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ABSTRACT

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ANALYTICAL PLANNING FOR UNIVERSITY LIBRARIES

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PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

This paper reports on a microeconomic analysis of the circulation function of libraries. This analysis derives average and marginal cost curves and least cost circulation policies. Circulation is only one part of total library operations and this analysis recognizes the existence and importance of other library functions. However, while the quantitative models presented in this paper are abstractions from reality, they do indicate the current state of the art in the application of operations research to library management.

ABSTRACT

A survey is made of the more important technological and managerial problems in the planning of university library services and recommendations are made for a positive program of innovation and development. Two approaches are explored in considerable detail. The first is the use of operations research models of the acquisition and storage functions. Elementary models and decision rules, based on the assumptions of exponential growth, independence of item usage, and obsolescence, are used to minimize average costs of circulation and to suggest more general models for library services.

This is an exploratory study of the problems in library planning. This report does not pretend to offer a definitive statement on the subject; it does not provide any well-tested models, data for the design of library systems, nor a thorough analysis of jointly dependent items. It does attempt an assessment of the current state of the art and to identify some promising and different directions for the development of planning criteria and techniques of analysis.

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I. INTRODUCTION

The development of libraries in the future depends heavily on the outcome of several important issues. The most frequently cited issue is the impact of the computer and the possible replacement of the book by some array of cybernetic devices in the long run - by the year 2000 or later. A considerable amount of basic research along these lines is focused on the medium range objective or creating large scale, computer based bibliographic control systems to monitor the flow of documents in conventional or miniaturized form. While this latter effort may be at least 10 to 20 years from fulfillment, its appeal is strong enough to retard experimentation in larger libraries with the kinds of automated document control systems for which the technical knowledge is available. Such short-range development is being pursued, however, in smaller, industrial-type special libraries. The reluctance of larger libraries to press for such experimentation deprives them of a valuable learning process and helps retard long-range developments.

For example, there is a serious lack of reliable data on the operational characteristics of libraries, and very few operational studies of the measurement of library performance or the derivation of a value structure necessary to compare available alternatives. There have been very few serious attempts to evaluate or implement the operations research that has been conducted in library systems. However, it is more likely that such work will be done in connection with a positive program of library automation and innovation than in a conservative, "wait and see" environment.

An important conceptual problem in library systems research is the relationship between storage and retrieval functions. The development of a "theory of libraries" around this issue is needed to give direction to long-run research and development efforts. More operational analysis is needed to clarify this issue and to provide the groundwork for an acceptable theory. The balancing of storage and retrieval services may be the most important factor influencing library's effectiveness, especially under conditions of exponential growth.

In addition, too little is known about the role of the user in library systems. Even where user needs can be aggregated and represented by institutional objectives, there is little understanding of the relation between library capabilities, user behavior, and institutional controls [Churchman 1968; Baker 1967] The initiative in clarifying this triangle of interests should be taken by the library itself, and it should begin by making a concentrated effort to re-examine and to develop new methods of communicating with users and with funders.

Program budgeting, supported by systems analysis, appears to be a promising approach toward the creation of a new kind of "language" for libraries to use in arguing their case with users and funders. This approach seems to be a necessary and desirable alternative to either placing libraries in direct economic competition with other services or maintaining them as a subsidized and privileged sector in the academic community [Keller 1969, Shishko 1968].

In most areas where a lack of research and development is found, it is not too difficult to prescribe suitable remedies. There is need for more operational experience with computer-based systems, more analysis of storage and retrieval operations, and more attention to the language and

procedures used in processing planning and control data. All of these efforts imply the diversion of a considerable portion of the library budget to research, planning, and control activities.

It is not easy, however, to prescribe a remedy for the lack of understanding in one critical area of library planning, namely the assessment of library performance. The approach taken by this paper has focused on the problems of measuring library costs and how such costs might be reduced by the choice of an operating policy. Decision rules are developed for minimizing average cost per unit of service, which is the type of cost-benefit ratio used in business analysis. While this approach can draw on a rich history of business practice and economic theory, and while it can provide a basis for the processing of much planning and control data. it seems to beg the all important question of benefit measurement. There is little comfort in noting that this same problem plagues the education field generally and in other public sector areas outside the market place.

It is unlikely that the general problems of welfare economics are going to be solved in the library field. However, some part of this solution has to be attempted within library circles in order to continue to justify the costs of conventional systems and the investment in innovation. The lack of suitable theoretical and empirical data on library benefits should not be used as an excuse to defer the study of library costs nor should the relative ease of getting at costs be used to hide the necessity of trying to get at the benefits. Just as the emphasis on the storage function pervades the analysis in this report and focuses attention on costs, it is likely that a similar analytic study of the retrieval function will lead to a better understanding of benefits. Such a study deserves further attention.

II LIBRARY MODELS AND POLICIES

1. Library Storage Models

Information storage theory is a relatively new and developing area of research. Libraries are among the world's oldest and largest information systems, and they provide a rich history of operational experience for the student of general information systems and a large working environment in which to test new design concepts. Although conventional libraries are essentially manual systems for the handling of mechanically-stored information in book form, many of their operating characteristics are readily transferable to more sophisticated systems using computers and microform storage devices. This is apparent in library operations research studies such as the work of P. M. Morse [1968] at M.I.T. Library operations research studies have concentrated on the problems of storing and using library materials, while library and information scientists have focused on problems of organizing and retrieving these materials according to their intellectual content. The latter problems seem to constitute a more difficult long-run research field, since the introduction of the newer methods of information storage preclude direct user access and require newer methods of obtaining remote intellectual access to the file.

Much of the operational analysis of libraries is related directly to the problem of library size, and the use of such options as depositories, inter-library loans, blanket orders, duplication, and compact storage, as means of optimizing library size relative to the observed usage of the library. Usually, the library under study is thought of as a member of a larger information network which permits local suboptimization without precluding the possibility of the user going elsewhere for information. A good prototype example of this kind of approach is the model proposed by P. F. Cole [1962], and refined by

M. K. Buckland and I. Woodburn [1968], by which it is shown that a 2,000 volume petroleum library can expect to satisfy the greatest number of user requests by subscribing to approximately 190 different journals or serials and holding them for about eleven years. Variations on this theme of "optimal library size" are seen in the study of depositories by Morse [1968] and W. C. Lister [1967] and the study of interlibrary loan by G. Williams [1968]. A more sophisticated approach is the fully stochastic model of H. M. Gurk and J. Minker [1968] which studies the effect of retention policies on the size of a data base for a computer utility.

The size of a library or data base seems to be the most important measure of its worth apart from its usage, since it suggests comprehensiveness or completeness of knowledge. This has long been the traditional measure of stature in library circles. The two important determining factors of size are the breadth of acquisition and the length of retention. These are also important factors in determining usage, along with the ease of access. While some models have been developed which concentrate on library breadth, (see Leimkuhler [1967, 1968]), the problem of retention time has been given the greatest attention. The storage cost models and storage policies developed below are intended to reveal some of the essential economic characteristics of information storage systems in an elementary way by developing decision rules which are both practicable and near-optimal.

The present study gives particular attention to phenomena of exponential growth and obsolescence in library materials and their effect on acquisition, storage, and circulation. Exponential growth is sometimes called the "law of libraries", and is the most common way of describing the so-called "information explosion". The first substantial study of library growth was made by Fremont Rider in the 1930's, who traced its course back to the earliest libraries and developed an authentic systems approach to the study of information storage

problems. He came to the conclusion "that no emendations in present library method alone were going to provide a sufficient solution to our growth problem," and sought a "new systhesis" of library functions which would be based on microform technology. A few years later, Vannevar Bush coupled microreproduction with computer technology as the basis for developing a new approach to the problems of information storage and retrieval. At about the same time in England, S. C. Bradford did the basic work on the need and feasibility of establishing national and international document control systems and services. Much of the subsequent work in library development can be traced to the efforts of these three pioneers.

The following models are more in line with the work of Rider. They focus on the library as an ongoing local information service agency which satisfies the demand for books and periodicals according to well established methods of acquiring and shelving these items. It is presumed that a library has already justified its existence as the best way to provide this kind of service and that readers are making worthwhile use of it. The models are intended to describe how this system functions in the simplest possible way in order to better understand how growth and scale of operation contribute to its cost and influence usage. This should provide a better basis for planning the future development of libraries, controlling growth, and fostering innovations.

2. Cost of Storing a Single Item

Recent studies of the cost of operating library-type information systems such as the work of Williams [1968], and R. Shishko [1968], suggest the following cost model for information storage systems for independent items:

$$K(t) = k_1 + k_2 t + k_3 u(t) \qquad (1)$$

Here $K(t)$ represents the total cost of holding one item for a period of t years; k_1 is the initial cost of acquiring the item; $k_2 t$ is the holding

cost which is linearly related to the retention period; and $k_3u(t)$ is the usage cost which is proportional to the number of uses made of the item during the period t. This model is consistent with those used by Lister [1967] and Buckland [1968], although their models included more terms in order to recognize other control variables. Equation (1) could be discounted in order to obtain its equivalent present value as was done in the study by Williams. Equation (1) is not supposed to represent the ordinary way in which the costs of libraries or other types of information systems are reported for either budgetary or cost control purposes. Rather, it is intended to express storage cost as a function of time and usage in the simplest possible manner. There is no theoretical reason, for example, for not including user costs in the parameters along with the direct and indirect costs of the storage system proper.

