A preschool curriculum for lower class children was developed based on Piaget's theory. Evaluation procedures were developed to parallel a Piagetian curriculum. According to Piagetian theory, the mechanism of classification is the coordination of the intensive and extensive properties of a group of objects. The ability to dichotomously classify all objects in a group occurs at a rather high developmental level, attained in four stages delineated by Piaget, and discussed at length in this report in relation to the development of logic. In a noncontrolled experiment, pre- and posttests were given to 23 disadvantaged white and black nursery school children using three sets of objects. These scores were compared with scores of 16 middle class nursery school children. The Piagetian exploratory method was used, allowing the examiner to help the child to understand the questions. It was concluded that curriculum significantly increased the children's ability to make both first and second dichotomies and to shift criteria. The middle class children's classificatory ability remained at the same level. Both groups progressed in their ability to give verbal justifications. With suggested modifications, the testing method seems valuable because it evaluates the ability to coordinate both intensity and extension. (NH)
A PIAGETIAN METHOD OF EVALUATING PRESCHOOL CHILDREN'S DEVELOPMENT IN CLASSIFICATION

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July, 1969

This is one of a series of reports describing exploratory research in progress at the Ypsilanti Early Education Program. The main objective of the Program is to develop a preschool curriculum for lower-class children based on Piaget's theory. The original plans for the evaluation of the curriculum called for the use of tests that were already in existence. However, as the work progressed, it became increasingly clear that, for the evaluation of a Piagetian curriculum, the available tests are inadequate even to meet the simple, basic requirements that were specified by Scriven (1967). The Program, therefore, added a new objective—the development of evaluation procedures that would parallel a Piagetian curriculum.

The framework and general procedures of evaluation were reported by Kamii (in press) in a chapter of a book on evaluation that puts the emphasis on the content of what the pupil mastered, rather than on psychometric data. The present paper focuses on one aspect of this framework, namely classification.

The first part of the paper will present the rationale for the method of evaluation. For this purpose, it will be necessary to discuss Piaget's theory of classification in some detail. The second part of the paper will present the data obtained with the techniques that have been developed. The findings will then be discussed, and the paper will conclude with an evaluation of the method and the technical modifications that were found to be necessary.

I. The Rationale for the Method

It was decided that the classification tasks would consist of a number of objects which the child would be asked to dichotomize twice or three times on the basis of different criteria. For example, with the 20
geometric shapes of two colors, two shapes, and two sizes that were used for one of the tasks, the child was asked to put "all those that go together" into one box, and "all the others that go together" into another box. After the first dichotomy, the child was asked to re-group the objects by using a different criterion. If a second dichotomy was successfully made, the child was asked for a third way of putting the shapes into the two boxes. The possibilities that were open to him were dichotomy by color, by shape, and by size. This decision to ask children to dichotomize objects and to shift criteria was made on the basis of Piaget's theory of classification and the normative data from Geneva which indicated the age when the various stages of classification are generally attained.

According to Piaget, the mechanism of classification is the coordination of the intensive and extensive properties of a group of objects. "Intensive properties" can be defined as a set of properties that are common to all members of a class and distinguish one class from another. Intensive properties are the qualitative aspect of classification, i.e., the criteria such as color, shape, and size that the child can use to decide that certain objects belong together. "Extensive properties", in contrast, refer to the quantitative aspect of classification, e.g., the notions of all, some, one, none, and whether there are more red circles or more circles.

In general, before the age of four, children cannot coordinate the intensive and extensive properties of objects. When presented with the geometric shapes mentioned above, and asked to put together "all the things that go together", they often make piles of randomly selected objects, such as the arrangement shown in Figure 1. In this arrangement, each pile consists of some of the squares, some of the circles, some of the blue ones, some of the red ones, some of the big ones, and some of the little ones. Not being able to put all of one kind in a pile, all of another kind in another pile, etc., indicates the child's inability to coordinate the intensive and extensive properties of a group of objects.

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Insert Figure 1 about here
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The very first correct coordination of intensive and extensive properties usually consists of many small collections, rather than a few
big ones. Thus, the children who are beginning to coordinate intension and extension are likely to make the following eight groups when given the geometric shapes and asked to put together "all the things that go together" (without any instruction as to how many groups he should make):

- All the big, red circles
- All the big, blue circles
- All the big, red squares
- All the big, blue squares
- All the small, red circles
- All the small, blue circles
- All the small, red squares
- All the small, blue squares.

The above coordination of intensive and extensive properties is perfect. All the big, red circles are in a group, all the big, blue circles are in another group, etc.

At a slightly more advanced level, the child usually makes a smaller number of groups, such as the following:

- All the big circles (red and blue)
- All the big squares (red and blue)
- All the little circles (red and blue)
- All the little squares (red and blue).

The above classification also shows perfect coordination of intensive and extensive properties of the objects, but at a level higher than in the previous situation. All the big circles are in a group, all the big squares are in a group, etc. Intension here has become more inclusive than in the previous situation, where the child merely matched identical objects.

At a still higher level, the child becomes capable of grouping objects into still larger groups, such as the following dichotomy:

- All the circles (red and blue, big and little)
- All the squares (red and blue, big and little)

In a dichotomy, intension is even more inclusive than when the objects are divided into four groups. For one group, the intensive property is "red and blue, big and little circles" and for the other, it is "red and blue, big and little squares".

