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ABSTRACT

This booklet has been written for parents and teachers who want to guide young children (ages 5-10) through informal activities involving mathematical concepts. The author feels that primary sensory experiences which lead to questions of size, number, shape, pattern, and movement will strengthen and prepare children for formal learning in school. The suggested activities deal with counting, shape, pattern, volume, figures, sets, arrangements, lines and games. (RS)

PONDERING —
PUZZLING —
PLAYING —

An Approach To Mathematics for The Young Child

INSTRUCTIONAL SERVICES

THE SCHOOL DISTRICT OF PHILADELPHIA

PONDERING, PUZZLING, PLAYING: AN APPROACH TO MATHEMATICS FOR THE YOUNG CHILD

Lore Rasmussen, Director
LEARNING CENTERS PROJECT

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PONDERING, PUZZLING, PLAYING: AN APPROACH TO MATHEMATICS FOR THE YOUNG CHILD

In the last few years, many mathematicians, school teachers, and psychologists have been pooling their knowledge of modern mathematics, methods of teaching, and learning theory in order to design a new curriculum from kindergarten through college. This large scale effort on behalf of mathematics education should bring exciting and great changes after so many generations of standing still. The object of the new curriculum is to get a child's mathematics anchored so he will understand it, not DREAD it. The method seems to be to help children to make mathematics come alive as a vital personal experience instead of as a memorized set of recipes which manipulate numbers, "letters," and figures.

The following are examples of experiences that could be provided for children ages 5-10. These experiences are not "schoolroom mathematics;" they are informal activities which strengthen children's faculties for observing and analyzing their world around them. They are pleasurable and recreational. Many have an element of humor, a light touch. They serve merely to orient the children to the type of thinking mathematics involves. This type of thinking is probing, questioning, testing, categorizing--things children do naturally and intuitively.

Probably it will be more difficult for the parent or teacher than for the child to "let himself go" and to draw with confidence and courage on his ability to relate and express his insights. Small children are easily absorbed and have a great power of persistence. What better evidence could there be of this than the fact that these children have mastered the pattern of our language with its vocabulary, grammar, intonation, and sentence structure. No one <u>taught</u> these children <u>formally.</u> Right in their own home, under their parent's feet, while the parents are busy preparing meals, caring for their physical needs, doing household tasks, CHILDREN LEARN TO COMMUNICATE in words their feelings, needs, and experiences.

The parents had not <u>studied</u> to teach a language. They <u>used</u> it unselfconsciously. They lived it. <u>Exactly the same job can be done for mathematics.</u>

It is in this spirit that the specific samples of informal activities which follow should be read. They are only a few illustrations of a wide range of possible fun. Other activities will suggest themselves to you; the job of teaching children the formal language and content of mathematics with its special notation computation techniques of arithmetic, the proofs of plane geometry, the formulas of algebra. (Schools must do and will do these in increasingly better ways.) However, parents and teachers can help prepare children and strengthen them for what they will learn later in school by providing the primary sensory experiences which build understanding of mathematical relationships. These experiences are with number, with measurement, with shapes; they are experiences in comparisons and contrasts; they are experiences in questioning and in discovering whether an answer works or if a better answer can be found.

Providing an environment where mathematical thinking comes alive for children cannot be done without one's self becoming involved and curious too! That's the beauty of it.

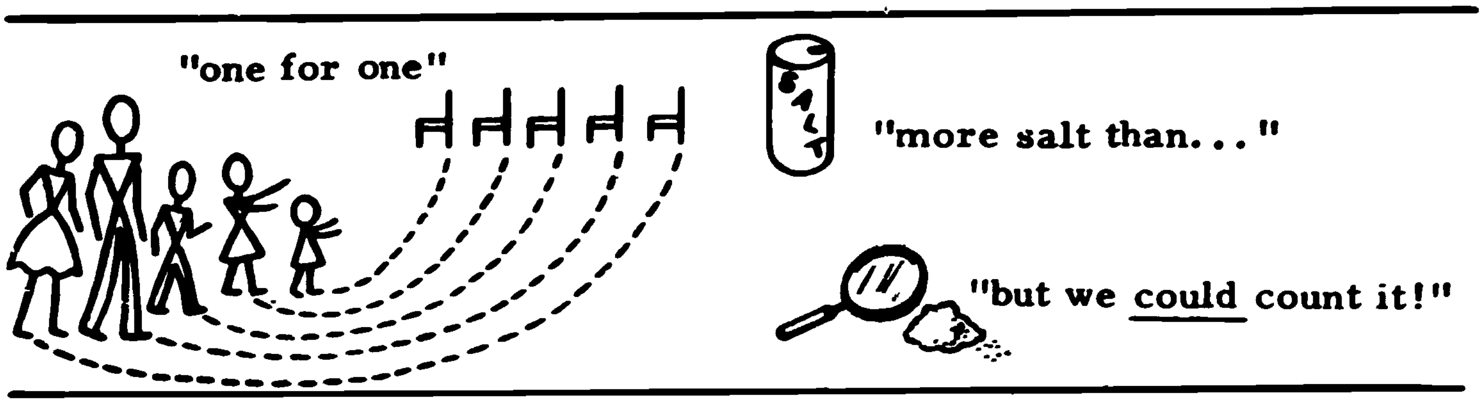
The purpose of most of the activities is to raise questions.

The answers given to children are not half as important as the questions raised. In fact, doors are often closed for children by too quick an answer when the need is to keep the doors open for new questions, new investigation. Thus, the first step is to know what questions to ask, then to conceive of plans which will move toward an answer. These plans may involve reasoning only, or materials which facilitate reasoning. Even if one finds no solution and ends up with nothing more than good questions and interesting plans and models, it's been worth it.