In his study of book use models, A. K. Jain [1967] described several models which express book usage as a function of age. In all of these models, the cumulative use, $u(t)$, increases monotonically with t, while $u'(t)$ decreases. The simplest of these models is the exponential case, that is:

$$u'(t) = re^{-bt} \quad (2)$$

$$u(t) = (r/b)(1 - e^{-bt}) \quad (3)$$

where r is a scale parameter associated with the instantaneous initial usage level and b denotes the instantaneous obsolescence rate. The ratio (r/b) is the limit of $u(t)$ as t approaches infinity and therefore, a measure of the lifetime usage of the item. Based on an extensive study of the M. I. T. Libraries, Morse [1968] proposed a usage model similar to that of equation (3) but including a constant or residual

use term which is independent of age, that is, the usage rate drops exponentially to a residual level. He showed that this model results from a simple Markov process for the change in usage from year to year.

By substituting equation (3) into equation (1), the total, marginal, and average costs as a function of holding time are obtained respectively, as follows:

$$K(t) = k_1 + k_2 t + k_3 (r/b)(1 - e^{-bt}) \quad (4)$$

$$K'(t) = k_2 + k_3 r e^{-bt} \quad (5)$$

$$\overline{K}(t) = (k_1/t) + k_2 + k_3 (r/bt)(1 - e^{-bt}) \quad (6)$$

Both the marginal cost and average cost of retention time diminish to the level k_2 as the holding period increases, and the total cost becomes increasingly linear with time.

3. Cost of Providing Uses of an Item

A more interesting and useful cost relationship is obtained by expressing the total cost as a function of the cumulative usage during the retention period. By inverting equation (3), one obtains the time required to provide the first u uses of an item in storage, that is:

$$t(u) = \ln(1 - bu/r)^{-1/b} = (-1/b)\ln(1 - bu/r) \quad (7)$$

By substituting equation (7) into equation (1), the total cost for providing the first u uses is defined as follows:

$$K(u) = k_1 - (k_2/b)\ln(1 - bu/r) + k_3 u \quad (8)$$

The marginal cost for providing the u^{th} service is approximately equal to

$$K'(u) = k_3 + k_2/b(1 - bu/r) \tag{9}$$

where it is assumed that the derivative of $K(u)$ approximates the finite difference, $K(u) - K(u-1)$. The average cost of providing the first uses of an item is defined by the equation:

$$\overline{K}(u) = (k_1/u) - (k_2/bu)\ln(1 - bu/r) + k_3 \tag{10}$$

While both the total cost and marginal cost of usage increase monotonically and quite rapidly with increased usage, the average cost decreases at first and then increases with usage.

The implications of equations (8), (9), and (10) can be more readily seen if they are expressed in terms of a relative measure of usage, x , which is the ratio of the cumulative usage over the lifetime usage, that is;

$$x = bu/r \tag{11}$$

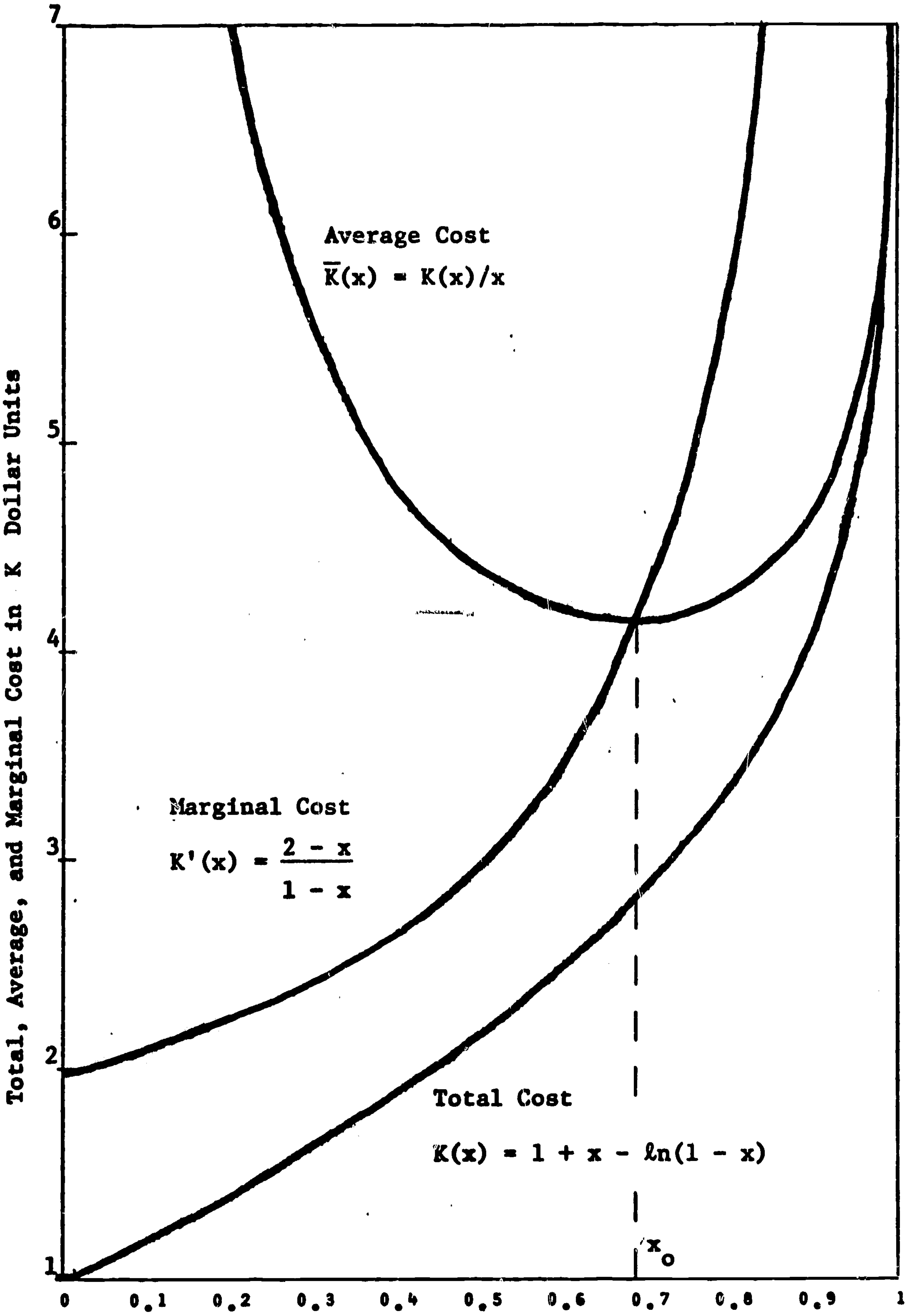
It is convenient also to define the parameters K_2 and K_3 as follows:

$$K_2 = k_2/b \tag{12}$$

$$K_3 = rk_3/b \tag{13}$$

where K_3 can be interpreted as the total lifetime usage cost of an item, and K_2 as the holding cost for a relaxation interval, $1/b$. By using these definitions, the equations for the total, marginal, and average cost of usage become:

$$K(x) = k_1 - K_2\ln(1 - x) + K_3x \tag{14}$$



Cumulative Usage, x , Fraction of Lifetime Use

Figure 1--Information Storage Costs of One Item as a Function of its Usage, When Parameters k_1 , k_2 , and k_3 Are Equal to K .

$$K'(x) = K_3 + K_2/(1 - x) \tag{15}$$

$$\overline{K}(x) = (k_1/x) - (K_2/x)\ln(1 - x) + K_3 \tag{16}$$

These relationships are plotted in Figure 1 to show their general shape and properties. The plotted values are based on the arbitrary assumption that k_1 , k_2 , and k_3 are of equal magnitude.

$$\overline{K}'(x) = \frac{-k_1}{x^2} + \frac{K_2}{x^2}\ln(1-x) + \frac{K_2}{x}\frac{1}{1-x} = 0$$

$$-k_1 + K_2\ln(1-x) + K_2x\frac{1}{1-x} = 0$$

$$\frac{k_1}{K_2} = \ln(1-x) + \frac{x}{1-x} \tag{17}$$

4. Storage Policies for a Single Item

The total cost function, $K(x)$, consists of a linearly increasing component and a logarithmically increasing component which are weighted with the time-cost for storage. When the time-cost parameter, K_2 , is relatively large, the total cost increases quite rapidly for higher values of x . This is reflected in the marginal cost which increases much faster than total cost. If regulations allow, it is reasonable to expect a library to discard an item before it has exhausted all of its potential usage in order to avoid the extremely high cost of continuing to hold the item indefinitely. In practice, it is more common for libraries to transfer infrequently used items to depositories unless assured of their availability in some other cooperating library. The experience with depositories has suggested that there is a significant cost associated with the selection and recording of such transfers. Much of this cost might properly be considered as an acquisition cost for the depository collection, although there would be some cost of changing records in the primary collection. The present model is not intended to account for all of the various options which are available to a library, although it could be expanded to include such options.

