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ABSTRACT

This monograph provides a theoretical substantiation for benefit-cost analysis and cost-effectiveness analysis in school system planning. Nine examples of decision-making situations are presented. A simple case of a single objective to be attained with one plan being selected from a set of proposed plans with no resource constraints evolves into a more complex and more realistic case with multiple objectives and several activities or programs to be chosen from a host of possibilities under resource constraints. Cases include not only a priori decision-making schemes but evaluation schemes as well. A summary of the nine cases offers specific recommendations about methods that can be used for generating and processing the kind of "data types" required for educational decision making. A 36-entry bibliography is appended. This document is a rewritten chapter of a dissertation entitled, "A Cost-Effectiveness Evaluation Approach to Improving Resource Allocations for School Systems" (EA 002 937). (DE)

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TECHNICAL PAPER

A COMPREHENSIVE THEORY OF COST-EFFECTIVENESS

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A COMPREHENSIVE THEORY OF COST-EFFECTIVENESS

Sanford Temkin

PREFACE

In the past few years countless authors have suggested, in one way or another, that cost-effectiveness methods be used as a basis for decision-making in education. As one sorts and analyzes these proposals and prescriptions two important inadequacies become evident:

1) Firm theoretical bases for these studies are lacking. Theory, in the domain of decision-making, should provide not only a basis for description and explanation but explicit statements of assumptions underlying the proposed rationale and methodology.
2) Little help is offered to the individual who wishes to select from the various economic based approaches an appropriate method to apply to a practical problem.

This monograph is written to meet needs arising from these inadequacies in the literature. It is anticipated that this monograph will be utilized by a limited technical audience who is interested in: general theory of school system planning, and the theoretical substantiation for benefit-cost analysis and cost-effectiveness analysis. Those who want to design realistic methods for school district planning may also find assistance here. Above all, this is directed to those who believe that meaningful change can be effected in schools by systematic and rational planning. Hopefully, better planning methods will be followed by higher levels of educational productivity for the educational dollar.

The bulk of this document is devoted to the <u>theoretical</u> development of

nine decision cases. A three page summary of the nine cases offers specific recommendations about methods which can be used for generating and processing the kind of "data types" required for educational decision-making.

An appendix provides a discussion of some combinatorial problems. A bibliography, which was used in developing many of the key concepts, is also included.

Numerous people have contributed to the synthesis of the ideas in this paper. It is not feasible to mention them all, but a few must be singled out for their impact on the author's thinking. I shall list these names and hope that each understands how much his help is appreciated: Professors Francis Brown, Morris Hamburg, Donald Morrison, and George Parks of the University of Pennsylvania; Professor Harold Goldman of Bucks County Community College; Paul Hood, Director of the Communication Program of the Far West Laboratory for Educational Research and Development; Roger Sisson and Robert Pritchard, consultants; and James Becker, Louis Maguire, Harris Miller, Joseph Mirsky, Frederick Tanger, and Jo Ann Weinberger, all of Research for Better Schools.

S. T.
April, 1970

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DECISIONS AND CHANGE: THE ROLES

The theory developed in this paper is quice general and may be applied to systematic study in many areas. The local school district has been selected as a situation to be used for specific comments to facilitate better communication.

There are several people who perform roles in the decision-making processes which are described. It is important for the reader to become acquainted with these roles so that their inter-relationships can be appreciated.

Decision-maker

First, there is the decision-maker. He is the person who is responsible for decisions. At times he may delegate his authority to another person, but the responsibility is ultimately and rightly his. The role is fitting for the superintendent of schools.

Educational Engineer

Second, we should understand the role of the educational engineer. He is responsible for designing programs which meet the "overall objectives" of the school district. Often the engineer may design several alternative programs each of which is designed to meet same set of targets or overall objectives. The superintendent may want to select the best alternative from the group supplied by the engineer. In formulating the alternatives, the engineer must assure that they are practical and can be implemented within the framework of available resources. A change may be technically sound, but when implementation personnel are not equipped with the understanding, training, and attitudes required to effect the change, there is little chance that the change will be effective.

Engineer versus Professor-scientist

The engineer's role is often taken by a committee of teachers, a high school principal, and, on occasion, the superintendent himself. Sometimes a school district hires a professor-scientist from a university. The professor-scientist is often interested in an experimental situation in order to obtain information to test his theory for subsequent journal publication. The school district's engineering needs, then, may not be consistent with the role played by the professor-scientist, although this is sometimes obscured.

Implementor

Another role is that of the implementor. He is responsible for implementing changes suggested by the decision-maker. He directs his staff, the implementation team. This implementation role is, in the opinion of the author, taken too much for granted by everyone, including those involved in implementation. The implementor is sometimes a school principal, a curriculum coordinator, a research office person, or the engineer himself.

Evaluator

The last role to be described in this introduction of characters is that of the evaluator. To understand the evaluation role one must first appreciate that there are many reasons for undertaking evaluation. Some of these are: 1) the funding agency insists that evaluation be conducted, 2) the decision-maker feels that evaluation is important, 3) the research office feels that evaluation is important, 4) the implementor feels that evaluation may be helpful in implementing the program in the next period, etc.

Evaluation is conducted for the purpose of decision-making. Evaluation may focus on providing information to help the engineer design better programs in the future, or to aid the decision-maker in his assignment of priorities or to assist the implementor in improving his implementation techniques. In addition it may indicate the need for better communications among decision-maker, engineer and implementor. If, however, the evaluation role does not enter into the decision-making process in specific, recognizable ways, then it is not consistent with the posture suggested in this paper.

TIME, ACTIVITY, AND LEVEL OF RESOURCES

The decision framework used in the development of theoretical arguments in this paper is an evolutionary one. Primitive cases introduce much of the terminology. As the cases become more realistic the terminology takes on added complexity. In order that the reader can appreciate the decision framework, some supporting concepts are now introduced.

Time Reference

Two time references are considered in this paper. An *a priori*, or prior, reference is required when the decision-maker wants to:

1) Decide among alternatives without considering an existing school system: this would be appropriate when considering designs for a completely new school system.

2) Decide among alternatives for an existing school system: this would be appropriate when considering changes for a school system.

An a posteriori, or evaluative, reference is used when the decision-maker wants to evaluate how well the school system did in relation to its stated objectives.

Activity

An activity-design is a plan for an activity. It is developed by the engineering function and lends itself to a priori decisions. Once an activity-design has been selected by the decision-maker and implemented, it generally will be evaluated. Evaluation of an activity-implementation is carried out to supply information useful for a new cycle of decisions.

Some activity-designs are very complex. When complexity mandates more detailed analysis, the notion of task is introduced. A set of tasks comprises a program-package in more complex instances. Some of these tasks are independent of performances of preceding tasks ("in parallel"), while other tasks are dependent on performances of preceding tasks ("in series").

Finally, the term activity is used in later sections of the presentation. Activity connotes the same meaning as program-package but is more meaningful given the context.

Level of Resources

Cost is viewed by the decision-maker in one of three ways. The level of resources is either unlimited, limited or known. Unlimited and limited refer to a priori situations and known refers to a posteriori situations.

THE NINE CASE FRAMEWORK: AN OVERVIEW

The cases described in this section, as indicated earlier, present an evolutionary framework. The apparent nature of this evolution is toward complexity; yet while this is true, there is a more fundamental flow. The underlying feature of the nine case structure is realism. Case 1 represents the simplest decision situation possible. It is also remote from the reality of school systems. Cases 8 and 9 represent complex situations. Their solutions represent, as will be seen, prescriptions for school district planning and decision-making.

Each case is developed and analyzed in the format which follows:

Case structure

1. Number of objectives (single or multiple).
2. Number of activities (single or multiple).
3. Time scale (*a priori*, i.e. activities are to be selected, or *a posteriori*, i.e., activities were selected and implemented and are to be evaluated).
4. Level of resources (unlimited or limited, in the *a priori* cases, or known in the *a posteriori* cases).

Case approach

1. Discussion of the approach taken to develop the *decision-variable*. The decision-variable enables the decision-maker to select the preferred course of action or plan. The preferred plan is expected to result in meeting more of the decision-maker's objective(s) than any other

alternative, given the case assumptions and conditions.

Case summary

Discussion of the flow of information required to support selection of the preferred alternative.

Much of the mathematics presented is used to provide inductive proofs consistent with the total approach which should be seen as a synthesis emerging from the simplest case -- Case 1. Figure 1 outlines the logic underlying the classification of the nine cases.

FIGURE 1

A CASE CLASSIFICATION OF LOGICALLY RELATED DECISION PROBLEMS

CASE	STRUCTURE	DECISION FRAMEWORK	RESOURCE LEVEL	COMMENT
1.	Single objective with set of proposed plans (activity-designs) for achieving the objective; one plan is to be selected.	<u>A priori</u>	Unlimited	A utopian research and development problem.
2.	Single objective with set of activity-designs; one to be selected.	<u>A priori</u>	Limited	The constraint limits admissible alternatives; but still no incentive to economize.
3.	Evaluation of Cases 1 and 2.	<u>A posteriori</u>	Known	A performance evaluation involving a partition of outcome space.
4.	Single objective with set of tasks; several to be selected as a program package.	<u>A priori</u>	Unlimited	A more complex version of Case 1; still utopian.
5.	Single objective with set of tasks; several to be selected as a program package.	<u>A priori</u>	Limited	The contraint limits admissible alternatives as in Case 2; still no incentive to economize.
6.	Evaluation of Cases 4 and 5.	<u>A posteriori</u>	Known	A more complex version of Case 3.
7.	Multiple objectives with multiple activities; several to be selected.	<u>A priori</u>	Unlimited	A much more complex version of Cases 1 & 4; the relative weight of objectives becomes important.
8.	Multiple objectives with multiple activities; several to be selected.	<u>A priori</u>	Limited	The general cost-effectiveness case; the only case meeting the necessary and sufficient conditions for cost-effectiveness analysis.
9.	Evaluation of Case 8.	<u>A posteriori</u>	Known	The general program evaluation case; provides cost-effectiveness evaluations for present year and inputs for next year's budget.

CASE 1

This situation is described by a single objective. The decision-maker must assess a group of alternative activity-designs and select one. These alternatives have been prepared by an expert, herein referred to as an engineer. The time scale is a priori and resources are unlimited.

The decision-maker is confronted with a "utopian R & D" situation and must select the optimal plan to meet the given objective. Optimization implies that some type of constraint is put on the decision-maker (if not, he would be involved in maximization). Some constraining factors are time (how much time will the decision-maker allow until he wants to see some results?), state of technology (does measurement methodology exist to enable the implementor to measure a critical variable?), and attitudes of personnel (did the engineer have sufficient commitment to the task of designing improved alternatives?).

Activity-design i has a distribution of performance estimates, K_i. This distribution is based on the engineer's perception of the performance after implementation and hence includes the engineer's biases.

The decision-maker's aim is to select that alternative which provides a preferred balance between estimated average performance and variability of performance, i.e., he is to use the mean-variance decision-criterion. The standard deviation, σ, has been used as a measure of risk in studies dealing with investment decision models.[1]

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[1]For example, see: Donald E. Farrar, The Investment Decision Under Uncertainty (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962), pp. 25-26.

Basic to the mean-variance criterion is the idea that there exists a relationship between risk, σ, and acceptable mean return, μ. Most decision-makers are risk avoiders. They require increasingly higher mean returns as risk is increased, i.e., they discount risk at a higher rate than mean return.

A certainty-equivalent function is an indifference curve comprised of combinations of $\{\mu, \sigma\}$ for which the decision-maker is indifferent. The certainty-equivalent is the value of mean return corresponding to zero risk for a given indifference curve. If the decision-maker is allowed to choose from the $\{\mu, \sigma\}$ points on a given curve, including the certainty-equivalent, he should be indifferent to all.