Children, on their own level of maturity, want to know more about the size of things, about the number of things, about the shape of things, the patterns made by many things together, and the movement and growth of things. These are all topics which mathematics investigates. Children learn best about them by seeing, touching, rearranging, comparing--EXPERIENCING.

Here, then, are a few examples of informal home and school activities:

The first illustration is in the form of a silly little talk which might have happened in a kitchen when two children were playing while the parent was cooking dinner. It's about developing their number sense, about more and less, and the fact that all things can be counted. The number zero is defined as "one less than one," and the idea of negative numbers is hinted at as "10¢ in debt." While no counting is involved, estimation is encouraged. Reasonable estimation is very important in mathematics. Parents can find many opportunities at home for practice:



A CONVERSATION ABOUT THE NUMBER OF THINGS

A home is full of collections of things. There is just one mother, but there are two parents. There are only 12 eggs in the egg box, but many more raisins in the raisin box.

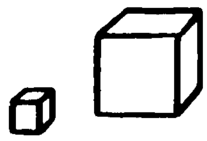
> "How many raisins are in the raisin box? You don't know? More raisins than eggs?"
>
> "Yes!"
>
> "More raisins in the raisin box than salt in the salt box?"
>
> "No! Lots less!"
>
> "How could we make sure?"
>
> "Count them!!"
>
> "Could we count the salt too?"

What about the number of live elephants in your house? None! Zero! Just one less elephant than mothers. Too bad.

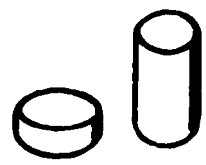
And how much has Bill left of his allowance? You say he owes Linda 10 cents! He has "minus 10" cents!!!

A "FEELING" GAME ABOUT THE SHAPE OF THINGS

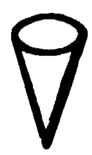
(For pre-school, primary, and intermediate ages)



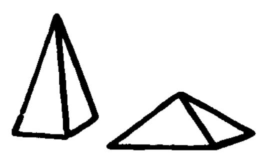
cubes



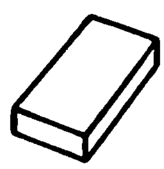
cylinders



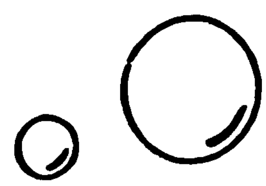
cone



pyramids



rectangular prism



spheres

You will need some props for this one. They are drawn for you above. They should be small enough to hold in one hand and feel with the other. If you do not have some of them, make them out of wood or cardboard. Get at least two sizes of each shape. The solids are hidden from the children and placed one at a time in their hands behind their backs.

"Tell what the object feels like without looking at it."

Here are two descriptions from small children that I would like to share with you:

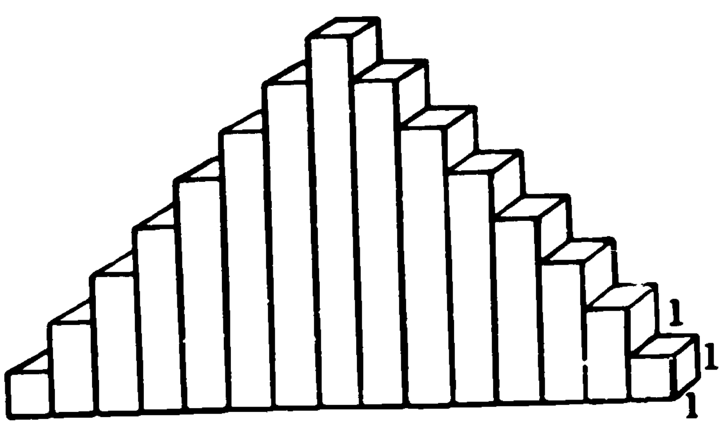
A six-year old describing a <u>cone:</u>

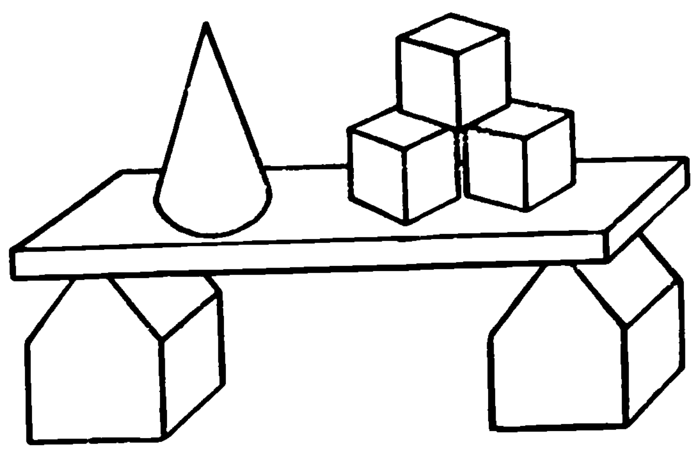
"It's a sharp point on the top and it's round and round and has a circle on the bottom."

A seven-year old describing a <u>cube:</u>

"A block with flat squares put together with sharp edges and sharp points in the corners."

You must be careful not to turn this into a game of names. The name "cube" or "cylinder" does not describe the shape, which the above quotations do. By the way, little children will call cylinders "rollers," and pyramids "steeples," but will also understand and gradually use the technical names when the names are casually introduced.





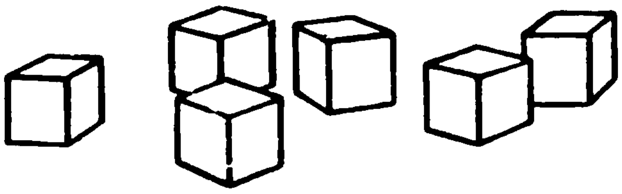
BUILDING WITH BLOCKS

(For all ages)

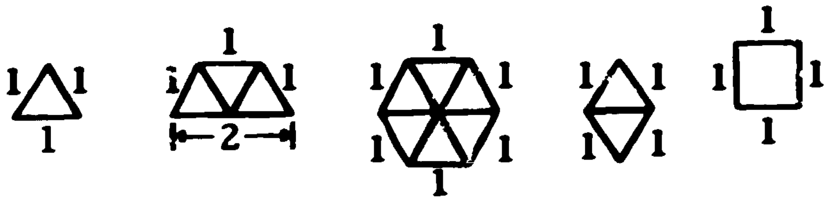
Another illustration of individual play or group play is centered around building with blocks. Many a home nursery is equipped with large-sized wooden blocks which are put away as "childish things" when school age approaches.