The above development can be summarized as an evolution from a stage in which there is a maximum of resemblance (intension) and a minimum of extension.
(each group consisting of a small number of elements), to a stage in which there is a minimum of resemblance and a maximum of extension. In the situation involving eight small groups, each group consists of identical objects (a maximum of resemblance and a minimum of extension). In the dichotomy, on the other hand, each group has a mixture of qualities, e.g., red, blue, big, and little circles (a minimum of resemblance), and as many as 10 pieces (a maximum of extension).

Intension and extension mutually depend on each other. The criterion selected for grouping the objects determines how many objects there will be in a group. How many objects to group together likewise determines the criterion that will be used.

Asking children to put all the objects into two boxes requires the highest level of coordination of intension and extension. In deciding to evaluate children's classificatory ability by asking them to make dichotomies, we were not worried about the possible feeling of failure the child might experience with this difficult task. According to Piaget, all children group objects at their own level in their own way. For example, those who can make four groups but not two would arrange the shapes as shown in Figure 2(a). Others at a still lower level might make the arrangement shown in Figure 2(b). As S. Papert, one of Piaget's collaborators, remarked, children always answer the question they ask themselves. Thus, if they are unable to understand the adult's request in the way the adult meant, they will interpret it in the only way they can. As Figures 2(a) and (b) indicate, all children at all levels put together in some way all the things that "go together".

The main point of the preceding discussion was to show that the ability to dichotomize objects is at a rather high level of development. The question that remains to be answered is the significance of this ability. In order to show that the ability to make dichotomies and shift criteria has implications far beyond the mere ability to group and re-group objects, it is necessary to review the developmental stages in some detail. This review will hopefully clarify the relationship between the classification tasks and the later ability to conduct controlled experiments and to make inferences.
Piaget delineated four major stages. The first three are described in Inhelder and Piaget (1964), and the fourth stage is discussed with numerous examples in Inhelder and Piaget (1958). The first stage is called that of "graphic collections", and extends from about 2½ to 5 years of age. The second stage, called the stage of "non-graphic collections", extends from 4 or 5 to 7 or 8 years of age, when the third stage, that of "classification", is attained. The stage of classification is an aspect of the period of concrete operations, when a dramatic number of abilities emerge (e.g., conservation, operational seriation, spatial operations, etc.). The fourth and final stage belongs to the period of formal operations, which is attained around age 12.

Stage I (graphic collections). Stage I is characterized by the child's making pictorial patterns. Figure 3(a) is an example of a graphic collection. The child was quite content after thus putting together "all the things that go together". For this child, there is not yet any intension or extension in a logical sense. The only primitive "extension" that is available to him is the spatial, or graphic, extension. Therefore, the child uses space, rather than any notion of "some" and "all", to put together "all the things that go together".

The stage-I child deals with only one object at a time. Thus, when he first begins to coordinate more than one element, he coordinates the intensive and extensive properties of only two or three objects. Figure 3(b) is an example of the emerging coordination, which can be inferred also from the slow, one-by-one process the child uses to put the shapes into the boxes. When viewed from the standpoint of a dichotomy, the classification shown in this figure cannot be said to indicate the coordination of intensive and extensive properties. However, within each small group, the coordination is perfect.

Similar thought processes can be seen more clearly in the "classification" shown in Figure 3(c) without the use of the boxes. Some articulate children explain how the shapes go together in this kind of arrangement by saying that shapes 1 and 2 are both squares, shapes 2 and 3 are both
Piaget theorizes that the reason for the stage-I child's inability to coordinate more than two or three elements is that his thought is static. The child who makes a line like the example shown in Figure 3(c) can think only of successive pairs in a static way. By the time he comes to the third element, he "forgets" how he grouped the first two elements. Since he "forgets" how he began his classification, he has no plan for the rest either. What Piaget called "hindsight", or "mobility of thought backward", refers to the child's ability to remember what he just did. The ability to plan how to group the rest of the items is called "foresight", "anticipation", or "mobility of thought forward". Foresight and hindsight develop hand in hand because if the child can remember what he just did, he can also use this knowledge to plan how to group the other objects. The children who grouped the objects as shown in Figures 3(b) and (c) are satisfied with what they did, and do not see any inconsistency in their logic because they can think only of two or three elements at a time.

**Stage II (non-graphic collections).** For the stage-II child, it is intension that determines extension. If redness is his criterion, for example, the child will keep collecting things until all the red ones are grouped together. The stage-II child can thus make dichotomies that evidence a high degree of coordinated intension and extension. The ability to make dichotomies is due to the increased mobility of his thought. He can now remember why he put elements 1 and 2 together, and uses this knowledge to anticipate how to group elements 3, 4, 5, . . . , 10, 11, 12... It is this hindsight which enables the child to continue the classification with foresight, thereby giving consistency to the intension. The stage-II child even actively looks for what he wants. Because the collections the child makes are no longer based on graphic considerations, stage II is called the stage of "non-graphic collections".

Although the stage-II child's coordination of intensive and extensive properties is solid enough to make dichotomies, it is not yet solid enough to evidence class inclusion. "Class inclusion" is the defining characteristic of stage III, and refers to the child's ability to make quantitative judgments (extension) concerning hierarchical classes.
With the objects shown in Figure 4(a), for example, the child is asked "Are there more red circles, or more circles?" The stage-II child's answer is "More red circles."

From the standpoint of intension, the stage-II child knows very well that some of the shapes in front of him are red circles, that others are blue circles, and that all are circles. From the standpoint of extension, however, the moment the stage-II child focuses on the "red circles", this group becomes separated from the whole, and the whole for him becomes reduced to the blue circles that remain. In fact, when the child answers, "There are more red circles," the examiner immediately asks, "than what?" The child's answer is likely to be "than blue ones".

