This paper reports on the answers indicated by research studies to six questions concerning instructional planning. 

1. When should we begin systematic instruction? Answer: Grade one or kindergarten.

2. How should the content be organized? Answer: A structured sequential development incorporating activities.

3. How should schools be organized? Answer: No conclusions.

4. Does Individually Prescribed Instruction increase achievement? Answer: No conclusion.

5. What benefits come from meaningful instruction? Answer: Greater retention and transfer, increased independence.

Is there research to guide us in deciding...

... when to begin systematic instruction?

There is general agreement today that we will begin to teach mathematics systematically in grade 1, if not in kindergarten, since it has been shown that children can and do learn a great deal about number in the early years.

... how to organize the content for instruction?

Children can learn through an "activity method," if activities are (1) carefully planned to include sequential development of mathematical skills and (2) accompanied by strong drill programs. However, a program stressing sequential development with activities incorporated to introduce and reinforce concepts is generally advocated today. Emphasis in content organization is thus placed on the structure of mathematics, with consideration given to learning levels of children.

... how to organize schools for instruction?

No general conclusion can be drawn from research regarding the relative efficiency of any one organizational pattern for mathematical instruction. Neither team teaching nor departmentalization nor self-contained classrooms nor any other pattern appears in, per se, increase pupil achievement in mathematics. Perhaps the most important implication from various studies is that good teachers are effective regardless of the nature of classroom organization.
Is achievement in mathematics increased by a program of Individually Prescribed Instruction?

No substantial evidence to date supports an affirmative answer to this question. When the Individually Prescribed Instruction (IPI) program of the Oakleaf Project is considered, achievement of pupils has generally been found to be approximately equivalent to that of pupils in non-individualized programs. The type of research design and the measuring instruments used undoubtedly contribute to this finding.

Is there research which identifies outcomes of programs of "meaningful" instruction?

Meaningful teaching generally leads to (1) greater retention, (2) greater transfer, and (3) increased ability to solve independently. Teachers should (1) use more materials, (2) spend more class time on development and discussion, and (3) provide short, specific practice periods. Higher achievement in computation, problem solving, and mathematical concepts has been found to occur when more than half of the class time was spent on developmental activities, with the remainder on individual practice.

Is there research which identifies outcomes of "modern" or "contemporary" programs?

Generally, "modern" programs are as effective as "traditional" programs in developing "traditional" mathematical skills. Evaluation of groups taught with School Mathematics Study Group (SMSG) materials indicates that these groups can be expected to understand mathematical principles better than those using conventional materials. No significant differences in computational skills were reported, though results may vary depending on the type of test.

It has been suggested that we can become so concerned with principles and properties that too little time is spent on computational practice and applications in social situations. Such practice and applications must be carefully planned...
Using Research: A Key to Elementary School Mathematics

PLANNING FOR INSTRUCTION

Is there research to guide us in deciding . . .

... when to begin systematic instruction?

With a few exceptions, there is general agreement today that we will begin to teach mathematics systematically in grade 1, if not in kindergarten. Forty years ago, however, this was a matter of great debate. It was argued that formal study should be deferred "until the child could understand more and had a need for using mathematics." Therefore, until at least the third grade, mathematics should be learned "incidentally," through informal, unplanned contacts with number.

Opponents argued that such delay was a waste of time. Data to support this were collected; for instance, Washburne (1928) found that pupils who began mathematics in either grade 1 or 2 made better mathematics scores in grade 6 than did pupils who began mathematics in grade 3.

On the other hand, Sax and Ottina (1958) found more recently that by seventh grade, there was no significant difference in computation scores. Meaning scores were higher for pupils in a
school in which formal instruction was deferred until fifth grade. However, with the emphasis today on teaching an increased amount of mathematics at any earlier age, the question of when to begin systematic instruction has not seriously been reopened.

During the 1930's there were many investigations of the effectiveness of "activity programs," planned to acquaint children with number as part of the environment. Generally research showed that mathematics could be learned through an "activity program," if (1) carefully planned to incorporate sequential development of mathematical skills and (2) accompanied by strong drill programs (e.g., Wilson, 1930; Harap and Maps, 1934; Wrightstone, 1935).

For years the work of Washburne (1928) and the Committee of Seven strongly influenced the sequencing of topics in the curriculum. This group of superintendents and principals in the midwest surveyed pupils to find when topics were mastered, and then suggested the order and mental age or grade level in which each should be taught.

With the curriculum reform movement which began in the 1950's, much reorganization of content has been suggested. Generally, various topics and patterns have been "tried out" to see if they could be taught at a proposed level: research reflects many such trials. Gagne has long been working on the development of hierarchies of learning tasks. Suppes (1969) is approaching the problem of organization and sequencing with the aid of computer-stored data on pupils' responses.

Educators have long searched for the "perfect" organizational pattern to meet individual pupil needs and increase achievement. A vast number of studies have been conducted to attempt to ascertain the efficacy or the superiority of departmentalization, team teaching, multi-graded, non-graded, or self-contained classrooms. However, attempts to isolate and measure the effects of any of these is extremely difficult, since factors such as content organization and teacher background interact with the pattern. The definitions of the various patterns also tend to overlap—what one person labels team teaching another defines as departmentalization, etc.

It is apparent from a review of the research that no general conclusion can be drawn regarding the relative efficiency of any one pattern for mathematics instruction. There appears to be no one pattern which, per se, will increase pupil achievement in mathematics. A proponent of any pattern can find studies that verify his stand. Achievement differences are affected more by other variables such as the mathematical background of the teacher, than by the organizational pattern. Perhaps the most important implication of the various studies...
Is achievement in mathematics increased by a program of Individually Prescribed Instruction?