Indifference curves are generally used to indicate combinations of two commodities for which a person, given his preferences, is indifferent. An indifference curve is negatively sloped reflecting that it takes an increased amount of one commodity to offset the loss of an amount of the other commodity. The slope of the curve at any point indicates the marginal rate of substitution of one commodity for the other. Two indifference curves cannot intersect since this would mean that there could exist two equally preferred points each with the same return but with different risk assignments.

How does this relate to the selection of the preferred activity-design? Assume that the engineer can provide, for each proposed activity-design i, expected performance, $\bar{K}_i$, and variance, S_i^2, where

$$\bar{K}_i = \int_{K_0}^{\lambda_i} k_i\, f(k_i)\, dk_i \qquad , K_0 \leq K_i \leq \lambda_i ,$$

$$S_i^2 = \int_{K_0}^{\lambda_i} (k_i - \bar{K}_i)^2 f(K_i)\, dK_i \qquad , K_0 \leq K_i \leq \lambda_i .$$

It is assumed that performance is defined over the range 0 to 1.0. This range indicates the degree to which the activity-implementation is expected to meet its objective. Initially, the decision-maker selects a minimum level of acceptable performance, K_0 . The λ_i upper limit is based on the engineer's estimate of a level of performance which "cannot" be surpassed were activity-design i to be implemented. Once K_0 has been established, the decision-maker considers those higher performance scores and their inherent risks which he prefers equally to a riskless K_0 . These points define an indifference curve which is independent of any data received from the engineer. That is, the curve is defined solely on the basis of the decision-maker's preference for various combinations of expected performance and risk. If the preferences of the decision-maker were not considered, then he would be unnecessary in Case 1 and the engineer could make the selection using his criteria.

Figure 2 indicates the general nature of the Case 1 decision-maker's indifference curve, denoted $I(K_0)$. The sketch describes a relationship of expected performance to variability for a decision-maker characterized by risk avoidance. Therefore, K_i must increase at a faster rate than S_i in order for the decision-maker to remain indifferent. K_0 is the certainty-equivalent value.

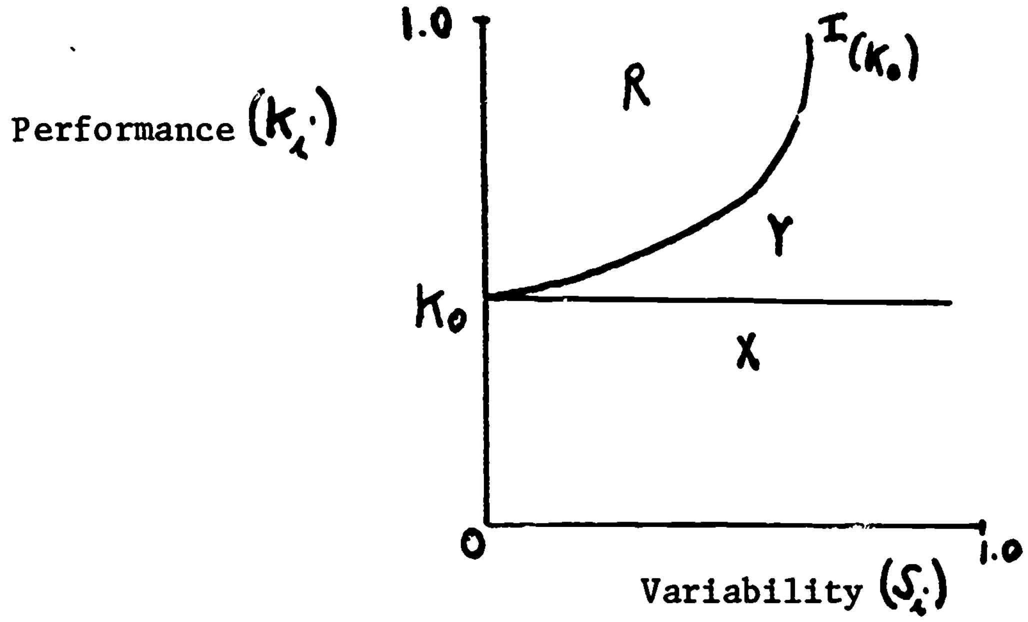

Figure 2.
Indifference Curve of K_0

The X region of Figure 2 contains points for activity-designs which have expected performance levels below the minimum acceptable level, K_0 . The Y region contains points that are acceptable with respect to expected performance, but all of these points are dominated by any point on $I_{(K_0)}$ having the same expected performance but a lower variability.

Figure 3 shows a group of indifference curves. The curve $I_{(K_0)}$ increases rapidly in the region $S_i = 0.5$, indicating the decision-maker's uneasiness over the high level of variability.

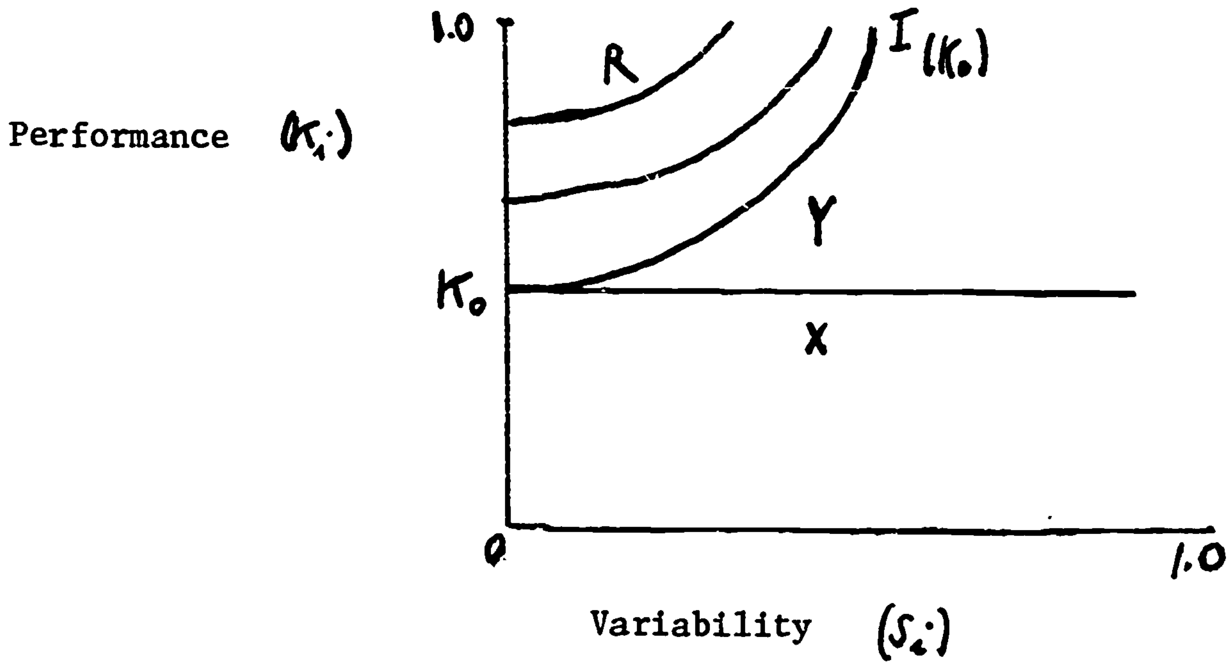

Figure 3. Several Indifference Curves

Suppose a parabola of form $S = a + bK + cK^2$ approximates the form of these indifference curves. Since three constants are to be determined, three sets of coordinates are required. For instance, $\{K_0 = 0.4, S_i = 0\}$,

$\{K_{i_1} = 0.6,\ S_{i_1} = 0.3\}$ and $\{K_{i_2} = 1.0,\ S_{i_2} = 0.6\}$ satisfy the equation $S_i = 0.10 - 0.75K_i + 1.25K_i^2$. In general, K_i values can be obtained for the quadratic form using the formula

$$K_i = \frac{-b \pm \sqrt{b^2 - 4c(a - S_i)}}{2c},\ c \neq 0.$$

Some checks on the equation would involve seeing that $0 \leq S_i \leq 1$ and $K_0 \leq K_i \leq 1$. The slope of the function can be interpreted as the rate of increase of K_i with S_i, that is $\frac{dK_i}{dS_i}$;

$$K = \frac{-b \pm \sqrt{b^2 - 4c(a-s)}}{2c} = \frac{-b}{2c} \pm \frac{1}{2c}\left[b^2 - 4c(a-s)\right]^{1/2}$$

$$\frac{dK}{dS} = \left(\frac{1}{2c}\right)\left(\frac{1}{2}\right)\left[b^2 - 4c(a-s)\right]^{-\frac{1}{2}}(4c)$$

$$\frac{dK}{dS} = \pm\left[b^2 - 4c(a-s)\right]^{-\frac{1}{2}}$$

Using the equation for dK/dS we find the rate of change in expected performance per unit of variability at the point $S_i = 0.5$ in the previous equation is 0.53 (-0.53 is not meaningful). This means that the rate of change of the curve when $S_i = 0.50$ is $0.53K_i$ per unit of S_i. At $S_i = 0.6$ the derivative is 0.57 indicating the increase in the response of K_i to changes in S_i.

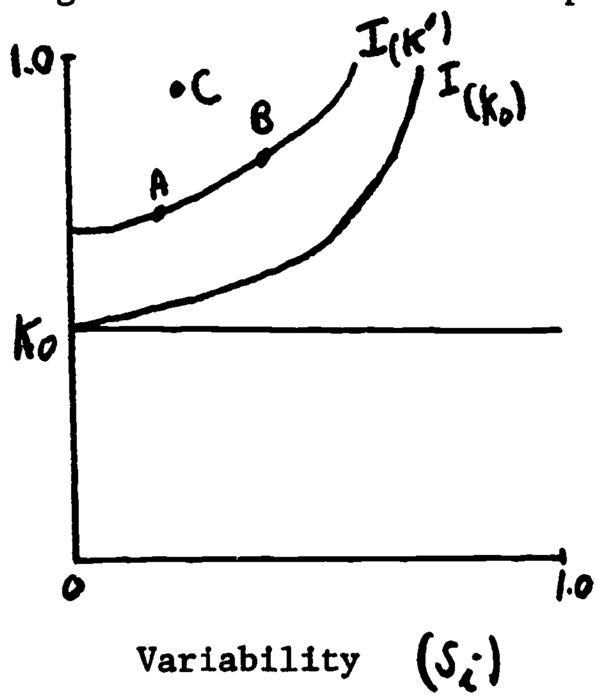

Figure 4.
Selecting a Preferred Point.

The logic that allows the preferred point to be selected is not complex. The ideal point is $\{K_i = 1, S_i = 0\}$. Figure 4 shows an indifference curve and three points. Points A and B are equally preferred since they lie on $I_{(K')}$. Point C is preferred to both A or B since it has a higher performance value than corresponding points on $I_{(K')}$ having the same level of variability as Point C. Point C is also preferred to any point on $I_{(K')}$ having the same level of performance as C since C has less variability. The preferred activity-design is the one with a point lying on or above the uppermost indifference curve.

In conclusion, for Case 1 the decision-maker should state his preferences as combinations of expected performance and variability. Then he should select that alternative which will provide the optimal combination of expected performance and variability according to his feelings.

Case 1 data types by source*

Source	Data Type
decision-maker	single objective
engineer (s)	activity-designs
engineer (s)	performance estimates
engineer (s)	distribution of performance estimates
decision-maker	minimum level of acceptable performance
decision-maker	indifference curve of K_0

*Implied in the development of Case 1 was a performance criterion since a prior condition for obtaining a performance estimate, K_i, is that a criterion exist. This criterion is prepared jointly by the decision-maker and engineer. They may be wise, however, to consult with users of the system and the implementor.

