This curriculum bulletin is the second part of "Mathematics--Pre-Kindergarten, Kindergarten, and Grade One." This is a developing curriculum program that incorporates the pre-kindergarten into the educational system and reorganizes mathematics materials in the early childhood years. The materials in this bulletin deal with numbers and operations with numbers, early levels of number-line concepts, geometric concepts, and fractional parts. Included also is a suggested plan for introducing topics and subtopics and a scope and sequence for the indicated grades. (RP)
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MATHEMATICS

PRE-KINDERGARTEN

PART TWO

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FOREWORD

This publication is the second part of the bulletin Mathematics: Pre-Kindergarten, Kindergarten, and Grade One. The bulletin is a part of the developing curriculum program that incorporates the pre-kindergarten into the educational system and reorganizes mathematics materials in the early childhood years.

The materials in the bulletin are a result of collaboration with staff working on various professional projects in the field and are specifically based on research and experimentation involving young children, their teachers, and supervisors. During the 1965-66 school year, 218 teachers used preliminary drafts of this publication with their children, submitted evaluations, and made suggestions for revision.

Part One of the program presented in a preceding bulletin deals mainly with sets and subsets, numbers in sets, and number names. Part Two of the program is presented in this bulletin and deals with numbers and operations with numbers, early levels of number-line concepts, geometric concepts and fractional parts. The material contained herein is intended to assist the teacher in developing in her pupils a firm foundation for future studies in the field of mathematics.

SEELIG LESTER
Deputy Superintendent of Schools

December, 1968
INTRODUCTION

When young children come to school for the first time, they need time to become adjusted to school conditions - to the presence of other children, to the class and school environment, to class and school routines, and most of all to the teacher. This is true also of young children when they return to school in the fall.

Those things children ordinarily do with relative ease and confidence at home might be accomplished only with difficulty and anxiety in school - such tasks as taking off or putting on outdoor clothing, hanging clothing, getting and replacing toys or supplies, eating, etc. Children are expected to behave in school in ways which are quite different from those they may have learned at home. The process of adjusting to diverse ways of doing things adds to a child's anxieties.

LEARNING MATHEMATICS IN SCHOOL RELATED TO EMOTIONAL DEVELOPMENT

Young children need to learn what is expected of them in school. How long it takes a particular child to become comfortably adjusted to school depends upon the simplicity or complexity of the classroom situation. It depends also upon the child's comfort or anxiety in school.

It is difficult for young children to develop mathematical ideas until they are able: 1) to be comfortable with the teacher and in the presence of other children in school, 2) to use materials purposefully and constructively, 3) to work and play independently, that is, without requiring the close presence of the teacher, and without requiring frequent encouragement or comforting by the teacher.

Observations of Children Made by the Teacher

The teacher observes each child at work and at play. The teacher notes whether a child knows what to do with his personal belongings and how he does this, whether he goes to a work-play center or needs to be guided there personally, whether he uses materials in a work-play center and how he handles these, whether he carries out instructions, whether he works or plays alone or with one or more other children and how he does this, whether he communicates with the teacher and how he does this (touching or signs or sounds or words), whether he communicates with other children and how he does this, whether he clings to or avoids the teacher, whether he seems to be comfortable in school. The teacher thinks about individual children as plans are made for helping them develop mathematical ideas.

Some Emotional Reactions to Learning in School

By the time a child enters school he has lived through a countless number and a great variety of experiences. A primary determinant of how a child will learn in school is his personal emotional reactions to his experiences. Many children seem to have developed personality structures so well integrated that under ordinary school conditions they are able to settle down and develop mathematical ideas relatively soon. Those who are girls are generally readily attuned to what the teacher expects of them. Boys for the most part need more time to adjust to a teacher's concept of acceptable behavior in school.

IV
Some children, in order to maintain some degree of self-esteem, may automatically push down feelings they believe are unacceptable to the teacher. During school hours these children use up much of their energy to push feelings down. They may succeed wholly or partially. When their efforts fail some children explode for little or no detectable reason. Most of the time, however, such children seem to have little or no energy. The teacher helps the child who believes his feelings are bad and, therefore, that he is bad 1) to accept his feelings, and 2) to learn how to express his feelings in ways that will not hurt himself or others. The teacher may say: "I know you're angry with John (implying that anger is an acceptable and recognizable feeling). But don't hit him (implying that this expression of anger is undesirable in the classroom). I'll help you find some clay (implying that pounding clay is one expression of anger that is desirable in the classroom)."

The teacher provides all children with opportunities to express feelings in ways that are constructive rather than destructive, that is, in ways that will not hurt the child himself, the teacher, or other children. Feelings may be felt and expressed through carefully supervised activities, such as painting (finger painting at first), clay modeling, water play, cutting little pieces of wood, sweeping, scrubbing with warm soapy water, hammering, putting hands into clay being molded, pounding a nail, arranging materials for a collage, handling beads or pennies or discs or other small objects. Eventually children will draw geometric figures and write mathematical symbols—these being controlled and sophisticated expressions of intellectual and emotional energy.

There may be a child who finds school so painful that he automatically shuts out from his consciousness most of the stimuli in the classroom. This is his way of protecting himself from being completely overwhelmed by his feelings from losing complete self-control. This child may not "hear" the teacher or another child. He may not speak in the classroom. This child needs close, warm, accepting, friendly encouragement by the teacher for some time before he will respond adequately to learning situations in the classroom.

PROVIDING FOR DIFFERENCES AMONG CHILDREN

Teachers use a variety of procedures to guide children at varying levels of readiness for development mathematical concepts. Work-play centers provide opportunities for the teacher to call attention to mathematical aspects of the materials being used and to do this at each child's level of understanding.

Developing Mathematics in Work-Play Centers

During the early weeks or months of the school year the Pre-Kindergarten and Kindergarten teachers emphasize mathematical ideas with one child at a time in a work-play center. Grade One teachers may work at this time with a small group of children. Later in the year the Pre-Kindergarten and Kindergarten teachers work with several children in a group, Grade One teachers with a larger group of children.
Some of the ways teachers reported that they developed mathematical ideas in work-play centers follow:

**Work-Play Centers in Pre-Kindergarten Classes**

If a child takes out materials and begins to use these I join him. I ask a question...I try to get around to as many children as I can.

As a child becomes interested in a game we work together. If it distracts him from his enjoyment of the game I do not continue. If he likes it, I work on.

Informally I take one child aside and we work with and talk about a number of blocks or buttons or cups...

I sit down at a table and start grouping nuts or wheels. One or two children come over to see what I'm doing. I ask a child to tell me what they are or how many...

I work with a child. Usually another child joins us. The original child may be no longer interested. I then suggest another activity to him.

I sit down with one child, or two children at most. I ask questions of one child at a time. I spend only a short time because the other children need help and supervision.

In a spontaneous manner, while one or two children are playing at a table and using table blocks, dough balls, cubes, dominoes, science materials...I encourage the development of mathematical concepts through this.

**Work-Play Centers in Kindergarten Classes**

(Kindergarten teachers reported procedures similar to those reported by Pre-Kindergarten teachers. Other emphases are reported here.)

Informally I approach a child or a group and ask how many are here or there...

I join the children at house play and ask at an appropriate time: "If two more people come today..."

A group of children play in the housekeeping area, another with blocks, another with puzzles...These lend easily to the teaching of mathematics.

There are many opportunities for mathematical experiences during any work-play period - handling scissors or brushes, using blocks, examining science materials, playing games...

There are many table games suited for mathematical learnings. If a child chooses to use them I may sit down with this child or a small group and devise games that we play. These may involve matching, telling how many...
I set up an activity and watch to see which children are interested...

Work-Play Centers in Grade One Classes

As children are playing I seek to heighten their awareness of mathematics. I try to arrange materials to focus on their own problems in order that they discover ideas themselves.

Sometimes I develop mathematical concepts informally with a group of children, sometimes formally.

Children signal to me to see their games or designs. They use mathematical terms. I ask questions...

House-play activities, such as setting a table for 6...Playing store, making transactions...Dominoes and other games, keeping score...Toys on shelves - top, bottom, middle, 6 puppets...Trucks or cars in a line...Arts and crafts...

Developing Mathematical Concepts in Preparing for Snack Time

Teachers at all three grade levels reported that preparation for snack time affords excellent opportunities for developing mathematical ideas and for doing this at children's individual levels of understanding. This includes placing chairs at places at a table, finding the place to sit, anticipating the number of items needed for the table, selecting enough items for the group, distributing items to children, checking the number before and after distribution, etc.

During the preparation for snacks period and during the period of cleaning up after snacks the teacher selects one or two children at a time for focusing on specific mathematical content. The social and emotional gains afforded by the snack period itself are thus not interfered with.

Widening of Differences Within a Class

As the year proceeds and as the mathematical and emotional needs of individual children are being met, the teacher observes that differences among the children in the class have become wider. This is a characteristic of good teaching, that it results in ever-widening differences among the children in the class.

PLANNING FOR TEACHING MATHEMATICS

The teacher plans for children to engage in experiences and to handle objects as part of such experiences. The teacher also plans to focus
children's attention on specified mathematical content arising from the materials of the experience.

Before children came to school they developed mathematical ideas through home-living experiences, particularly by handling objects in such experiences. But no two children could have had identical experiential backgrounds. The teacher, therefore, plans for children to engage in home-like experiences in school and to use familiar objects. This provides for some commonality of experiences and materials in the classroom.

Suggested Plan for Teaching Topics 1–6

The mathematics bulletin is organized around six broad topics. The first three topics are presented in Part One, p. 2-67. The last three topics are presented in Part Two (this part), p. 68-219.

A general plan for introducing, emphasizing, and continuing the development of the six topics throughout the school year follows:

**Beginning of the School Year.** Introduce Topic 1.

Introduce Topic 2.

**Fall.** Introduce Topic 3.

EMPHASIZE TOPIC 2.

EMPHASIZE TOPIC 3.

Continue to develop aspects of Topic 1.

**Winter.** Introduce Topic 4.

EMPHASIZE TOPIC 3.

EMPHASIZE TOPIC 4.

Continue to develop aspects of Topics 1 and 2.
Early Spring. Introduce Topic 5.

EMPHASIZE TOPIC 4.

EMPHASIZE TOPIC 5.

Continue to develop aspects of Topics 1, 2, and 3.

Late Spring. Introduce Topic 6.

EMPHASIZE TOPIC 4.

EMPHASIZE TOPIC 5.

EMPHASIZE TOPIC 5.

EMPHASIZE TOPIC 6.

Continue to develop aspects of Topics 1, 2, or 3.

See also Suggested Plan for Introducing Topics and Sub-topics in Mathematics, p. xi.

Organization of Each of the Six Topics

For each topic all three grades are presented together on facing pages—the lefthand, even-numbered pages for the Pre-Kindergarten and Kindergarten, the righthand, odd-numbered pages for Grade One. Thus a teacher in any one of these three grades can readily find mathematics, materials, and activities appropriate for every child in the classroom.

Preliminary Statement

Each topic is introduced with two Preliminary Statements, one for the Pre-Kindergarten and Kindergarten, the other for Grade One. These present:

Mathematics for the Teacher. This includes mathematics pertinent to the topic. The mathematics is described in simplified form.

Mathematical Emphases for Each Grade. The teacher will find it profitable to consider the emphasis for each of the Pre-Kindergarten, Kindergarten, and Grade One, those described on lefthand pages and those described on righthand pages.

IX
General Suggestions Relative to Teaching the Mathematics of the Topic. The results of research with young children and the experiences of teachers are summarized. Suggestions for developing unifying mathematical ideas are outlined and illustrated.

Lists of Materials for Developing the Topic. This includes a list of essential materials and a list of additional materials. Essential materials include those considered to be especially important for young children's personal and mathematical growth. It is recommended that these be ordered for every classroom.

Concepts of Position, General Size, Length, Shape, Weight, Temperature, Time, Set and Subset, and Quantity. Children develop these concepts throughout the year.

Outlines of Contents for the Topic. The teacher will find it profitable to consider the contents for all three grades.

Contents, Teacher Preparation, and Pupil Activities

For each topic, following the Preliminary Statement, are suggested teaching procedures. These include suggestions for teacher preparation, content organized around pupil experiences, and descriptions of pupil activities for each of the Pre-Kindergarten, Kindergarten, and Grade One (on facing pages). The teaching procedures presented are suggested only. The teacher will adapt procedures to the needs of the children in the classroom.
# Suggested Plan for Introducing Topics and Sub-Topics in Mathematics: Pre-Kindergarten, Kindergarten, Grade One

<table>
<thead>
<tr>
<th>Sub-Topic**</th>
<th>Pre-Kindergarten</th>
<th>Kindergarten</th>
<th>Grade One</th>
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</thead>
</table>

** See pages VIII and IX, "Suggested Plan for Teaching Topics 1-6," which includes a plan for introducing, emphasizing, and continuing the development of each topic.

** For titles of Sub-Topics see Scope and Sequence, beginning on page XII.
SCOPE AND SEQUENCE FOR MATHEMATICS: PRE-KINDergarten AND KINDergarten

Suggested teacher preparation, pupil activities, mathematical terms, and illustrations may be found for each of the items below on pages indicated in parentheses. Pages 4-67 are in Part One, Pages 68-219 are in Part Two.

TOPIC 1. NUMBER NAMES AROUND US: READINESS FOR NUMBERS

Pre-Kindergarten

1. Number Names in Classroom, School, and Neighborhood (Sept. or Oct.)

Sees, hears, and repeats number name on classroom door; may see other number names in classroom (p.4,6)

Sees, hears, and repeats number names for school and nearest exit, may see other numerals around the school (p.6)

On a class trip sees, hears, and says number names in the neighborhood (p.8)

2. Number Names from Personal Data (Sept. or Oct.∗)

Knows it is his birthday, repeats number name for his age, hears other children’s ages (p.8)

Observes his house number; repeats number name for his address, e.g., forty-one or four-one; repeats sets of digits for pronunciation (p.10)

3. Number Names in Stories and Rhymes (Sept. or Oct.∗)

Notes title, listens to story or rhyme, may join in reciting (p.10)

Kindergarten

1. Number Names in Classroom, School, and Neighborhood (Sept.∗)

Observes and names number names on classroom doors and around the classroom (p.4,6)

Observes, hears, and repeats number names for school and exits, learns number name for school (p.6)

On a class trip looks for number names in the neighborhood, locates and learns his own house number (p.8)

2. Number Names from Personal Data (Sept.∗)

Realizes it is his birthday, observes numerals for his age and for other children’s ages, compares ages (p.8)

Observes numerals for addresses, may learn his address, repeats sets of digits, sees number names for measures (p.10)

3. Number Names in Stories and Rhymes (Sept.∗)

Notes title, listens to story or rhyme, may recall number names and some rhymes (p.10)

∗Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XII
SCOPE AND SEQUENCE FOR MATHEMATICS: GRADE ONE

Suggested teacher preparation, pupil activities, mathematical terms, and illustrations may be found for each of the items below on pages indicated in parentheses. Pages 4-67 are in Part One, Pages 68-219 are in Part Two.

TOPIC 1. NUMBER NAMES AROUND US: READINESS FOR NUMBERS

Grade One

1. Number Names in Classroom, School, and Neighborhood (Sept.*)

Looks for number names around the classroom, learns room number, reads and compares numerals around the classroom, finds pages in a book, may be able to recognize numerals on a chart (p.5,7)

Locates number names for school, learns number name for school, locates and may be able to read number names around school, names number names on experience charts, identifies numerals on clock (p.7)

On class trip locates number names around the neighborhood, names some of these (p.7,9)

2. Number Names from Personal and Class Data (Sept.*)

Tells his age, identifies numeral for his age, observes monthly calendar, learns day and month for his birthday, notes day and month of holidays and other birthdays, draws birthday card (p.9)

Learns his address; identifies his address on absence card or from envelopes brought from home; compares numerals for addresses; learns apartment number; may be able to learn Zip Code number and telephone number; reads numerals for attendance, amount of money collected, measures, calendar (p.9,11)

3. Number Names in Stories and Rhymes (Sept.*)

From the title anticipates number of things probably involved, may describe parts of story, recites rhymes (p.11)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.
TOPIC 2. ONE-TO-ONE CORRESPONDENCE BETWEEN SETS OF OBJECTS

Pre-Kindergarten

1. A Set of Objects for a Set of Children (Sept. or Oct.*)

- Observes there are 2 (or more) items of clothing for 2 (or more) children (p.16)
- Matches 1 or 2 (or more) food items with 1 or 2 (or more) children (p.18)
- Observes parts of one's body in mirror; hears and repeats number of eyes, heads, arms, etc. (p.18, 20)
- Observes parts of the body, one part for one child or puppet or doll, e.g., one child has one nose (p.20)
- Observes parts of the body, two parts for one child or doll, e.g., another child has two eyes, hands, feet, etc. (p.20, 22)

2. Matching a Set of 2 or 3 Objects with a Set of Related Objects (Sept. or Oct.*)

- Observes sets of 2 (or more) buttons and buttonholes on open coat, observes matching (p.22, 24)
- Observes sets of 2 (or more) scissors and slots, containers of milk and straws, etc., observes matching (p.24)
- Observes in a dramatization 3 bears and bowls, 3 pigs and houses, etc., observes matching (p.24, 26)
- Selects and names 2 or 3 party items to match those in a model set, rearranges his items and realizes number does not change (p.26, 28)

Kindergarten

1. A Set of Objects for a Set of Children (Sept.*)

- Observes there are 2 or 3 (or more) items of clothing for a set of 2 or 3 (or more) children (p.16)
- Matches 2 or 3 (or more) food items with a set of 2 or 3 (or more) children (p.18)
- Observes parts of one's body in mirror; names number in each set observed (p.18, 20)
- Observes parts of the body, one-for-one, e.g., 2 or 3 (or more) children, collectively, have 2 or 3 (or more) heads, noses, mouths, etc. (p.20)
- Names number of parts of the body, two-for-one, e.g., number of eyes, etc., for one child; number of eyes, etc., for two children, collectively (p.20, 22)

2. Matching a Set of 2 or 3 or More Objects with a Set of Related Objects (Sept.*)

- Perceives and names number (without counting) in sets of 2 or 3 (or more) buttons and buttonholes, observes matching (p.22, 24)
- Perceives and names number (without counting) in sets of 2 or 3 (or more) scissors and slots, etc., observes matching (p.24)
- Perceives and names numbers (without counting) in sets in a dramatization, observes matching (p.24)
- Selects and names 3 or more party items to match those in a model set, rearranges his items and realizes number does not change (p.26, 28)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.
TOPIC 2. ONE-TO-ONE CORRESPONDENCE BETWEEN SETS OF OBJECTS

Grade One

1. A Set of Objects for a Set of Children (Sept.*)

Names number in a set of 3 or 4 (or more) items for snack time and a set of 3 or 4 (or more) children. Observes one-to-one correspondence, knows number of items he will receive and checks, compares his set with that of another child (p.17,19)

Observes set of 3 or 4 (or more) children (and puppets, etc.) and names number; observes parts of the body and names total number of heads, noses, eyes, etc. (p.19)

Names number in sets of 3, 4, or more items to be distributed (quoits, crayons, etc.) and children - one item for one child; names number in sets of 4, 6, 8, or more items and children - two items for one child (p.19,21)

2. A Set of Objects with a Set of Related Objects (Sept.*)

Names number in sets of 3 or 4 (or more) items for painting and related painting items (jars and brushes, etc.), realizes that sometimes it does and sometimes it does not matter how items are matched (p.21,23)

Names number in sets of 3 or 4 (or more) related construction items of varying sizes - one-for-one (nuts and bolts, dolls and dresses, etc.); realizes it does matter how items are matched (p.23,25)

Names number in sets of 3 or 4 (or more) related construction items, two-for-one; thinks out relationships (p.25)

Plays "Guess Which Hand" game, places markers for guesses, realizes next guess may be either correct or incorrect (probability), names number of markers in each line (p.25,27)

Uses worksheet with drawings of related objects, matches related objects, writes numerals (p.27,29)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XV
TOPIC 3. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS WITHOUT COUNTING

Pre-Kindergarten

1. Concept of Pair (Sept. or Oct.*)
   Observes pair of symmetrical items of clothing or body parts, sees there are two, may observe differences between items (p.42,44)

2. Working and Playing with Sets of One, None, Two (or More) Objects (Oct. or Nov.*)
   Looks for sets of one, two, and nonexistent objects in house-play center, rearranges 2 objects and observes there are still two (p.44,46)

   Feels, names objects, and names number of 1 or no or 2 (or more) objects in mystery boxes and bags (p.46,48)

   Looks for sets of objects around the classroom or on trips, names objects, may be able to name number (p.48,50)

3. Using and Observing Sets of Two Objects, Subsets Within Sets (Oct. or Nov.*)
   Observes sets of 2 instruments in a rhythm band, observes number in subsets and set (p.50,52)

Kindergarten

1. Concept of Pair (Sept. or Oct.*)
   Observes pair of symmetrical items of clothing or body parts, observes differences, may be able to see lefthand one and righthand one (p.42,44)

2. Working and Playing with Sets of Zero, One, Two, Three (or More) Objects (Oct. or Nov.*)
   Looks for and names number in sets of 1, 2, 3 (or more), or nonexistent objects (a zero number) in house-play center; rearranges 2 or 3 objects and realizes there are still 2 or 3 (p.44,46)

   Feels, names objects, and names number of 0, 1, 2, or 3 (or more) objects in mystery boxes and bags, compares numbers in two bags (p.46,48)

   Looks for sets of objects around the classroom or on trips, names objects, may be able to name number (p.48,50)

3. Using and Observing Sets of Two or Three Objects, Subsets Within Sets (Oct. or Nov.*)
   Names numbers in sets of 2 or 3 instruments in a rhythm band, names numbers in subsets, conserves number in set (p.50,52)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XVI
Grade One

3. A Set of Tallies for a Set of Objects or Children (Oct.*)

Names number in a set of objects - puppets, beanbags, markers in "Guess Which Hand" game, etc.; observes tallying; may be able to tally objects himself (p.29,31)

Names number in a group of children, may be able to tally children, compares number of tallies with number of children (p.29,31)

TOPIC 3. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS WITHOUT COUNTING

Grade One

1. Sets and Subsets: Zero Through Five Objects (Oct.*)

Observes number of items in a pair of scissors (blades), glasses (lenses), pliers, etc. (p.43)

Names number of items in 2 (or more) pairs of boots, rubbers, etc. (p.43,45)

Sorts and classifies science items into sets of from one through four (or more) items, names numbers in sets and subsets, rearranges items in a set and conserves number in set and subsets (p.45,47)

Patterns sets of discs, through 4 in a set; delineates subsets and names numbers; conserves numbers; draws sets and writes numerals (p.47,49)

Arranges and rearranges sets of cubic blocks, through 5 (or more) in a set; conserves numbers in set and subsets (p.49,51)

Patterns sets of discs, through 5 in a set; delineates subsets, names numbers, conserves numbers; compares sets; uses numeral card (p.51,53)

Uses worksheet with drawings of sets of objects, through 5 (or more); writes numeral for number in each set; writes numerals for subsets (p.53,55)

2. Sets and Subsets: Six Through Ten Objects (Nov. or Dec.*)

Names number in a set of 6 eggs in a carton (or toy milk bottles), indicates subsets, conserves number in set (p.55,57)

Names number in a set of 6 discs, patterned; arranges set and indicates subsets; makes drawings of set and subsets (p.57,59)

Names number in a set of 7 discs, patterned; arranges set and indicates set of 6 and an odd one; makes drawings and indicates set and odd one; uses numeral card (p.57,59)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XVII
<table>
<thead>
<tr>
<th>Pre-Kindergarten</th>
<th>Kindergarten</th>
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<tbody>
<tr>
<td>Parks or docks 2 toy vehicles, names number in set and subsets, rearranges</td>
<td>Parks or docks 3 toy vehicles, names number in set and subsets, rearranges</td>
</tr>
<tr>
<td>vehicles and conserves number in set (p.52,54)</td>
<td>vehicles and conserves number in set and subsets (p.52,54)</td>
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<tr>
<td>4. Working and Playing with Sets of From None Through Three (or More)</td>
<td>4. Working and Playing with Sets of From Zero Through Four (or More) Objects</td>
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<tr>
<td>Objects (Nov. or Dec.*)</td>
<td>(Nov. or Dec.*)</td>
</tr>
<tr>
<td>Orders set of 1 or 2 or 3 or no bottles of milk (or cans of food), checks</td>
<td>Orders set of zero, 1, 2, 3, or 4 (or more) food items, checks order, names</td>
</tr>
<tr>
<td>order, names number delivered (p.54,56)</td>
<td>number delivered (p.54,56)</td>
</tr>
<tr>
<td>Names number in sets of 1 or 2 or 3 (or more) bowling pins (or other tall</td>
<td>Names number in sets of 1 or 2 or 3 or 4 (or more) bowling pins, names number</td>
</tr>
<tr>
<td>objects), names number down (p.56)</td>
<td>down (p.56)</td>
</tr>
<tr>
<td>Feels and names number of 1 or 2 or no or 3 (or more) objects in mystery</td>
<td>Feels and names number of from zero</td>
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<tr>
<td>boxes and bags, checks</td>
<td>through 4 (or more) objects in mystery</td>
</tr>
<tr>
<td>(p.56,58)</td>
<td>boxes and bags, checks (p.56,58)</td>
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<tr>
<td>Locates sets of objects around the classroom or on trips, names number,</td>
<td>Locates sets of objects around the classroom or on trips, names number,</td>
</tr>
<tr>
<td>compares numbers in two sets (p.58)</td>
<td>compares numbers in two sets (p.58)</td>
</tr>
<tr>
<td>5. Using and Observing Sets of Two or Three Objects, Subsets (Dec.*)</td>
<td>5. Using and Observing Sets of Two, Three, or Four Objects, Subsets (Dec.* )</td>
</tr>
<tr>
<td>Observes numerousness in set of cubic blocks, in games names number in set of</td>
<td>Observes numerousness in set of cubic blocks; in games names number in set</td>
</tr>
<tr>
<td>2 or 3 and subsets (p.58,60)</td>
<td>of 2, 3, or 4 (or more) and subsets (p.58,60)</td>
</tr>
<tr>
<td>Observes numerousness of objects in science collections and other objects,</td>
<td>Observes numerousness of objects in science collections and other objects,</td>
</tr>
<tr>
<td>sorts, names number in set of 2 or 3 and subsets (p.60,62)</td>
<td>sorts, names number in set of 2 or 3 or 4 and subsets, conserves numbers</td>
</tr>
<tr>
<td>(p.60,62)</td>
<td>(p.60,62)</td>
</tr>
<tr>
<td>Observes items for collages; selects 2 or 3, names items and number in sets</td>
<td>Names items for collages; selects 2 or 3 or 4, names number in set and subsets</td>
</tr>
<tr>
<td>and subsets (p.62,64)</td>
<td>(p.62,64)</td>
</tr>
<tr>
<td>Observes spots on a domino, names number on each part and on entire domino</td>
<td>Names number of spots on a domino – each part and entire domino (p.64,66)</td>
</tr>
<tr>
<td>(p.64,66)</td>
<td></td>
</tr>
</tbody>
</table>

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.*

XVIII
Grade One

Names number in a set of 8 discs, patterned; arranges set and indicates doubles; makes drawings; uses numeral card (p.57,59).

Names number in a set of 9 discs, patterned; arranges set and indicates 8 and an odd one; makes drawings; uses numeral card (p.57,59).

Names number in a set of 10 discs, patterned; arranges set and indicates doubles; makes drawings; uses numeral card (p.57,59,61).

Thinks out number of discs, not patterned, in sets of from zero through 10 (p.7,61).

Uses worksheet with drawings of sets of geometric figures, through 10, emphasizing doubles and an odd one; writes numerals (p.63).

3. Sets Through Twenty or More (Dec. or Jan.*).

Names number of discs or pennies in a set of 11; arranges set and indicates subsets of 10 and 1, and of 5 and 5 and 1; numeral (p.63,65).

Names number of discs or pennies in a set of 12; arranges set and indicates subsets of 10 and 2, numeral (p.63,65).

Names number of discs or pennies in a set of 13; arranges set and indicates subsets of 10 and 3, numeral (p.63,65).

Names number of discs or pennies in a set of 14; arranges set and indicates subsets of 10 and 4, numeral (p.63,65).

Names number of discs or pennies in a set of 15; arranges set and indicates subsets of 10 and 5, numeral (p.63,65).

Names number of discs or pennies in a set of 16; arranges set and indicates subsets of 10 and 6, numeral (p.63,65).

Names number of discs or pennies in a set of 17; arranges set and indicates subsets of 10 and 7, numeral (p.63,65).

Names number of discs or pennies in a set of 18; arranges set and indicates subsets of 10 and 8, numeral (p.63,65).

Names number of discs or pennies in a set of 19; arranges set and indicates subsets of 10 and 9, numeral (p.63,65).

Names number of discs or pennies in a set of 20; arranges set and indicates subsets of 10 and 10, numeral (p.63,65).

Uses worksheet with drawings of sets of geometric figures, through 20 (or more), emphasizing 10 and a number (or 20 and a number) (p.65,67).

* Suggested time for introducing sub-topic. Continues to develop some items during succeeding months.

XIX
TOPIC 4. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS:
COUNTING, COMBINING, AND SEPARATING

Pre-Kindergarten

1. Counting 2 and 4 Objects, Pattern- ing; Observing Last Set (Jan. *)

Repeats jingles; names set and number in set of 1, 2, or 3 objects (p.84)

Observes set of 4 house-play objects, may be able to name number, observes counting and patterning, observes last set (p.86,88)

Observes set of 4 science items, names set and number if he can, observes counting and patterning and makes a move, observes last set (p.88,90)

Names set and number in set of 4 toy vehicles, observes shapes of parking lot and spaces, with help counts the 4 vehicles, counts and parks 2 cars, observes subsets (p.90,92,94)

Sorts construction material by color; selects 4 and counts, patterning; names number in subsets, conserving number in set (p.96,98)

2. Counting 3 and 5 Objects, Pattern- ing; Observing Last Set (Jan. *)

Names set and number in set of 3 science items, buttons, cubic blocks, etc.; counts, patterning; observes last set (p.98)

Names set and number in set of 5 small objects; counts, patterning; observes last set and subsets (p.98,100)

Names number and counts 5 pennies, patterning; observes last set; exchanges coins (p.102)

Kindergarten

1. Counting 2, 4, and 6 (or 8) Objects, Pattern- ing; Observing Last Set (Jan. *)

Repeats jingles; names set and number in set of 1, 2, 3, 4, or more objects (p.84)

Names number in set of 4 house-play objects; counts and patterns 4 or 2 objects, observes even arrangement of last set (p.86,88)

Names set and number in set of 4 science items, counts and patterns, observes pattern of last set (p.88,90)

Names set and number in sets of 4 and 6 toy vehicles; counts, observes shapes of parking lots and spaces, observes subsets (p.90,92,94)

Sorts construction material by color; selects 4 and counts; selects 6 and counts; observes patterns of last sets; names number in subsets (p.96,98)

2. Counting 3 and 5 (or 7) Objects, Pattern- ing; Observing Last Set (Jan. *)

Names set and number in sets of 3 science items, buttons, cubic blocks, etc.; counts, patterning; observes pattern of last set and subsets (p.98)

Names set and number in sets of 5 small objects; counts each, patterning; observes pattern of last set and subsets (p.98,100)

Appreciates values of 1 penny, 5 pennies, etc.; names number and counts 5 pennies, patterning; observes pattern of last set; exchanges coins (p.102)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.
TOPIC 4. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS: COUNTING, COMBINING, AND SEPARATING

Grade One

1. Counting Forward and Backward by Ones and by Twos, Patterning Through 20 or More (Jan.*)

Perceives number in 6, 7, 5, etc. (10 or under) wheels or buttons or other objects; counts forward by ones, patterning; studies last set (p.85,87)

Perceives number in 10 or less wheels or buttons or other objects; counts forward by ones, patterning; studies last set; counts patterned objects backward by ones (p.85,87)

Perceives number in 9, 5, 6, etc. (10 or under) discs; counts forward by ones, patterning; studies last set; writes numerals (p.85,87)

Perceives number in 9, 5, etc. (10 or under) discs; counts forward by ones, patterning; studies last set; counts patterned discs backward by ones; writes numerals (p.85,87)

Perceives number in an even number of wheels, buttons, or discs, 10 or under; counts forward by twos, patterning; studies last set; counts patterned objects backward by twos; writes numerals (p.87,89,91)

Perceives number in an odd number of wheels, buttons, or discs, 9 or under; counts forward by twos, patterning; studies last set; counts patterned objects backward; writes numerals (p.91)

Counts discs by ones through 10, 9, 11, etc. (20 or under), patterning; studies last set; counts patterned discs backward; uses pattern cards; uses worksheets (p.93,95,97,99,101,103)

Counts discs by twos through 10, 12, 8, 14, 11, etc. (20 or under), patterning; studies last set; counts patterned discs backward by twos; uses pattern cards and worksheets (p.93,95,97,99,101,103)

Counts pennies by ones and by twos through 10, 11, 12, etc. (20 or under); exchanges coins; writes symbols (p.93)

Reads numerals forward and backward on a clock, on a calendar, pages in a book; worksheets (p.103,105)

*Suggested time for introducing s::t-topic. Continue to develop some items during succeeding months.