From the viewpoint of microeconomic analysis, a policy for limiting the retention time of an item and therefore limiting its usage should be based on a consideration of both the costs and the benefits incurred or avoided by the policy. An optimal economic policy should seek to expand service as long as the marginal benefits are of greater value than the marginal costs. If the resources are available, then all services should be expanded to the same point of zero marginal net benefit. If resources are limited, then the service should be expanded to the point where the marginal net benefit is the same for all costs, since otherwise the costs could be

reallocated so as to increase the total net benefit. In order to apply these optimality principles directly, one needs to evaluate the benefits derived from item usage in a manner which is directly comparable to the cost measurements. However, the direct measurement of the economic value of the benefits of information retrieval is an extremely difficult, if not impossible, task, and indirect methods are the only recourse.

An alternative approach to the establishment of storage policies is to choose that retention period which minimizes the average cost of usage. In addition to the practical advantage of being based on the direct measurement of costs only, this policy has economic attributes which recommend it as a near-optimal solution with regard to user benefits. There is good reason to suppose that the marginal and average benefits from item usage are relatively constant from the standpoint of anticipating such benefits for the purpose of establishing a policy. Furthermore, marginal benefits of an activity should be greater than the marginal cost of that activity for the venture to be initiated. In the absence of binding resource constraints and given constant marginal benefits, an initially feasible activity should be continued until the point where average costs are minimized. This is a relatively conservative approach to the problem which is not at all unreasonable when there is almost complete ignorance about the relative worth of the benefits derived from item usage.

There is a well-established economic thesis which holds that the long-term tendency in competitive production is for the producers to be driven to the point of zero net surplus profit, that is, where average cost equals average revenue. While the situation in information storage is not directly analagous, it seems to be quite similar in that there are usually alternative information sources available to the user, and these alternatives will be exercised as long as they can do so at less cost. The competitive interaction of users should tend to match benefits with costs.

5. Minimization of the Average Cost of Usage

A storage policy based on the minimization of the average cost of usage is relatively easy to implement on the basis of cost information alone. Since the average cost achieves a minimum value when it is equal to the marginal cost, a decision rule can be easily obtained by equating equations (15) and (16) and solving for x as follows:

$$\text{Min } \overline{K}(x) \Rightarrow k_1/K_2 = \ln(1 - x) + x/(1 - x) \quad . \tag{17}$$

This decision rule is evaluated in Table 1 where the relationship is shown between the parametric ratio (k_1/K_2) and the value of x which minimizes average cost. By referring to equation (3), it is possible to translate this decision rule into the holding times which minimize average cost as follows:

$$\text{Min } \overline{K}(x) \Rightarrow k_1/K_2 = e^{bt} - 1 - bt \tag{18}$$

where b is the obsolescence rate and bt expresses holding time in the number of relaxation intervals. By expressing holding time this way, it is possible to demonstrate the effect of the decision rule on holding time using equation (7). This is done in Table 1.

An approximate version of the decision rule can be obtained by expanding the exponential term in equation (18) and ignoring all but the first three terms in the expansion. This leads to the simpler rule:

$$\text{Min } \overline{K}(x) \Rightarrow t_h = \sqrt{2k_1/b^2K_2} \tag{19}$$

Table 1. Values of Relative Usage, Holding Times, and Costs which Minimize the Average Cost of Usage for an Item

Fraction of Total Life-time Usage x	Holding Time (relaxation intervals) bt	Ratio of Cost Parameters k_1/K_2	Economic Holding Time as Computed from Equation (19) $bt_h = \sqrt{2k_1/K_2}$
0.1	0.11	0.01	0.11
0.2	0.22	0.03	0.23
0.3	0.36	0.07	0.38
0.4	0.51	0.16	0.56
0.5	0.69	0.31	0.78
0.6	0.92	0.58	1.08
0.65	1.05	0.81	1.27
0.70	1.20	1.13	1.50
0.75	1.39	1.61	1.80
0.80	1.61	2.39	2.19
0.85	1.90	3.77	2.75
0.90	2.30	6.70	3.67
0.95	3.00	16.00	5.66
0.99	4.61	94.40	9.72

where t_h denotes a holding time which effects an approximate minimization of the average cost of usage. This version of the decision rule has some intuitive appeal because of its similarity to the economic lot-size formula of inventory theory. The control parameter t_h can be called the "economic holding time" for an information system. It can be seen in Table 1 how t_h tends to overestimate the time required to minimize average cost especially at unusually large values of the ratio (k_1/K_2). Equation (19) implies that the economic holding time will change as the square root of changes in the cost of acquisition and storage or changes in the obsolescence rate. As the cost of acquisition, k_1, decreases the holding time will decrease, and as the storage cost, k_2, decreases, the holding

time is increased. An increase in the obsolescence rate will decrease the holding time and will decrease the total usage obtained from the item since, in equation (17), an increase in b decreases the parameter K_2 , which decreases both the ratio (k_1/K_2) and the minimizing value of relative usage, x .

It is interesting to note that the decision rule establishes the holding time independently of the usage parameter, k_3 . In fact, if only acquisition and time dependent costs are considered, the holding time would be the same. The interesting point is that it is reasonable to argue that almost all of the costs of operating a library can be allocated between these two cost categories, since most of the labor cost in libraries is expended for professional or semi-professional personnel who in many ways represent as much of a system investment as do the purchase price of the materials. Almost all categories of library cost correlate closely with the size of the collection and/or the acquisition rate of new materials. Even the acquisition costs are correlated closely with size, because of the steady exponential growth patterns which are characteristic of large libraries. Some, but certainly a small part, of direct library expense does vary directly with usage, as in the operation of reserve book rooms where items circulate with a very high frequency. If it is valid to consider storage system costs as being represented by the parameters k_1 and k_2 **only**, then it would seem worthwhile to consider the cost parameter k_3 as being representative of the cost to the user in obtaining information from the system. Equations (8) or (14) would then represent the combined total cost to both the patron and the storage system for providing uses from an item, and the decision rule would determine the holding time which minimizes the combined average cost per use.

6. Cost of Storing Many Items in a Growing Library

The storage model for a single item does not take into account the effect of collection growth on the allocation of cost among the various items in a library. In practice, new items are arriving at a fairly constant rate (the so-called law of libraries) and these items compete with older items for space and file maintenance. Under the assumption of declining use with age, the newer items will displace the older items because of the relative cheapness of their cost per use.

The following model is intended to demonstrate this effect by expanding the single item model to include exponential growth of a collection of items which have similar usage patterns. Since costs will be developed on a usage basis, those user or library costs which are directly related to the level of usage are omitted from the analysis to simplify the model. The use cost parameter, k_3, could be included and would become a constant added to the total cost per use.

It has been observed by Dunn [1967] and others that large university libraries have been growing in an exponential manner for a long time at a relatively constant rate. The data suggests a simple growth model of the form:

$$N_t = N_o e^{-at} \tag{20}$$

where N_t defines the size of the collection at a time t years in the past relative to the present size, N_o. The parameter "a" is a constant instantaneous growth rate, i.e., the ratio $-N_t'/N_t$. If the symbol α denotes the annual acquisition rate, then it is related to the instantaneous rate by the relation

$$\alpha = (N_{t+1} - N_t)/N_t = e^a - 1 \tag{21}$$

or

$$a = \ell n(1 + \alpha) \quad . \tag{22}$$

It also can be shown that

$$a = (\ell n\ 2)/t_d = 0.693/t_d \tag{23}$$

where t_d denotes the doubling period or constant time it takes for the collection to double in size.

Dunn's study of the costs of operating large university libraries suggests a simple cost model of the form:

$$K_t = k_1 A_t + k_2 N_t \tag{24}$$

where K_t denotes the annual cost rate at a time t years ago. This is defined in terms of the annual number of acquisitions, A_t, at that time multiplied by a unit cost k_1, and the size of the collection at that time multiplied by a unit cost k_2 per volume per year. Since the annual number of acquisitions per year is related to the size of the collection by the annual acquisition rate, α, it is possible to rewrite equation (24) in the following ways:

$$K_t = (k_1\alpha + k_2)N_t = (k_1\alpha + k_2)N_o e^{-at} \tag{25}$$

or in terms of the annual number of acquisitions, as:

$$K_t = [k_1 + k_2(1 + \alpha)/\alpha]A_t = [k_1 + k_2(1 + \alpha)/\alpha]A_o e^{-at} \quad . \qquad (26)$$

These equations assume that the unit cost coefficients, k_1 and k_2, remain constant over time, which is not very likely over any long period of time. A relatively simple and effective way to compensate for the change in unit costs over time, is to assume that these costs have been increasing at the same constant rate over time. If the instantaneous rate of change in the value of k_1 and k_2 is some constant k, then equations (25) and (26) should be modified as follows:

$$K_t = (k_1\alpha + k_2)N_o e^{-(a+k)t} \qquad (27)$$

and

$$K_t = [k_1 + k_2(1+\alpha)/\alpha]A_o e^{-(a+k)t} \qquad (28)$$

or simply

$$K_t = K_o e^{-(a+k)t} \qquad (29)$$

where K_o is the current annual cost rate. Thus, library costs are increasing faster than library size and both increase exponentially.

