This report represents a survey of some of the school mathematics contests which are conducted in the United States. The report is based on information obtained from fifty-nine national, state, regional, and local level contests. National contests are described separately. Brief accounts of certain school mathematics contests, the extent of such contests, contest procedures and practices, and advantages and disadvantages for establishing a mathematics contest are considered. A bibliography of articles which provide information about mathematics contests is included. Names and addresses of organizations which sponsor the contests reported in this study are also provided. (FL)
A REPORT

SCHOOL MATHEMATICS CONTESTS

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
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Preface

This report represents a compilation of information on some of the school mathematics contests in the United States. A questionnaire was used as the basic vehicle for collecting the information. Other data solicited included sample copies of rules, regulations, procedures, constitutions, and contest questions.

Originally, notices requesting information as to the whereabouts of contests were sent to knowledgeable people in the field of school mathematics, including state supervisors of mathematics, local supervisors of mathematics, officers of Affiliated Groups of the National Council of Teachers of Mathematics, and various other individuals. In addition, a notice was printed in the December 1965 issue of the Bulletin for Leaders, published by the Council.

The response to the request was excellent. As a result, questionnaires were sent out to ninety-three individuals identified as being affiliated with a contest. From this group fifty-nine usable replies were obtained, representing individual contests on a state, regional, or local level. The main portion of this report is based on information concerning these fifty-nine contests. National contests are described separately.

No claim is being made that this represents an accurate survey of school mathematics contests in the United States, since it is known that there are a number of contests on which no information was received. However, it does represent a reasonable indication of the scope, purpose, and practice of contests as a medium of enrichment and motivation in the field of school mathematics. Hopefully, this report can serve as background information and as a general reference for others who are interested in contests.

I wish to take this opportunity to express my gratitude to each of the many people who contributed to this report, and especially to the individuals who took the time and effort to submit information about their contests. Without their excellent cooperation and willingness to share their information, the report would not have been possible.

Howell L. Gruver

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Growth and Extent of School Mathematics Contests

Mathematics contests do not represent a new method of enrichment for the school mathematics program in this country and abroad. Contests date back some seventy years in Hungary and some forty years in Russia. In this country, the contest of the Interscholastic Mathematics League of New York City traces its origins to the early 1920's, when it began as an algebra test. This is perhaps the longest continuously operating contest in the United States.

The University of Texas Interscholastic League sponsors two areas of mathematics competition as part of a broad, statewide program of interscholastic academic and athletic competition. One of these is the Number Sense Test, which began in 1924. The second, the Slide Rule Contest, was added in 1943.

In recent years there have been strong indications that contest activity is increasing, both in the number of contests and in the number of students participating. A study of contests in 1956 dealt with forty-three contests involving approximately 4,200 schools, and it was estimated that there were probably some sixty or more contests in operation at that time.¹

The questionnaire used for this present report asked for data for the school years 1960/61 to 1964/65 that would be used to indicate increased contest participation or the lack of it. The resulting totals are approximations, since the data were not reported on several replies and were estimated on several others. Some of the results include the following:

1. Of the fifty-nine contests reported, thirty-two were begun during or after 1960. Approximately 1,500 schools were involved in the contests.

in operation in the school year 1960/61. Approximately 4,500 schools were involved in the contests in operation in the school year 1964/65.

2. The number of students participating more than doubled. Approximately 40,000 students participated in the contests in operation in the school year 1960/61 and more than 80,000 students in the 1964/65 school year.

3. Contest activity is concentrated heavily at the senior high school level, senior high schools being defined as schools that include Grades 10, 11, and 12. However, there are indications that contest activity is increasing at other levels, especially in the junior high school. This activity includes both separate contests and provision for participation in existing senior high school contests by lower grade levels.

4. The data indicated that contests are largely a public school activity. Approximately ten times as many public schools as parochial and independent private schools participated in contests.

No attempt has been made to assess the effect of the National High School Mathematics Contest on state, regional, or local contests. National and international participation in this contest, a project of the Mathematical Association of America, has increased tremendously since its inception in the spring of 1958: from 80,000 participants in more than 2,600 high schools to more than 200,000 in some 5,500 high schools. A few more details of this contest are described in a later section.

To summarize briefly, indications are that there have been marked increases in the number of mathematics contests, the number of participating schools, and the number of participating students.

Although some students probably participate in more than one contest, it seems reasonable to estimate that, with the inclusion of the MAA National Contest, there are more than a quarter of a million participants annually in school mathematics contests in the United States.
T MAY be quite presumptuous to attempt to categorize the reported contests, since each is unique in one or more aspects. However, for purposes of description, an attempt will be made. Most of the contests can be generally described by one of the following patterns: the single-test, short-answer contest; the mathematics league; the mathematics field day, incorporating a variety of competitive mathematical activities; problem-solving competition; and the mathematics fair.

THE SINGLE-TEST, SHORT-ANSWER CONTEST

This consists of a single test composed of a number of questions or problems, usually of an objective nature. The test is completed at one sitting within a specified time limit and is given only once during the school year.

The number of problems varies widely, but generally ranges from fifteen to fifty. The problems are designed to be answered by short responses. These may include true-false and/or multiple-choice answers.