As the children grow, blocks take on new meaning. Put more care in their selection but--surround the children with blocks of various related shapes and sizes. The children are old enough now to manipulate small blocks on a table surface where they can be observed closely and used deliberately. These blocks can reveal a great deal of solid geometry arithmetic relationships; they can become models for arithmetic and geometric progressions and allow insight into area and volume measurement.

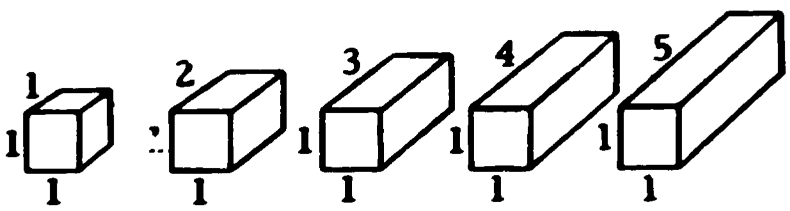
The sketches appearing below illustrate some possible useful sets of blocks. Measurements may be in inches or centimeters and are only suggested. Be sure that you provide several hundred pieces for a set.



cubes

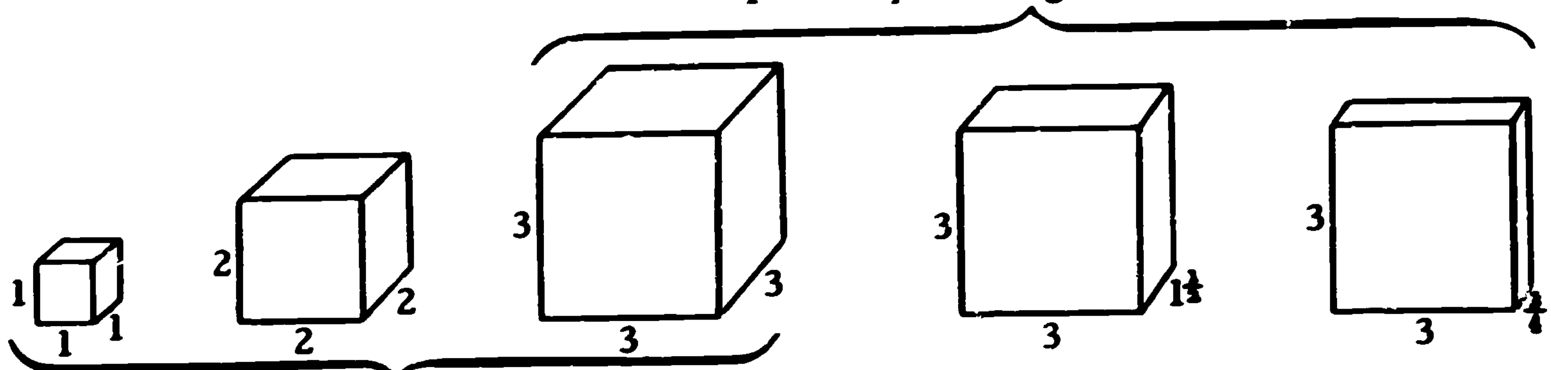


wooden tiles which coordinate

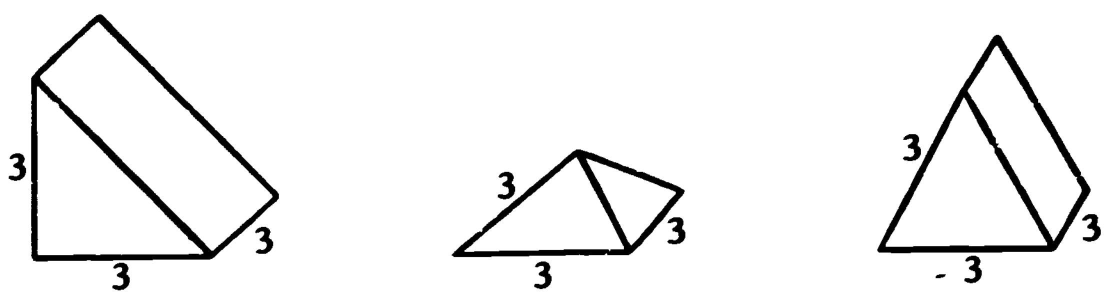


blocks where one dimension increases

repeatedly halving one dimension



cubes where edges increase



different kinds of triangular prisms and pyramids

Blocks show the patterns things make when they are put together. Children will observe the relationship of one block to another if the collection is carefully selected and carefully made. If not homemade, they can be purchased through some educational supply houses.

The children should have free range in this play. Since the blocks of a set are scaled to one another, eventually many questions will be posed and answered through building.

Examples of questions:

Is there a block that is as long as these two?

These three?

PROVE IT.

Is there a block that is half as long as this one?