No substantial research evidence has been reported to date to support an affirmative answer to this question. It refers to the project on Individually Prescribed Instruction (IPI) originated as a cooperative venture of the University of Pittsburgh's Learning Research and Development Center and Oakleaf Elementary School of the Baldwin-Whitehall School District of Pittsburgh.

In a recent Progress Report on IPI (1969) it was concluded that "on standard achievement tests IPI pupils do as well as non-IPI pupils." No claim is made for higher achievement on the part of IPI pupils. For instance, at the third, fourth, and fifth grade levels Fisher (1968) found no significant achievement differences under three instructional treatments: (1) IPI, (2) "programmed learning instruction," and (3) "standard classroom instruction."

In an inconclusive investigation of IPI effects among low, average, and high ability fourth, fifth, and sixth grade pupils, Deep (1967) questioned the appropriateness of standardized tests for measuring achievement within the IPI context. Other assessment problems and instructional factors associated with IPI have also been studied. Findings from such research and evaluation have been used to revise the program.

Is there research which identifies outcomes of programs of "meaningful" instruction?

[For "discovery-oriented instruction," see Bulletin A-3.]

Earlier in this century, it was doubted that children needed to understand what they learned. It was enough if they developed high degrees of skill. To take time to give explanations and develop understanding was deemed wasteful, besides being perplexing to the learners.

Then came the realization that certain things were to be gained if content made sense to the learner. When mathematics is taught according to the mathematical aim, learning becomes meaningful; when taught according to the social aim, significant. Children do not necessarily acquire meanings when they engage in social activities involving mathematics. Significant mathematical experiences need to be supplemented by meaningful mathematical experiences.

Dawson and Ruddell (1955) summarized studies, such as those by Swenson, Anderson, Howard, and Brownell and Moser, which were concerned with various aspects of meaning. They concluded that meaningful teaching generally leads to: (1) greater retention, (2) greater transfer, and (3) increased ability to solve independently. They also suggested that teachers should (1) use more materials, (2) spend more class time on development and discussion, and (3) provide short, specific practice periods.

Studies since that date have supported these findings. Greathouse (1966), for instance, found that groups taught by a
group-oriented meaningful method achieved more than those taught by individually-oriented meaningful methods, but each achieved more than a group taught by a drill-computation method. Miller (1957) found that "meaning" methods were more effective for most computational areas and for understanding of the principles of mathematics. The "rule" method, however, seemed more effective for low IQ children.

To determine how the use of class time affects achievement, Shipp and Deer (1960) compared four groups, in which 75%, 60%, 40% or 25% of class time was spent on group developmental work while the remainder was spent on individual practice. Higher achievement in computation, problem solving and mathematical concepts was obtained when more than half of the time was spent on developmental activities.

In replications of this experiment, Shuster and Pigge (1965) and Zahn (1966) used other time allocations. They confirmed the finding that when the greater proportion of time is spent on developmental activities, achievement is higher.

Hopkins (1966) compared two fifth grade groups which spent 50% time on meaningful activities and 50% time (1) on practice or (2) in informal investigations of more advanced concepts. No significant differences between computation scores for the two groups were found, but significant differences on understanding measures occurred. Hopkins concluded that the amount of time spent on practice "can be reduced substantially and still retain equivalent proficiency in arithmetic computation." If activities are carefully selected, understanding can be increased.

Payne (1965) summarized several studies and reported that "modern" programs were as effective as "traditional" programs in developing "traditional" mathematical skills; this is supported by more recent studies. There is evidence that "modern" materials are appropriate for a wide range of student abilities.

One phase of the National Longitudinal Study of Mathematical Abilities (NLSMA) compared achievement patterns based on 38 measures over a three-year period, for programs in grades 4-6 represented by six textbook series—three "modern" and three "conventional" (Carr and Weaver, 1969). The conjecture that achievement patterns would be more similar within textbook classifications ("modern" and "conventional") than across classifications was not supported consistently by actual findings of the investigation. It was not uncommon to identify subtests on which large differences in achievement existed among the "modern" textbook groups and also among the "conventional" textbook groups. Furthermore, the findings did not agree consistently with the hypothesis that "modern" and "conventional" texts could be distinguished on the basis of achievement level associated with particular subtests—although there was a trend in support of this conjecture.
Many persons feel that the School Mathematics Study Group (SMSG) has had the greatest impact on the curriculum of any experimental program. Certainly much research and evaluation has been concerned with the SMSG materials.

Hungerman (1967) and Grafft and Ruddell (1968) compared sixth grade classes who had studied the SMSG program during grades 4, 5, and 6, with classes who had studied a conventional program. Grafft and Ruddell reported that the SMSG group understood principles of multiplication better than did the conventionally taught groups, while no significant differences in computation were found. Hungerman found that achievement data significantly favored non-SMSG groups on a test of conventional arithmetic, and the SMSG group on a test of contemporary mathematics. Several other researchers reached this same conclusion in studying other "modern" programs.

Sloan and Pate (1966) studied teaching strategies, reporting that more SMSG teachers than teachers of "traditional" mathematics used analysis and comprehension questions, eliciting spontaneous responses, and developing content. The non-SMSG teachers used recall and recognition questions to a greater degree than any of the other questions they might have used.

One caution is included in several reports: we can become so concerned with principles and properties that little or no opportunity is given pupils to practice computation or apply mathematics in social contexts. Such practice and applications must be planned for...

List of Selected References


Shipp, Donald E. and Deer, George H. The Use of Class Time in Arithmetic. Arithmetic Teacher 7: 117-121; March 1960.


---

Project on Interpreting Mathematics Education Research Center for Cooperative Research with Schools 302 Education Building The Pennsylvania State University University Park, Pennsylvania 16802

---

Nonprofit Org. U. S. Postage P A I D Permit No. 1 University Park, Pa.