subject. You will say, "here is a mathematical idea that appeals to grown-up mathematicians. Now what is there in the life of a child that will make this idea important to him?" And that is an act of humanization.
If we urge the prospective trainer of teachers to do the research that he will have to do in order to come out with the right kind of degree, if we urge him to concentrate on problems of curriculum development, then I don't think this research experience will be harmful. In fact, it will be essential. There is also an area for research in classroom organization. I think, for example, that we are now faced with problems of what to do with educational technology. These are complicated problems. People think they have a panacea in each new invention that comes to the market place. We need people to worry about how to organize these various aspects of educational technology to produce the best kind of education. So I think that if one broadens the notion of research and gives as much credit for creativeness in proposing a didactic organization of a piece of subject matter as one gets for proposing a Latin square, then I think that research experience can be beneficial to the person who is going to train teachers.

I now would like to make one or two remarks about teaching and personality. Bunnie is a cool guy and he doesn't realize how much of his personality goes into his teaching. I wonder if he has video tapes of himself? I think that personality is very important in making effective teachers and to tell you the truth, it worries the hell out of me. I know we have the need for lots and lots and lots of teachers and there are a good many of them who won't have good personalities for teaching. But that's if you think of teaching in some sort of conventional mode. For example, you may think of teaching this way: there is one adult and twenty-five to fifty kids in front of him, and he conducts the class as he would a symphony orchestra, and each answer is used in constructing the next question. If you think of that kind of teaching, I'm afraid we'll never be able to duplicate that on any kind of large scale and what's more, you probably shouldn't because this teaching smacks of paternalism. It is the teacher who is doing all the work and it is the teacher who is figuring out the next best question to ask. That kind of teaching probably has the least effect on children. Once the support of that teacher is missing, once child has to go on his own, who is it that asks the next right question? Who is
it that avoids saying, "You are wrong," but instead asks the next question which capitalizes on the error? The teacher, mainly through his personality, acts as a readily available answer book. There is a constant check the child has as he looks at the teacher. There is also a constant flow of information from the child to the teachers. Now I would like to be able to say that we could train teachers to look at this flow of information coming from the child, interpret it, and know what to do next. But I'll bet you find very few people who can be successfully trained to do that. Instead I think that we need to look at teaching as a system in which children begin to develop the equipment they need to learn things for themselves. But to do so you have somehow to get rid of the paternalism that characterizes most teaching and to equip children with the tools and the understandings and the self-esteem they need to give them a feeling, first of all, that they can learn things by themselves and secondly, that it's a good thing, it's an interesting thing, to learn by yourself.

In the last fifteen years, we've had lots of fun in reforming curriculum—especially at the secondary school level—and even in the elementary school. It was very interesting to play around with various structural organizations of the subject. You know we did sort of impress each other with how clever a definition we could give for prime number or for function. That was great fun. It was too bad that somebody with a great deal of wisdom did not say, "Let's take all these guys and put them on an island somewhere, have them produce units and entertain each other with them. And let the rest of the country get on with the problem of education."

I think when we talk in terms of trying to get little children to understand the structure of knowledge, it's a bunch of nonsense. I think the only kind of people who appreciate the fact that knowledge has structure and that there are various kinds of knowledges are people who have had a great deal of experience with unstructured knowledge. These usually are adults and they can charm each other with the fact that you can make sense out of this big blooming confusion
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that we have all been living in. I don't think it makes pedagogical sense to say that we can cut out the confusion for the kids and start them out right away on the right track. I just don't feel that is sensible and if you live for any period of time with six year olds, if you watch them, you will be convinced, as I'm being convinced, that you can't do these kinds of things with little children.

There comes a time in the child's life when structure may make sense, but it's not anywhere near age six or seven, eight or nine or ten or eleven or twelve. I am beginning to feel that maybe it's not even sensible in high school. I think teachers themselves need to know things about structure, of course. But there is a great danger that if you press on these issues in the training of teachers, they will think, "Now all I have to do is figure out a baby version of this and I will have the right thing to do for the children in my class." There is a great deal of danger in that and so I think it's politically dangerous to put a great deal of stress in the training of teachers on the structural organization of the discipline.

Well, I hope that that will provoke some discussion. Thanks.
I was afraid that I would have to follow Professor Beberman. As I feared, he has already said many of the things that I've written down here to remind myself that I wanted to say. So, I'll cut my remarks quite short because I think you've already heard a lot of words. I don't mean to say that the previous two speakers have been wordy but I have a hunch that you may be bursting with questions and comments yourselves.

You know it's a funny thing about words. We hear thousands and millions of them, but most go in one ear and out the other and we don't remember too many of them. I want to tell you a story about some words—just a few that I heard four or five years ago. I have heard a lot of words since, but these words stuck with me. I was out supervising a student teacher who was thrashing through some lesson and having a horrible time. The cooperating teacher came out and said to me, and these are the words I remember: "What the heck are you guys doing up there in teaching these people methods classes? This guy doesn't know anything." Obviously I would remember it because I was the one who taught the methods course. I don't remember my answer to him but I do remember that it bothered me. The reason that it bothered me most of all was because here was a man who was just as responsible for training this teacher as I was and he was giving up his responsibility or refused to accept it in the first place. What I am trying to say here is that it's too easy for the teachers in the field to pass the buck to college professors of education saying, "What the heck are you doing in those methods classes? Why aren't you teaching people how to teach mathematics?" I think that we have to impress upon the teachers in the field that it is part of their professional obligation to help in the training of teachers. And so, if you accept this particular thesis then what you are doing in your job, if you are training teachers, has
relevance because in effect you are training teachers of teachers.

I guess I have to tell another story on myself. So often in my work with student teachers in the field training experience, all the suggestions and tips I give seem to be of little use. The teacher goes on teaching just the way he wants to and seems to ignore the things I tell him. When you think about it, if he ignores the things that we tell him and goes on teaching the way that he wants, chances are that 95% of the time he is teaching in the manner in which he was taught. So, in a very real sense the teacher is the person who trains the future teacher. The teacher takes his models of teaching behavior from the persons who taught him. I think it is very important that somehow or other we get this across to our students who are going into teaching for the first time. They should be aware that they, perhaps in an unobtrusive way, are going to be doing the teaching of future teachers and it behooves them to teach in the best way possible.

Now, just let me add a corollary to this because I have a little hobby horse I like to ride. Part of the job of teacher training comes in the very beginning when we recruit people to come into teaching in the first place. And where do we get these people from? Well, maybe we sit in our academic offices and our campuses and wait for kids to come to us and say, "Dear Sir or Dear Madam, I am interested in becoming a math teacher. Can you tell me about the program and what I have to do?" I think part of our problem is that when we say we have poor teachers, it is because we are not attracting the good people in the first place. We have to actively go out and recruit mathematics teachers. I think at the time the students get to the college and university campuses it is almost too late to recruit them. What we need to do, is to get out and recruit in the junior high schools and the high schools. You know, it doesn't hurt to go out to a bright young personable mathematics student and say, "Have you ever thought about teaching mathematics?"

This is where the training of mathematics teachers begins, I think.

Let me just talk about one more thing--the recruitment of teacher educators--people like ourselves who are going to teach or train prospective elementary
teachers. Where do we get these people from? To start with, we get these people from the ranks of those who are already teaching elementary school or perhaps people who are teaching in secondary school. In other words, we look for the person who has taught, thinking that experience counts. We must get someone who knows what it's like—has really been in there and had the experience! Maybe he hasn't had ten years of experience—maybe he's only had one year of experience ten times. I'm not talking about going into the high school and elementary classroom—I'm talking about people who are going to come into the ranks that I think most of us occupy. I think a good place to look for these people and to encourage these people, it's again a recruitment idea, is within the ranks of those we are training now to be mathematics teachers. Say, "Look, young man or young lady, have you thought about going straight on through working for an advanced degree preparing to train future teachers instead of going out teaching in the elementary school?" For I think this business of experience is one of these myths we hang on to far too long. I think we can theorize about the teaching act and here's where I agree with Professor Smith, that it is very possible to train a person to analyze and to criticize and to look at the teaching act without ever having done teaching himself. I think it is a perfectly possible thing to do. And you want to look for the bright, young people who have got years of productivity in front of them rather than these old timers like myself into the field of math education.

There is one further point which came to my mind and which may be the genesis for discussion. Both the previous speakers talked about training teachers. I guess I've talked about it myself. Now we seem to be worried about teaching. I wonder whether we should rather be worrying about learning. I have a hunch the two things are not one and the same. If we worried more about how kids learn and what kinds of atmosphere promotes this learning we might be better off. Professor Blevin said I touched on this—he said we want to look for the person and create this atmosphere. But sometimes I think the best teacher in the sense of the teacher who can promote learning, is the person who says nothing. I would love to hear a
tape, a classroom recording, with nothing going on except the busy noise of kids
learning and manipulating and doing things by themselves.

I promised I would keep my remarks short and I will. Thank you for the
opportunity of talking with you this afternoon.
There are leaders in mathematics education. But what kind of people are they, and what about this type of person in the future? Let me say in the first place that no one can act as a generator of classroom mathematics teachers who is not at all times fully cognizant of the existing knowledge of the mental development and behavior of the students which he has to teach, in complete command of the nature of contemporary mathematics, and of the purposes and goals to be attained by its study. One does not usually ask what kind of a person you expect to have at the end of an educative process. What is your goal? What are the targets you are going to shoot at? I would say it is a person with the capability of demonstrating at any time an efficient processing of mathematical knowledge and conceptual thinking into the minds of the children. That is a big order.

I have frequently seen many attempts to relegate the training of teachers and the teaching that had to be done in the classroom to a scientific, machine-like, technical process and seldom, if ever, has it ever succeeded. I believe teaching is as much an art backed up by intelligence and understanding as it is a technical, scientific procedure.

Now, I want to speak about another kind of leadership. This is the kind of leadership that brings new knowledge of mathematics to bear on what goes on in the schools of the city, the state, or the nation, or in the colleges of education at our universities; one that develops new knowledge in pedagogy, new procedures for teaching, new ideas about classroom organization, and one that even dares to innovate new goals or new objectives for education in mathematics. Who are the persons to do this? Where do they come from? How do they arise? What need exists for them? How do they function and from what base? All of these questions must be given immediate attention, if we sense in any way, that our schools in the United States should produce graduates that excel, or at least equal, the
educational products of other countries. No matter how well teachers are trained in existing curriculums and current theories of learning, the teaching of mathematics in the schools will become anachronistic unless the teachers of teachers are tuned-in to what the leaders and their research results are saying, and are able to translate the most promising elements of this knowledge speedily into action in the schools.

What Constitutes Leadership?