The stage-II child's inability in class inclusion is due to the lack of mobility of thought. His thought is mobile enough to make dichotomies, but not enough to have class inclusion. For him, once the whole is separated into two parts, it is destroyed, and only the parts exist. He is able to think of the whole and the parts consecutively, but not simultaneously.

Stage III (classification). The stage-III child's mobility of thought enables him to separate the whole into parts and still hold the whole in mind. It is for this reason that the stage-III child knows that there are more circles than red circles. In Piaget's theory, classification is not considered to be "classification" until extensive properties are thus coordinated hierarchically with intensive properties. Any grouping activity that the child engages in prior to having class inclusion is considered to be the making of collections--not classification.

The stage-III child is able not only to engage in class inclusion but also to shift criteria and engage in multiple classification, which includes intersections and matrices. These terms are clarified schematically in Figure 4.

While the matrix requires him to make two dichotomies (by color and by shape) simultaneously, the shifting of criteria from color to shape allows him to make the dichotomies consecutively. Shifting is, therefore,
easier than making matrices. This relative ease is the reason for including the task of shifting in the curriculum and evaluation procedures for preschool children.\(^9\)

The reason for the stage-III child's ability to handle hierarchical classes, intersections, and matrices can be explained in terms of an aspect of mobility of thought, to which Piaget and Inhelder referred as the ascending method and the descending method. "Ascending" is illustrated by the arrows in Figure 5.

Insert Figure 5 about here

It was stated earlier that very young children tend to make many small collections, and gradually become able to make fewer and larger collections by combining the small ones.\(^10\) In stage I, the child can coordinate only two or three elements at a time. Therefore, he cannot possibly be expected to go up in the hierarchy of classes to make a dichotomy. The stage-II child is able to make a dichotomy because his thought has become supple enough and mobile enough to make this ascent. However, his thought is not yet reversible, and when he dichotomizes the objects by the ascending method, he can move in only one direction, and cannot hold in his mind the subclasses that he has just combined. In other words, the stage-II child cannot ascend and descend in the hierarchical structure simultaneously.

In stage III, the child's thought has become mobile enough to retain the subclasses while combining them into a larger class. Therefore, the stage-III child can ascend and descend in the hierarchical structure simultaneously. When his thought can thus move quickly in both directions, it becomes possible for the child to abstract a criterion instantly. The ability to shift criteria comes from this mobility of thought that has become so quick that it no longer looks like a process. Intension and extension are coordinated instantly.

When thought can move fast and simultaneously in both directions, it becomes possible for the child to coordinate the intensive and extensive properties of an entire hierarchy of classes. It also becomes possible for him to make intersections because he can think simultaneously of
"all the circles" and "all the red shapes" (ascending) and "some of the circles" and "some of the red shapes" (descending). In making a matrix, the child can treat the columns as the subclasses of each row, and the rows as the subclasses of each column. Multiple classification is thus made possible by the simultaneous operation of the ascending and descending methods.

**Stage IV (the period of formal operations).** The importance of evaluating the child's ability to make dichotomies and to shift criteria was discussed above in terms of their indispensability for the achievement of class inclusion and multiple classification. In order to point out the eventual long-range significance of these abilities, it will now be shown that class inclusion and multiple classification are in turn indispensable for the attainment of formal operations in adolescence.

While concrete operations structure only the concrete, empirical data, formal operations represent reasoning with the structures that are achieved in the period of concrete operations. The adolescent becomes able to reason with categories and propositions. This new system of logic can be seen in a variety of forms, such as the adolescent's ability to perform controlled experiments and to compare probabilities. Formal operations are thus operations on concrete operations. An experiment will be cited below to illustrate the indispensability of classification for the attainment of formal operations.

In the experiment (Inhelder & Piaget, 1958, Ch. 3), the child is given a number of metal rods; weights of 100, 200, and 300 grams; and the apparatus shown in Figure 6. The experimenter shows to the child how to adjust the screws to make the rods stay horizontally in the apparatus and to adjust their lengths. The experimenter also demonstrates how the weights can be screwed onto the other end of the rod to make it bend. The rod sometimes bends a little bit, and at other times it bends enough to touch the surface below. The child's tasks are (a) to find out experimentally the variables that influence the flexibility of the rods and (b) to prove his conclusion.

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Insert Figure 6 about here
As can be seen in Figure 6, the rods differ in material (brass and steel), thickness, and cross-sectional shape (round, square, and rectangular), and their length can be varied. In order to find out the factors that make the rod bend, the child needs to use a multiple classificatory scheme. The child in the period of concrete operations can easily classify the rods by any of the attributes, and he can name all the attributes of each rod. ("It's yellow, long, thin, and round.")

However, in experimenting with the objects, the stage-III child compares any rod with any other rod, thinking at the most of two or three attributes at a time. Thus, he is likely to pick up a long, thin, square, steel rod, and compare it with a long, thick, round, brass rod. The stage-IV child, on the other hand, compares each rod with one that is identical except for thickness if thickness is the hypothesis he wants to verify. The stage-IV child's inquiry is thus systematic and proceeds with one delimited question at a time. This systematic process of formulating, verifying, and reformulating hypotheses is called hypothetico-deductive thinking.

The proof that the child considers to be necessary and conclusive also reveals his logical system. Below are examples of the ways in which a 9-year-old, an 11-year-old, and a 16-year-old responded to the request, "Could you show me that a thin one bends more than a thick one."