These models do not pretend to take into account any major innovations in methods and facilities for making inputs or maintaining collections. Such changes would be likely to cause jumps in the cost patterns with

the exponential pattern resuming thereafter. Also, changes in the acquisition rate would cause exponential shifts in the growth patterns. However, the larger libraries have been growing steadily for a very long time, (F. Ryder said it is so since colonial days) and there has been relatively little change in the basic technology up to the present time.

7. Exponential Obsolescence and Circulation

As in the case of a single book, it has been observed that the usage rate of collections of books declines steadily from the time of their acquisition or publication. It has also been observed that the total usage or circulation of a collection tends to be proportional to the size of the collection over periods of time. If it is assumed that all items in the collection follow the simple exponential obsolescence pattern defined by equations (2) and (3) and that the collection grows in an exponential manner, then a fairly simple model of library circulation can be developed which agrees with both of the above observations.

If the instantaneous input to a collection at a time t years ago is weighted with its usage rate at the present time, a measure of the contribution of that input to current circulation is obtained. If these contributions are summed or integrated over the last t years of growth, a measure is obtained of that part of the total circulation rate which is due to items acquired over the past t years. Let v_t denote the current circulation rate due to items which are less than t years old, then with reference to equations (2) and (20), this is defined as follows:

$$v_t = \int_o^t N_t' \, u_t' \, dt = \int_o^t rN_o e^{-(a+b)dt}$$

$$v_t = rN_o[a/(a+b)] \, [1 - e^{-(a+b)t}] \qquad (30)$$

where r is a scale parameter or the instantaneous initial usage rate of a new item. Its numerical value can be related to the average annual usage rate of the item during its first year since publication or acquisition.

As the period of collection growth over which v_t is computed becomes large, the total current circulation rate due to acquisitions during the period t approach a limiting value, which is defined by V_o where

$$V_o = rN_o a/(a + b) \quad . \tag{31}$$

Equation (31) states that the total current circulation rate is directly proportional to the current size of the collection, N_o ; and the proportionality constant is a simple function of the rates of growth, obsolescence, and initial use. Figure 2 shows this pattern in the growth and circulation of the Purdue University Libraries as observed by Leimkuhler [1966].

The ratio of equations (27) to (31) defines a cost per unit of circulation which can be denoted by $\overline{K}(V_o)$ and derived as follows:

$$\overline{K}(V_o) = K_o/V_o = (a + b)(k_1\alpha + k_2)/ra \quad . \tag{32}$$

Since the cost was assumed in equation (27) to increase exponentially over time because of the growth of the collection and because of external increases in the cost of inputs, the average cost per circulation will also increase because of the latter factor, i.e.,

$$\overline{K}(V_t) = \overline{K}(V_o)e^{-kt} \quad . \tag{33}$$

Here $\overline{K}(V_t)$ denotes the average cost per circulation at a time t years in the past.

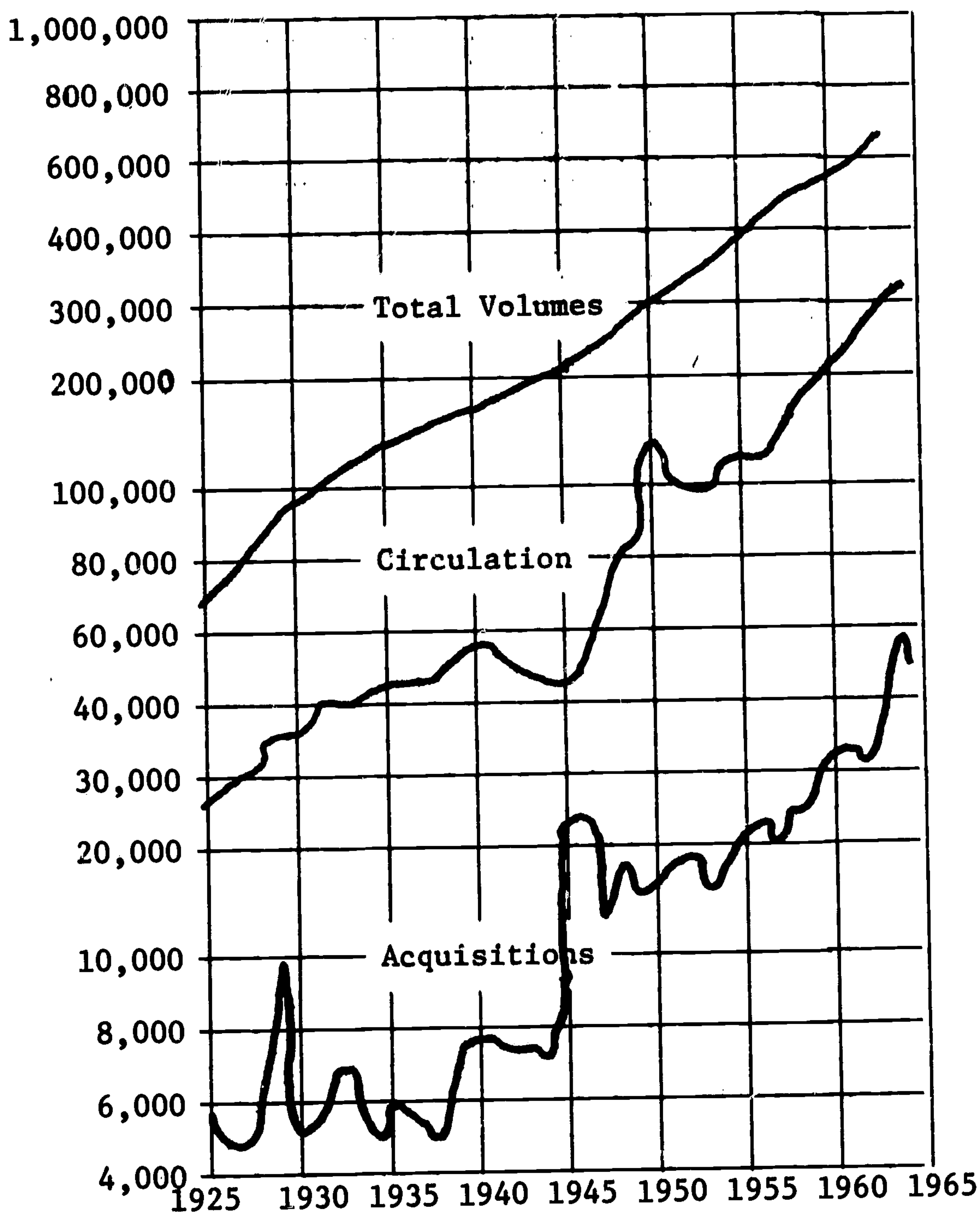


Figure 2: Purdue libraries

8. Library Cost When All Items Are Held for a Limited Time

In the previous sections it was assumed that all items were held indefinitely in the library. Many libraries find it desirable to discard inactive material or to transfer such material to a depository. Items to be discarded are selected on a basis of judgment, circulation histories, and age; or some combination of these factors. The age rule is perhaps the easiest to apply and is used commonly in the storage of periodicals and other serials where age is a major factor in identifying the item and in determining relative shelf location. The selection and discarding of little used items can be expected to add a certain amount of extra cost to the operation of a library. However, if it is done in a routine manner these costs are likely to be partly proportional to the input cost and partly proportional to the storage cost. That is, they would be reflected in increases in the unit costs, k_1 and k_2 , which are used in equation (24). For this reason, an additional cost term can be avoided in the cost model for a library with limited retention time.

If all items are held for a period of t years and then sent elsewhere, the cost of operating the library could be expressed in the same form as equation (24) , i.e.,

$$K(n_t) = k_1A + k_2n_t \tag{34}$$

where n_t denotes the number of volumes acquired which are less than t years old and A is the annual number of acquisitions. The variable n_t is defined by

$$n_t = N_o - N_t = N_o(1 - e^{-at}) \quad . \tag{35}$$

Here, N_o and N_t are defined in the same way as in equation (20). The circulation which is generated by these items which are less than t years old is defined by equation (30) as the variable v_t. By referring to equation (31), this can be written as

$$v_t = V_o[1 - e^{-(a + b)t}] \quad . \qquad (36)$$

By solving equations (35) and (36) simultaneously so as to eliminate the argument t, the circulation of the restricted collection can be defined directly in terms of the number of items held, i.e.,

$$v(n) = V_o[1 - (1 - n/N_o)^{1 + b/a}] \quad . \qquad (37)$$

The inverse relation, which is used below, is

$$n(v) = N_o[1 - (1 - v/V_o)^{a/(a + b)}] \quad . \qquad (38)$$

The subscript t has been dropped from the variables n and v for simplicity, since they imply a value of t according to equations (35) and (36).