XXI
Pre-Kindergarten

3. Perceiving Doubles, Within 4 Objects (Jan.*)

Sorts vehicles by number of wheels, observes sets of wheels on four-wheeled vehicles, perceives number in set and in subsets of 2 and 2 (p.104)

Observes set of 4 toy milk bottles in a rack or eggs in a box for 6; takes away by twos and makes observations; counts, patterning; indicates subsets of 2 and 2 (p.106,108)

Names number in sets of 4 science materials, collage materials, cubic blocks, etc. - patterned and unpatterned; indicates subsets of 2 and 2 (p.108,110)

Observes dominoes and names number on as many halves as he can, finds the doubles ones and twos, names number on each half and on both halves together of doubles (p.110, 112)

4. Subsets, Additions, and Subtractions Within 2 and 3 Objects (Feb.*)

Names number in sets of 1, 2, and no house-play objects or toy vehicles; names numbers in subsets and in set before and after rearranging set (conservation); thinks out number if he adds one, or if he takes away one (p.114,116)

Names number in sets of 1, 2, and no science materials; indicates subsets before and after rearranging set; thinks out number if he adds one or more, or if he takes away one or more (p.116,118,120)

Names number in sets of 1, 2, 3, and no cubic blocks or construction materials; compares numbers in two sets and names numbers; thinks out number if he takes away one or two and then adds the number (inverse operations)(p.120,122,124,126)

Kindergarten

3. Perceiving Doubles Within an Even Number of Objects, Through a Total of 6, or More (Jan.*)

Sorts vehicles and names number of wheels; names and conserves number on four-wheeled vehicles in set and in two subsets, same number in each subset (p.104)

Indicates sets of 4 and of 6 toy milk bottles or eggs; indicates subsets of 2 and 2, of 3 and 3; takes away by twos and makes observations; counts, patterning (p.106,108)

Names number in sets of 2, 4, 6 (or 8) small objects - patterned and unpatterned; indicates doubles (p.108,110)

Names number if he can on domino halves and both halves together; finds doubles and names number on each half of the zeros, ones, twos, threes, or more, and on both halves together (p.110,112)

4. Subsets, Additions, and Subtractions Within 2, 3, and 4 Objects (Feb.*)

Names number in sets of zero, 1, 2, and 3 house-play objects or toy vehicles; names numbers in pairs of subsets and in set before and after rearranging set, (conservation and commutation); thinks out number if he adds one or two, or if he takes away one or two (p.114,116)

Names number in sets of zero, 1, 2, 3 and 4 science materials; names numbers in pairs of subsets before and after rearranging set (conservation and commutation); thinks out number if he adds 1, 2, 3, or 4, or if he takes away 1, 2, 3, or 4 (p.116,118,120)

Names number in sets of zero, 1, 2, 3, and 4 or more cubic blocks or compares numbers in two sets; names numbers in pairs of subsets and in the set; thinks out number if he takes away 1 or 2 or 3 or 4 and then adds the number (inverse operations)(p.120,122,124,126)

* Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.
Grade One

2. **Doubles and Near-Doubles, Through a Total of 10 or 11, or More (Feb.*)**

Patterns and indicates doubles within sets of 4, 2, 6, 8, 10 (or 12) buttons or discs; names doubles and number in the set; perceives and names doubles in sets of unpatterned objects, conserving number in set; uses pattern cards with doubles; uses worksheets with doubles (p.105,107,109,111)

Uses buttons or discs to pattern a set of doubles and another set with one more, e.g., a set of 4 and a set of 5, 6 and 7, 8 and 9; names numbers in doubles and near-doubles conserving number in set; uses pattern cards; uses worksheets, with doubles and near-doubles (p.107,109,111)

Perceives number in a set of 4 discs in a row or unpatterned, thinks out number if he doubles the set of 4, thinks out number left if he takes away one-half of the set of 8, thinks out number if he doubles the set of 4 and adds one; proceeds similarly beginning with a set of 5 or 3 or 2 or 6 discs in a row or unpatterned (p.111,113)

3. **Subsets Within 10; Adding and Taking Away 1, 2, 3, and 4 Within a Total of 10 Objects, or More; Commutation (Feb.*)**

Strings 10 beads of two colors; moves without counting 5, 6, 4, 10, etc.; studies subsets of 5 and 5; thinks out number if he adds 5 to 5 and then if he takes away the 5 from the 10 (inverse operations) writes sentences; proceeds similarly with other subsets within 10 beads, e.g., 6 and 4, etc.; may be able to use 10 beads of one color and proceed as with beads of two colors (p.115,117,119)

Adds one to a set of 6, 8, 7, etc., discs, patterned and unpatterned; takes away one from a set of discs, patterned and unpatterned, adds one to a set of beads and then takes one away (inverse operations); adds and takes away 2 with discs and beads (p.119,121,123)

Adds 1 to a number (under 10) and then adds to 1, e.g., 6 and 1 are 7, 1 and 6 are 7 (commutation) using discs, dominoes, pattern cards, and sentences (p.123,127,129)

Adds 1 to a number (under 10) and then takes away the 1, e.g., 7 and 1 are 8, 8 take away 1 are 7 (inverse operations) using discs and beads (p.123,125,127,129)

Adds 2 to a number and then adds to 2 (commutation); adds 2 to a number and then takes away the 2 (inverse operations); uses procedures as for preceding two items (p.123,125,127,129)

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*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.*

XXIII
Pre-Kindergarten

5. Thinking Out Result of Adding One and Taking Away One Within 5 Objects (Feb.*)

Names number if he can in sets of patterned toy cars or spaces for cars, cubic blocks, or other small objects, from none through 5; thinks out number if one is added or taken from a set (p.126,128)

Names number if he can in sets of patterned pennies from none through 5; thinks out number if one penny is added or taken from a set; exchanges 5 pennies for a nickel (p.128,130,132)

Kindergarten

5. Thinking Out Result of Adding One and Taking Away One Within 6 Objects (Feb.*)

Names number if he can in sets of patterned toy cars or spaces, cubic blocks, or other small objects from zero through 6; thinks out number if one (or 2) is added or taken from a set (p.126,128)

Names number if he can in sets of patterned pennies from zero through 6; thinks out number if one penny is added or taken from a set; appreciates he needs 4 or 5 or 6 pennies to make a purchase; exchanges 5 pennies for a nickel; may be able to exchange 10 pennies for 2 nickels (p.128,130,132)

TOPIC 5. NUMBER LINE CONCEPTS: EARLY LEVELS OF DEVELOPMENT

Pre-Kindergarten

1. Perceiving at Once or Thinking Out the Number in a Line of Objects - Through 4 (Mar.*)

Arranges, observes, and may be able to locate lines of 2, 3, or 4 objects; observes directions; encircles each set; names each set and number of objects if he can (p.152,154)

Conserves number in lines of 2 or 3 objects perceived from different positions, after 2 objects are interchanged, after change in position of line (p.154,156,158)

Sorts objects into sets of 1, 2, 3, or 4, and aligns; thinks out the number in lines of 3 or 4 objects by first perceiving all but one (p.158,160,162)

Thinks out the number in lines of 4 objects by first perceiving doubles, conserves number in the set (p.162,164,166,168)

Kindergarten

1. Perceiving at Once or Thinking Out the Number in a Line of Objects - Through 6 or More (Mar.*)

Arranges, observes, and locates lines of 2 - 6 objects; observes directions; encircles each set; names each set of objects; names number if he can (p.152,154)

Conserves number in lines of 2 or 3 or 4 objects perceived from different positions, after 2 objects are interchanged, after change in position of line (p.154,156,158)

Sorts objects into sets of 1, 2, 3, 4, or 5, and aligns; thinks out the number in lines of 4 or 5 objects by first perceiving all but one (p.158,160,162)

Thinks out the number in lines of 4 and 6 objects by first perceiving doubles, conserves number in the set (p.162,164,166,168)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XXIV
Grade One

Adds 3 and takes away 3 from sets of discs and beads and other materials, relating to adding and taking away 2 (associative principle and inverse operations); adds 4 and takes away 4 similarly (p.131)

Adds 3 or 4 to a number and then adds to 3 or 4 (commutation); subtracts 3 or 4 (p.131)

4. Counting Forward and Backward by Fives and Tens - Through a Total of 20 or More Objects (Feb.*)

Counts 10, 15, 20, or more pennies and beads by fives forward, patterns, counts backward; exchanges pennies for other coins (p.133)

Counts 20 or more pennies and beads by tens forward, patterns, counts backward; exchanges pennies for other coins (p.133)

TOPIC 5. NUMBER LINE CONCEPTS: EARLY LEVELS OF DEVELOPMENT

Grade One

1. Perceiving at Once or Thinking Out the Number in a Set of Objects in a Line - Through 10 or More (Mar.*)

Locates, observes, and arranges sets of 2 - 10 objects in vertical and horizontal lines, names each set of objects, names number, without counting, if he can (p.153)

Classifies objects into sets, arranges objects in vertical and horizontal lines (p.155)

Conserves number in each line of 3 or 4 or 5 objects after change in position of line (p.157)

Conserves number in each of 2 subsets and in the set of 3 or 4 or 5 objects arranged in lines after the two lines of objects are combined into one line (p.157)

Thinks out the number in lines of 4, 6, 8, and 10 objects by first perceiving doubles (p.159)

Thinks out the number in lines of 5, 7, and 9 objects by first perceiving doubles (p.159, 161, 163)

Thinks out the number in lines of 5 objects by first perceiving the number in each of two subsets (p.163, 167, 169)

Thinks out the number in lines of 6 objects by first perceiving the number in each of two subsets (p.163, 165, 167, 169)

Thinks out the number in lines of 7 objects by first perceiving the number in each of two subsets (p.165, 167, 169)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months. XXV
Pre-Kindergarten

Locates, observes, and compares two lines of 1, 2, 3, or 4 related objects by number (p.170, 172,174)

Observes and names each object and each line of objects, may be able to perceive relationships between the two lines of objects (p.174)

2. Concepts of Length and Weight; Using Rectangular Building Blocks of 4 Lengths (Apr. *)

Compares lengths of sets of 2 objects, including rectangular building blocks of 4 lengths (p.174,176)

Estimates comparative weights of sets of 2 objects of different lengths; holds objects, one in each hand, to check (p.174,176)

Develops additions and subtractions within 2 using sets of rectangular building blocks, one set twice as long as the other (p.176,178,180)

Develops some additions and subtractions within 4 using sets of rectangular building blocks, one set four times as long as the other (p.176,178,180)

Kindergarten

Locates, observes, and compares two lines of 1, 2, 3, 4, or 5 related objects by number (p.170,172,174)

Observes and describes each object and each line of objects, indicates relationships between the two lines of objects (p.174)

2. Concepts of Length and Weight; Using Rectangular Building Blocks of 4 Lengths (Apr. *)

Compares lengths of sets of 2 or 3 objects, arranges objects in order of length (p.174,176)

Estimates comparative weights of sets of 2 objects of different lengths, holds objects, one in each hand, to check; estimates and checks weights of 2 objects of different compositions or contents (p.174,176)

Develops additions and subtractions within 4 using sets of rectangular building blocks, one set four times as long as the other (p.176,178,180)

Develops some additions and subtractions within 8 using sets of rectangular building blocks, one set eight times as long as the other (p.176,180)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XXVI
Grade One

Thinks out the number in lines of 8 objects by first perceiving the number in each of two subsets (p.167,169)

Thinks out the number in lines of 9 objects by first perceiving the number in each of two subsets (p.167,169)

Thinks out the number in lines of 10 objects by first perceiving the number in each of two subsets (p.167,169)

Compares two lines of 1 - 10 related objects by number, counts the objects in each line and compares by number (p.171,173)

Describes characteristics of each object in a line, and of each line of objects; compares characteristics of the two lines of objects indicating relationships (p.171,173,175)

2. Concepts of Lengths and Weights: Using Rectangular Table Blocks and Tagboard Strips of 10 Graduated Lengths (Apr. *)

Compares lengths of sets of 2, 3, 4, or more objects; arranges objects in order of length; selects 2 rectangular table blocks or tagboard strips, one double the length of the other (p.175,177,179)

Estimates comparative weights of sets of 2 objects, holds objects one in each hand to check; estimates comparative weights of sets of 3 objects, holds two objects at a time one in each hand to check (p.175,177)

Develops additions and subtractions involving doubling or taking away one half within 10, using scored rectangular table blocks or tagboard strips (p.179)

Develops additions and subtractions involving adding 1, adding to 1; adding 1, taking away 1; using scored blocks or tagboard strips (p.179,181)

Develops additions and subtractions involving adding 2, adding to 2; adding 2, taking away 2; adding 3, adding to 3; adding 3, taking away 3; adding 4, adding to 4; adding 4, taking away 4; using scored blocks or tagboard strips (p.179,181)

* Suggested time for introducing sub-topic. Continue to develop some items during succeeding months. XXVII
Pre-Kindergarten

3. Measuring the Length of an Object by Matching with Unit-Objects Placed End-to-End (Apr.*)

Strings straws, names number; places toy cars along an object, names number of cars needed; places rectangular building blocks along a shelf, names number of blocks needed (p.180, 182)

Kindergarten

3. Measuring the Length of an Object by Matching with Unit-Objects Placed End-to-End (Apr.*)

Strings straws, names number; places toy cars, dowel sticks, pipe cleaners, etc., along an object, names number needed; places rectangular building blocks along a shelf or along an object, names number of blocks needed (p.180, 182)

TOPIC 6. GEOMETRIC CONCEPTS AND FRACTIONAL PARTS: EXPLORATION AND EXPERIMENTATION

Pre-Kindergarten

1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves (May*)

Observes and feels beans or other large seeds, perceives numerousness of objects arranged in 2 heaps (p. 192, 193)

Divides heaps of seeds into halves, conserves quantity (p. 194)

2. Sharing a Single Object and a Set of Objects: One Half (May*)

Judges where to cut food or clay ball into halves, cuts, evaluates halves (p. 196)

Divides 4 nuts into halves, names number in each half; divides 6 nuts into halves, names number in each half; divides 4 and 6, or more, wrapped fruit or cookies into halves, names number in each half (p. 196, 198)

Kindergarten

1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves and Fourths (May*)

Observes and feels beans or other large seeds, perceives numerousness of objects arranged in 2 heaps, judges which heap has more or less objects or particles (p. 192, 193)

Divides heaps of seeds or sand into halves, divides each half into halves, conserves quantity (p. 194)

2. Sharing a Single Object and a Set of Objects: One Half (May*)

Judges how to share food or clay ball with another child, cuts, evaluates halves (p. 196)

Divides 6 nuts or wrapped fruit or cookies into halves, names number in each half; divides 8 and 10 objects into halves, names number in each half; divides 5 and 7 and 9 nuts or wrapped fruit or cookies into halves, judges how to share the extra one (p. 196, 198)

*Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XXVIII
Grade One

3. **Measuring Lengths of Familiar Objects Using a Rectangular Table Block or Tagboard Strip as a Unit of Length (Apr.* )**

Places unscored blocks or tagboard strips along an object to measure its length, names number needed, uses ordinal number names to indicate blocks or strips used (p.183)

Places unit-blocks or unit-strips along an object to measure its length, names number needed; places multiple-unit-blocks or multiple-unit-strips along an object, names number of units needed (p.183)

Uses a single unit-block or unit-strip to measure the length of an object (p.183)

**TOPIC 6. GEOMETRIC CONCEPTS AND FRACTIONAL PARTS:**
**EXPLORATION AND EXPERIMENTATION**

Grade One

1. **Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves, Fourths, Eighths, and Thirds (May*)**

Perceives numerousness of particles of seeds in heaps; divides heaps into halves, divides each half into halves for fourths, divides each fourth into halves for eighths (p. 193)

Handles sand, observes and feels particles, perceives numerousness of particles, explores with dividing heaps into thirds (p.195,197)

Follows directions and completes worksheets (p. 195,197)

2. **One Half and One Fourth of a Single Object and a Set of Objects (May*)**

Divides food into halves, then each half into halves for fourths; folds paper into halves and into fourths, then cuts designs at folds, labels halves and fourths (p. 197,199)

Divides sets of objects into halves, then divides each half into halves again for fourths (p. 199)

Follows directions and completes worksheets (p. 199)

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* Suggested time for introducing sub-topic. Continue to develop some items during succeeding months.

XXIX
Pre-Kindergarten

3. Comparing Capacities of 2 Cylindrical Containers, One Half of a Container, Measuring Capacities (June)

Fills and empties 2 containers of different sizes, compares contents, conserves quantity; compares heights of water levels and weights, and temperatures of contents (p. 198, 200, 202)

Fills 2 containers of different capacities, each one half full; compares contents; measures capacities using a non-standard unit of measure (p. 202, 204)

4. Perceiving Some Properties of Rectangular, Cylindrical, and Spherical Objects; Concepts of Circular; Concepts of Time and Direction (June)

Observes rectangular objects; observes and feels surfaces, corners, edges; compares 2 objects; hears names of objects (p. 204, 206, 208)

Observes cylindrical objects and feels the flat surfaces and the curved surface; observes spherical objects, feels curved surface, rolls it; compares a cylindrical object with a spherical object (p. 208, 210, 212, 214)

Makes circular movements, makes clockwise movements; observes hands and numerals on the clock; observes plan of play park (maze), walks to areas, notes turns (p. 214, 216, 218)

Kindergarten

3. Comparing Capacities of 3 Cylindrical Containers, One Half of a Container, Measuring Capacities (June)

Compares contents of 3 containers of different sizes, conserves quantity; compares heights of water levels, and weights and temperatures of contents (p. 198, 200, 202)

Fills 3 containers of different capacities, each one half full; compares contents; measures capacities using non-standard units of measure (p. 202, 204)

4. Perceiving Some Properties of Rectangular Cylindrical, and Spherical Objects; Concepts of Circular; Concepts of Time and Direction (June)

Observes rectangular objects; observes and feels and may be able to count surfaces, corners, edges; compares objects; identifies objects (p. 204, 206, 208)

Selects cylindrical objects and feels 2 flat surfaces and the curved surface; selects spherical objects, feels curved surface, rolls it; compares cylindrical objects with spherical objects (p. 208, 210, 212, 214)

Makes circular movements, makes clockwise movements; observes directions of hands and numerals on the clock; observes plan of play park (maze), walks to areas, counts turns (p. 214, 216, 218)
Grade One

3. **Comparing Capacities of Cylindrical Containers; One Half and One Fourth of the Capacity of a Container; Measuring Quantities (June)**

Comparing capacities of 3 or \( \frac{1}{4} \) containers of different kinds; conserves quantity; compares heights (lengths) of water levels, weights of containers, temperatures of contents; may be able to read the temperature (p. 199, 201)

Fills 3 or \( \frac{1}{4} \) containers of different kinds, each one half full; fills containers, each one fourth full; tips containers one half or one fourth full; observes water levels; measures capacities using non-standard units and a standard cup as units of measure (p. 203, 205)

4. **Rectangular Objects and Rectangles; Spherical and Cylindrical Objects and Circles; Concepts of Time and Direction (June)**

Locates and observes rectangular objects; perceives some properties of rectangular objects, including cubic objects; draws rectangles; perceives some properties of rectangles, including squares (p. 205, 207, 209)

Perceives some properties of spherical and cylindrical objects; compares spherical, cylindrical, and rectangular objects; observes hemispherical objects, discs, and wheels; draws and compares circles, radii, chords (p. 209, 211, 213, 215)

Reads and interprets a calendar; observes hour hand and tells time by the hour, near the hour, between 2 hours; observes plan of a neighborhood (maze), indicates directions, counts turns (p. 215, 217, 219)
TERMINOLOGY: PRE-KINDERGARTEN, KINDERGARTEN, GRADE ONE

Terms of position, general size, length, distance, capacity, shape, temperature, time, weight, quantity, set and subset are listed below for each of the six topics. The teacher uses these terms as children focus attention on the mathematical aspects of each of the topics. Children also use some of these terms.

Pre-Kindergarten and Kindergarten

<table>
<thead>
<tr>
<th>Topic</th>
<th>Grade One</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> up-down</td>
<td><strong>1.</strong> Those used previously and the following:</td>
</tr>
<tr>
<td>top-bottom</td>
<td>front-back</td>
</tr>
<tr>
<td>larger-smaller</td>
<td>higher-lower</td>
</tr>
<tr>
<td>faster-slower</td>
<td>under-over</td>
</tr>
<tr>
<td>older-younger</td>
<td>largest-smallest</td>
</tr>
<tr>
<td><strong>2.</strong> off-on</td>
<td><strong>2.</strong> Those used previously and the following:</td>
</tr>
<tr>
<td>different-same</td>
<td>top-bottom-middle</td>
</tr>
<tr>
<td>largest-smallest</td>
<td>more-not as many (fewer)</td>
</tr>
<tr>
<td>more-not enough</td>
<td>the same number</td>
</tr>
<tr>
<td><strong>3.</strong> set</td>
<td><strong>3.</strong> Those used previously and the following:</td>
</tr>
<tr>
<td>whole set</td>
<td>subset</td>
</tr>
<tr>
<td>part of the set</td>
<td>double</td>
</tr>
<tr>
<td>no or none or zero</td>
<td>near-double</td>
</tr>
<tr>
<td>other number names</td>
<td>even number</td>
</tr>
<tr>
<td>how many</td>
<td>odd one</td>
</tr>
<tr>
<td>more-fewer</td>
<td></td>
</tr>
<tr>
<td>both-pair</td>
<td></td>
</tr>
<tr>
<td>on-off</td>
<td></td>
</tr>
<tr>
<td><strong>4.</strong> in the set</td>
<td><strong>4.</strong> Those used previously and the following:</td>
</tr>
<tr>
<td>in the subset</td>
<td>in-out (of the set or subset)</td>
</tr>
<tr>
<td>more-less</td>
<td>above-below-between</td>
</tr>
<tr>
<td>add-take away</td>
<td>odd one</td>
</tr>
<tr>
<td>set of objects is even</td>
<td>even number</td>
</tr>
<tr>
<td>extra one</td>
<td>odd number</td>
</tr>
<tr>
<td>square</td>
<td>right-left</td>
</tr>
<tr>
<td>rectangle</td>
<td></td>
</tr>
<tr>
<td>filled-empty</td>
<td></td>
</tr>
</tbody>
</table>

XXXII
5. in a line
   longer—shorter
taller—higher
   as long as
on top of—in front of
heavier—lighter
weigh
rectangular block
cubic block
top—bottom—between

6. heap—pile
   half of—halves of—one half of
fourth of—fourths of—one fourth of
full—empty
jargy—glassful
hotter—colder
rectangular—cylindrical—spherical
farther—nearer
circular
daytime—nighttime
on time—late
tomorrow—yesterday

5. Those used previously and the following:
   longest—shortest—second longest—second shortest
tallest—highest
upward—downward
backward—forward
to the right—to the left
same length—different length
twice as long—four times as long
one half as long—one fourth as long
ordinal number names
pencil length (and other objects)
unit length
unit square
heaviest—lightest
weight

6. Those used previously and the following:
eighth of—eighths of—one eighth of
third of—thirds of—one third of
handful—bowlful—panful
warmer—cooler
rectangle—circle
inside—outside
straight line—curve
morning—noon—afternoon—night
day of the week—days of the week
name of current month—names of
some months
day—week—month
time by the hour—time near the hour

XXXIII
### Suggested Materials for Mathematics: Pre-Kindergarten, Kindergarten, Grade One

<table>
<thead>
<tr>
<th>Topics</th>
<th>Grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acorns (See Science objects)</td>
<td>all</td>
</tr>
<tr>
<td>Age cards, badges, hats, crowns, etc. (Prepared by the teacher)</td>
<td>1,5,6</td>
</tr>
<tr>
<td>Animals, toy</td>
<td>2,3,5</td>
</tr>
<tr>
<td>Bags, mystery (See Mystery bags and boxes)</td>
<td>6</td>
</tr>
<tr>
<td>Balloons</td>
<td>3,5,6</td>
</tr>
<tr>
<td>Balls, spherical - 2 colors</td>
<td>4,5,6</td>
</tr>
<tr>
<td>Beads, various shapes and colors</td>
<td>3,5,6</td>
</tr>
<tr>
<td>Bean bags or erasers</td>
<td>2,5</td>
</tr>
<tr>
<td>Blocks, building</td>
<td>3,4,5,6</td>
</tr>
<tr>
<td>Blocks, cubic</td>
<td>3,4,5,6</td>
</tr>
<tr>
<td>Blocks, rectangular table - 10 lengths - scored (See also Tagboard strips)</td>
<td>5,6</td>
</tr>
<tr>
<td>Board, steel (For magnetized discs)</td>
<td>3,4</td>
</tr>
<tr>
<td>Boats, toy (See Vehicles)</td>
<td>1</td>
</tr>
<tr>
<td>Bottles, toy (See Milk bottles)</td>
<td>1,2,3,4,5,6</td>
</tr>
<tr>
<td>Bowling pins and ball</td>
<td>3,4,5</td>
</tr>
<tr>
<td>Bowls, 3 sizes for 3 bears (See also House-play objects)</td>
<td>2,5,6</td>
</tr>
<tr>
<td>Box for eggs (See &quot;Eggs&quot; and egg cartons)</td>
<td></td>
</tr>
<tr>
<td>Box with slots for scissors (See Scissors)</td>
<td></td>
</tr>
<tr>
<td>Boxes, crayon (See Containers)</td>
<td></td>
</tr>
<tr>
<td>Boxes, mystery (See Mystery bags and boxes)</td>
<td></td>
</tr>
<tr>
<td>Box lids (See Snack items)</td>
<td></td>
</tr>
<tr>
<td>Buttons (See Markers, and Wheels)</td>
<td></td>
</tr>
<tr>
<td>Calendar, monthly</td>
<td>1,4,6</td>
</tr>
<tr>
<td>Cans (See also Containers)</td>
<td>2,3,4,5,6</td>
</tr>
<tr>
<td>Cards, age (See Age cards)</td>
<td>all</td>
</tr>
<tr>
<td>Cards, numeral (See Numeral cards)</td>
<td></td>
</tr>
<tr>
<td>Cards, pattern (See Pattern cards)</td>
<td></td>
</tr>
<tr>
<td>Carriages, doll (See Vehicles)</td>
<td></td>
</tr>
<tr>
<td>Cars, toy (See Vehicles)</td>
<td></td>
</tr>
<tr>
<td>Cartons (See Containers)</td>
<td></td>
</tr>
<tr>
<td>Chairs and tables, in the classroom</td>
<td>2,4,5</td>
</tr>
<tr>
<td>Charts with numerals (Prepared by the teacher)</td>
<td>1,5</td>
</tr>
<tr>
<td>Checkers (See Wheels, etc.)</td>
<td>all</td>
</tr>
</tbody>
</table>

XXXIV
<table>
<thead>
<tr>
<th>Topics</th>
<th>Grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay ..................................................................................</td>
<td>2,4,5,6</td>
</tr>
<tr>
<td>Clips or clothespins for pairs ...........................................</td>
<td>3</td>
</tr>
<tr>
<td>Clock, real ...........................................................................</td>
<td>1,4,6</td>
</tr>
<tr>
<td>Clothing children's and dolls' - boots, caps, coats, ......</td>
<td>2,3,4,5</td>
</tr>
<tr>
<td>gloves, hat, mittens, shoes (See also Pairs)</td>
<td>all</td>
</tr>
<tr>
<td>Construction material - cloth, paper, felt, oilcloth, ...</td>
<td>1,2,3,4,5,6</td>
</tr>
<tr>
<td>pipe cleaners, felt-tipped pen, etc., - for age cards,</td>
<td>all</td>
</tr>
<tr>
<td>apples, men, cards, collages, display boxes, dogs or</td>
<td></td>
</tr>
<tr>
<td>pigs and houses, geometric models, kittens and mittens, masks,</td>
<td></td>
</tr>
<tr>
<td>maze, party baskets or favors, paper</td>
<td></td>
</tr>
<tr>
<td>dolls and clothing, puppets, toy animals or vehicles, snowmen,</td>
<td></td>
</tr>
<tr>
<td>umbrellas, etc.</td>
<td></td>
</tr>
<tr>
<td>Containers - plastic jars with covers, boxes, cartons, ...</td>
<td>2,4,5,6</td>
</tr>
<tr>
<td>beakers, pitcher</td>
<td>all</td>
</tr>
<tr>
<td>Containers of milk ..........................................................</td>
<td>2,3,4,5,6</td>
</tr>
<tr>
<td>Copper or steel bar (See Science objects)</td>
<td>all</td>
</tr>
<tr>
<td>Cotton or other light material (See Science objects)</td>
<td></td>
</tr>
<tr>
<td>Crayon boxes (See Containers)</td>
<td></td>
</tr>
<tr>
<td>Crayons (See also Pencils)</td>
<td>2,3,4,5,6</td>
</tr>
<tr>
<td>Cubic blocks (See Blocks)</td>
<td>all</td>
</tr>
<tr>
<td>Cups, paper</td>
<td>2,4,5</td>
</tr>
<tr>
<td>Cups, toy (See House-play objects)</td>
<td></td>
</tr>
<tr>
<td>Cymbals (See Rhythm instruments)</td>
<td></td>
</tr>
<tr>
<td>Digits, lists - for enunciation (Prepared by the teacher) .......</td>
<td>1</td>
</tr>
<tr>
<td>Dimes (See Money)</td>
<td>1</td>
</tr>
<tr>
<td>Discs, magnetized</td>
<td>3,4,5,6</td>
</tr>
<tr>
<td>Discs, paper</td>
<td>4,6</td>
</tr>
<tr>
<td>Discs, plastic</td>
<td>2,3,4,6</td>
</tr>
<tr>
<td>Dishes, toy (See House-play objects)</td>
<td></td>
</tr>
<tr>
<td>Doll carriage (See Vehicles)</td>
<td></td>
</tr>
<tr>
<td>Dolls and doll dresses - real</td>
<td>2,3,5</td>
</tr>
<tr>
<td>Dolls and doll dresses - paper</td>
<td>2,3,5</td>
</tr>
<tr>
<td>Dominoes</td>
<td>3,4</td>
</tr>
<tr>
<td>Dowel sticks</td>
<td>4,5</td>
</tr>
<tr>
<td>Drums (See Rhythm instruments)</td>
<td>PK, K</td>
</tr>
<tr>
<td>&quot;Eggs&quot; and egg cartons</td>
<td>3,4,6</td>
</tr>
<tr>
<td>Envelope with address (Prepared by the teacher) ....................</td>
<td>1,4,6</td>
</tr>
<tr>
<td>Eyeglasses, pair (See Pairs)</td>
<td>all</td>
</tr>
<tr>
<td>Felt or flannel board and cutouts</td>
<td>2</td>
</tr>
<tr>
<td>Food items (See Snack items)</td>
<td>1</td>
</tr>
<tr>
<td>&quot;Footsteps&quot;</td>
<td>3,5</td>
</tr>
<tr>
<td>Formboard (Cardboard formboards may be prepared by ...</td>
<td>2,5</td>
</tr>
<tr>
<td>(See also Pairs)</td>
<td>all</td>
</tr>
</tbody>
</table>

XXXV
<table>
<thead>
<tr>
<th>Topics</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Games and puzzles</td>
<td>1,4,5,6 all</td>
</tr>
<tr>
<td>Gloves, pair (See Pairs)</td>
<td>all</td>
</tr>
<tr>
<td>Gravel</td>
<td>5,6   all</td>
</tr>
<tr>
<td>Hoops or laces</td>
<td>4,5   all</td>
</tr>
<tr>
<td>Horseshoes (See Quoits)</td>
<td>all</td>
</tr>
<tr>
<td>House-play objects - cups and saucers, bowls, pots</td>
<td>2,3,4,5,6 all</td>
</tr>
<tr>
<td>and pans, utensils</td>
<td></td>
</tr>
<tr>
<td>Gloves, pair (See Pairs)</td>
<td>all</td>
</tr>
<tr>
<td>Gravel</td>
<td>5,6   all</td>
</tr>
<tr>
<td>Hoops or laces</td>
<td>4,5   all</td>
</tr>
<tr>
<td>Horseshoes (See Quoits)</td>
<td>all</td>
</tr>
<tr>
<td>House-play objects - cups and saucers, bowls, pots</td>
<td>2,3,4,5,6 all</td>
</tr>
<tr>
<td>and pans, utensils</td>
<td></td>
</tr>
<tr>
<td>Jars (See Containers)</td>
<td>all</td>
</tr>
<tr>
<td>Laces</td>
<td>2,3,4,5 all</td>
</tr>
<tr>
<td>Lima beans (See Science objects)</td>
<td></td>
</tr>
<tr>
<td>Magnifying glass</td>
<td>6     all</td>
</tr>
<tr>
<td>Markers - buttons, discs, shells, wheels, etc. (For &quot;Guess Which Hand&quot; or probability game)</td>
<td>2,5 all</td>
</tr>
<tr>
<td>Mats, plastic or paper - for arranging discs or other</td>
<td>2,3,4,5,6 all</td>
</tr>
<tr>
<td>small objects, for science displays, for snacks, etc. (Prepared by the teacher)</td>
<td></td>
</tr>
<tr>
<td>Milk bottles and rack, toy</td>
<td>3,4   all</td>
</tr>
<tr>
<td>Milk containers (See Containers of milk)</td>
<td></td>
</tr>
<tr>
<td>Mittens, pair (See Pairs)</td>
<td></td>
</tr>
<tr>
<td>Mirror, full-length</td>
<td>2     PK, K</td>
</tr>
<tr>
<td>Money - pennies, nickels, dimes</td>
<td>1,2,3,4,5,6 all</td>
</tr>
<tr>
<td>Mystery bags and boxes (Prepared by the teacher)</td>
<td>3,4,5,6 all</td>
</tr>
<tr>
<td>Napkins, paper (See Snack items)</td>
<td></td>
</tr>
<tr>
<td>Nickels (See Money)</td>
<td>all</td>
</tr>
<tr>
<td>Numeral cards (Prepared by the teacher)</td>
<td>3,4,5 1</td>
</tr>
<tr>
<td>Numerals for identification - door, exits, toys, etc. (Prepared by the teacher)</td>
<td>1 all</td>
</tr>
<tr>
<td>Nuts (See Science objects)</td>
<td></td>
</tr>
<tr>
<td>Nuts and bolts (See Bolts and nuts)</td>
<td></td>
</tr>
<tr>
<td>Pails and shovels</td>
<td>2,5   PK, K</td>
</tr>
<tr>
<td>Painting materials - brushes, jars of paint, finger</td>
<td>2,4,5,6 all</td>
</tr>
<tr>
<td>paints, easel, smocks, etc.</td>
<td></td>
</tr>
<tr>
<td>Pairs - boots, gloves, shoes, mittens, binoculars</td>
<td>3,5 all</td>
</tr>
<tr>
<td>tongs, pliers, tweezers, eyeglasses</td>
<td></td>
</tr>
<tr>
<td>Paper, heavy - for toy cars, etc. (See also Mats)</td>
<td>4 all</td>
</tr>
<tr>
<td>Party baskets (See Construction material)</td>
<td></td>
</tr>
<tr>
<td>Pass, room (See Room pass)</td>
<td></td>
</tr>
<tr>
<td>Paste pots</td>
<td>3,4,5 all</td>
</tr>
<tr>
<td>Pattern cards (Prepared by the teacher)</td>
<td>3,4,5 1</td>
</tr>
<tr>
<td>Pebbles (See Science objects)</td>
<td></td>
</tr>
<tr>
<td>Pencils (See also Crayons)</td>
<td>2,3,4,5,6 all</td>
</tr>
<tr>
<td>Pennies (See Money)</td>
<td></td>
</tr>
</tbody>
</table>

XXXVI
<table>
<thead>
<tr>
<th>Topics</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pictures for clothing hook identification (May be .........) 2</td>
<td>PK, K</td>
</tr>
<tr>
<td>prepared by the teacher)</td>
<td></td>
</tr>
<tr>
<td>Pipe cleaners (See Construction material)</td>
<td></td>
</tr>
<tr>
<td>Plaques or plates with numerals (See Numerals for</td>
<td></td>
</tr>
<tr>
<td>identification)</td>
<td></td>
</tr>
<tr>
<td>Plant pots and plant dishes (See Science objects)</td>
<td></td>
</tr>
<tr>
<td>Plastic containers (See Containers)</td>
<td></td>
</tr>
<tr>
<td>Plates, paper or plastic (See Snack items)</td>
<td></td>
</tr>
<tr>
<td>Pots and pans, toy (See House-play objects)</td>
<td></td>
</tr>
<tr>
<td>Puppets (See Construction material)</td>
<td></td>
</tr>
<tr>
<td>Puzzles (See Games and puzzles)</td>
<td></td>
</tr>
<tr>
<td>wuoits or horseshoes ...........................................</td>
<td></td>
</tr>
<tr>
<td>Records of children's addresses, heights, etc...............</td>
<td></td>
</tr>
<tr>
<td>(Prepared by the teacher)</td>
<td></td>
</tr>
<tr>
<td>Rectangular building blocks (See Blocks, building)</td>
<td></td>
</tr>
<tr>
<td>Rectangular table blocks (See Blocks, rectangular table)</td>
<td></td>
</tr>
<tr>
<td>Rhythm instruments - drums and drum sticks, cymbals, .......</td>
<td></td>
</tr>
<tr>
<td>2,3,4,5,6</td>
<td></td>
</tr>
<tr>
<td>triangles, maracas, sand blocks, etc.</td>
<td></td>
</tr>
<tr>
<td>Room pass ..................................................................</td>
<td></td>
</tr>
<tr>
<td>Saucers, toy (See House-play objects)</td>
<td></td>
</tr>
<tr>
<td>Science objects - acorns, corks, dried beans, nuts, .......</td>
<td></td>
</tr>
<tr>
<td>3,4,5,6</td>
<td></td>
</tr>
<tr>
<td>peach pits, plant pots and plant dishes and plants,</td>
<td></td>
</tr>
<tr>
<td>shells, wood, metal, sponges, seeds, twigs and</td>
<td></td>
</tr>
<tr>
<td>leaves, sand and earth, sawdust</td>
<td></td>
</tr>
<tr>
<td>Scissors and box with slots (Box prepared by the ..........</td>
<td></td>
</tr>
<tr>
<td>2,3,5</td>
<td></td>
</tr>
<tr>
<td>(Prepared by the teacher)</td>
<td></td>
</tr>
<tr>
<td>Shelves - bookcase, closet ......................................</td>
<td></td>
</tr>
<tr>
<td>Shoes, pair (See Pairs)</td>
<td></td>
</tr>
<tr>
<td>Signs with numerals (See Numerals)</td>
<td></td>
</tr>
<tr>
<td>Snack items - boxes of cereal, cans of food, carrot .......</td>
<td></td>
</tr>
<tr>
<td>2,3,4,5,6</td>
<td></td>
</tr>
<tr>
<td>sticks, cookies, fruit, napkins, plates, pudding,</td>
<td></td>
</tr>
<tr>
<td>straws, trays or box lids (See also Containers of milk)</td>
<td></td>
</tr>
<tr>
<td>Spot cards (See Pattern cards)</td>
<td></td>
</tr>
<tr>
<td>Stencils (See Worksheets)</td>
<td></td>
</tr>
<tr>
<td>Straws (See Snack items)</td>
<td></td>
</tr>
<tr>
<td>Tagboard strips, 10 lengths, scored on one side ...........</td>
<td></td>
</tr>
<tr>
<td>(Prepared by the teacher)</td>
<td></td>
</tr>
<tr>
<td>Thermometer, wall..................................................</td>
<td></td>
</tr>
<tr>
<td>Tools .......................................................................</td>
<td></td>
</tr>
<tr>
<td>Toy animals (See Animals)</td>
<td></td>
</tr>
<tr>
<td>Toy milk bottles and rack (See Milk bottles)</td>
<td></td>
</tr>
<tr>
<td>Toy vehicles (See Vehicles)</td>
<td></td>
</tr>
<tr>
<td>Trays or box lids (See Snack items)</td>
<td></td>
</tr>
<tr>
<td>Topics</td>
<td>Grade</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Vehicles, toy - airplanes, boats, buses, doll........................... 1,3,4,5,6</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
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<td></td>
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<td></td>
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MATHEMATICS

PRE-KINDERGARTEN, KINDERGARTEN, AND GRADE ONE

Suggestions for the Pre-Kindergarten and Kindergarten are on lefthand, even-numbered pages. Suggestions for Grade One are on righthand, odd-numbered pages.

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PRE-KINDERGARTEN AND KINDERGARTEN

TOPIC 4. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS:
COUNTING, COMBINING, AND SEPARATING

PRELIMINARY STATEMENT

Pre-Kindergarten and Kindergarten children observed and arranged sets of objects as part of their work and play activities as Topics 2 and 3 were developed. Pre-Kindergarten children were encouraged to perceive and name the number in sets through three objects, and to perceive subsets within two objects. Kindergarten children were encouraged to perceive and name the number in sets and subsets through four objects, at least.

Counting was deferred until this topic (Topic 4) so that children would be able to perceive and name the number in successive sets of patterned objects, and to study the last set of objects after counting. (The conventional method of counting objects is not taught since it is a shortcut method which does not relate successive number names to successive sets of objects. For example, conventionally we say "one" while looking at and moving one, then we say "two" while looking at and moving one, then we say "three" while looking at and moving one, etc.)

Before developing counting with a child or a small group of children the Pre-Kindergarten and Kindergarten teacher may wish to evaluate the child's understanding of the concept of set, including the number in a set. One procedure of interest to young children is the use of jingles which encourages children to hunt for, to discover, and to identify sets.

A jingle might direct a child to name the set, e.g.: Here are three. What do you see? (Balls or 3 balls, etc.) A jingle might direct a child to find sets with a specific number of objects, e.g.: Come and see! Look for three! (Children point to sets of 3.) A jingle might include number names in order and then direct a child to find sets with that number of objects, e.g.: One-two-three. Where are three? (The teacher does not "count out" objects, one by one.)

Children who are able to perceive and name the number in a great variety of sets of objects may be ready to count such sets. Before a child counts he should always think out (not guess) how many objects there are in the entire set.

Pre-Kindergarten and Kindergarten children learn to count and pattern an even number of objects first, beginning with a set of 4 objects. They think out the number, pattern the objects moved, and name each successive set. They check the number in the last set of patterned objects with the number thought out before counting.

Children at this level count objects as part of their work and play activities. The teacher observes children as they work or play, then calls their attention to a set of objects and teaches a child or a small group of children how to count the objects.

(Continued on Page 70)
GRAD ONE

TOPIC 4. PERCEIVING THE NUMBER IN A SET AND ITS SUBSETS:
COUNTING, COMBINING, AND SEPARATING

PRELIMINARY STATEMENT

In Grade One for this topic the teacher helps children continue the development of sets and subsets introduced in Topics 2 and 3 for this grade. As these earlier topics were developed children made observations relative to sets and subsets within numbers through 10, e.g., all subsets within the numbers through 6, some subsets within the numbers 7 through 10. Children also made observations relative to numbers through 20 or more, e.g., a subset of 10 and a subset of one or two or three, etc.