In any profession or any large industrial or scientific enterprise, I think you will grant that it is innovation and adaptation of new knowledge to the changing needs of society, and its emerging culture, that constitutes development. No large organization, no industry, no scientific organization will prosper and flourish unless it innovates, develops, and creates. The persons who engage with real force and responsibility in these activities are those who exert leadership in education. I can say that what the University of Illinois Committee on School Mathematics has done over the last fifteen years is an example of what I mean by innovation, exerting force and pressure and developing new goals, new targets and new ideas in the field of education. The degree to which persons act responsibly and succeed in making amelioration is a measure of their leadership. The objectives of educational development, as stated by a special committee of the Scientific and Technical Personnel Division (1) of the Organization for Economic Cooperation and Development adequately express the tasks of leadership activity. They are:

a) To devise methods by which educational targets can be established and achieved,

b) To find the resources needed for this purpose,

c) To determine the optimum ways in which these resources can be utilized.

Thus, leadership in educational development implies the determination of objectives and the translation of these targets into operational realization in terms of the entire structure of our educational system.
I want to note at once that there is a great difference between providing schools (which is an ongoing process in a very traditional sense at present) and providing a pertinent educational program. It is easy to build schools, enroll more pupils, train teachers in the ordinary way of teaching, and supply books and materials. But it is much more difficult to decide what these schools should do to bring about a desired end product of the whole educational program. And should we try to define an end product? What is it that we are trying to produce in the twelve to sixteen years of schooling? I would say resourceful, competent citizens who realize their own potentials, and contribute accordingly to the development of their own community.

How well do we succeed in this? How well are we creating a mathematical program for them to do it? The really truly important questions for us or for any mathematical leaders are: what mathematics shall the schools be teaching? I would like to say parenthetically there is so much mathematics at present that mathematical knowledge alone cannot serve as a means of building a curriculum. One has to make a choice. What mathematics? To what students should they be teaching it? All? Some? Maybe none? For how long a period should they be teaching it and to what depth? How can this mathematics be most effectively learned? How can teachers be educated and continuously informed? Education is a profession in which teachers should be educating themselves continuously throughout the entire life of the profession. How do you educate them to do this, and, above all, how can you insure, to the greatest extent possible, that what is taught to children today is what the children will need tomorrow when they enter the society in which they shall live as adults?

What we teach need not be relevant to the child's felt needs. What we teach him should be relevant to what he must have when he enters society as an adult to carry on the great tradition of moving humanity forward. I am saying that in leadership it is the mathematics curriculum, its purposes and the goals to be achieved, and the teaching process that are the important educational activities.
In the past curricula and teaching methods have adjusted eventually to new knowledge. It usually took about two-to three-hundred years before some new idea in education finally found its way into the school. You will grant that today this process is much too slow. There is need now for deliberate action to hasten and facilitate curriculum adjustment and the doing of this requires a new kind of leadership.

In this connection, it is well to note that today education is no longer a privilege or a luxury for the few, but it is an important right for all youth so they may participate in the economic development of our country. Not only do we measure how much is paid into our educational enterprise, but we also estimate the amount of the gross national product that education contributes to economic growth. We can, for the purpose of comparison, make an analogy of the educational output to industrial production.

a) There is raw material. This consists of the minds of our students, which we must accept in the various qualitative intellectual abilities in which they exist. There is a discipline, the subject matter, the substance, which we can select as we desire. It must, however, be of the most recent design of the highest quality, and selected for producing the ultimate specified product we believe that future society will need.

This substance must be supplied by mathematicians and scientists working at the frontiers of their field. They are persons who know the best in quality and design, and the type of mathematics that will be of use and have value in the years ahead. Thus leadership must necessarily include the research mathematicians who feel a responsibility to society in what they are doing.

b) There is a processing of this substance into the minds of the students—the whole machinery of educating which consists of all kinds of materials for instruction, the operator of the process called a teacher, and a technical (and even artistic) procedure called a system of instruction. This processing or engineering, applied to delicate human minds, must be of the highest competency and precision,
for no matter how good the substance, if the machining is poorly done, the product will be inferior--of poor quality. In creating this machinery, and keeping it up to date we need to consult the learning psychologist and mathematical educator who are at the forefront in dealing with research theory and practical applications of learning theory. Of no less importance is the devoted, intelligent classroom teacher who is constantly seeking to create new procedures in her day-by-day teaching to improve the product under her construction. These persons are engaged in leadership activities.

c) Finally, there is the product, ten to twelve or more years in the making. The product will be of varying degrees of excellence, but it should not be an inferior one. Briefly, we expect it to be a person who has within his mind an understanding of the nature of contemporary mathematical thought, adequate basic and utilitarian mathematical knowledge, who can relate this thought and knowledge to the general daily affairs of man and who has built into himself the capacity for continued learning of the subject, if he chooses to do so.

The development of a mathematically literate person is thus an economic process. The subject matter is the hard core of this process. Its relevance to general educational goals and the achievement of these goals through psychologically sound teaching procedures will determine the efficacy of the entire process. As society changes, when advances in knowledge and new insights into the manner of its acquisition by human minds are made, there must be (a) a change in curriculum so as to adjust the content of teaching to the latest developments in the state of mathematics and (b) an adaptation of the presentation of the subject to the particular exigencies of the type and level of schooling where it is to be taught. These changes and adaptations will be in continuous operation.

Who is to do all this? Those charged with leadership.

Leaders and Leadership

Who are the people who have been saying the curriculum shall be this, the
teaching shall be done this way, and here are the books? These are the leaders in mathematical education.

As one examines the activities for the improvement of mathematical education around the world, until quite recently, there usually appeared in each country one or several persons to whom the leadership in these reforms is attributed. A look at these men and their educational careers should shed some light on how leadership develops. In England today the name of Bryan Thwaites as the prime reformer is unquestioned. He was a lecturer in applied mathematics at a technical university. Having read the report of the Royamount Seminar, he approached the Nuffield Foundation for financial support to instigate reform in mathematics teaching, surrounded himself with a small group of aroused and interested teachers, began writing and experimenting (and advertising what his group was doing) and soon had underway a School Mathematics Project as the largest reform study in Great Britain. He knew because he read in the literature, he had drive, energy, and a fine attractive personality, and the quality of attracting capable teachers and inspectors to his cause. Thwaites never had a course in psychology, methods, history or philosophy of education.

In Belgium, a mathematician of note at the Free University of Brussels, and at the same time a senator in the state government took the chance of ruining his professional reputation by developing and teaching a course in modern mathematics to normal school students preparing to teach primary school children. To his surprise, he discovered that what had been done for these teachers, when properly applied, worked equally well with the elementary school children. He then established, with the help of the government a Center for Mathematical Pedagogy, where he and his wife, Frederica, have developed an ultra-modern course in abstract mathematics for the high school. They are now doing the same for the elementary school. Both these persons are exceptionally creative—Papy in mathematics, his wife in pedagogy. Their work is the basis for setting the new curriculum in mathematics for the entire country of Belgium. Papy is pure
mathematician, who never studied education--but read it and disagreed with much that he read. He is world renowned today as an educator!

In France, the reform movement is credited largely to four men and one woman. The woman, Lucien Felix, is a brilliant mathematician, a graduate of the L'Ecole Normal Superieure, the next to highest degree in France (the State Doctorate is the only higher degree). She was a teacher in the Lycée Fontainne for girls and the only woman recognized by Bourbaki. Her books on modern mathematics started the French reform. The men are all mathematicians of the highest rank. Professor Gustave Choquet --a Bourbaki--who rewrote the mathematics program for the Sorbonne (called Bébé Bourbaki), who in two years failed so many students, that he literally forced the preparatory schools (Elementaire, Superieure) to write a new curriculum of the contemporary variety. Professor Dieudonné who at the Royamount Seminar shouted "To hell with Euclid" and "Death to the triangle" is today causing Euclid to disappear rapidly as the way of studying geometry (space). Professor Revuz and Professor Lichnerowicz, both of the Bourbaki School, led the Association of Professors of Mathematics in Public Institution into their present reform activities. All these men are pure mathematicians with no background in educational study. They are all hailed as great teachers as well as leaders in mathematical education.

In Russia, the names associated with leadership in Mathematical Education are those of the great mathematicians who have taken time out of their university work to enter into the high school and elementary school to teach and to create new programs. We may mention names such as Gnedenko, Kolmogorov, Markuschewitch, Jaglom, Chintsin, and the like. They work with educators and psychologists at the Academy of Pedagogical Sciences.

And so it goes in all the countries: Freudenthal in the Netherlands, Pedro Abellanos in Spain, Mario Villa in Italy, Djuro Kurepa in Yugoslavia, Akizuki in Japan. All of the men who were the leaders, who developed new programs, who insisted on changes in programs in the past ten years or up to two or three years
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ago were pure mathematicians with no background in psychology or education whatsoever. I didn't say they were teachers, however. I said they were producing the program. Now there is a reason for this predominance of mathematicians. The universities were charged with the production of pure and applied mathematicians. In our country the universities must produce mathematicians for industry, for research, for technology, for university professors, and for research at the university. They found that the preparatory students who came from the secondary school into the university were ill prepared. This happened at Illinois you remember. In 1951 or 1952 the school of engineering listed one-hundred four skills that the high school students did not have, and they had better get it if they intend to study engineering. Now this was not a plea for modern or contemporary programs, but it was a charge that the secondary schools were not doing their jobs. So as leaders they stepped into the educational world at the lower level. They told the persons what kind of mathematics would be needed and should be taught. The remedy was not always in good pedagogical taste and at times it was not always effective. Look at some of our own USA efforts to get better mathematical programs and see what is still being taught in the high school! In this first period of reform, mathematical educators and the teachers, not knowing sufficiently about new mathematics or the new terms in mathematics or new structures in mathematics, kow-towed to the authoritarianism of the mathematician. We listened, we were afraid, even when doubtful to question it, and we didn't raise our voices in protest. We tried to do something that they requested us to do. And I would say that the result has not been bad.