9. Minimization of Average Cost Per Use

According to equation (34) the library cost increases linearly with the size of the active collection, and according to equation (37) the circulation increases at a decreasing rate with the size of the active collection. This pattern is sketched in Figures 3(a) and 3(b), and is a typical kind of benefit cost relationship for a productive activity. In determining an optimal or near-optimal point for the planned operating

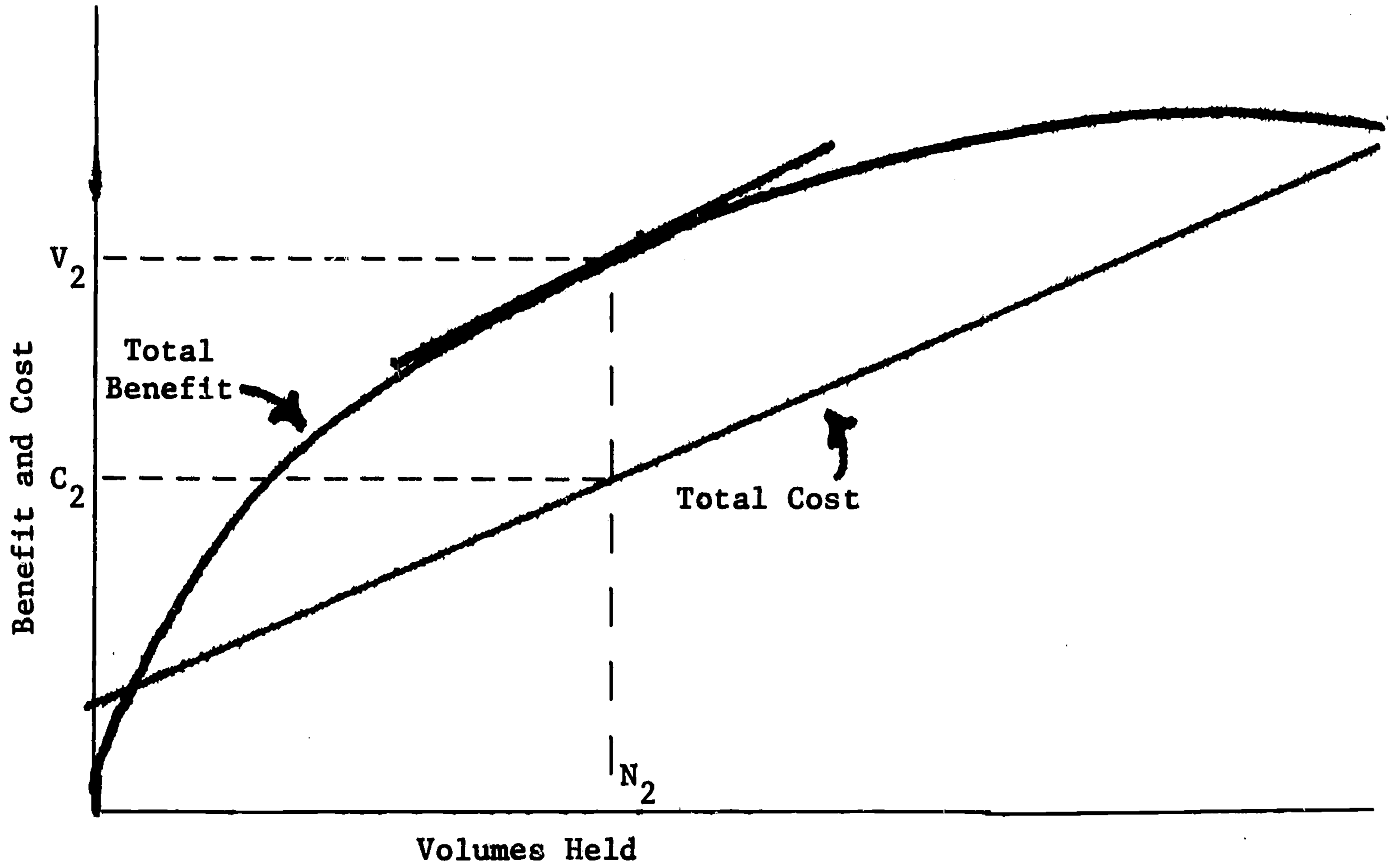


Figure 3(a): Benefit and Cost vs. Holdings

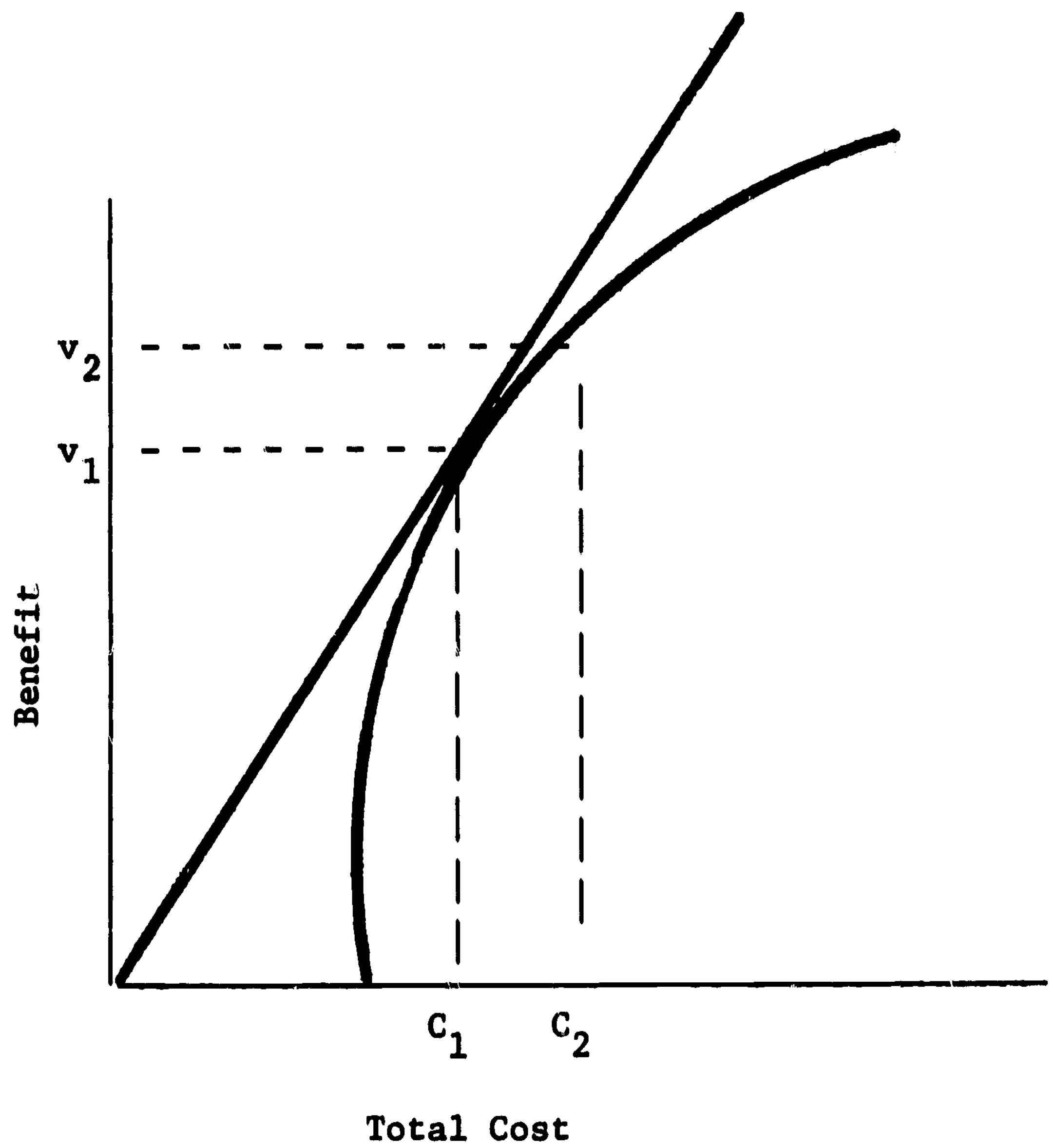


Figure 3(b): Benefit vs. Cost

level of the activity, it is well to consider the two possibilities v_1 and v_2 identified in Figure 3(b). The point v_1 represents the point of minimum average cost per unit of circulation or the maximum average number of circulations per dollar spent. The point v_2 locates a higher level of activity where the marginal gain in circulation is less than the marginal cost. If the variable v were measured in dollars of net benefit, this would be a true optimum point, but since this is not the case there is no way of knowing the wisdom of choosing v_2 as the operating point. With v_1 there is the advantage of minimizing the possible diseconomies of the activity by establishing a technically efficient level of operation which can be controlled.