Counting was deferred until this topic so that children would be able to establish, with confidence, number perception with respect to sets of objects (without counting.) Some children may have learned to count using a rote shortcut method. Children who count by rote especially need to be encouraged to see the relationship between the number name and the number in a set. For example, a child who is learning to count objects (unlike an adult who counts) says each number name after he has completed a move and as he perceives the number of objects. He arranges the objects he moves in patterns so that he and other children can readily perceive the number in the successive sets (subsets of the entire set).

The counting procedures developed in this topic lay a basis for children's learning what addition and subtraction are. As children count forward by ones they are actually adding one to successive sets of patterned objects. As children count backward by ones they are actually taking away one from successive sets of patterned objects.

Grade One children begin this topic by counting objects by ones through 6 or 8 or 10 or 7 or 9, etc. Children think out the number, pattern the objects moved, name each successive set (subset), then study the last set. (See counting-by-ones procedure illustrated on page 70.) Children check the number found with the number thought out before counting. They observe whether the set has an even number of objects or an odd number of objects, as illustrated:

Even Number

Odd Number

"eight"

"nine"

(Continued on Page 71)
Illustrations of a way in which a Kindergarten child, or a mature Pre-Kindergarten child counts six cups follow - his moves, his successive patterns, and the number names he uses. Note that the patterned cups touch.

Teacher focuses attention on a selected set of 6 objects that are not patterned, e.g., 6 cups on the house-play table. Children think out how many there are.

First Move. Child moves one cup to the left. He perceives there is one (cup in the set he moved). Then he says "one."

Second Move. Child moves another one and places it under the first one. He perceives there are now two (cups in the set moved), then says "two."

Third Move. Child moves another one and places it as shown. He perceives there are now three, then says "three."

Fourth Move. Child moves another one and places it as shown. He perceives there are now four, then says "four."

Fifth Move. Child moves another one and places it as shown. He perceives there are now five, then says "five."

Last Move. Child moves the last one and places it as shown. He perceives there are now six, then says "six."

(Continued on Page 72)
Children who have developed proficiency in counting by adding one to successive patterned sets learn to "undo" the process of counting forward by taking away one, that is, by counting backward. Illustrations of a way in which a child counts six buttons backward follow: (Note that the patterned buttons touch.)

Child first counts the objects forward—moving the objects to the left, patterning them, and naming the number, as illustrated, beginning on facing page 70. He perceives his last set of six patterned objects and says "six."

First Move. Child moves one button back (to the right). He perceives that he still has five (buttons). He says "five."

Second Move. Child moves another button back. He perceives he still has four (buttons). He says "four."

Third Move. Child moves another button back. He perceives he still has three (buttons). He says "three."

Fourth Move. Child moves another button back. He perceives he still has two (buttons). He says "two."

Fifth Move. Child moves another button back. He perceives he still has one (button). He says "one."

Last Move. Child moves the last button back. He perceives he now has a zero number of buttons. He says "zero."

(Continued on Page 73)
After a set of objects has been counted children study the last set of patterned objects, e.g., a set of 4 toy saucers. Pre-Kindergarten and Kindergarten children observe that the set of saucers is even (that is, that its pattern is even). Kindergarten children also observe that the pattern is in the form of a square. For example, they observe the square shape of construction paper cut and placed under the set of 4 saucers; as illustrated:

Kindergarten children also count and pattern 6 objects. They observe that the set of 6 objects is even, and that the pattern is in the form of a rectangle, as illustrated for 6 wheels:

Pre-Kindergarten and Kindergarten children now count an odd number of objects and study the last set of patterned objects. They count 3 objects, 5 objects, or more. Children pattern the objects as they move objects and count. They observe the last set. They perceive that the last set has an extra one (an odd one). They perceive the set of 2 in the set of 3, that there are 2 and 1 more in the set of 3. They perceive the set of 4 in the set of 5, that there are 4 and 1 more in the set of 5.

As children study each last set (after counting has been completed) they perceive numbers in subsets, conserving the number in the set.
The same set of objects may be moved forward and then backward by several children (in a group). Every child needs numerous opportunities to think out the number in a set, to count the objects forward by adding one to successive patterned sets, and to "undo" this process by counting backward.

Children who are able to count objects by ones learn to count by twos forward and backward through 10, or 8, or 6, etc. Children count forward by twos, using the procedure they used for counting by ones - first thinking out or estimating the number, then moving and patterning the objects two at a time, then perceiving and naming the number, then studying the last set (perceiving numbers in subsets, yet conserving number in set).

The first move is illustrated for counting 8 discs forward. Note that the patterned discs touch. Child moves two discs (to the left). He perceives there are two (discs in the set he moved). Then he says "two."

After the 8 discs have been counted by twos each child compares the number in the last set with the number he thought out before counting. He also examines the last set to note whether there is an even or odd number, to note the 4 twos in 8, and to observe the rectangular shape of the set of patterned discs.

A child now counts the 8 discs backward by twos, as illustrated for the first move:

Children also count an odd number of objects by twos forward and backward. They examine the final set after forward counting, e.g., 7: noting the odd number, noting the 3 twos and 1 more in 7.

Children learn to count forward by ones and by twos through 20, or 14, or 12, or 16, etc. They separate the set of 10 from the remaining number, e.g.: "ten and four" "fourteen"

Children study the last set of patterned objects after counting 10 or more objects. They compare the number of objects in the last set with the number they thought out before counting. They note whether the set has an even number or odd number of objects. Illustrations follow:

(Continued on Page 75)
Pre-Kindergarten and Kindergarten children learn to perceive doubles in a set of 4 objects. They name these as: 2 here and 2 here, or 2 and 2 more, or 2 and 2 in 4. Children can perceive doubles in a set of 4 around the classroom: wheels on toy vehicles, blocks on a table, cups in a table setting, etc. Some of these objects may be patterned; others may not be patterned. As children construct collages or other decorations they may show doubles in 4 or other numbers.

Some children may be able to see doubles within 4 in more than one way, as illustrated:

Kindergarten children and mature Pre-Kindergarten children also learn to perceive doubles in sets of 6 objects, or more. They name these as: 3 here and 3 here, or 3 and 3, or 3 and 3 in 6. Sets of six objects may be placed or arranged by the teacher or by children, e.g.: milk containers in a carrier or on a tray, napkins in a table setting, eggs in a carton, paper cups on a mat in the center of a table for children's snack time, etc. Illustrations follow.
Children also observe the number of twos in 10, in the number exceeding 10, and in the entire number. For example, for the number of twos in 16: There are 5 twos in 10; there are 3 twos in 6; there are 8 twos in 16. Mature children may be able to count more than 10 objects backward by ones and by twos.

After children have learned to count sets of pennies with efficiency they learn to exchange pennies for other coins. For example, a child counts 7 pennies, then exchanges 5 pennies for a nickel. Or, he counts 14 pennies and exchanges 10 pennies for a dime or 2 nickels. After a child makes an exchange he checks to see if he still has the amount of money he counted (conservation of money value).

Grade One children observe successive numerals (number names) on the clock, calendar, pages in a book. They observe the direction of the numerals on a clock (clockwise). They observe the left-to-right and top-to-bottom directions of numerals on a monthly calendar. They observe the even-numbered pages (left-hand) and the odd-numbered pages (right-hand) in a book.

Grade One children now study doubles and their relationships to near-doubles. A suggested sequence follows:

1. Perceiving and naming doubles within an even number of patterned and unpatterned objects - through a total of 10, or more (e.g., 4 and 4 in 8, or 2 fours in 8)

2. Perceiving and naming doubles within an even-numbered set of objects, and near-doubles within the next larger odd-numbered set - through a total of 10 or 11, or more.

3. Doubling a number of objects laid in a row, or a set of unpatterned objects - through a total of 10, or more (e.g., 4 and 4 more are 8, or 4 and 4 are 8)

4. Finding one half and then taking away one half of a number of patterned and unpatterned objects - through a total of 10, or more (e.g., one half of 8 are 4; 8, take away 4, are 4)

(Continued on Page 77)
Children now study specific numbers of objects — through 3 in the Pre-Kindergarten, through 4 in the Kindergarten. Children study sets and subsets within these numbers, additions within these numbers, and subtractions within these numbers. Objects may be patterned or unpatterned.

Sets of 3 or 4 objects around the classroom that are often patterned by children, yet are often unpatterned, include: house-play objects, toy vehicles, science materials, cubic blocks, construction materials, etc.

A suggested sequence for developing a study of numbers follows for the Pre-Kindergarten (Pre-K) and Kindergarten (K):

1. Subsets within 2, adding and taking away within 2 (Pre-K and K)
2. Subsets within 3, adding and taking away within 3 (Pre-K and K)
3. Subsets within 4, adding and taking away within 4 (K)

A child names the number in a set, e.g., "two." He arranges and rearranges the set in several ways, noting the subsets for each arrangement, as illustrated for 2 toy pans in the house-play area.

Children also add and take away within the set of objects, e.g., 3. For example, John has one block. The teacher asks: How many blocks will you have if you take two more? John thinks this out (3). John then demonstrates this by adding the two blocks. He may then also count the three objects by moving and then naming the number. Similarly, John has three blocks. The teacher

(Continued on Page 78)
5. Doubling a number of objects laid in a row, or a set of unpatterned objects then adding one — through a total of 11, or more (e.g., 4 and 4 and 1 are 9, 4 and 5 are 9)

Fifteen of the basic addition facts may be developed through emphasis on doubles and near-doubles within a total of 10 objects or less. (Within a total of 18 objects 25 facts may be developed through doubles and near-doubles)

The following is clarification for the teacher's information only. "Doubles" refers to specific subsets within an even number, e.g., the doubles in 4 are twos, the doubles in 6 are threes. The term "a double" refers to the even number itself, e.g., 4 is a double, 6 is a double. Six has threes as doubles; four has twos as doubles.

As children learn to perceive doubles within an even number of patterned objects they are also learning addition, subtraction, multiplication, and division facts. For example, for a set of 6 there are 3 and 3 in 6 (addition), there will be 3 left if you take 3 from the 6 (subtraction), there are 2 threes in 6 (multiplication), in 6 there are 2 threes (division).

Children now study the number in each of two subsets within one row of 10 beads. Children string beads, 5 beads of one color and 5 beads of a contrasting color. They attach the string of beads to a weaving board. (See p.115.) Many Grade One children will need to use beads of two colors for all or most of Grade One. Some children, however, will develop proficiency with beads of two colors; these children will re-string the row, using beads of only one color.

Children use their string of beads to show and to develop sets and subsets within 10, one set of subsets at a time, e.g.: 5 and 5 in 10; later, 6 and 4 in 10; etc. Some illustrations follow for 7 and 3 in 10.

1. Children move 7 beads to the left on their frames. They observe the 7 moved and the 3 that have not been moved, as illustrated for beads of 2 colors:

   ![Diagram of beads showing 7 and 3]

2. Children move 7 beads to the left. They think out how many they will have if they add 3 beads.

3. Children move all 10 beads to the left. They think out how many there will be if 3 beads are taken away (moved to the right).

4. The teacher may cover the beads at the right or at the left. Children think out how many beads are covered.

Etc.

(Continued on Page 79)
asks: How many blocks will you have if you put two back (in the box). John thinks this out (1). John then demonstrates this by taking two blocks away from his set of three, then observing how many he has left.

Children learn that the order in which they see two specified subsets in a set of 2 or 3 objects does not affect the number. (Commutative principle in addition) This is illustrated for the subsets of 2 black buttons and 1 white button within a set of 3 buttons, e.g.: 2 and 1 are 3, 1 and 2 are 3.

Children learn to add one, at first to sets of patterned objects, later to sets of unpatterned objects. First they anticipate how many there will be if one is added to a set: For example, John sees that there are 2 milk containers on a tray. The teacher asks how many he will have if he adds one. Then John adds the one and observes he was right. The teacher may describe his action as: 2 and 1 more are 3, or 2 and 1 are 3.

Pre-Kindergarten children learn to add one to sets through a total of 5 objects, e.g.: 4 and 1, 2 and 1, 3 and 1, 1 and 1. Kindergarten children learn to add one to sets through a total of 6 objects, or more.

Children learn to take away one, at first from sets of patterned objects, later from sets of unpatterned objects. This is much more difficult than adding one, since the original set cannot be seen after taking away. Children first think

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Grade One children learn to add and to take away 1, then 2, as they use objects through a total of 10, or more. For example, children use patterned discs for adding and for taking away 1. Later they use beads. Then they use patterned discs for adding and for taking away 2; later beads. Illustrations follow for developing 6, take away 2.

1. Using Patterned Discs
   a. [Discs image]
   6
   b. [Discs image]
   6, take away 2, 4 are left.

2. Using Beads
   a. [Beads image]
   b. [Beads image]
   take away 2, 4 are left.

The teacher will observe that subtraction (taking away) is much more difficult than addition. In addition the original set is maintained, and after adding, can be perceived as a subset. In subtraction the original set is not maintained and cannot be readily perceived. This is illustrated for 4 and 1 (addition) and for 5, take away 1 (subtraction):

1. 4 and 1
   a. [Discs image]
   4 (the original set)
   b. [Discs image]
   4 and 1 are 5.

2. 5, take away 1
   a. [Discs image]
   5 (the original set)
   b. [Discs image]
   5, take away 1, leaves 4.

Children learn to derive an unknown fact from a known fact. For example, a child who can add one to a number can learn to add one and one more, that is, to add 2 to the number. (This is an application of the associative principle in addition.) Also, a child who can take one from a number can learn to take away one and one more, that is, he can subtract 2 from the number.

The teacher may wish to demonstrate a relationship between addition and subtraction (principle of inverse operations), as illustrated for 6, take away 2:

Using Discs
1. [Discs image]
   4 and 2 are 6.

(Continued on Page 81)
out how many there will be if one is taken away from a set. For example, Susan sees that there are 3 milk containers in a tray. The teacher asks how many there will be if Susan takes one off. Susan then takes the milk container and observes whether she was right. The teacher may describe Susan’s action as: 3, take away 1, leaves 2.

Pre-Kindergarten children learn to take away one from sets of 2, or 4, or 5, or 3 objects. Kindergarten children learn to take away one from sets through a total number of 6 objects, or more.

The teacher works with one child at a time or a small group of children for most of the activities suggested for these grades. This is necessary so that children can move and pattern materials, can see all sets clearly, can learn to see subsets, and can express themselves often. Thus the needs of individual children may be adequately met.

The objects children use for developing this topic should be relatively small and compact. This is important for counting since it is necessary for children to move and pattern the objects, and to perceive the number in the sets of patterned objects. It is important also that small and compact objects be used for combining and separating since the numbers of objects in sets and in subsets need to be readily perceived. Suggested materials follow:

**Essential Materials:** Jingles about sets around the classroom; hoop, yarn, string, or lace; toy dishes and other small house play objects; science materials - wheels, corks, pebbles, etc; toy cars; milk containers, tray, napkins, paper cups, mats; construction paper, colored paper discs; buttons; toy milk bottles and rack; dominoes; cubic blocks; pennies; nickels....

**Additional Materials:** Toy pots and pans; party baskets; shells or nuts; toy airplanes or other vehicles; colored paper or felt cut in various shapes; "eggs" and egg carton; clay and dowels....

Children continue to develop concepts with respect to position, general size, and quantity as they think about sets and subsets. The teacher and many children continue to use terms such as those suggested for Topics 1, 2, and 3. Other terms suggested are: in the set, in the subset, more-less, add-take away, set of objects is even, square, rectangle, extra one, filled-empty.
Using Discs (Cont.)

2. 

Using Beads (Cont.)

6, take away 2, 4 are left.

4 and 2 are 6.

Children learn to apply the commutative principle in addition. Children are applying this principle when they change the order of two sets of objects to find the total number more easily. For example, a child has learned to add 1 to sets of various sizes. Now he learns to move the set of 1, thus changing the order of the two sets, as illustrated for using 6 and 1 discs:

1. 

2. 

Similarly children learn to add 2, then to add to 2. Children who are able to add and take away 1 and 2, and to add to 1 and 2, now learn to add 3, take away 3, add to 3, add 4, take away 4, and add to 4.

Children count pennies by ones, recording the total number, e.g., 8 cents. Then they arrange the pennies in sets of 5 each, or 10 each. They count these again by fives or tens, and check the total number with the number recorded. Children may also exchange sets of pennies for nickels or dimes, again checking with the amount recorded.

Children count beads by fives and by tens, forward and backward. The teacher makes sure that children indicate the appropriate set of beads for each number name, that is, as a child says "fifteen" he indicates 15 beads, not 5.

Objects that are small and compact are best for developing counting, combining, and separating in Grade One. Suggested materials follow:

**Essential Materials:** Wheels or buttons; discs; steel board and magnetized discs; pennies, nickels, dimes; beads of 2 colors, laces, and weaving boards; dominoes; construction material for pattern cards and numeral cards, gummed paper discs; worksheets, stencils, duplicator; clock; calendar....

**Additional Materials:** Toy cups and saucers, milk containers, bottle caps, acorns and other nuts, checkers, one-inch paper discs, cubic blocks....

Children continue to develop concepts with respect to position, general size, and quantity as they think about sets and subsets, adding, and taking away. The teacher and children continue to use terms such as those suggested for the Pre-Kindergarten and Kindergarten, and those suggested for Topics 1, 2, and 3 for Grade One. Other mathematical terms suggested are: in-out (of the set or subset), above-below-between, odd one, even number, odd number, right-left.
An outline of the contents suggested for the Pre-Kindergarten and Kindergarten follows:

1. Counting an Even Number of Objects; Making Observations About the Last Set of Patterned Objects - Through 4, Through 6 or More
   
   Using Jingles to Name a Set and the Number in the Set - Through 3, Through 4 or More
   Counting House-Play Objects - Through 4
   Counting Science Items - Through 4
   Counting Toy Vehicles - Through 4, Through 6
   Counting Construction Materials - Through 4, Through 6 or More

2. Counting an Odd Number of Objects; Making Observations About the Last Set of Patterned Objects - Through 5, or More
   
   Counting Science Items, Buttons, Cubic Blocks, Etc.
   Counting Pennies, Exchanging for a Nickel

3. Perceiving Doubles Within an Even Number of Objects - Within 4, Within 6 or More
   
   Sets of Wheels on a Four-Wheeled Vehicle
   Sets of Toy Milk Bottles in a Rack or Eggs in a Box for 6
   Sets of 2, 4, 6, or More Small Objects - Science Materials, Collage Materials, Cubic Blocks, Etc.
   Sets of Doubles on Dominoes

4. Discovering Subsets, Additions, and Subtractions Within a Specified Number - Within 2, 3, and 4 (or more) Objects
   
   Sets of House-Play Objects or Toy Vehicles - Through 2, Through 3
   Sets of Science Materials - Through 2, Through 4
   Sets of Cubic Blocks or Construction Material - Through 3, Through 4 or More

5. Thinking Out the Result of Adding One or Taking Away One - Through a Total of 5 Objects; Through a Total of 6 or More Objects
   
   Sets of Toy Cars, Cubic Blocks, or Other Small Objects - Through a Total of 5; Through a Total of 6 or More
   Sets of Pennies, Exchanging for Nickel - Through a Total of 5; or Through a Total of 6 or More

Pre-Kindergarten and Kindergarten teachers will find it profitable to read at least the Preliminary Statement for Topic 4, Grade One, on odd-numbered pages 69-83.

Pre-Kindergarten and Kindergarten teachers will consider the activities suggested for Topic 4 on the following lefthand, even-numbered pages for both the Pre-Kindergarten and Kindergarten. Teachers at either of these levels can then select activities which are appropriate for their children.

(Continued on Page 84)
An outline of the contents suggested for Grade One follow:

1. Counting Forward and Backward by Ones and by Twos - Through a Total of 20 Objects, or More
   - Counting Wheels or Buttons and Discs, Forward and Backward by Ones - Through a Total of 10
   - Counting Wheels, Buttons, or Discs, Forward and Backward by Twos - Through a Total of 10
   - Exchanging Pennies for Other Coins; Counting Backward
   - Numerals on Clock, Calendar, Pages in a Book

2. Doubles and Near-Doubles - Through a Total of 10 or 11, or More
   - Perceiving Doubles and Near-Doubles Within Sets of Buttons or Discs - Through 10 or 11, or More
   - Doubling, Taking Away Half, Doubling and Adding One to a Set of Buttons or Discs - Through a Total of 10 or 11, or More

3. Subsets Within 10; Adding and Taking Away 1, 2, 3, 4 Within a Total of 10 Objects, or More; Commutation
   - Subsets Within 10 Beads - Two Colors, One Color
   - Adding and Taking Away 1 and 2 From Sets of Discs and Beads - Through a Total of 10, or More
   - Adding 1 and Adding to 1; Adding 2 and Adding to 2; Subtracting 1 and 2 - Using Discs, Dominoes, Pattern Cards, and Sentences
   - Adding and Taking Away 3 and 4 From Sets of Discs and Beads - Through a Total of 10, or More
   - Adding 3 and Adding to 3; Adding 4 and Adding to 4; Subtracting 3 and 4 - Using Discs, Dominoes, Pattern Cards, and Sentences

4. Counting Forward and Backward by Fives and Tens - Through a Total of 20 or More Objects
   - Counting Pennies and Beads by Fives Forward and Backward - Through 20, or More
   - Counting Pennies and Beads by Tens Forward and Backward - Through 20, or More

Grade One teachers will find it profitable to read the Preliminary Statement for the Pre-Kindergarten and Kindergarten on facing, lefthand pages 68-82. They will also consider the contents and pupil activities suggested for these earlier grade levels as well as those suggested for Grade One on the following righthand, odd-numbered pages.

(Continued on Page 65)
CONTENTS, TEACHER PREPARATION, AND PUPIL ACTIVITIES

1. Counting an Even Number of Objects; Making Observations About the Last Set of Patterned Objects - Through 4, Through 6 or More

Suggestion: Introduce in the winter. Develop with some children to the end of the school year. See Item 4.1, page XI.

Using Jingles to Name a Set and the Number in the Set - Through 3, Through 4 or More

Teacher Preparation: Composes jingles, such as those described in the Preliminary Statement for the Pre-Kindergarten and Kindergarten on page 68, paragraph 4. Other illustrations follow. You and you. Where are two? Let's have fun. Where is one?....Plans to use jingles which will direct children to find sets, to name each set of objects, and to name the number of objects in each set....Provides blocks, pans, hoops or yarn or string or laces for children to delineate each set by encircling....Plans to use terms of position, general size, and quantity....

Pre-Kindergarten Activities

(Naming set; naming numbers 1, 2 or 3; naming both set and number in set)

Child listens to jingle....
Carries out direction given in jingle: may name the set (pans), or name the number in the set (2), or name both the number and the set (2 pans)....Encircles set with yarn or hoop or string or lace....

Kindergarten Activities

(Naming set; naming numbers 1, 2, 3, 4, or more; naming both set and number)

Child listens to jingle and carries out directions....
Plays "hunting game," finding sets as indicated in jingle....Encircles each set with yarn or hoop....May play "hunting game" with another child....

Additional Activities: Carries out direction in jingle to indicate color of set, position of set, name and color of items in set (red car, blue car, yellow car)....

(Continued on Page 86)
CONTENTS, TEACHER PREPARATION, AND PUPIL ACTIVITIES

1. Counting Forward and Backward by Ones and by Twos — Through a Total of 20 Objects, or More

Suggestion: Introduce in the winter. Develop with some children to the end of the school year. See Item 4.1, Grade One, page XI.

Counting Wheels or Buttons and Discs Forward and Backward by Ones — Through a Total of 10

Teacher Preparation: Plans to observe a group of children preparing for a construction activity...Plans to focus attention on forward counting of 6 or 7 or 5 or 8, etc., wheels or buttons or other small objects, using the procedure illustrated on page 70...Plans to emphasize backward counting, using the procedure illustrated on page 71, beginning with 6, 8, 5, 9, 10 objects...Plans also for each child in a group to use discs for counting forward and backward...Plans for children to have some experience in repeating and writing numerals forward and backward, but not as a substitute for the counting of objects...Plans to focus attention on terms of position, general size, shape, and quantity...

Grade One Activities

(Thinking out the number in sets of construction objects, later discs — 6, 7, 9, 10, etc.; counting by ones forward; studying last set: subsets, conservation of number in set, even number, odd number; counting backward; learning number names forward and backward)

Child thinks out or estimates the number in his set of 6 small construction objects...Counts construction objects forward — moves each to the left, patterns successive sets...Studies the last set of patterned objects, noting rectangular shape of the pattern, the 2 threes, the 3 twos, the even pattern...Uses construction paper 3" by 2", and places the 6 objects on it to indicate rectangular shape...Learns that 6 is an even number...

Thinks out the number in his set of 7 construction objects...Counts forward...Studies last set of patterned objects, noting the 6 and 1 more, the twos and one more...Uses construction paper 3" by 2" and places 6 of the 7 objects on it...Observes rectangular shape...Places odd object...Learns that 7 is an odd number...

Proceeds similarly with sets of 5, 9, 10, etc....

(Continued on Page 87)
Teacher Preparation: Plans to observe children in house play area....Plans to call attention to sets of 4 objects being used by children - 4 toy cups or 4 saucers or 4 bowls or 4 milk containers, etc....Plans clearly to delineate the set (using yarn or a lace, etc.) or to have a child delineate the set.... Plans for children to think out the number of cups or saucers, etc., if they can....Plans for children to observe counting as the teacher uses the procedure illustrated on page 70 (First Move through Fourth Move)....Plans for Kindergarten children to count the 4 objects if they can or at least 2 of them....Plans for children to study the last patterned set, as for 4 cups on a table....

Plans to use terms of position, general size, and quantity....

**Pre-Kindergarten Activities**

(Thinking out number in sets of 4 house play objects, observing teacher count, repeating number names for successive sets, observing pattern of last set of objects)

Child observes set of 4, may be able to think out and name number or hold up 4 fingers....

Observes teacher count numerous sets of 4 objects....Later, may be able to repeat successive number names himself after teacher has patterned successive sets of objects....

**Kindergarten Activities**

(Thinking out number in set of 4 house play objects, observing teacher count 4 objects, counting 4 or 2 objects, studying pattern of last set of objects)

Child thinks out and names the number in the delineated set of 4....

Observes teacher count the 4 objects, repeating number names as these are used by teacher....

Counts a set of 4 himself, or at least 2 of the 4 objects....Observes other children count....

(Continued on Page 88)
May be ready to count construction objects backward. (See p. 71.)
First counts his set of 5, or 3, or 4, or 7, or 8, or 9, or 10 objects forward and studies the last set of patterned objects.
Then counts construction objects backward. Observes he has no objects left when backward counting is completed.

Observes set of 9 discs. Thinks out whether 9 is an even or odd number.
Counts his set of 9 discs forward. Studies last set of 9 patterned discs, noting the odd one, the rectangular pattern of 8 and 1 more, the 4 and 4 and 1. Realizes that 9 is an odd number.

Counts his set of 9 discs backward. Observes he has no objects left when he has completed counting backward. Demonstrates to other children counting forward and backward with magnetized discs.

Counts his set of 5, of 6, of 7, of 10 discs forward and backward and proceeds as with his set of 9 discs (preceding paragraph).

Plays game involving saying number names forward. Dramatizes countdown of rocket launching (cardboard rocket). Repeats number names backward from 6, or 7 or 10, etc.

Writes number names forward in a column from 1 through 7, or 8, etc. In a second column writes number names backward, beginning with 7, or 8, etc. Reads each column of numerals downward, then upward.

Additional Activities: Thinks out the number, counts forward and backward, and studies last set of milk containers, cups, saucers, mats, science material.

Counting Wheels, Buttons, or Discs, Forward and Backward by Twos - Through a Total of 10

Teacher Preparation: Plans for each child in a group to use construction materials and discs to count an even number of objects by twos. Plans also for children to demonstrate counting by twos using magnetized discs on a steel board. Plans for children to have some experience in repeating and

(Continued on Page 89)
### Pre-Kindergarten Activities (Cont.)

- Observes last set of patterned objects...
- Hears teacher show and use the word "even"....

### Kindergarten Activities (Cont.)

- Studies last set of patterned 4 objects after they have been counted....
- Observes even arrangement of objects....

### Additional Activities: Patterns and counts (if he can) pots and pans after a "cooking" activity, party baskets....

### Counting Science Items - Through 4

**Teacher Preparation:** Plans to observe children as they handle or experiment with wheels or corks or pebbles or small pieces of wood or metal.... Plans to call attention to a set of 4 and plans for a child to delineate the set.... Plans for children to think out and name the number.... Plans for children to observe counting.... Plans for each Pre-Kindergarten child to make at least one move in counting a set and to name the number in each successive set.... Plans for each Kindergarten child to count numerous sets of 4 objects.... Plans for children to study the last set.... Plans to use terms of position, general size, and quantity....

<table>
<thead>
<tr>
<th>Pre-Kindergarten Activities</th>
<th>Kindergarten Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Thinking out number in sets of 4 science items, observing teacher count, moving and patterning at least one object in a set, naming numbers in successive sets, observing last set of patterned objects)</td>
<td>(Thinking out number in sets of 4 science items, observing teacher or children count, counting 4 objects, studying last set of patterned objects)</td>
</tr>
<tr>
<td>Child observes set of 4 science items, names these, names the number if he can....</td>
<td>Child thinks out and names the number in set of science items....</td>
</tr>
<tr>
<td></td>
<td>Observes several children (or teacher)</td>
</tr>
</tbody>
</table>

(Continued on Page 90)
Teacher Preparation (Cont.)

writing numerals forward and backward...Plans to focus attention on terms of position, general size, shape, and quantity...

Grade One Activities

(Thinking out the number in sets of wheels, buttons, other construction objects, or discs - 4, 6, 8, 10; counting forward by twos; studying the last set; counting backward by twos; repeating and writing number names by twos, beginning with an even number)

Child thinks out the number in his set of objects...Thinks out whether 4 is an even or odd number...Counts the objects forward by twos - moves each set of 2 to the left, patterns successive sets...Studies last set of 4 patterned objects, noting square shape of pattern, the 2 twos, the even pattern...Realizes that 4 is an even number...

Counts the 4 objects backward - moves away two at a time, names number remaining...Observes he has no objects left...

Thinks out the number in his set of 6 objects...Thinks out whether 6 is an even or odd number...Counts the objects by twos, moving twos and patterning successive sets...Studies the last set of patterned objects, noting the 3 twos, the even pattern, the rectangular shape of pattern...Proceeds similarly with 8 objects, later with 10 objects...

Counts each set of 6, or 8, or 10 objects backward, moving away two at a time and naming the number remaining...Observes he has no objects left...

Demonstrates counting by twos forward and backward, using sets of 4, 6, 8, and 10 magnetized discs...

Repeats number names forward by twos through 8, or 6, or 10, etc....Repeats number names backward by twos from 8, or 6, or 10, etc....

 Writes number names by ones in a column through 8, or 6, etc. Encircles all even numbers...In a second column writes number names by ones, beginning with 8, or 6, etc. Encircles all numerals for even numbers, as illustrated. Reads all encircled number names in each column downward and upward...

1 6 1 8
2 5 2 7
3 4 3 6
4 3 4 5
6 1 6 2
(Continued on Page 91)
Pre-Kindergarten Activities (Cont.)

Observes teacher count several sets....Says number names as teacher counts....Makes one of the moves himself....

Observes last set of patterned objects....May be able to use the word "even"....

Kindergarten Activities (Cont.)

count the 4 objects....

Counts sets of 4 objects himself—moving, patterning, naming number, etc.

Studies last set of patterned objects....Observes that the pattern is even....Observes the shape of paper the teacher places under the objects (square)....

Additional Activities:
Sorts, patterns, and counts science objects collected on a trip or brought to school by teacher or children—shells, chestnuts, acorns, walnuts, etc....

Counting Toy Vehicles—Through 4, Through 6

Teacher Preparation: Plans to call attention to a set of 4 toy cars being used by children; later to a set of 6 cars being used by Kindergarten children....Plans to help children count the cars....

(Continued on Page 92)
Teacher Preparation: Plans for each child in a small group to use his own discs to count forward and backward by twos, through an odd number of discs – through 5, 7, 9, 3....Plans for children to demonstrate counting by twos using magnetized discs on a steel board....Plans for children to have some experience in repeating and writing numerals forward and backward, but not as a substitute for the counting of objects....

Grade One Activities

(Thinking out number in sets of 5, 7, 9, and 3 discs; counting forward by twos; studying last patterned set; counting backward by twos; repeating and writing number names forward and backward, indicating twos beginning with an odd number)

Child thinks out the number in his set of 5 discs....Thinks out whether 5 is an even or odd number....Counts the discs by twos....Studies last set of 5 patterned discs....Realizes that 5 is an odd number....

Counts the 5 discs backward by twos....Realizes he has 1 disc left when backward counting by twos is completed....

Demonstrates counting forward and backward by twos through an odd number of magnetized discs....

Repeats number names forward by twos, beginning with an odd number....
Repeats number names backward by twos, beginning with an odd number....

Writes number names by ones in a column through 7, or 9, etc. Encircles all numerals for odd numbers....In a second column writes number names by ones, beginning with an odd number. Encircles all numerals for odd numbers, as illustrated. Reads all encircled number names in each column downward and upward.....

\[
\begin{array}{cccc}
1 & 7 & 5 & 9 \\
2 & 6 & 6 & 8 \\
3 & 5 & 7 & 7 \\
4 & 4 & 8 & 6 \\
5 & 3 & 9 & 4 \\
6 & 2 & \text{etc.} & \text{etc.} \\
7 & \text{1} & \text{etc.} & \text{etc.} \\
\end{array}
\]

Writes numerals for all odd numbers forward and backward in two columns...

\[
\begin{array}{cccc}
1 & 9 & \text{1} & \text{7} \\
3 & 7 & \text{3} & \text{5} \\
5 & 5 & 5 & 3 \\
\text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} \\
\end{array}
\]

Additional Activities: Counts forward and backward, by twos, pairs of mittens, boots, etc....

(Continued on Page 93)
Cut sheets of newsprint paper for marking parking lots - 8" by 8" for Pre-Kindergarten, also 12" by 8" and 16" by 8" for Kindergarten. Outlines 4-inch squares to indicate parking spaces....Plays games for children to dramatize the parking of the toy cars....Plans to use terms of position, general size, shape, and quantity....

Pre-Kindergarten Activities

(Thinking out and naming number in sets of 4 cars making at least 2 of the 4 moves independently as he counts 4 objects, observing shape of parking lot and parking spaces)

Child observes set of 4 toy cars, names these (cars), and names the number ("four").....

Kindergarten Activities

(Thinking out number in sets of 4 cars and 6 cars, counting 4 cars and 6 cars, observing shapes of parking lots and parking spaces, observing number of spaces filled and not filled)

Child names the number in a set of 4 toy cars....Counts the cars....Proceeds similarly with 6 toy cars....

(Continued on Page 94)
Counting Discs and Pennies by Ones and by Twos Through 20 or More; Exchanging Pennies for Other Coins; Counting Backward

Teacher Preparation: Plans for activities in which children pay for cookies, milk, etc. Plans for each child to count discs and pennies by ones and by twos and to exchange pennies for other coins. Plans games. Prepares pattern cards with sets of one-inch gummed discs, all one color, and numeral cards, as illustrated.

Plans first to emphasize the numbers 10, 11, and 12. Plans then to emphasize the numbers 13, 14, 15, etc. - one at a time. Plans for children to count pennies by ones and by twos through 10, 11, 12, 13, 14, 15, etc., and to exchange pennies for dimes.

Grade One Activities

(Buying milk or cookies, etc.; counting discs and pennies by ones and by twos through 10, 9, 11, 12, etc.; studying last set; exchanging pennies for other coins and vice versa.)

Child selects as many discs as he has fingers on both hands, or toes on both feet. Counts the 10 discs - patterns those as he counts, studies the last set. Counts discs backward. Proceeds similarly with 20 discs.

Brings in money for a contribution, to pay for a week's milk, to make a purchase, or just for fun. Records the number of pennies. Exchanges a dime for 10 pennies. Counts the 10 pennies forward and backward by ones and by twos. Reads and writes the symbols 10 cents, and 10¢. Counts and patterns 11 discs or pennies forward and backward...Reads and writes the symbols.

Uses 10 and 12 pennies and discs as with 10 and 11 described in the preceding two paragraphs.

Uses 10 and 13 pennies and discs as with 10 and 11.

Proceeds similarly with 10 and 14, 10 and 15, etc.

(Continued on Page 95)
Pre-Kindergarten Activities (Cont.)

Counts the 4 cars with the teacher's help, making at least 2 of the moves independently.

Observes shape of 8" by 8" parking lot and parking spaces (square). Indicates the number of parking spaces (No counting).

Selects as many cars as he needs to fill the parking spaces. Places one car in each parking space. Tells the number of cars he has parked (No counting).

Selects 2 cars. Counts these as he places them into parking lot. Observes number of parking spaces occupied. Observes number of parking spaces unoccupied.

Additional Activities: Counts other toy vehicles, such as trucks, airplanes, boats, etc. Parks trucks in a "garage", docks boats at a "pier", places planes in a "hangar".

Kindergarten Activities (Cont.)

Observes 8" by 8" parking lot. Names the number of parking spaces (No counting). Counts 4 cars as he places these in parking places. Observes that he cannot place 6 cars in this parking lot. Observes shape of parking lot and parking spaces (in the shape of a square).

Observes 12" by 8" parking lot. Names the number of parking spaces (No counting). Counts 4 cars as he places these in parking spaces. Observes the number of parking spaces occupied and unoccupied. Proceeds similarly with 6 cars - counting cars as he places these in parking spaces, later observing number of occupied and unoccupied spaces. Observes shape of parking lot (rectangular) and of parking spaces (square).

Uses 16" by 8" parking lot - observing number of spaces. Counts 4 cars as he places these in parking spaces, later observing number of spaces occupied and unoccupied. Counts 6 cars as he places these in parking spaces, later observing number of spaces occupied and unoccupied. Observes shape of parking lot and of parking spaces.

(Continued on Page 96)
Thinks out the number of discs or pennies the teacher tosses on a table - 5, 10, 6, 9, 8....Counts and patterns each set of discs and studies last set.... Counts the discs backward....