Within the last decade, from 1960 up to the present day, a new type of leadership has gradually emerged. During the nineteenth century and the first decade of the twentieth century the authors of mathematics textbooks were mathematicians. Since the textbook was written to some curriculum, usually prescribed by the college as entrance requirements, the textbook writers exerted leadership and direction in school mathematics. Thus men of the type of Davies (Professor
at West Point) Granville (at Pennsylvania), Wells, Durell, Hart, Stone (all college professors) and the like decided what and how arithmetic--algebra, geometry, and trigonometry should be taught. The entire school curriculum was directed toward college entrance requirements for the subsequent continued study of mathematics. The influx of great masses of students into the secondary school in the twenties and thirties changed this situation. The mathematicians could not write books for this large non-college intending segment of the school population. The new books were written by Virgil Mailory, William Betz, W. D. Reeves, Ralph Schorling, John Clark (arithmetic) and many other mathematical educators. These persons, while trained to a certain degree in mathematics had a large part of their graduate study given over to education--its history, philosophy, psychology, and methodology--and secured the first doctorates given in the new field of mathematical education. The pedagogy of most of these books had real positive effects on changing the character of teaching in the classroom. The mathematical quality of their textbooks was quite a different thing, and frequently open to deserved criticism. The effect of these people and their books was so great, that in 1933, the College Entrance Examination Board gave up its dictation of the high school curriculum, reserving only the right to keep its testing program. The latter was also modified to test ability in, rather than acquisition of, mathematical knowledge. It should be noted that in 1954 the CEEB rescinded this position saying it could no longer tolerate the poor preparation of candidates taking their examinations. In 1955, it appointed a Commission on Mathematics to study the state of mathematical education in the USA and give a directive to the Board.

It is the composition of this Commission that indicates a recognition of the new kind of leadership now appearing in many countries of the western world. The original commission consisted of seven pure mathematicians, two college mathematics educators, and four classroom teachers; its executive committee contained one member from each classification. The written report was in
large measure the work of a secondary school teacher who had completed all requirements for a Ph.D. in pure mathematics. In fact the mathematicians played far more the role of an advisory and consulting subset of the commission while the subset of educators and teachers provided the constructive and innovating force.

When the New Commission for the Improvement of Mathematical Education in the Scandinavian countries was organized, four persons, one from each of the Nordic countries, became the essential leaders. They were Kay Piene, a mathematical educator in charge of teacher training in mathematics for Norway; Bent Christiansen, head of the department of mathematics in the Royal College of Education in Copenhagen, and outstanding as a mathematical educator of Denmark; Dr. Simila of Helsinki; and the executive secretary of the Commission, Matts Hästad, in charge of mathematics education in the Swedish ministry of education. All these persons were well versed in mathematics, as well as in philosophy, psychology and methodology.

In England after the first wave of reform had passed, a number of other groups sprang into action and are leading England to more realistic programs. They are Cyril Hope, a lecturer in mathematics and an educator of the Worcester Training College, who sparked the Midlands Mathematics Project which has received praise for its pedagogical aspects; Frank Gorner, Deputy Head of the College of Education of Manchester University, who with a group of volunteer workers is creating a contemporary program with a non-rigourous approach to accommodate a far greater segment of the secondary school population. To these names we can add that of Geoffrey Matthews who is leading reform in elementary school mathematics, Skemp at Leicester, Dienes, formerly of Leicester, and now in Canada who are applying psychological principles of learning in developing new curriculums.

In a similar way in Belgium, Willy Servais, Chairman of the National Committee on School Mathematics and President of the Belgium Association of
Mathematics teachers is an educator—not a pure or research mathematician. Pickert and Hans-Georg Steiner are leading Germany to a genuinely modern program in mathematics and both these men are heading mathematical didactic seminars in their universities. Krygowska, a woman educational psychologist, with an added degree in mathematics, is Poland's leader in today's school curriculum reform. Seeking an explanation for the emergence of these new leaders, one readily notices a difference between the mathematical educators of the earlier part of this century and these new leaders, namely, that they are highly trained in current mathematics, and speak and hold their ground in dialogue with the pure mathematicians. They do not accept the authority of the mathematicians in relation to curriculum and methodology, who in fact are not authorities in these matters, however great they are in their research in mathematics.

Another difference in display of effective leadership can also be seen in two important conferences and their reports. One, in the USA, a report of a Cambridge, USA conference, prepared by a small group of mathematicians (there were no educators or teachers in the group, and only one psychologist) is called Goals for School Mathematics. So eminent was this group that the report was prefaced by a foreword by the then United States Commissioner of Education who, not knowing what the report said, nor what consequences it could have if ever initiated in this century, supported the mathematical authority from on high. One paragraph from the Introduction may give an indication of the attitude of these men (they were twenty-eight men and but one woman).

"We made no attempt to take account of recent researches in cognitive psychology. It has been argued by Piaget and others that certain ideas and degrees of abstraction cannot be learned until a certain age. We regard this question as open, partly because there are cognitive psychologists on both sides of it, and partly because the investigations of Piaget, taken at face value, do not justify any conclusions relevant to our task. The point is that Piaget is not
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a teacher but an observer -- he has tried to find out what it is that children understand, at a given age, when they have been taught in conventional ways."

It is safe to say that with the exception of a very few teachers -- a handful who have earned the Ph.D. in mathematics and who have continued their study of recent research in mathematics -- the report in its subject matter curriculum description is totally incomprehensible. It cannot have, and has not had, any effect in recent reforms in the USA or elsewhere in the world and it is now six years since that report appeared.

In the same year, 1963, five months later, another conference was held in Athens, Greece, under the auspices of O.E.C.D. This conference was attended by thirteen mathematicians, and twenty-six mathematical educators. The title of the report of the conference is Mathematics Today -- A Guide for Teachers. It is written and addressed to those who would lead in reform of mathematics instruction using an experimental approach for so doing. The report is the basis for the continued reform movements throughout the twenty-two countries of the participants. The effect of this report is being felt worldwide, due to the fact that the group of mathematical educators held serious dialogue with the mathematicians. The mathematicians gave the essential subject matter that must be learned, they showed the new conceptions of mathematics and its structure, and the educators showed how this information could be put into a sequential curriculum and taught by appropriate procedures that take cognizance of what is known today about learning at the various stages of intellectual development.

These two contrasting examples of leadership in curriculum development show the future role to be played by the academic community in this vital aspect of education. To them is entrusted not only the custody and promotion of inherited and new knowledge but also the planning and developing of means for the acquisition of this knowledge by those who constitute the next generation of adult workers -- those now in school. Recent curriculum improvement projects
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abroad and at home have drawn heavily on specialists (leaders) from higher education, who were the only ones able to restore the connection between school teaching of the subject and the advanced state that mathematics has reached in the past few decades. Today the curriculum development in mathematics in the USA is located in, and heavily staffed by institutes of higher education—not institutes of mathematics.

Problems for Leaders

What are the tasks ahead of us in mathematics that leaders will have to tackle—to experiment with—and to give tentative solutions to? The number of particular tasks is too large to enumerate, but it is possible to state some broad general areas in which the special problems will lie.

1. The New Role of Mathematics in School Education

Today enrollment in the elementary and secondary schools is essentially 100% of the age group. The objectives we had in our teaching only a few years ago are superseded today because of (a) the pace of social and economic changes, and (b) the ways in which science and technology are changing the environment. Both of these events have created a serious gap between what we are now teaching in the schools and what will be required by society from its members in the twenty-first century. Traditional mechanisms for updating the content and methodology of school mathematics are inadequate in face of the problems raised by the rapid progress in increased mathematical knowledge. Programs in school mathematics can no longer be thought of as education for passing examinations and for subsequent admission to higher education. Indeed, the needs of the new mass-clientele must be reconciled with the intellectual elitist requirements of higher education and with the labor market requirements for qualified manpower. Training for citizenship requires the development of individual aptitudes and interests, so that children may play the role in their society that suits them best. Such a curriculum is necessarily a general curriculum for all children offering differentiation according to individual abilities and interests. The
consequences of this evaluation in objectives becomes most critical in the junior high school years. Here provisions for the needs of workers in a modern labor market must aim at a broader background than present general courses do. It requires the introduction of new mathematics with new approaches. Leaders must know the new role that mathematics must play.

2. Deciding Policy and Objectives in Mathematics Education

I heard not a word about our objectives in mathematical education this afternoon. Yet in the past, thirty, forty years ago, it was decided that before you could build a curriculum, you had to know what kind of an end product you would like to have. Today we look at the mathematics and say we are going to build this knowledge, and we seldom, if at all, consider what our end product should be.

So educational change will continue. You can't stop it. The growing activities in all educational movements, the development of new media for learning, the accomplishments of recent scientific endeavors indicate changes in all aspects of education. While no one could forecast with accuracy the needs of the world in the year 2001, we can assume that certain trends will continue and that we can predict to some extent the mathematical understanding that will have to be acquired by the population. This knowledge must be learned from the ages four to twenty years. We can assume that education will become the most important part of all our national activities with by all means the largest part of the budget. People laugh when I say this sometimes, but I am convinced that the greatest part of the federal budget and state budgets must be given over to education. It is the important thing for developing an economy and developing and maintaining an affluent and happy society. We can also assume that the end of full-time formal education, (age 20, 22, 24) will be followed by a life-long continued training and education. In fact, we can assume that knowledge alone will be unable to act as a basis for instruction in mathematics. A different task, one of intellectual formation, based on more general concepts of cognitive think-
ing, must replace present day special concepts and specific behavioral goads as targets for mathematical education. And to set up special, specific goals, special specific tasks and to train teachers to teach these to the children would be a tremendous mistake because ten years from today most of those special skills and tasks will be outmoded and useless. We must teach, not for the special, but for the more general adaptation of the intellect, and intellectual work and activity, to the changes that will come about in our society as we move forward. This means that leaders in mathematical education must take the initiative in anticipating new needs, preparing new teacher education programs, new materials, as well as new techniques, to meet these needs.

3. Mathematical Education Must be Relevant Education

The varied objectives of mathematical education must be adjusted to the rapid socio-economic changes on the one hand and to the evolution of knowledge in the subject on the other hand. The growing need for trained manpower, especially in the sciences, and the increasing awareness of the part school education can play in this training by insuring high levels of competence, has raised with a new intensity the question of the relevance of what we are teaching. Increasingly the problem of selection of subject matter must be resolved in terms not of how much but what and how. The perspective of life-long learning introduces an important element in the curriculum maker's choice in deciding what is relevant and what is inessential for teaching in school mathematics.

Relevance of what is taught takes on an even more significant aspect when we consider that what is taught to a child entering school this year and continuing for the next twelve to sixteen years must suffice for a working life that may last a further half century! Many, if not most, of these pupils will be living the prime of their working and contributing lives the first three decades of the 21st Century. Our mathematics program must, of course, relate the subject matter to the pupil's needs, but it must also be made relevant, so far as we can determine
to the social, economic, political, and technological needs of the culture of the coming century.

4. **Flexibility and Efficiency in Mathematical Education**

The 'how' of teaching and the nature of classroom techniques employed in mathematics classrooms have for centuries undergone little change. The image of a teacher lecturing to a class of children who take notes, do their exercises, and repeat what they learned, is as valid today as it was 300, or even 2000 years ago. Most new methods, and use of new media are of very recent origin, and they are used largely because of outside and industrial promotion and not by educational design. Nor has the teaching assimilated the findings of child psychology and the psychology of learning concerning the conditions under which certain materials and presentations should be made to correspond to the stages in the child's mental development.