This type of contest has several advantages: (1) It lends itself to large numbers of participants. (2) It can be administered over a large geographical region with relatively little difficulty, since it is not necessary to bring contestants together at one central contest site. The test can be given in a number of regional sites—or in individual schools, for that matter. (3) Grading procedures are relatively simple, since the answers are usually either right or wrong. If facilities are available, it can be machine-graded. (4) Fewer people per number of contestants are needed to administer it. This is not to imply, however, that administration of this type of contest is a simple job. Because of its very nature there are usually a large number of students participating, and many people are needed to administer it. One of the striking features about any of the reported contests is the number of teachers, professors, supervisors, and others who volunteer their time and effort to assist in conducting a contest.
School Mathematics Contests: A Report

The above description of this type of contest is general, and several variations were reported.

One of these is to have the contest consist of two parts, a preliminary test and a final one. Students who score well in the preliminary test are invited to participate in the final one, given at a later date. This may be more difficult, and it may include problems requiring more perception on the part of the students.

Another variation is to substitute for a test that is uniform for all grade levels two or more tests designed for different grade levels. Even where only one test is given to all students, grading and scoring provisions often take different grade levels into account.

Sample Problems

1. Let the operation \(^*\) be defined for the set of odd integers by
\[ a \ast b := (a + b) - 1. \]
Which of the following is false?

a. \(5 \ast 3 = 7\).

b. The set of odd integers is closed with respect to \(\ast\).

c. There is no identity element for this system.

d. The operation \(\ast\) is associative in the set of odd integers.

e. The operation \(\ast\) is commutative in the set of odd integers.

2. Which of the following is not equivalent to the remaining four?

a. \(\log_4 8\)  
b. \(\log_{10} 81\)  
c. \(\log_9 27\)  
d. \(\log_e 125\)  
e. \(\log_{10} 216\)

3. The ratio of the areas of two concentric circles is 1:3. If the radius of the smaller is \(r\), then the difference between the radii is best approximated by

a. 0.41\(r\)  
b. 0.73\(r\)  
c. 0.75\(r\)  
d. 0.73\(r\)  
e. 0.75\(r\)

4. The number of degrees in one interior angle of a regular polygon is \(x\). The number of sides of the polygon in terms of \(x\) is

\[ \frac{x + 360}{180}, \quad \frac{180}{x}, \quad \frac{360}{180 + x}, \quad \frac{360}{x}, \quad \frac{360}{180 - x}. \]

5. \(A\) varies directly as the cube root of \(B\), and \(B\) varies inversely as the sixth power of \(C\). Then \(C\) varies inversely as what power of \(A\)?

a. \(\frac{1}{2}\)  
b. 2  
c. 18  
d. \(\frac{1}{6}\)  
e. \(\frac{1}{3}\)

6. If \(x^3 - kx + 8\) is exactly divisible by \(x - 4\), the value of \(k\) is

a. 2  
b. 66  
c. 6  
d. -6  
e. 18

7. A square \(PQRS\) has side \(PQ\) along the \(x\)-axis (\(Q\) to the right of \(P\)): vertex \(R\) is on the line \(2x + y - 12 = 0\), and vertex \(S\) is on the line \(x - y - 1 = 0\). What is the perimeter of the square?

a. \(8\)  
b. 4  
c. \(\frac{16}{3}\)  
d. 6  
e. 8
8. The sides of a right triangle are represented by three consecutive even integers. The smaller acute angle is bisected and the bisector, produced to the opposite side. Find the length of the longer segment into which that side is divided by the bisector.
   a. $3\frac{1}{2}$  b. $2\frac{1}{2}$  c. 4  d. 3  e. $1\frac{1}{2}$

9. The difference of the squares of two odd numbers is always divisible by
   a. 3  b. 5  c. 6  d. 7  e. 8

10. If $x$ and $\log_{10} x$ are real numbers and $\log_{10} x < 0$, then
    a. $x < 0$  b. $-1 < x < 1$  c. $0 \leq x \leq 1$  d. $1 < x < 0$  e. $0 < x < 1$

11. In the figure at the right, $AC$ is a diameter of circle $B$, with $DC = 2$ and $AB = 5$. How long is $DE$?
   a. 4  b. $\sqrt{10}$  c. 10  d. 16  e. Not enough information is given.

12. Let $n = x + y^{(x+y)}$. Find $n$ when $x = 1$ and $y = -1$.
    a. 0  b. 1  c. 2  d. -1  e. Indeterminate

13. If the radius of a circle is increased by 1 unit, the ratio of the new circumference to the new diameter is
    a. $\pi + 2$  b. $\frac{2\pi + 1}{2}$  c. $\frac{2\pi - 1}{2}$  d. $\pi$  e. $\pi - 2$

14. Two equilateral triangles are inscribed in and circumscribed about a given circle. The ratio of the area of the inscribed triangle to that of the circumscribed triangle is
    a. 1:4  b. 1:3  c. $\sqrt{3}:1$  d. $\sqrt{3}:2$  e. 1: $\sqrt{3}$

15. The first three terms of an arithmetic progression are $x - 1$, $x + 1$, and $2x + 3$, in the order shown. The value of $x$ is
    a. 2  b. 0  c. 2  d. 4  e. Undetermined

16. The figure at the right is constructed of four semicircles, two of which are tangent at their midpoints. If the diameter of each of the semicircles is one unit, then the area in square units of the interior of the figure is
    a. $\pi$  b. $\pi \sqrt{2}$  c. $\frac{\pi}{4}$  d. 1  e. None of these
THE MATHEMATICS LEAGUE

The mathematics league is similar in nature to interscholastic athletic leagues. Each school is represented by a team of students, usually called "mathletes," and each team competes with all the other teams in the league. The league is usually made up of schools within the same city or county school division. Schools already members of an interscholastic league, or schools located within a restricted geographic area. In theory this type of activity could be conducted on a statewide basis, but in practice this seems not to be done.