Thinks out the number of discs or pennies in a mystery box or can....Thinks out the number in a cloth or paper mystery bag....Lifts and shakes box or can or bag, feels discs in bag....Writes numeral for number....Counts and patterns each set of discs and studies last set....Checks number counted with number thought out before counting....Counts the discs backward....

mystery can mystery box mystery bag

Thinks out the number in a riddle....Indicates the number by patterning discs....

<table>
<thead>
<tr>
<th>I know a number.</th>
<th>I know a number.</th>
</tr>
</thead>
<tbody>
<tr>
<td>It comes after 10.</td>
<td>It comes before 10.</td>
</tr>
<tr>
<td>It is an even number.</td>
<td>It is an odd number.</td>
</tr>
<tr>
<td>It is smaller than 14.</td>
<td>It is larger than 2.</td>
</tr>
<tr>
<td>What is it? (12)</td>
<td>What is it? (3, 5, 7, or 9)</td>
</tr>
</tbody>
</table>

Places a set of pattern cards in order - from 0 through 10, from 10 through 15, from 4 through 9, from 9 through 4, by twos through 8 or 10 or 16, backward by twos from 8 or 10 or 16....Matches numeral cards with pattern cards....

Observes a set of pattern cards placed in order by the teacher....Observes whether one is missing....Observes a set of pattern or numeral cards placed out of order by the teacher....Places cards in order forward and backward....

Observes a set of boys holding pattern cards and notes whether the cards are in order or one is missing - forward or backward....Observes a set of girls holding numeral cards and notes whether the cards are in order or if one is missing....Observes whether each girl goes to stand beside the boy with the matched pattern card....

(Continued on Page 97)
Counting Construction Material — Through 4, Through 6 or More

Teacher Preparation: As preparation for construction activities places an assortment, by color, of one-inch gummed paper discs on a mat....Plans for children to sort discs by color....Plans for one child at a time to select 4 discs of one color and to count these....Plans for children to select 4 discs of two colors, to count these, and to note subsets in last set of patterned objects....Plans for Kindergarten children to select 6 discs — one color, later of 2 colors....Plans to use terms of position, shape, and quantity....

Pre-Kindergarten Activities

(Sorting discs by color; selecting 4 discs — one color, two colors; counting discs; observing pattern of last set of discs)

Child sorts discs by color....Selects 4 discs of one color....Counts discs....Observes last set of patterned objects — notes set is "even", notes each disc is in the shape of a circle....

Selects 4 discs of two colors (various combinations)....Counts....Observes last set of patterned discs....Observes number in subsets by color....Conserves number in set...

Kindergarten Activities

(Sorting discs; selecting 4 and 6 discs of one color, of 2 colors; counting; studying pattern of last set of discs)

Child sorts discs by color....Selects 4 discs of one color....Counts these....Observes pattern of last set of objects....Notes pattern is even and that the paper placed under the set is in the shape of a square....

Selects 4 discs of two colors...Observes number in subsets...Conserves number in set...Counts...Observes that pattern of last set of objects is even and in shape of a square.

(Continued on Page 98)
Additional Activities: Uses checkers and paper discs as with plastic discs and pennies, preceding....

Teacher Preparation: Prepares worksheets to be duplicated....Plans to emphasize recognition of the number in a set of patterned objects through a total of 20 or more, thinking out the number in sets of unpatterned objects, counting, even numbers, odd numbers, the next number, the number after, the number (s) between, the number after the next number, the number before the preceding number....Plans directions appropriate for children using the worksheets.... Plans to read directions to children....

Parts of some worksheets are shown here for illustrative purposes only.

1. Numbers in Sets of Patterned Objects

<table>
<thead>
<tr>
<th>How many?</th>
<th>How many in the set?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Patterned Object" /></td>
<td><img src="image2" alt="Patterned Object" /></td>
</tr>
<tr>
<td><img src="image3" alt="Patterned Object" /></td>
<td><img src="image4" alt="Patterned Object" /></td>
</tr>
<tr>
<td><img src="image5" alt="Patterned Object" /></td>
<td><img src="image6" alt="Patterned Object" /></td>
</tr>
<tr>
<td><img src="image7" alt="Patterned Object" /></td>
<td><img src="image8" alt="Patterned Object" /></td>
</tr>
<tr>
<td>Color 4 buttons in every set:</td>
<td>Color 10 buttons in every set:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Draw discs.</th>
<th>Draw more discs if needed.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image9" alt="Drawn Discs" /></td>
<td><img src="image10" alt="Drawn Discs" /></td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Color 10 discs in every set.</td>
<td>Color 10 discs in every set.</td>
</tr>
</tbody>
</table>

(Continued on Page 99)
Pre-Kindergarten Activities (Cont.)

Selects 6 discs of one color, later of 2 colors. . . . Counts. . . . Studies last set of patterned objects — even, in shape of rectangle.

Kindergarten Activities (Cont.)

Counts an Odd Number of Objects; Making Observations About the Last Set of Patterned Objects — Through 5, or More

See Item 4.2, page XI.

Counting Science Items, Buttons, Cubic Blocks, Etc.

Teacher Preparation: Plans to focus attention on a set of 3 or 5 small objects being used by a child or small group of children — the number in the set, counting the set, observing the pattern of the last set. Plans to use language such as, extra one, odd one, 2 and 1 more, 4 and 1 more, one more than 2 or 4, one less than 4 or 6, 1 and 1 and 1 more, 2 and 2 and 1 more, a double and one more, etc.

Pre-Kindergarten Activities

(Thinking out the number in various sets of 3 and 5 small objects in each set; observing pattern of the set of 3 or 5 objects after counting)

Child indicates the number in sets of 3 corks, pebbles, buttons, blocks, etc. . . . Counts each set or observes another child as he counts. . . . Observes the pattern of each set of objects after counting.

Kindergarten Activities

(Indicating the number in sets of 3 and 5 objects, counting, observing patterns of sets of 3 and 5 objects after counting)

Child indicates the number in sets of 3 small objects, counts, observes pattern of objects after counting.

Additional Activities: Sorts and counts black and white buttons, or buttons with 2 holes and with 4 holes. . . . Sorts and counts other small objects for collages — pebbles, shells, etc.

2. Counting an Odd Number of Objects; Making Observations About the Last Set of Patterned Objects — Through 5, or More

See Item 4.2, page XI.

Counting Science Items, Buttons, Cubic Blocks, Etc.

Teacher Preparation: Plans to focus attention on a set of 3 or 5 small objects being used by a child or small group of children — the number in the set, counting the set, observing the pattern of the last set. Plans to use language such as, extra one, odd one, 2 and 1 more, 4 and 1 more, one more than 2 or 4, one less than 4 or 6, 1 and 1 and 1 more, 2 and 2 and 1 more, a double and one more, etc.

Pre-Kindergarten Activities

(Thinking out the number in various sets of 3 and 5 small objects in each set; observing pattern of the set of 3 or 5 objects after counting)

Child indicates the number in sets of 3 corks, pebbles, buttons, blocks, etc. . . . Counts each set or observes another child as he counts. . . . Observes the pattern of each set of objects after counting.

(Continued on Page 100)
2. Numbers in Sets of Unpatterned Objects

- Draw a line around 4.
  - How many?
  - Color sets of 4.
- Draw lines around doubles.
  - How many?
  - Use 2 colors to show doubles.
- Draw lines around all sets of 5.
  - How many?
  - Color sets of 5.

3. Even and Odd Numbers

- How many?
  - Color sets of even numbers red.
- How many?
  - Color all sets of 10.

4. The Next Number (or One More)

- How many?
  - Draw a set with one or more.
  - Color all sets of 10.
(Continued on Page 101)
Child thinks out the number in sets of 5 small objects....Indicates the number....Counts each set or observes counting....Observes pattern of each set of objects after counting - not even, an extra one....

Child indicates the number in sets of 5 small objects....Counts each set....Observes pattern of each set of objects after counting....

Uses terms: extra one, odd one, 2 and 1 more, 4 and 1 more....

Observes the set of 4 within sets of 5 patterned objects....

Observes the set of 2 within sets of 3 patterned objects....Observes the set of 4 within sets of 5 patterned objects....

(Continued on Page 102)
5. **The Number Before (or One Less)**

<table>
<thead>
<tr>
<th>How many?</th>
<th>Draw a set with one less.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Set 1]</td>
<td></td>
</tr>
<tr>
<td>![Set 2]</td>
<td></td>
</tr>
<tr>
<td>![Set 3]</td>
<td></td>
</tr>
</tbody>
</table>

Color all sets of 10

6. **The Number Between**

<table>
<thead>
<tr>
<th>Fill in spaces.</th>
<th>Fill in spaces.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Set 1]</td>
<td>![Set 2]</td>
</tr>
<tr>
<td>![Set 3]</td>
<td>![Set 4]</td>
</tr>
<tr>
<td>![Set 5]</td>
<td>![Set 6]</td>
</tr>
</tbody>
</table>

7. **Missing Numerals**

<table>
<thead>
<tr>
<th>Fill in spaces.</th>
<th>Fill in spaces.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Set 1]</td>
<td>![Set 2]</td>
</tr>
<tr>
<td>![Set 3]</td>
<td>![Set 4]</td>
</tr>
<tr>
<td>![Set 5]</td>
<td>![Set 6]</td>
</tr>
</tbody>
</table>

(Continued on Page 103)
Counting Pennies, Exchanging for a Nickel

Teacher Preparation: Plans a class trip to the Post Office to purchase a 5¢ stamp for a birthday or Valentine card to a sick child...Plans before the trip for every child to count the 5 pennies, to observe the pattern after counting, and to exchange the 5 pennies for a nickel....

Pre-Kindergarten Activities

(Observing set of 5 pennies thinking out the number; counting these, observing last set of patterned objects; exchanging for a nickel; observing purchase of stamp and posting of card)

Child recognizes the 5 pennies as coins to use for making a small purchase...Thinks about the number...Counts...Observes the pattern of the set after counting....

Recognizes a nickel...Exchanges for the 5 pennies....

Observes purchase of a 5¢ stamp with the nickel...Observes stamp being placed on card...

Kindergarten Activities

(Naming number in set of 5 pennies; counting; observing final pattern; exchanging for 1 nickel, anticipating purchase of stamp and posting of card)

Child appreciates value of one penny, 5 pennies, 1 nickel, etc....Names number in set of 5 pennies...Counts...Observes last set....

Selects a nickel from among other coins....Exchanges for the 5 pennies....

Observes purchase of stamp and placement on card...

(Continued on Page 104)
Grade One Activities

Child examines worksheet....Thinks out what to do....Reads directions or listens to teacher read these....Follows directions....Evaluates....

Numerals on Clock, Calendar, Pages in a Book

Teacher Preparation: Locates a large clock (real)....Prepares or mounts a large class calendar for the month....Selects books with clearly marked numerals for page numbers....Plans to emphasize numerals in order - forward and backward - on the clock, calendar, pages in a book....Plans for children to observe whether the shorter hand on clock is before or after 9 or 10 or 11 (o'clock)....Plans for children to observe numeral for the day of the month, for yesterday, for tomorrow....Plans for children to observe even- and odd-numbered pages (lefthand and righthand pages)....Plans for children to prepare bookmarks....Plans to prepare worksheets....Plans to use terms of position, general size, shape, and time....

Grade One Activities

(Learning sequence of numerals on the clock, calendar for the month, pages in a book; relating shorter hand on clock and numerals to time before or after the hour; relating numerals on calendar to day of the month; finding specified pages in a book)

Child observes and reads numerals on a real clock - forward beginning with 1 or other numeral, backward beginning with 12 or other numeral....Observes numeral(s) indicated by shorter hand....Uses terms: before, after, between, shorter, longer....

Observe and reads numerals on calendar for the month - forward for a week, backward for a week....Observe numerals for the day of the month, yesterday, tomorrow, next Sunday, last Sunday....Uses terms: today, tomorrow, yesterday....

Observe numerals for page numbers in a book....Notes whether page 1 is indicated....Finds pages 2, 3, 4, etc....May discover that numerals for even numbers are always on lefthand pages, and that numerals for odd numbers are always on righthand pages....Places his bookmark to indicate the last page read....

Examines worksheet....Thinks out what to do....Reads directions or listens to teacher read these....Follows directions....Evaluates....

(Continued on Page 105)
3. **Perceiving Doubles Within an Even Number of Objects**
   - Within 4, Within 6 or More

See Item 4.3, page XI.

**Sets of Wheels on a Four-Wheeled Vehicle**

**Teacher Preparation:** Plans to observe children as they play with small toy vehicles....Plans to call attention to four-wheeled vehicles....Provides materials for constructing vehicles....Plans for children to count out 4 wheels - moving, patterning, observing last set....Plans to use terms of position, general size, shape, and quantity....

**Pre-Kindergarten Activities**

(Observing wheels on small 4-wheeled vehicles, perceiving number in set and subsets)

Child observes small toy vehicles in the classroom....Sorts these by number of wheels....Observes wheels on 4-wheeled vehicles - number in the set, position and number in the subset....Hears teacher say and repeats: 2 and 2, front and back, this side and that side....

**Kindergarten Activities**

(Sorting out small 4-wheeled vehicles, studying number in sets and subsets of wheels)

Child sorts small toy vehicles by number of wheel....Studies wheels on several vehicles....Names number in the set and in subsets front and back, one side and the other side....Uses terms: 2 in front and 2 in back, 2 on this side and 2 on that side, 2 and 2 in 4, 2 twos in 4....

Selects and counts wheels he needs for constructing a vehicle....Observes the wheels after constructing the vehicle....

Uses wheels in play or construction....

**Additional Activities:** Observes sets and subsets in larger vehicles with 4 or more wheels - doll carriage, cars and trucks on the street....

(Continued on Page 106)
Grade One Activities (Cont.)

What is the time?

Fill in spaces.

Fill in spaces.

Fill in spaces.

Fill in spaces.

2. Doubles and Near-Doubles - Through a Total of 10 or 11, or More

See Item 4.2, Grade One, page XI.

Perceiving Doubles and Near-Doubles Within Sets of Buttons or Discs - Through 10 or 11, or More

Teacher Preparation: Plans to observe a group of children preparing for a construction activity....Plans also for each child in a group to use discs ....Plans to focus attention on doubles within a set of patterned and unpatterned objects - a set of 2, 4, 6, 8, 10, or more....Plans to use terms of position, general size, shape, and quantity....

(Continued on Page 107)
Pre-Kindergarten and Kindergarten: Topic 4

Sets of Toy Milk Bottles in a Rack or Eggs in a Box for 6

Teacher Preparation: Plans to observe children as they use a toy wooden milk bottle rack with wooden milk bottles.... Plans for dramatizations involving milkman and delivering milk.... Plans to call attention to sets of 4 for Pre-Kindergarten, 4 and 6 for Kindergarten and doubles (as subsets) within 4 and 6.... Provides a lace or length of yarn for children to delineate each set by encircling.... Plans to use terms of position, shape, and quantity....

Pre-Kindergarten Activities

(Observing set of 4 milk bottles in a set of 6, observing subsets of 2 and 2)

Child indicates 4 milk bottles, using one hand to indicate all 4.... Uses two hands to indicate 4 bottles (2 and 2)....

Encircles 4 bottles using a lace or yarn....

Takes out of rack 4 bottles, 2 at a time.... Observes how many empty spaces are in the rack (no counting).... Compares number of empty spaces with number of bottles out of rack.... Observes how many bottles are still in the rack....

Kindergarten Activities

(Observing set of 4 milk bottles and a set of 6; observing subsets of 2 and 2 in 4, and 3 and 3 in 6)

Child indicates 4 milk bottles, using one hand; then uses 2 hands to indicate subsets of 2 and 2.... Removes 4 bottles, 2 at a time.... Compares set of 4 spaces with set of 4 bottles.... Observes set of 4 and set of 2 still in rack.... Patterns 4 bottles indicating subsets of 2 and 2....

Replaces the bottles.... Observes 6 bottles.... Indicates subsets of 3 and 3 in the set of 6....

(Continued on Page 108)
Grade One Activities

(Perceiving and naming doubles within sets of patterned and unpatterned construction objects or discs - even numbers through 10 or more)

Child patterns 4 buttons or discs....Thinks out the doubles: 2 and 2 in 4; 2 twos in 4....Indicates doubles using a lace....Names doubles, conserving number in set...

Uses doubles in a set of 4 unpatterned objects (arranged on a table or tossed by the teacher)....Indicates doubles using a lace....Proceeds similarly with 2 objects....

Patterns 6 objects and names doubles: 3 and 3 in 6; 2 threes in 6....Indicates doubles using a lace....Names doubles in a set of 6 unpatterned objects.... Indicates doubles using a lace.... Names doubles, conserving number in set...

Proceeds with 8 objects as with 6....Proceeds with 10 or more objects as with 6....

4 and 4 in 8
2 fours in 8

5 and 5 in 10
2 fives in 10

6 and 6 in 12
2 sixes in 12

Uses pattern cards with patterned gummed paper discs....Identifies doubles ....Names doubles and the number in the set....

Arranges pattern cards with doubles in order forward (2, 4, 6, etc.) or backward (6, 4, 2)....Names the number on each card....Discovers he is counting by twos forward or backward....

Teacher Preparation: Plans to relate near-doubles to doubles within sets of patterned and unpatterned construction objects or discs....

1 and 1 in 2. 1 and 2 in 3; 2 and 1 in 3.
2 and 2 in 4. 2 and 3 in 5; 3 and 2 in 5.
3 and 3 in 6. 3 and 4 in 7; 4 and 3 in 7.
4 and 4 in 8. 4 and 5 in 9; 5 and 4 in 9.
5 and 5 in 10. 5 and 6 in 11; 6 and 5 in 11.

Plans to use terms of position, general size, shape, and quantity....

(Continued on Page 109)
Pre-Kindergarten Activities (Cont.)
Counts any patterns the 4 bottles he
has taken out.... Observes subsets of
2 and 2.... Uses lace or yarn to dif-
ferentiate subsets....

Plays role of milkman delivering 1
or 2 or 3 or 4 bottles of milk from
the rack of 6.

Kindergarten Activities (Cont.)
May be able to count the 6 bottles - re-
moving, patterning, observing last set....

Plays role of milkman delivering milk....
Observes number of bottles removed and
the number left....

Additional Activities: Observes sets, and subsets as doubles, in a box of 4
or 6 "eggs" (empty shells)....

Set of 2, 4, 6, or More Small Objects - Science Materials, Collage Materials,
Cubic Blocks, Etc.

Teacher Preparation: Plans to call attention to a set of objects a child has
used or is using, e.g.: A child has been using a set of science objects and
has replaced the set on the science table. A child has completed a collage.
A child has arranged his cubic blocks in sets of 4, etc.... Plans to emphasize
the doubles in sets of 4 patterned and unpatterned objects in the Pre-Kinder-
garten; of 2, 4, 6, or more objects in the Kindergarten.... Plans to use terms
of position, general size, shape, and quantity....

(Continued on Page 110)
Grade One Activities

(Comparing a double with its next larger near-double: 4 with 5, 6 with 7, etc.; perceiving and naming doubles and near-doubles - within sets of patterned and unpatterned objects through 10 or 11 or more, conserving number in set)

Child patterns a set of 4 buttons or discs (a double)....Patterns another set containing the next higher number (5)....Observes the even-numbered set (the double) and the odd-numbered set (the near-double)....Proceeds similarly with sets of 2 and 3, 6 and 7, 8 and 9, 10 and 11....

Patterns a set of 4 buttons or discs and a set of 5....Thinks out the doubles in 4 and the near-doubles in 5: 2 and 2 in 4, 3 and 2 in 5....Rearranges the odd one in 5 and names near-doubles: 2 and 3 in 5....

Child patterns a set of 4 buttons or discs and a set of 5....Patterns another set containing the next higher number (5)....Observes the even-numbered set (the double) and the odd-numbered set (the near-double)....Proceeds similarly with sets of 2 and 3, 6 and 7, 8 and 9, 10 and 11....

Patterns a set of 4 buttons or discs and a set of 5....Thinks out the doubles in 4 and the near-doubles in 5: 2 and 2 in 4, 3 and 2 in 5....Rearranges the odd one in 5 and names near-doubles: 2 and 3 in 5....

2 and 2 in 4. 3 and 2 in 5 2 and 3 in 5

Names and indicates doubles in a set of 4 unpatterned objects....Names and indicates near-doubles in a set of 5 unpatterned objects....

2 and 2 in 4. 3 and 2 in 5 2 and 3 in 5

Proceeds with a set of 6 and a set of 7 buttons or discs as with sets of 4 and 5....Proceeds with sets of 8 and 9 as with sets of 4 and 5....Proceeds with sets of 10 and 11 as with sets of 4 and 5....

Uses pattern cards....Selects all cards with a double and all cards with a near-double....Matches card showing a double with card showing the next higher near-double, e.g.: 6 with 7....Names doubles and near-doubles, e.g.: 3 and 3 in 6, 4 and 3 in 7, 3 and 4 in 7....

Teacher Preparation: Prepares worksheets to emphasize doubles and near-doubles....

<table>
<thead>
<tr>
<th>How many?</th>
<th>Draw a set with one more</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Doubles" /></td>
<td><img src="image2" alt="Near-doubles" /></td>
</tr>
</tbody>
</table>

(Continued on Page 111)
**Pre-Kindergarten Activities**

(Observing 2 and 2 in sets of 4 science objects, collage materials, cubic blocks - sets of patterned and unpatterned objects)

Child observes set of 4 patterned objects, names the number, and sees the subsets 2 and 2...
Selects a set of 4 objects, patterns these and observes the 2 and 2...

Observes a set of 4 unpatterned objects, names the number, and sees the subsets of 2 and 2...

**Kindergarten Activities**

(Observing doubles in sets of 2, 4, 6, or more small objects - sets of patterned and unpatterned objects)

Child names the number in a patterned set of 4 objects, names the number in the subsets of 2 and 2...Names the number in a set of 4 unpatterned objects, observes and names the number in the subsets of 2 and 2...Takes away 2 and notes he has 2 left...

Names the number in a set of 2 objects and the subsets of 1 and 1...

Names the number in a set of 6 patterned objects and the subsets of 3 and 3...Takes away 3 and notes he has 3 left...May be able to name the number in a set of 6 unpatterned objects and the subsets of 3 and 3...

---

**Sets of Doubles on Dominoes**

**Teacher Preparation:** Plans to observe a child work or play with a set of dominoes...Plans to call child's attention to sets of spots on each half of a domino and to name the number if he can...Plans to have child "find a double" and to name the number on each half if he can, and on both halves together if he can...

(Continued on Page 112)
Grade One: Topic 4

Teacher Preparation (Cont.)

Grade One Activities

Child thinks out what to do....Writes numerals, draws lines to show subsets, draws circles, etc., where needed....Evaluates....

Additional Activities: Selects and matches dominoes showing doubles and near-doubles....

Doubling, Taking Away Half, Doubling and Adding One to a Set of Discs - Through a Total of 10 or 11, or More

Teacher Preparation: Plans to focus the attention of a group of children on 1) perceiving the number of discs arranged in a row or unpatterned (e.g., 4) 2) doubling the number (4 and 4 are 8), 3) then taking away one half of the number (8, take away 4, are 4), 4) then doubling and adding one to the number (4 and 4 and 1 are 9)....Plans to use terms of position and quantity....

Grade One Activities

(Perceiving the number of discs in a row or unpatterned, through

(Continued on Page 113)
Pre-Kindergarten Activities

(Observing spots on dominoes, naming number on each half if he can, selecting doubles, naming number in each half and on both halves together if he can)

Child observes spots on dominoes....Names number on as many halves as he can....Names number on both halves together if he can....Selects doubles.

Names number on each half and both halves together if he can....Lears that 1 and 1 are 2....Lears that 2 and 2 are 4....

Kindergarten Activities

(Observing spots on dominoes and naming number on each half and both halves together if he can, selecting doubles, naming number on each half and on both halves together if he can)

Child names number on as many domino halves as he can....Names number on both halves together if he can....Selects doubles and names number on both halves together if he can....Lears that zero and zero are zero, 1 and 1 are 2, 2 and 2 are 4, 3 and 3 are 6....

Additional Activities: Models clay domino blocks, making spots for doubles with a dowel....
Grade One Activities (Cont.)

5 or more; thinking out the number if the set is doubled; thinking
out the number if one half of the discs are taken away; thinking
out the number if the set is doubled and one more is added)

Child names the number in a set of 4 discs arranged in a row....Thinks out
the number if the set of 4 is doubled....Doubles the set....Names doubles:
4 and 4 are 8....Proceeds similarly with a set of unpatterned discs....

Step 1. or 4

Step 2. "If 4 discs are added, there will be 8."

Step 3. or 4 and 4 are 8

Observes the 8 discs....Thinks out the number in one half of the set of 8
disks....Uses lace to show one half....Thinks out how many there will be
left if one half are taken away....Names numbers as he takes away the 4 discs:
8, take away 4, are 4....

Step 1. or 8

Step 2. "There are 4 discs in one half" (of the 8 discs).

Step 3. or One half of 8 is 4

Step 4. "If 4 are taken away, there will be 4 left."

Step 5. or 8, take away 4, are 4

Names the number in a set of 5 discs arranged in a row and unpatterned....
Thinks out the number if the set is doubled, then halved, then doubled and
one added....Proceeds as with a set of 4 discs, in the three preceding
paragraphs....Proceeds similarly with a beginning set of 3 or 2 or 6 discs
....

(Continued on Page 115)
4. Discovering Subsets, Additions, and Subtractions Within a Specified Number - Within 2, 3, and 4 (or more) Objects

See Item 4.4, page XI.

Sets of House-Play Objects or Toy Vehicles - Through 2, Through 3

Teacher Preparation: Plans to observe children as they use house-play objects - sets of toy cups, saucers, bowls, toy milk bottles, milk containers.... Plans also to observe children as they play with toy vehicles.... Plans for Pre-Kindergarten and Kindergarten children to focus attention on the numbers in a set and its subsets, and on additions and subtractions, within a total of 1 and 2 objects.... Plans for Kindergarten children also to focus attention on a total of 3 objects.... Plans to use terms of position, general size, quantity, set and subsets....

Pre-Kindergarten Activities

(Naming number in sets of house-play objects or vehicles - none, one, two; observing subsets and naming number; adding and taking away within 1 and 2 objects)

Child names the number of one and two and no house-play objects.... Names the number in sets of vehicles....

Points out subsets of 1 and 1 in various sets of 2 house-play objects and of 2 vehicles.... Names numbers as he points to subsets and set, as for 2 cups, one red and one green: 1 and 1, 2....

Kindergarten Activities

(Naming number in sets of zero, one, two, and three unpatterned house-play objects or vehicles; observing subsets and naming number; adding and taking away within a total of 1, 2, or 3 objects)

Child names the number in sets of 0, 1, 2, and 3 house-play objects.... Names the number in sets of vehicles, e.g., zero airplanes, 3 cars....

Points out subsets in various sets of 2 objects.... Names numbers in subsets and set - 1 and 1 in 2.... Exchanges the 2 objects and names number in subsets and set. Observes that the numbers are the same....

Points out pairs of subsets in various sets of 3 objects.... Names numbers in subsets and sets as illustrated for 1 and 2 in 3, and for 2 and 1 in 3....

(Continued on Page 116)
3. Subsets Within 10; Adding and Taking Away 1, 2, 3, 4 Within a Total of 10 Objects, or More; Commutation

See Item 4.3, Grade One, page XI.

Subsets Within 10 Beads - Two Colors, One Color

Teacher Preparation: Plans for each child in a group to string his own set of 10 beads on a lace - 5 of one color, 5 of a contrasting color....Plans for children to attach lace to a weaving board, 12 inches across and 9 inches high.

The beads on the lace are placed across the entire front of the board. The lace is then pulled through the two holes from the back to the front and pulled tightly. The lace is then tied in the back....Plans to work with a group of children as they perceive the number and move specified sets of beads - 5, 4, 6, 3, 7, 2, 8, 1, 9, 10, 0....Plans for children who can readily perceive and move any number of beads to perceive and name the number in each of two subsets....

<table>
<thead>
<tr>
<th>5 and 5 in 10</th>
<th>6 and 4 in 10</th>
<th>7 and 3 in 10</th>
<th>8 and 2 in 10</th>
<th>9 and 1 in 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 and 5 in 10</td>
<td>4 and 6 in 10</td>
<td>3 and 7 in 10</td>
<td>2 and 8 in 10</td>
<td>1 and 9 in 10</td>
</tr>
</tbody>
</table>

Grade One Activities

(Stringing 10 beads of 2 colors on a lace and attaching to weaving board; perceiving and moving numbers of beads without counting; perceiving and naming the numbers in sets of two subsets within 10)

Child strings 10 beads on a lace. - first 5 beads of one color, then 5 beads of a contrasting color....Attaches his string of beads to a weaving board....Moves his 10 beads to the right of his frame....Observes the number of each color, e.g.: 5 red and 5 green....

Moves to the left, rapidly and without counting, numbers of beads indicated by the teacher - 5, 6, 4, 3, 7, 2, 8, 1, 9, 10, 0 ("Move 5").

Studies subsets of 5 and 5 as follows: Moves 5 beads to the left....Perceives the 5 beads at the left....Perceives the 5 beads still at the right....Names number in subsets and set: 5 and 5 in (or are) 10....

(Continued on Page 117)
Pre-Kindergarten Activities (Cont.)

Exchanges the 2 caps and names numbers in the subsets and set: 1 and 1, 2....

Proceeds similarly with pairs of subsets in other sets of 2 objects....

Thinks out how many he will have if he adds 1 to his set of one object....Add the one object and observes he has 2....

Thinks out how many he will have if he adds none to his set of 1 or 2 objects....

Thinks out how many he will have if he takes away 1 from his set of 1 or 2 objects....Take away one object and observes he has 1 left....

Thinks out how many he will have if he takes away none from his set of one or two objects....

Proceeds similarly with adding and taking away from a variety of sets of objects....

Sets of Science Materials – Through 2, Through 4

Teacher Preparation: Plans to observe children as they use science materials....Plans for Pre-Kindergarten and Kindergarten children to focus attention on the number in a set and its subsets, and on additions and subtractions, within a total of 1 and 2 objects....Plans for Kindergarten children also to focus attention on a total of 3 and 4 objects....Plans to use terms of position, general size, quantity, set, and subset....
Grade One Activities (Cont.)

Thinks out how many he will have if he adds 5 beads: If I add 5 I'll have 10....Adds the 5 beads, leaving a space to show the addition....Says 5 and 5 are 10....

Moves all 10 beads to the left....Thinks out how many there will be if 5 are moved away (to the right)....Moves the 5 beads and says: 10, take away 5, are 5....

Observes teacher move 5 beads to the left....Observes that the beads at the right are covered....Think out how many beads are at the right....Names numbers in subsets and set: 5 and 5 are 10....

Observes and writes sentences, such as:

5 and 5 are 10
10, take away 5, are 5

May also observe and write sentences using "shorthand", such as:

\[5 + 5 = 10\] (read: 5 and 5 are 10)
\[10 - 5 = 5\] (read: 10, take away 5, are 5)

Studies another set of subsets, e.g., beginning by moving 6 beads and observing the 4 beads at the right....Proceeds as with 5 and 5 in 10....

Proceeds similarly with 7 and 3 are 10, 8 and 2 are 10, 9 and 1 are 10, 4 and 6 are 10, etc.

Teacher Preparation: Evaluates the ability of each child to move specified numbers of beads of 2 colors with facility and to name the number in subsets and set....Plans for children with skill in using 10 beads of 2 colors to re-string their beads using only one color....Plans to proceed as with beads of 2 colors....

Grade One Activities for Mature Children

(Stringing 10 beads of one color on a lace and attaching to weaving board; perceiving and moving numbers of beads without counting; per-

(Continued on Page 11f)
Pre-Kindergarten Activities

(Naming number in sets of science materials - none, one, two; observing and naming numbers in subsets of one, and one and in set of 2; adding and taking away within a total of 1 and 2 objects)

Child names the number in sets of one, two, and no science objects. ...Points out subsets of 1 and 1 and names numbers in subsets and set: 1 and 1, 2....

Exchanges the 2 objects and names numbers in subsets and sets....

Thinks out how many he will have if he adds one to his set of one object....Adds the one object and observes he has two....

Thinks out how many he will have if he adds none to his set of 1 or 2 objects....

Thinks out how many he will have if he takes away 1 from his set of 1 or 2 objects....Takes away one

Kindergarten Activities

(Naming number in sets of zero, one, two, three, and four unpatterned science materials; naming numbers in pairs of subsets and in the set; naming doubles in 4; adding and taking away within a total of 1, 2, 3, and 4 objects)

Child names the number in sets of 0, 1, 2, 3, and 4 science objects....

Points out or uses lace to indicate pairs of subsets in sets of 2 and 3 objects....Names numbers in set and subsets: 1 and 1 in 2, 2 and 1 in 3, 1 and 2 in 3....

Points out or uses lace to indicate pairs of subsets in various sets of 4 objects....Names numbers in subsets and sets - 2 and 2 in 4, 3 and 1 in 4, 1 and 3 in 4, 2 twos in 4....

Thinks out how many he will have if he adds 1 to his set of zero, 1, 2, or 3 objects....As he adds one object he names the numbers: 1 and 1, 2; 2 and 1, 3; zero and 1, 1; 3 and 1, 4....

Thinks out how many he will have if he takes away 1 from his set of 1, 2, 3, or 4 objects....As he takes away one object he names the numbers: 4, take away 1, 3; 3, take away 1, 2; and 1, take away 1, zero; 2, take away 1, 1....

Thinks out how many he will have if he adds 2 objects to his set of zero, 1, or 2 objects....As he adds 2 objects he names the numbers: 2 and 2, 4; 1 and 2, 3; zero and 2, 2....

Thinks out how many he will have if he takes away 2 from his set of 2, 3, or 4 objects....As he takes away 2 objects he names the numbers: 4, take away 2, 2; 2, take away 2, zero; 3, take away 2, one....

May be able to think out how many he will have if he adds 3 or 4 or takes away 3 or 4 from his set of objects....As he adds or takes away 3 or 4 he names the numbers: zero

(Continued on Page 120)
Grade One Activities for Mature Children (Cont.)

Child strings 10 beads all of one color on a lace, and attaches to weaving board....

Proceeds as with beads of two colors - identifying and moving numbers of beads, perceiving and naming subsets within 10....

Wrote sentences about subsets and a set of 10, e.g.: 7 and 3 are 10; 10, take away 3, are 7; etc.

Adding and Taking Away 1 and 2 From Sets of Discs and Beads - Through a Total of 10, or More

Teacher Preparation: Plans for each child in a group to use discs to add 1 to sets of 5, 7, 5, 8, etc....Plans for children also to take away 1 from sets of discs....Plans for children to add 1 and take away 1 from sets on beads....Plans also for children to use magnetized discs on a steel board.... Plans to demonstrate using one-inch beads on a rod at least 25 inches long ....Plans to use pattern cards....

Grade One Activities

(Adding 1 to sets of patterned and unpatterned discs - to 6, 8, 7, 5, 9, etc.; taking away 1 from sets of patterned and unpatterned discs; adding 1 and taking away 1 from sets of beads)

Child plays "Add One" game with another child (or teacher). He patterns and names a set of discs, e.g., 6. Second child says the "Add One" sentence: 6 and 1 are 7. He may demonstrate by adding the disc. If second child's sentence is correct it is his turn to pattern a set, e.g., 8 or 7 or 5, etc. First child then says this "Add One" sentence. Etc.

Step 1. Objects presented 6
Step 2. Child thinks 6 1
"6 and 1 are 7."

Uses pattern cards with gammed discs....Selects a card with 5 discs and a card with 1 disc....Thinks out the combined number and says the sentence: 5 and 1 are 6....Proceeds similarly with sets of 7 and 1, 9 and 1, etc....

Thinks out the number, without counting, in a set of unpatterned discs (tossed by the teacher)....Says the "Add One" sentence....

Proceeds with "Take Away One" game as with "Add One" game....Thinks out the number in a set of unpatterned discs and says "Take Away One" sentence.

Step 1. Objects presented 8 1
Step 2. Child thinks 8 1
"8, take away 1, are 7."

Step 1. Objects presented 7 1
Step 2. Child thinks 7 1
"7, take away 1, are 6."

(Continued on Page 121)
Pre-Kindergarten Activities (Cont.)

object and observes he has none left, or one left, as illustrated for 2, take away 1:

Pre-Kindergarten Activities

Kindergarten Activities (Cont.)

and 3, 3; zero and 4, 4; 1 and 3, 4; 3, take away 3, zero; 4, take away 4, zero; 4, take away 3, 1....

1 and 3 are 4.

Thinks out how many he will have if he takes away none from his set of 1 or 2 objects....

Set of Cubic Blocks or Construction Materials - Through 3, Through 4 or More

Teacher Preparation: Plans to observe children as they use cubic blocks or construction materials....Plans for Pre-Kindergarten and Kindergarten children to observe and name the number of objects in a set and its subsets, to compare the numbers in two sets of objects, to take away from a set and then to add the same number (application of the principle of inverse operations), within a total of 1, 2, or 3 objects....Plans for Kindergarten children also to focus attention on 4 objects or more....Plans to use terms of position, general size, quantity, set, and subsets....

(Continued on Page 122)
Uses beads to show adding 1 to 5, 4, 9, etc. Leaves a space to indicate adding....Says the "Add One" sentence: 5 and 1 are 6....

"5 and 1 are 6"

Uses beads to show taking away 1. Leaves space to show taking away....Says the "Take Away One" sentence: 6, take away 1, are 5....

Step 1.

"6"

Step 2.

"take away 1, 5 are left"

Teacher Preparation: Plans for each child in a group to use discs and beads to add 2 and to take away 2....Plans to use pattern cards....

Grade One Activities

(Adding 2 and taking away 2 from even numbers of patterned discs, from odd numbers of patterned discs, from even and odd numbers of unpatterned discs, from beads; relating adding 2 and taking away 2 to adding 1 and taking away 1)

Child engages in "Add 2" game with another child (or teacher)....Patterns an even number of discs, e.g., 6....Thinks out and says "Add 2" sentence: 6 and 2 are 8....Uses pattern cards to combine an even number of discs with 2 discs....Proceeds similarly with "Take Away 2" game involving an even number of discs....