A 9-year-old (in the period of concrete operations) Places 200 grams each on a long thin rod, and a short thick rod. No amount of help enables him to see that his proof is not a proof.

An 11-year-old (not quite in the period of formal operations) Places 100 grams on a round, steel, long, thick rod, and 200 grams on a rod identical with the first except for its thinness. The experimenter then only says, "I would like you to show me only that the thin one bends more than the wide one. Is that way right?" The child replaces the 100-gram weight with a 200-gram weight.

A 16-year-old (in the period of formal operations) Immediately picks up two rods that are identical except in thickness.
It can be said from the above discussion that the characteristic of the stage-III child is his inability to hold "all other factors" constant. The reason for this inability is that he does not yet have the combinatorial system. He deals with empirical facts only as facts, rather than as an aspect of all the possible combinations.

The concrete system structures simple tables of associations such as the following matrix:

\[
\begin{array}{c|c}
\text{r} & \bar{r} \\
\hline
x & r.x \\
\bar{x} & \bar{r}.x
\end{array}
\]

where \(x = \text{it bends}\), and \(\bar{x} = \text{it does not bend}\).
\(r = \text{it is thin}\), and \(\bar{r} = \text{it is not thin}\).

The stage-III child implicitly makes this kind of matrix and arrives at the conclusion, "The thinner the rod, the more it bends." However, he does not view this \(2 \times 2\) combination as a part of all the possible combinations, of which there are 64.\(^{12}\) There are \(2^6 = 64\) possible combinations in the experiment because the following six dimensions are involved:\(^{13}\)

- \(p = \text{it is made of brass}\). \(\bar{p} = \text{it is not made of brass}\).
- \(q = \text{it is long}\). \(\bar{q} = \text{it is not long}\).
- \(r = \text{it is thin}\). \(\bar{r} = \text{it is not thin}\).
- \(s = \text{it is round}\). \(\bar{s} = \text{it is not round}\).
- \(t = \text{it has a heavy weight on it}\). \(\bar{t} = \text{it does not have a heavy weight on it}\).
- \(x = \text{it bends}\). \(\bar{x} = \text{it does not bend}\).

The holding of "all other factors" constant can be represented in the following way: \((p.q.r.s.t.x)\) and \((\bar{p}.\bar{q}.r.s.t.\bar{x})\).\(^{14}\) This ability to hold "all other factors" constant indicates an understanding of the fact that unless they are controlled, the effect \(x\) could result from causes other than \(r\). The adolescent can thus bring many classes and relations into a single whole. He can create a system of interpretation that is much broader than the establishment of causal relationships by simple associations.
The importance of the coordination of intensive and extensive properties was recognized a long time ago by Aristotle (McKeon, 1941), who wrote about syllogisms and classified the various types of fallacious arguments. An example of an error in making an inference is "Things made of wood float. Chairs are made of wood. Chairs float. Metal chairs will float likewise."

Since classification and logic are involved in all areas of knowledge, their importance in mathematics, science, social studies, etc., can hardly be overemphasized.

The above discussion of the developmental stages may have been too long. It was presented in detail in hopes of giving to the reader an appreciation of the fact that the preschooler's ability to coordinate the intensive and extensive properties of a small group of simple objects has significance far beyond the ability to make dichotomies and shift criteria.

II. The Method of Evaluation and the Findings

Three sets of objects were used to evaluate the children's classificatory ability. The items included in each set will be listed first in the following discussion. The examiner's procedure and scoring criteria will then be described. The results obtained from the administration of pre- and post-tests will be presented at the end of this section.

A. Materials

Set 1 ("crayons and books")
- 2 red crayons
- 2 yellow crayons
- 2 red spiral notebooks (5" x 3")
- 2 yellow spiral notebooks (5" x 3")
- 2 flat boxes, about 6" x 9"

Set 2 ("balls and shoes")
- 2 white styrofoam balls (1½" diameter)
- 2 black rubber balls (3/4" diameter)
- A pair of white plastic toy shoes (less than 1" long)
- A pair of black shoes, identical with the white ones except for the color
- 2 flat boxes, about 6" x 9"
Set 3 ("geometric shapes" cut out of construction paper)

- 3 red circles, 25 mm in diameter
- 2 blue circles, 25 mm in diameter
- 2 red circles, 50 mm in diameter
- 3 blue circles, 50 mm in diameter
- 2 red squares, 25 mm x 25 mm
- 3 blue squares, 25 mm x 25 mm
- 3 red squares, 50 mm x 50 mm
- 2 blue squares, 50 mm x 50 mm
- 2 flat boxes, about 6" x 9"

The reason for using the geometric shapes was that they had been used in Geneva by Piaget and his associates, and normative data were already available. The other two sets of materials were assembled because of the feeling that (a) geometric shapes might have no meaning to disadvantaged children, and (b) the number of objects and criteria in the shape-sorting task might be too difficult for disadvantaged children.

B. Procedure

Piaget's exploratory method of questioning children was used. This method differs considerably from the psychometric method. In the psychometric method, the examiner is required to follow a standard set of procedures specified in the manual, without any deviation. The wording of a question cannot be changed, and the number of times the instruction can be repeated is specified. In the "exploratory method", on the other hand, the examiner has an outline and a hypothesis in mind at all times, and he tests these hypotheses by following the child's train of thought in a natural, conversational way. The examiner uses his ingenuity to make himself understood by the child in any way possible.