Leaders must determine how to break the rigidity of existing educational procedures and to establish a flexibility for innovation and experimentation under controlled conditions. Mathematics education must be made more responsive to change—especially change in educational technology. One problem is to speed up the slow rate of diffusion of new curricula, new techniques, and new practices. A type of flexibility of immediate concern relates to insuring that students are not placed too soon in irreversible lines or course sequences. We must be concerned with guiding pupils into instruction that fits their relative state of knowledge and learning so that they can continue on in their study. We must keep the doors open for continued study and that means not placing students into general, slow-learning, culturally deprived, etc. courses in which further study in mathematics at a later age becomes almost impossible. The mathematics curriculum must allow flexibility of all kinds. Leaders must propose means for such flexibility.
Removing Ignorance through Appropriate Research

Educational change is an exceptionally complex undertaking and like all social change, has at times to be done hurriedly on the basis of inadequate information. Bluntly, little is still known about the many basic questions of mathematics education today, the training of teachers, the teaching-learning situation, the ideal type of mathematics.

There has been much research in mathematical education and there is a spate of it going on today. But all of this research has suffered from two fundamental things:

a. Research in education has never been fully recognized as a discipline of academic standing and is consequently insufficiently supported. One real difficulty is that it is very hard to find sufficiently and ably trained researchers to work in this field. To do research work of an empirical nature with observations, and with the use of the best mathematical statistical procedures requires a good deal of training. We have some competent researchers, but not many.

b. In the second place, the results of almost all our education research have little practical use. Most of them are neutral. It does not matter whether you teach this method or this method; they probably learn just as well. If you use \( a + b = c \) or you write \( \frac{c}{b} \) they get the answer easier with \( a \) and \( b \) in a column than they do on a line. Now where do you use that? I suppose you ought to draw the conclusion, therefore, throw out all line addition and put in only column addition. And if you do this what are you going to do when you come to equations \( ax^2 + bx + c = 0 \)? Do you write \( \frac{bx}{c} \)? You see, it is the fact that a lot of this research is trivial, and this hurts us. I would say that the results have seldom formed a basis for policy in making new programs. In fact, most policy making is the result of decisions from persons in the field of pure mathematics.

A strong stimulus for research in education has resulted from the
importance our society attributes to education as a relevant factor in economic
growth. Now ahead of us lies the task of using this stimulus to promote basic
research on the teaching-learning process so that it can serve as the center of
gravity of decision and policy making in the formation of objectives, in subject
matter selection, in developing the technology of teaching-learning, and in pro-
ducing a good product—the literate mathematical mind that fits into the society
in which it must contribute and live.

Qualities of Leadership

It is difficult to state specific qualities of those who must lead the pro-
fession of mathematics education to improved conditions that promise more
relevant and more efficient results. Leaders have arisen in varied positions
with varied backgrounds. It will be sufficient that some of the qualifications
that follow should be possessed by one who would contribute to the advancement
of society through leadership in mathematical education,

1. A person who has studied mathematics well into the Ph. D. require-
ments in pure mathematics—and continues his study of mathematics the rest of
his educational career. He must be able to speak with mathematicians on the
state, direction and trend the subject is taking. He must be able to read and see
what mathematical research that may be of significance for revision of school
mathematics. Let me cite an example. Fifteen years ago one would ask why
must anyone who is concerned with elementary or high school teaching be
knowledgeable about abstract algebra, groups, rings, fields, vector space. 
Does not this belong in the university study for Ph. D.'s? Today groups, rings,
fields, and even linear algebra are becoming the foundational study in the
secondary school. Why? Because there were some people who could speak
about this mathematics to the mathematicians and then relate it and see how it
can be brought down into the schools.

In the same sense I would say that the leaders today ought to be able to
speak to mathematicians on categorical algebra. Now I want to confess that two years ago I heard the word "categorical"--I didn't know what it was. But I made it my business to know what a category is, what a functor is, how categories are related by functors and one or two examples. And I would like to say that it may be just possible that ten or twenty years from today, as groups, rings, and fields come down into the high school, categorical algebra, which is now the field of researchers or of the upper eschelon in the Ph. D. level, may come down into the university study. This is what I mean by being able to speak and talk to the mathematicians so they will at least respect you. They won't expect you to know all they know.

2. A person who has at least two years of university study in several fields of science, especially physics, chemistry, biology, geology, and the behavioral sciences. Without this you hardly know just where and how mathematics enters into the problems in these fields.

3. A person who has taught mathematics to children a minimum period of five years. There is no substitute for genuine classroom teaching at the secondary or elementary school level.

4. A person who has a strong psychological and pedagogical training, leading to a Ph. D. or Ed. D. in Education. For a holder of the Ph. D. in pure mathematics, to be a leader, he must balance his preparation with this pedagogical requirement in psychology and pedagogy.

5. A person who is a member of the leading mathematical societies and associations, both in pure mathematics and in education, both nationally and internationally.

6. A person who can work with and be respected by both the mathematical and the educational world.

7. A person who is associated with the graduate division of an outstanding university having a graduate school of education.

8. A person who can speak one or several foreign languages, is
acquainted with the mathematical education leaders of other nations, and can read the journals written in these foreign languages.

9. A person who is engaged in curriculum and/or practical research in mathematical education and is training researchers in this same area.

10. A person who can adequately serve on national committees charged with giving direction to mathematical education for all levels of ability.

11. One who has contributed significantly by writing research articles, books, reports, and other literature related directly to curriculum development and teaching-learning in the field of mathematics.

12. One who can speak to school teachers of mathematics and the teachers of mathematics teachers in a language that is easily understood and with mutual respect for each other's contributions.

In conclusion may I state a personal point of view born from experience during the last twenty years of involvement with leaders in many countries and their mode of operation. It is this: leadership in the USA can no longer be left to chance and spontaneity. Leadership in science education among the highly developed countries of the world now has a very significant implication, not only for placing their country first in world leadership, but even more so for the economic stability and enlightenment of all nations. For the United States of America leadership must now be sought out, centered, and used to give all the communities of the country the best information on mathematical education that is available. There is need for a few centers or locations where all the activities mentioned earlier can be carried on and where the mathematical community may look for guidance, direction, and decision making in the improvement and adaptation of their mathematics programs. This is not to stifle or deny the advantage of individual initiative which must always be encouraged to come forth and to thrive. However, such individuals can, by attaching themselves to one of the several centers, best reap the success of their own endeavors.
To aspire to leadership or to be enlisted, because of native endowment and cultural development, requires on the part of the individual a responsibility of service to society and a foreswearing of many personal, family, and social enjoyments. It requires exceedingly high scholarship, complete professional involvement, the risk of isolation from the "vie courant," the responsibility of decision making, and the abuse of those who disagree. In fact, it requires a "hero." Ben Johnson, the great English poet, is reputed to have said, "Claret is the liquor for boys, Port is for men, but he who aspires to be a hero must drink the "Eau de vie," Brandy. Now the time has come for Brandy.
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I've never been a discussant before so I didn't really know how to prepare. I assumed that you would sit and listen and made notes on all the mistakes that the speaker made and when it was over you would correct him and sit down. Since I disagree with so many people about so many things I thought that would be very easy to do. But as I sat here and listened I found that I had nothing to quarrel with. I found myself agreeing with just about everything.

There is one thing that was said, however, that touched a nerve and I am going to comment on that briefly. There was a reference to a mathematician who worked with a group of teachers, and he did it in secret lest his colleagues learn of this activity which he felt would reflect rather badly on him. I think what we are being told here is something which is quite important for us to think about. It is, that a gulf does exist between research mathematicians and the educational community at large. I don't know whether that gulf has always existed—I rather doubt that it has. I think that it is probably of somewhat modern origin. Even if that gulf had existed earlier, it was probably of less importance than it is today because the world was moving much slower. Suppose that a research mathematician discovered something new and important. Over a period of years his students learned it, passed it on to more students and finally it got into graduate courses. Then, bit by bit it seeped down into the undergraduate curriculum. And when it came to the attention of the educational community they began to act on it. But I think that the world now is beginning to move at such a pace that we do not have the time to wait for this process to happen in the traditional way. I think it is vitally important that we find means of bridging this gulf between research mathematicians and the educational community at large. So I am particularly interested in the remarks that Mr. Fehr made at the beginning of his talk—that we do need a corps of highly qualified, well trained people who can serve to establish lines of
communication between the research community and the educational community at large.

I think that we all know that there is historical information that is of pedagogical value. We all too often lose sight of the fact that history is being made today. So I think that it would be extremely valuable to the educational community at large if every research mathematician surrounded himself not only with a satellite of research students but also with a satellite of students whose concern was not with the creation of new mathematics but with understanding the situation and the context in which the mathematics of today is being created. To record and report on events. To make some kind of judgment and assessment of what is going on and to make this kind of information available to people of less mathematical sophistication. It seems to me that this kind of communication is really essential for those of us who are interested in mathematics in order to gain the information with sufficient rapidity, to bring forth the kinds of programs that will keep mathematics education current.
I assume that I should add a secondary school administrator's point of view to the presentation by Dr. Fehr and, to some degree, that of Dr. Smith. In "attacking" (I shouldn't use that word right off), in responding to Dr. Fehr's remarks tonight from the frame of reference of being in high school work, from being the recipient of the programs developed by research mathematicians and mathematics educators, and from being in schools that employ the teachers being trained in colleges today, I feel a little bit disturbed by some of the remarks that were made. I also am dismayed by what was left unsaid.

Right now some of the curriculum research and development is too advanced for what we are doing in high schools to be practical to us. Furthermore, there are so many proposals that we certainly cannot handle all of these products. Every institution has something that they have just developed, and they want us to move from this program to that program to the next program immediately. The fact is that we have not had time to evaluate the outcomes of the new curriculum which we incorporated the last time. Furthermore, the people who are developing programs are giving them to us in a frame of reference which we cannot duplicate in at least some high school districts. It is impossible for us to incorporate without extensive modifications some of the programs at the University of Illinois Laboratory School, for instance, because we do not have their facilities. On the other hand, the people developing new curricula are somewhat behind in their assessments of what is really taking place in the high schools. The realities of what is actually being done in the high schools should be examined as they start developing their programs.

What kinds of instructional materials and programs are being used in the
high schools? What kinds of moods or what kinds of fads seem to be permeating high school teaching? Through an examination of them, curriculum developers could straighten out some of the things we might be doing wrong. Conversely, they could incorporate into what they are doing some of the things that seem to be successful. If a close look were to be taken at what is being done in the high schools it would be found that one of the problems facing teachers is "how to develop on-the-spot curriculum." Despite the fact that there is a proliferation of new materials and new methodology, much of it just doesn't seem to quite fit into the needs and pressures which occur in specific elementary and high schools across Illinois and around the nation. Consequently, the high school teacher is having to develop his own curriculum, and this is something I would like to talk about at a later date.