CASE 2

This case differs from Case 1 in one important respect: resources are limited. Again there is a single objective with multiple activity-designs from which only one is to be selected in an a priori framework. The decision-maker will select that activity-design which promises the highest certainty-equivalent he can afford. When he does not elect to behave in this manner he has implicitly introduced a second objective. It may be a personal objective such as the desire to be efficient under any circumstances, or he may entertain some vague notion of future possibilities developing for investment and hence want to save funds. But the point remains, however, that in Case 2 he has no alternative objectives in competition with the attainment of the single, given objective.

The decision rule is to select the activity-design represented by a point on or above the highest indifference curve, provided that it falls within the limitations of his budget. Instead of treating the three variables (mean, standard deviation and cost) in a three dimensional drawing, the decision graph may be viewed as two dimensional by using the certainty-equivalent in place of K_i and S_i . Figure 5 shows certainty-equivalent values on the ordinate and estimated costs associated with each of the admissible activity-designs on the abscissa. Costs are assumed to be point estimates with no accompanying measure of variability. The decision-maker should, therefore, be cognizant of this, since activity-implementations with prior cost estimates less than but near C_{MAX} may exceed C_{MAX} . When there is some budgetary leeway he may not be troubled by the possibility of exceeding C_{MAX}. In realistic instances the decision-maker tries to provide for such contingencies.

Admissible alternatives, in Figure 5, are represented by points for which $C_i \leq C_{MAX}$, and $K_i \geq K_o$. In order to select the preferred alternative, the decision-maker selects the point having the highest certainty-equivalent within the region $C < C_{MAX}$. If there were two or more candidate points having "equally highest" certainty-equivalents, then the decision-maker would select the least-cost point, although according to the strict structure and assumptions of Case 2 he would be unable to use a partially unexpended budget. The net result of adding the budget limitation is that points such as B in Figure 5 are eliminated from consideration. In the Figure 5 example, the decision-maker would be compelled to select Point C over Point B if Point A did not exist, indicating the potential importance of a budgetary restriction.

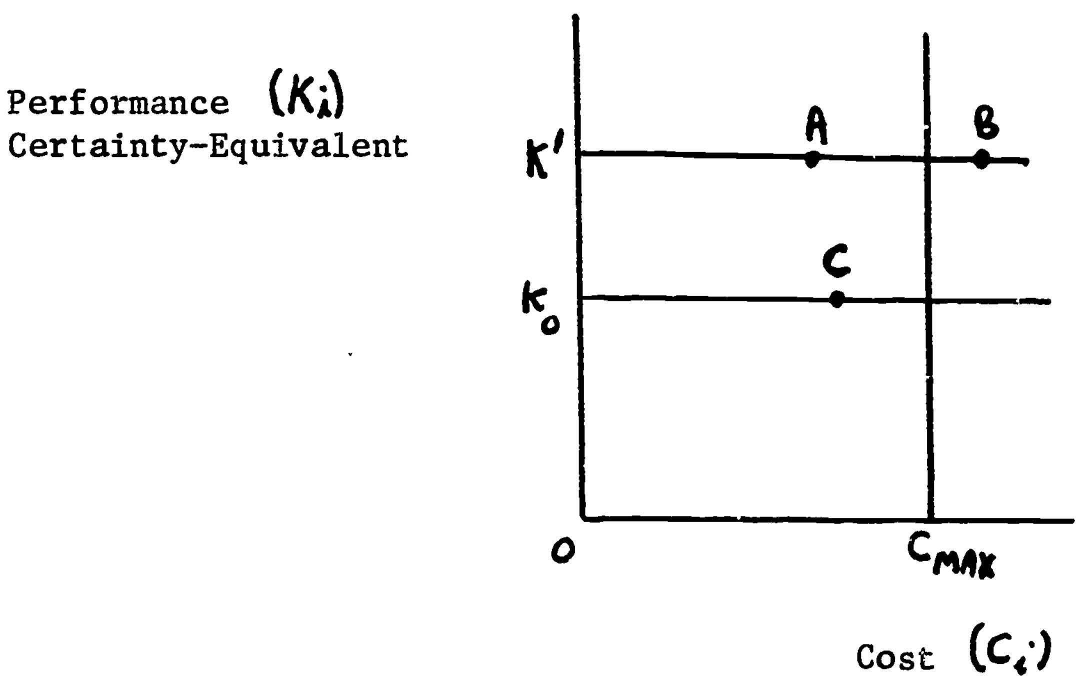

Figure 5. Selecting a Preferred Point with a Cost Constraint

In summary, for Case 2 the decision-maker should follow the guidelines established in Case 1 with regard to stating his preferences. Then he should select that admissible alternative which will provide the optimal combination of expected performance and variability. The cost constraint determines which activity-designs, of those promising $K_i \geq K_o$, are admissible.

Case 2 data types by source*

Source	Data Type
presumably imposed on the decision-maker	cost constraint, C_{MAX}
engineer and implementor in concert	cost estimates, C_i

---

*Only new data types are included. Those which were defined in Case 1 are not repeated here.

CASE 3

Situations which are characterized by the structures of Case 1 and 2 result in the selection and implementation of an activity-design. Soon the decision-maker is faced with an evaluative problem -- given that the activity has been conducted, how well did it do? Case 3 is the a posteriori evaluation of Cases 1 and 2.

At first it might appear that the problem should be structured as a test of a statistical hypothesis. This is not required, however, since possible performance outcomes can be partitioned without statistical consideration.

Figure 6 shows the probability distribution of performance, $f(K_i)$, as perceived, a priori, by the engineer.

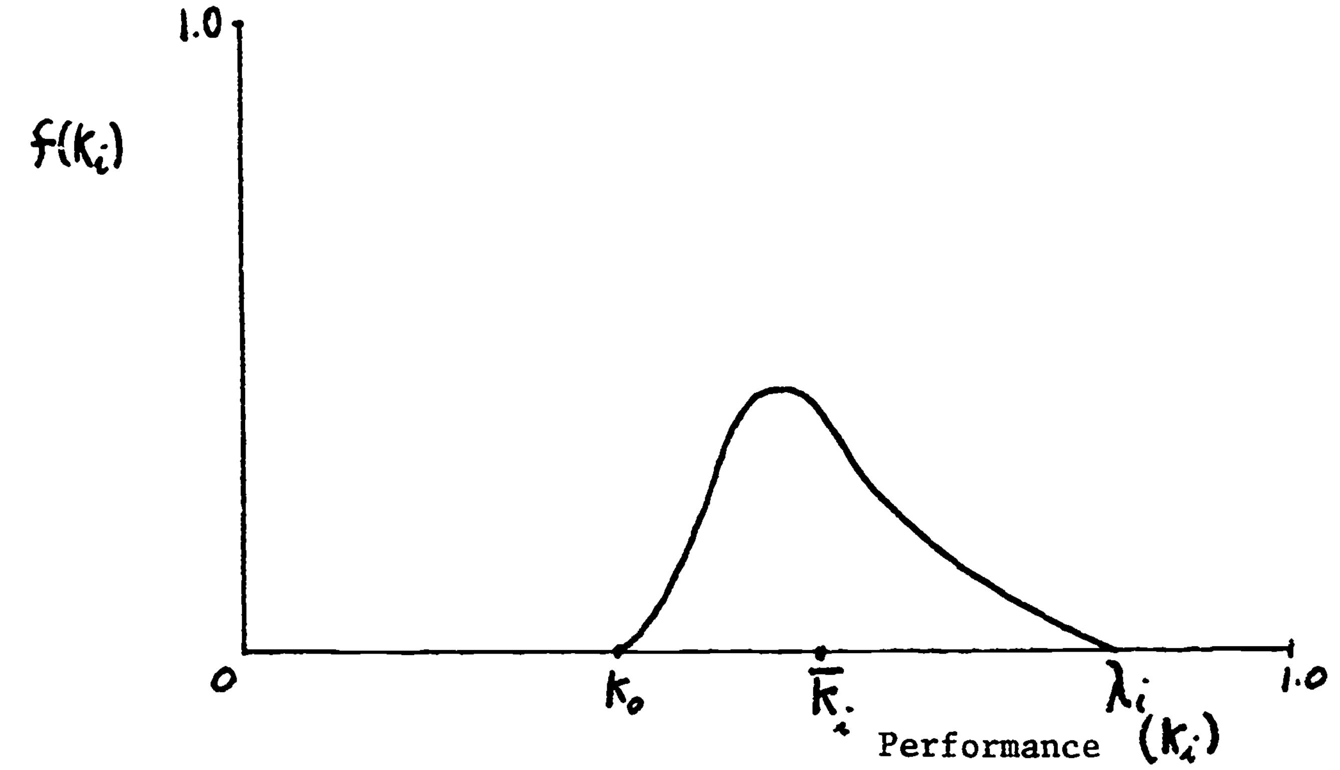

Figure 6. Prior Distribution of K_i

When actual performance, K_i^*, falls below the level of K_0, i.e., $0 \leq K_i^* < K_0$, the engineer has not necessarily made a miscalculation. This performance outcome could be attributable to factors which he could not possibly predict a priori. The decision-maker may re-evaluate the engineer's ability to design and estimate; the engineer may evaluate his methods.

It should be noted, in a precautionary sense, that K_0 is a certainty-equivalent and the probability of $K_i^* < K_0$ is dependent in part on the density of K_i^*. The density of K_i^* exists in the same sense that the sampling distribution of the sample mean exists even though only a single outcome, $\bar{X}_i$, is generally observed. It should be added, of course, that much less is known about the density of K_i^* than the density of $\bar{X}_i$.

If actual performance, K_i^* , falls between K_o and λ_i , as the engineer predicted prior to implementation, then it is within the range deemed acceptable. The engineer should, nevertheless, evaluate his methods to be sure that he has minimized the probability of getting the right answer for the wrong reason.

If actual performance exceeds the upper boundary of the prior distribution $(i.e., \lambda_i < K_i^* \leq 1.0)$, then the engineer may have miscalculated on the favorable side. This prior "pessimism" on the part of the engineer may have ruled out other activity-designs which should have been selected for implementation. The decision-maker should re-evaluate the engineer's ability to estimate and the engineer should evaluate his methods.

The decision-maker should understand that the main benefit derived from evaluating actual outcomes is discovering improved methods for

1. designing future activities,
2. estimating the potential of future designs, and
3. implementing future activities as they are designed.

Implementing a newly designed activity often requires a "change technology" that is not understood. An activity-design may be perfect with respect to technical design, yet its implementation may be difficult due to inadequacies in training, support systems, and attitudes. Good communication between engineer and implementation team during the activity-design phase as well as the implementation phase will improve chances of obtaining high performances.

Case 3 data types by source*

Source	Data Type
engineer in consultation with implementor	distribution of K_i , $f(k_i)$
decision-maker	evaluation decision-rules corresponding to partition of $f(k_i^*)$
evaluator in consultation with implementor	performance outcome, k_i^*

*Only new data types are included. Those which were defined in previous cases are not repeated here.

CASE 4

Cases 4 and 5 closely parallel Cases 1 and 2. A difference exists in that Cases 4 and 5 require the decision-maker to select a package of tasks in order to achieve his single objective.

Case 4, then, considers a decision-maker pursuing a single objective and having unlimited resources. The framework is <u>a priori</u> and he wants to select a program package consisting of a group of tasks.