The competition consists of a series of meets during the school year. The number of meets reported ranged from three to eleven per year. At each meet approximately four to eight problems are presented, one problem at a time. Each member of each school team attempts to solve the problem individually. Each problem must be solved within a specified time limit. Time limits vary according to the difficulty of the problem but range roughly from two to ten minutes. At the end of the time limit students must stop working on the problem whether they have finished or not. The papers are collected and scored as soon as possible so that a running score can be kept throughout the meet. Usually the same number of points is assigned to each of the individual problems. A team's score for any particular problem is the product of the number of points assigned to the problem times the number of team members correctly solving the problem.

There is usually a few minutes' break between successive problems. During this time contestants can relax, papers can be graded, and team sponsors may make team substitutions for the next problem. No substitutions may be made after the students begin working a problem.

Each school team consists of a specific number of students, usually about five. Many more students from a school may participate in a meet when substitutions are made between problems. Team selection is a matter for the individual school, but however selected, these students will be some of the top mathematics students in the school. Quite often, the team activity is one of the functions carried out by the mathematics club in the school.

It is usually necessary for each school to have a teacher to serve as sponsor or coach, with responsibility for organizing and carrying out team activities. In addition to the sponsor’s responsibilities at meets, there is usually some provision for periodic team practice between meets. Practice may consist of solving problems similar in nature to those used at a meet, and under similar conditions. Problems from previous contest sessions are often used. Reports indicated league members practice weekly or oftener, as a general rule.

It is not necessary for all participating teams to gather at one central loca-
Types of Contests

In a meet to minimize travel and for other reasons, the meet may be conducted at several host schools within the league area. It is, of course, necessary for the overall meet to be conducted on the same date and at approximately the same time during the day. Since the running time for any one meet is only about an hour to an hour and a half, it can conveniently be scheduled in the afternoon after school or at night.

Following a meet, all materials are returned to one person, usually called the coordinator, chairman, or secretary, who has the responsibility for tabulating both team and individual scores. As soon as all materials are in and checked, the chairman sends out an announcement that includes the results of the meet just completed and the cumulative scores of the teams. The chairman also has many other duties. Generally, he is the one with overall responsibility for conducting the league activity. The functions of the chairman can also be performed by a committee, but the majority of mathematics leagues indicated that such duties were done by an individual.

Some of the leagues reported that for the last meet of the year all participating teams gather at one central location. Following the meet itself, there is a banquet at which the final team standings and the individual student winners are announced. Awards to the top teams and students are presented, and the year’s activities are concluded with a festive air.

Sample Problems

1. A particle is placed on the parabola \( y = x^2 - x - 6 \) at a point \( P \) whose ordinate is 14. It is allowed to roll along the parabola until it reaches the nearest point \( Q \) whose ordinate is -6. Find the horizontal distance traveled by the particle. Time: 4 minutes.

2. With \( AB \) as radius and \( A \) as center, \( CBD \) is drawn. With \( BA \) as radius and \( B \) as center, \( CAD \) is drawn. If \( AB \) is 10 inches long, find the area of \( CBDA \). Time: 10 minutes.

3. At what times will there be a right angle between the minute and hour hands on a clock, between the hours of 7:00 p.m. and 8:00 p.m.? Time: 5 minutes.

4. A sequence of numbers of which 30 is the first number of the sequence is determined in the following manner:

Add all the whole number divisors of 30 which are less than 30 to obtain the next number. Thus the whole number divisors of 30 are 1, 2, 3, 5, 6, 10, 15. These have a sum of 42 which is the next number in the sequence. The first three numbers of this sequence are 30, 42, 54.

Using this method of finding the numbers of the sequence, find the eighth number in this sequence. Time: 5 minutes.
School Mathematics Contests: A Report

5. A cube 4 inches on each edge is painted on all faces. If the cube is cut into one-inch cubes, how many of the one-inch cubes will have no paint on any of their faces? Time: 2 minutes.

THE MATHEMATICS FIELD DAY

Another contest pattern includes several types of competitive mathematical activities. Sometimes named by such colorful expressions as Chalk-Talk Derby, Dual Dig, Power Tests, Leapfrog Relay, Rapid Transit, and Mad-Hatter Marathon. Also included are mathematically related games.

Such a contest consists of three or four of these activities. It is held once a year and is usually an all-day affair. Typically, a school enters a five-man team, and individuals on the team participate in the different activities. The team score is the sum of the scores of individual participants. Individual awards are given to the winners of the different events.