Currently, our secondary schools are loaded with students who have a mathematical sophistication we did not have when we went to high school. It would take most secondary teachers too long to become as knowledgeable as all students in all areas of mathematics. That is all right, because the majority of high schoolers are not of this ilk. The majority of students are not thinking of becoming mathematicians. Many of them are taking just enough mathematics to get into college. Therefore, we must develop our educators so they will understand that they most likely will be teachers of mathematics, but not teachers of mathematicians.

In solving to some degree the drop-out problem, we now have students in school who would not have been with us in the past, and so we are dealing with a number of phase-out students. They are sitting there in class but, for all practical purposes, they have dropped out. Some of the programs that are being developed now and some of the proposals that Dr. Fehr made tonight—and the techniques and skills and the task orientation programs in teacher education that Dr. Smith talked about this afternoon—would almost leave the instruction of these
students out of the picture completely. That would be fine, except it cannot be done! We have these children in high school and we have to do something with them.

I would hope that the Triple T program we are talking about, that this conference is all about, would provide some leadership toward providing solutions to such problems. I would like to see the people who go into the Triple T program become leaders in this direction. However, I do not think they will become the leaders by necessarily following the thrust of the programs presented by Dr. Fehr.

The leadership of teacher trainers should not consist of people who are looking for cut-and-dried teaching techniques to pass on to their charges. They must come into the program with something that has not been mentioned to any great extent at all—the humanizing effects of teaching—and pass these on to future secondary school mathematics teachers.

Future teachers are going to have to take care of all the students I have mentioned—those who are not really interested in mathematics but who will become leaders in other fields, and those students who may not be leaders at all. These students must be taught and understood by their teachers. I do not believe that secondary school teachers will reach these students by adopting the attitude (and I hope the teacher trainers who are selected for this program will not teach with this attitude), "I'll teach, and you learn—or be damned!" Such students do not care that much for mathematics and the teachers will lose them. A second group is going to be lost unless they are provided with personalized techniques and personalized approaches to teaching. The third group that will be lost without the humanizing approach is the kind who are good mathematics students—and some of these intend to become secondary mathematics teachers themselves.

Right now some of the teachers we are getting—and I am sure some of you in secondary education and perhaps in elementary education would say the same thing—have the ambition to be teacher trainers in the future. Their original intent was to stay in secondary education. The reason for the change in intent is
that they have been working under college teachers with the "I teach, you learn" philosophy for a period of time and they have seen his technique. (It was mentioned this afternoon by Dr. Moser that you teach following the model you have seen taught, and that has impressed you.) The prospective high school teacher had been working with people who were teaching and not really reflecting upon the learner, and he noticed how well he got along. Well, why shouldn't he, working with college people of such high caliber? Then somewhere in his training program the prospective teacher is sent out to look into some high schools. He sees that you cannot use this technique with high school students, that high school students are not simply there following theory of learning courses. (Maybe someone should develop a course in theory of unlearning, or not learning, because many of these students do not follow the pattern described in theory courses at all.) If they would pay attention, the teacher could say, "Well, fine, I can see that you understand and I can now proceed to the next step." But the students are not paying any attention. As a result of this and similar experiences, the prospective secondary teacher becomes a candidate for college teaching where he feels he can place the responsibility for learning solely on the students' shoulders.

Therefore, I would hope that the people you choose for the Triple T program will become leaders in education, in mathematics education, and that they provide this leadership by developing good curriculum, by responding to good curriculum and transmitting it to their charges, and by passing on to their students something concerning the humanizing effect of teaching. Otherwise we are going to lose a lot of good elementary and secondary mathematics teachers before they arrive on our campuses.

In the program that finally develops from the Triple T, I would hope that one idea which will be examined is the development of specialists in mathematics teaching. We can no longer continue to develop all of our teachers to be the Renaissance mathematics teacher—the teacher who can teach everything to every
student in the high school. Perhaps the Triple T program could be used as the instrument to look in this direction. Dr. Fehr talked about the large amount of material that has to be studied today to become knowledgeable in the mathematical field, and I am convinced that there are few people who will become high school teachers who can master it all.

Within the development of their program, the Triple T planners might be able to do something about specialization in mathematics teaching. Why not work with some prospective teachers so they will become specialists in teaching children of specific ability areas such as the gifted or the slow learner. Why not train some of the future mathematics educators in the satellite possibilities that go with teaching. For example, there are so many new techniques made possible through developments in audiovisual materials and equipment and data processing, that perhaps we should attempt to train some of our people as specialists in these areas. Then, instead of worrying about the use of the audio-visual director in the high school, a staff member aware of the problems encountered in teaching mathematics could, as part of his assignment, assist his fellow mathematics teachers in their development of teaching aids and in the use of the newer methods and media.

Finally, when the Triple T program becomes fully operational, I hope that the people involved in "running the show" will consider some of the people in secondary education as collaborators within the development and implementation of the program. I would like them to consider the fact that people in the secondary school are educational leaders, too, and can provide a necessary dimension to the successful outcome of this venture.
Dr. Fehr has given a nice accounting of the role that mathematicians have played in the curriculum reform movement. They have performed an amazing test of the consistency between the logical organization of mathematics and the psychological development of the student. Their primary role has been that of reworking the theory presented through the school materials to make it logically and thereby perhaps even psychologically sound. The premise that the psychological and logical aspects are closely inter-related seems to me to be a good one; and apparently it has paid off. This task of clarifying the theory of school mathematics, is a relatively simple and remedial one. However, it does not produce guidelines for future improvements.

We are here to consider questions of leadership, and in connection with Dr. Fehr's paper, I wish to raise the question, "Leadership in what direction, and by whom?" It would seem that we must give increased attention to the improvement of mathematics programs at the level of the schools themselves. We might ask, then, of the needs for leadership in that dimension and how to produce it.

Also, we might note that most of the teacher's professional education is acquired while he is in-service, much of it within the school system proper. What about leadership in improving in-service education? Or again, we might look at the question of teacher education in general.

This Colloquium is directed at the level of Trainers of Teacher Trainers. In a recent meeting in Washington I heard talk of $T^4$ and $T^5$ levels. One does not have to go far on this $T$, $T^2$, $T^3$, ..., $T^n$, ... chain to lose track of the functions that such persons might be performing, let alone the directions in which they might be taking us. It is evident, though, that substantial improvements are needed on existing levels and that the hierarchy must be extended. From where will come
the leadership required to take substantial progress along these lines?

The matter of direction of leadership is geared closely to the directions in which the educational programs in and about mathematics are to take in the future. Those that create the programs will operate in a vacuum unless we can produce appropriate leadership at all supporting levels, starting with the identification and preparation of those to operate at the grass roots level within the school and college teaching staffs.

Relevant to this level, Dr. Shull raised the vital point of "purpose" with respect to schools adjusting from one program to another. It is reminiscent of a talk Dr. Feinstein gave some years back entitled, "What Are We Trying To Do?" which, in the present setting would be phrased, "Where Are We Trying To Go, And Why?" This, it seems to me, is the fundamental question that must be continually reviewed by all persons in leadership roles. And all too few seem to be concerned with it.

As an example of the directions in which such considerations might lead, I call your attention to the growing demands of communities for "humanizing" education if you prefer, for making it "more relevant." This I take to mean recasting curricula and pedagogy so that their benefits are more readily recognized by the student as fruitful, personal gains. (In considering this direction, I am admittedly stepping outside the popular image of mathematics departments of the technologically oriented university.)

Dr. Fehr indicated earlier that he distinguishes between training and education. I support that position. But as we look to the future only with respect to the content or "facts" of mathematics, are we really looking at education or are we focusing on a highly specialized program of training? Granted that the content objectives of mathematics education are the easiest ones on which to get a hold, but will we continue to be satisfied by paying only lip service to the broader aspects that really justify mathematics education for all? Will we continue to
focus only on utilitarian mathematics or will we progress also in the direction of clarifying and achieving those humanistic objectives of education in mathematics to which we have long subscribed?

At present, the humanistic aspect of mathematics curricula is very deficient. For years we have endorsed it in the schools; difficult computation and complex Euclidean proofs were long justified on the grounds that they helped improve the students' 'thinking power.' If today's social interaction is an index of the soundness of that rationale, it appears to have been a very poor one. Correspondingly, meager provisions in this direction have been popularized through the SMSG "column defense" of programs for solving linear equations. This striving for increased analytic ability is perhaps the major justification for including the computer in the school mathematics programs. The highly formal approach used in secondary school and college, where authors frequently and mistakenly equate formalism with rigor (and heuristicism with nonsense) apparently is being justified also on these grounds.

At the 1966 AMS-MAA Annual Meeting I was fortunate to be present, in a service capacity, at a closed press conference where R. Anderson, E. G. Begle, Morris Klein, Edwin Moishe and H. O. Pollak aired their views on the modern curriculum of the schools. I was struck at the time by a statement made by Moise in a last ditch effort to save a point. It went roughly, "We are not only trying to get kids to do mathematics like mathematicians do; we want them to think like mathematicians think." I am sorry that it came out in this way, for this is perhaps the most crucial issue of the liberal arts aspect of mathematics. That remark rings out a modern interpretation of philosophers' assertions from the ancients on. Education in mathematics can provide not only pragmatically useful information. Its more worthy contribution lies in what it can and should provide the student in developing his intellectual abilities.

Frankly, I doubt that the SMSG approach, or more generally that of the
formalists, contributes much more to the student in this direction than did exceedingly difficult "reckoning" years ago. In fact, such pure formalism does more to obscure the nature of mathematics than it does to clarify it. Most "mathematical thinking," like any other thinking, is done informally in variously refined stages of precision. If we are to expect such transfer of mathematical methodologies on a broad front, the objectives must be exhibited as being the end results of a developmental process, and the students must themselves become involved in the development.

We have a long way to go in this direction. The typical curriculum is still geared almost exclusively to technology-oriented students. The rest of the students, who constitute the majority, are justified in objecting to having to study mathematics at all. Those objections will continue to be well founded until the mathematics program is adjusted to contribute substantially and recognizably to general education.

We have yet to determine what there is in "...the way mathematicians think" that is desirable for all. At least it includes a facility with theorizing, the kind of mental activity that all of us are forced to use informally in day to day activities, where abilities to do so effectively are best incubated in the simple and precise framework of mathematics.

If education in mathematics (as opposed to "mathematical mechanics") is supposed to contribute substantially to one's capabilities as a citizen and to the fullness of life as a human being, then in this respect we are doing a meager job at best. Even doctoral students, in approaching their thesis topics, all too often are called upon for the first time to actually do mathematics, and most have never seen mathematics being done. The credentials to that point certify only that they were successful participants in the course-examination-grade game.