For example, before asking the child to group the objects, the examiner must ascertain that the child notices all the pertinent attributes that are involved in the classification. To insure that the child has noticed size differences, for instance, the examiner might pick up a big and little red circle, and ask, "Are these the same? . . . How are they different?" If the child does not understand this question, the examiner might try "How are these not the same?" If this question produces no response, the next
attempt might be "Are these the same size?" If this question still does not work, the examiner might try "Is this one just as big as this one?"

In other words, part of the exploratory method attempts to find out how much help the child needs to be able to answer the questions.

The general outline of the procedure will be presented below in terms of the third set of objects, i.e., the geometric shapes. (The procedure for the other two sets is the same except for the objects used and the fact that only one shift is possible instead of two.) As stated above, the first thing the examiner does is to ascertain that the child has noticed all the attributes of the objects that will be needed to make the dichotomies (i.e., color, size, and shape). The second task for the child is to make a "free" classification. This classification is called "free" because the child is free to use the entire table surface to make any grouping in any way he likes, rather than having the structure of a dichotomy imposed on him. The third, fourth, and fifth tasks are to make the first, second, and third dichotomy with the boxes.

1. Establishing the perceptual discrimination and vocabulary associated with the relevant attributes

   The examiner places all the geometric shapes randomly on the table and asks, "Tell me what you see." If the child does not spontaneously say, "red", "blue", "circle", "square", "big", and "little", the examiner uses the above exploratory method to find out how much help the child needs to answer the question correctly.

   If there is any question about the child's perceptual discrimination or ability to point to the correct object, the examiner teaches these skills.15 The classification task is not begun until the examiner is satisfied that the child has mastered the necessary prerequisites.

2. Free classification

   The examiner says, "Put together all those that you think go together." If the child does not respond, the examiner says, "Put together all those that are alike." If the child still does not respond, the child is told to "put together all those that are like each other."

   When the child has finished, the examiner asks for an explanation, e.g., "How did you know to put them together like this?"
3. First dichotomy

The examiner mixes all the shapes, puts the two boxes in front of the child, and says, "Now, I want you to put them into two bunches, one bunch in this box, and another bunch in this box.

When the child has finished, the examiner asks for an explanation. He points to one box and says, "Can you explain to me why all these go together?"

4. Second dichotomy

If the child has made a dichotomy, the shapes are again mixed, and the examiner reviews the first dichotomy. He then asks for a different way to dichotomize the objects. ("Last time, you put all the X's together and all the Y's together. This time, I want you to make two bunches in another way. Put one kind in here, and another kind in here in a different way.")

In spite of the review, some children repeat the first dichotomy, in which case the examiner allows the child to finish what he started doing. If the examiner feels that the child might be able to shift into another criterion, he might say, "That's fine. You put all the red ones together and all the blue ones together. Now, I want you to think of another way we can make two bunches. Put all the ones in here that are alike, and all the others that are alike in this box."

When the child has finished his second dichotomy, he is again asked for a justification.

5. Third dichotomy

If the child has made two correct dichotomies, the shapes are again mixed. The first two dichotomies are then reviewed, and a third dichotomy is requested.

As usual, when he has finished, the child is asked to justify what he did.

C. Scoring criteria and data analysis

A form was made to facilitate the taking of the protocol of what the child did, especially how he arranged the objects spatially.
Only the perfect dichotomies were considered in the present analysis. Any dichotomy the child did not make before was considered acceptable as long as (a) the intensive and extensive properties were coordinated, and (b) no element was left out of the boxes. The following were the possible dichotomies:

Set 1: All the red ones, and all the yellow ones
       All the crayons, and all the books
Set 2: All the black ones, and all the white ones
       All the balls, and all the shoes
Set 3: All the red ones, and all the blue ones
       All the circles, and all the squares
       All the big ones, and all the little ones.

Some children made dichotomies in the "free" situation, before the boxes were given to them. Needless to say, these were scored as one of the dichotomies the child made. Below are examples of the borderline cases which were considered to be acceptable.

1. A perfect dichotomy, in which the objects are arranged to make a pleasing pattern
2. A perfect dichotomy, in which the subgroups are separated (such as the arrangement in Figure 2 (a)).
3. An imperfect initial coordination, which is later corrected by the child when he is asked for a justification.

The examiner tried in every way possible to obtain a justification of the dichotomies, and even asked, "What do you call these?" In harmony with these efforts to give every benefit of the doubt to the child, the scoring was also done with liberal criteria. Anything that implied the intensive property was accepted as long as it corresponded to reality. For example, "These are green and these are red" was accepted because it implied color. (However, "circles and squares here, circles and squares here," was not accepted when the dichotomy was by color.) "These are circles" was also accepted because it implied shapes. "This is red, and this is red, and this is red, and this is red, ...." was also accepted.17

The children's performance on the three tasks was analyzed separately. For each task, the percent of those who made "at least one
dichotomy" and "at least two dichotomies" was computed. Thus, the group of children who went on to make a second dichotomy was included in the group of those who made "at least one dichotomy".

D. Samples

Since the project was engaged in exploratory work both in curriculum development and in the development of evaluation procedures, it did not feel ready for a controlled experiment. To obtain some general idea of the effects of the instruction and the appropriateness of the method of evaluation, the pre- and post-tests were administered to a sample of the children in the Program. This set of scores was compared with those obtained from a group of middle-class children attending a cooperative nursery school.

The population of the Early Education Program consisted of "disadvantaged" children defined as meeting at least one of the following criteria: (a) Receiving public welfare assistance, (b) having a parent who has completed ten years of schooling or less, and (c) having the head of the household in an unskilled occupation. A sample of 23 children was selected from this population. It consisted of equal numbers of white boys, black boys, white girls, and black girls, and represented the Stanford-Binet IQ range of the 80 children in the Program at the time of entry. (Mean = 91.1, S. D. = 14.3). Three children were taken from each of the eight classes.