CHALK-TALK DERBY

This is an original oral presentation of a particular topic, prepared and given by one of the members of a school team. Several months before the contest, the contest authorities announce the topics for the chalk-talk part of the coming contest. In some contests the Chalk-Talk Derby consists of successive elimination rounds, and the students have to be prepared to deliver talks on three or four different topics. Some of the topics named in samples of these contests were "Vectors," "Inverse Functions," "Prime Numbers," "The Structure of Proof," "Fibonacci Numbers," and "Theory of Equations." The talks are delivered to a panel of judges, and the judges may ask the speaker questions at the conclusion of the talk. Usually there is a time limit of approximately five minutes.

DUAL DIG, POWER TESTS, AND LEAPFROG RELAY

Two students from each school team participate in this phase of the contest, which consists of the solution of sets of problems. Two general modes of operation were reported.

In one of these, each of the two students works a different set of problems. At the end of a specified time period, the two students exchange papers; and each student is allowed to go over his partner's paper, correcting and completing problems if necessary. During this period, the two students may collaborate on their work by showing each other what they are doing and by notes, but no talking is allowed. This latter period also has a time limit. The score for the two-man team is the total score on both papers.

In the other mode of operation, each of the two students receives the same set of problems. The two students are seated together; they may work together, or they may work separately and compare results. Any communica-
Types of Contests

Sample Problems

1. Indicate \((1 - 2i)^{-1} \cdot (i + 2)^{2}\) in the form \(a + bi\) where \(a\) and \(b\) are real numbers.

2. Simplify:

\[
\frac{\sin \frac{\pi}{6} + 2 \cos \frac{4\pi}{3}}{\sec \frac{3\pi}{4} \csc \frac{11\pi}{6}}
\]

3. Evaluate \(\log_{27} 81^{1/12}\).

4. Express the total surface area of a cube as a function of its diagonal \(D\).

5. Solve for \(x\) the following inequality:

\[
\frac{x - 1}{2x + 1} < 2.
\]

6. A fly on a sheet of coordinate paper starts at the point \((3, 4)\), proceeds by a straight line to the nearest point on the unit circle with center at the origin, and then follows the small arc of the circle to the point \((1, 0)\). How far has the fly walked?

7. Find all pairs of integers \((x, y)\) such that \(x^2 + 2x + y^2 = 4\).

8. \(A\) and \(B\) have two dollars each. A coin with probability \(1/3\) of coming up heads is tossed. If heads shows up, \(B\) pays \(A\) one dollar. If tails shows up, \(A\) pays \(B\) one dollar. What is the probability that \(A\) will win both of \(B\)'s dollars?

9. Find the relation between \(a\), \(b\), and \(c\) if one zero of the quadratic polynomial \(ax^2 + bx + c\) is \(n\) times the other.

10. A chemist's measuring glass is conical in shape. If it is 8 cm. deep and 3 cm. across the mouth, find the distance on the slant edge between the markings for 1 cc. and 2 cc. Give an answer in simplest radical form.

Rapid Transit, Mad-Hatter Marathon, Ciphering

This phase of the contest involves rapid computation and estimation. In some contests this phase is open to all students; in others, it is restricted to designated team members. Problems may be shown on a screen by a projector, read aloud, shown by flashcard, or written on a blackboard. The problems are presented rapidly, and probably not every student will be able to solve every problem. Practices vary as to whether and how individual scores are incorporated into the team score.

Sample Problems

1. What is the value of \(\sin 17\pi\)?

2. Between which two consecutive integers does \(-\sqrt{100}\) lie?

3. If the diameter of a steel ball bearing is twice that of one which weighs 500 grams, how many kilograms does the larger one weigh?
School Mathematics Contests: A Report

4. Two pennies are tossed. What is the probability of getting at least one head?
5. What is the value of $e^{\text{ln} x}$?
6. What is the value of $\ln e^x$?
7. Solve the inequality $125 > 625^x$ for $x$.
8. If $\tan t = 1$, find the value of $\sec^2 t$.
9. The cost of gasoline in Mexico is 86 centavos per liter. There are 100 centavos to a peso and one peso is worth 8 cents. A liter is .95 quarts. How many cents per gallon are you paying?
   a. 24.5    b. 26    c. 29    d. 31
10. If 3 girls use 3 jars of eye shadow in 3 days, how many jars of eye shadow will 12 girls use in 6 days?

Mathematically Related Games

A variety of games that students can play is an additional phase of the contest. Some of the games named were Three-dimensional Tic-tack-toe, Hex, Nim, Five-in-a-Row, Tactics, and Pentamino Hunt. As many as four such games may be included in one contest. All games are played by two people. Some type of elimination system, usually the pyramid system, is used to declare a winner in each game. Practices vary as to whether any team member can play or whether only designated students on the school team can play. Where the games are open to any team member, the contest schedule is arranged so that the team members have ample opportunity to participate in the games without interfering with the other phases of the contest in which they may be entered. Practices vary as to whether the game scores are included as part of the team score.

Three-dimensional Tic-tack-toe

This game, which helps develop three-dimensional imagery and adds fun when mathematicians get together, is similar to the well-known tic-tack-toe, a 3-in-a-row game.