I think it is extremely important that we give increased attention throughout our programs to exposing mathematics as an art of conceptual analysis,
specialized by method, but not by content. Here, I tend to agree with many mathematicians that one probably cannot formally communicate the nature of mathematics to one who has never done mathematics. To humanize mathematics, we must engage the students at a very early age in doing mathematics, not merely in absorbing the mathematics that others have done. This would have a double advantage. First, it would justify mathematics as a required part of education. Secondly, it would provide students with a more valid basis for determining whether or not to pursue mathematics in preparation for advanced study.

Presently, such decisions are based solely on experience with engineering oriented, utilitarian mathematics through the calculus. In moving in these directions, we might do well to look at the work of R. L. Moore, and Polya, and the discovery method used by Robert Davis wherein even young children are led into doing mathematics. Whatever may be the short-comings of these efforts in terms of the content-oriented curriculum, it is fair to say that those students have a broader perspective than those who presently see no creativity in mathematics.

Expansion of the scope and purpose of education in mathematics, in these or comparable directions, would require a great deal of leadership at many levels. Who are such leaders to be and who is going to prepare them? Is such guidance to come from mathematics educators, from mathematicians, from educators, psychologists or philosophers? At present, this kind of background is not likely to be found much below the Ph. D. level in mathematics. But these new leaders would also have to be able to view the question of what education in mathematics should be doing for people as people as opposed merely to what it contributes to their ability to grasp or contribute to technology.

As for the supply of such leaders, we look most naturally to university mathematics departments or to departments of mathematics education. The latter seems to be the most unlikely choice, since we have failed to attack even
the problems in mathematics education in "...the way mathematicians think." Neither critical attitudes nor systematic approaches prevail in the field. So far, we have not endeavored to clarify our basic concepts (definitions (?)) or to establish their interrelationships (theorems (?)). The fact that mathematics education is partially empirical by no means excuses our failure to consolidate information in the field into systematized bodies of knowledge. Nor is it likely that a field which has failed to work analytically in its own area could produce leaders to generate such analytic abilities in others— at least not until some innovations such as those suggested by Professor Smith, in his paper, are forthcoming.

The situation is not much more promising for mathematics departments. It is becoming more widespread for universities to encourage mathematicians to exert leadership in improving education; occasionally even by recognizing such achievements as being comparable to productivity in research. It would be rare, however, for a mathematics department to endeavor to prepare future leaders in educational reform as such.

The problem seems to lie in the fact that university departments are rather tightly categorized, and the preparation of future leaders falls in the proper domain of none of them. Unless some institutions make unusual formal provisions, it would seem that the leaders will continue to appear on the scene as "strays" from mathematics or mathematics education.

Doubtless there are many other new directions for leadership, perhaps some that are even more pressing. My concern is that if we tend to focus, as Dr. Fehr seems to have done in his paper, on improving program content for technically oriented students, then we may fail to consider the other dimensions of mathematics education and in doing so fail to provide the leaders who can attend to them.
"PREPARATION FOR EFFECTIVE MATHEMATICS TEACHING IN THE INNER CITY"

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When Dr. Kenneth Travers called to ask if I would be willing to talk on the topic, "Educating for Effective Mathematics Teaching in the Inner City," I replied that I had had some experience teaching inner city youth and in working with inner city teachers, but I did not consider myself an authority on the subject. He very graciously said that he thought I could do the job and hung up before I had a chance to say, "no." This is one of the main reasons, I think, why I am here today.

My first step in preparation for this talk was to search the literature, particularly the literature of the past few years, to determine what research studies have been conducted and what we have learned from them. I discovered that very little is known about inner city youth, how they learn mathematics, what are the characteristics of good teachers in urban centers and the related educational problems. Much needs to be done in this extremely difficult area of teaching inner city youngsters.

The first point I would like to make is that as a result of the experiences which students have prior to becoming teachers, and their differing biological makeup, they enter universities and schools of education with widely diverse abilities. And this is, of course, what makes our job as teachers and teachers of teachers so difficult. Students differ in their ability to recall and understand the mathematics they have already learned, the speed and accuracy with which they learn new mathematics, the strategies of inquiry which they have formed and their desire to learn new mathematics. (1)

These differences become even more pronounced as our students become teachers and are confronted with a host of problems that they have never discussed in their education classes. The locale of the school, the educational resources
available to the teacher, the previous experiences of the learner, the educational
and moral leadership of the administrators in the community, the nature of the
subject matter—are just some of the variables influencing the learning situation.

As has been brought out repeatedly in the previous talks, we have tried
for years in institutions of higher learning to educate prospective teachers as if
they were being prepared to teach now, forgetting, as Dr. Fehr so adequately
pointed out, that most of these youngsters will be spending the major part of their
lives in the 21st Century. We are forgetting that it is not adequate just to equip
them for their present teaching, in spite of that, we are preparing most of our
teachers as if they are all going to be doing the same kind of work when they begin
to teach. I am pleased, of course, that this conference has seen fit to recognize
that the problems of teaching in the inner city are to a considerable extent differ-
ent from the problems of teaching in the more affluent suburbs. For the purposes
of our discussion I am going to divide my remarks into two chief categories.
Unfortunately, the union of these sets is not disjoint, but I hope I can keep the
overlap to a minimum and talk about two needs of the inner city teachers of which
we, as teachers of prospective teachers should be aware, (1) mathematics back-
ground and (2) professional competencies.

Mathematics Background

The recommendations of the CUPM(2) with regard to grades K-6 (and for
the benefit of some of you who perhaps are not overly familiar with this, let me
remind you what they are: twelve semester hours of mathematics for level 1; six
hours in the development of real numbers; three hours devoted to algebra; and
three hours devoted to geometry) were designed to serve as a minimum require-
ment for teachers of elementary school mathematics. I do not propose to list
here the specific requirements, the specific mathematical understandings which
teachers should possess. I will briefly touch on a few of these requirements a
little later, but the specific understandings which teachers should possess I
would rather not discuss at this time. You could do much better, I think, by consulting various of the CUPM publications. However, what was intended to serve as a minimum level of competence has become in many instances an upper bound for teacher preparation. In a study conducted by Paige and Beattie (3) in 1967, the investigators found that colleges in general were upgrading requirements in mathematics for prospective leaders although not up to the CUPM recommended twelve semester hours. As a result of the CUPM regional meetings, activities, and publications this study showed fourteen states had changed certification requirements (nine of these had no previous requirement) but eighteen states still had no mathematics requirement for certification. In some cases mathematics and science requirements were lumped together further obscuring the real problem. In 1967 I believe only one state had fully complied with the twelve hour recommendation of the CUPM. So we see that we cannot expect CUPM or any other mathematics organization to pave the way with roses. Local action is far more desirable.

The trend in the last decade as far as course requirements and certification laws are concerned makes it imperative that the content courses for teachers be taught by persons who know some mathematics and are aware of the problems teachers in inner city schools face. In an unpublished study conducted by the CMPTESM(4) in 1967, in only fourteen percent of the colleges and universities in the state of Illinois were the mathematics content courses for elementary education students taught by members of the mathematics department in the respective institutions. While the dangers of turning loose a research oriented Ph. D. on a class of defenseless girls are many, the prospect of a professor of Philosophy of Education teaching them mathematics is just as dreary. Thus the teachers of the teachers of mathematics themselves need very special qualifications.

Further, it would be desirable (were it realistic) to select for inner city schools students who have some proficiency in mathematics. Studies continue to
show that prospective teachers are earning higher scores on examinations of a content nature, but most of them still score below the median for eighth and ninth grade pupils studying algebra. Reys(5) found that only sixty percent of the items pertaining to the real number system were answered correctly by a group of elementary education majors. This same group missed eighty percent of the items on graphs and functions. Yet, how can we impose additional qualifications on prospective teachers of ghetto youth when we are already having an extremely difficult task of attracting capable teachers to our inner city schools?

I wish to list under the heading of "Mathematical Background," however, a plea for including in every prospective teacher's background a good treatment on Socratic, heuristic, inductive, discovery-oriented learning, problems, puzzles and devices. I use the word "discovery" with a great deal of care because much controversy exists concerning the phrases "discovery teaching," "discovery learning." A very good discussion of discovery learning is contained in the chapter in Keislar and Shulman's book, Learning by Discovery.(6) However, I am hoping the younger persons in the audience have not seen some of the discovery devices I am going to show you now.

"Discovery Oriented" Types of Activities

Will you pretend to be fourth-grade children for just a few minutes? I am going to write a succession of numbers and see if you can tell what should come next. In the first column at the right, I will start with "1", "3", "5", "7", "9". What comes next?

"11". You are a very intelligent group of fourth graders. I will finish the column with "13" and "15". (Let me state that in a serious mathematical sense, one really can not tell what number comes next. If these numbers are to form
sequences then the general term must be specified.) In the second column from the right I will start with "2", "3", "6", "7". What comes next? Those of you who said "10", "11", "14", "15" were thinking of the same simple pattern I was. In the first column we write one number, skip the next, write a number, skip the next, etc. In the second column, we start with "2", write two numbers, skip two numbers, write two numbers, skip two numbers, etc. In the third column from the right, we start with "4", write four numbers, skip four numbers, write four numbers. The fourth column from the right, we start with "8" and write eight numbers in succession. This same pattern could be used to extend the matrix to include numbers from one through thirty-one by including a fifth column headed "16", and to numbers one through sixty-three by a sixth column headed "32", etc.

I have done this with first and second grade children in the Doolittle School, Einstein and Pershing and several schools on the south side of Chicago where teachers said their pupils could not add, could not do anything. To use this game you direct a child, "Take a number from one to fifteen and do not tell me what it is." Then you ask, "Is it in the first column, second column, etc.?

Notice there are several things that come out of a device of this kind. First of all, it is what I call a "binary scrambler." It is one way we can force the first fifteen of the natural numbers to take one a binary designation because if you were to pick, say 13, "13" in binary designation would be "1101", would it not? If you look carefully you will see that the scheme I used to construct the little matrix represents 13\text{ten} as 1101\text{two}, and 13 = 1 + 4 + 8. All one does is add the numbers at the top of the columns where the answer given is yes. The sum of those numbers, of course, is the number one has in mind.
Here is a second device. Pick any six natural numbers, say 3, 5, 8, 4, 9 and 7. Then we add: $8 + 4 = 12$, $9 + 7 = 16$, and $3 + 5 = 8$. Then we add 16 and 12 to get 28; 16 and 8 to get 24; 8 and 12 to get 20. Now $12 + 24 = 36$, $16 + 20 = 36$, and $8 + 28 = 36$. Why are the encircled numerals all alike?