Previously, there was an implied assumption that, in the interval k_0 to λ_i , changes in worth to the decision-maker were proportional to changes in performance. That is,

$$\frac{\Delta K_{AB}}{\Delta w_{AB}} = \frac{\Delta K_{BC}}{\Delta w_{BC}} = \cdots = \frac{\Delta K_{XY}}{\Delta w_{XY}}\text{, where,}$$

ΔK_{AB} = the change in performance from K_A to K_B , where both points are between K_0 and λ_i , and

Δw_{AB} = the change in worth associated with ΔK_{AB}.

It may also be assumed that below K_0, the minimum level of acceptable performance, the same performance-worth relationship exists but the decision-maker feels that an inadequate level of worth would result for performance in this interval. The same reasoning holds for $K_i^* > \lambda_i$. This assumption is carried through Case 6.

The reason for introducing the concept of worth, w_{iJ}, is that the decision-maker may feel that various tasks within a program package do not contribute equally to the overall objective. Task worth will be used to weight task performance, and expected weighted performance will serve as the decision-variable.

The situation confronting the Case 4 decision-maker is that he must select a package of tasks. Some task outputs may be independent of outputs of previous tasks. Other tasks may rely heavily on outputs of previous tasks. The decision-maker may ask his engineers to rearrange tasks in order to design an improved package. It was mentioned earlier that the decision-maker may perceive the various tasks as contributing unequally to the overall objective. The problem, therefore, is to select the program-package promising the largest expected worth. This problem may be exceptionally difficult since it not only involves a choice of tasks but also arrangement of tasks within the package.

Theoretically, the engineer would structure a multivariate joint density function describing the outcomes and associated probabilites for each program package,

$$f(K_{i1}, K_{i2}, \ldots, K_{im}).$$

For the distribution above, $K_{0_{iJ}}$ would refer to the minimum acceptable performance for task J of activity-design i, and λ_{iJ} would refer to the "maximum performance possible" for task J of program package i.

Then the engineer would be able to derive marginal density functions by integrating the joint density with respect to the other variables over their full ranges (e.g. $K_{0_{iJ}}$ to λ_{iJ} for the general term). For instance, the marginal density of task 1 is

$$f_1(K_{i1}) = \int_{K_{0_{i2}}}^{\lambda_{i2}} \int_{K_{0_{i3}}}^{\lambda_{i3}} \cdots \int_{K_{0_{im}}}^{\lambda_{im}} f(K_{i1}, K_{i2}, \ldots, K_{im})\, dK_{i2}\, dK_{i3} \cdots dK_{im}\,; \quad f_J(K_{iJ}) \geq 0.$$

Also the joint distribution of tasks 1 and 2 is

$$g_1(K_{i1}, K_{i2}) = \int_{K_{0_{i3}}}^{\lambda_{i3}} \int_{K_{0_{i4}}}^{\lambda_{i4}} \cdots \int_{K_{0_{im}}}^{\lambda_{im}} f(K_{i1}, K_{i2}, \ldots, K_{im})\, dK_{i3}\, dK_{i4} \cdots dK_{im}\,; \quad f_J(K_{iJ}) \geq 0.$$

If the outcomes of task 2 are independent of task 1 outcomes, then their joint density function would be equal to the product of their marginal densities.[2] From $g_1(K_{i1}, K_{i2})$ the engineer would obtain the conditional density of task 2 outcomes given those of task 1 by regarding K_{i1} as a constant over its range of values (for that matter, over any part of its range).

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[2] In a general sense, independence can be assumed when the joint density can be factored into marginal densities, each involving only one variable and with the limits of each not involving another of the variables.

If the range of K_{i1} is restricted then the conditional density of K_{i2} will be limited by the range of K_{i1}. The conditional density of K_{i2} is defined as

$$h_2(k_{i2} \mid k_{i1}) = \frac{g_1(k_{i1}, k_{i2})}{f_1(k_{i1})}, \quad f_1(k_{i1}) > 0.$$

As was indicated earlier, the engineer could, if the decision-maker requested, answer such questions as "what is the probability that performance in task 2 will take on a value $\bar{K}_{i2}$ or more given that performance in task 1 was also $\bar{K}_{i1}$ or more?" This would be structured as

$$h_2(k_{i2} \geq \bar{K}_{i2} \mid k_{i1} \geq \bar{K}_{i1}) = \frac{\int_{k_{i1}=\bar{K}_{i1}}^{\lambda_{i1}} \int_{k_{i2}=\bar{K}_{i2}}^{\lambda_{i2}} g_1(k_{i1}, k_{i2})\, dk_{i1}\, dk_{i2}}{\int_{k_{i1}=\bar{K}_{i1}}^{\lambda_{i1}} f_1(k_{i1})\, dk_{i1}}; \quad f(k_{i1}) > 0$$

And if the sequential outcomes are independent, then

$$h_2(k_{i2} \geq \bar{K}_{i2} \mid k_{i1} \geq \bar{K}_{i1}) = f_2(k_{i2} \geq \bar{K}_{i2})$$

The preceding type of analysis would be possible from the multivariate density level down to the univariate level if the engineer were capable of

providing a distribution of such complexity. In all fairness to the engineer, however, it is assumed for the remainder of this analysis that he cannot develop distributions more complex than bivariate densities.

Since the decision-maker wants to be able to compare alternative program packages so as to select that package promising the highest expected worth, he must consider several things. He has the engineer derive the marginal probability density for each task from the bivariate density, linking each task with the preceding task. The implicit assumption is that dependency is an appropriate consideration for adjacent tasks only. This means, for instance, if task 2 outcomes depend on those of task 1 the dependency would be ignored by this analysis. A more general case would involve considering all task combinations. In this instance, C_2^m relationships would result,[3] but as will be seen later, expected worth is a function of the marginal distribution alone. Consequently, the extension is only appropriate when considering particular probability questions. Then he assigns a worth value w_{ij} to each task, where $0 \leq w_{ij} \leq \theta_{ij}$. Lastly, he must consider the relationship of w_{ij} to performance outcomes, K^*_{ij}, of the respective tasks.

The procedure involves taking the respective bivariate densities and integrating out the marginal probability density, $f_j(K_{ij})$, for each task. Then $f_j(K_{ij})$ is transformed into $W_j(w_{ij})$, and the expected worth for the entire package can be found by aggregating over the respective tasks in the package. For the sake of illustration, let a three task situation be described for program package i : $g_1(K_{i1}, K_{i2})$ and $g_2(K_{i2}, K_{i3})$.

[3] Appendix A discusses combinatorial aspects of the selection problem.

When these bivariate densities are factored into their component probability densities

$$g_1(k_{i1}, k_{i2}) = f_1(k_{i1}) f_2(k_{i2})$$
$$g_2(k_{i2}, k_{i3}) = f_2(k_{i2}) h_3(k_{i3} | k_{i2})$$
$$= f_3(k_{i3}) h_2(k_{i2} | k_{i3}).$$

The transformation of k_{iJ} into w_{iJ} is a linear mapping in all instances, where a and b are psychological parameters.

$$w_{i1} = b_{i1} k_{i1} + a_{i1} \quad , \quad 0 \le w_{i1} \le \theta_{i1} \quad ,$$

$$w_{i2} = b_{i2} k_{i2} + a_{i2} \quad , \quad 0 \le w_{i2} \le \theta_{i2} \quad ,$$

and

$$w_{i3} = b_{i3} k_{i3} + a_{i3} \quad , \quad 0 \le w_{i3} \le \theta_{i3} \quad .$$

Solving for k_{iJ} in terms of w_{iJ}, we obtain

$$k_{i1} = \frac{w_{i1} - a_{i1}}{b_{i1}} \quad , \quad b_{i1} > 0 \, ,$$

$$k_{i2} = \frac{w_{i2} - a_{i2}}{b_{i2}} \quad , \quad b_{i2} > 0 \, ,$$

and

$$k_{i3} = \frac{w_{i3} - a_{i3}}{b_{i3}} \quad , \quad b_{i3} > 0.$$

The differentials of k_{iJ} are

$$dk_{i1} = dw_{i1}/b_{i1} \quad ,$$
$$dk_{i2} = dw_{i2}/b_{i2} \quad ,$$
$$dk_{i3} = dw_{i3}/b_{i3} \quad .$$

Since the outcomes of task 2 are assumed in our current example, to be independent of task 1 outcomes, the expected worth integration involves obtaining the marginal densities, transforming performance into worth, and generating the first moment. The marginal density of K_{i1} is $f_1(k_{i1})$, and it is a probability distribution. That is

$$\int_{K_{o_{i1}}}^{\lambda_{i1}} f_1(k_{i1})\,dk_{i1} = 1 \quad , \; f_1(k_{i1}) \geq 0$$

The marginal density of w_{i1} is $W_1(w_{i1})$, and it is a probability distribution. The limits of integration for W_1, are not, in general, 0 and θ_{i1}, since the scale of K_{i1} ranges from 0 to 1 with $K_{o_{i1}}$ and λ_{i1}, generally, well within this interval. That is

$$\int_{w_{i1}=b_{i1}K_{o_{i1}}+a_{i1}}^{w_{i1}=b_{i1}\lambda_{i1}+a_{i1}} W_1(w_{i1})\,dw_{i1} = 1 \qquad , \; W_1(w_{i1}) \geq 0$$

The expected worth, $\bar{w}_{i1}$, of task 1 is

$$\bar{w}_{i1} = \int_{b_{i1}K_{o_{i1}}+a_{i1}}^{b_{i1}\lambda_{i1}+a_{i1}} w_{i1}\,W_1(w_{i1})\,dw_{i1} \,.$$

The identical procedure for task 2 will yield $\bar{w}_{i2}$, but task 3, which is related to task 2 in terms of performance outcomes, requires a consideration of the conditional relationship. The expected value of K_{i3} given a particular

value of K_{i2} is an arithmetic mean. If, however, the expectation of K_{i3} over the entire range of K_{i2} is desired, then this is a function of K_{i2} and what is needed is the expectation of the distribution of expectations. That is:

1) $E(Y|x)$ is in general a function of x. Let $u(x)$ denote this function and $h(x)$ be the probability density of x. Also X and Y denote the random variables and x and Y values of the random variables.

2) $$E[E(Y|X)] = E[u(x)]$$

$$= \int_{-\infty}^{\infty} u(x)\,h(x)\,dx = \int_{-\infty}^{\infty} E(Y|x)\,h(x)\,dx$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} Y g(Y|x)\,dY \right] h(x)\,dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Y g(Y|x)\,h(x)\,dY\,dx$$

$$= E(Y) = \bar{Y}$$

The proof is taken from Mood and Graybill[4] and it shows that the conditional expectation of $Y|x$ is equal, in general, to the mathematical expectation of the margiral density of Y. Consequently, $\bar{w}_{i3}$ is equal to the expected value of $W_3(w_{i3})$. From this example it is seen that, in general, the expected

[4] Alexander M. Mood and Franklin A. Graybill, Introduction to the Theory of Statistics, 2nd edn. (New York: McGraw-Hill Book Company, Inc., 1963), p. 118.

worth of program package i is the sum of the expected worths for the tasks in the package. That is,

$$\sum_{J=1}^{m} E\left[W_J(w_{iJ})\right] = \sum_{J=1}^{m} \int_{w_{iJ}=b_{iJ}K_{0iJ}+a_{iJ}}^{w_{iJ}=b_{iJ}\lambda_{iJ}+a_{iJ}} w_{iJ}\, W_J(w_{iJ})\, dw_{iJ} \qquad ; W_J(w_{iJ}) \geq 0.$$

As in Case 1, the decision-maker compares alternative packages and selects the one with the preferred value of the decision variable.[5] In this case, the decision variable is overall expected worth since he is unconstrained by resources and does not have alternative outlets for expenditures. He compares overall expected worth for the respective alternatives and selects the package with highest expectation.