If this block is my wood unit,

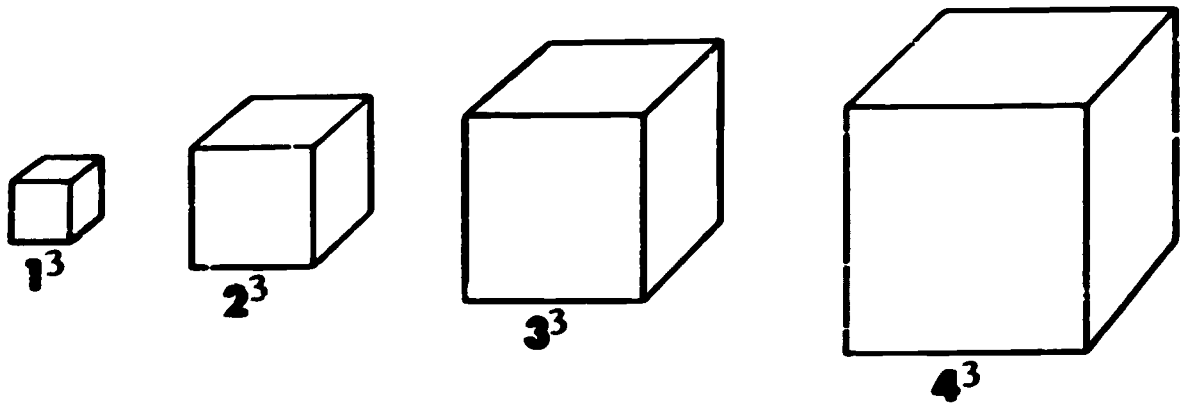
what is this block? (two units)

This block? (four units)

This block? (ten units).

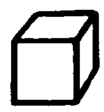
<u>PROVE THAT YOUR CHOICES ARE CORRECT.</u>

How many little blocks would build the biggest block?



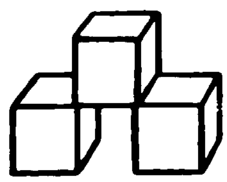
How many little blocks would build one layer of the big block?

one



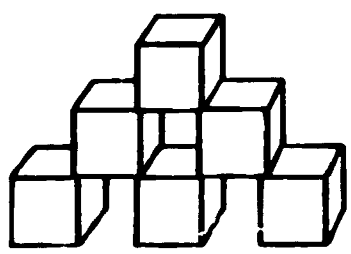
I had one block.

three



Then I built a triangle tower with 3 blocks, and

six



then one with 6 blocks.

?

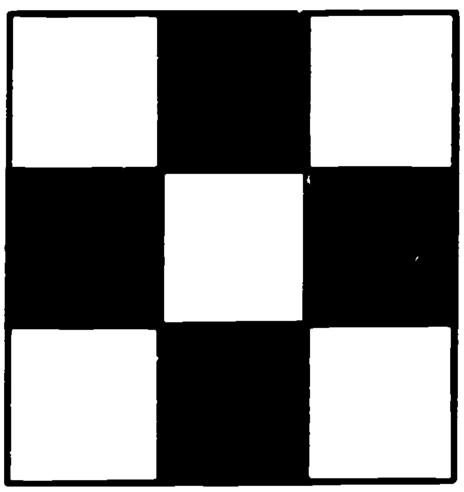
How many blocks will be needed to build the next three higher triangle towers?

Keep on making them grow and note how many more blocks you need each time.

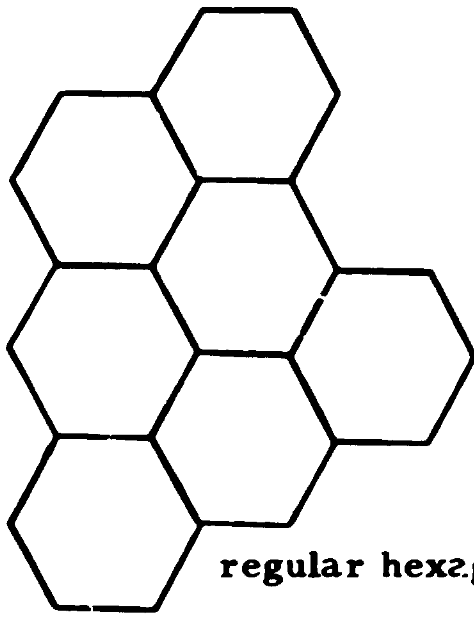
There is an inexhaustable number of problems and challenges children or family groups can pose for themselves in block play.

TILING

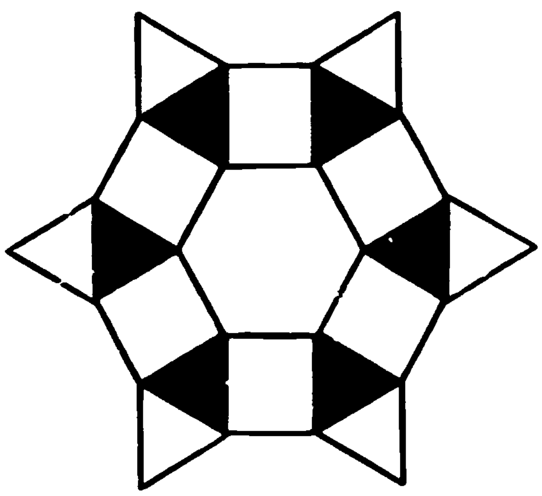
Did you know that the Greeks supposedly got a great deal of inspiration for plane geometry from their contemplative viewing of the beautiful tiled floors which graced their public buildings? Why not include tiles in your collections and tiling as one of the play activities of your home or classroom? Not all regular shapes will tile a plane surface; only certain combinations will work. The study of angles will provide an answer.