Before pupils can discover why this device works, they must be able to add. Furthermore, when children are confronted with situations of this kind they want to learn many mathematical ideas and skills they had never cared about before. As you can see, the explanation for this device is rather simple—all you are really doing is totalling the six natural numbers you began with.

These little devices which camouflage or obscure an obvious situation and force children to look for patterns and generalizations are desirable things to try. What is more they put the burden of much of the so-called "discovery" process on the learner. Max Beberman and others have pointed out that the difficulty of much of "discovery learning" is that it is the teacher who is doing the "discovering" and in too many instances the one who is doing the learning.

What we are looking for are devices which will transfer the responsibility for the learning to the learner. Here are more of these. Start with a 3x3 magic square. Fill in three of the cells as in Figure 3. Then ask the pupils to fill in each cell so that the sum of every column, every row, every diagonal will be 15, using each of the first nine natural numbers only once. By using the numbers two through ten, or three through eleven, etc. you can make the sums various natural numbers.
Russian Peasant multiplication (which all of you know is called bourgeois American multiplication in Russia) gives children the chance to extend concepts of binary numeration to ordinary multiplication. Imagine we wish to multiply thirteen by twenty three. We successively divide by two in the left-hand column, throwing out the remainders if the division is uneven. In the right-hand column we double each entry. Whenever an even number appears in the left-hand column we rule out the row. Six is even so we rule out six and forty six. We add all the numbers remaining in the right column and get two-hundred ninety nine. These are the sorts of things I mean by "discovery" type devices and there are hosts of them.

I think every prospective teacher who is going to be working in inner city schools ought to start collecting various kinds of discovery problems. Here is another kind of problem. A man drives up a one-mile hill at an average speed of thirty miles per hour. When he gets to the top, he decides he has been moving too slowly and wants to travel down the other side of the hill, also one mile in length, fast enough to average sixty miles per hour for the complete trip. If you give this problem to youngsters they will immediately tell you that if he goes up at thirty miles per hour and down at ninety miles per hour, he will average sixty miles per hour. Therefore, he cannot possibly average sixty miles per hour. Children need to know that there are problems incapable of solution.

To keep motivation high, crypto puzzles like SPIRO x 7 = AGNEW are of great help. The counterfeit coin problem--eight coins, one of which is off-weight--is typical of a whole class of weighing problems, which can be used effectively to attract the disinterested.

Arithmetic with frames, \( \lfloor \) least integer, \( \lceil \) greatest integer: devices as popularized by my colleague, David Page, add to the store of discovery...
oriented devices every teacher must have. His Standstill Points, Surface Area
with Blocks, and many other devices, should challenge most youngsters.

Problems based on number theory hold a fascination for children. Take
any number represented by a 3-digit numeral, say 658, and to this numeral append
the same digits, getting a 6-digit numeral: 658658. Now proceed to show that
this number 658658 is divisible by 7, 11, and 13. I tried it with a group of fifth-
grade suburban pupils; it was unfortunate that it was a group of suburban children.
One little boy after about one-half hour said, "Well, if it is divisible by 7, 11 and
13, then it must be divisible by 1001." I asked, "How did you get that?" He
replied, "If it is divisible by 7 and 11, it has got to be divisible by 77. If it is
divisible by 7, 11, and 13, then it has got to be divisible by 77 times 13 or 1001. If
you take 658 and multiply it by 1001, this first "one" gives you the 658, and that
second "one," all it does is gives the 658, but it just pushes it over three places.
That is why when you take any 3-digit number and multiply it by 1001, you get a 6-
digit number of the 3-digit pattern repeated." This boy is thinking mathematically.

Devices, puzzles, games, tricks to heighten interest may be used to illus-
trate principles, to demonstrate that mathematics may be worthwhile and enjoyable,
but do not take the place of a good understanding of the real number system.

With the recent trend toward a laboratory approach in the teaching of
mathematics, it would be helpful for teachers to learn to make homemade levels,
transits, angle measures, sextants and the like. For certain kinds of learning
situations, involving the student in the construction of devices he will use to
develop some generalization or principle is extremely desirable.

Professional Competencies

The teacher of mathematics of inner city youth must have infinite patience
and perseverance. Ghetto youth do not seem to learn mathematics as well as their
suburban counterparts—if standard evaluative instruments which we use have any
reliability and validity. Let me give you one specific instance of what I am talking
Of the freshmen who entered the University of Illinois in the fall of 1967, the mean ACT math score for the negro students was 21.6, but the mean score for the total class was 27.3\(^{8}\), almost a stanine difference. This is indeed significant by any statistical criteria. I am not asserting that the difference in the two means is solely a result of racial differences --although Arthur R. Jensen\(^{9}\) claims that intelligence is a matter, in many instances, of such differences. Too many variables enter into the computation of ACT scores, but the difference in the means is still quite disturbing. From my own experience as a consultant to the Doolittle Project\(^{10}\) and other experiments in inner city education, too many teachers tend to give up, to cop out. Many of them told me that teaching mathematics to all negro youngsters was an impossible task. Clearly any teacher who wishes to succeed in teaching mathematics to inner city youth will require googols of patience.

The second quality which is essential for teachers of inner city youth is exceptional ability to communicate, and I am using the word "communicate" in its fullest sense. Teachers need to be able to use language --verbal, symbolic, abstract --to help the child arrive at patterns and generalizations in his mathematics learning. Engelmann\(^{11}\) has called attention to the fact that language is not a substitute for effective teaching experiences and situations but skillful communication makes for a lessening of ambiguity in learning situations which employ no language. In 1938, I participated in an experiment conducted in several south side schools of Chicago using the McDade non-oral technique for teaching mathematics. For an inexperienced teacher who enjoyed talking (the Achilles tendon of our profession), I found my situation almost unbearable. No communication except phrases such as "right," "wrong," "try it again," was permitted. The experiment showed the technique to be a failure.

How do colleges and universities improve student's abilities to communicate? I am not sure I know. I am simply calling attention to the fact that the more
difficult the learning situation, the more necessary are effective communication skills. Robert Davis calls this "the rhetoric gap."

Our potential teacher of mathematics needs to have some course work in learning theory, particularly as it applies to mathematics. She needs to know some basic information and concepts of learning and to be able to read the research in learning experiments which is beginning to appear in journals teachers usually read.

Every teacher whether or not she is aware of it has some psychology of learning which motivates her teaching. What sort of a learning model does a teacher follow who makes children write addition and multiplication facts hundreds of times? What psychology of learning motivates the teacher at evaluation time to ask the children to repeat on exams the very same information she has previously given them and in her own language if possible?

A brief treatment of the work of Piaget is more desirable. Piaget has written so many books, has had so many disciples and interpreters that one hardly knows where to begin a study of his ideas. I would suggest Hans Furth's book entitled Piaget and Knowledge as a good introduction to Piaget. Piaget is "a biologist concerned with the problem of the adaptation of a species to its environment." (12) How different his approach is to that of Jensen! To "prove" that compensatory and other educational programs being tried in ghetto schools are failing Jensen uses gain or lack of gain in I.Q. (13) Jensen would have most black youngsters learn only Level I learnings—trial-and-error learning, object memory, and other simple associative learnings. Level II learnings—complex conceptual learnings—would be left for the intellectually capable, a judgment made chiefly on the basis of I.Q. If teachers are fed this God-given concept of intelligences in their professional courses is it any wonder that many say black children cannot learn mathematics? Teachers need to see the differences in approach of Robert Gagne, John Carroll, Ben Bloom with that of Jerome Bruner, Piaget,
Beberman and others. At this point I would like to raise several questions which indicate directions in which we must be thinking. Does mathematics learning proceed sequentially? Does breaking a subject into small units and testing frequently help a child over any hump that gets in his way? Of what significance is Piaget's conversation principle? Does Gagné's learning hierarchy principle (learning is an ascending staircase of ordered levels or blocks. One cannot take on the fourth level without having learned the third) lend itself more readily to inner city youth studying mathematics than the ideas of Bruner? What role does failure play in the education of ghetto youth? Perhaps we ought to work toward Glasser's idea of schools without failure! Can it be that the mathematics programs need a greater relevance for pupils both in and out of school? Is a laboratory approach to learning the answer to learning problems encountered by big-city youth?

Well, these are questions I am raising with you. I do not have the answers. Now let me quickly suggest five or six problems to think about, and then I will have said more than I should have.

I think we should make use of the skillful teachers of inner city youth by bringing them into the colleges and universities of the nation. This is a little hard for many of our stuffy universities. I am suggesting that this is possible. The SEEK project in New York which I visited two years ago employed teachers who had no degrees in some instances, not even a baccalureate. And these people were members of the faculty of New York University, had the same rank as their counterparts in the university, and drew the same pay as people with Ph. D's.

The SEEK project showed that such kinds of employment of city teachers is possible—that is, of teachers who are skillful in working with inner city youth.

My second point is this: let us give assistance to the inner city teachers. Several people have mentioned this lack of articulation. We do not have the help...
that we need. Let us give the city schools a lot of help. We know so little about the education of inner city youth that it would be miraculous if we did a good job of preparing our teachers so that when they got into their teaching they needed no help in-service. In other words, I am saying that as teachers of teachers let us not forsake our former students. Let us not forsake them once they get into the teaching field because they need help much more desperately after they start to teach than before. I think this is one of the problems—that we ought to have a continuing kind of evaluation, an ongoing program where once teachers start to teach we have a kind of feedback from them so we will know the ways in which they think we shortchanged them and failed to prepare them.

Let us search for leaders to function at the local school level. This is one serious gap. It is one thing to talk about the leadership up on high (and I think we have plenty of that and it is pretty good in most instances), but I think where we are hurting is in the area of leaders at the local levels—leaders who can come into the schools and help the teachers at the local level with the problems they are facing. Let us search for such people and when we find them, let us employ them.

Fourthly, let us form in the universities of the state and nation groups of leaders who are available at all times to serve as resource persons to assist inner city schools where teachers get into difficulty. This could be almost like the system of lecturers employed by the Illinois Section of the MAA in which very competent persons go into the high schools and lend their enthusiasm and knowledge to those who are studying mathematics.

Next, let us encourage universities and colleges to embark on experimental projects such as the Elementary School Mathematics Advisor Training Project. Programs of this kind take up some of the slack left by the pre-service, in-service gaps and other aspects of the total education program. In other words, I am proposing that we hit this problem on all fronts—pre-service, in-service, after in-service. Let us hit it on as many approaches as we can.
Let us put to work some of the best brains we can find to create text materials which will be suitable for use by inner city teachers. Inner city youth lack the communicative skills—reading level, vocabulary level, symbolism—of their more favored suburban youth. Materials suited for suburban schools do not usually fit the inner city schools.