"10, take away 2, are 8"

Thinks out the number in a set of unpatterned discs (tossed or arranged by teacher) without counting, e.g., 7....Thinks out the number and the sentences if 2 are added, e.g.: 7, 8, 9; 7 and 2 are 9. Or, directly, 7 and 2 are 9....Proceeds similarly with taking away 2 from a set of unpatterned discs....Proceeds similarly with other numbers of unpatterned discs before adding or taking away 2....

"5 and 2 are 7"  
"7, take away 2, are 5"

(Continued on Page 123)
### Pre-Kindergarten Activities

(Naming number in sets of unpatterned cubic blocks or construction materials - none, one, two, three; comparing 2 sets; naming numbers in subsets and set; adding and taking away within a total of 1, 2, and 3 objects)

Child names the number in sets of 1, 2, 3, and no blocks or construction materials....

Compares number in 2 sets of objects: 2 with 1, 3 with 1, 2 with 3, 1 with 1, 2 with 2, 3 with 3.... Names number in each set....Indicates whether number of objects is the same or different in the 2 sets....Indicates which set has more....May be able to indicate how many more....Exchanges the 2 sets and compares numbers....

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### Kindergarten Activities

(Naming number in sets of zero, one, two, three, and four or more unpatterned cubic blocks or construction materials; comparing 2 sets; naming numbers in pairs of subsets and in the set; naming doubles in 4; adding and taking away within a total of 1, 2, 3, and 4 objects)

Child names the number in sets of 0, 1, 2, 3, 4, or more blocks or construction materials....

Compares number in 2 sets of objects: 2 with 1, 3 with 2, 4 with 3, 3 with 1, 4 with 2, 4 with 1, 1 with zero, 2 with zero, 3 with zero, 4 with zero.... Names number in each set....Indicates which set has more....Indicates how many more....Exchanges the 2 sets and compares numbers....

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(Continued on Page 124)
Grade One Activities (Cont.)

Names number in a set of beads moved to the left, e.g., 7. Thinks out the number and the sentence if 2 are added. Then adds the 2. Thinks out the number and the sentence if 2 are taken away. Proceeds similarly with adding 2 and taking away 2 from other numbers.

![Diagram of beads moved to the left](image)

Adding 1 and Adding to 1; Adding 2 and Adding to 2; Subtracting 1 and 2 - Using Discs, Dominoes, Pattern Cards, and Sentences

Teacher Preparation: Plans for each child in a group to use discs, dominoes, pattern cards, and sentences for adding 1 and for adding to 1. Plans for children to add 2 and then to add to 2. Plans to relate adding 1 and 2 to subtracting 1 and 2 using discs and beads.

Grade One Activities

(Using discs, dominoes, and pattern cards to add 1, add to 1, add 2, add to 2; using discs and beads to relate adding 1 and 2 to subtracting 1 and 2)

Child uses discs to add 1 and to add to 1, as illustrated for 6 and 1 are 7, and 1 and 6 are 7.

Step 1. Adding 1:

![Discs illustration](image)

"6 and 1 are 7."

Step 2. Adding to 1:

![Discs illustration](image)

"1 and 6 are 7."

Proceeds similarly with adding 1 to other numbers, and then reversing the subsets: 5 and 1, 1 and 5; 7 and 1, 1 and 7; etc.

Selects a domino with 1 as a subset. Says sentence, e.g.: 4 and 1 are 5. Turns domino so that the one is at the left. Says sentence: 1 and 4 are 5.

![Domino illustration](image)

"4 and 1 are 5."

Selects pattern cards, one card with 1 gummed disc on it. Says sentences, e.g.: 6 and 1 are 7. Reverses cards. Says sentence: 1 and 6 are 7.

![Pattern cards illustration](image)

"6 and 1 are 7."

"1 and 6 are 7."

(Continued on Page 125)
Pre-Kindergarten Activities (Cont.)

Points out or uses lace to indicate pairs of subsets in sets of 2 and 3 objects....Names numbers in set and subsets: 1 and 1, 2; 2 and 1, 3; 1 and 2, 3....

Thinks out how many he will have if he takes away 1 from his set of 2 objects....Takes away 1 and observes he has 1 left....Adds the 1 and observes he now has 2 again....Proceeds similarly with taking away 1 from a set of 3 objects, then adding the 1....

Thinks out how many he will have if he takes away 2 from his set of 3 objects....Takes away 2 and observes he has 1 left....Adds the 2 and observes he now has 3 again....

Kindergarten Activities (Cont.)

Uses lace to indicate pairs of subsets in sets of 2, 3, and 4 objects....Names numbers in set and subsets: 1 and 1 in 2, 2 and 1 in 3, 1 and 2 in 3, 2 in 4, 2 twos in 4, 3 and 1 in 4, 1 and 3 in 4....

Thinks out how many he will have if he takes away 1 from his set of 1, or 2, or 3, or 4 objects....Takes away 1 and observes how many he has left....Adds the 1 and observes he has the original number of objects again....

Thinks out how many he will have if he takes away 2 from his set of 2, or 3, or 4 objects....Takes away 2 and observes how many he has left....Adds the 2 and observes he has the original number of objects again....

"3"

"3 and 1 are 4"

Thinks out how many he will have if he takes away 3 from a set of 3, or 4 objects....Takes away 3 and observes how many he has left....Adds the 3 and observes he has the original number of objects again....

Proceeds similarly with taking away 4 from a set of 4 objects, then adding the 4....

(Continued on Page 126)
Grade One Activities (Cont.)

Uses discs to show relationship between adding 1 and taking away 1. Begins with taking away 1 or adding 1:

1. 

2. 

(Taking away the 1 added)

8, take away 1, are 7

3. 

(Returning the 1 subtracted)

7 and 1 are 8

Etc.

Uses beads to show relationship between adding 1 and taking away 1. Begins with adding 1 or taking away 1:

1. 

8, take away 1, are 7

2. 

Returning the one subtracted

7 and 1 are 8

3. 

Taking the one away again

8, take away 1, are 7

Etc.

Proceeds similarly with adding 1 and taking away 1 from other numbers:

6 and 1 are 7; 7, take away 1, are 6
9, take away 1, are 8; 8 and 1 are 9

Etc.

Uses discs, dominoes, and pattern cards to add 2 and to add to 2. (as with adding 1 and adding to 1): 

6 and 2 are 8, 2 and 6 are 8
4 and 2 are 6, 2 and 4 are 6
8 and 2 are 10, 2 and 8 are 10

5 and 2 are 7, 2 and 5 are 7
3 and 2 are 5, 2 and 3 are 5
7 and 2 are 9, 2 and 7 are 9

Etc.

Uses discs and beads to show relationship between adding and taking away 2 (as with adding and taking away 1):

(Continued on Page 127)
Additional Activities: Plays a hand-guessing game with teacher or another child. A known number of buttons or other small objects is used, e.g., 3. Teacher separates the 3 buttons to hold in both closed hands. Child guesses how many teacher holds in each hand.

5. Thinking Out the Result of Adding One or Taking Away One - Through a Total of 5 Objects; Through a Total of 6 or More Objects

Sets of Toy Cars, Cubic Blocks, and Other Small Objects - Through a Total of 5, Through a Total of 6 or More

Teacher Preparation: Plans to call attention to toy cars being used... Constructs "garages" which will hold 2 or 3 or 4 or 5 or 6 or more cars... Plans for Pre-Kindergarten children to use 2-car, 3-car, 4-car, and 5-car garages....

Plans for Kindergarten children to use "garages" from a 1-car garage through a 6-car garage, or more... Plans to observe children as they "drive" cars into a garage or out of a garage... Plans to ask questions which encourage children to anticipate how many there will be when one is added to the number of cars in the garage or to those parked outside of the garage....

Plans to call attention to cubic blocks being placed on a card or into boxes or box covers, and to encourage children to anticipate the number if one is added or one is taken away... Plans a game involving adding a block or taking one away....

Plans to call attention to other activities involving adding one or taking one from a set of other small objects - buttons, spools, clay balls, etc... Plans to use terms of position, general size, and quantity....

Pre-Kindergarten Activities

(Naming number of toy cars, cubic blocks, or other small objects - none, one, two, three, four, or five; indicating how many there will be when one is added or one is taken from a set of cars, blocks, or other objects)

Kindergarten Activities

(Naming number in sets of small objects, spaces in "garages" or boxes; naming how many there will be when one (or 2) is added or taken away)

(Continued on Page 128)
Grade One: Topic 4

Grade One Activities (Cont.)

6 and 2 are 8; 8, take away 2, are 6
4 and 2 are 6; 6, take away 2, are 4
8 and 2 are 10; 10, take away 2, are 8

3 and 2 are 5; 5, take away 2, are 3
7 and 2 are 9; 9, take away 2, are 7
1 and 2 are 3; 3, take away 2, is 1

Teacher Preparation: Plans to present mathematical problems involving commutation in addition, e.g. How many are 6 and 2? How many are 2 and 6? ... Plans for children to demonstrate problems using discs, dominoes, and pattern cards ... Plans for children to say sentences ... Plans to prepare sentences with frames for children to complete, e.g.: 6 and 2 are □, 6 and □ are 8 ... Prepares worksheets to be duplicated, such as those illustrated.

Fill in spaces.

Grade One Activities

(Demonstrating commutation problems using discs or dominoes or pattern cards, and saying sentences; completes written sentences; follows directions on worksheets)

Child uses discs or dominoes or pattern cards to demonstrate problems and says sentences for adding 1 and adding to 1, for adding 2 and adding to 2. The latter is illustrated.

Problem | Using Discs | Sentence
--- | --- | ---
1. Show 6 and 2 How many are there? | 6 and 2 are 8. | 8
2. Now show 2 and 6. Now how many are there? | 2 and 6 are 8 | 8

Writes numerals in frames for sentences prepared by the teacher...Uses discs or dominoes or pattern cards to demonstrate each problem...

(Continued on Page 129)
Pre-Kindergarten Activities (Cont.)

Child names number of spaces for cars in 2-car, 3-car, 4-car, and 5-car garages. "Drives" cars in and out of garages....

Thinks out number if one more car is "driven" into garage.... Thinks out number if one car is "driven" out of garage.... Uses terms such as: 3 and 1 are 4; 4, take away 1, leaves 3; etc....

Thinks out how many cubic blocks can be put into boxes or on cards.... Thinks out how many there will be if one is added or taken away....

Plays "What Happened" game with teacher: Observes number in a set of blocks, e.g., 4. Closes his eyes. Teacher adds a block or takes one away. Child opens eyes and sees what happened.

Kindergarten Activities (Cont.)

Child names number of cars and number of spaces in each "garage" from a one-car garage through a 5-car garage or larger.... "Drives" cars in and out of garages, noting the number in the garage, anticipating number if one (or 2) more is driven in or out....

Thinks out how many cubic blocks can be put into boxes or box lids of various sizes.... Places blocks into box, noting number in the box, thinking out number if one more is put in.... Removes blocks, noting number in box, anticipating number if one (or 2) more is removed....

Plays "What Happened" game with another child.

Additional Activities: Anticipates how many buttons or spools or clay balls there will be if one (or two) is added or taken away....

Sets of Pennies, Exchanging for Nickel - Through a Total of 5; Through a Total of 6 or More

Teacher Preparation: Plans a class trip to a local store to make a 5¢ (or 6¢ or 4¢, etc.) purchase - decoration for a get-well card and an envelope, etc.... Makes arrangements for the trip with local storekeeper.... Before the trip plans to emphasize adding one to sets of pennies or taking one from sets of pennies through a total of the number needed for the purchase.... Plans for children to exchange 5 pennies for 1 nickel....

(Continued on Page 130)
1. Frame at end of sentence
(for sum)

- 3 and 2 are __
- 2 and 3 are __
- 5 and 2 are __
- 2 and 5 are __
- Etc.

Uses worksheets....Thinks out what to do....Follows directions....Evaluates....

---

Teacher Preparation: Plans to present problems involving related additions and subtractions, e.g. How many are 7 and 2? How many are 9, take away 2?...
Plans for children to demonstrate problems using discs, and for children to say sentences....Plans to prepare sentences with frames for children to complete, e.g. 7 and 2 are __; 9, take away 2, are __....

Grade One Activities
(Using discs to demonstrate related additions and subtractions, saying sentences; completing written sentences)

Child uses discs to demonstrate problems and says sentences for adding and taking away 1, and for adding and taking away 2....The latter is illustrated.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Using Discs</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Show 7 and 2. How many are there?</td>
<td>![Discs]</td>
<td>7 and 2 are 9.</td>
</tr>
<tr>
<td>2. How show 9, take away 2. How many are left?</td>
<td>![Discs]</td>
<td>9, take away 2 are 7.</td>
</tr>
</tbody>
</table>

Writes numerals in frames for sentences prepared by the teacher....Uses discs to demonstrate each problem....

<table>
<thead>
<tr>
<th>Problem</th>
<th>Using Discs</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Frame at end of sentence (for sum or remainder)</td>
<td>![Discs]</td>
<td>7 and 2 are __</td>
</tr>
<tr>
<td>9, take away 2, are __</td>
<td>9, take away __, are 7</td>
<td></td>
</tr>
<tr>
<td>5 and 2 are __</td>
<td>7, take away 2, are __</td>
<td></td>
</tr>
<tr>
<td>Etc.</td>
<td>Etc.</td>
<td></td>
</tr>
</tbody>
</table>

(Continued on Page 131)
May plan for a "Are There Enough Pennies?" game. The purchase to be made is discussed; the pennies needed are patterned and placed on a small mat in the center of a table, e.g., 4. A child is asked to take 2 pennies from a box or bag and then is asked if he has enough to make the purchase. He is then asked how many he will have if he gets one more. Teacher and child proceed similarly until he has 4 pennies and realizes this is the number he will need....

This game may be used also for taking one penny from a set by planning a 2-cent purchase and starting a child with a set of 4 or 5 pennies.

May plan a "Penny Guessing Game" to be played by the teacher and a child. Teacher places a penny in one hand; child guesses which hand. For each correct guess child gets the penny, anticipating each time how many he will have. He patterns his pennies until he has 5. For each incorrect guess the teacher takes the penny. For each set of 5 pennies child or teacher exchange for a nickel. Game proceeds until the child or the teacher get a nickel, or a nickel and a penny, etc....
Adding and Taking Away 3 and 4 From Sets of Discs and Beads – Through a Total of 10, or More

Teacher Preparation: Plans for those children who have learned to add 2 and take away 2 to use discs and beads to add 3 to sets of 3, 4, 1, 5, 7, 6, etc. Plans to relate adding 3 to adding 1 and 2. Plans for children also to use pattern cards. Plans for mature children to proceed similarly with adding and taking away 4.

Grade One Activities

(Adding 3 and taking away 3 from even numbers of patterned discs, from odd numbers of patterned discs, from unpatterned discs, from beads; relating adding 3 to adding 1 and 2; relating taking away 3 to taking away 1 and 2; adding 4 and taking away 4, as with adding and taking away 3)

Child uses patterned discs in "Add 3" and "Take Away 3" games – patterning even numbers of discs first. Thinks out the number in sets of unpatterned discs, e.g., 7, then the number if 3 are added or taken away, e.g.: 7, 8, 10; or 7, 6, 4. Uses beads to add and take away 3. Uses pattern cards to show adding 3.

Proceeds similarly with adding 4 and taking away 4.

Additional Activities: Uses buttons, wheels, or science materials.

Adding 3 and Adding to 3; Adding 4 and Adding to 4; Subtracting 3 and 4 – Using Discs, Dominoes, Pattern Cards, and Sentences

Teacher Preparation: Plans for those children who have learned to add 2 and to add to 2 (pages 127, 129) to use discs, dominoes, pattern cards, and sentences for adding 3 and adding to 3. Plans to relate adding 3 to subtracting 3 using discs and beads. Plans to develop adding 4, adding to 4, and subtracting 4 with mature children.

Grade One Activities

(Using discs, dominoes, and pattern cards to add 3, add to 3; using discs and beads to relate adding 3 to subtracting 3; proceeding similarly with adding 4, adding to 4, and subtracting 4)

Child proceeds as with adding 1 and 2, adding to 1 and 2, and subtracting 1 and 2 (odd-numbered pages 123-129).

(Continued on Page 133)
"Penny Guessing Game" may be adapted to subtracting one from a set. Child begins with a nickel. He exchanges the nickel for 5 pennies. He loses a penny for each incorrect guess, anticipating each time how many he will have left. Game proceeds until child has lost all his pennies.

<table>
<thead>
<tr>
<th>Pre-Kindergarten Activities</th>
<th>Kindergarten Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Naming number in sets of patterned pennies, from one through five)</td>
<td>(Naming number in sets of patterned pennies, from a zero number through 6 or more)</td>
</tr>
<tr>
<td>(Thinking out the number if one penny is to be added to sets of pennies of various sizes, thinking out number if one penny is to be taken from sets of pennies of various sizes, exchanging 5 pennies for a nickel)</td>
<td></td>
</tr>
<tr>
<td>Child realizes that he needs 5 or 4 or 3 pennies to buy a decoration....</td>
<td>Child appreciates that he needs 4 or 5 or 6 or more pennies to make a purchase....</td>
</tr>
<tr>
<td>Patterns sets of pennies through 5 and names the number in each.... Names the number in sets of 1, 2, 3, 4, and 5, pennies....</td>
<td>Patterns sets of pennies through 6 or more and names number in each.... Names number in sets of pennies from zero through 6 or more....</td>
</tr>
<tr>
<td>Plays games involving adding one to sets of pennies of various sizes.... Plays games involving taking away one from sets of pennies of various sizes...</td>
<td>Plays games involving adding one to sets of pennies of various sizes through a total of 6 or more pennies.... Plays games involving taking one from sets of pennies of various sizes through a total of 6 or more pennies....</td>
</tr>
<tr>
<td>Exchanges 5 pennies for a nickel....</td>
<td>Exchanges 5 pennies for a nickel.... May be able to exchange 10 pennies for 2 nickels....</td>
</tr>
<tr>
<td>On class trip observes purchase made for 4 pennies, or 5 pennies, or 1 nickel, etc....</td>
<td>On class trip may be able to make the purchase....</td>
</tr>
</tbody>
</table>

(Continued on Page 134)
4. Counting Forward and Backward by Fives and Tens - Through 
   a Total of 20 or More Objects

   See Item 4.4, Grade One, page XI.

Counting Pennies and Beads by Fives Forward and Backward - Through a Total of 20, 
or More

Teacher Preparation: Plans for activities in which children bring money to 
school, e.g., contributions for Junior Red Cross, purchasing bulbs....Plans 
for children to exchange nickels and dimes for pennies, to arrange pennies 
in sets of 5 each, and to count the pennies by fives - forward and backward....
Plans for children to exchange 5 pennies for 1 nickel, 10 pennies for 1 
dime....

Plans for each child in a group to re-string his row of 10 beads for 10 beads, 
all of one color....Plans for children to learn to identify sets of 5 without 
counting....Plans for children to string at least one more row of beads, all 
one color....Plans for children to count sets of 5, 10, 15, 20, or more beads 
forward and backward....

   Grade One Activities

   (Counting pennies by fives forward and backward; exchanging coins; 
counting beads by fives forward and backward)

Child observes his own coins for contribution or purchase, or observes those 
of another child....Exchanges coins for pennies....

Arranges 15 pennies in sets of 5 each....Counts pennies forward by fives ...
Patterns the fives....Counts pennies backward by fives....Exchanges pennies 
for 3 nickels....Exchanges pennies for 1 dime and 1 nickel....Counts other 
sets of pennies forward and backward....

Re-strings his row of beads of two colors for 10 beads, all of one color....
Learns to identify sets of 5 beads....Strings a second row of beads, all one 
color....Identifies fives....

Counts sets of 10, 15, 20, or more, beads forward by fives....Counts sets of 
beads backward by fives....

Counting Pennies and Beads by Tens Forward and Backward - Through a Total of 20, 
or More

Teacher Preparation: Plans for children to arrange sets of 20 or more 
pennies by tens (2 fives), and to count these forward and backward....Plans 
for children to exchange sets of pennies for other coins....Plans for children 
to count beads by tens forward and backward....

   Grade One Activities

   (Counting sets of pennies by tens forward and backward; exchanging 
pennies for other coins; counting beads by tens forward and backward 
through 20 or more)

Child proceeds as with counting pennies and beads by fives.

(Continued on Page 135)
PRE-KINDERGARTEN AND KINDERGARTEN

TOPIC 5. NUMBER LINE CONCEPTS: EARLY LEVELS OF DEVELOPMENT

PRELIMINARY STATEMENT

Pre-Kindergarten and Kindergarten children have been developing set concepts from the beginning of the school year (from Topic 2 on). The teacher has been observing children as they handled objects in their work and play activities. The teacher has been focusing the attention of a child or a group of children on sets of objects, asking questions or making comments which would encourage children to perceive and to think about sets. As the teacher emphasizes this topic (Topic 5) children continue to develop set concepts as they make observations about sets of objects arranged in lines.

Set concepts continue to be developed by Pre-Kindergarten and Kindergarten children through mathematical experiences such as the following: Children delineate and name sets of objects arranged in lines. They name the number in a set of objects in a line, conserving the number regardless of how the objects are perceived. They observe subsets in a set, yet maintain the idea of the set, e.g., they perceive the number of objects in each subset yet they conserve the number in the set. They also conserve the number in the set when the objects are rearranged. They think out the number if two sets of objects in lines are to be combined; they think out the number remaining if part of a set is to be taken away. (See also p. 135, Items 1-6.)

Children delineate and name sets of objects arranged in lines. A child may use a hoop or a lace to delineate a set of toy vehicles, milk containers, etc. The teacher also indicates the line by moving a hand along the line of objects in a vertical direction for objects that are stacked, in a horizontal direction for objects along the surface of the table.

Children themselves build or arrange objects in a line as part of their work or play activities. A child may build or arrange a set of objects upward from the floor or table top (in a vertical line). Examples of such sets of objects in lines are: a tower of blocks, a stack of cups or saucers or bowls, a stack of mystery boxes or boxes of crayons, a stack of books or games or puzzles, a stack of jars, etc. Illustrations of such lines of objects follow:

(Continued on Page 136)
GRADE ONE

TOPIC 5. NUMBER LINE CONCEPTS: EARLY LEVELS OF DEVELOPMENT

PRELIMINARY STATEMENT

Grade One children have been developing set concepts from Topic 2 on, as well as in the Pre-Kindergarten and Kindergarten. (See p. 134.) As the teacher emphasizes this topic (Topic 5) children continue to develop set concepts using objects arranged in a line. Among the set concepts developed by children as Topics 2, 3, 4, and 5 are emphasized are the following:

1. The idea of a set of objects as a collection of objects. Children locate sets of objects. They name the objects (members) in the set. They think out the common property of the objects in the set and name the set of objects (blocks, boxes, toys, science objects, etc.). They arrange each set in a pattern (Topics 2-4) or in a line (Topic 5).

2. The number property of a set of objects including the idea of a zero number of objects in a set (empty set). Children learn ways to think out the number in a set of patterned objects (Topics 2-4) and in a set of objects in a line (Topic 5). They name the ordinal position of each object in a line. Children learn ways to compare the number in two sets of objects. They think out logical relationships within the sets of objects to be compared.

3. The idea of the conservation of the number in a set of objects. Children arrange and rearrange the objects within a set conserving the set and the number of objects in the set.

4. The idea of subsets within a set of objects. Children separate a set (actually or mentally) into subsets. They determine the number of objects in each subset, conserving the number in the set.

5. The idea of set union. Children learn ways to determine the number in the combination of two (or more) sets of objects.

6. The idea of set separation. Children learn ways to think out the number of objects remaining if a subset is taken away.

As children locate, arrange, or name a set of objects in a line and the number in such a set the teacher differentiates between a set of objects arranged in a line vertically (upward from a table top or the floor) and a set of objects arranged in a line horizontally (along the surface of the floor or table top). Children are encouraged to observe a line of objects from various positions - from the front, from the back, from the right, from the left, from above, etc.

Concepts of vertical and horizontal are contrasted through an activity such as the following: A child stands a doll on the floor or table top in a vertical position, then lays the doll down in a horizontal position (along the surface of the floor or table top).

(Continued on Page 137)
A child may build or arrange a set of objects along the surface of the floor or table top (in a horizontal line). Examples of such sets of objects in lines are: a set of cars end-to-end, a set of blocks end-to-end as a road, a set of dolls aligned on a shelf, a set of beads on a lace, a set of books standing on a shelf, etc. The teacher moves a hand along each line of objects and says, e.g.: a line of cups, a line of boxes of crayons, a line of bolts, a line of mystery boxes, a line of beads, etc.

Young children are likely to believe that any change in arranging a set of objects changes the number. The teacher helps children continue to develop the idea of the conservation of the number in a set - through 3 objects in the Pre-Kindergarten, through 4 objects in the Kindergarten. (See also Topic 3.) Procedures which will help children conserve the number in a set of objects arranged in a line follow: (The teacher discourages counting the objects by ones.)

1. Changing direction in which the line of objects is perceived

The teacher observes that a child has arranged objects in a horizontal or a vertical line. The teacher asks how many there are "this way," moving a hand in one direction - left to right or bottom to top. The teacher then asks how many there are "this way," moving the hand in the opposite direction - right to left or top to bottom. (No counting) The child observes that the number of objects does not change. Illustrations follow:

2. Changing position from which the line of objects is perceived

A child changes his own position, observing the line of objects from the front, from the right, from the rear, from the left, from a corner, etc. He notes that the number does not change.

3. Interchanging two objects in a line of three or more

The teacher observes that a child has arranged a set of objects in a row, or in a tower or stack. The teacher asks how many there are. (No counting) The teacher then interchanges two of the objects and asks again how many there are. The teacher may continue with other interchanges. The child observes and realizes that the number remains the same, regardless of the order of the objects.

(Continued Page 138)
Children locate in the classroom, or sort out and arrange, sets of objects in a line. As children sort out objects the teacher notes levels of development. Some of these follow: All objects in a line are of the same kind and color. Objects in a line vary in shape or size or color. The objects in a line vary but all have a common function or use, e.g., tools.

Children name the sets of objects in lines. The teacher again notes levels of development, as follows: One object is named, e.g., car. Each object is named, e.g., car, car, bus, car. The set of objects is named by a commonly used term, e.g., blocks. The set of objects is named more specifically, e.g., rectangular blocks. The set of objects is named by some common function or use, e.g., toys, tools, vehicles, clothing, furniture, dishes.

Children continue to develop the idea of the conservation of the number in a set - through 5 or 6 objects in a line. The teacher covers each set after brief exposure in order to discourage counting by ones. After a change has been made the teacher notes whether any child has difficulty in perceiving and conserving the number of objects in a set. Illustrative changes follow: (See also facing pages.)

1. Changing position of a line of objects from vertical to horizontal and vice versa, or from one horizontal position to another

2. Combining two lines of objects, an equal number in each line (doubling); separating a line of objects into two lines, an equal number of objects in each (halving)
4. Changing position of a line of objects from vertical to horizontal, or from horizontal to vertical

The teacher observes that a child has arranged a line of 2 or 3 or 4 objects. The teacher asks how many there are. (No counting) The teacher then rearranges the objects - a stack to a row, or a row to a stack - and asks again how many there are. The child arranges and rearranges objects himself. He observes and realizes that the number remains the same.

Children learn to think out the number in a line of objects (stacked or arranged horizontally) - through 4 objects in the Pre-Kindergarten, through 6 or more objects in the Kindergarten. When the number of objects in a line is small and when the objects are small and compact children readily perceive the number at once. The teacher encourages children to sort a miscellaneous collection of objects into sets of the same kind, to arrange these in lines, and to name the sets and numbers.

Children learn to think out the number in a line of objects by perceiving the number in all but one of the objects in the set. Then they add the one. For example, the teacher asks a Kindergarten child how many there are in a line of 5 objects (3 or 4 objects of a Pre-Kindergarten child). If the child hesitates, or appears to count, the teacher covers one object (at either end), asks how many there are now, then uncovers the one object and asks how many there are now.

Children also learn to think out the number in a line of objects by perceiving doubles within the line of 4 objects in the Pre-Kindergarten, of 6 objects in the Kindergarten. For example, the teacher asks a child how many there are in a line of 4 familiar objects. If the child hesitates, or appears to count, the teacher separates the line of objects into doubles and asks how many there are in each subset. The teacher then closes the line of objects and asks how many there are in the set. This is illustrated for a line of 4 toy cars:

(Continued on Page 140)
Grade One children learn to think out the number in a line of objects by perceiving doubles within the line - through a total of 10 objects, or more. First, the teacher tests each child's ability to perceive and to name the number in a set of 1, of 2, of 3, of 4 objects. The teacher also tests understanding of a zero number of objects by asking questions, such as: How many elephants (are standing) on this table?

Children learn to perceive doubles within a line of 4 objects, then 6 objects, then 8 objects. For example, the teacher asks a child how many there are in a line of 6 objects. If the child hesitates, or appears to count, the teacher separates the line into doubles, asks how many there are in each subset, then closes the line of objects and asks how many there are in the set. (See page 140 for illustrations showing such separation of 4 toy cars into doubles.) The teacher also asks the child to separate the line into doubles himself, or to point out the doubles, as illustrated for 6 paper cups on a tray.

The teacher proceeds cautiously with doubles within 8 objects. The teacher first makes sure that each child readily perceives the number in a line of 4 objects without counting by ones.

Each child uses his string of 10 beads (now all beads of one color) to think out the number in a line of 10 by perceiving doubles. Children also learn to perceive doubles within a line of 10 objects, other than beads, e.g.: 10 pennies, 10 milk containers, 10 paper cups, 10 cubic blocks.

Children learn to think out the number in a line of 5, of 7, and of 9 objects by first perceiving doubles (largest doubles) and then noting there is one more. For example, in a line of 7 discs the child perceives the doubles 3 and 3, then adds the one. The teacher may assist a child by first covering the one object.

Children also complete sentences or write sentences to reinforce the concept of doubles and near-doubles, e.g.: 2 and 2 are ____, ____ and 3 are 6, 3 and 3 and ____ are 7, etc.

Grade One children also learn to think out the number in a line of objects by perceiving numbers in two subsets - through a total of 10 objects, or more. The teacher first presents a line of 4 or 5 objects. Children think out the number in the set. Then they indicate two subsets by using both hands or by using a lace to separate the set into subsets. They make discoveries and write sentences, such as: 3 and 2 are 5. (Or, 3 + 2 = 5)

The teacher presents a line of 6 objects and proceeds as with lines of 4 or 5 objects.

(Continued on Page 141)
From time to time the teacher focuses attention on a set of objects in a line larger than 4 for Pre-Kindergarten children, and larger than 6 for Kindergarten children. The teacher observes and evaluates each child's approach as he thinks out the number in such a set.

Children compare two lines of objects - through 4 objects in a line in the Pre-Kindergarten, through 6 objects in a line in the Kindergarten. Children compare the two lines of objects by number to find out whether they match in number, or whether one line has more or fewer objects than the other. Children first think out the number of objects in each line, remember these numbers, then determine whether the numbers are the same or different. They match the objects in a one-to-one correspondence and find out how many more or less in one line than the other. (See also Topic 2.) Illustrations follow for cups and saucers:

Children also count the objects in each of the two lines of objects. They remember the numbers counted and think out whether these are the same or different. When counting objects a child always designates a set of objects for each number name. For example, when he says "four" he designates four objects, not one. An illustration follows:

Children learn to think out and to express logical relationships within the two lines of objects to be compared. Children first make observations about each of the objects in a line, e.g.: This cup (or, the first cup) is red. This cup (or, the second cup) is blue. The children who are able to characterize individual objects, then characterize and name the set of objects in a line, e.g.: cups, or bars, or paper cups, or blocks, or beads, or dolls, etc. Children then characterize and compare the two lines of objects, e.g.: These are cups and these are saucers. These cups are red but these are blue. These are tea cups and these are paper cups. These are nuts; these are bolts. Dolls, doll dresses. Containers of milk and children.

(Continued on Page 142)
Before presenting lines of 7 or 8 or 9 or 10 objects the teacher evaluates each child's ability to think out the number in each set and its subsets. For 10 objects children use their string of beads.

Grade One children compare two lines of objects, through 10 objects in a line. Children compare the two lines by number. They first think out the number in each line by perceiving doubles or other subsets. Thus children determine whether the two sets of objects in lines are equivalent. Children check their thinking by matching the two lines of objects in a one-to-one correspondence. (See also Topic 2.)

Children also count the objects in each line as another way to compare the numbers. Children continue to designate a set of objects for each number name. See illustration on facing page 140.

Children think out and express logical relationships within the two lines of objects to be compared. Children make as many observations as they can about each object in a line (each member of the set). The teacher lists as statements the characteristics observed, e.g., for the first jar of paint on an easel (the lefthand one): It has red paint. It is made of plastic. You can see through it. After two or more jars of paint have been analyzed children note which statements are the same and which are different. For example, all of the jars are made of plastic; you can see through all of them, but the colors of paint vary.

The teacher may also present true and false statements. For example: The third jar has green paint. (False) All of the jars are round. (True) None of the jars has paint. (False)

The teacher may help mature children reason based on a given postulate, e.g.: If all of our jars are made of plastic what is this jar made of? If we use only red, blue, and yellow paint, and the first jar has red paint, and the second jar has blue paint, what color paint should we put into the third jar?

Children also characterize and compare two lines of objects. The teacher observes degrees of precision in making comparisons, e.g.: coats-hats, girls' coats and hats; cups-saucers, red and green cups-red and green saucers; etc.
Through this topic children develop concepts of length. Although lengths are emphasized, concepts of weight are also developed. The teacher uses comparative terms in referring to lengths or weights, e.g.: longer (not long), shorter, heavier, lighter. The concept of length or weight (or size, temperature, time, or capacity) always implies comparison with a pre-determined unit of length. Thus, a pumpkin is not big; rather it is larger than some unit with which it is compared, such as an orange or a smaller pumpkin.

The teacher makes comments or asks questions about comparative lengths or weights of two of the objects children are using in a work or play activity - rectangular blocks in the blockbuilding area, dolls in the house-play area, tumblers at water play, copper tubing in the science area, etc.

When two lengths are to be compared the teacher moves a hand along each of the lengths, so that children clearly differentiate lengths from overall sizes (surfaces or volumes). For example, the teacher moves a hand along the heights of two dolls and asks: Which doll is longer this way? Which doll is shorter? Or, the teacher moves a hand along the heights of two tumblers or towers of blocks and asks: Which is longer this way? The child also moves his hand along each of the lengths so that he can "feel" each length, as well as perceive it.

Children are encouraged first to estimate comparative weights of two objects before they hold or lift the objects. At first children will learn to estimate weights in relation to sizes of the objects, e.g., assuming that the larger or longer object is also the heavier. To develop this relationship children compare weights of 2 dolls or toy animals, 2 building blocks, 2 toy vehicles, 2 cans or boxes, 2 balls, etc. Each child checks his estimate by holding an object in each hand and sensing which is heavier or lighter. The teacher may dramatize this by having a child stand with arms outstretched holding two objects (as if he were a balance scale), as illustrated:

Later, children learn to study the objects more closely to estimate comparative weights. Thus children compare weights of a rubber ball with a clay ball, a container full of milk with an empty milk container, a box full of books with an empty box, a container filled with gravel with a container filled with cotton or other light material, etc.

(Cont···ed on Page 144)
Grade One children continue to develop concepts of length and weight through this topic. As in the earlier grades the teacher uses comparative terms in referring to lengths and weights. (See p.142, first paragraph.)

When two (or more) lengths are to be compared the teacher continues to move a hand along each of the lengths or lines of objects in order to emphasize lengths (rather than overall sizes). Although the word "length" actually refers to a measure of an object along a line in any direction, e.g., the width of a block as a length, the teacher emphasizes the longest length at this level. The teacher may mark a "chalk line" on each object to indicate its length, as illustrated:

At this level all of the objects in the lines to be compared will be of the same length. Each of these lengths may thus be considered a unit of length, as illustrated for lines of rectangular-block-lengths:

Grade One children continue to estimate comparative weights of two objects before they hold the objects to sense the weights kinesthetically. Children estimate comparative weights of familiar objects around the classroom and check weights by holding or lifting the objects. Among such objects will be work-play area materials; pencils, crayons, and chalk; and rectangular table blocks (cubic blocks, Stern blocks, Cuisenaire blocks, etc.). The teacher also encourages careful consideration of what the two objects to be compared are made of (copper or wood, aluminum or steel, etc.), whether either of the containers is full or empty, and what the containers are filled with (one filled with gravel and one with chalk, one filled with magnetized discs and one with cotton, one filled with books and one with clothing, etc.).

Mature Grade One children are encouraged to estimate comparative weights of 3 objects, and to "weigh" these by holding the objects, two at a time. (Thus children develop beginning concepts of transitivity: the first object is heavier than the second object, the second object is heavier than the third object, therefore, the first object is heavier than the third object.)

Rectangular table blocks and tagboard strips of 10 graduated lengths are particularly useful in Grade One for comparing lengths, and for developing additions and subtractions through a total of 10 units. These are illustrated:

(Continued on Page 145)
Pre-Kindergarten and Kindergarten: Topic 5

The rectangular building blocks are particularly useful in the Pre-Kindergarten and Kindergarten for comparing lengths, and for developing additions and subtractions through a total of 4 units. These blocks are all 21/4 inches high and 1 3/8 deep. They are of four lengths as illustrated:

Children use a variety of objects, including the rectangular building blocks, to make comparisons of length. A suggested sequence follows:

1. Children make explorations using objects of varying lengths, e.g., building with blocks, arranging cans or boxes, cutting dowel sticks, arranging jars and tumblers after water play.

2. Children look for and locate another object to match a particular object in length, or one longer or shorter, e.g., another 11" rectangular block for the road, a longer bolt, a higher can.

3. Children classify a set of objects by length, e.g., stacking the rectangular building blocks by length, placing dowel sticks in boxes by length, storing cans on a shelf by length (height).

4. Children compare two rectangular building blocks by length – one length twice (or one half) the length of the other, one length four times (or one fourth) the length of the other. Children make explorations as they build with the blocks. The teacher asks questions, such as:

   How many blocks this long (5 1/2") do you need to cover this block? (11" block)?
   How many blocks this long (5 1/2") do you need to cover this block? (22" block)?