In conclusion, Case 4 has introduced several features into the analysis. The certainty-equivalence structure was abandoned due to advantages gained by using probability distributions for multivariate situations. In addition, mathematical expectation played a critical role because it was necessary to reflect the unequal potential contribution of tasks. The decision variable, weighted expected performance, was obtained by means of transformation. While the transformation was treated as linear, the analysis is not restricted to

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[5] Implied in the summation of $\overline{w}_{iJ}$ is an assumption that the scale of w_{iJ} is an interval scale. This means the scale is unique up to a positive linear transformation. That is, a new scale, Y, can be made by a linear transformation of the X scale $(Y = a + bX ; b > 0)$ without distorting the underlying relationships. See, for instance, Torgerson, Warren, Theory and Methods of Scaling, (New York: John Wiley & Sons, Inc., 1958), pp. 19-20.

linear transformations. The perspective needed to transform performance into worth, in a given situation, can only be supplied through an understanding of the real importance of performance outcomes in relation to the desired outputs of the enterprise.

Case 4 data types by source*

Source	Data Type
decision-maker in consultation with implementor and engineer	performance to worth transformation
engineer	joint and marginal probability task performance densities
decision-maker and engineer	minimum acceptable task performance
engineer providing calculations	expected worth of program package i

*Only new data types are included. Those which were defined in previous cases are not repeated here.

CASE 5

This case introduces a cost restriction into the structure of Case 4. The decision-maker attempts, as in Case 4, to achieve a single objective by selecting one program package in an a priori framework.

Again, as in Case 2, the decision-maker does not encounter an economic problem even though he has a cost constraint. The reason for this conclusion is that he has no alternative uses for unexpended resources, since he has only one objective. The only restriction which the constraint places upon the decision-maker is that the package he selects must have an expected cost

which is less than, or equal to, his budget amount. Hence the program-package selected may have a lower expected worth than the program-package selected in Case 4. This argument is consistent with the results of Cases 1 and 2.

Figure 7 is structured similar to Figure 5 (Case 2). A difference between the two figures is that Figure 7's ordinate is expected worth, while Figure 5's ordinate is certainty-equivalent. The reasoning is perfectly analogous. Admissible points lie in the region where $C_i \leq C_{MAX}$ and $\sum_{j=1}^{m} \bar{w}_{ij} \geq \bar{\bar{w}}(K_o)$. Before continuing, it is appropriate to define $\bar{\bar{w}}(K_o)$. In Case 4 it was seen that each task had a minimum acceptable performance level which was set by the decision-maker <u>and</u> the engineer. This value, $K_{o_{ij}}$, was actually a concept carried forward from the certainty-

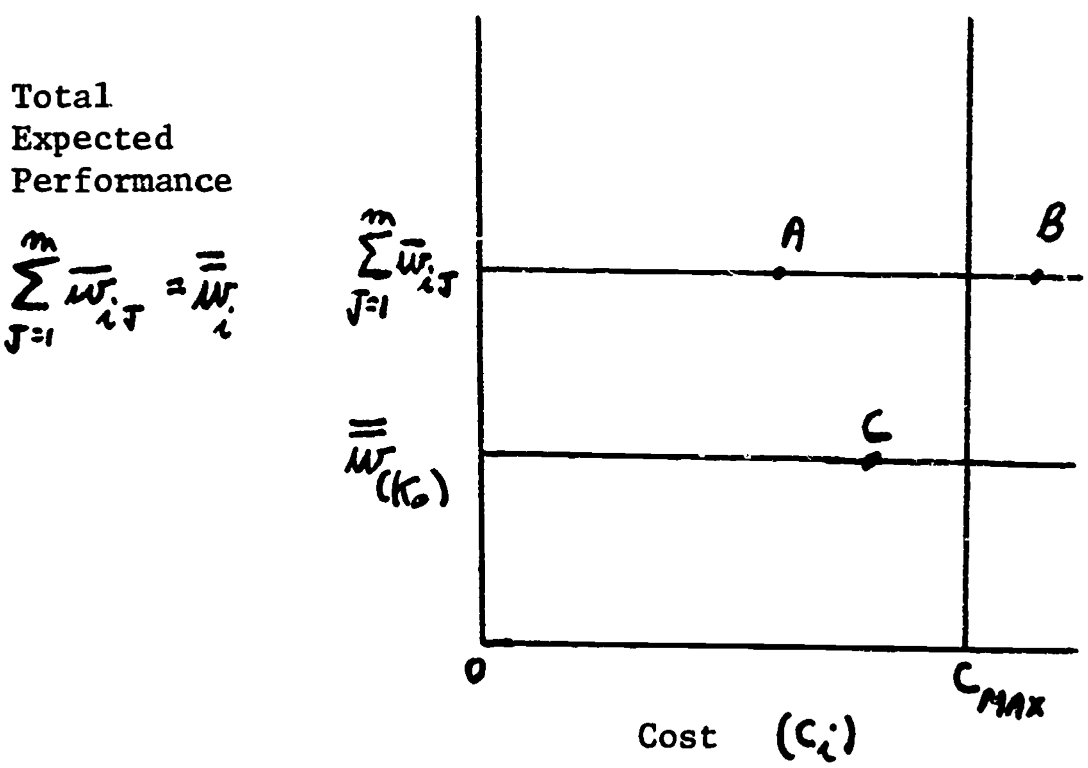

Figure 7. Selecting a Preferred Package with a Cost Constraint

equivalent analysis of earlier cases. At a higher order level there is still an overall K_0 which the decision-maker sets. It was assumed that $\frac{w_{0ij} - a_{ij}}{b_{ij}}$ was the transformed value of K_{0ij}. Therefore, $\sum_{j=1}^{m} w_{0ij}$ is assumed to be equivalent to the worth corresponding to the overall K_0. This implies that $\sum_{j=1}^{m} w_{0ij}$ is a constant for all packages, although the condition is not necessary for the analysis.

An additional complication arises in that the decision-maker considers the advisability of allocating the budget among tasks of the preferred program-package. The position taken by the author is that allocation of the budget among tasks belongs in the domain of the implementor since he, alone, has responsibility for implementation. The implementor's major problem is one of accounting and control. It is true that he also has a problem of allocating his budget among tasks. Presumably, he has the available cost information prepared by the engineer. It is suggested, in this light, that the engineer should involve the implementor in estimating task cost figures.

A special instance of Case 5 is found in situations where the decision-maker pursues a single objective and the various tasks result in production of a homogeneous output. For example, consider a planning problem in which the decision-maker wants "to maximize the number of high school graduates" subject to a budgetary constraint. He will consider alternative institutions which produce high school graduates in the same way we have treated tasks. His problem may lend itself to linear programming solution.[6]

[6] James F. McNamara, Pennsylvania Department of Education, is credited with pointing out the applicability of linear programming to the special instance of Case 5.

In summary, Case 5 parallels Case 2. The cost constraint is seen by the decision-maker as a restriction on the admissible program-packages. Expected program-package worth is the decision-variable taking into account the differential ability of tasks to contribute to their end product.

The only new data type is expected task cost, C_{ij} . It is supplied by the engineer, hopefully, in consultation with the implementor.

CASE 6

Situations which are characterized by the structures of Cases 4 and 5 result in the selection and implementation of a program-package. At the end of a period of time the decision-maker is faced with an evaluation problem -- given that a package has been implemented the results should be evaluated. Case 6, therefore, is the a posteriori evaluation of Cases 4 and 5.

The rationale employed in this evaluation is similar to that of Case 3. The purpose of evaluation is to improve the decision-making process through insights into ways to improve the engineering and implementation of program-packages.

The present case has two levels of evaluation. At a higher order or overall level the decision-maker wants to evaluate the overall performance and compare it to K_0. It is allowable to think in terms of either performance or worth depending on the type of question being considered. If an overall comparison is required, it is probably more satisfying to transform produced worth, based on the observed performance outcomes of the respective tasks, into an overall performance variable, K_i^* . Two values satisfying this

relationship are available from the analysis. They are

overall performance	produced worth
K_o	$\bar{\bar{w}}_{(K_o)} = \sum_{J=1}^{m} w_{o_{iJ}} = \sum_{J=1}^{m} b_{iJ} K_{o_{iJ}} + a_{iJ}$
λ	$\bar{\bar{w}}_{(\lambda)} = \sum_{J=1}^{m} w_{\lambda_{iJ}} = \sum_{J=1}^{m} b_{iJ} \lambda_{iJ} + a_{iJ}$

The points connecting $\sum_{J=1}^{m} w_{iJ}^{*}$ and K_i^{*} are required of the decision-maker in order for him to be able to make the overall evaluation. For the most part this evaluation is a comment on the manager of the implementation team. The engineer also is interested in the outcomes, since it was his design that was implemented. The interrelationship of responsibilities can cause problems for all parties involved. Therefore, the emphasis should be on improvement of methods rather than assignment of blame.

The second level of evaluation focuses on performance of the respective tasks. Here the emphasis is directed more to the engineer's estimating methods, the decision-maker's worth assignments, and the members of the implementation team. These evaluations are similar to those of Case 3 and are not repeated here.

A digression of possible interest is in order. Each task has its own probability density. Since only one outcome is observed, the distribution of outcomes for task J is $f_J(K_{iJ})$. This, however, is from an a priori point of view. The fact that some "impossible" outcomes can occur, i.e.

$K^*_{ij} > \lambda_{ij}$ or $K^*_{ij} < K_{0_{ij}}$, is a clear indication that the notion of a posterior distribution of K^*_{ij} is appropriate. Bayesian extensions to analysis of the nine cases are not developed, but further work within this framework could include such analysis.

CASE 7

The structure of Case 7 introduces multiple objectives which will be referred to as overall system objectives. The decision-maker is faced with the a priori selection of program-packages, herein to be referred to as activities, with unconstrained resources.

For the first time, there are alternative competing objectives but since there is no restriction on level of resources, the decision-maker will select a system of activities which, when implemented, should contribute as much as possible to the overall system objectives. As was implied in Cases 1 and 4, the limitations on ultimate system performance are a function of the engineer's imagination, the state of the technological arts, and the abilities of the implementation team.

Let $O_A, O_B, \cdots, O_i, \cdots O_R$ denote the overall system objectives. Since activities selected will ultimately be evaluated on the basis of how well they contribute to these objectives, it is necessary

to assign values, indicating relative importance,[7] to them.

These measures of relative importance afford the decision-maker the opportunity to consider trade-offs among overall system objectives. The common frame of reference or dimension is, of course, value. Assumptions underlying this assignment of values are identical to those for assignment of worth in Case 4. This value function supplants the worth function which was used to weight outcomes, so that a common frame of reference may be developed. The value function, developed in Case 7, provides a uniform basis for assessing outputs prior to, as well as subsequent to, implementation of activities. These values are denoted $V(O_A), V(O_B), \ldots, V(O_i), \ldots, V(O_k)$.

Each activity has potential for contributing to one or more of the overall system objectives. This indicates that the value of overall objective i, $V(O_i)$, is the maximum that can be produced by the system with respect

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[7] Russell Ackoff, et al., show that for systems with two mutually exclusive, totally exhaustive objectives, the maximization of expected value is accomplished by maximizing performance. (Actually they label performance as efficiency and define expected value as $\sum_J E_{iJ} V_J$, where E_{iJ} is performance on objective J by method i, and V_J is the value of objective J.) An extensive proof is given for the case with three or more objectives. This maximization requires performance and values.

Russell L. Ackoff, Shiv K. Gupta, and J. Sayer Minas, Scientific Method, (New York: John Wiley & Sons, Inc., 1962), pp. 93-97.