Finally, let us improve the articulation between the colleges, inner city high schools and elementary schools, so that we can share our common knowledge and begin to attack our common problems and arrive at, if not total solutions, some partial solutions of our problems. We have yet to do this. I think you cannot mention a single organization in the state of Illinois which is really doing all of this. Although we do have some organizations which are attacking parts of the problems, not all of the problems are being faced.

Well, I have enjoyed talking to you very much and I hope you have gleaned something from the ideas I have tried to convey.
REFERENCES


2. Committee on the Undergraduate Program in Mathematics, Course Guide for the Training of Teachers of Elementary School Mathematics, Berkeley, California, Revised 1968.


4. For a brief discussion of the CMPTESM see the Mathematics Monthly, November, 1968, p. 1046. CMPTESM is an acronym, Committee on Mathematical Preparation of Teachers of Elementary School Mathematics.


It may be helpful at this stage of the colloquium to think of what has happened and where we have gone. We need the forefront of pedagogy and we need technology to assist us in taping and analyzing our teaching. We all need it. We need to have the prototype of a leader spelled out for us. We can all try to attain it. And then we come to another need. The Triple T program needs the city and we need them. And we are so pleased at the prospects of a partnership between the universities and the cities. We in the inner-city need all the help we can get.

Let me first fill in a little the picture of the under-achiever, or what I call the "mathematically under-developed." Let us look at the mean achievement of the inner-city youngsters as they are coming through the elementary grades. At third grade they are about one-half year behind intermediate achievement. (and I will still pay some lip service to the limitations of testing) At sixth grade they are already about one year behind. At eighth grade they are about a year and a half to two years retarded in arithmetic achievement. A large proportion of this group is coming to high school absolutely unprepared to take algebra. And the numbers are growing and growing. By the time these students are in secondary school they have taken another way out. They have gone in for absenteeism.

We have another problem. What about some of the methods that we have to develop for the teacher of this group? The young teacher fresh from methods courses comes in and starts a beautiful lecture. What happens? This group of under-achievers all know something and they all don't know something. So whatever you say in an expository method suddenly meets resistance. We have to suggest limiting that methodology in teaching. We have to start tailoring much more individual approaches. We need much individualization of material. Let us hope that Dr. Feinstein gave us the incentive to try to develop more such material.
I was pleased to hear reference to Piaget. By the time we are in secondary with these youngsters, trying to visualize is a huge task. How can we visualize 2x3, say, when we do not know about 2 and we do not know about 3? Then we are going to operate on them—say 2x3. May I give a message to the teachers of teachers? Please, even if your college does divide methodology for those people who decided to be secondary teachers only and those who decide to be elementary teachers only, let us slip in for secondary teachers a lot of the necessary elementary methods. Even if you are at the secondary level, go back and demonstrate and show in every way possible. The other day I saw a teacher demonstrating using a place value board. It was one of these that has beads that flop over, and suddenly I was thinking that by the hearing, "one, two" even the sound of the flopping helped. Let us take this methodology and let us not forget it.

There is one other point that Dr. Feinstein did not make directly. For this teacher of inner-city youngsters, we need to do everything we can to have teaching humanized. Everybody has his own definition of humanization. I would like to suggest that the schools of education have an oath which is analogous to the oath taken by the physician. The oath is "Let us use what testing and guidance shows us. Let us recognize differences. Let us take that extra little time."

I would like to enlarge the definition of inner-city beyond Black. I was in an inner-city school the other day that was the most terrific polyglot that you can imagine. I had time to help this teacher go to the folders and see some of the backgrounds. We had Black, we had Indian, we had Chinese. We had Puerto Rican who did not speak English and some who did. We had a young boy whose folder showed that he was in this class because of a language difficulty. He was Norwegian. He probably knew the mathematics. We must enlarge our concept of the inner-city.

Then, let us think a little about where the teacher of teachers operates. The obvious place is in the school of education and the universities. Next down
Mrs. Lucile B. Jautz

the line we have all kinds of in-service. I think Dr. Feinstein was representing something I always talk about -- the vending machine approach. Sometimes as a consultant I feel that a vending machine approach would provide some of his quickie motivating ideas. Sometimes there is a need for interesting skill sheets that you can develop ahead of time, some new ways of drilling -- skill drill, or some sheet that will help us just as a quick handout, to suggest, say, another way of coming at equivalent fractions. In our vending machine we also need diagnostic approaches and diagnostic testing. Let us try to take care of those 25 to 30 individuals in front of us, with all different degrees of failing to understand mathematics. Let us tailor our techniques. Let us be sure of all the possible diagnostic approaches we can use and take it from there.

In our bag of tricks for the teachers of the inner-city youngsters, we need all of the audio-visual devices we can think of. We can use prepared overhead transparencies. We can use our overhead as a change of pace. We can include films and filmstrips. And we can attend professional meetings such as those of the Illinois Council of Teachers of Mathematics. For instance, there will be a film showing this evening which will include a very interesting film on the laboratory approach. It has to do with discovering similarities in prisms. It is a new film that tailors itself to the lab approach. I think the laboratory approach has much to be said for it. It provides visualization and individualizing tasks.

We have some successful programs in our inner-city schools. A desk calculator approach can be very successful. I am always singing the praises of it because I have seen it operate in a suburb of Chicago with some youngsters, that I would call comparable to about the best inner-city we can give you. These youngsters are coming into the high school level with sixth grade and below ability, and through a desk calculator approach are getting an answer, a right answer, that they never saw before. Gradually they can be led through all the trouble spots and weaned from the desk calculator into a textbook situation and
Then, somewhere along the line I noticed we had not talked much of computer assisted and programmed learning, such as the PLATO program here at the University of Illinois. In a diagnostic approach to arithmetic, the SRA program that was tried out in several of our inner-city elementary schools is worth knowing something more about. It was strictly a drill diagnostic technique. The writers of the program started out with fourteen or fifteen kinds of addition items. Then they would start the youngsters supposedly in the middle of the program and if they missed, they backtracked, were drilled, and retested repeatedly. We found that fifteen steps were not enough. We began to see that ordinary addition is a complex hierarchy. An understanding of these hierarchies is necessary if we are to do effective individualizing.

These are but some of the problems facing us in secondary schools. My colleague, Mr. Maher, will discuss the situation at the elementary level.
COMMENTS ON I. K. FEINSTEIN'S PAPER
"PREPARATION FOR EFFECTIVE MATHEMATICS TEACHING IN THE INNER CITY"

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Let me start off by saying that as a result of listening to the speakers of yesterday and, in part, this morning, I find myself in a little dilemma. It is somewhat like the three fellows who went down to the dock to take a trip overseas. In their excitement they arrived at the dock several hours too early, so they went into the corner pub. After imbibing for quite a while, one of them noticed the boat pulling out. So quickly they filed off and when the gangplank was being pulled in, two of them got on and one fell in the drink. An officer standing by, after fishing this fellow out, said, "Well, that is a pretty good average, two out of three." He said, "Pretty good average my foot, they came down here to see me off!"

This is where I find myself. I am not in the swim here because the inner-city has many more problems than were touched on here. As far as I can see, the mathematical problem is not much different, shall I say, than science, language arts, social studies, and so on. In regard to Dr. Feinstein's remarks, let me get the equipment situation cleared up. The inner-city schools have far more equipment than any of our schools in our so-called better neighborhoods. I have been working in one of the more affluent districts this last couple of weeks and we are lucky if we find one overhead and one projector. But in the inner-city they have probably one overhead projector for every room, or if they are badly off, one for every two or three rooms. And this goes all the way down the line. So do not get the impression they do not have things to work with. However, after having them now for several years, it is likely that much of the equipment is inoperable because of the many things that happen in the inner-city. Much of the equipment is broken, deliberately. Much of it is stolen and cannot be replaced or has not been replaced. So these things of course do limit us and some of the
equipment that the government did share with the city, unfortunately, has limited usefulness. For instance, there are some single concept machines that use eight millimeter filmstrips or cartridge. But most of the cartridges today are made for super eight filmstrips so these machines are outdated before they have been used.

Well, let us go back to the problems of the inner-city. I think one of the major problems is attitude, not only of the teachers themselves, but of the children and of the parents. A number of teachers, I find, take the position that these children cannot be taught. This was touched on by the previous speakers. This attitude probably is more prevalent among some younger teachers than it is among the experienced teachers. And in the inner-city, unfortunately, we are getting more and more of the newer, inexperienced teachers. So the attitude of the teacher in many cases has to be changed. And if the attitude of the teacher will change, probably the attitude of the children will change.

The attitude of the parents, of course, is very important also. At the preschool level we do start to involve the parents. We call this our child-parent center program. Many of you probably are familiar with these centers. The children there go through kindergarten and first grade so the parents must be involved. That is, the child cannot be enrolled unless the parent is willing to come in frequently and participate in the program. And, of course, this is not only in mathematics, but the entire curriculum. This program has been very successful thus far.

But it does not go far enough and soon they have the crowded conditions of the regular schoolroom. It is a lot different when these people from particular classes that are blessed with twenty to twenty-five children are put in the regular school at the second grade level where the classes probably number upward from thirty-five to forty-three. Coming from a more selected basis in the beginning, they go into the second grade with all other kinds of problems. And with children that are not really ready for the second grade, they are in with children
that are probably in need of social adjusting. All these are confronting the teacher so that they have to consider these pupils as well as the so-called normal children. This, of course, goes on in many, many rooms in the inner-city where there are also EMH pupils. We do have special rooms for EMH, although we do not have enough. And consequently these people who are waiting for space are in the regular classroom and they take a lot more attention than the teacher can really afford.

Another big problem is the inner-city's transiency of the children themselves. It seems that they are moving constantly. I was talking to a little girl the other day who was eleven years old, I believe, and had been in eight schools. Anyone who has been in eight schools in that short a time, even with the best teacher in every school, would still be behind. Still another problem is that of truancy. We have much more of that in the inner-city than in the rest of the city. It is probably just as great among Appalachian whites which is another inner-city area.

So there are many more problems than just the teachers being knowledgable about mathematics. I think in general most of them have a pretty good content background. It is true they need other methods of showing and interesting the children. Dr. Feinstein showed what could be used and the equipment they have lends itself to this. Consequently, the teachers do have equipment, they do have knowledge, but they must have the attention of the children before they can do any teaching. This is well illustrated by the brilliant new Educational-TV show for pre-school children with which you are probably familiar. "Sesame Street," as you know, makes use of many unique and interesting techniques of gaining attention and reenforcing learning. But this is sometimes hard to do in the classroom with all the other obstacles.

I think that is all I will say. I hope you have some questions that we can discuss.
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