Players alternately play cross and naught in the squares in the layers of a $4 \times 4 \times 4$ cube until one player gets four in a row in some direction—horizontally, vertically, or diagonally. Each $4 \times 4$ square in the diagram represents a layer of the $4 \times 4 \times 4$ cube. In the illustration, X has played an aggressive game and has had three 3-in-a-row threats blocked. However,
he has two 3-in-a-rows ready to be completed on his next turn. Meanwhile, 0 has had two 3-in-a-row threats blocked but has just completed a 4-in-a-row and wins! (See the filled-in naughts.) The student should examine the illustration to see all of these different ways 4-in-a-row can be made.

**Five-in-a-Row**

Players alternately play cross and naught in the squares of a sheet of graph paper. The first to get five in a row is the winner.

![Graph Paper with Five-in-a-Row Diagram]

**Pentamino Hunt**

Each player selects one of the twelve pentaminos pictured, indicates his choice to his opponent, then places his pentamino (oriented in any way he chooses) in his own square, not letting his opponent see what he has done.

Player X then fires salvo Number 1, consisting of eight shots, by calling out coordinate letter and number, for example, D7, A3, G6, etc. Both players record 1’s (ones) in the squares indicated on Y’s square, and Y states the number of hits that have been made on his pentamino. Similarly, Player Y fires salvo Number 1, and both players record 1’s in squares indicated on X’s square.
School Mathematics Contests: A Report

Players then alternate with salvo Numbers 2, 3, 4, etc., of eight shots each, with salvo numbers being recorded in squares indicated. Play continues until both pentaminoes have been completely covered. A player’s score is the sum of the salvo numbers that cover his opponent’s pentamino. Low score wins. (In the example shown, Y’s score is nine, X’s score is eleven.)

**Hex**

Players alternately play cross and naught on a lattice of hexagons as shown. The winner is the one who first marks a connected line of adjoining hexagons starting at one of the two sides labeled with his symbol and ending at the other. Notice that the corner hexagons can be considered as belonging to either of two adjacent sides.

In the game shown here, 0 can force a win.

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OPPONENT’S SQUARE

In the game shown here, 0 can force a win.
THE MATHEMATICAL PROBLEM-SOLVING CONTEST

Only two contests of this pattern were reported. However, they are described here, with additional details under the Problem-Solving Program and Mathematics Competition (see page 20), because of the potential of this type of contest, the large-scale coverage it affords, and its proven success in other countries.

The competition consists of the solution of a series of problem sets over the school year. A unique feature of this type of contest is that the students are not rigidly bound by a short time factor. The student has approximately a month to work on each set of problems. Thus, the student has time to think over the problems extensively and to research them.

For example, a student receives a set of problems on October 1, to be returned by November 1. On November 1 he receives a second set of problems—these to be returned by December 1—and either at that time or soon thereafter receives the solutions to the first set so that he can examine them while the problems are still fresh in his mind.

A particular advantage of this contest is that it is not necessary to bring the students together for it. The sending and receiving of problem sets is handled through the mails. (However, high-scoring students can be invited to some central location at the end of the contest for appropriate activities.)

The problems are purposely designed to be solved without extensive coursework background. Rather, they are designed to require perception, insight, and ingenuity on the part of the students (although obviously, students who are farther along in their study of mathematics have an advantage with their additional background and experience).

Reports indicated that students are free to use reference materials and other aids and to discuss the problems with other students. However, any work turned in must be the personal work of the competing student.

Many of the problems require proofs or demonstrations that the student must show on the problem. Even when a problem may be answered by a numerical response, the student is strongly encouraged to include all his work on the problem.

**Sample Problems**

1. A bowl contains 20 objects numbered 1 to 20. What is the probability that any two of these objects chosen from the bowl will have numbers that are relatively prime? (Note that 1 is considered to be relatively prime to all other numbers.)

2. A man takes a walk on a flat plane according to the following pattern:
   a) He walks due north however long he wishes.
   b) He walks in any direction except south however long he wishes.
School Mathematics Contests: A Report

c) He repeats Step (a) and Step (b) in that order as often as he wishes, as long as he does not cross his path. Prove that it is impossible for the man to return to the place where he started.

3. There are $n$ people in a room, and each one has shaken hands with at least one other person in the room. Show that there are at least two people who have shaken hands the same number of times.

4. Take any number in base 7. Rearrange the digits (some of the digits can remain in the same position). Prove that the difference between the original number and the rearranged number is divisible by 6.

5. Four houses that form the vertices of a square are to be linked by the shortest system of roads such that by traveling only on the roads it is possible to reach any house from any other. Show that the system of roads consisting of the diagonals is not the shortest one.

6. Two regular hexagons of side $s$ are so placed that their overlapping area is a regular dodecagon. What is the area of this dodecagon?

7. Can a solid cube measuring one foot on a side be constructed from bricks $2 \text{ in.} \times 4 \text{ in.} \times 8 \text{ in.}$? Show that your answer is correct.

8. If $x$ and $y$ are chosen to be integers, then $2x + 3y$ is divisible by 19 if and only if $5x - 4y$ is divisible by 19.

9. There are three locks and four keys. There is one key for each lock but not that must be made with the keys so that the three locks may be opened? What is the average number of tries is $S$ the largest?