In addition, the above book presents a method for allowing a single decision-maker to assign weights to objectives. Other methods for eliciting subjective assignments of weights may be found in:

1. Von Neumann, John and Morgenstern, Oskar, Theory of Games and Economic Behavior, 2nd edn., (Princeton, N.J.: Princeton University Press, 1947).
2. Helmer, Olaf, Convergence of Expert Consensus Through Feedback, (Santa Monica: The Rand Corporation, September 1964).

to O_i . If all overall objectives were produced perfectly, the system would have produced the maximum value possible, i.e., $\sum_{i=A}^{\ell} v(o_i)$

The extent to which a particular activity contributes to a particular overall system objective is a function of two variables -- potential for contribution and actual activity performance. That is, $E^*_{mi} = f_{mi}\left[R^*_{mi}, v(a_{mi})\right]$

where E^*_{mi} = the effectiveness of the m^{TH} activity toward the i^{TH} overall system objective,

$v(a_{mi})$ = the maximum value that the m^{TH} activity could produce with respect to the i^{TH} overall system objective,

and $\hat{R}^*_{mi}$ = the adjusted performance score.

The maximum value that the m^{TH} activity could produce with respect to the i^{TH} overall system objective is a function of how important that activity is relative to the other activities aiming at O_i. Here the decision-maker makes a relative value assignment such that the sum of the relative weights is equal to $v(o_i)$. (Were there only one overall objective, there would be no need for $v(a_{mi})$ and Case 7 could be reduced to Case 4.) The distribution of overall value to activities is represented as follows:

Activity directed at O_i	Relative importance	Maximum value possible
a_{1i}	$\ell(a_{i1})$	$\ell(a_{1i})v(o_i) = v(a_{1i})$
a_{2i}	$\ell(a_{i2})$	$\ell(a_{2i})v(o_i) = v(a_{2i})$
$\vdots$	$\vdots$	$\vdots \quad \vdots$
	$\sum^{m} \ell(a_{mi}) \leq 1.0$	$\sum^{m} \ell(a_{mi})v(a_{mi}) \leq v(o_i)$

Based on this distribution of potential value, $V(O_i)$, to activities, it is now possible to determine effectiveness at the activity level. To do this several performance concepts need definition.

The decision-maker and the engineer establish a criterion that can be used to assess the package's performance. Then they list the possible outcomes and relate these possibilities to index scores for the criterion. For instance, suppose that an activity was directed to "getting eligible people to pass Examination Five." One criterion might be the number of eligible people passing the examination expressed as a percentage of the total number of eligible people. The index could be formed by the percentage points. In this context, K^*_{mi} is the index score (percentage) reflecting the performance of package toward overall system objective i . The decision-maker, however, may not feel that each percentage point is entitled to one percent of the potential value of a_{mi}. In this case, K^*_{mi} is transformed into $\hat{K}_{mi}$. The relationship between K^*_{mi}, $\hat{K}^*_{mi}$ and $V(a_{mi})$ is depicted in Figure 8. When $\hat{K}^*_{mi}$ is multiplied by $V(a_{mi})$ the result is E^*_{mi} or effectiveness. Effectiveness is defined as weighted performance. That is, $E^*_{mi} = \hat{K}^*_{mi} \cdot V(a_{mi})$.

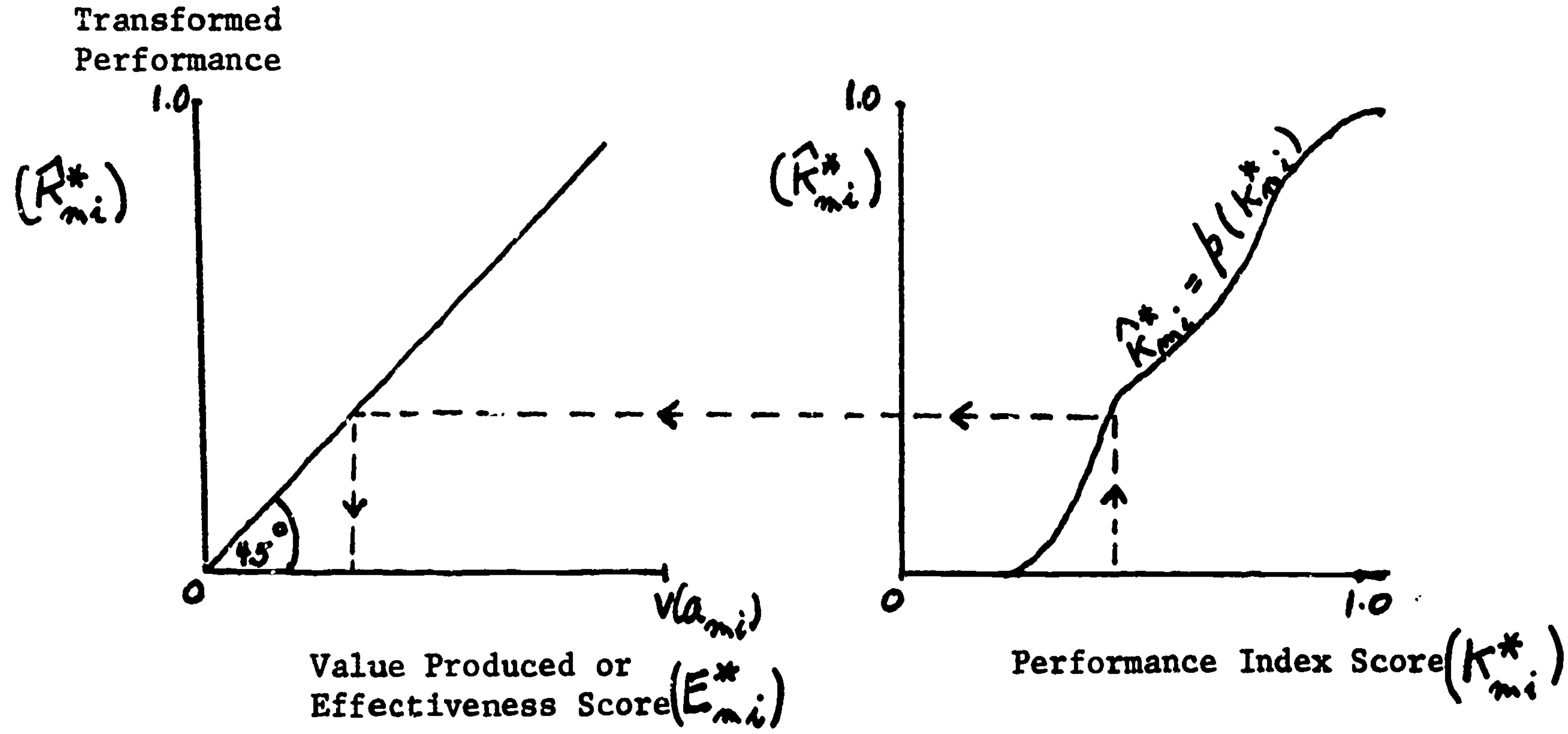

Figure 8. Transformation of Performance Index Scores into Effectiveness Scores

The right hand section of Figure 8 shows the relationship between the criterion index scores, K^*_{mi}, and the decision-maker's adjustments, $\hat{K}^*_{mi}$. A special case occurs when $K^*_{mi} = \hat{K}^*_{mi}$ over the complete range from 0 to 1.0. In general, $\hat{K}^*_{mi} = p(K^*_{mi})$, where the function p denotes the perceptions that underlie the transformation.

The left hand section of Figure 8 shows the way $\hat{K}^*_{mi}$ is used to generate value produced or effectiveness. The reflection line joins the points $\{E^*_{mi} = 0, \hat{K}^*_{mi} = 0\}$ and $\{E^*_{mi} = v(a_{mi}), \hat{K}^*_{mi} = 1\}$, and specifies that the change in effectiveness with respect to a unit change in performance is constant. The relationship is

$$E^*_{mi} = \hat{K}^*_{mi}\, v(a_{mi})$$

and the change in effectiveness for a small change in performance is

$$\frac{dE^*_{mi}}{d\hat{K}^*_{mi}} = v(a_{mi}),$$

with the differential of effectiveness, dE^*_{mi}, equal to $d\hat{K}^*_{mi}\, v(a_{mi})$.

Now that effectiveness has been defined as weighted performance, two simple ideas should be considered.

1. Are large effectiveness scores always preferred?

2. What is the maximum effectiveness score?

In order to conclude that the decision-maker prefers any larger E score to any smaller E score, several comments are necessary. First, the relative values assigned to the overall system objectives represent a set of targets for

the decision-maker. If a unit of value is equally preferred, no matter what objective is being produced, the decision-maker clearly prefers a larger to a smaller E in any case. If a unit of value of one objective is preferred to a unit of value of another objective then the assignment of values requires revision. It is, of course, possible to constrain the production of an overall objective in such a way that some production is preferred over other production. For example, the decision-maker could specify that overall objective i must be produced at some minimum level or better, i.e.

$\sum_{m} \hat{K}^{*}_{mi} \, v(a_{mi}) \geq 0.62 \, [v(o_i)]$. This type of production constraint could have efficiency implications for the system. It would have the same effect on the system operations as a policy, since a policy can inhibit functioning with respect to given criteria in selected instances.

A production function shows technical possibilities relating alternative inputs to the output or commodity under consideration. For instance, test score achievement of a school district may be viewed as the commodity under consideration. Conceptually, this could appear as

$$T = f(X, Y, Z),$$

where T is test score mean,

X is a vector of non-school variables,

Y is a vector of school variables,

Z is a vector of teacher variables.

The search for an optimal solution for the Case 7 type of problem depends on such factors as the nature of the production function, the costs of production, performance, and values.

The maximum E score is the sum of the valuations assigned to the overall system objectives, i.e., $\sum_{i=A}^{R} V(O_i)$. This is readily seen by inspecting the E^* function,

$$E^*_{mi} = \hat{k}^*_{mi} \; V(a_{mi}) \; ,$$

and observing that when $\hat{k}^*_{mi}$ is equal to 1.0 (the maximum value since $0 \leq \hat{k}^*_{mi} \leq 1.0$), $E^*_{mi} = V(a_{mi})$. Therefore, under conditions of perfect performance, $\hat{k}^*_{mi} = 1.0$, $\sum_m V(a_{mi}) = V(O_i)$, and over all objectives the maximum $E^* = \sum_{i=A}^{R} V(O_i)$.

In this situation, as in Case 4, it is within the scope of the argument to treat performance as a random variable. This procedure is now outlined.

The engineer provides the probability density of K_{mi} as in earlier instances. In an a priori sense, the distribution $f_{mi}(k^*_{mi})$ is identical to $f_{mi}(K_{mi})$. Two transformations are required. The first allows $\hat{k}^*_{mi}$ scores to substitute for k^*_{mi} scores. Since a simple multiplicative form has been used to relate E^* to its components, the second transformation is constant for all such relationships and the probability density of E^*_{mi} can be inferred from $f_{mi}(k^*_{mi})$. That is,

$$E^*_{mi} = f_{mi}\left[\hat{k}^*_{mi}, V(a_{mi})\right]$$

$$E^*_{mi} = \hat{k}^*_{mi} V(a_{mi})$$

Since $V(a_{mi})$ is a constant for all $\hat{k}^*_{mi}$, the expected value, $\bar{E}^*_{mi}$ is given by

$$\bar{E}^*_{mi} = \bar{V} = \int_{\hat{K}^*_{mi} v(a_{mi}) = 0}^{\hat{K}^*_{mi} v(a_{mi}) = v(a_{mi})} \hat{K}^*_{mi} v(a_{mi}) f_{mi} \left[\hat{K}^*_{mi}, v(a_{mi}) \right] d\hat{K}^*_{mi} v(a_{mi}) ,$$

$$\bar{E}^*_{mi} = \bar{V} = v(a_{mi}) \int_{\hat{K}^*_{mi} = 0}^{\hat{K}^*_{mi} = 1} \hat{K}^*_{mi} f(k^*_{mi}) d\hat{K}^*_{mi} .$$

Case 7 introduced many of the key concepts which serve as a foundation for the more realistic Cases 8 and 9. The co cept of overall system objective was introduced into Case 7. With this notion we are able to recognize that conflict may appear within an organization as a function of the multiple and competing objectives. In order to explicitly acknowledge differences among the overall objectives, the decision-maker assigned weights reflecting their relative importance.