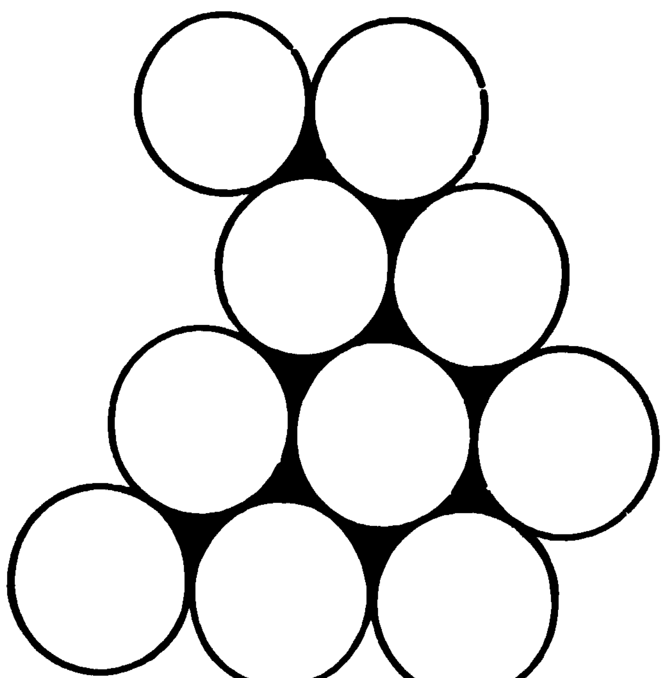
squares



regular hexagons



regular hexagon
equilateral triangles
squares



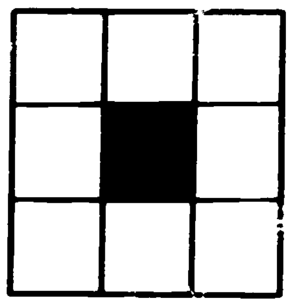
circles

You will need collections of squares, triangles, hexagons, discs (pennies will do) and some small trays for space to tile. (Bathroom tiles will work well and come in these shapes.) This activity is mainly unguided play in making designs. Triangles, hexagons, and quadrilaterals are the only shapes that will each alone tile a surface and will make interesting combinations when used together. Their edges should be of equal length.

For children who are of intermediate school age or older, problems may be more structured.

Here are some questions:

If I surround a square by other squares of the same size, I can make a larger square:

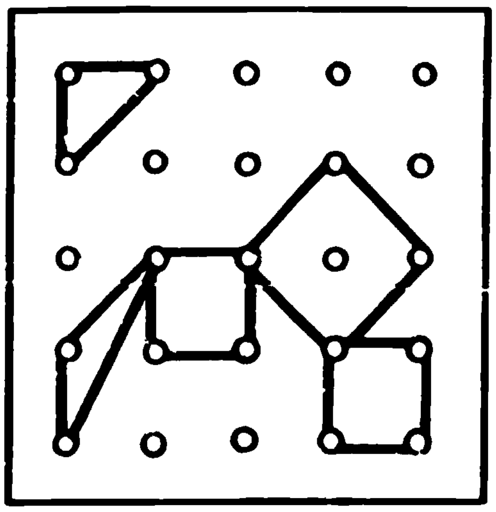


Can I make larger triangles if I surround an equilateral triangle by other triangles like it? Can I make a square or a regular hexagon out of equilateral triangles alone? Can I make a hexagon out of little regular hexagons? Can I make a square out of hexagons? Can I make a square out of hexagons and equilateral triangles? Can I make a hexagon out of little hexagons and equilateral triangles?

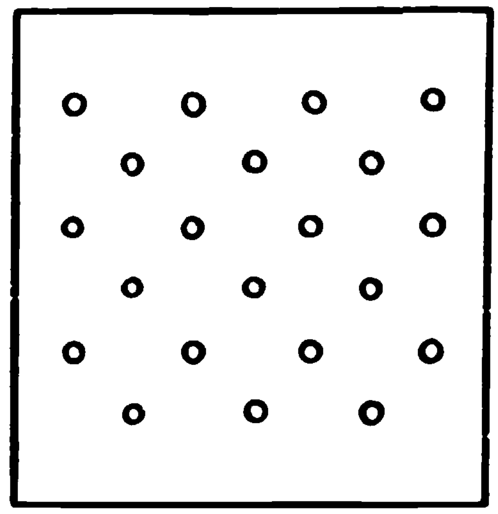
More geometry — this time with pegboards and nailboards. (This activity is for all ages.)

What better use could there be for assorted colored rubber bands, pegboards and nail-studded boards than letting the children explore simple geometric shapes and line designs?

All you need are pieces of pegboard and dowels or boards studded with stationary upholstery tacks. (The latter is less frustrating for the smaller children.)



or



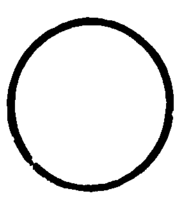
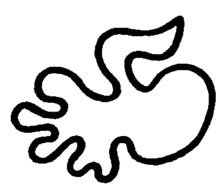
These are so easy to handle for young children who cannot draw straight lines accurately.

How quickly a triangle is transformed into a quadrilateral or into a pentagon! How quickly we have a model of intersections, parallel lines and diagonals!

DOODLES

I think we have had enough ideas about geometry of the usual kind. Let's turn away from angles, straight or curved lines, size or shape to look at doodling with closed figures. Mathematicians investigate these also, and look for other common properties than those mentioned previously.

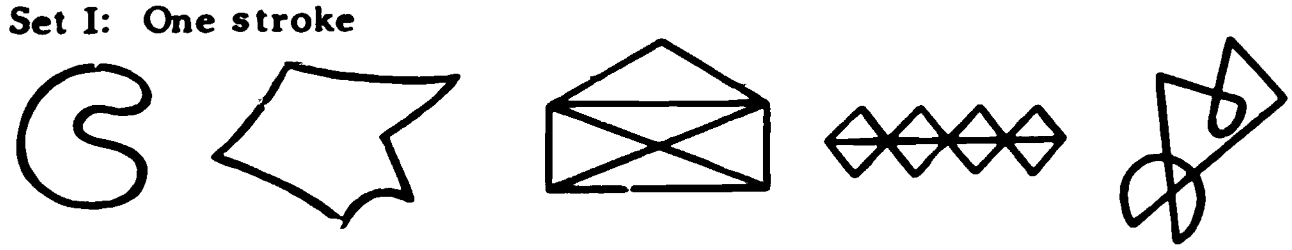
A 7 or 10 year old won't be half as surprised as you if you tell them that