THE MATHEMATICS FAIR

Mathematical projects and activities have long been included as one of the branches of science in a science fair. This participation is expected to continue in the future. Additionally, however, some communities are conducting separate fairs with the emphasis on mathematics only. The pattern of a mathematics fair closely resembles that of a science fair. It is assumed that the reader is familiar with this nationally popular activity; but additional information can be found on page 29. Below are sample math fair projects:

Mathematics and Music
Fibonacci Numbers
Relativity
Diophantine Equations
Programmed Politics
A Study of Spirals
Convex Smooth Curves
Polynomial Base Conversion

Mathematics of Symmetry
Topology
Fermat's Last Theorem
Four-Dimensional Cube
The Mathematics of Checkers
Unusual Characteristics of a
Negatively Based System
A Projective Geometry

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Examples of Contests

The following are brief accounts of a few of the contests that were reported.

THE UNION COUNTY, NEW JERSEY, REGIONAL HIGH SCHOOL DISTRICT MATHEMATICS CONTEST

The three high schools and the board of education of the Union County, New Jersey, Regional High School District No. 1 sponsor an annual contest for students of the high schools. In 1964/65 there were 171 students who participated.

The contest is a fifty-question test, and the students have two hours to work. The questions are short-answer questions. Most of the questions are problems that can be answered by numerical answers, and a few are multiple choice. There are three separate competitions, one for students studying Algebra I, a second for students studying Geometry, and a third for students studying Algebra II. A separate test is developed and given for each of the three levels.

Participants for the contest are selected by the mathematics staffs of each of the three schools. Generally, the criterion for selection is performance in the student's present course in mathematics.

The mathematics teachers of the high schools and the county coordinator of mathematics handle all aspects of the contest. These include preparing the tests, administering them, and grading them. The board of education of the school district finances the contest.

Awards are given to the first-, second-, and third-place winners in each school for each of the three levels of competition. Awards are in the form of medals, certificates, and appropriate mathematical publications.

THE MONTGOMERY COUNTY INTERHIGH MATHEMATICS LEAGUE

The Montgomery County, Maryland, Interhigh Mathematics League began operation in 1962/63. At present, all public senior high schools in the county
and one parochial high school participate in the league. Approximately two hundred students participate in the league activities. Participating schools operate the league cooperatively, with assistance from the central administrative office.

The competition consists of a series of meets during the school year, the number of meets being decided by a representative committee at the beginning of each school year. Although the number of meets per year may vary, approximately ten meets per year are usually held.

Each school is represented by a team of not more than five students to participate in any given problem, but additional students may participate in a meet if the composition of a team is changed between problems. The matter of team selection is left to each individual school. Some schools operate their team through a math club or through the Mathematics Honor Society. Other schools hold trial meets and select the top scorers to represent the school at each meet.

The county is divided into five regions to facilitate the holding of meets, but all schools gather at one central location for the first and last meets of the year. Within each region, the role of host school is rotated among the member schools from meet to meet. The meets are almost always held in the afternoons after school.

Five problems are presented at each meet. The time allowed for working each problem ranges from two to seven minutes, so that the total working time at each meet is approximately twenty minutes. The problems can generally be answered numerically or with other types of short answers. They are selected from the general areas of arithmetic, algebra, geometry, and trigonometry at a level of difficulty so that at least 80 percent of them can be answered by tenth-grade students.

A very important role in the administration of the league is that of the county coordinator. A secondary-mathematics staff member who is not associated with any one school and preferably is located in the county central office is designated as coordinator. The duties of the coordinator include selecting problems for each meet, preparing and distributing contest materials, maintaining individual and team records, reporting contest results, and being the final arbitrator on any protested problem.

Also important are the faculty sponsors in each school. In addition to the responsibilities associated with sponsoring a team, they are responsible for directing meets, grading problems, keeping time, and sending materials and results of each meet to the coordinator. Throughout the year the coordinator maintains cumulative scores on both individual students and school teams.

The winning team is awarded a trophy donated by the county mathematics teachers association. All participants receive certificates, and individual student winners receive appropriate awards, such as mathematical books and
Examples of Contests

slide rules, which are donated by various businesses and individuals. Other costs of operating the league, such as providing materials, are absorbed by the individual schools and the central office.

THE WISCONSIN MATHEMATICS CONTEST

This contest is sponsored by the Wisconsin section of the Mathematical Association of America. (The contest is distinct from the National High School Mathematics Contest of the MAA and the NCTM.)

The Wisconsin Mathematics Contest has grown from a single contest examination taken by some 400 students about ten years ago to a contest of much greater scope. Latest figures available at the time of this report are that over 24,000 students in about 360 high schools took a preliminary contest examination. Of these contestants more than 1,100 qualified for an essay-type examination at regional centers.

As indicated, the contest consists of two parts, the preliminary and the final. The preliminary is a seventy-five-minute examination consisting of questions and problems with multiple-choice answers. Questions on the preliminary examination are designed to cover content through the first year of algebra and the first semester of geometry. Students take the exam in their individual schools. Teachers in the individual schools grade the exam with an answer key, supplied with the exam, and report the scores to the contest headquarters. After the scores have been received, a lowest score for entrance to the final contest exam is determined. This is set so that between 1,000 and 1,200 students qualify for the final exam.