A set of coefficients was derived in order to determine the relative ability of an activity to contribute to overall objectives.

Several performance concepts were defined in order to translate performance criterion outcomes into a performance index, expressing the degree to which task performance was obtained.

Lastly, an effectiveness variable was defined. It combined performance index score with potential to contribute, yielding value produced.

As the situations become more realistic it becomes increasingly difficult to specify who supplies various data types. Consequently, let it suffice to say that useful data are usually not derived "in a vacuum" e.g. a research office which tells a program director: "you need this information, so here it is." Unless those parties who "need" the data believe they need it <u>and</u>

want to use it, data will be derived "in a vacuum."

CASE 8

Case 8 represents an extension of Case 7 under a budget constraint. Here the decision-maker pursues a set of overall school system objectives by means of selecting a group of activities in an a priori framework. This is the first and only a priori cost-effectiveness case for school systems. Analysis of single objective situations is sub-optimal. Unfortunately, sub-optimal decision models are expedient and familiar. Comprehensive or system-wide models are virtually nonexistent in local school districts.

Cost-effectiveness is considered by the author to be the general way to approach school district planning and decision-making. Let us see why.

Benefit-Cost Analysis is applicable to the selection of projects from a set of alternatives when investment is clearly the spirit of the decision, and when inputs and outputs can be fairly well measured by dollar units. The method derives from the criterion of present value of net benefits as a basis for comparing proposed investment alternatives.[9]

If the decision-structure is suitable to the design of a totally new system with a remote time horizon, then "benefit-cost analysis" may be appropriate. On the other hand, when the decision-structure is suitable for planned and incremental improvement of an existing system with a time dimension weighted heavily toward the present and immediate future, then cost-effectiveness methods are appropriate. Cost-effectiveness, as a school district planning tool, is further justified by its superior ability in handling intangible and

[9]Clear statements of alternative benefit-cost criteria may be found in: McKean, Roland N. Efficiency in Government through Systems Analysis (New York: John Wiley & Sons, Inc., 1958).

incommensurable outputs which derive from the educational processes.

Previous developments have provided clues as to the necessary and sufficient conditions for economic analysis. Case 1 was not an economic situation because costs were not involved. In Case 2 it was observed that even the presence of a cost constraint was not sufficient to suggest economic analysis since the decision-maker was interested in a single objective and he therefore had no alternative outlets for funds that might be saved due to economic considerations. Case 5 could, in the special instance for which linear programming was applicable, use economic analysis, but this was largely a comment on level of application. That is, the engineer could use linear programming to sort alternative packages for the decision-maker,but at the higher order level of the decision-maker there is one objective and a constraint. The higher order problem is solved without recourse to the linear programming model or any other form of economic analysis.

Consequently, it is not until Case 8 that the decision-maker finds budgetary restrictions and multiple outlets (objectives) competing for this budget. The necessary and sufficient conditions for cost-effectiveness analysis to be appropriate are:

1. More than one alternative outlet for the available resources.
2. Less available resources than are required to satisfy all of the alternatives. A budget constraint is not sufficient unless it forces the curtailment of at least one activity, e.g., a budget constraint of $1,000,000 would probably not restrict vacation plans of a married couple.

Case 8 borrows the structure and developments of Case 7 and adds to these the analysis of cost. The critical relationships are among performance, cost,

and non-cost factors. That is,

$$K^*_{mi} = K(C_{mi}, G_{mi})$$

where:

K^*_{mi} = the performance index score,

C_{mi} = the estimated cost for activity m as a producer of O_i (this estimate is not treated as a random variable),

and

G_{mi} = the non-cost factors associated with the performance of activity m as a producer of O_i. While these could be viewed as a function of C_m, they are assumed to be given and adequate in the analysis. They include the human factors, such as disposition toward the tasks, level of managerial know-how, level of skills, and the technological factors.

Assume that there are overall system objectives valued as $V(O_A), V(O_B), \ldots, V(O_R)$. To interject some realism into the analysis, assume that the production of O_A is specified by the decision-maker to be at least θ_A. The decision-maker has his engineers develop cost-effectiveness curves for each proposed activity. Figure 9 shows a hypothetical curve prepared by the engineer. This function indicates that C_1 dollars must be spent before any production

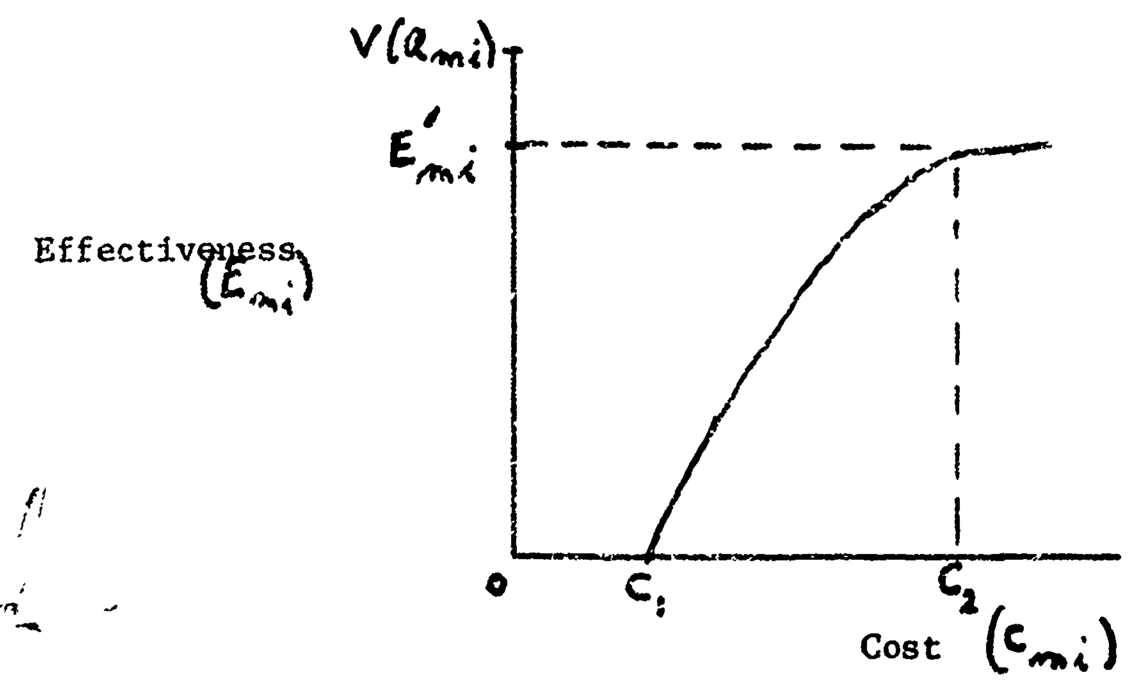

Figure 9. An A Priori Cost-Effectiveness Curve.

is realized. When C_2 dollars are spent the marginal productivity response becomes almost zero at the level of E'_{mi}. This implies that the G_{mi} factors are restricting further productivity. Implicit in the construction is a series of relationships. They are the relationships between

1. C_{mi}, K^*_{mi} (Given G_{mi}),

2. K^*_{mi}, $\hat{K}^*_{mi}$

and 3. $\hat{K}^*_{mi}$, E^*_{mi}.

If the system had been in operation for a period, then evaluation could be conducted for the purpose of obtaining cost-effectiveness points. These points would reflect the productivity of the system as it is presently defined. Allocations for the next period could then be based on these considerations.

The actual solution to the problem involves non-linear programming with three sets of inputs: 1) the functions relating effectiveness to cost, e.g., $E_{mi} = f_{mi}(C_{mi})$, 2) the specifications (policy constraints) telling how much of which objectives should be produced, e.g., $E_A \geq \theta_A$, and 3) the budget constraint, i.e., $\sum_m C_{mi} \leq B$. The programming problem would involve a numerical analysis of the functions. This could be done by taking the derivative of effectiveness with respect to cost at, for example, $100 intervals for each proposed activity. A list of slopes would be generated which could be combined for all activities. Production inequalities (those specifying production must be at least θ_A) could be satisfied first. Once

this was done,the remainder of the problem would consist of a search for the maximum productivity for the unencumbered funds. This is not as simple as the procedure just described implies,because the decision-maker cannot purchase the increment of effectiveness related to the expenditure of $100, in the interval $2,400 to $2,500, unless he has also agreed to purchase the first $2,400 of the activity's production.

Case 8 represents a situation which is faced by school district chief administrators. The prescription emerging from detailed analysis of this recurrent situation is rational planning.[10] Case 8 introduces the concept of efficiency into the model. The efficiency variable is defined as effectiveness per unit of cost.

The principal data files of Case 8 can be described as follows: three are structural (overall objectives, activities, and coefficients which link activities to overall objectives); two are judgmental (performance criteria and value assignments); and two are outcome files (performance and costs). It may be noted parenthetically that experience in this kind of planning with schools has suggested three other files: pupil needs assessment, socio-political survey information, and an alternatives file used by "engineers" within the school district.

Case 9 allows for evaluation of the school district after the school year. Realistically, Case 9 feeds information to Case 8 in an iterative process.

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[10]A method has been developed called "Comprehensive Planning" and is based on deterministic operationalization of Cases 8 and 9.

CASE 9

The final case considers the evaluation of Cases 7 and 8. It is the general problem faced by the decision-maker who has an ongoing system of activities and wants to evaluate current operations so as to prescribe subsequent action.

The evaluation of Case 7 (no constraint on resources) is similar to Cases 5 and 3 and is not repeated here. The one difference is that the system of Case 7 has multiple objectives and therefore one would encounter more of the same comparisons.

Analysis of Case 9 leans heavily on the concepts and constructs of Cases 7 and 8. Overall system objectives and values provide the production target. What information is required in order to determine how well the system produced? In an overall sense it is the sum of the production (effectiveness) scores for the various activities. This total compared to the maximum score possible rates the system in terms of relative effectiveness. That is,

$$REL(E^*) = \frac{\sum_{i=A}^{R} \sum_{m} \hat{K}^*_{mi}\, v(a_{mi})}{\sum_{i=A}^{R} \sum_{m} v(a_{mi})}, \quad \hat{K}^*_{mi} = 1 \text{ in the denominator,}$$

$$= \frac{\sum_{i=A}^{R} \sum_{m} \hat{K}^*_{mi}\, v(a_{mi})}{\sum_{i=A}^{R} v(o_i)}$$

The relative productivity of the system toward overall system objective i is

$$REL(E^*_i) = \frac{\sum_{m} \hat{K}^*_{mi}\, v(a_{mi})}{v(o_i)}, \quad v(o_i) > 0 .$$

Overall evaluations do not provide sufficient information to evaluate for the purpose of making future-period decisions. Presumably, decisions made in the present for the next period should incorporate the same kind of analysis as was used in Case 8; that is, a systematic evaluation of alternative possibilities so as to arrive at a combination of activity expenditure levels which will lead to a higher level of productivity.