this circle  and that doodle  are really very

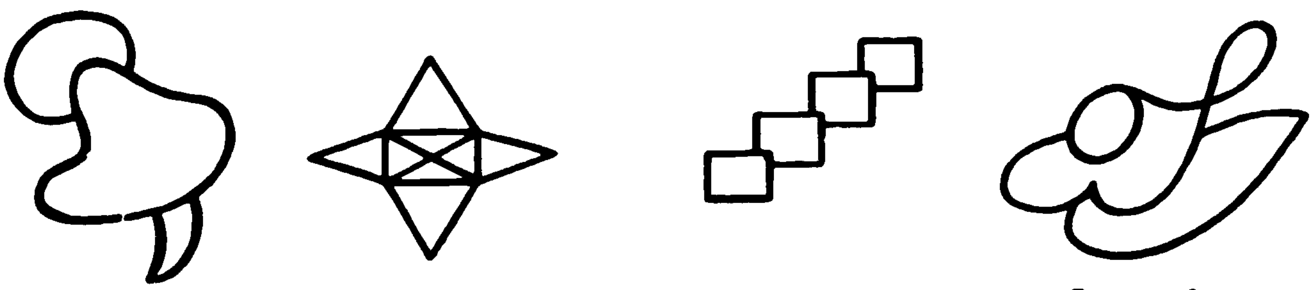
similar mathematically. They are both simple closed curves. The children can easily imagine them to be string and even the doodle to be rearranged as a circle.

Look at the closed figures below. All those in the first row can be drawn in one stroke, without lifting up the pencil or retracing any part. The ones in the second row cannot be drawn that way. WHY?

Set I: One stroke



Set II: More than one stroke

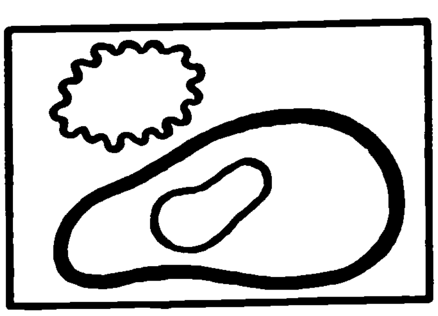


2 strokes | 2 strokes | 3 strokes | 2 strokes

(I'll give you a hint. Observe closely the types and the number of junctions in the drawings in Set I and Set II. These differ and in the difference lies the answer.)

Now that we have begun to draw with pencil and paper, let's continue and draw pictures that show relationship of things. Size and shape do not matter. Everyone can try his hand. Simple as they appear, these are tools of mathematical logic.

FIRST PUZZLE:



The three loops on the left stand for three different kinds of things: furry animals, rabbits, birds.

Which one represents bird?

Which one represents furry animals?

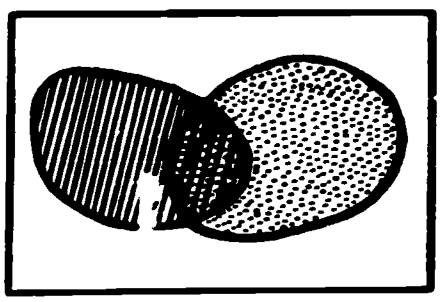
Which one represents rabbits?

Put a loop in for bears. Make a loop for fish. Make a loop for feathery animals.

In order to be sure you did it right, here are the answers: The thin-line loop is for rabbits. The larger loop around it is for furry animals -- of course, the bears would go inside that one, but not where the rabbits are. The wiggly-line loop shows the birds and the feathery animals would be the same as the birds since all birds belong to the feathery animals, and all feathery animals are birds. The poor fish could go any place where the picture is empty.

Here is another example:

SECOND PUZZLE:



The shaded part contains all the potatoes in the world. The dotted part contains all the mashed foods in the world. What is the dotted and shaded part?

(MASHED POTATOES)

Make up many of these yourself and let the children make them too. In no time, they will be your favorite tools to explain anything from evolution to chemical synthesis.

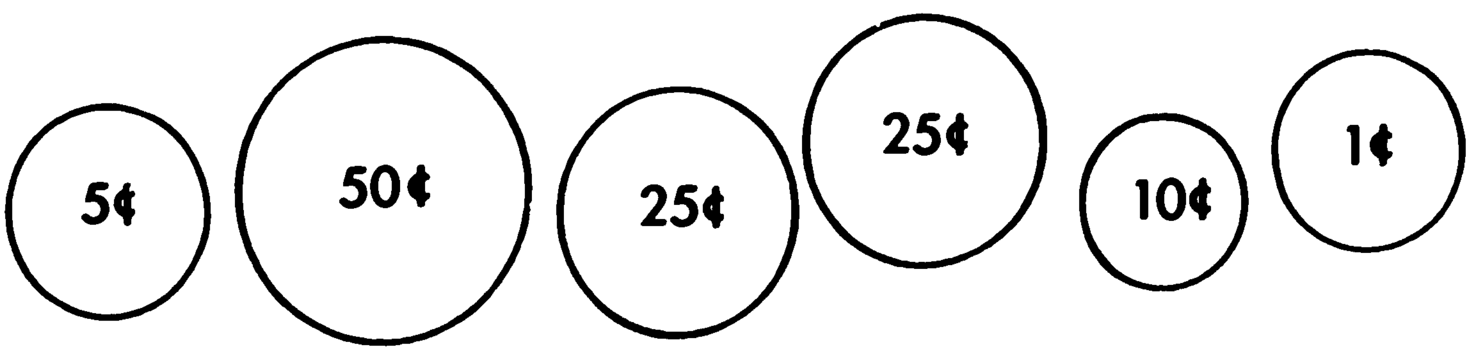
IN HOW MANY DIFFERENT WAYS . . . ????