The final exam is a ninety-minute exam consisting of four essay-type questions involving proofs and the solution of problems. These problems may include content up to the early part of second-semester geometry. Qualifying students take the exam at some twenty-five or more regional test centers. The exam is graded by teams of four or five people for each of the questions. The graders are college faculty members, assisted by graduate students in mathematics. A prescribed score is awarded for each essentially correct answer. Extra credit may be awarded for evidences of such things as unusual insight and generalization of results.

The competition in this contest is individual only. There are no school team scores. Awards are given to the top fifty contestants in the final exam and to the student in each school who makes the best score on the preliminary exam.

THE GREATER SAN DIEGO MATHEMATICS FIELD DAY

This event is sponsored jointly by the Greater San Diego Mathematics Council and San Diego State College. All high schools in San Diego County and Imperial County are invited to participate. Over the past few years, more
than thirty schools and approximately 160 students have participated annually.

The contest consists of four different competitive activities. These are Dual Dig, Chalk-Talk Derby, Rapid Transit A and B, and Individual Games. Each participating school may send a team composed of not more than five students. Ordinarily, the team of five will consist of two members for the Dual Dig, one member for the Chalk-Talk Derby, and one member for each of the two Rapid Transits. The member entering the Rapid Transit B must be a sophomore. No other grade restrictions are made as to the makeup of the team.

The Dual Dig involves two members of a team, both members receiving the same set of problems. The two-man team receives a single sheet on which to record the agreed-upon answers. The two members may work individually and then check each other's work, or they may work together on each problem.

The Chalk-Talk Derby involves a single member of the five-man team, and he must be prepared to give a talk and to answer questions on each of three mathematical topics that have been chosen and announced several months in advance of the competition. After the talks, which are to be about five minutes long, the judges are free to ask questions on the topic. In Rounds 1 and 2 each contestant appears before two different groups of judges. The combined scores from each round determine the winners who will compete in the finals, Round 3.

Although the topics are announced in advance, the order in which they will be presented is not announced until the day of the contest. Thus, each speaker must prepare presentations on the three topics. At the 1966 contest, the subjects were (1) "The Structure of Proof," (2) "Periodicity and Circular Functions," and (3) "Combinatorial Analysis (Permutations and Combinations) and Probability."

Rapid Transit A and B are contests involving rapid calculation and estimation. Only a brief time is allowed for the students to work the problem before the next problem is presented. Problems are generally presented so rapidly that not every student will be able to work every problem. Separate sets of problems are presented in Rapid Transit A and Rapid Transit B, since Rapid Transit B is open only to sophomores.

Individual Games are a group of four mathematically related games in which individual students play against each other. The games are Three-dimensional Tictactoe, Hex, Tactics, and Pentamino Hunt. Any member of a school team may enter the games. The overall contest schedule is arranged so that everyone can compete in one or more of the games. Winners are determined by successive eliminations.

The complete contest is an all-day affair. The last competitive feature is Round 3 of the Chalk-Talk Derby, held in an auditorium before all other participants and observers. An awards ceremony concludes the day.

Individual awards, such as mathematical books, are given to the first four
Examples of Contests

winners in each event. Team awards are given to the first three teams in each division (there are two divisions based on school enrollment). There is also a sweepstakes award, a traveling trophy, for the best team participating.

THE TENNESSEE ANNUAL STATEWIDE HIGH SCHOOL MATHEMATICS CONTEST

The Tennessee Mathematics Teachers Association sponsors this contest. Prior to 1957 several independent high school mathematics contests were held in various localities in the state. The Tennessee Mathematics Teachers Association effected a merger of these contests and began to sponsor a contest open to all public, private, and parochial high schools in Tennessee. In 1964/65 some 4,050 students from 382 schools participated in the contest.

The contest is a forty-question test and the students have eighty minutes for the work. All questions are in multiple-choice form, and for the most part they are problems that require solution. There are four divisions of competition: Algebra I, Algebra II, Geometry, and Comprehensive. The divisions for Algebra I, Algebra II, and Geometry are open to students who are currently enrolled in the corresponding course, if they have not had more than nine months of instruction in that subject. The Comprehensive division is restricted to seniors. A separate test is provided for each of the four divisions. The number of students from any school who may enter each division of the contest is determined by a formula based on enrollment in the corresponding course.

The method of selecting contestants is determined by the individual schools. However, it is recommended that the schools conduct special examinations for the selection of students who will represent the school and that these examinations be open to all eligible students who wish to compete.

The contest is financed by contributions from several business and industrial firms and by payment of an entrance fee for each contestant. Local contest supervisors are requested to seek payment of these fees from general school funds.

To conduct the contest, the state is divided into regions, each headed by a regional chairman. Test centers are located at twenty-two colleges within the respective regions. Individual high schools are assigned to a test center. Each test center is headed by a chairman who is responsible for the activities at the center, including supervision of grading the papers. He forwards to the regional chairman the papers of the top ten contestants in each division.

The regional chairman, in turn, is responsible for supervision of regrading all papers he receives from the test centers. He forwards to the state chairman of the examination committee the papers of the top ten contestants in each division from his region. The state chairman sees that these papers are checked once more and then announces the state winners.
School Mathematics Contests: A Report

Recognition of achievement is quite extensive. Included are certificates of participation to all who take part in the contest and—for each division—certificates of merit to the first three place-winners at each test center, plaques to the first three place-winners in each region, and plaques to the first three place-winners for the state as well as to the schools they represent.