In the middle-class nursery school, there were only 18 four-year-old children, and two of them left before the post-test. Although this was a small sample, it provided data on the upper developmental limits of four-year-old children from highly advantaged homes.

E. Results

1. Ability to make dichotomies and to shift criteria

Table I presents the children's ability to make dichotomies at the beginning and the end of the year. Figure 7 gives the same information in graphic form.

Insert Table 1 and Figure 7 about here
It can be seen from this table and figure that the Early Education children made marked progress in (a) ability to make the first dichotomy where this ability was low on the pre-test, and (b) ability to shift criteria. The former can be seen in the task involving geometric shapes. The percent of children in the Program who could make at least one dichotomy with the 20 shapes increased from 63 to 100.

The most striking progress was made in the ability to shift criteria. The percent of children in the Program who made a second dichotomy increased from 8 to 57, from 13 to 69, and from 8 to 52 on the three respective tasks. On the pre-test, the Early Education children's ability to shift was at a level very similar to that of the Coop children. On the post-test, however, the children in the Program far surpassed the middle-class group.

The conclusion that can be made of the middle-class children's classificatory ability is that they remained at the same level over the year. In none of the tasks did they show any increase in ability to shift to a second criterion.

2. The intensive properties chosen by the children

Table 2 shows the relative frequency with which the children selected the various types of criteria as a basis for their dichotomies. It can be seen from this table that the intensive property selected with overwhelming frequency by the middle-class children was color, both on the pre-test and the post-test.

In contrast, the lower-class children selected color proportionately less often on the pre-test, and the gains they made in ability to shift appears to have been mainly in their ability to abstract criteria other than color. The percent of the lower-class children who used criteria other than color on the three respective tasks increased from 30 to 70, from 43 to 87, and from 11 to 35. These increases were compared with the corresponding figures for the middle-class group, which are from 17 to 19, from 11 to 31,19 and from 8 to 9.)
3. Ability to verbally justify a dichotomy

Insert Table 3 about here

Table 3 presents the number and percent of dichotomies that the children made but could not justify verbally. At first glance, it can be concluded from this table that the lower-class children were less able to justify their dichotomies than the middle-class group, both on the pre-test and the post-test. A closer examination and interpretation of this table will be presented in the next section.

III. Discussion of the Data

The conclusion which can be drawn from the findings is that, in the area of classification, the instruction given in the Program seems to have helped the children. The children in the Program were behind their middle-class peers on the pre-test, particularly when the task was very difficult (geometric shapes). However, on the post-test, they far surpassed the middle-class group.

A full description of the curriculum that brought about this change is beyond the scope of this paper, but it is important to point out that the focus of teaching was on the process of coordinating intension and extension, rather than on the final product the child made. The instruction encouraged the child to choose his own criterion and to use it consistently.

It can be seen from Table 3 that the progress the middle-class children made during the year was in the ability to give verbal justifications. This group did not grow in classificatory ability on the performance level. It can be concluded from these data that by the end of the year, their verbal ability caught up with the performance level that they had shown on the pre-test. The lower-class children in the Program, too, went up in verbal ability, to the level of their pre-test ability to make dichotomies, and slightly beyond. The difference between the two groups can be said to be the fact that only the Early Education children grew beyond this growth in verbalization. The ability to give verbal justifications thus seems to come after the ability to make dichotomies.
The minimal role language plays in children's ability to classify can also be seen in the scores obtained by the two groups on the Peabody Picture Vocabulary Test. The lower-class children surpassed the middle-class group in ability to shift criteria, but their mean PPVT IQ was only 95 compared to the 109 IQ of the middle-class group.

Although colors and shapes were taught in the Program, the test items were selected with a view to avoiding the objects that the teachers used in their teaching. To the best of the authors' knowledge, spiral notebooks and tiny toy shoes were not used by the teachers, but at clean-up time the children did put away all their crayons. The three tasks described in this paper can thus be said to get at the process of coordinating intensive and extensive properties, although the memory of specific learned content was also involved. (Plans to change the items will be discussed in the next section.)

It must be emphasized, in the light of Kohlberg's article (1968), that the gains reported in this paper are only short-term gains. The Program has no illusions about the necessity of continuing the same kind of teaching in kindergarten, first grade, and beyond, if the children's logic is to develop fully in the long run into the concrete and formal operations that were described earlier in the paper. All that can be said now is that the instruction increased the probability of the children's growth into class inclusion, multiple classification, and hypothetico-deductive thinking. After all, in a Piagetian developmental sense, each stage is a necessary but not a sufficient condition for the attainment of the next stage. Being able to coordinate intension and extension to the extent of shifting criteria increases the probability that the child will achieve the next stage. Such an attainment, however, is not insured.

IV. Discussion of the Techniques

The general format of the procedure seems appropriate and sensitive to the type of development the curriculum desires. However, some specific techniques were found to require modification. One set of problems was found in the objects that were used. The other problem
concerns the procedure, i.e., how to communicate to the child that we want him to regroup the objects in a different way.

A. The Objects: The ceiling provided by them and their appropriateness

The ceiling provided by the first two sets of objects (crayons and books, and balls and shoes) may be too low for the post-test of the children in the Early Education Program. The possibility of a third dichotomy must, therefore, be explored.