Upon substituting equation (38) into equation (34) library cost can be written as a function of circulation, i.e.,

$$K(v) = k_1A + k_2N_o[1 - (1 - v/V_o)^{a/(a + b)}] \quad . \qquad (39)$$

Since the average cost, $K(v)/v$, per unit of circulation achieves a minimum value at the point where it equals the marginal cost, this point can be found by setting the derivative of equation (39) equal to the average cost and solving for the level of circulation, v_o, which is found to be defined as follows:

$$v_o = V_o(1 - [k_2/(k_1\alpha + k_2)]^{1 + a/b}) \quad . \qquad (40)$$

By substituting this value for v_o into equation (36), the holding time, t_o, which minimizes the average cost per unit of circulation is:

$$t_o = (1/b)\ 2n(1 + k_1\alpha +/k_2) \quad . \tag{41}$$

Also, the size of the active collection, n_o , which minimizes average cost can be obtained by substituting equation (40) into equation (38). This yields the relationship

$$n_o = N_o(1 - [k_2/(k_1\alpha + k_2)]^{a/b}) \quad . \tag{42}$$

The ratio n_o/N_o is the fraction of total past acquisitions which are retained in the active collection, and the ratio $(1 - n_o/N_o)$ measures the percentage reduction in annual storage costs due to the disposal of inactive items.

As an example of the implications of this model, consider a collection which is growing at a rate of about 5 percent per annum and is subject to an obsolescence rate of approximately 0.05 . Also, assume that the cost of a new acquisition is about 20 times the cost of holding an item one year. Under these conditions, items should be held in the active file for about 13 years, which accounts for about half of the total past acquisitions and about three-fourths of the total potential circulation from past acquisitions. This would imply also that at an optimal level of operation two-thirds of the total cost would be devoted to the acquisition or input effort and only one-third to the retention or storage effort.

10. Retention of Inactive Items and Age Rule Depositories

The above model focuses on holding time as the control variable in storage and emphasizes the economic advantage from discarding inactive materials. Because of this, it appears to ignore some important practical

questions about providing access to relatively inactive items. An important assumption in the model is that all items in the collection can be represented by a single exponential obsolesence pattern. If different items have different obsolesence rates they should be retained for different periods in order to minimize their average cost per use. According to equation (41) the optimal holding time is directly proportional to the inverse of the obsolescence rate, i.e., if the rate is doubled then the holding time should be halved and vice versa. In practice, it can be observed that technical libraries which are said to exhibit the highest obsolescence in their usage are also more likely to pursue a vigorous discarding policy and are more likely to think in terms of a limited size for the total collection. The opposite is supposedly true for libraries in the humanities.

An obvious way to increase the optimal retention time for an item is to reduce the unit cost parameter, k_2 , which includes all library costs other than those connected with the input of new items. Equation (41) indicates that the effect of reducing k_2 is not nearly as direct as the effect of decreasing the obsolescence rate. In practice, libraries have attempted to reduce holding cost in two principal ways: by increasing the size of collections and achieving certain economies of scale, and by use of depository or overflow libraries where inactive items can be shelved more compactly and in less expensive space. These methods tend to increase the cost of access for both the user and the library. From the standpoint of the storage model, it is important that the reduction in space cost isn't cancelled out by the increase in retrieval cost. Shishko [1968] seems to indicate that this may be the case with the use of depository-type storage at M. I. T. There appears too little evidence that the use of this kind of facility can achieve dramatic reductions in cost, although

there is need for more hard data to support this conclusion.

The same problem of increasing retrieval costs while reducing space costs appears to be delaying the acceptance of miniaturization as a long run solution to the problem of providing access to inactive materials. A more successful working solution is reported in England in the establishment of a national lending library service to back up local libraries. This service is provided on a formal pricing basis with established costs of time for the user and of money for the borrowing library. The formal pricing structure helps to both insure the recovery of cost and to control the demand so that a more stable utilization of materials results. The report by Williams argues for the establishment of similar lending services in the U. S. A. on a regional or national basis.

11. Age Rule Depositories

Where space is at a premium, depositories offer a rational approach for suboptimizing the problem of storage. Two commonly used rules for selecting items for storage are the age rule and the current usage rule. The latter rule can be based on the time since last use or the average usage over a specified period. These rules may be used in various combinations, and may be modified according to certain characteristics of the material, such as language, and the opinion of users or librarians. The age rule is the simplest rule to apply and is often used to select journals for storage.

Under the assumption of exponential obsolescence and growth, equation (37) provides a simple relationship between the size and the activity of an age rule depository. If the symbol m is used to denote the proportion of the collection stored in the depository, and w denotes the proportion of total usage from the depository, then

$$w = m^{1 + b/a} \quad . \tag{43}$$

If the obsolescence rate b is equal to the accession rate a, then w is the square of m; and if b is greater than a, w is a higher power of m, that is, w is considerably smaller since we are dealing in fractions. As the library grows in total size, if the retirement age is kept constant, the fractions w and m remain constant although both are growing exponentially in absolute terms.

The above model can be used to show the effect of restricting the size of an active collection to a predetermined level. When this level is reached, the depository would begin to grow exponentially, and the

fraction of the collection stored after t years would be defined by:

$$m_t = 1 - e^{-at} \quad . \tag{44}$$

The fraction m_t grows at a decreasing rate until virtually all of the collection is stored in the depository, assuming there is no change in the policy. If the items exhibit simple exponential obsolescence and are selected on a basis of age, then the retirement age, d_t , would have to diminish as time goes on and would be defined as follows:

$$d_t = -(1/a)\ln m_t = -(1/a)\ln (1 - e^{-at}) \quad . \tag{45}$$

The fraction of total library usage generated by the depository would increase exponentially according to the relationship

$$w_t = m_t^{1 + b/a} = (1 - e^{-at})^{1 + b/a} \quad . \tag{46}$$

If the parameters a and b are equal, equation (46) reduces to the simpler relationship

$$w_t = m_t^2 = 1 - 2e^{-at} + e^{-2at} \quad . \tag{47}$$

This equation is evaluated in Table 2 for various values of t .

Table 2: Depository Size and Usage when the Active Collection is Held at a Fixed Level and the Rates a and b Equal 0.05 .

Year after start of Depository, t	Rel. Size of Depository, m_t	Rel. Usage of Depository, w_t	Retirement Age, d_t
1	0.05	0.003	60 yrs.
5	0.12	0.01	42
10	0.39	0.15	19
15	0.53	0.28	13
20	0.63	0.40	9
25	0.73	0.53	6
50	0.92	0.85	2

Table 2 demonstrates the effect of limiting the size of active collections. It is interesting to observe the delayed effect on the usage of the depository as compared with rapid initial growth of the percentage of volumes in the depository. The size of the depository is only dependent on the growth rate of the collection, while the use of the depository is dependent on both the growth rate and the obsolescence rate. The values in Table 2 hold for the case where an age rule is used and the rates are equal to 0.05; but the pattern would be much the same for other rates and selection rules. In the long run the depository would eclipse the active collection. It is of interest to note here a conviction held in special library circles that technical libraries tend to maintain a rather stable size over time, some say at about 20,000 volumes. This is done by dis-

carding inactive items, and would imply that the discards would eventually exceed the active collection by a considerable amount. Since these discarded items are likely to be needed at some time in the future, the fixed size technical library would need to be backed up by a large depository library. Industrial libraries tend to use university libraries for this purpose, and this is one of the roles of the National Lending Library in Science and Technology in England.

It is unlikely that the simple exponential model is a sufficient explanation of the actual patterns of inactivity of library materials, or that the age rule would work well without some compensating modifications. Trueswell [1964], Lister [1967], and Morse [1968] have shown that usage rules do a better job of selection than do age rules. Jain [1968] has examined retirement policies based on the probability of item inactivity, and Gurk and Minker [1968] developed a model to estimate the storage requirements for a data bank where retention is a function of age and usage. Such models are facilitated by computer-based circulation control systems.

12. Acquisition Delays and Item Usage

The larger part of library operating costs goes into the initial cost of adding new items to the collection rather than retention. The larger part of input cost is the cost of selection and processing rather than the purchase price of materials. While the high cost of adding new materials is a major concern in library budgeting, of more importance to library users is the long time delay in making new items available for perusal. Morse [1968] has given considerable attention to the urgency of making new items available during the first few years of publication when demand is at a peak level. A closely related problem is how to identify quickly those items which need duplication, since any long delay loses most of the advantage in duplication and magnifies the disadvantages. Quick accession provides an important opportunity for increasing the benefits to be gained from a library item, especially if it can be done without a substantial increase in the accession cost. Some university faculty members identify accession delay as the most important deficiency in current library service.

The significance of acquisition delays can be easily seen by considering the case where a library is growing exponentially and the input delay time is constant. Let h denote the input delay time for the ordering, receiving, and cataloging processes, i.e., the time from publication to first use, and let $I(h)$ represent the inprocess inventory of all such items. Then $I(h)$ can be defined by the relationship

$$I(h) = N(1 - e^{-ah}) \qquad (48)$$

where N is the size of the inprocess and shelved collection and a is the growth rate. Since N is growing exponentially, $I(h)$ is also growing exponentially as long as h is held constant. This is shown graphically in Figure 4.