   Each child thinks out the number, then checks his perception by using the blocks.

5. Children use rectangular building blocks to think out additions and subtractions within a total of 4 or more unit-lengths. Illustrations follow for using a 22" block and 5 1/2" blocks:

   Tr. Which block is longer?
   Child points.

   Tr. Which block is shorter?
   Child points.

(Continued on Page 146)
Grade One: Topic 5

1. Rectangular table blocks scored in square units, 10 graduated lengths

2. Rectangular table blocks, unscored, 10 graduated lengths

3. Tagboard strips scored in units on one side, 10 graduated lengths (units in one-inch squares)

The teacher prepares a master sheet (stencil) completely ruled with one-inch squares. Sheets of tagboard are cut to fit and the squares are duplicated on the tagboard. Each child gets at least one tagboard sheet of squares, at least 7 squares in one direction and at least 13 squares in the other direction.

Children prepare their own strips. They outline or color strips, beginning with the longest (the ten-strip, then the nine-strip, etc.) ending with the shortest (the one-square). For each child there will be a minimum of the following tagboard strips: 1 ten-strip, 1 nine-strip, 1 eight-strip, 1 seven-strip, 1 six-strip, 2 five-strips, 2 four-strips, 3 three-strips, 5 two-strips, 10 one-squares.

Children use a variety of objects and lines of objects, including the graduated rectangular table blocks and tagboard strips, to make comparisons of length. For some time children make explorations individually or in small groups with blocks or strips. Children who work in a small group share their observations and discoveries with one another. The teacher makes comments or asks questions to further mathematical thinking as children handle their materials (blocks or strips). A suggested sequence follows:

1. Children compare lengths of familiar objects, e.g., 4 pencils, 2 through 10 unmarked rectangular strips. They name the number of units in each block or strip, table blocks or tagboard strips (unmarked side). Children check their judgments of comparative lengths by placing the objects in order of length.

2. Children compare lengths of 2 or 3 or more lines of objects, and of 2 or 3 or more scored rectangular table blocks or tagboard strips (scored side). Children think out the number of units in each line. They arrange the lines of objects, blocks, and strips in order of length.

(Continued on Page 147)
Tr. How many of these shorter blocks will you need to cover this longer block?

Child thinks out the number. Then he places 4 shorter blocks on top of, in front of, or behind the longer block to check.

Tr. If you move two of the four blocks down from the top (or up from in front of, etc.) to the front of the longer block, how many will be left on top? (4, take away 2, are 2)

Child thinks out the number. Then he moves 2 blocks and observes number left.

Tr. If you move these two blocks down from the top to the front, how many will there be in front? (2 and 2 are 4.)

Child thinks out the number. Then he moves the 2 blocks, and observes number in front now.

Tr. If you move 2 of the 4 blocks in front to the top, how many will be left in front? (4, take away 2, are 2.)

Child thinks out the number. Then he moves the blocks, and observes number left.

The teacher proceeds similarly with taking away 1 or 3 blocks from the 4, and with adding 1 or 3 blocks to 3 or 1. Children may also make explorations using the 22" block and 2 1/2" blocks (additions and subtractions within 8).
3. Children arrange in order of length 10 unscored rectangular table blocks or tagboard strips (unscored side) of varying lengths. (See second set, p. 145.)

Children arrange in order of length 10 scored rectangular table blocks or tagboard strips (scored side) of varying lengths. (See first set, p. 145.) Children name the number of units in each line (9, 8, 7, etc., or 1, 2, 3, etc.).

4. Children select two unscored rectangular table blocks or tagboard strips (unscored side), one block or strip double the length of the other. Illustrations follow:

Children select two scored rectangular table blocks or tagboard strips (scored side), one double the length of the other. They name the number of units in each block or strip. Illustrations follow:

5. Children combine two scored rectangular table blocks or tagboard strips (scored side), both blocks or strips the same length. (Doubles) Children name the number of units in each block or strip and in the combination as illustrated:

Children cover or take away one block or strip, and say the sentence, as illustrated:

6. Children combine two scored rectangular table blocks or tagboard strips (scored side) to show adding 1 and adding to 1, adding 2 and adding to 2, adding 3 and adding to 3, adding 4 and adding to 4, near-doubles. Children say sentences. Illustrations follow for adding 3 and adding to 3: (These also show near-doubles.)

Children cover or take away one block or strip, and say the sentence for subtracting 1 and 2, 3 and 4.

(Continued on Page 149)
In the Pre-Kindergarten and Kindergarten children measure the length of an object by placing unit-objects end-to-end to match the length of the object to be measured. For example, a child may measure the length of a 22" block using a 5 3/4" block as a unit of measure. He places four 5 3/4" blocks in front of (or on top of) the 22" block (to match the 22" block). The child perceives that it takes 4 of the shorter blocks to match (measure) the longer block.

Children also use blocks to measure the lengths of the shelves in a bookcase. One child may use cubic blocks as units of length to measure the length of one shelf. Another child may use blocks 5 3/4" long as units to measure the length of a second shelf. A third child may use blocks 11" long to measure the length of a third shelf. Children think out the number of blocks used to measure the lengths of each of the three shelves. They realize that where a longer block was used (as a unit of length) there aren't as many.

The teacher works with one child at a time or a small group of children for most of the activities suggested for these grades. This is necessary so that children can see all of the objects in a line, can learn to perceive subsets, can compare objects by length, can use objects to measure length, and can express themselves often. Thus the needs of individual children can be adequately met.
Grade One children learn to measure lengths of familiar objects, e.g., pencils, chalk, paper, books, a lace, a child's arm, etc. Children use the length of an unmarked rectangular table block or tagboard strip (unmarked side) as a unit of length (preferably not the unit-block or tagboard square).

Children learn to measure length by placing the lefthand end of the unit-block or strip to match the lefthand end of the object to be measured for length. At first children use enough unit-blocks or strips to match the object to be measured. This is illustrated for using a 3-unit strip (unmarked side) as a unit to measure the length of a pencil.

![Diagram of a pencil and unit blocks](image)

"A little more than 2 units long."

Children measure the length of an object using unit-blocks or strips of varying length. They conclude that the longer the unit the fewer the number of units, and vice versa.

Children now use the unit-block or the unit-tagboard square for measuring lengths of objects. Then they use a multiple unit-length, scored face showing. Children add sets of unit-lengths as they measure the length of an object, as illustrated for using sets of 3 unit-lengths to measure the length of a pencil. The child concludes that the pencil is a little longer than 6 unit-lengths.

![Diagram of a pencil and multiple unit blocks](image)

"A little more than 6 units long."

Mature children may learn to use a single unit-block or square to measure-off the length of an object. They mark the object as they measure, as illustrated:

![Diagram of a pencil and a single unit block](image)

(Continued on Page 151)
Pre-Kindergarten and Kindergarten: Topic 5

Following are lists of materials appropriate for developing this topic in Pre-Kindergarten and Kindergarten classes:

**Essential Materials:** Boots in pairs, hoops or laces, toy vehicles, "footsteps," beads of varying shapes on a lace, drums, boxes or cans of food, containers of milk, books or games or puzzles on a shelf or table, bolts and nuts, dowel sticks, rectangular building blocks, cubic blocks, mystery boxes, crayon cans, jars, science objects, dolls and toy animals, balls, bean bags, toy cups and saucers, snack items, tools, chairs, clay, shelves in a bookcase or closet...

**Additional Materials:** Cymbals, children in a line, wedgies, automobiles parked on a street...

As children focus attention on the mathematical aspects of this topic they develop concepts of position, general size, length, shape, weight, and quantity. The teacher and some children use terms such as those suggested for Topics 1, 2, 3, and 4. Other terms used by the teacher are: in a line, longer-shorter, taller-higher, as long as, on top of - in front of, heavier-lighter-as heavy as, weigh, rectangular block, cubic block. Some of these terms may be used by the children.

An outline of the contents suggested for the Pre-Kindergarten and Kindergarten follows:

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1. **Perceiving at Once or Thinking Out the Number in a Set of Objects in a Line - Through 4, Through 6 or More**

   - Delineating and Naming Sets of Objects in Lines
   - Conservation of the Number in Lines of Objects - Through 3, Through 4
   - Thinking Out the Number in a Line of Objects by Perceiving All But One - Through a Total of 4, Through a Total of 5 or More
   - Thinking Out the Number in a Line of Objects by Perceiving Doubles - Within 4 Objects, Within 6 Objects
   - Comparing Two Lines of Objects, Through 4 or 6 in a Line - Number, Logical Relationships

2. **Concepts of Length and Weight; Using Rectangular Building Blocks of 4 Lengths**

   - Comparing Lengths and Weights of Two Objects
   - Using Rectangular Building Blocks for Developing Additions and Subtractions Within a Total of 4, or More, Unit Blocks

3. **Measuring the Length of an Object by Matching with Unit-Objects Placed End-to-End**

(Continued on Page 15x)
Following are lists of materials appropriate for developing this topic in Grade One classes:

**Essential Materials:** Doll, children's clothing, books, containers of milk and straws, nuts and bolts, plant pots and plant dishes, tables and chairs, paper dolls and dresses, cubic blocks, drums, paper cups, bean bags or erasers, jars, mystery boxes, "footsteps," painting materials, crayons, paste pots, pencils, chalk, science objects, copper or steel rod or bar, wooden or aluminum pole or bar, gravel or magnetized discs and cotton or other light material, beads of 2 colors on a lace and weaving board, rectangular table blocks of 10 graduated lengths — scored and unscored, tagboard strips of 10 graduated lengths scored on one side, construction material, worksheets, stencils, duplicator...

**Additional Materials:** Cymbals, quoits, hoops, in the classroom; automobiles in a line on a street; people in a line at a supermarket; buildings in a row; cans or boxes on a shelf in a store...

As children focus attention on the mathematical aspects of this topic they continue to develop concepts of position, general size, length, shape, weight, and quantity. The teacher and some children use terms such as those suggested for Topics 1, 2, 3, and 4, as well as those suggested for the Pre-Kindergarten and Kindergarten. Other terms used by the teacher are: longest—shortest—second longest—second shortest, tallest—highest, upward—downward, backward—forward, to the right—to the left, same length—different length, twice as long—four times as long, one half as long—one fourth as long, pencil length (and other objects), unit length(s), unit square, weight. Some of these terms may be used by the children.

An outline of the contents suggested for Grade One follows:

1. **Perceiving at Once or Thinking Out the Number in a Set of Objects in a Line — Through 10, or More**
   - Sorting, Locating, and Naming Sets of Objects in Lines
   - Conservation of the Number in Lines of Objects — Through 5 or 6
   - Thinking Out the Number in a Line of Objects by Perceiving Doubles — Through a Total of 10 Objects, or More
   - Thinking Out the Number in a Line of Objects by Perceiving Numbers in Two Subsets — Through 10 Objects, or More, in the Set
   - Comparing Two Lines of Objects, Through 10 in a Line — Number, Logical Relationships

2. **Concepts of Length and Weight; Using Rectangular Table Blocks and Tagboard Strips of 10 Graduated Lengths**
   - Comparing Lengths and Weights of Sets of Objects
   - Using Rectangular Table Blocks and Tagboard Strips for Developing Additions and Subtractions within a Total of 10 blocks

(Continued on Page 153)
Pre-Kindergarten and Kindergarten teachers will find it profitable to read at least the Preliminary Statement for Topic 5, Grade One, on odd-numbered pages 135-153.

Pre-Kindergarten and Kindergarten teachers will consider the activities suggested for Topic 5 on the following lefthand, even-numbered pages for both the Pre-Kindergarten and Kindergarten. Teachers at either of these levels can then select activities which are appropriate for their children.

CONTENTS, TEACHER PREPARATION, AND PUPIL ACTIVITIES

1. Perceiving at Once or Thinking Out the Number in a Line of Objects - Through 4, Through 6 or More

Suggestion: Introduce early in the spring. Develop to the end of the school year. See Item 5.1, page XI.

Delineating and Naming Sets of Objects in a Line

Teacher Preparation: Provides hoops or laces for children to use for delineating a set by encircling... Makes materials available for children to use in work or play, e.g., house-play objects, toy vehicles, snack items, mystery boxes and small objects, boxes of crayons or puzzles or games, bolts and nuts, tools, rhythm instruments, painting materials, blocks, beads and laces... Provides for sets of objects in vertical or horizontal lines... Plans to use terms such as, in a line, set, more, less...

Pre-Kindergarten Activities

(Arranging or building a set of objects in a line; encircling set; naming set of objects; naming number if he can)

Child uses work or play materials and may arrange these in a line... Observes teacher move a hand along the line...

Names the set of objects if he can, e.g., drums, cars, blocks... If the number is small may be able to name the number...

Kindergarten Activities

(Arranging or building or locating a set of objects in a line; encircling set; naming set of objects; naming number if he can)

Child arranges work or play materials in a line... Observes teacher move a hand along the line...

Names the set of objects... Names the number if he can...

Locates lines of objects around the
Grade One teachers will find it profitable to read the Preliminary Statement for the Pre-Kindergarten and Kindergarten on facing, lefthand pages 134-152. They will also consider the contents and pupil activities suggested for these earlier grade levels as well as those suggested for Grade One on the following righthand, odd-numbered pages.

CONTENTS, TEACHER PREPARATION, AND PUPIL ACTIVITIES

1. Perceiving at Once or Thinking Out the Number in a Set of Objects - Through 10 or More

See Item 5.1, page XI.

Suggestion: Introduce early in the spring. Develop to the end of the school year.

Locating and Naming Sets of Objects in a Line

Teacher Preparation: Notes whether there are sets of objects in the classroom - in vertical lines and in horizontal lines...Plans for children to locate lines of objects, to name the sets, and to name the number of objects in each line...Plans for children to classify objects into sets and to arrange the sets in vertical and in horizontal lines, e.g., to stack the books or the cubic blocks on a table, to stack crayon boxes or the mystery boxes on a shelf, to stack the paste pots in a closet, to arrange the milk containers in a row on a tray, to place the paint jars along the easel ledge, to arrange the shells in a row on a science table, to lay the scissors in a row on a tray, to place the chairs in a row for a game, to re-string the beads on a lace...Plans to use terms of position, quantity, and set...

Grade One Activities

(Locating, classifying, and arranging sets of objects in vertical and in horizontal lines; naming sets of objects, naming number if he can without counting)

Child locates in the classroom sets of objects arranged in lines (vertical or horizontal)...Names each set of objects, e.g., blocks or (better) rectangular blocks...Names or estimates (not guesses) the number of objects in a line. Hears teacher ask: Is it more than 5?

Places a block in a vertical position with its longest length up from the floor or table top...Builds with cubic blocks in a stack (vertically)...Observes teacher indicate with a hand the vertical line upward and downward...

(Continued on Page 155)
Pre-Kindergarten Activities

Observes another child's line of objects... Observes teacher indicate the line with a hand... Hears teacher name the set of objects... May be able to name the number, or hears teacher name the number of objects in the line...

May be able to locate a line of objects, encircle the set, name the set of objects, name the number...

Hears and may use terms: in a line, set of, more than, less than...

Kindergarten Activities

classroom or on a trip... Encircles each set of objects in classroom... Names each set of objects... Names each number if he can...

Realizes that a line of objects may be along the table or floor surface, or upward in a stack or tower...

Uses terms: in a line, set of, more than, less than...

Additional Activities: On a trip observes lines of children in the cafeteria, lines of cars on the street...

Conservation of the Number in Lines of Objects - Through 3, Through 4

Teacher Preparation: Plans to focus the attention of a child or a small group of children on a line of objects arranged or built by a child at work or play - 2 or 3 objects in the Pre-Kindergarten, 3 or 4 objects in the Kindergarten... Plans to emphasize 1) direction in which the line is perceived, 2) position from which the line is perceived, 3) interchanging two objects in a line, 4) changing the position of the line... Plans to encourage children to arrange and re-arrange lines of objects themselves... Plans to note and to observe which children appreciate that the number remains the same regardless of how the objects in a line are perceived or arranged... Plans to use terms of position, general size, length, shape, and quantity...

Pre-Kindergarten Activities

(Names number in line of 2 or 3 objects perceived in different ways; names number before and

Kindergarten Activities

(Names number in line of 2 or 3 or 4 objects perceived in different ways; names number before and after 2 objects in the

(Continued on Page 156)
Grade One Activities (Cont.)

Places the block in a horizontal position with its longest length along the surface of the floor or table top. Builds in a horizontal line. Observes teacher indicate with a hand the horizontal line from left to right, from front to back, and other directions.

Classifies objects into sets and arranges these into stacks (vertical line) ...Names sets of objects...Names number...Proceeds similarly with horizontal lines...

Observes lines of objects arranged by other children...Names each set of objects...Names or estimates number...

Uses terms: set of, in a line, more than, less than, up and down...

Additional Activities: On a trip locates and names sets of objects in lines - buildings, cars, people, stacked cans in a store...Names or estimates number...

Conservation of the Number in a Set of Objects - Through 4 or 5 Objects in a Line

Teacher Preparation: Notes whether there are sets of objects in the classroom which can be arranged in both vertical and horizontal lines, e.g., drums or cymbals or saucers, crayon or mystery boxes, books or games or puzzles, paint jars or milk containers, building blocks, bean bags or erasers...Plans to find out which children are able to conserve the number regardless of the position of the line of objects. For example, the teacher conceals a row of objects with a card or folder, exposes the objects briefly and asks how many there are; conceals the objects and rearranges them into a stack, exposes objects and asks how many; conceals objects and rearranges in a different horizontal line, exposes objects and asks how many...Plans for children to arrange and rearrange a set of objects into lines themselves...

Step 1

Step 2

(Continued on Page 157)
Pre-Kindergarten Activities

after 2 objects in the line are interchanged, names number before and after the position of the line is changed

Child names the number "2" or "3" as the teacher moves hand across a line of cups, containers of milk, mystery boxes, drums, games...Names number as teacher moves hand in the opposite direction...Names set of objects, e.g., cups...Proceeds similarly with other lines of objects in horizontal or vertical positions...

Observes a line of objects and names number and set, e.g., 3 drums...Changes his position, observes line, and names number...Changes his position again and names number...Proceeds similarly with other lines of objects...

Observes a horizontal line of 3 objects. Names number and set of objects...Observes teacher interchange 2 of the objects and names number...

Observes a vertical line of 3 objects. Names number and set of objects...Observes teacher interchange 2 of the objects and names number...Proceeds similarly with horizontal and vertical lines of objects he arranges and rearranges himself.

Kindergarten Activities

line are interchanged, names number before and after the position of the line is changed

Child names the number "2" or "3" or "4" as the teacher moves hand across a line of objects in one direction...Names number as teacher moves hand in the opposite direction...Names set of objects...Proceeds similarly with other lines of objects in horizontal or vertical positions...

Observes lines of objects from several positions and names numbers...Realizes number remains the same...

Observes a horizontal or vertical line of 3 or 4 objects. Names number and set of objects before and after 2 objects have been interchanged. Realizes number is the same...Proceeds similarly with lines of objects he arranges and rearranges himself...

(Continued on Page 158)
Plans also to find out which children are able to conserve the number in the set and in each of its subsets when two lines of objects are combined into one line. For example, the teacher arranges two rows or stacks of objects, asks how many in each. The teacher combines these into one row or stack and asks a child to indicate the subsets, the number in each, and the total number...

plans to proceed similarly with other objects, other positions, and other combinations, e.g.: 2 and 3, 4 and 1, 1 and 4, 2 + 2, 3 + 1, 1 + 3.

### Grade One Activities

(Naming number before and after change in position of a line of 3, 4, or 5 objects; naming number in each of two lines of objects, a total of 4 or 5 in both lines; conserving number in each subset and in the set when the objects are combined into one line)

Child names the number in a stack of 3 or 4 objects (exposed briefly)...Names number when the objects are rearranged in a row...Names number when the objects are rearranged in a different horizontal line...Aligns and realigns 3 or 4 objects himself...Realizes that arranging a set of objects in a different kind of line does not change the number...

Names number in a line of 5 objects (exposed briefly)...Names number after the objects are realigned...Names number after the objects are aligned in a still different way...Aligns and realigns 5 objects himself...Realizes that the number is conserved.

Names number in each of two lines of objects, a total of 4 objects in both lines, e.g., 3 and 1...After the two lines are combined names number in each subset and in the entire line, e.g., 3 and 1 are 4...Makes arrangements within 4 objects himself...Realizes that the numbers are conserved...

Names number in each of two lines of objects, a total of 5 objects in both lines, e.g., 3 and 2...After the two lines are combined names number in each subset and in the entire line, e.g., 3 and 2 are 5...Makes arrangements within 5 objects himself...Realizes that the numbers are conserved.

Additional Activities: Locates objects in the classroom to align and realign, e.g., cymbals, quoits, hoops, discs, puzzle boxes...Observes differences in

(Continued on Page 159)
Pre-Kindergarten Activities

Observes a horizontal line of 2 or 3 objects. Names number and set of objects...Observes teacher change the line of objects to a vertical line (stack), or to a horizontal line in another position. Names number...Proceeds similarly with lines of objects he arranges and rearranges himself...

Kindergarten Activities

Observes a horizontal line of 3 or 4 objects. Names number and set of objects...Observes teacher change the line of objects to a vertical line (stack), or to a horizontal line in another position. Names number. Realizes number is the same...Proceeds similarly with lines of objects he arranges and rearranges himself.

Thinking Out the Number in a Line of Objects by Perceiving All But One - Through a Total of 4, Through a Total of 5 or More

Teacher Preparation: Plans to find out each child's ability to perceive the number in lines of 1, 2, and 3 objects, e.g., toy cups or saucers or pans, toy cars, cubic blocks, rectangular building blocks, nuts or bolts, wheels, containers of milk, mystery boxes, books or puzzles or games, rhythm instruments, beads on a lace, chairs, coats or caps, dolls or toy animals, dowel sticks, boxes or cans of food, jars of paint or brushes, quoits or horseshoes, science objects, tools, wedgies...Plans for children to sort out objects into sets of 2 or 3 objects, arranging these in lines...Plans to use terms of position, general size, shape, and quantity...

Pre-Kindergarten Activities

(Naming number in sets of 1 or 2 objects, naming sets of objects, sorting and aligning sets of 2 objects)

Child names the number and the set of objects in a variety of lines of 1 or 2 objects, e.g.:

Kindergarten Activities

(Naming number in lines of 1, 2, and 3 objects; naming sets of objects; sorting and aligning sets of 2 or 3 objects)

Child names the number and the set of objects in a variety of lines of 2 or 3 objects...

(Continued on Page 160)
Grade One Activities (Cont.)

appearance when such objects are stacked and when arranged in a row...Realizes that the numbers are conserved...

Thinking Out the Number in a Line of Objects by Perceiving Doubles - Through a Total of 10 Objects, or More

Teacher Preparation: Plans to test each child's ability to perceive at once and to name the number in sets of familiar objects arranged in stacks and in rows, from zero through 4 in a line...Plans to emphasize doubles in lines of 4, 6, 8, and 10 objects...Plans to use terms of position, general size, length, shape, and quantity...

Grade One Activities

(Naming number in lines of 2, 3, and 4 familiar objects; naming number in sets of zero and of one object; perceiving and naming number in doubles within lines of 4, 6, and 8 discs or other objects, and in the entire line; perceiving and naming number in doubles within a row of 10 beads of one color, and in lines of 10 familiar objects)

Child confidently names the number in stacks or rows of 2, 3, and 4 objects (exposed briefly)...Names number in a set of zero objects, e.g., elephants in the closet, principals in the room...Names number in a set of one object...

Observes teacher separate a stack or a row of 4, of 6, of 8 discs or other familiar objects into doubles by using a lace...Using a lace separates lines of 4, of 6, of 8 objects into doubles himself and names number in each subset and in the entire set, e.g., 2 and 2 in 4, 3 and 3 in 6, 4 and 4 in 8...Learns to perceive doubles without separating lines of objects...

"4" \[=\] "4" \[=\] "4 and 4 are 8."

Explores with his string of 10 beads, all one color...Confidently finds doubles (no counting)...Names number in each subset of doubles and in the set (line)...

"5 and 5 are 10"

Separates stacks or rows of 10 or more discs or other familiar objects into doubles...Names number in each subset and in the entire set, e.g.: 5 and 5 are 10, 6 and 6 are 12...

Teacher Preparation: Plans to emphasize doubles and one more in lines of 5, 7, and 9 objects...

(Continued on Page 161)
Pre-Kindergarten Activities

2 beads, 2 blocks, 2 books, 2
toys, 1 can, 1 chair, 2 pup-
pets, 2 containers of milk, 2
cups, 1 doll, 1 drum, 2 cookies,
2 games, 2 mystery boxes, 2
paint jars, 1
car...

Sorts objects into sets of 1 or
2 each... Arranges sets of ob-
jects in stacks and in rows...
Names number and set of objects
in each line...

Kindergarten Activities

Sorts objects into sets of 1 or 2 or 3...
Arranges sets of objects in stacks and in rows...
Names number and set of objects in
each of these lines...

May use terms of position,
general size, shape...

Teacher Preparation: Plans to help children think out the number in a line of
3 objects in the Pre-Kindergarten, in a line of 4 or 5 or more objects
in the Kindergarten, by first perceiving the number in all but one object...
Plans to use terms of position, general size, length, shape, and quantity...

Pre-Kindergarten Activities

(Thinking out number in lines
of 3 and 4 objects when one of
the objects is first covered)

Names number in a line of 3
objects if he can... Observes
teacher cover one object, names number visible (2), ob-
erves teacher uncover the object and names number (3)...
Plans to use terms of position, general size, length, shape and quantity...

Grade One Activities

(Perceiving largest doubles and one more in lines of 5, 7, and 9 discs or beads or other objects arranged in lines; naming number in each subset and in the set; writing sentences)

Child perceives doubles in a line of 4 objects, names number in each subset and in set: 2 and 2 in 4 (or 2 and 2 are 4)...Observes teacher add one object, still perceives doubles, and one more, names number in each subset and in set...Observes teacher write sentence: 2 and 2 and 1 are 5 (perhaps, also: 2 + 2 + 1 = 5)...May write sentence himself...

Proceeds similarly with doubles in a line of 6 objects, then a line of 7 objects...Proceeds similarly with lines of 8 and 9 objects...

Teacher Preparation: Prepares worksheets to be duplicated, emphasizing doubles and near-doubles (doubles and one more), as illustrated.

(Continued on Page 163)
Pre-Kindergarten Activities

Proceeds similarly with other lines of 3 objects...

Sorts objects into sets of 1 or 2 or 3 each and arranges in lines...Names number and set of objects in each line...

May use terms of position, general size, length, and shape...

Names number in a line of 4 objects if he can...Observes teacher cover one object, names number visible (3)...Observes teacher uncover the object, names the number (4)...Proceeds similarly with other lines of 4 objects...

Sorts objects into sets of 1 or 2 or 3 or 4 each and arranges in lines...Names number and set of objects in each line...

Kindergarten Activities

Proceeds similarly with other lines of 4 objects...

Sorts objects into sets of 1 or 2 or 3 or 4 each and arranges in lines...Names number and set of objects in each line...

May use terms of position, general size, length, and shape...

Names number in a line of 5 objects if he can...Observes teacher cover one object, names number visible (4)...Observes teacher uncover the object, names number (5)...Proceeds similarly with other lines of objects...

Sorts objects into sets of 1 or 2 or 3 or 4 or 5 each and arranges in lines...Names number and set of objects in each line...

Thinking Out the Number in a Line of Objects by Perceiving Doubles - Within 4 Objects, Within 6 Objects

Teacher Preparation: Plans to help children develop some procedures for perceiving doubles within a line of objects...Plans to arrange lines of 4 objects in twos by color with Pre-Kindergarten children...Plans to separate 4 objects by twos with Kindergarten children...Plans to observe which children conserve the number in the set...Plans to use terms of position, general size, length, shape, and quantity...

Pre-Kindergarten Activities

(Perceiving doubles within a line of 4 objects arranged in twos by color, indicating the doubles, naming number in each subset, conserving number in the set of 4)

Observes sets of 4 objects in lines, e.g.: 2 black and 2 white buttons, 2 red and 2 green beads, 2 red and 2 yellow toy cars, 2 vanilla and 2 chocolate cookies...Names number in each subset

Kindergarten Activities

(Perceiving doubles within a line of 4 objects, all objects of the same kind; indicating the doubles when subsets are separated; naming number in each subset, conserving number in the set)

Names number and set of 4 objects arranged in a line...Observes teacher separate the objects into twos (twos), by moving 2 objects (later, by using a lace)...Names number in each subset...Conserves number in the set...Proceeds similarly with other

(Continued on Page 164)
Grade One Activities (Cont.)

Child examines worksheet, and thinks out what he should do...Listens to directions...Follows directions...Evaluates work...

Thinking Out the Number in a Line of Objects by Perceiving Numbers in Two Subsets
Through 10 Objects, or More, in the Set

**Teacher Preparation:** Arranges 5 familiar objects in stacks or rows...Plans to focus attention on the number in the set and in each of the subsets, separating subsets with a lace...Plans for each child to use his own set of 5 discs to find subsets...Plans to write sentences and to have children write sentences...Plans to proceed with lines of 6 objects as with lines of 5 objects (preceding)...Plans to write sentences and to have children write sentences...

**Grade One Activities**

(Perceiving and naming the number in lines of 5 objects and in subsets within 5, perceiving and naming the number in lines of 6 objects and in subsets within 6, writing sentences as sets and subsets are perceived)

Child names the number in a line of 5 familiar objects, such as books, cubic blocks, boxes of crayons, mystery boxes, containers of milk, paste pots, jars of paint, discs, pennies...Names the number in each subset and in the set as the teacher or a child uses a lace to separate the set into subsets...Proceeds similarly with other lines of 5 familiar objects...

Uses his own set of 5 discs to align, to separate into subsets, and to assist him in writing sentences...

\[
\begin{align*}
\text{1 and 1 are 5.} & \quad \text{1 and 4 are 5.} & \quad \text{2 and 2 and 1 are 5.} \\
\text{3 and 2 are 5.} & \quad \text{2 and 3 are 5.} & \\
\end{align*}
\]

Proceeds with lines of 6 familiar objects and with lines of 6 discs as with lines of 5 objects (preceding).

\[
\begin{align*}
\text{3 and 3 are 6.} & \quad \text{5 and 1 are 6.} & \quad \text{1 and 5 are 6.} \\
\end{align*}
\]

(Continued on Page 165)
Pre-Kindergarten and Kindergarten: Topic 5

Pre-Kindergarten Activities

in response to a question such as: How many black buttons? ... Names number in the set...May be able to name number and the set, e.g.: 4 buttons...

Kindergarten Activities

lines of 4 objects...

Step 1. "4 drums"

Step 2. "2" "2"

Teacher Preparation: Plans to continue development of doubles within a line of objects...Plans to separate 4 objects by twos with Pre-Kindergarten children...Plans to help Kindergarten children indicate doubles within 4 and 6 objects in a variety of ways...Plans to use terms of position, general size, length, shape, and quantity...

Pre-Kindergarten Activities

(Perceiving doubles within a line of 4 objects after separated into twos, naming number in each subset, conserving number in the set of 4)

Kindergarten Activities

(Perceiving doubles within a line of 4 objects, doubles indicated in various ways; perceiving doubles within a line of 6 objects; naming number in each subset and in set)

(Continued on Page 166)
Teacher Preparation: Plans to evaluate each child's ability to perceive and to name the number in a variety of lines of 5 and of 6 objects, and to perceive and name the numbers in subsets within 5 objects and within 6 objects...Plans to proceed with lines of 7 objects only with those children who are ready to develop subsets within 7 objects...Plans to proceed similarly with subsets within lines of 8 objects, with subsets within lines of 9 objects, with subsets within lines of 10 objects...Plans to use beads on a lace (all one color) with those children ready to develop subsets within lines of 10 objects...Plans to use terms of position, general size, length, shape, and quantity...

Grade One Activities

(Naming numbers with little hesitation in lines of 5 and 6 objects and in subsets within 5 and within 6 objects, perceiving and naming numbers in lines of 7 objects and in subsets within 7 objects, perceiving and naming numbers in lines of 8 objects and in subsets within 8 objects, perceiving and naming numbers in lines of 9 and 10 objects and in subsets within 9 or 10 objects)

Perceives at once and names the number in a variety of lines of 5 and of 6 objects...Perceives with little hesitation and names numbers in subsets within lines of 5 and of 6 objects...Uses terms of position, general size, length, shape, and quantity.

Thinks out the number in a line of 7 familiar objects, such as books, cubic blocks, containers of milk, magnetized discs, pennies...Names the number in each subset and in the set as the teacher or a child indicates these using a lace or separating the objects...Observes teacher write sentences...Proceeds similarly with many other lines of 7 objects...

Uses his own set of 7 discs to arrange in a line and to separate into subsets...Refers to his discs as he writes sentences...

6 and 1 are 7.
5 and 2 are 7.
4 and 3 are 7.
1 and 6 are 7.
2 and 5 are 7.
3 and 4 are 7.

(Continued on Page 167)
Pre-Kindergarten Activities

Names number in a line of 4 objects if he can, e.g.: blocks, cups, saucers, drums, chairs, dolls, science objects...Observes teacher separate the line of objects into doubles...Names number in each subset...Observes teacher combine the subsets...Conserves number in the set...May be able to name number and the set in a line of 4 objects, e.g.: 4 cups...

Step 1

"4"

Step 2

"2" "2"

Step 3

"4"

Proceeds similarly with other lines of 4 objects - naming number in each subset, conserving number in the set...

Kindergarten Activities

Names number and the set in a line of 4 objects...Separates the line of objects into doubles by moving 2 objects...Names number in each subset, conserving number in the set...Separates the line of objects by using a lace or a dowel stick...Names number in each subset and in set...Proceeds similarly with other lines of 4 objects and with lines of 6 objects...

Names number and the set in a line of 4 objects...Uses hands or fingers to indicate doubles...Names number in each subset and in the set...Proceeds similarly with other lines of 4 objects and with lines of 6 objects...

"3 and 3, 6"

"3 and 3 are 6."

"6 cars"

(Continued on Page 168)
Develops some proficiency in perceiving and naming the number in lines of 7 objects, and in perceiving and naming the numbers in subsets within 7 objects...

Thinks out the number in a line of 8 familiar objects...Names the number in each subset and in the set as subsets are indicated...Oberves teacher write sentences...Proceeds similarly with other lines of 8 objects...

Uses 8 discs to arrange in a line, to separate into subsets, and to write sentences: 4 and 4 are 8. 7 and 1 are 8. 1 and 7 are 8. 6 and 2 are 8. 2 and 6 are 8. 5 and 3 are 8. 3 and 5 are 8.

Thinks out the number in a line of 9 familiar objects...Names numbers in subsets as these are indicated...Oberves teacher write sentences...Proceeds similarly with other lines of 9 objects...

Uses 9 discs to arrange in a line, to separate into subsets, and to write sentences: 8 and 1 are 9. 1 and 8 are 9. 7 and 2 are 9. 2 and 7 are 9. 6 and 3 are 9. 3 and 6 are 9. 5 and 4 are 9. 4 and 5 are 9.

Thinks out the number in a line of 10 objects, e.g.: 10 beads on a lace all one color, 10 cups in a row, 10 cubic blocks in a row or stack...Names numbers in subsets as these are indicated...Oberves teacher write sentences...

Uses 10 discs and his row of 10 beads, all one color, to separate into subsets and to write sentences: 5 and 5 are 10. 9 and 1 are 10. 1 and 9 are 10. 8 and 2 are 10. 2 and 8 are 10. 7 and 3 are 10. 3 and 7 are 10. 6 and 4 are 10. 4 and 6 are 10.

Teacher Preparation: Prepares worksheets to be duplicated, emphasizing the number in a set and in each of its subsets - within 5, 6, 7, 8, 9, and 10...

Some worksheets are shown for illustrative purposes only:

<table>
<thead>
<tr>
<th>Color to show subsets.</th>
<th>Color to show subsets.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 and 1</td>
<td>3 and 3</td>
</tr>
<tr>
<td><img src="5" alt="Image" /></td>
<td>![Image](How many?)</td>
</tr>
<tr>
<td>1 and 4</td>
<td>5 and 1</td>
</tr>
<tr>
<td>![Image](How many?)</td>
<td>![Image](How many?)</td>
</tr>
<tr>
<td>3 and 2</td>
<td>1 and 5</td>
</tr>
<tr>
<td>![Image](How many?)</td>
<td>![Image](How many?)</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>

(Continued on Page 169)
Step 2.

May be able to perceive doubles within a line of 4 objects without separating subsets or using hands or fingers...

Additional Activities: Thinking out the number on a line of wedgies in a display box...

(Continued on Page 170)
### Grade One Activities (Cont.)

<table>
<thead>
<tr>
<th>How many?</th>
<th>Do not count.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Discs" /></td>
<td><img src="image2.png" alt="Discs" /></td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How many?</th>
<th>Do not count.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Discs" /></td>
<td><img src="image2.png" alt="Discs" /></td>
</tr>
</tbody>
</table>

- Draw discs. Color to show subsets.
- 4 and 1
  ![Discs](image1.png)
  How many?
  1 and 4
  How many?
  etc.

<table>
<thead>
<tr>
<th>How many lines?</th>
</tr>
</thead>
<tbody>
<tr>
<td>//</td>
</tr>
<tr>
<td>//</td>
</tr>
<tr>
<td>//</td>
</tr>
<tr>
<td>Go on.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How many?</th>
<th>Do not count.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Discs" /></td>
<td><img src="image2.png" alt="Discs" /></td>
</tr>
</tbody>
</table>

- Draw discs. Color to show subsets.
- 3 and 3
  ![Discs](image1.png)
  How many?
  5 and 1
  How many?
  etc.

- Color to show subsets.
- 4 and 4
  ![Discs](image1.png)
  How many?
  7 and 1
  ![Discs](image2.png)
  How many?
  1 and 7
  etc.

---

**Grade One Activities**

Child examines worksheet and thinks out what he should do...Listens to directions...Follows directions...Evaluates...