Let's turn the impulse of children to "do it their way" into mathematical fun by playing many, many games of "IN HOW MANY DIFFERENT WAYS can we . . . . . ?????"

Following are samples of such games with mathematical implications:

About Change:

(This game gets you in the spirit of exploring a subject fully.)



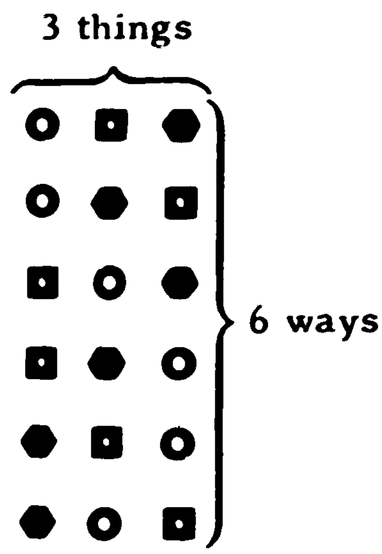
Jingle the change in your pocket, let's say 36¢, and ask: "In how many ways could I have that much money in my pocket? Could I have it in one coin? Two? . . . . Ten coins? Write down the ways from the fewest coins to the most."

About Arranging Things in a Row:

Here are three things: 

In how many ways can we rearrange a given number of things in a row?

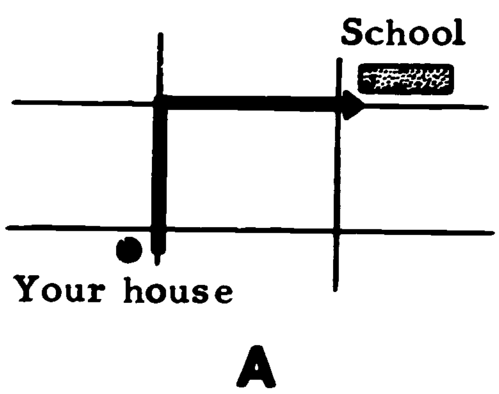
For 3 things, there are six ways:



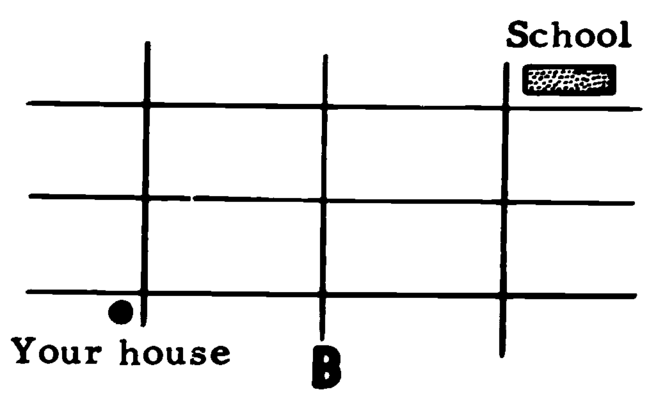
But how many ways are there for one, two, three, four . . . . things?

About Roads and Shortest Paths:

Here are two maps showing streets from "your house" to "school":



(number of shortest paths)

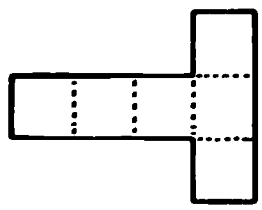


(number of shortest paths)

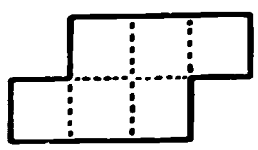
One "shortest path" from home to school is drawn in. How many "shortest paths" are there on map A? On map B?

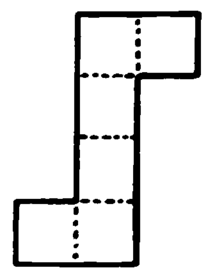
Now try to find all the paths on a larger map. Continue until you can predict the number of choices you will have.

<u>About Folding a Pattern into a Cube</u>

This pattern  will fold into a cube when creased on the dotted lines. One of the following two will fold similarly; the other will not:

Which is which?

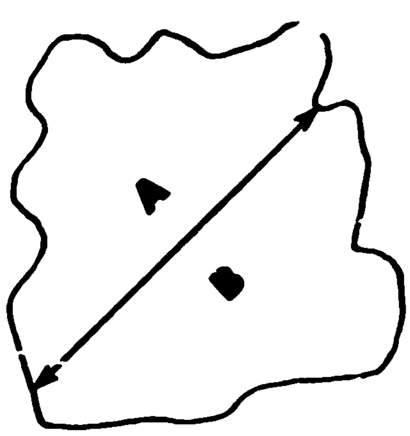




Spend some leisure time finding many other patterns which will fold into a cube. (Every square must be connected by at least one common edge to another square.) Find a system that tells you when you have found them all.

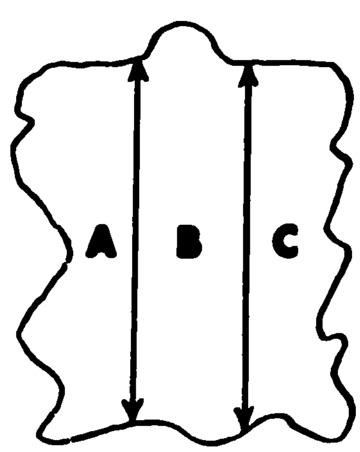
<u>About Lines and Regions:</u>

What are the most regions into which a given number of lines can divide the plane?