There is reason to suspect that equation (48) actually understates the size of the inprocess inventory since in order to maintain a constant input time it is necessary to expand the capacity of the operations at the same rate as the work load increases. Dunn [1967] indicates that the expansion in library labor is at a somewhat slower rate than the growth in volumes acquired. This could indicate either that the work capability is not keeping pace with the load or that labor efficiency is being increased because of economies of size or the introduction of new methods. A second reason for expecting equation (48) to underestimate the size of the inprocess inventory is the presence of random and periodic variations in the work load. Under conditions of peak loading and ever-increasing scale, one would expect the variations in loading to cause considerable interference and congestion in processing which should add to the size of the inprocess stock.

Under the assumption of a common exponential obsolescence pattern for the collection, a fixed input delay interval would cause a "loss" in a certain amount of potential usage which would be a fixed proportion of the actual circulation. This proportion, q, can be defined as follows:

$$q = e^{(a + b)h} - 1 \quad . \qquad (49)$$

For relatively small values of the exponent, $(a + b)h$, the value of q is approximately equal to the value of the exponent. For example, if

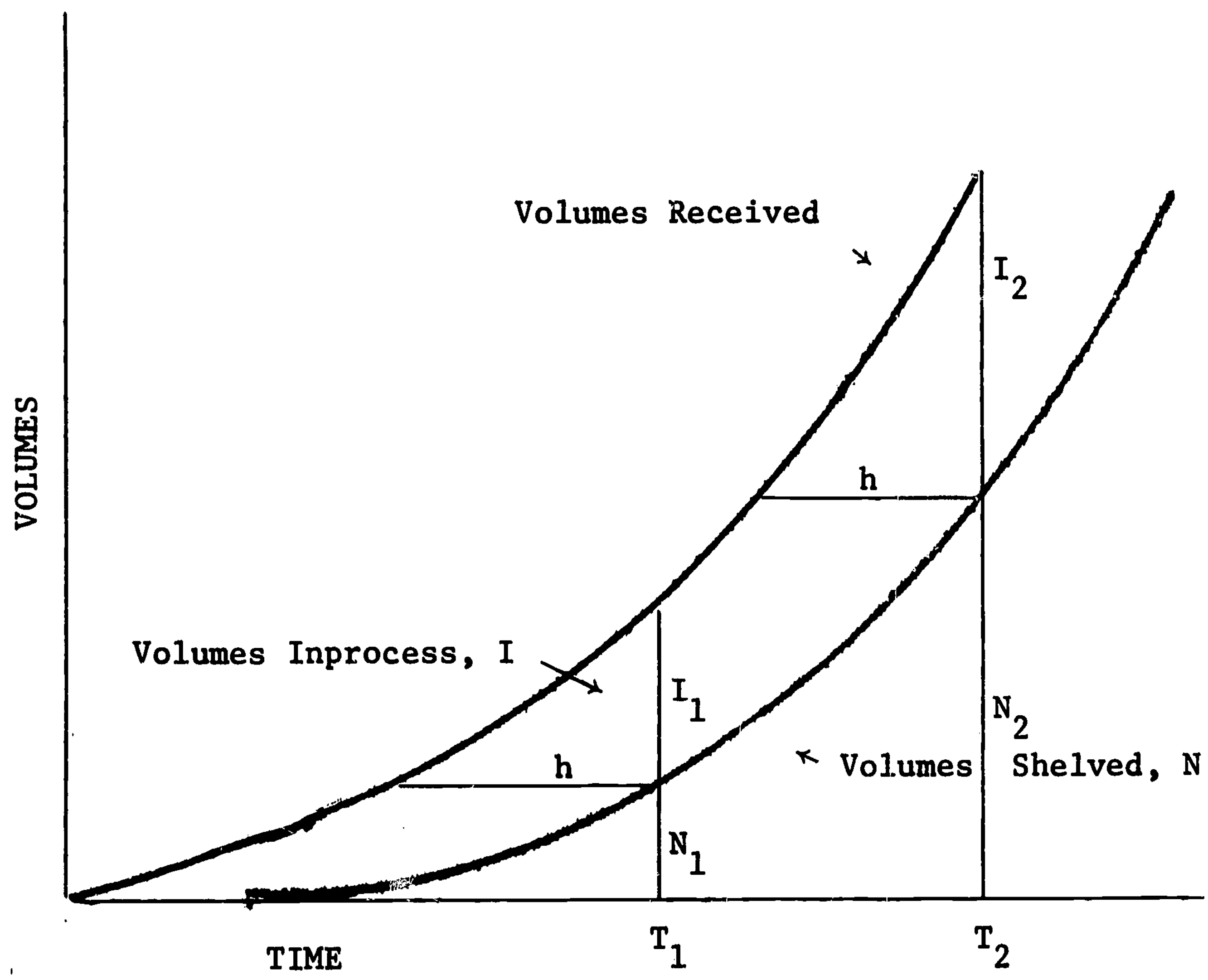


Figure 4: Inprocess Inventory Due to a Fixed Accession Delay Time, h , When a Collection Grows Exponentially

a and b both equal 0.05 and h equals 0.5 years, then q is approximately five percent of actual usage. This is likely to be a gross underestimate of the actual case, however, since Morse has shown that the obsolescence rate of new materials is considerably higher than 0.05 and can even be as high as 0.50 for some scientific items. This would indicate that the ratio q could be as high as 0.30 or almost a third of the actual usage, if h is about six months. Morse found that 80 percent of the total ten year usage of a book occurs in its first three years in the library. He urges the development of methods of quickly identifying and acquiring original and duplicate copies of high use items as a particularly important way to improve library effectiveness.

13. Circulation Interference and Usage

An important factor contributing to user cost and to the level of usage of a collection is availability of items which have been added to the collection but may be missing from the shelf for a variety of reasons. Various people have estimated that the probability of a requested item being on the shelf at the time of request varies from 0.50 to 0.67 in many libraries, although some librarians say this is as high as 0.75. This does not mean that only that proportion of the collection is on the shelf, but that proportion of the requested items are available. Thus the measure is heavily weighted by the absence of the more popular items. Buckland and Hindle [1969] have argued that the unavailability of popular and well used items not only inconveniences those patrons requesting them, but creates a shelf bias toward inactive and perhaps less relevant materials for browsing and for those patrons who must use the material available. The two basic methods of increasing the availability of a collection are duplication and tighter circulation control.

Duplication of popular items increases the availability of the items but actually decreases the circulation rate per volume of the library. While duplicates may be acquired at less cost than original items, because they have already been cataloged, it is unlikely that this reduction in cost is sufficient to prevent duplication from increasing the average cost per use for the collection. Leimkuhler [1966] developed an elementary queuing-type model which shows the effect of duplication on circulation. In general, the model shows that two copies of an item can never succeed in doubling the circulation rate per volume. For example, if a single copy is used so that it is available only half the time, then duplication will only increase the total circulation for the two volumes sixty percent,

which reduces the average circulation rate twenty percent for the two volumes. At the same time, the availability rate for the title would be increased from 0.5 to 0.8 by duplication according to the model. This model can also be used to show the effect of duplication among branch libraries, which in effect divides the demand between two locations. In general, it can be shown that duplication at two locations yields less circulation per volume than does duplication at a single location. It also fails to produce as great a level of availability at the two locations than at a single location. This model is particularly interesting because it demonstrates a case where there is a trade-off relationship between user and library cost, that is, the user cost in terms of availability is reduced by increasing the cost per use to the library.

Buckland and Hindle [1969] in their study of availability concluded that improved circulation control may be a better approach than duplication in centralized libraries. Circulation control can range from no-loan policies, through various limited loan policies with recall and renewal options, to unlimited loans. They found that when loans are made for periods of a week or more, the return of the material and its probability of renewal was independent of the loan period. This suggests that usage can be confined to a relatively short period without penalizing most users. Buckland and Hindle used a simulation model to show that a good policy for a library to follow is to restrict loan periods to one week for the most popular items in the collection, and to allow end-of-term loans for the balance of the collection. The latter policy has the advantage of simplifying the date control and notification procedures. The simulation results indicate that this policy should produce a relatively high level of general

availability and a low collection bias when about ten percent of the collection is placed on a short loan status.

It is interesting to note that under the assumptions of exponential growth and obsolescence, the most popular ten percent of the collection would be those items acquired within approximately ten years, if the acquisition rate is about five percent per annum. Actually, Buckland recommends monitoring the average number of recorded uses per year as a guide to the selection of short loan period items. These results seem to agree substantially with the finding of Morse [1968] from his extensive study of the M. I. T. Library. Morse takes an analytic approach rather than a simulation approach by drawing heavily on the theory of stochastic process and queues. He suggests the use of a one week period for popular items, and also notes the importance of giving greater attention to the time it takes to reshelve these items and to replace missing items. In general, the evidence indicates that the popular items in a library, which are likely to be the newer items and a relatively small proportion of accumulated holdings, should be handled in a different manner than the rest of the collection. Such a division in the document control procedures can greatly increase the effectiveness of a library without increasing library cost excessively. The implementation of such a policy is much easier if a computer-based data processing system can be used to control document circulation and location. A more interesting question is whether the benefits to be gained from a more sophisticated document control system can be sufficient to underwrite the cost of introducing mechanized data processing.