(Continued on Page 171)
Pre-Kindergarten and Kindergarten: Topic 5

Comparing Two Lines of Objects, Through 4 or 6 in a Line - Number, Logical Relationships

Teacher Preparation: Plans for children to locate in the classroom sets of related objects, such as nuts and bolts, toy cups and saucers, plant pots and plant dishes, containers and covers, buttons and buttonholes (on a coat), containers of milk and straws, puppets and puppet hats, dolls and doll dresses, pails and shovels, scissors and slots in a box, forms and spaces in a formboard, brushes and jars of paint, 2 lines of markers (for probability game), 2 lines of cubic blocks (built by 2 children)... Plans to arrange sets of objects into two lines, or for a child to arrange into two lines objects he is using at work or play... Plans for children to compare the sets by number - how many in each line of objects, is the number the same or different, which line has more or less objects... Plans for children to count the objects in each line and again to make comparisons... Plans for children to characterize individual objects in a line, to characterize and name the set of objects in a line, then to characterize the two lines of objects... Plans to use terms of position, general size, length, shape, and quantity...

<table>
<thead>
<tr>
<th>Pre-Kindergarten Activities</th>
<th>Kindergarten Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Locating sets of related objects; observing lines of related objects; compares by number two lines of objects - 1,2,3, or 4 in each line; counting the objects and again comparing the two sets by number; naming each object in a line; naming the set of objects; naming the two sets of objects)</td>
<td>(Locating sets of related objects; observing lines of related objects; comparing by number two lines of objects - 1,2,3,4,5, or more in each line; counting the objects and again comparing the two sets by number; naming each object in a line; naming the set of objects; naming the two sets of objects)</td>
</tr>
</tbody>
</table>

Child looks for two sets of objects that go together... Observe teacher arrange these into two lines, one line beneath the other...

Child locates sets of related objects... Arranges these into two lines or observes teacher or another child do this, one line beneath or beside the other...

Observe whether the numbers in the two sets are the same or different... Observe which set has more or less objects... Names the number in each line of objects...

Names number in each line of objects... Compares the two sets by number - number the same or different, which set has more or less objects...

Counts the objects in each line, pointing to a set of objects for each number name... Remembers the number of objects counted for...

Counts the objects in each line, indicating a set of objects for each number name... Compares the two sets by number...

(Continued on Page 172)
Comparing Two Lines of Objects, Through 10 in a Line - Number, Logical Relationships

Teacher Preparation: Plans to arrange in 2 lines sets of objects, such as those listed on page 170...Plans for children to make comparisons - whether the 2 sets match in number (are equivalent) or not, how many more or less in the first line than in the second, how many more are needed in the second line for the sets to match...Plans for children to compare numbers in two lines of objects, such as 5 with 5, 5 with 4, 5 with 3, 5 with 2, 5 with 1, 6 with 6, 6 with 5, 6 with 4, 6 with 3, 6 with 2, 6 with 1, 7 with 7, 7 with 6, 7 with 5, etc....Plans for children to count the objects in each line and again to make comparisons...Prepares worksheets to be duplicated...Plans to arrange in lines objects which have some common characteristics and some differences. Plans for children to characterize each object in a line, to name the set of objects, and to compare two lines of objects...Plans to use terms of position, general size, length, shape, and quantity...

Grade One Activities

(Comparing two lines of objects by number - 5 in one line with 5 or less in the other, 6 in one line with 6 or less in the other, 7 in one line with 7 or less in the other, etc.; counting the objects and again comparing; describing characteristics of each object in a line and of the line of objects; comparing characteristics of two lines of objects)

Child perceives and names number in each of 2 lines of objects, 5 objects in one line...Observes whether the lines match in number of objects, how many more or less in one line than in the other, how many more are needed in the line with less objects...Counts the objects in each line, indicating a set of objects (not a single object) for each number name...Compares the two sets by number...

Proceeds similarly with 5 objects in one line and other numbers of objects in the other line...Compares number in a stack of objects with the number in a row...

Uses his own discs to make comparisons...

Proceeds with 6 objects in one line and 6 or less objects in the other line, as with 5 objects in one line (preceding 3 paragraphs)...Proceeds similarly with 7,8,9, and 10 objects in one line...

Uses 2 rows of beads, all one color, to make comparisons using 10 objects in one row, as illustrated for comparing 10 with 4:

(Continued on Page 173)
Pre-Kindergarten Activities

each line...Compares the two sets by number...

\[
\begin{align*}
\text{"one"} & \\
\text{"two"} & \\
\text{"three"} & 
\end{align*}
\]

Plays "Guess Which Hand" (probability) game with the teacher (p.28,30)...Observes which line of markers has more or less...Names number of markers in each line...

Kindergarten Activities

Plays "Guess Which Hand" (probability) game with the teacher (p.28,30)...Compares numbers in the two lines of markers...

Observes other lines of related objects and compares by number...

(Continued on Page 174)
Plays "Guess Which Hand" (probability) game with another child (See p. 25, 27, 28, 30)... Compares numbers in the two lines of markers...

Follows directions and completes worksheets, as illustrated:

<table>
<thead>
<tr>
<th>How many more are needed?</th>
<th>Underline the set with less.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Image" /></td>
<td>7 plant pots</td>
</tr>
<tr>
<td></td>
<td>4 plant dishes</td>
</tr>
<tr>
<td></td>
<td><strong>How many less?</strong></td>
</tr>
<tr>
<td></td>
<td>8 pails</td>
</tr>
<tr>
<td></td>
<td>3 shovels</td>
</tr>
<tr>
<td></td>
<td><strong>How many less?</strong></td>
</tr>
<tr>
<td></td>
<td>etc.</td>
</tr>
</tbody>
</table>

Observes characteristics of each objects in a line... Describes the first object (lefthand one), then the second object, etc.... Observes the teacher list as statements the characteristics of each object as described by various children, e.g.:

- **First Jar of Paint**
  - It has red paint.
  - It is round.
  - etc.

- **Second Jar of Paint**
  - It has blue paint.
  - It is round.
  - etc.

- **Third Jar of Paint**
  - It has green paint.
  - It is round.
  - etc.

Thinks out whether a statement made by the teacher about an object or a line of objects is true or false, e.g.: The first jar is made of glass. (False) None of these jars has red paint. (False) One of these jars has red paint. (True)

Observes line of objects... Describes or names the set of objects...

Thinks out "if-then" reasoning problems presented by the teacher, e.g.: If this is the first jar, then which jar is this? (Second) If all of the jars are cylindrical in shape, then what is the shape of this jar? (Cylindrical)

(Continued on Page 175)
Pre-Kindergarten Activities

Observes lines of related objects...
...Names each object...Names each line of objects...Names both lines of objects if he can...Proceeds similarly with other lines of related objects...

Kindergarten Activities

Observes 2 lines of related objects...
Describes each object and each line of objects...Hears other descriptions...
Describes both lines of objects, indicating relationships...Proceeds similarly with other lines of related objects...

2. Concepts of Length and Weight: Using Rectangular Building Blocks of 4 Lengths

Comparing Lengths and Weights of Two Objects

Teacher Preparation: Plans for children to compare lengths of two objects, such as jars, cans, plants, building blocks, bolts, boxes for games or puzzles, books, dolls, dowel sticks, boots or shoes, strings of beads...Plans to indicate each length by moving a hand along the height or depth of the object...Plans for children to compare weights of two objects differing in length or overall size...Plans for children to compare weights of two objects differing in composition or contents...Plans to use terms of position, general size, shape, and comparative terms of length and weight (longer, not long; heavier, not heavy)...

Pre-Kindergarten Activities

(Observing 2 objects of different lengths, locating an object to match one in length, comparing 2 rectangular building blocks by length, observing 2 objects of different lengths and holding these to find comparative weights)

Child uses objects of varying lengths at work or play...

Kindergarten Activities

(Locating 2 objects of different lengths; matching 2 objects by length; arranging a set of objects by length; comparing 2 rectangular building blocks by length; estimating comparative weights of 2 objects of different lengths; testing weights by holding; estimating weights of 2 objects of different composition or contents, testing weights by holding)

Child makes explorations using objects of varying lengths - blockbuilding, arranging objects on shelves by height, cutting dowel sticks...

(Continued on Page 176)
Grade One Activities (Cont.)

Locates two sets of related objects...Arranges these in two lines...Describes the two lines of objects...Hears other children describe the same lines of related objects...Proceeds similarly with other sets of related objects...

2. Concepts of Lengths and Weight; Using Rectangular Table Blocks and Tagboard Strips of 10 Graduated Lengths

See Item 5.2, page XI.

Comparing Lengths and Weights of Sets of Objects

Teacher Preparation: Arranges sets of 2, 3, 4, or more objects for children to compare lengths, e.g., pencils, jars, boots or rubbers, books or boxes, rectangular table blocks or tagboard strips...Indicates the longest length of each object by marking a "chalk line" along the object (See p. 143)...Arranges sets of 2 or 3 objects to use for children to compare weights, e.g., rectangular table blocks of different lengths, iron pipe with a wooden pole, box full of books with a box full of clothing...Plans to use terms of position, general size, and shape, and comparative terms of length and weight...

Grade One Activities

(Comparing lengths of 2 or 3 or 4 or more familiar objects, arranging objects in order of length; comparing lengths of 2 or 3 or 4 or more rectangular table blocks or tagboard strips, arranging in order of length; selecting two blocks or strips, one double the length of the other; estimating comparative weights of 2 or 3 objects, testing weights by holding objects 2 at a time)

Child observes chalk lines drawn by the teacher to indicate the longest lengths of a variety of objects...Observes teacher move a hand along these lengths...

Compares lengths of 2 objects...Compares lengths of 3 objects...Arranges these in order of length...Proceeds similarly with 4 or more objects...Uses terms: longest, shortest, second longest, second shortest, pencil length, book length...

(Continued on Page 177)
Pre-Kindergarten Activities

Observes teacher indicate lengths of 2 objects... Observes which is longer and which is shorter... Proceeds similarly with other objects...

Locates an object as long as the one he has, e.g., a block, a jar, a dowel stick... Observes teacher indicate the lengths... Observes they are the same length... Proceeds similarly with other objects...

Compares 2 rectangular building blocks by length: 5 1/2" with 22", 11" with 22", 22" with 22"...

Counts out how many of the shorter blocks he will need to match the longer block... Checks his perception by using blocks...

Observes 2 objects of different lengths... Thinks out which is heavier... Holds the objects, one in each hand to check...

Kindergarten Activities

Locates 2 objects of different lengths... Observes teacher indicate the lengths... Uses terms: longer, shorter... Proceeds similarly with other sets of 2 objects...

Matches 2 objects by length, e.g., 2 blocks, 2 jars, 2 dowel sticks... Observes teacher indicate the lengths... Uses term: as long as... Proceeds similarly with other sets of 2 objects...

Compares sets of 2 rectangular building blocks by length: 5 1/2" block with the other blocks, 22" block with the other blocks, 11" block with the other blocks...

Counts out how many of the shorter blocks he will need to match the longer block... Checks his perception by using blocks...

Estimates comparative weights of 2 objects of different lengths... Holds the objects, one in each hand to check...

Estimates comparative weights of 2 objects of different compositions or contents, e.g., a plastic ball with a clay ball, an empty milk container with a full one... Holds the objects to check...

Using Rectangular Building Blocks for Developing Additions and Subtractions Within a Total of 4, or More, Unit Blocks

Teacher Preparation: Plans for children to select rectangular building blocks from among the blocks (See page 144.)... Plans for children to make explorations with these blocks... Plans for children to use blocks of two lengths, one twice the length of the other - to explore with these blocks, to make comparisons, and to develop additions and subtractions within 2... Plans for children then to use blocks of two lengths, one four times the length of the other - 5 1/2" blocks and 22" blocks, or 2 1/2" blocks and 11" blocks... Plans to use terms of position, length, and shape...

(Continued on Page 178)
Grade One Activities (Cont.)

Compares lengths of 2, then 3, then 4, etc., unscored rectangular table blocks or tagboard strips (unscored side)...Arranges these in order of length...Uses terms: tallest, highest, block length, strip length...

Compares lengths of 2, then 3, then 4, etc., scored rectangular table blocks or tagboard strips (scored side)...Arranges these in order of length...

Selects two familiar objects, or unscored rectangular table blocks or tagboard strips (unscored side), one object double the length of the other...May be able to proceed similarly with two objects, one 4 times the length of the other...Uses terms: twice as long, one half as long, four times as long, one fourth as long...

Estimates comparative weights of two objects...Holds, one in each hand, to check...Proceeds similarly with other sets of two objects...

Estimates comparative weights of three objects...Holds two objects at a time, one in each hand, to check...Uses terms: heaviest, lightest, second heaviest, second lightest...

Using Rectangular Table Blocks and Tagboard Strips for Developing Additions and Subtractions Within a Total of 10 Blocks

Teacher Preparation: Provides rectangular table blocks for demonstration or for children to use...Checks children's tagboard strips to find out if each child has a minimal supply (See Item 3, page 145.)...Provides other objects to arrange in lines, e.g., cubic blocks, books, discs, containers...Plans for children to compare lengths and to think out the number of units in each of 2 or 3 or more lines of objects, and of scored rectangular table blocks or tagboard strips (scored side)...Plans for children to think out the number of units if 2 scored blocks or tagboard strips are combined...Plans for children to think out the number of units if 1 scored block or tagboard strip is covered or taken away...Plans for children to develop additions within 10 and subtracting 1, 2, 3, and 4 with 10...Plans to use terms of position, length, and share...

(Continued on Page 179)
Pre-Kindergarten and Kindergarten: Topic 5

Pre-Kindergarten Activities

(Observing differences in lengths of rectangular building blocks; making explorations; comparing 2 blocks, one twice as long as the other; developing additions and subtractions within 2; comparing 2 blocks, one four times as long as the other; developing some additions and subtractions within 4)

Child makes explorations using rectangular building blocks of 4 lengths...Observes differences in lengths...Hears teacher use terms of length...May use some terms himself...

Observes 2 blocks, one twice as long as the other...Indicates which block is longer and which block is shorter...Thinks out how many of the shorter blocks he will need to cover the longer block...Places the 2 shorter blocks on top of the longer block...Places the shorter blocks in front of the longer block...

Uses 3 blocks of 2 lengths, one twice as long as the other, e.g.: two 5 1/4" blocks and one 11" block...Covers the longer block with 2 shorter blocks...Moves one shorter block down and observes there is one block left on the longer block...Observes one shorter block up and one shorter block down...Moves the other block down and observes 2 blocks down...Proceeds similarly with blocks of other sizes, e.g.: two 11" blocks and one 11" block, two 2 1/4" blocks and one 5" block...

Kindergarten Activities

(Selecting rectangular building blocks; arranging blocks in 4 piles by size; making explorations; selecting blocks of 2 lengths, one twice as long as the other, making comparisons; developing additions and subtractions within 2; selecting blocks of 2 lengths, one four times as long as the other, making comparisons; developing additions and subtractions within 4)

Child selects rectangular building blocks from among the blocks...Arranges rectangular blocks in 4 groups by length...Makes explorations...Compares blocks by lengths...Uses terms: longer-shorter, taller-higher, as long as, rectangular block, in a line...

Selects blocks of 2 lengths, one twice as long as the other...Learns that he will need 2 shorter blocks to match the longer block...Places 2 shorter blocks to match the 11" block...Selects a block twice as long as the longer block and makes comparisons...Realizes that the formerly longer block is now the shorter block when compared with a still longer block...

Uses 3 blocks of 2 lengths, one twice as long as the other...Realizes that 2 of the shorter blocks will cover the longer block. Covers the longer block with 2 shorter blocks...Realizes that if he moves one block down there will be one block left on the longer block. Moves block and observes one block up and one block down...Realizes that he moves the other block down there will be 2 blocks down. Moves block...Makes other moves and observations...Proceeds similarly with other sets of blocks...

(Continued on Page 180)
Grade One Activities

(Comparing lengths of 2 or 3 or more lines of objects, naming number of units in each; comparing lengths of 2' or 3' or more scored rectangular table blocks or tagboard strips, naming number of units in each, arranging in order of length; arranging in order of length from 3 to 10 unscored blocks or tagboard strips; selecting 2 unscored blocks or strips, one double the length of the other; selecting 2 scored blocks or strips, one double the length of the other, and naming number of units in each; combining 2 scored blocks or strips, naming the number in each and in the combination, covering or taking away one and naming the number left; combining or separating other scored blocks and strips to show the following additions and subtractions within 10: adding 1 or 2, taking away 1 or 2, adding to 1 or 2, adding 3 or 4, taking away 3 or 4, adding to 3 or 4)

Child compares lengths of 2 or 3 or 4 or more lines of objects, e.g.: rows of cubic blocks, stacks of books...Names number in each line...Compares lengths of 2 or 3 or 4 or more scored rectangular table blocks or tagboard strips (scored side)...Names number in each block or strip...Arranges in order of length...Arranges in order of length 2 or 3 or 4 or more unscored blocks or strips...(See page 145.)

Selects 2 unscored blocks or strips, one double the length of the other...May be able to select 3 unscored blocks or strips, the second double the length of the first, the third double the length of the second...Arranges in order of length...

Selects 2 or 3 or 4 scored blocks or tagboard strips, one double the length of the preceding one, e.g.: 1 unit, 2 units, 4 units, 8 units; 3 units, 6 units...Arranges in order of length...Names number of units in each...

Selects 2 scored blocks or tagboard strips, both the same length...Names number of units in each and in the combination...Covers or takes away one block or strip...Says the sentence for subtracting...

(Continued on Page 181)
Pre-Kindergarten Activities

Step 1. "2 blocks"

Step 2. "one block up"

Step 3. "one block down"

Uses 5 blocks of 2 lengths, one block 4 times as long as the other 4 blocks, e.g.: one block 22" long and 4 blocks 5½" long...Proceeds as indicated on page 146...Proceeds similarly with one block 11" long and 4 blocks 2½" long...

Teacher Preparation: Provides materials for children to use as units of length, e.g., 1"-2" straw lengths, 3"-4" dowel sticks, drum sticks, toy car lengths, rectangular building blocks...Plans for children to make bracelets using straw lengths...Plans for children to measure lengths of familiar objects by matching with a number of objects used as units of length...

(Continued on Page 182)
Grade One: Topic 5

Grade One Activities (Cont.)

Step 1.  

"5 and 5 are 10 (units)."

Step 2.  

"10, take away 5, are 5 (units)."

Selects 2 scored blocks or tagboard strips to show adding 1 and adding to 1, e.g.: 4 and 1, 1 and 4, 5 and 1, 1 and 5, 6 and 1, 1 and 6, 7 and 1, 1 and 7, 8 and 1, 1 and 8, 9 and 1, 1 and 9...Names number of units in each and in the combination...

Step 1.  

"5 and 1 are 6 (units)."

Step 2.  

Moves the unit - square.

"1 and 5 are 6 (units)."

Selects 2 scored blocks or tagboard strips to show adding 1 and taking away 1...Says the sentences...

Step 1.  

"5 and 1 are 6 (units)."

Step 2.  

"6, take away 1, are 5 (units)."

Proceeds similarly with adding 2 and adding to 2, e.g.: 3 and 2 are 5. 2 and 3 are 5, 4 and 2 are 6. 2 and 4 are 6. Etc...Proceeds similarly with adding 2 and taking away 2, e.g.: 3 and 2 are 5. 5, take away 2, are 3. 4 and 2 are 6. 6, take away 2, are 4. Etc....

Proceeds similarly with adding 3 and adding to 3, with adding 3 and taking away 3.... Proceeds similarly with adding 4 and adding to 4, with adding 4 and taking away 4...

3. Measuring Lengths of Familiar Objects Using a Rectangular Table Block or Tagboard Strip as a Unit of Length

Teacher Preparation: Provides a variety of familiar objects for children to measure, e.g., pencils, laces, boots or rubbers, "footsteps," cans or jars or boxes, books or games or puzzles, rectangular table blocks, paper, cards or charts...Provides rectangular table blocks for demonstration or for children to use as units to measure length...Checks children's tagboard strips to be used as units to measure length...Plans for children to use blocks or tagboard strips to measure lengths of familiar objects...Plans to use ordinal number names and terms relative to measuring length...

(Continued on Page 183)
Pre-Kindergarten Activities

(Stringing straws 1" to 2" long for a bracelet or necklace, naming number of straws; placing toy cars along the length of a block or book or box, naming number of cars; placing blocks along the length of a longer block or shelf or box, naming number)

Child strings 3 or 4 or 5 straws, cut into 1" or 2" lengths for a bracelet or necklace...Names number of straws...

Places toy cars on a block or along the edge of a table...Names number of cars...Hears teacher say: 4 cars long...

Observes set of 4 building blocks, such as the 5½" blocks...Arranges blocks side by side along a shelf beginning at the left...Hears teacher say: about 4 blocks long...May be able to use blocks to measure other lengths...

Selects a set of rectangular building blocks, e.g.: a set of 5½" blocks...Places such blocks side by side on a shelf to measure the length of the shelf...Places blocks side by side along the edge of a table to measure the length of the table...Uses terms: 4 blocks long, more than 5 blocks long...

Kindergarten Activities

(Stringing straws for a bracelet or necklace, comparing length of his with that of another child, naming number in each, counting if necessary; placing dowel sticks, pipe cleaners, or drum sticks along the length of a table top, a road of building blocks, a sheet of newprint paper; using blocks as units of length to measure the length of another block, a shelf, a table top, etc.)

Child cuts straws in lengths of approximately 1" or 2"...Strings 3 or 4 or 5 or more of these for a bracelet or necklace...Compares length of his with that of another child...Names number in each, counting if necessary...

Places toy cars or dowel sticks or sticks for drums or pipe cleaners along the length of a table top, or along a road of blocks, or along a sheet of newprint paper...Names number of units of length...Says: 5 cars long...

(Continued on Page 184)
Grade One Activities

(Measuring lengths of familiar objects; using sets of unscored table blocks or tagboard strips as units; using sets of unit-blocks or unit-strips; using sets of multiple-unit-blocks or multiple-unit-strips; using a single unit-block or unit-strip; using ordinal number names to indicate position of unit-blocks or unit-strips)

Child uses a set of unscored blocks or tagboard strips as units to measure the length of a familiar object. Places enough blocks or strips (all the same length) to match the length of the object to be measured...Uses ordinal number names to indicate first, second, third, etc., unit-block or unit-strip placed on or beside the object measured...Selects a set of shorter blocks or strips as units to measure the length of the same object...Realizes he needs more of the shorter unit-blocks or unit-strips to match the length of the object...

Proceeds similarly with measuring lengths of other familiar objects using unscored blocks or tagboard strips as units...Uses terms: 3 unit-blocks long, 4 unit-strips long, and ordinal number names...

Step 1. [Diagram of 2 blocks]

"The drumstick is 2 strips long."

Step 2. [Diagram of 4 blocks]

"The drumstick is 4 strips long."

Uses a set of unit-blocks or unit-strips (the shortest blocks or the shortest strips) to measure the length of a familiar object. Places enough blocks or strips to match the length of the object to be measured...Uses ordinal number names and measurement terms...Proceeds similarly with measuring other familiar objects...

Uses a set of scored blocks as multiple-unit-blocks, or scored tagboard strips as multiple-unit-strips to measure the length of a familiar object. Places enough blocks or strips to match the length of the object to be measured...Indicates the combined number of units in all blocks or strips as the measurement of the object...

[Diagram of scored strips]

Proceeds similarly with measuring lengths of other objects using scored blocks or tagboard strips as multiple-unit-blocks or multiple-unit-strips.

Uses a single unit-block or unit-strip to measure the length of an object...Begins at the end of the object, measuring-off and counting-off the number of units...May mark the object as he measures (See page 149).

(Continued on Page 185)
PRE-KINDERGARTEN AND KINDERGARTEN

TOPIC 6. GEOMETRIC CONCEPTS AND FRACTIONAL PARTS: EXPLORATION AND EXPERIMENTATION

PRELIMINARY STATEMENT

This topic deals for the most part with non-numerical aspects of mathematics. Pre-Kindergarten and Kindergarten children are encouraged to make explorations, observations, and experimentations with a variety of materials. The teacher guides children's observations so that they may develop concepts of numerosness of very small objects; concepts of fractional parts of bulk, single objects, and sets of objects; concepts of capacity and conservation of quantity; geometric concepts; concepts of weight, temperature, time, and direction.

Children have been developing geometric concepts and fractional parts, using materials suggested for this topic, since the beginning of the year. They have been making observations as they explored with bulk, liquid, and objects of different shapes. At this time (Topic 6) the teacher plans to emphasize and help children integrate the mathematics suggested for this topic.

A variety of activities are suggested for developing the mathematics of this topic. Some children may need to develop most of these activities. Other children may need to develop fewer activities or other activities than those described here.

Children perceive the numerosness of very small objects and particles in heaps, e.g., a boxful of dried beans or a bagful of peanuts in shells or a pailful of sand in a heap. Children observe that the objects or particles are too numerous for them to count.

Children divide heaps of beans or gravel or sand into halves to share with another child. The teacher observes how each child divides the material into halves. Kindergarten children divide each half into halves again to share with 3 other children. The teacher observes which children realize that after dividing they now have 2 or 4 parts, that each part is one half or one fourth of the original whole, and that the parts put together make a heap like the original heap (conservation of quantity).

Children divide a single object, such as an apple, orange, pie, or cake, into halves to share with another child. They also learn to share with another child a set of objects, such as 8 or 10 or more peanuts, buttons, pennies, or paper discs. The teacher observes which children realize that there are 2 parts of (about) the same size, that each part is one half of the original object or set of objects, and that the two parts put together are the same size or number as the original object or set of objects (conservation of quantity).

(Continued on Page 186)
GRADE ONE

TOPIC 6. GEOMETRIC CONCEPTS AND FRACTIONAL PARTS: EXPLORATION AND EXPERIMENTATION

PRELIMINARY STATEMENT

Grade One children continue to develop non-numerical aspects of mathematics, such as those outlined for the earlier grades in the Preliminary statement on facing left-hand pages, beginning on page 184. Children continue to explore, observe, and experiment with a variety of materials. They continue to develop concepts of numerosness of very small objects and particles in heaps; concepts of fractional parts; concepts of capacity and conservation of quantity; geometric concepts; concepts of weight, temperature, time, and direction.

Children perceive the numerosness of very small objects and particles in heaps, e.g., grains, seeds, sand, earth, sawdust. Children may use a magnifying glass to observe the particles more clearly. Children judge how many particles there are in a heap of grass seed or in a bowl of uncooked cereal. Judgments will be very rough (a whole lot, a million, a thousand, etc.) but judgments should indicate numerosness. Children make an effort to count the particles; they learn that counting so many particles is very difficult or impossible.

Children compare the number of small objects or particles in two heaps. First they think out which has more or less, then they study the heaps carefully to check. Illustrative comparative heaps are: 2 heaps of gravel, a heap of sunflower seeds and a heap of marigold seeds, a heap of sand and a heap of earth, a handful of sand and a pailful of sand, a tall narrow jarful of sand and a low wide jarful of sand.

Children experiment with dividing a heap into fractional parts. First they divide the heap into halves, then into fourths by dividing each half into halves, then into eighths by dividing each fourth into halves. Later, children divide a heap into thirds.

Children perceive the number of parts into which each heap was divided, whether these parts are all approximately equal in size (i.e., number of particles). They realize that the quantity of the original whole was conserved.

Children divide a single object into halves, then into fourths by dividing each half into halves. Children also divide a set of objects into halves, then into fourths. They observe and name the number 1) in the set, 2) in each half, and 3) in each fourth. They realize that the number of objects in the two halves or four fourths is the same as the number in the original set (conservation of quantity).

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Children make explorations as they pour bulk or liquid in and out of transparent or translucent containers, cylindrical in shape. Children compare capacities of cylindrical containers after considerable experimentation. They learn that the quantity does not change when poured from one container into another container of a different size (conservation of quantity). Children develop concepts of length as they compare heights (levels) of contents in containers. They develop concepts of weight as they lift containers, 2 at a time. They develop concepts of temperature as they compare temperatures of contents.

Children learn to fill containers one half full. They observe the level of the contents in each container. They learn that one half of one container is not the same quantity as one half of a container of a different size.

Children learn to measure quantities of bulk or liquid by using non-standard units of measure, e.g., a jar, a toy cup, a toy pan, a can, a pitcher.

All year long, as children used building blocks, boxes, bottles, balls, bowls, cans, etc., in work and play activities they may have sensed or perceived some of their characteristics. For this topic, the teacher stimulates further perception of differences and likenesses among such objects of different shapes. Thus, children perceive some properties of rectangular, cylindrical, and spherical objects. Children may be guided to observe that cubic (rectangular) blocks have surfaces (faces) all of which look alike. Some Kindergarten children may observe that rectangular blocks, which are not cubic blocks, have some surfaces that look alike (opposite surfaces – front and back, left and right, top and bottom). Children continue to develop concepts of position, general size, length, and weight as they work and play with rectangular, cylindrical, and spherical objects.

Heretofore, children may have sensed the concept of circular as they painted rings ("circles") at the easel, as they marched in a circular path, as they noticed the rim of the clock face. Through this topic children continue to develop the concept of circular through activities such as: moving an arm in a large circular path, moving a finger along the rim of a saucer or a cup, observing the path of a pointer as it progresses around the clock face from the numeral "1" through the numeral "12," etc. Children also develop concepts of time and direction.

For children to develop non-numerical concepts of mathematics, such as those indicated in the preceding paragraphs it is essential that the teacher provide
Grade One children continue to compare capacities of cylindrical containers. They compare sizes and shapes of several containers (transparent or translucent). They make explorations using sand or water as contents. For example, children prepare to pour the contents of a smaller container into a larger container. Children estimate level of contents in the larger container, pour, and make observations.

Children fill to the same level 2 containers of the same size and shape — later, of different sizes or shapes. They observe heights (lengths) of contents and judge comparative quantities in the 2 containers; they check by pouring the contents of each container into cups, all cups the same kind.

Children select 3 or 4 containers with covers, all containers of the same size and shape. They pour in bulk or liquid to the same level in each container. Children estimate levels (heights or lengths) of contents if one jar is turned upside down, another jar on its side, another jar tipped. They turn the jars and make observations. Through explorations such as these and others children continue to develop concepts of conservation of quantity.

Children continue to develop concepts of length as they compare heights of contents in containers (level). They continue to develop concepts of weight as they lift containers with or without contents. They develop concepts of temperature as they compare temperatures of the contents of containers, e.g., colder water, hotter cocoa, cooler orangeade, etc.

Children learn to fill containers one half and one fourth full. They judge quantities of contents in various containers as a little more than one half full, a little less than one fourth full, nearly full, etc. They judge what the level (height or length) of the contents will be if the upright (Continued on Page 189)
space, time, and opportunities for each child to experiment with bulk and liquid, to build with blocks, to sense kinesthetically the notion of circular, to share fruit and other objects, to make observations with respect to position, general size, length, weight, temperature, and time. Each child will engage in activities, such as: handling, feeling and sharing bulk; dividing objects and sets of objects for sharing with another child; playing with water, handling containers, pouring liquid in and out of containers, observing level of contents; handling, observing, and sorting simple objects of different shapes; moving the body, arms, and fingers on a circular path; etc.

The teacher uses language that is mathematically correct. For example, "rectangular block" is used, never "long square block" (which is mathematically incorrect). The terms "one half," "half," or "halves" are always related by the teacher to the objects divided, for example, "one half of the heap of sand," "one half of this bottle," etc. The teacher accepts children's language, however, casually replacing incorrect terms by correct terms. For example, a child may refer to the cylindrical box as "the round one." The teacher may say: Yes, it is round here; it's a cylindrical box. The non-technical term "round all around like a ball" is mathematically correct and may therefore be used at this level. The term "one quarter" may be confused with the coin and is better replaced with "one fourth."

The teacher makes comments somewhat casually or asks questions as a child or a small group of children are working and playing. For example the teacher may approach a child who is sitting or standing in a large carton. The teacher may ask one or more of the following questions: Is there room in there for Timothy? Does your house have any walls? How many? Can you touch your ceiling? Where is the floor? Do you need something to sit on? Where did Tommy put his chair (block)? Is the chair in a corner of the room? Is the chair in the middle of the room? Do the corners look the same outside your house? What shape is your house? Yes, it has square corners; it is a rectangular house.

Following are lists of materials appropriate for developing this topic in Pre-Kindergarten and Kindergarten classes:

**Essential Materials:** Dried beans or rice or barley, sunflower or sweet pea or nasturtium seeds, sand, apples or oranges, peanuts in shells or buttons or wheels or pennies, jars of different sizes, beakers, pitcher, balls, beads, building blocks, cubic blocks, bowls, cans, clock, milk containers, envelop, books, games or puzzles, rhythm instruments, toy vehicles, wheels...

**Additional Materials:** Magnifying glass, quoits, thermometer...

As children focus attention on the mathematical aspects of this topic they develop concepts of position, general size, length, shape, weight, temperature, time, and direction. The teacher and some children use terms such as those suggested for Topics 1-5. Other terms used by the teacher are: heap-pile, half of (an apple)-halves of-one half of, fourth of-fourths of-one fourth of, full-empty, jarful-glassful, hotter-colder, rectangular (block)-cylindrical-spherical, farther-nearer, circular, daytime-nighttime, on time-late, tomorrow-yesterday. Some of these terms may be used by the children.

(Continued on Page 190)
container or the container on its side is filled one half or one fourth full. Children tip a container to observe level of contents. Children fill several containers of different sizes and shapes one half or one fourth full, first estimating levels of contents. Children learn that the concept of one half refers to one half of something for example, that one half of a smaller container is different from one half of a larger container (or one half of an apple, or one half of a heap).

Grade One children measure capacities of containers, first using non-standard units, such as: handful, scoopful, paper cupful, etc. Later, children use a standard cup for measuring capacities. Children learn to estimate the number of cupfuls needed before they begin to measure.

Grade One children perceive properties of rectangular, spherical, and cylindrical objects; also of rectangles and circles. Children compare rectangular objects of different dimensions, cubic rectangular objects with non-cubic rectangular objects, cubic objects of different sizes, etc. Children compare cylindrical objects of different dimensions, cylindrical objects with rectangular objects, cylindrical objects with spherical objects, etc. Children draw rectangles around the bottom of rectangular objects. They draw circles around the bottom of cylindrical objects. They make observations about rectangles and circles. Children also classify objects arranging these in order of size and kind.

Children continue to develop concepts of time and direction. They observe the clock face, noting position and direction of the hour hand. Children use language, such as: after 11 o'clock, between 11 and 12 o'clock, about halfway between 11 and 12 o'clock, etc. (It might be desirable for this grade to paint the minute hand white or to paste white paper over it.)

The teacher may arrange on the classroom floor a model of a park or of streets and buildings in a project or neighborhood area. Children indicate directions to walk starting at various points.

Following are lists of materials appropriate for developing this topic in Grade One classes:

**Essential Materials:** Grains, grass or magnolia seeds, sand or earth, magnifying glass, apples or oranges, peanuts in shells or buttons or wheels or pennies, jars of different sizes with covers, beakers, pitcher, table blocks, cubic blocks, balls, beads, cans, clay, milk containers, books, games or puzzles, rhythm instruments, toy vehicles, construction paper...

**Additional Materials:** Sawdust, quoits, thermometer...

As children focus attention on the mathematical aspects of this topic they develop concepts of position, general size, length, shape, temperature, time, and direction. The teacher and some children use terms such as those suggested for Topics 1-5, as well as those suggested for the Pre-Kindergarten and Kindergarten. Other terms used by the teacher are: eighth of, eighths of, one eighth of; third of, thirds of, one third of; handful, bowlful, panful; warmer, cooler; rectangle, circle; inside, outside; straight line, curve; morning, noon, afternoon, night; day of the week, days of the week; name of current month, names of some months; day, week, month; time by the hour, time near the hour. Some of these terms may be used by the children.

(Continued on Page 191)
An outline of the contents suggested for the Pre-Kindergarten and Kindergarten follows:

1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves, into Halves and Fourths
   - Observing, Feeling, and Perceiving Numerousness of Very Small Objects in Heaps
   - Dividing Heaps of Very Small Objects into Halves, into Halves and Fourths

2. Sharing a Single Object and a Set of Objects (One Half)
   - Sharing a Single Object
   - Sharing a Set of Objects

3. Comparing Capacities of Cylindrical Containers; One Half of the Capacity of a Container; Measuring Quantities
   - Making Explorations; Comparing Capacities, Conserving Quantity;
   - Comparing Lengths, Weights, and Temperatures
   - Filling Containers One Half Full; Measuring Capacities by Using Non-Standard Units of Measure

4. Perceiving Some Properties of Rectangular, Cylindrical, and Spherical Objects; Concept of Circular; Concepts of Time and Direction
   - Perceiving Some Properties of Rectangular Objects
   - Perceiving Some Properties of Cylindrical and Spherical Objects
   - Concept of Circular; Concepts of Time and Direction

Pre-Kindergarten and Kindergarten teachers will find it profitable to read at least the Preliminary Statement for Topic 6, Grade One, on odd-numbered pages 185-191.

Pre-Kindergarten and Kindergarten teachers will consider the activities suggested for Topic 6 on the following lefthand, even-numbered pages for both the Pre-Kindergarten and Kindergarten. Teachers at these levels can then select activities which are appropriate for their children.
An outline of the contents suggested for Grade One follows:

1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves, Fourths, Eighths, and Thirds

   Perceiving Numerousness of Very Small Objects or Particles in Heaps; Dividing Heaps into Halves, Fourths, and Eighths
   Dividing Heaps into Thirds

2. One Half and One Fourth of a Single Object and a Set of Objects

   One Half and One Fourth of a Single Object
   One Half and One Fourth of a Set of Objects

3. Comparing Capacities of Cylindrical Containers; One Half and One Fourth of the Capacity of a Container; Measuring Quantities

   Comparing Capacities, Conservation of Quantity; Comparing Lengths, Weights, and Temperatures
   One Half and One Fourth of the Capacity of a Container; Measuring Capacity
   Using Non-Standard and Standard Units of Measure

4. Rectangular Objects and Rectangles; Spherical and Cylindrical Objects and Circles; Concepts of Time and Direction

   Rectangular Objects and Rectangles
   Spherical and Cylindrical Objects and Circles
   Concepts of Time and Direction

Grade One teachers will find it profitable to read the Preliminary Statement for the Pre-Kindergarten and Kindergarten on facing, lefthand pages 184-190. They will also consider the contents and pupil activities suggested for these earlier grade levels as well as those suggested for Grade One on the following, righthand, odd-numbered pages.
CONTENTS, TEACHER PREPARATION, AND PUPIL ACTIVITIES

1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves, into Halves and Fourths

Suggestion: Introduce late in the spring. Develop to the end of the school year. See Item 6.1, page XI.