A serious problem which should have been obvious from the beginning was the fact that a pair of shoes belongs together in a way that is different from how balls can be grouped together. Also, by their very nature, crayons suggest color. For these reasons, the use of the first two sets of objects will be discontinued.

The first two sets of objects yielded essentially the same statistical results. Therefore, only one set will be used to replace them. After much search, a kit containing the following eight objects has been assembled.  

| Big pink comb | Big pink toothbrush |
| Big blue comb | Big blue toothbrush |
| Small pink comb | Small pink toothbrush (transparent and having a bird at the end) |
| Small blue comb | Small blue toothbrush (transparent and having a bird at the end) |

In an effort to evaluate the children's ability to coordinate intension and extension, the evaluation procedure under-emphasized looking into the range of intensive properties that preschool children abstract. Thus, the tasks used in 1968-69 limited the information obtained to color, shape, function, and size. Plans for 1969-70 include the addition of a task similar to Sigel's sorting test (Sigel, 1967).

The items will be the following:

| Straw | Car | Pencil | Cup | Sail boat |
| Spoon | Dog | Glass | Horse | Tablet |
| Train | Fork | Cow | Baby bottle | Knife |
The procedure will include showing to the child one object at a time and asking him to find "everything that can go with this one."

B. The communication of the examiner's request to shift criteria

The procedure described in this paper is open to the criticism that perhaps some of the children did not understand what the examiner wanted when he said, "I want you to put them in the boxes in another way... in a different way... one kind in this box and another kind in this box... ."

The solution to this problem may be the demonstration of "shifting" before the first kit is presented to the child. With an easy kit that does not involve the criteria that can be applied in the test situation, the examiner will demonstrate what he means by "put them together in another way."

In conclusion, the method described in this paper seems to show encouraging results. It evaluates the growth of children's classificatory ability not from the standpoint of intension alone, as most object-sorting tests have done (Charlesworth, 1968; Goldstein & Scheerer, 1953; Sigel, 1967), but from the standpoint of the ability to coordinate intension and extension. Another feature is that it views the coordination of intension and extension at the preschool level as a beginning of a long sequence of development in logic. It suggests to makers of educational tests and to teachers a new perspective for assessing and developing a basic aspect of children's intelligence.
References


Footnotes

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1. The work described in this paper was supported by Title III of the Elementary and Secondary Education Act of 1965 (No. 67-042490). The opinions expressed herein, however, do not necessarily reflect the position or policy of the funding agency, and no official endorsement by the Office of Education should be inferred. The authors are grateful to H. Sinclair of the University of Geneva for the many ideas she contributed to this paper.

2. The examples given in this paper were taken from the Early Education Program.

3. Many stage-I children make this kind of linear arrangement without even coordinating two elements. They pick up any object and make a line with them without any attempt to find similarities among objects.

4. Strictly speaking, the use of the word "forget" here is inappropriate because the stage-I child intuitively puts two elements together without any intention of using the same criterion consistently. Saying that the child remembers or forgets a criterion is adultomorphic and attributes to the child an intention that he does not have.

5. In stage I, intension and extension are not yet differentiated. Extension, in the form of spatial configurations, determines intension.

6. Before asking this question, the examiner ascertains that the child knows that the shapes are all circles, and that some of them are red circles and others are blue circles.

7. In making intersections, the stage-III child coordinces "all the circles" with "all the red shapes", and puts in the intersection "some of the circles" and "some of the red shapes".

8. In making a matrix, the child coordinates "some of the circles", "some of the squares", "some of the red ones", and "some of the blue ones".

9. In the authors' opinion, a preschool program should enable children to make dichotomies at the stage-II, pre-operational level, and slightly beyond, but it should not push children into class inclusion and multiple classification.
10. Inhelder and Piaget state that this evolution is generally true, but that the descending method is sometimes seen first, depending on the objects used, the degree of differences among them, and whether or not the child is asked to classify by touch alone or to anticipate his classification before touching the objects.

11. For further detail, see the following cases in Inhelder and Piaget (1958): BAU (p. 50), DUR (p. 57), and DEI (p. 60).

12. The combinatorial system does not imply that the child is conscious of all the 64 possible combinations, or that he knows how many possible combinations there are. The adolescent simply uses the combinatorial system because it is available to him when he needs it.

13. The six dichotomies use the language of classes and ignores serial correspondences for purposes of simplification. Some of the factors are also omitted in this discussion for purposes of simplification (e.g., the rectangular cross-sectional shape).

14. 

"(p.q.r.s.t.x)" means "It is made of brass, it is long, it is thin, it is round, it has a heavy weight on it, and it bends." "(p.q.r.s.t.x)" means "It is made of brass, it is long, it is not thin, it is round, it has a heavy weight on it, and it does not bend."

15. For example, the examiner might show a big red circle and a little red circle to the child, bring the big one closer to him, and say, "This one is big. Show me another circle that's just as big as this one." It is of significance to note that all this teaching does not seem to help the child who cannot make a dichotomy. Any verbalization which differentiates the objects is accepted throughout the session, such as "wheel" for "circle" and "window" for "square".

16. Free classification is more difficult for the child than being asked to put one kind of objects in one box and another kind in another box. This difficulty is precisely the reason for asking for a free classification. The examiner's purpose is to find out what structure the child creates when it is not given to him.

17. This language is similar to the stage-I child's way of thinking—dealing with only one thing at a time. There may be stages in the verbal justification children give, and the sequence may be the same as that of cognitive development. After the first stage of saying, "Because this
is red, and this is red, and this is red, . . ." the child appears to say, "Because these are red." Still later, he appears to say, "Because they are all red."