14. Generalized Models of Library Costs and Benefits

The above models have been deliberately simplified to facilitate the analysis of certain operating policies and to help develop a sound approach to the study of library costs. Operating cost is defined as the sum of three component costs which relate to the acquisition, storage, and circulation functions of a library. A more general cost model should recognize all of the important options which are available in the exercise of these three functions. A useful approach to the identification of these options is to divide them between those options which are related to the scale of an activity and those which are related to how the activity is performed. This is the distinction in microeconomics between how a product is made and how much of the product to make. The first is a more technical question and depends on the ways in which inputs can be combined to produce a product and the supply of the inputs. The second question is more of a marketing problem and depends on the answers to the first question and the demand for the product. In practice, of course, this division is not so neat, nor is a manager free or willing to explore all possible options.

In the above models, the principal scale factors were the level of acquisitions, the volume of stored documents, and the amount of usage. When the assumptions of exponential growth and obsolescence are made, it is possible to tie these three together in a deterministic manner, so that the specification of one level determined the others. The methods for performing the three functions or the technology factors were reflected in the cost coefficients, k_1 , k_2 , and k_3 ; and also in such control variables as the holding time, t_h , the accession delay time, t_a , and the user access time, t_u . Most of the above analysis focused on the single control variable, holding time.

A generalized model of library costs could be written in the following manner:

$$C(S,T) = C_1(S_1,T_1) + C_2(S_2,T_2) + C_3(S_3,T_3) \tag{50}$$

where S designates scale variables and T designates technology variables, and the subscripts 1, 2, and 3 designate acquisition, storage, and usage functions. The cost, $C(S,T)$, of operating a library with a particular technology, T, at a scale, S, is the sum of the three component costs, C_1, C_2, and C_3. The complete definition of the technology of a library in a manner which reveals all of the possible factors which might be varied so as to reduce costs, is a very difficult task. Normally, one focuses on a few variables at a time and assumes that all other factors are fixed and can be represented by appropriate parameters and functional relationships. It appears that the technology factors of interest are those which have either a space or time dimension, such as acquisition, storage, and retrieval time, or shelf space, reading space, and user proximity. There are many others which are of interest and even these might be subdivided into smaller time or space components.

Scale factors and measures can also become quite involved and complicated, when one wants to define and manipulate them in an analytic manner. Scale is a relative measure and the base chosen is important. For example, the acquisition effort might be described by the actual number of items acquired or in terms of the potential number which might have been acquired, or in terms of the actual number screened. Similarly, usage might be measured in terms of actual uses, actual requests, or potential requests. Collection size could refer to those on hand, those on hand and in use, and those held relative to what might be held. Besides these distinctions,

there is the difficult problem of aggregation and the avoidance of adding dissimilar things. From some viewpoints, everything in a library is a "one of a kind" item, every use made is a unique entity, and every acquisition is a special problem. Such distinctions greatly enlarge the problem of analytic representation and force the use of computers and computer methods to handle the sheer bulk of the analytic detail required.

The problems of cost measurement and cost modeling in libraries, for all of their difficulty, are not nearly as formidable as the problems of benefit measurement and modeling. The same distinction made between scale factors and technical factors in the modeling of costs seems to apply to the measurement of benefits. Library benefit can be thought of as a difference measure between user reward and user cost to obtain the reward. User rewards relate to user needs which are outside the direct control of a library, although the library may have an influence in determining which needs the users seek to satisfy. Library benefit is more directly related to the library's ability to reduce the cost to users of need satisfaction. This is done by increasing the probability of satisfaction and reducing the time and effort required. The latter element of user cost, time and effort, is related closely to the technological design of the library. The level of satisfaction is the scale component in user cost, and is related to the scale of the library. Thus, it is argued that the larger the collection in a field, the greater the likelihood of its satisfying requests, although it might require more time and effort from the user. Furthermore, there are likely to be diminishing returns with increased size due to obsolescence and increased "scattering" of references in peripheral locations. How to strike meaningful balances between availability, collection size, and access time is one of the most difficult questions in library

planning. How to develop an analytic solution to this problem remains as a difficult but important task for library systems analysis.

15. Summary Observations

Chapter II has made the following major points:

a. The models presented in this report explore the phenomena of exponential growth and obsolescence in library materials and their effect on acquisition, storage and circulation.

b. Assuming linear storage, acquisition, and usage costs; and exponential decay in the book usage rate, the total, marginal, and average costs of library operations as a function of the holding time is derived.

c. This total cost is then transformed from a function of holding time to a function of cumulative usage. Once again, analytical expressions of total, marginal, and average costs of library operations are derived.

d. Assuming single item library holdings, the holding time which minimizes the average cost of system operation is developed.

e. This optimal holding time is independent of usage and varies proportional to the square root of changes in the cost of acquisition and storage.

f. This analysis is then extended to a collection of similar items which is growing at an exponential rate.

g. The ensemble analysis is then specialized for the case in which all items are transferred to a depository after a fixed number of years in the collection. Using this model, an analytic derivation of the holding time which minimizes the average cost per unit circulation is performed, and the size of the minimal cost collection is developed.

h. Under the assumption of exponential obsolescence and growth, we derive the relationship between library and depository size and usage. The size of the depository is only dependent upon the growth rate of the collection, while the use of the depository is dependent on both the growth rate and the obsolescence rate.

i. The single item collection model is then extended to include a constant delay time of acquisition and we give analytically the size of the inprocess inventory.

j. The evidence indicates that circulation interference can be alleviated by handling the more popular item in the library in a more restricted manner including allowing a shorter circulation period.

k. A formalism for a more general cost-benefit analysis is proposed in the last section, but the question of how to develop an analytic solution to this problem remains both a difficult and an important task for library systems analysis.

BIBLIOGRAPHY

Baker, N. R. [1947] "Toward an Analytic Framework for Service Organizations," Paper, 14th International Meeting, Institute of Management Sciences, Mexico City.

Buckland, Michael K., and Hindle, M. [1969] Paper presented at Conference on Planning of Library Services, University of Lancaster, England, July, 1969.

Buckland, Michael K., and Woodburn, Ian [1968] "Some Implications for Library Management of Scattering and Obsolesence," Occasional Papers No. 1, University of Lancaster Library, England ED 022 502.

Buckland, Michael K., and Woodburn, Ian [1968] "An Analytic Approach to Duplication and Availability," Occasional Papers No. 2, University of Lancaster Library, England ED 022 560.

Churchman, C. W., [1968] "Management Information Systems," Chapter 7, in The Systems Approach, Dell Publishing Co., New York.

Cole, P. F. [1962] "A New Look at Reference Scattering," J. of Documentation, Vol. 18, No. 2, pp. 58-64.

Dunn, Oliver C., Serbert, W. F., and Seheuneman, J. A. [1967] The Past and Likely Future of 58 Research Libraries 1951-1980: A Statistical Study of Growth and Change, Purdue University Libraries.

Gurk, H. M., and Minker, J. [1968] Storage Requirements for Information Handling Centers, Tech. Report No. 68-61, University of Maryland Computer Science Center.

Jain, A. K. [1967] "A Statistical Study of Book Use," Thesis, Purdue University, PB 176 525.

Keller, J. E. [1969] Program Budgeting and Cost Benefit Analysis in Libraries, College and Research Libraries, Vol. 30, No. 2, pp. 156-160.

Leimkuhler, F. F. [1966] Systems Analysis in University Libraries, College and Research Libraries, Vol. 27, No. 1, pp. 13-18.

Leimkuhler, F. F. [1967] "The Bradford Distribution," J. of Documentation, Vol. 23, No. 3, pp. 197-207.

Lister, W. C. [1967] "Least Cost Decision Rules for the Selection of Library Materials for Compact Storage," Thesis, Purdue University, PB 174 441.

Meier, R. L. [1961] "Efficiency Criteria for the Operation of Large Libraries," Library Quarterly, Vol. 31, pp. 215-234.

Morse, P. M. [1968] Library Effectiveness: A Systems Approach, M.I.T. Press.

Morse, P.M., and Elston, C. [1969] "A Probablistic Model for Obsolesence," Operations Research, Vol. 17, No. 1, pp. 36-47.

Shishko, R. [1968] "The Applications of Systematic Analysis to the M.I.T. Libraries," Essay, Department of Political Science, Massachusetts Institute of Technology. SEE ALSO Raffel, Jeffrey A., and Shishko, Robert [1969] "Systematic Analysis of Library Policies: An Application of Cost Benefit Analysis to the M.I.T. Libraries," The M.I.T. Press.

Trueswell, Richard W. [1966] Determining the Optimal Number of Volumes for a Library's Core Collection, Libri, Vol. 16, No. 1, pp. 49-60.

Williams, G. [1968] Library Cost Models: Owning Versus Borrowing Serial Publications, Report, Center for Research Libraries, Chicago, PB 182 304.