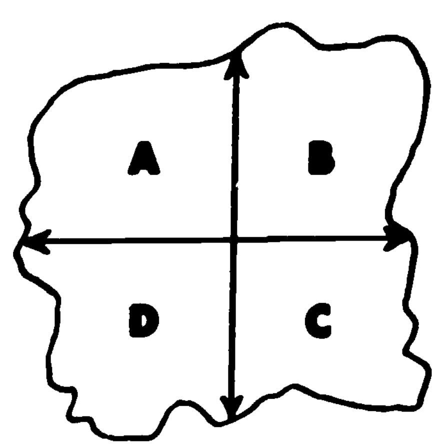
1 line, 2 regions

(the most)



2 lines, 3 regions

(<u>not</u> the most)



2 lines, 4 regions

(the most)

Find the answers for 3 lines, 4 lines, 5 lines . . . 23 lines . . . A table like this will help you get the pattern:

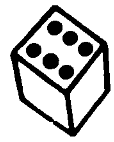
# of lines	creates # of regions
0	1
1	2
2	4
3	
4	
n	

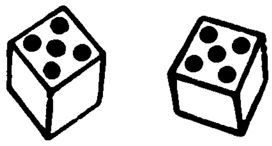
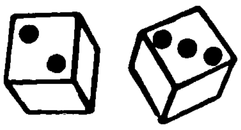
Games of Chance and Games of Strategy:

(Suitable for intermediate ages and above)

Mathematicians are constantly analyzing games from "winning hands in poker" to "best opening moves in chess" and collecting data which is valid in many areas of science.

Here are sample games of chance through which we learn to predict the probability of an occurrence:

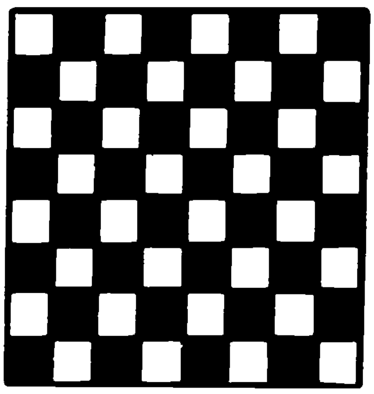
•The probability of throwing a  with one die,

or  ;  with two dice.

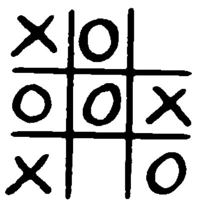
•The probability of throwing "heads" with one coin, two heads with two coins; three heads with three coins.

•The probability of both players simultaneously turning up an ace, an eight, or any two like cards when each has a well-shuffled half deck of cards face down. (Children call this game "War.")

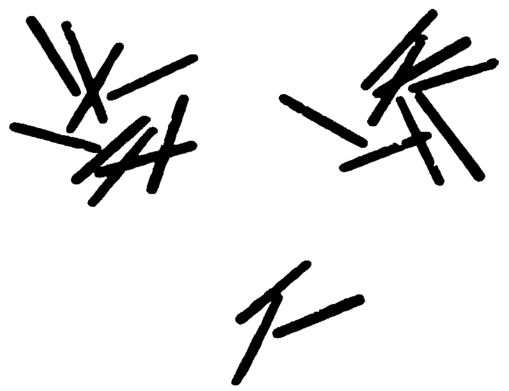
Now for some games of strategy through which we learn to plan a best attack to a successful solution of a problem. In this case, "winning" is the solution. Sometimes "first player" has an advantage; sometimes "second player." (This should be studied carefully.)



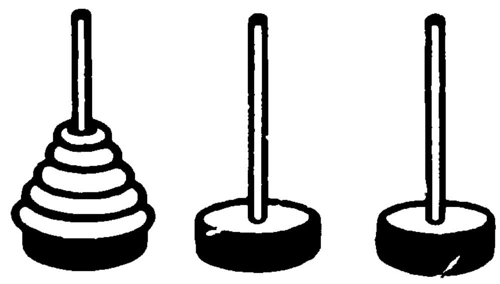
Checkers; Chess; Go; Nine-Man Morris; all of these fall into this category.



Two-dimensional and three-dimensional Tic-Tac-Toe.



Nim, a partner game with sticks or pennies (game analyzed in the binary system of numeration, the system electronic computers use).



Tower Game, a solitary game of transferring rings from one place to another.

Easily written mathematical explanations for the above as well as many other games are available in paperback books on mathematical recreations.

These suggestions could go on and on, but they would not strengthen the basic message: With sufficient thought, patience, humor and imagination, you can create an atmosphere where numbers and shapes have meaning and logical thinking is encouraged.

Too long has mathematics been taught by rote and feared by too many. Our elementary school teachers, through no fault of their own, have also had little mathematics study in their educational training. Now the teaching profession is trying to correct this and courses for teachers are being designed to acquaint them with a variety of mathematical topics interpreted in the light of modern mathematics.

For our children and the generations to come, computational skill and speed alone will be of little use. Already in our lifetime, we have seen adding machines and computers take over arithmetic functions that humans performed before. When was the last time that you yourself multiplied a four-digit number by a four-digit number, or divided a fraction by a fraction (unless it was to help a child with his homework)? Surely the children need to know why these are possible. They will need mathematics to design machines; but the routine mechanical arithmetic problems -- the machines which they designed, built, programmed and ran -- will solve for them speedily and without human error.

It is only recently that I discovered that many mathematically creative and talented adults, using mathematics in their scientific occupations, were and are poor computers. Some of them almost failed sixth grade arithmetic. They balked at the dull, repetitious mechanics of the subject as it was presented to them. Fortunately something, somehow, happened to them a few years later to give them the inspiration to look for meaning and structure. Most adults were not that lucky or perhaps not that gifted. They are those who feel blocked, afraid, and regretful.

This is why I suggest that you do not <u>teach</u> directly what you feared and did not understand. Instead, begin your own reconditioning by wanting to know (perhaps for the first time) in a very simple way, together with the children, about the size of things, the number of things, the shape and pattern of things.

Play with the children in the framework I suggested. Learn to observe, to compare and to question.

Have fun!

Sincerely yours,

Lore Rasmussen

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