18. One of the 24 children was eliminated from the sample, as he turned out to be untestable and the only one of the 80 children who was disturbed enough to require psychiatric care.

19. It is of significance, from the standpoint of children's ability to shift criteria, to note that when the percent of middle-class children who made the balls-shoes dichotomy thus increased by 20%, the corresponding percent of color dichotomy decreased by 20% (from 89 to 69). Such a phenomenon of rigidity was not observed among the children in the Early Education Program.

20. A general description of the curriculum can be found in Kamii and Radin (in press) and Sonquist, Kamii, and Derman (in press).

21. The following figures on the Early Education children support this statement:

<table>
<thead>
<tr>
<th>Total number of dichotomies made on the pre-test</th>
<th>Total number of dichotomies justified on the post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crayons and books</td>
<td>24</td>
</tr>
<tr>
<td>Shoes and balls</td>
<td>24</td>
</tr>
<tr>
<td>Geometric shapes</td>
<td>18</td>
</tr>
</tbody>
</table>

22. This is not an ideal kit but the best that could be assembled. It is easy enough to find a set of objects that provides for one shift, but extremely difficult to find objects with just the right shade of color, size, etc., to allow for two or more shifts.
Table 1
The Children Who Made at Least One, Two, or Three Dichotomies on the Pre- and Post-Tests

<table>
<thead>
<tr>
<th></th>
<th>At least 1 dichotomy</th>
<th>At least 2 dichotomies</th>
<th>3 dichotomies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>post-test</td>
<td>Pre-test</td>
</tr>
<tr>
<td>Crayons and books</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>100%</td>
<td>94%</td>
<td>6%</td>
</tr>
<tr>
<td>Early Educ.</td>
<td>92</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>Balls and shoes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>94</td>
<td>94</td>
<td>6</td>
</tr>
<tr>
<td>Early Educ.</td>
<td>88</td>
<td>100</td>
<td>13</td>
</tr>
<tr>
<td>Geometric shapes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>94</td>
<td>87</td>
<td>6</td>
</tr>
<tr>
<td>Early Educ.</td>
<td>63</td>
<td>100</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Coop</th>
<th>Early Educ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>Post-test</td>
<td>Pre-test</td>
</tr>
<tr>
<td>N</td>
<td>18</td>
<td>16</td>
</tr>
</tbody>
</table>
Table 2
The Various Criteria Selected to Make Dichotomies

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Group</th>
<th>Color (N)</th>
<th>Other than color</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crayons-books</td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td></td>
<td>Coop (N=18)</td>
<td>89</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Pre-test (N=16)</td>
<td>94</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Early Educ.</td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td></td>
<td>Pre-test (N=23)</td>
<td>74</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Post-test (N=23)</td>
<td>87</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Early Educ.</td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td></td>
<td>Pre-test (N=23)</td>
<td>61</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Post-test (N=23)</td>
<td>85</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>Geometric</td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td></td>
<td>Coop (N=18)</td>
<td>83</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Pre-test (N=16)</td>
<td>75</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Early Educ.</td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td></td>
<td>Pre-test (N=23)</td>
<td>48</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Post-test (N=23)</td>
<td>83</td>
<td>35</td>
</tr>
</tbody>
</table>

The numbers indicate, in percent, the proportion of times the criterion was used out of all the possible times it could have been used by the group at a given time. For example, 16 of the 18 Coop children dichotomized the crayons and books by color on the pre-test. Since the possibility of using color existed for all 18 children, the percent is 89. By the time of the post-test, the sample of middle-class children was reduced to 16, and 15 of these children made a color dichotomy with the same objects. The percent, therefore, is 94.
Table 3
The Dichotomies Made but Not Verbally Justified

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test</th>
<th></th>
<th>Post-Test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total no.</td>
<td>Dichotomies not</td>
<td>Total no.</td>
<td>Dichotomies not</td>
</tr>
<tr>
<td></td>
<td>dichotomies made</td>
<td>justified</td>
<td>total number</td>
<td>justified</td>
</tr>
<tr>
<td>Crayons-books</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>19</td>
<td>6</td>
<td>32</td>
<td>18</td>
</tr>
<tr>
<td>Early Ed.</td>
<td>24</td>
<td>11</td>
<td>46</td>
<td>36</td>
</tr>
<tr>
<td>Balls-shoes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>18</td>
<td>7</td>
<td>39</td>
<td>16</td>
</tr>
<tr>
<td>Early Ed.</td>
<td>24</td>
<td>12</td>
<td>50</td>
<td>39</td>
</tr>
<tr>
<td>Geom. shapes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop</td>
<td>18</td>
<td>6</td>
<td>33</td>
<td>16</td>
</tr>
<tr>
<td>Early Ed.</td>
<td>18</td>
<td>7</td>
<td>39</td>
<td>38</td>
</tr>
</tbody>
</table>

Percent of total:
- Crayons-books: 5%
- Balls-shoes: 25%
- Geom. shapes: 0%
Fig. 1. Collections in Which Intension and Extension Are Completely Uncoordinated

Fig. 2. The Beginning of Coordination
Fig. 3. Examples of Graphic Collections
Fig. 4. Class Inclusion, Intersection, and a Matrix
All the shapes

Fig. 5. The Ascending Method
Fig. 6. Apparatus Used by the Child to Test the Flexibility of the Rods
Fig. 7. The Children Who Made at Least One, Two, or Three Dichotomies on the Pre- and Post-Tests