Observing, Feeling, and Perceiving Numerousness of Very Small Objects in Heaps

Teacher Preparation: Provides materials, such as a boxful of dried lima beans or navy beans, several packets of sunflower or sweet pea or nasturtium seeds, a boxful of rice or barley, a pailful of sand...Plans to work with one child or a small group of children at a time...Plans for children to feel and observe the small objects or particles, to observe their numerousness in a heap, to judge which of two heaps has more or less small objects...

Pre-Kindergarten Activities

(Exploring with bulk; feeling and observing small objects; perceiving numerousness; observing material arranged into 2 heaps, same objects in each)

Child explores with bulk, e.g., sunflower seeds or dried lima beans...Takes a handful, handles, and feels the small objects...Sees the individual objects...

Observes a heap of small objects...Perceives there are individual objects...Perceives numerousness of the small objects...

Observes 2 heaps of beans or seeds...Judges which heap has more or less small objects or whether the number is about the same...

Kindergarten Activities

(Exploring with bulk and arranging in heaps; feeling and observing small objects and perceiving numerousness; judging which of 2 heaps has more or less small objects or particles)

Child handles bulk, e.g., nasturtium seeds, dried navy beans, rice or barley...Observes separate objects...Arranges in a heap...Observes numerousness of small objects...

Handles and feels sand...Observes separate particles...Observes numerousness of particles...Arranges in a heap...Observes numerousness of particles...

Observes 2 heaps of seeds or beans or rice or sand and judges which has more objects...Observes a heap of sand and a heap of rice or beans and judges which has the most particles...

(Continued on Page 194)
1. Perceiving Numerousness of Very Small Objects in Heaps; Dividing Heaps into Halves, Fourths, Eighths, and Thirds

Suggestion: Introduce late in the spring. Develop to the end of the school year. See Item 6.1, page XI.

Perceiving Numerousness of Very Small Objects or Particles in Heaps; Dividing Heaps into Halves, Fourths, and Eighths

Teacher Preparation: Provides materials, such as a boxful of rice or barley or buckwheat, packets of grass or magnolia seeds, a pailful of earth, milk containers for planting, magnifying glass...Plans to work with a group of children at a time...Plans for children to handle the material, to feel the particles and observe their numerousness in a heap...Plans for children to divide a heap of grains or seeds or earth into halves, then each half into halves for fourths, then each fourth into halves for eighths...Plans for children to plant seeds...

Grade One Activities

(Perceiving numerousness of particles in heaps; dividing a heap into halves, then into fourths, then into eighths)

Child handles grains or seeds and observes the particles...Tries to count and realizes there are too many particles...Handles earth and observes the particles, using a magnifying glass...Perceives numerousness of particles...

Divides heap into halves...Realizes he has 2 heaps now of (approximately) equal sizes and that each is called "one half"...Realizes that the 2 halves together are the same quantity as the original heap...

Divides each half into halves...Realizes he has 4 heaps now of (approximately) equal sizes and that each is called "one fourth"...Realizes that the 4 fourths together are the same quantity as the original heap...

Divides each fourth into halves...Realizes he has 8 heaps now of equal sizes and that each is called "one eighth"...Realizes that the 8 eighths together are the same quantity as the original heap...

Uses terms: half, halves, fourth, fourths, eighth, eighths...

Plants seeds...Observes plants as they grow...

(Continued on Page 195)
Pre-Kindergarten Activities

Hears teacher use terms: heap, pile, boxful, bowlful, pailful, handful... Additional Activities: Uses magnifying glass to examine small objects or particles...Tries to count the objects in a heap, realizes there are too many...

Kindergarten Activities

May be able to use terms: heap, pile, boxful, bowlful, pailful...

Dividing Heaps of Very Small Objects into Halves, into Halves and Fourths

Teacher Preparation: Provides packets of seeds, soil, and milk containers (upper part cut off)...Plans for children to divide a heap of seeds and a heap of soil into halves...Plans for kindergarten children to divide halves into halves again...Plans for children to plant seeds...

Pre-Kindergarten Activities

(Judging how to divide a heap into halves, observing there are now 2 heaps, realizing that the 2 heaps together have the same quantity as the original heap)

Child judges how to share a heap of seeds with another child...Divides heap into halves...Observes there are now 2 heaps...Realizes that the 2 halves have the same quantity as the original heap...Shares the halves with another child...

Kindergarten Activities

(Dividing a heap into halves; dividing each half into halves, realizing that the 4 heaps together have the same quantity as the original heap)

Child divides a heap of seeds into halves...Observes there are now 2 heaps...Realizes that the original quantity hasn't changed...Divides each half into halves...Observes there are now 4 heaps...Realizes that the original quantity hasn't changed...Shares fourths with 3 other children...

Pre-Kindergarten Activities

Divides heap of earth into halves...Shares the halves with another child for planting...

Kindergarten Activities

Divides heap of earth into halves, then each half again into halves...Shares the fourths with 3 other children for planting...

Hears teacher use terms: 2 halves, 1 half...

May be able to use terms: 2 halves, 1 half, 4 fourths, 1 fourth...

(Continued on Page 196)
Dividing Heaps into Thirds

Teacher Preparation: Places a heap of sand on a table...Plans for children, in small groups, to handle the sand - to feel particles, to observe particles using a magnifying glass, to perceive numerousness of particles...
Plans for children to explore with dividing the heap into thirds...

Grade One Activities

(Handling sand, feeling and observing particles, perceiving numerousness of particles, exploring with dividing a heap of sand into thirds)

Child handles sand and feels the particles...Uses a magnifying glass to see the particles...Tries to count the particles and realizes there are too many...Perceives numerousness of the particles...

Explores with ways to divide a heap of sand into thirds...Realizes that all 3 heaps must be of (approximately) equal quantities...

Experiments with dividing a heap into thirds until he develops some proficiency ...

Teacher Preparation: Prepares worksheets such as those illustrated:

Draw a line to show halves.

etc.

Draw lines to show fourths.

etc.

(Continued on Page 197)
2. Sharing a Single Object and a Set of Objects (One Half)

See Item 6.2, page XI.

Sharing a Single Object

Teacher Preparation: Provides apples or cupcakes or balls of clay, and a small knife...Plans to work with 2 children at a time...Plans for each child to judge where to cut a single object so he can share it with the other child...Plans for child to cut the object into halves...Plans to use terms, such as: about half, halves, nearly half (not "exactly") half ...

Pre-Kindergarten Activities

(Judging where to cut an object into halves, cutting object, evaluating halves)

Child observes single object, e.g., apple or cupcake...Judges where to place knife to cut it into halves...Cuts object...Observes the halves...Shares object with another child...

Proceeds similarly with ball of clay...

Kindergarten Activities

(Judging how to share an object with another child, cutting object, evaluating halves)

Child judges how to share a single object, e.g., apple or cupcake...Cuts the object into halves...Evaluates the halves...Shares object with another child...

Proceeds similarly with ball of clay...

...May be able to cut each half into halves and share the fourths with 3 other children...

Sharing a Set of Objects

Teacher Preparation: Provides a bagful of peanuts or other nuts in shells...Prepares packets of dried fruit or small cookies...Plans to work with 2 children at a time...Plans for each child to divide into halves a set of 4 or 6 or 8 or 10 nuts or packets of food and to share these with another child...

Pre-Kindergarten Activities

(Dividing 4 or 6 into halves, naming number in each half)

Kindergarten Activities

(Dividing 6 or 8 or 10 into halves, naming number in each half; dividing 5 or 7 or 9 into halves)

(Continued on Page 198)
Draw lines to show eighths.

Grade One Activities
Child follows directions and completes worksheets.

2. One Half and One Fourth of a Single Object
   and a Set of Objects
   See Item 6.2, page XI.

One Half and One Fourth of a Single Object

Teacher Preparation: Provides fruit, cupcakes, or sandwiches, and a knife
...Plans for children in groups of 2 to share food by dividing into halves
...Plans for children in groups of 4 to share food by dividing into fourths
...Prepares paper models of circles (discs), squares and rectangles (rectangular solids)...Provides scissors...Plans for children to fold models into halves or into fourths, then to cut a design at a fold or corner to illustrate symmetry...Prepares worksheets...

Grade One Activities

(Dividing food into halves, then each half into halves for fourths; folding paper into halves and into fourths, cutting designs at folds, labelling halves and fourths)

Child selects a single object to share with another child, e.g., an apple, a cupcake, a sandwich...Cuts object into halves...Evaluates the halves...Realizes that the 2 halves together are the same size as the original object...Use terms: half, 2 halves...

Selects fruit, cake, or sandwich to share with 3 other children...Cuts object into halves, then each half into halves again...Evaluates the fourths...Realizes that the 4 fourths together are the same size as the original object...Uses terms: 1 fourth, 2 fourths, 3 fourths, 4 fourths...

Selects from among a heterogeneous set of paper cutouts (circular, square, and rectangular), several of each shape...Folds one kind of cutout, e.g., discs, in half...Cuts a design at the fold or at a corner...Opens, observes the 2 halves, observes symmetrical design...Proceeds similarly with other cutouts...Colors and labels 1 half and 2 halves...

Selects cutouts, several of each shape...Folds each in half, then in half again...Cuts a design at a fold or at a corner...Opens, observes the 4 fourths, observes symmetrical design...Colors and labels 1 fourth, 2 fourths,

(Continued on Page 199)
Pre-Kindergarten and Kindergarten: Topic 6

Pre-Kindergarten Activities

Child observes heap of 4 nuts and names the number...Uses hands to divide the heap into halves...
Names number in each half...Shares set of nuts with another child...

Proceeds similarly with a set of 6 nuts...

Proceeds similarly with sets of 4 or 6 or more wrapped fruit or cookies...

Kindergarten Activities

Child names number in a heap of 6 nuts...Divides heap into halves...Evaluates...Names number in each half and in both halves together...Shares set of nuts with another child...

Proceeds similarly with sets of 8 and 10 nuts...

Proceeds similarly with sets of wrapped fruit or cookies...

Proceeds similarly with 5 or 7 or 9 nuts or fruit or cookies...Judges how to share the extra one...

3. Comparing Capacities of Cylindrical Containers; One Half of the Capacity of a Container; Measuring Quantities

Making Explorations; Comparing Capacities, Conceiving Quantity; Comparing Lengths, Weights, and Temperatures

Teacher Preparation: Provides transparent cylindrical containers of a variety of kinds and sizes, containers such as jars, tumblers, measuring cups, beakers. Provides a pitcher and water or other liquid or sand. Plans for children to work individually or in pairs to explore with filling and emptying containers. Plans to work with children to compare capacities. Plans for children to compare heights of contents in containers, weights of containers, temperatures of contents. Plans to use terms, such as full, empty, jarful, glassful, longer, shorter, heavier, lighter, warmer, cooler.

Pre-Kindergarten Activities

(Observing containers of different sizes, filling and emptying, comparing contents)

Child explores with water or sand; pouring in and out of 2 containers of different capacities...Makes observations...Hears teacher use

Kindergarten Activities

(Comparing containers of different sizes, comparing contents)

Child explores with water or sand; pouring in and out of 3 containers of different capacities...Makes observations.... May be able to use full,
Grade One Activities (Cont.)

3 fourths, and 4 fourths...

Follows directions and completes worksheets.

One Half and One Fourth of a Set of Objects

Teacher Preparation: Provides peanuts or other nuts, or wraps raisins or other dried fruit into packets... Plans for children to divide sets of nuts or packets into halves and into fourths... Prepares worksheets...

Grade One Activities

(Dividing sets of objects into halves, then each half into halves for fourths)

Child observes set of 8, 12, 16, or 20 nuts or packets and names number, or estimates number... Uses both hands to divide into halves... Names number in each half and in original set... Uses both hands to divide each half into halves again... Names number in each fourth and in the original set...

Follows directions and completes worksheets.

3. Comparing Capacities of Cylindrical Containers; One Half and One Fourth of the Capacity of a Container; Measuring Quantities

See Item 6.3, page XI.

Comparing Capacities, Conservation of Quantity; Comparing Lengths, Weights, Temperatures

Teacher Preparation: Provides transparent containers of various capacities, pitcher, sand, water or other liquid... Plans for children to observe containers and to compare capacities... Plans for children to compare heights (lengths) of levels of contents in containers, to compare weights of containers, to compare temperatures of contents in containers, to observe a large wall thermometer... Plans to use terms of capacity, length, weight, and temperature....

Grade One Activities

(Comparing capacities of a variety of kinds of containers, comparing heights of levels of contents, comparing weights of containers, comparing temperatures of contents, observing thermometer)

Child selects 3 or ¼ containers and compares... Explores, using sand or water or other liquid as contents... Pours sand or liquid from one container into another....

Selects ¼ containers of different capacities... Fills all ¼ containers with water to the same level... Checks heights (lengths) of water levels along sides of containers... "Weighs" containers by holding in hands 2 at a time... Compares weights and quantities... Uses terms: most, more, least, less, heavier, heaviest, lighter, lightest... Selects a second set of ¼ containers, all the same size... Pours water

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Pre-Kindergarten Activities

terms: almost empty, more, less....

Fills 2 containers of the same capacity to the same level, as illustrated in A and B following .... compares quantities....

Selects 2 containers of different capacities, e.g. containers C and D....Pours contents from container A into container C, and from B into D....Compares quantities by pouring contents back and forth from C to A D to B, A to C, B to D....May be able to conserve quantity....Selects 2 containers of the same capacity ....Pours liquid in one....Observes, along side of container, height (length) of contents....Pours liquid in the other, observing height (length) of contents....Compares....

Kindergarten Activities

empty, almost empty, more, less....

Fills 3 containers of the same capacity to the same level, as illustrated in A, B, and C following .... Compares quantities....

Selects 3 containers of different capacities, e.g., containers D, E, and F....Pours contents from container A into D, from B into E, from C into F ....Compares quantities....Reverses the process by pouring back into A, B and C, then again into D, E, and F....May be able to conserve quantity....Selects 3 containers of the same capacity....Pours liquid in one....Observes, along side of container, height (length) of contents....Proceeds similarly with the other 2 containers....Compares heights (lengths) of contents in the 3 containers....

(Continued on Page 202)
from first set of 4 containers into the second set of 4 containers...Observes heights (lengths) of water levels along sides of containers... "Weighs" containers...compares weights and quantities....

Step 1.

Step 2.

Step 3.

Pours water back into the first set of 4 containers, all containers of different capacities....Observes heights (lengths) of water levels along sides of containers ... "Weighs" containers...Compares weights and quantities...Touces water in all containers and compares temperatures....

Observes classroom wall thermometer....Observes cylindrical tube, fluid, level of fluid, and numerals...Learns that fluid rises when the air becomes warmer, and vice versa....May be able to read the temperature....

(Continued on Page 203)
Pre-Kindergarten Activities

Selects 2 containers of the same capacity....Pours liquid in one....Observes, alongside of container, height (length) of contents....Pours liquid in the other, observing height (length) of contents....Compares....

Selects 2 containers of the same capacity....Pours water in each....Holds one container in each hand to "weigh"....Experiments with different quantities of liquid....Compares temperature of contents....Uses terms: heavier, lighter, warmer, cooler....

Kindergarten Activities

Selects 3 containers of the same capacity....Pours liquid in one....Observes, alongside of container, height (length) of contents....Proceeds similarly with the other 2 containers....Compares heights (lengths) of contents in the 3 containers....

Selects 2 or 3 containers of the same capacity....Pours water in each....Holds two containers at a time, one each hand, to "weigh"....Experiments with different quantities of liquid....Compares temperatures of contents....Uses terms: heavier, lighter, heaviest, lightest, warmer, cooler, warmest, coolest....

Filling Containers One Half Full; Measuring Capacity by Using Non-Standard Units of Measure

Teacher Preparation: Provides transparent containers of various capacities, pitcher, liquid....Plans for children to engage in activities in which they fill containers one half full....Plans for children to compare one half the capacities of 2 or 3 containers, conserving quantities....Plans for children to measure capacities using non-standard containers, such as toy cups, tumblers, other small containers....Plans to use terms, such as 4 toy cups full, one half of the jar....

Pre-Kindergarten Activities

(Filling 2 containers, each one half full; comparing contents; measuring capacities of containers using a non-standard unit of measure)

Child fills a container one half full using water or other liquid, observes height (length) of the column of contents, evaluates....Uses the liquid to water a plant or to drink....Proceeds similarly with other containers....

Kindergarten Activities

(Filling 3 containers, each one half full; comparing contents and conserving quantities; measuring capacities of containers using non-standard units of measure)

Child fills a container one half full....Evaluates....Adjusts....Uses the liquid in an activity - snacks, house play, dramatic play....Proceeds similarly with other containers....

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One Half and One Fourth of the Capacity of a Container; Measuring Capacity Using Non-Standard and Standard Units of Measure

Teacher Preparation: Provides a variety of transparent containers, a pitcher, water or other liquid, standard transparent cups....Plans for children to fill containers one half and one fourth full....Plans for children to compare contents of several containers of different capacities that have been filled to one half or to one fourth full....Plans for children to use non-standard units and a standard cup for measuring capacities of containers....

Grade One Activities

(Filling containers one half full and one fourth full, using non-standard units to measure capacities, using a standard cup to measure capacities)

Child observes a container....Thinks out level of contents if a container is to be filled one half full....Puts sand or liquid in....Observes level of contents ....Evaluates and adjusts....Proceeds similarly with other kinds of containers ....

Thinks out level of contents if a container is to be filled one fourth full (one half of one half)....Puts sand or liquid in....Observes level of contents ....Evaluates and adjusts....Proceeds similarly with other kinds of containers ....

Selects 3 or 4 containers of different capacities....Pours liquid into one container to one half full....Observes level of liquid, evaluates, and adjusts....Proceeds similarly with a second container....Compares liquid levels in the two containers....Evaluates again to see if each is filled to one half full....Fills third container to one half full....Compares liquid levels of all 3 containers ....Evaluates again to see if each is filled to one half full....Proceeds similarly with a fourth container....Concludes that "one half" refers to one half of something....

Selects 3 or 4 containers of different capacities....Fills each to one fourth full, proceeding as for filling containers one half full in preceding item....Concludes that "one fourth" refers to one fourth of something....

(Continued on Page 20f)
Pre-Kindergarten and Kindergarten: Topic 6

Pre-Kindergarten Activities

Observes 2 containers of different capacities....Fills each one half full of water....Compares contents of the 2 containers....May be able to realize that each container is still one half full (conservation of quantity)....

Uses a small container to measure the capacities of larger containers ....Proceeds similarly with another non-standard unit of measure....

Makes play dough using: 1 container of flour, 1 container of salt, 1 container of water....

Kindergarten Activities

Selects 3 containers of different capacities....Fills each one half full of water....Compares contents of the 3 containers....Observes each is still one half full (conservation of quantity) ....Realizes the "one half" refers to a specific container.

Uses a non-standard container to measure capacities of containers of various sizes....Proceeds similarly with other non-standard containers as units of measure....Makes play dough using 1 container each of flour, salt, and water ....

4. Perceiving Some Properties of Rectangular, Cylindrical, and Spherical Objects; Concept of Circular; Concepts of Time and Direction

See Item 6.4, page XI.

Perceiving Some Properties of Rectangular Objects

Teacher Preparation: Provides large cartons, rectangular building blocks, cubic blocks, mystery boxes and other boxes, books, construction paper, scissors, magazines....Plans to emphasize properties of rectangular objects with children who are playing with boxes or blocks, using books, constructing collages....Plans to use terms,such as rectangular, cubic, corner, edge, surface....

Pre-Kindergarten Activities

(Observing rectangular objects, perceiving some properties, hearing names of objects)

Child observes a large carton.... Pretends it is a house....Crawls into carton....Observes ceiling, floor, and corners....Crawls out of carton ....Observes roof, sides, and corners of the house....

Kindergarten Activities

(Observing rectangular objects, perceiving some properties, identifying objects)

Child observes sizes of several cartons....Selects one carton to be an engine of a train....Arranges other cartons in a train....Takes his turn sitting in first carton as engineer or in another carton as a passenger ....Observes inside and outside surfaces, corners, and edges of cartons....

(Continued on Page 206)
Selects 4 containers with covers, all containers of the same capacity and shape....Fills each with water one half full....Compares levels of contents....Arranges containers as follows: one upright, one upside down, one on its side, one tipped....Oberves water levels....

Proceeds as with the preceding 4 containers but fills each to one fourth full....Oberves water levels.

Selects a container to fill with sand....Selects a unit of measure, e.g., handful, toy cupful, bowlful, small container full, standard cupful....Estimates how many units of measure he will need to fill the container, records his estimate....Fills container, counting number of units of measure....Compares with his estimate....Empties container and proceeds similarly using another unit of measure....

Selects several containers of different capacities....Selects a unit of measure for filling containers with water or other liquid....Estimates how many units of measure will fill each container, records estimates....Fills each container, counting number of units as he fills....Compares with his estimate....Proceeds similarly using another unit of measure....Uses a standard measuring cup to help in the preparation of juice or pudding....

4. **Rectangular Objects and Rectangles:** Spherical and Cylindrical Objects and Circles; Concepts of Time and Direction

See Item 6.4, page XI.

**Rectangular Objects and Rectangles**

Teacher Preparation: Provides unscored rectangular table blocks of different sizes, tagboard strips of different lengths, cubic blocks, books, rectangular boxes, milk containers, cards or signs, construction paper....Plans for children to perceive properties of rectangular objects, including cubic objects....Plans for children to perceive properties of rectangles, including squares....Plans to use terms, such as rectangular, cubic, cube, corner, edge, surface, line, right angle, side....Plans to use other terms, rectangular and square, to indicate boundaries of figures, not for objects that can be seen or held....

(Continued on Page 207)
Pre-Kindergarten Activities

Observes rectangular building blocks of 4 different sizes....Holds a block....Feels surfaces, corners, edges....Compares two blocks, observes differences and similarities....

Selects set of rectangular blocks from among the building blocks....Holds a block....Feels surfaces, corners, and edges....Tries to count these....Compares blocks, observes differences and similarities....

Observes set of cubic (rectangular) blocks all same size....Holds a block....Feels surfaces, corners, edges....Compares 2 cubic blocks....Stacks cubic blocks....

Observes set of cubic (rectangular) blocks, all same size....Holds a block....Feels and tries to count surfaces, corners, edges....Realizes that all surfaces are the same size....Compares with other cubic blocks....Compares with rectangular building blocks....Stacks blocks by kind....

(Continued on Page 208)
Grade One Activities

(Observing rectangular objects, perceiving some properties of rectangular objects, drawing rectangles and squares, perceiving some properties of rectangles and squares)

Child observes and handles unscored rectangular table blocks...Studies one block and perceives surfaces (6), corners (8), and edges (12)...Counts these....Perceives that opposite surfaces are the same size....Studies another block, and proceeds similarly....Compares blocks....

![Rectangular Blocks]

Observes and handles tagboard strips, unscored side....Readily perceives 4 of the 8 corners, 4 of the 12 edges, and 2 of the 6 surfaces....May be able to perceive other corners and surfaces....Compares strips....

Observes and handles cubic table blocks....Perceives corners, edges, and surfaces....Counts these....Sees that all 6 surfaces are the same size....Sees that all 12 edges are the same length....Compares cubic blocks....

Locates rectangular objects around the classroom, e.g.: boxes, cards, signs, table tops, desk tops, milk containers, blocks, books, paper, pads....Perceives surfaces, corners, edges....Identifies objects....Compares rectangular objects ....

Selects a rectangular object, e.g.: mystery box, book, crayon box....Traces around the object on a sheet of construction paper....Removes object and observes drawing of 4 lines (line segments)....Moves finger along the lines....Compares lengths of opposite lines (sides of rectangle)....Perceives square (right)angles....Uses the term, rectangle....

![Drawing a Rectangle]

Selects a cubic block....Traces block on paper....Perceives drawing of 4 lines (line segments)....Compares lengths of sides of square....Perceives square angles....Uses term, square....

(Continued on Page 209)
Pre-Kindergarten and Kindergarten: Topic 6

Pre-Kindergarten Activities

Feels large beads in a mystery box....
May be able to pick out a cubic bead or other rectangular bead....

Kindergarten Activities

Feels large beads in a mystery box....
Picks out all rectangular beads including cubic beads....

May be able to locate rectangular objects in a play area or around the classroom....

Observes cutouts from a magazine of rectangular objects....May be able to arrange some of these on construction paper as a collage....

Observes cutouts of a variety of rectangular shapes....May be able to arrange some of these on construction paper as a collage....

Locates rectangular objects around the classroom....

Locates pictures of rectangular objects in a magazine....Cuts these out....Arranges on construction paper collage....

Selects several from a variety of cutouts of rectangular shapes.... Arranges on construction paper as a collage....

Perceiving Some Properties of Cylindrical and Spherical Objects

Teacher Preparation: Provides cylindrical objects, such as drums or tambourines, cylindrical building blocks, cylindrical containers, and cylindrical foods with ends cut off - sausage or cucumbers....Plans for children to perceive some properties of cylindrical objects....Provides spherical objects, such as balls and beads and provides nearly spherical foods - apples, oranges, onions, grapes, cherries, tomatoes....Plans for children to perceive some properties of spherical objects....Provides rectangular objects, such as those used previously....Plans for children to compare rectangular, cylindrical, and spherical objects....

Pre-Kindergarten Activities

(Perceiving some properties of cylindrical objects and of spherical objects, comparing cylindrical with spherical objects)

Kindergarten Activities

(Perceiving some properties of cylindrical and spherical objects, comparing cylindrical with rectangular objects, comparing cylindrical with spherical objects)

(Continued on Page 210)
Grade One Activities (Cont.)

Draws several rectangular figures, including squares, on construction paper....Compares lengths of sides of figures....Observes square (right) angles....Compares squares with other rectangles....

Uses the corner of a 3" x 5" card to measure square (right) angles in a variety of figures....Labels figure with all 4 sides the same length "square"....Labels all figures with 4 square (right) angles "rectangle"....

Spherical and Cylindrical Objects and Circles

Teacher Preparation: Provides spherical objects, cylindrical objects, rectangular objects, and rings or discs....Plans for children to perceive some properties of a spherical object and to compare it with a rectangular object....Plans for children to perceive some properties of a cylindrical object, and to compare it with a rectangular and a spherical object....Plans for children to move a finger around the rim of a cylindrical container and to observe circular shape....Plans for children to cut a spherical object into halves, to observe the flat circular surface and to move a finger around the outside of the flat surface....Plans for children to locate circular objects, such as a wheel, and to trace a finger around the circular part....Plans for children to draw circles, radii, chords, and to make comparisons....Prepares worksheets....

Grade One Activities

(Perceiving some properties of spherical and cylindrical objects; comparing spherical, cylindrical, and rectangular objects; observing a hemispherical object, its flat surface, and the circular outside of the surface; observing discs and wheels and the circular outside; drawing circles, radii, and chords and making comparisons)

Child observes spherical, cylindrical and rectangular objects, such as balls or beads, containers, unscored rectangular table blocks or boxes....Perceives some properties of the spherical object, e.g., can roll, round all around, no flat surface....Compares spherical object with the rectangular object....

(Continued on Page 211)
Pre-Kindergarten Activities

Child participates in a rhythm band....
Selects a drum or tambourine or tom tom....
Observes circular striking surfaces at top and bottom....
Observes that it can roll....
Hears teacher use term: cylindrical....

Plays a rolling game with the cylindrical block....
Stands block on the floor....
Observes and feels flat surfaces....
Rolls block....
Observes and feels curved surface....
Hears teacher use term: cylindrical block....

In house-play center observes cylindrical objects, such as: cans, oatmeal boxes, plastic containers, jars....
Observes and feels flat surfaces....
 Rolls object....
Observes and feels curved surface....

Observes other cylindrical objects in the classroom, e.g., unsharpened cylindrical pencil, unused chalk....
Makes cylindrical objects, using clay....

Observes on a tray cylindrical ingredients for an antipasto salad, e.g., cucumber, bologna, salami....

Kindergarten Activities

Child participates in a rhythm band....
Selects drum, tambourine, and tom tom....
Observes that all of these instruments have 2 flat and circular striking surfaces....
Observes the curved surface....
Uses term: cylindrical....

Plays a rolling game with the cylindrical block....
Observes and feels the two circular flat surfaces....
Observes and feels the curved surface....
Uses terms: surface, cylindrical block....

In house-play center observes cylindrical objects....
Observes and feels the two circular flat surfaces....
Observes and feels the curved surface....

Observes other cylindrical objects in the classroom....
Makes cylindrical objects, using clay....

Observes on a tray cylindrical foods e.g., cucumber, bologna, salami....
Observes that each slice, after cutting, is also cylindrical in shape....

(Continued on Page 212)
Perceives some properties of the cylindrical object, e.g., can stand, 2 flat circular surfaces, one curved surface, can roll....Compares cylindrical object with the rectangular object....Compares cylindrical object with the spherical object....Locates and names other rectangular, spherical, objects around the classroom....Uses terms: rectangular, spherical, cylindrical....

Observes the circular rim of a cylindrical container....Moves a finger around the rim....Moves his arm in the air in a circular path to describe the rim....

Cuts a spherical object, such as an orange or an apple, into halves....Observes flat circular surface of the half-apple (hemisphere shape)....Moves a finger around the outside of the flat surface....Moves his arm in the air in a circular path to describe the outside of the flat surface....

Locates and observes circular objects around the classroom, e.g., wheels on a toy vehicle, quoits, discs, magnifying glass lens, buttons, pennies....Moves a finger around each rim....Moves his arm in the air in a circular path to describe a rim....

Draws a circle....Estimates its center, and places a dot....Draws two more circles and places dots to indicate centers....Selects his best "circle" and tells why it is best....

Traces around a wheel or button or disc, or around the circular top of a bowl or a cup or a saucer or a cylindrical object....Places a dot to indicate the center of traced circle....Compares this circle with his best freehand "circle"....Tells why the traced circle is better....

Traces two circles, one touching the other....Indicates centers by placing dots....

(Continued on Page 213)
Pre-Kindergarten Activities

Plays a game with another child, using a large rubber ball, e.g., "Pass the Ball" or "Ball-Rolling Game".... Handles ball, bounces it, rolls it.... Observes and feels the curved surface.... Learns teacher use term: spherical....

Observes spherical objects in a box, e.g., balls of various sizes, marbles, beads.... Handles each, feels curved surface, rolls it....

Observe on a tray foods for a salad, e.g., apples, grapes, oranges, plums, cooked peas, cooked green beans, bananas, cherry tomatoes.... Indicates those which are spherical or nearly spherical....

Kindergarten Activities

Plays a game with another child or with several children, using a large rubber ball.... Handles, bounces, and rolls ball.... Observes and feels the curved surface.... May use the term: spherical....

Places in a box a number of spherical objects.... Handles, feels curved surfaces, and rolls each....

Observe on a tray foods for a salad.... Names those which are spherical or nearly spherical.... Notes differences and similarities between two spherical foods....

(Continued on Page 214.)
Grade One: Topic 6

Grade One Activities (Cont.)

Traces two circles, one intersecting the other....Indicates centers by placing dots....

Uses buttons of two sizes, traces concentric circles....Indicates center....

Draws freehand, or traces circles to make a design or to represent an animal or snowman....

Uses rim of a cup to trace a circle. Indicates center....Draws another circle freehand about the same size as the traced circle....Indicates center....Compares the two circles....Draws 3 or 4 radii in each circle....Compares lengths of radii....Realizes that the radii of the traced circle are equal lengths....Discusses why this is not so in his freehand circle....

Traces a circle and indicates center....Draws 4 parallel chords....Realizes that chords closer to the center are longer than the others

(Continued on Page 215)
### Pre-Kindergarten Activities

- Observes a cylindrical and spherical object... Compares....
- Plays with clay... Makes cylindrical and spherical objects....

### Kindergarten Activities

- Selects a cylindrical and a rectangular (or cubic) block... Compares... Selects a cylindrical and spherical object.... Compares....
- Using clay makes cylindrical and spherical objects....

#### Concept of Circular; Concepts of Time and Direction

**Teacher Preparation:** Plans for children to make circular movements, such as outlining a ring in a painting or drawing, moving arms in a circular path, indicating paths around finger and wrist and neck and ankle, marching in a circular path... Provides circular objects for children to outline... Plans for children to make circular movements in a clockwise direction... Plans to call attention to position and direction of hands and numerals on the clock... Plans a maze to be placed on the classroom floor... Plans for children to indicate directions on the maze... Uses terms, such as circular, daytime, nighttime, on time, late, tomorrow, yesterday....

### Pre-Kindergarten Activities

- (Making circular movements, making clockwise movements, observing hands and numerals on the clock, noting directions on a maze)

- Child observes a ring ("circle") on his or another child's painting.... Moves his brush or finger around the outline.... Moves his finger and arm in a circular path....

- Moves a finger around a bracelet on his wrist, a ring on his finger, a necklace on his neck....

- Moves a finger around circular objects, e.g., around the top of a cylindrical container, around the rim of a saucer or a cup....

### Kindergarten Activities

- (Making circular movements, making clockwise movements, observing direction of hands on clock, observing direction of numerals on the clock, noting directions on a maze)

- Child observes a ring ("circle") on a painting.... Moves his finger or brush around the outline.... Notes that he can go "around and around".... Moves his arm in the air in a circular path....

- Outlines on himself or another child a bracelet, a ring, a necklace, an anklet, a belt... Makes similar movements in the air....

- Outlines circular objects, e.g., top of cylindrical container, rims of cups and saucers, quoits, pennies or discs... Makes similar movements in the air....

(Continued on Page 216)
Grade One Activities (Cont.)

....Draws a chord through the center....Realizes this is the longest chord....

Uses worksheets asking him to use crayon of a certain color to outline all rectangles or squares or circles, to draw a line from the word to the figure it names, to draw the next two figures (given 1 larger, 1 smaller, 1 larger, 1 smaller circle or square or rectangle), to place a dot in each circle where he thinks the center is, to draw a circle around each square....

Concepts of Time and Direction

Teacher Preparation: Plans for children to use a monthly calendar and to develop concepts of day, week, and month, and to learn names of the days of the week, the current month, some other months....Plans to cover the minute hand with white paper for children to focus attention on the hour hand....Plans for children to develop concepts of time - to observe direction of numerals and hands on the clock, and to interpret time on and near the hour and between 2 hours....Prepares worksheets dealing with numerals on the clock and with time telling by the hour, by the half hour, between two hours....Prepares a maze to be placed on a table or on the floor....

Grade One Activities

(Reading and interpreting a calendar; telling time by the hour, near the hour, between 2 hours; noting directions on a maze)

Child locates a monthly calendar in the classroom....Finds the name of the current month....Observes and reads names of the days of the week....Finds name of the current day....Finds numeral for the current day of the month....

Uses calendar to find name of tomorrow's day....Finds name of yesterday's day....Finds numeral for tomorrow's day of the month....Finds numeral for yesterday's day of the month....Finds names of other designated days....Finds numerals for other days of the month....

Observes numerals on a clock....Reads the numerals from 1 through 12....Observes direction of the numerals....Moves a hand and arm, clockwise, in the direction of the numerals, beginning with 6, with 3, with 9, with other numbers....

Observes hour hand....Observes it is the shorter hand....Observes that the minute hand is covered with white paper....Observes direction of the hands on the clock....

(Continued on Page 217)
Pre-Kindergarten Activities

Stands with other children in a circle
....Moves with the children in a circular path....

Observes clock....Observes the 2 hands and the numerals....Observes teacher move a finger or pointer around the clock face - clockwise....Moves arm in a clockwise path in the air....

Kindergarten Activities

Stands outside, inside, or in the center of a circle in song-plays: Blue-bird, Five Little Chickadees, Did You Ever See a Lassie?....

Observes hands and numerals on the clock....Observes direction of numerals from "1" to "12" as teacher points....Names numerals as teacher points....Observes direction minute hand moves....Watches minute hand move 1 minute and 5 minutes....Moves arm in a clockwise circular path in the air....

(Continued on Page 218)
Grade One Activities (Cont.)

Observes position of the hour hand on the clock for events that occur regularly on the hour, e.g., 9 A.M. bell, 3 P.M. bell....Observes position of hour hand for other events that occur on the hour, e.g., TV programs, time to leave for a trip....Tells time by the hour....

2 o'clock

6 o'clock

4 o'clock

Observes position of the hour hand when it is before or after the hour....Tells time, e.g.: after 2 o'clock, before 4 o'clock, nearly 6 o'clock....

Observes position of hour hand when it is half way between 2 hours....Tells time, e.g.: between 2 and 3 o'clock, half way between 5 and 6 o'clock....

Follows directions on worksheets....

(Continued on Page 216)
Pre-Kindergarten Activities

Observes plan of play park (maze) on floor....Observes walks....Observes swing area....Observes slide area....Locates the entrance to the park....Steps along walks to the swing area....Notes that he makes turns....Walks back out, noting turns....Walks to slide area, noting turns....Returns....Observes other children walk in the "park" and notes directions....

Kindergarten Activities

Observes plan of play park (maze) on the floor....Observes walks....Observes swing area and slide area....Locates entrance....Walks to swing area, counting number of turns he makes....Returns to entrance, counting number of turns....Walks to slide area, counting number of turns....Returns counting number of turns....Observes other children walk in the "park" and observes directions....May suggest equipment for other 3 play areas for the park....
Observes plan of a neighborhood (maze) on a table top....Observes streets....Locates school building, apartment building, and store....Locates the entrances (or exits)....Selects an entrance and indicates how he would walk to the school building....Indicates when he turns right or left....Counts turns....Tries the other entrance, indicates directions, and counts turns....Notes which entrance leads to the fewer turns....Uses terms: left, right, forward....

Proceeds similarly with ways to reach the apartment building....

Proceeds similarly with ways to reach the store....

Observes the other 3 areas....Suggests what might be built in these areas, e.g., a parking lot, a ball park, a park, another apartment building....
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