This calls for an assessment of the cost-effectiveness relationship for each ongoing activity. An assumption is made that activities will be continued although it is not necessary for the analysis (the reason for this assumption is to avoid the consideration of newly developed alternative activities). A cost-effectiveness curve is obtained for each activity by utilizing the curve developed to plan the activity in Case 7. It is indeed possible that the implementation staff may have contributions and modifications to make for these prior curves.

Figure 10 shows a cost-effectiveness curve for activity a_{mi}. The curve has been revised from the a priori relationship presupposed by the analysis of Case 8. Several aspects of the relationship warrant attention. With regard to costs, C^*_{mi} is the amount of resources actually expended and C_{mi} the level

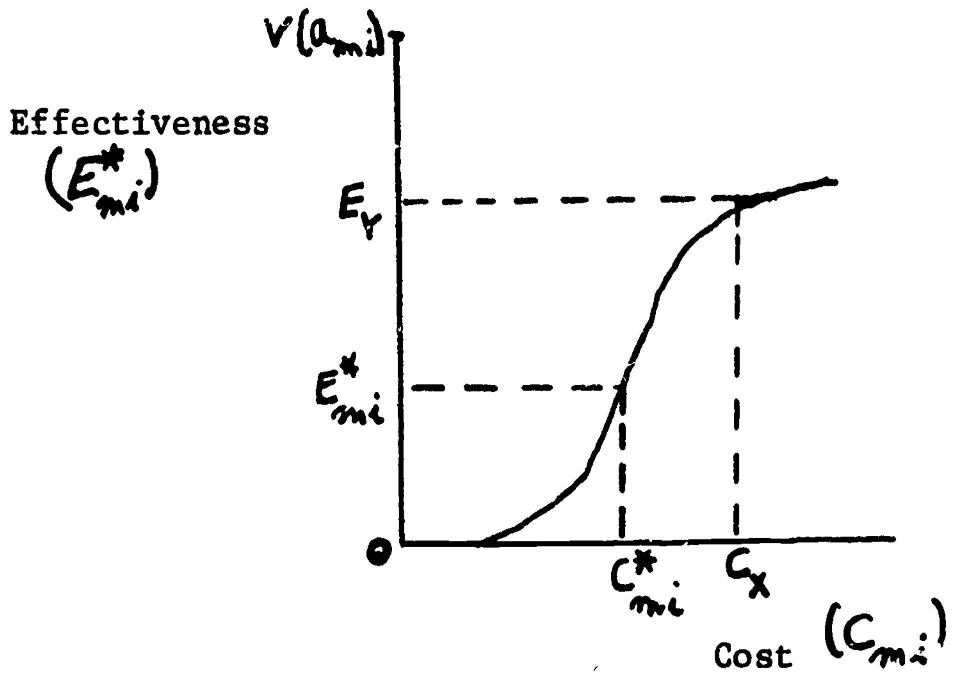

Figure 10. Revised Cost-Effectiveness Curve.

allocated to the activity to pursue overall objective i. Practically, $C_{mi} - C^*_{mi}$ will probably be close to zero since administrators are reluctant to return unexpended resources. Organizations who are able to make administrators comfortable with returning resources that cannot be spent wisely will benefit substantially. This problem is probably more acute in public sector organizations. If the activity were a developmental activity, then the expenditure of $C_X - C^*_{mi}$ additional dollars would allow productivity to rise to E_Y (assuming the adequacy of the revised $E^*_{mi} = f_{mi}(C_{mi})$). If the activity were, on the other hand, concerned with the maintainance of an ongoing activity, then it would require C_X dollars to produce at a level of E_Y. Since the purpose of this analysis is to evaluate ongoing educational systems, the developmental activity problem is disregarded. It must be underscored, however, that incremental planned change, based on incremental movement from presently defined activities, is probably not going to meet all school district needs. Alternatives to the present system must also be included in the planning process.

An inspection of the cost-effectiveness curve such as the one in Figure 10 yields a point $\{C^*_{mi}, E^*_{mi}\}$ representing the level of productivity during the current year. The implementation staff should have a reasonably good idea of how production could be changed were small changes in level of activity suggested. They may be less certain concerning large changes from the present expenditure level. The engineer, on the other hand, has studied the production function and, consequently, is more aware of the

process that relates the inputs and outputs than the implementation staff. Consequently, the engineer should be available for consultation (if not direct involvement) in the process of assessing changes in productivity were costs to be varied. The difficulty is that the engineer is not responsible for subsequent implementation and maintenance of the activity. This suggests that those responsible for these evaluations should consult the engineer.

At any rate, the implementation staff should prepare a schedule that lists expenditure levels above and below C^*_{mi} as well as the estimated responses in production associated with the changes from C^*_{mi}. These are analogous to the $\frac{dE_{mi}}{dC_{mi}}$ values generated in Case 8.

The same type of evaluative process is instituted as in Case 8. There is, however, the problem of what will be referred to as human factors, H. When resources are to be taken away from one activity and given to another based on cost-effectiveness considerations, then the decision-maker should be painfully aware of such human elements as loss of morale. While no attempt is made to study or measure H, it does figure into the decision process. In general, if

$$\frac{\left(+\Delta K_{mi_{(t+1)}}\right)\left(v(a_{mi})_{(t+1)}\right)}{\Delta C_{ni_{(t+1)}}} > \frac{\left(-\Delta K_{ni_{(t+1)}}\right)\left(V(a_{ni})_{(t+1)}\right)}{\Delta C_{ni_{(t+1)}} - H_{ni}}$$

then resources will be diverted from activity a_{ni} to activity a_{mi}. In this inequality the symbols are defined as

$\Delta K_{mi(t+1)}$ = the change in performance in year $t+1$ associated with the cost increment if it would be added to a_{mi}.

$V(a_{mi})_{(t+1)}$ = the decision-maker's assignment of potential value to activity a_{mi} as it would contribute to overall objective i.

$\Delta C_{ni\,(t+1)}$ = the increment of resources that is being considered for transfer from a_{ni} to a_{mi}.

H_{ni} = a dollar unit assessment of the human factors associated with the loss of $\Delta C_{ni\,(t+1)}$ dollars by the administrator of activity a_{ni} and the gain of these dollars by the administrator of activity a_{mi}

METHODOLOGICAL SUMMARY

Cases 8 and 9 have drawn from the other cases in order to provide a general planning, evaluation, decision framework which is applicable to ongoing school systems. Certainly, there has been no implication that the models developed in this monograph are understandable to or useable by school administrators. To the contrary, the Case 8 - 9 model is far from an operational reality capable of being implemented in school districts.

Before we add a few final comments it is appropriate to summarize the 9 Cases. Case 1 provided a starting point in terms of certainty-equi .lence and produced a criterion for selecting an activity directed toward one objective with no constraining factors. Case 2 instituted a cost-constraint and showed that a possible result could be the elimination of the preferred activity of Case 1 because it costs too much. Case 3 indicated that the evaluation of Cases 1 and 2 was not essentially statistical in nature. It stressed that the purposes of evaluation were derived from the need for improvement in future engineering design and estimation, decision-making and implementation.

Case 4 considered multiple activities functioning to meet a single shared objective. The decision-maker's need for assigning weights to outcomes according to their worth was indicated. Case 5 brougat costs into the Case 4 context. The implication of a cost constraint was that some packages, deemed acceptable on performance grounds, should be eliminated from consideration by the decision-maker. Cost versus

effectiveness was still not necessary for analysis since there was only a single objective in the Case 5 structure. It was seen, however, that linear programming, and other programming models, was an admissible methodology for lower order problems which the engineer must solve. Once the engineer makes his selections, the decision-maker does not face an economic problem. The evaluation of Cases 4 and 5 in Case 6 was similar to Case 3 with the emphasis again placed on evaluation for decision-making and future improvement in operations.

A set of overall objectives was introduced in Case 7. The decision-maker considered how to select a set of activities to produce these objectives. Productivity was defined as a function of performance and potential value. Case 8 provided a cost-effectiveness framework to facilitate decisions among proposed activities designed to meet the set of overall objectives. Benefit-cost analysis was seen as being appropriate when investment was clearly the spirit of the analysis. It was underscored that the decision-maker only has an economic problem when he pursues at least two objectives and has a level of resources that restricts at least one activity that could be conducted.

Case 9 tied the cost-productivity relationships which had been evaluated for the current year and related them to the budgetary process for the next year (really a new Case 8). The development of cost-effectiveness curves to evaluate ongoing operations was central to relating Case 9 evaluations to Case 8 decisions. Perhaps the most critical and least performed role is that of educational engineering - - the function which describes alternative processess which relate inputs to outputs in education.

A few final comments are required. The treatment in this monograph has run rampant over many real world technical difficulties. There are many problems associated with measurement of key variables including educational processess, performance criteria, costs, and value assignments. These are confounded by a great deal of uncertainty associated with "change." Think, for example, of the problems of convincing a school district business manager that it would be beneficial for his superintendent (and him) to change their line-item accounting system to a system which tracks costs by objective, program and activity. And if the business manager saw that this was good, how could he effect the required changes and still maintain legal and financial control.

The Administering for Change Program of Research for Better Schools has been working on making this model operational for the better part of four years. The method which has evolved is called "Comprehensive Planning." It is an attempt to put the Cases 8 - 9 iterative process into a form which may be used by managers of school districts to facilitate their planning efforts. The most difficult problems arise out of change itself and getting people to deal with change. Until change technologies are more clearly specified, there are serious doubts about the feasibility of prescriptions such as Comprehensive Planning.[11]

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[11] Temkin Sanford, Comprehensive Planning for School Districts, American Education Research Association Convention, "Planning and Managing Changes in Local School Districts Symposium," Minneapolis, March 1970, (Mimeographed.)

Appendix: Combinatorial Aspects of Project Selection[1]

Suppose the educational engineer submits n feasible projects $(1, 2, \cdots, i, \cdots n)$ to the decision-maker. Each project is accompanied by an estimate of cost, C_i, and its expected contribution to each overall objective, V_{iJ}. Objectives are denoted: $A, B, \cdots, J, \cdots, R$.

The estimated total contribution of all projects is

$$\hat{V} = \sum_J \sum_i V_{iJ}$$

The problem is to select the set of projects which maximizes $\hat{V}$ subject to $\hat{C} \leq B$. The symbol $\hat{C}$ is total cost and B is budget. Structuring the problem as we have oversimplifies it in several respects. First, probabilistic considerations for C_i and V_{iJ} have been ignored. Second, complex projects must be evaluated in terms of tasks and often sub-tasks. Third, situations are complicated by constraints, such as time, in addition to budgeting restrictions.

If we can purchase projects in continuous amounts, e.g., 10.0% of project 1 and 40.5% of project 2, standard linear programming techniques are applicable. If, as is more common, we have to either purchase or not purchase a project in its entirety, then we have a combinatorial problem. Actually, the case is usually in between; we can select a project at a few discrete levels but not as a continuous variable. This, then, is still a combinatorial problem.

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[1]Major credit for this Appendix belongs to Roger L. Sisson.

For example, if there are 20 feasible projects, there are $2^{20}-1$ or over 1,000,000 possible combinations. A combinatorial problem can be set up as an integer linear programming problem.[2] It is also possible to approximate the optimal set by sampling from the set of possible sets. The best set from a sample of a few hundred sets can be shown to have a performance which is close, in a probabilistic sense, to the best possible set.[3]

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[2] Weingartner, H. Mathematical Programming and the Analysis of Capital Budgeting. (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963).

[3] See: Sisson, et al. "The Project Concept in Planning, Programming and Budgeting," Journal of Socio-Economic Planning Sciences (a forthcoming issue).
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