THE PROBLEM-SOLVING PROGRAM AND MATHEMATICS COMPETITION

St. Mary’s College and the California Mathematics Council—Northern Section jointly sponsor these activities. The operation of the contest centers in St. Mary’s College. The program operates mainly in Alameda County and Contra Costa County, but some schools from other counties also participate.

Two phases are involved. First, there is the problem-solving program carried out during the school year. This program is conducted simultaneously in two sections, one for students in Grades 7–9 and one for students in Grades 10–12. For each of these two sections, four problem sets are sent out during the year at approximately six-week intervals. Solutions to the problems are usually due one month after the problems are sent out. Also, immediately following the due date, complete solution sets are sent to the participating schools so that students may check their results while the problems are still fresh in mind. During the following two weeks, the students' solutions are checked, points are assigned to the students, and a report of students’ ratings is prepared. This information and the next set of problems are then sent out.

Each set of problems has ten problems with a maximum score of 100 points for each set, and a total possible score of 400 points for the year. During the course of the year, students who earn at least 100 points are eligible to participate in the second phase of the competition, called the Mathematics Competition. In the 1965 contest, approximately 1,500 students participated in the problem-solving program. Of these, about 500 students were eligible to participate in the competition, and some 400 took the examination.

The Mathematics Competition is an examination in mathematics held at St. Mary’s College in the spring. There is a separate examination for each of the two sections, Grades 7–9 and Grades 10–12. Final winners are determined on the basis of the results of this examination.

THE LONG ISLAND MATHEMATICS FAIR

The Long Island Mathematics Fair is a major annual event of Nassau County and Suffolk County. The fair is sponsored by the mathematics teachers associations of the two counties, along with several business and industrial organizations. Students in Grades 8–12 in schools of the two counties are eligible to compete. In 1965, 290 students from thirty-four schools prepared projects and participated in the fair.
Examples of Contests

Each contestant prepares a mathematics project for judging. The project may be a lecture, an exhibit, or a combination of these, with a paper on his topic (one copy to each judge). Lectures are limited to fifteen minutes for presentation to the judges; additional time is allowed for questioning by the judges. The work of the project should be based on the student's research, but it does not necessarily have to be original. It is sufficient for the student to master a topic worthy of his grade level.

Normally this is a two-day affair, beginning on Friday afternoon and concluding on Saturday afternoon. There are three rounds of judging, and the winners in each round go on to the next round before a different panel of judges. The criteria for judging include (1) creativity, originality, and ingenuity; (2) effort, mathematical thought, depth of development, and validity of conclusion; and (3) thoroughness, organization, and clarity.

Obviously, many judges are required for a competition of this type and magnitude. Area high school mathematics teachers, college mathematics professors, and mathematicians from area businesses and industries are invited to serve as judges.

An awards assembly is the concluding feature of the fair. All participants receive certificates of participation. The winners of the first, second, and third rounds for each grade level (the third being the final round) receive bronze, silver, and gold medals, respectively. As a feature of the awards assembly, a prominent mathematician is invited to deliver an appropriate address on mathematics to the assembled students, parents, and others.
This section describes some of the operational features of the contests on which reports were received.

CONTEST SPONSORS

The sponsoring organizations for contests are widely varied in character. The three types of sponsors mentioned most frequently are professional organizations of teachers of mathematics, universities and colleges, and participating schools themselves. Others listed include business and industrial firms, such as insurance companies and newspapers; civic and professional clubs; local school boards; and high school and college mathematics clubs.

Joint sponsorship was reported frequently. In this situation the sponsors are usually a school-related organization, such as a professional organization of mathematics teachers, and one or more non-school-related organizations, such as business firms. The professional organization of mathematics teachers handles the organizational and operational part of the contest, while the other sponsor supplies financial support. This seems to be a fairly common pattern.

OBJECTIVES OF CONTESTS

Objectives of contests, according to practically all of the reports received, include stimulating interest in mathematics, giving recognition to outstanding mathematics students, recognizing and encouraging exceptional mathematical talent, and providing for healthy academic competition. A major effort is made to create an atmosphere of enjoyment and accomplishment during a contest, while retaining the stimulus of competition.

An additional objective, on the part of some sponsoring colleges, is the
Contest Procedures and Practices

hope of encouraging the better students to enroll at the college upon graduating from high school.

Other objectives that were listed include the following: encouraging independent study of mathematics, providing an opportunity to review high school mathematics, providing practice in speaking to an audience, implementing radio broadcast lessons, encouraging skill in mental computation, and encouraging interest and excellence in mathematics for capable young women.

NATURE OF THE CONTESTS

The most pertinent decisions to be made are those determining the nature of each contest: How will the contest questions be constructed? Will the students compete as teams or as individuals? Will different grade levels compete with each other, or will there be a separate section for each level?

TYPES OF QUESTIONS

The most popular type of question or problem used in contests is one to which the response is some type of short answer. Of the fifty-nine contests covered in this report, fifty-one had short-answer questions for part or all of the contest. The majority of these questions require a numerical answer. In some cases the questions are in multiple-choice or true-false form. The relative ease of preparing and administering contests based on this form of question, and especially of grading the short answers, would seem to account for its popularity.

Other types of questions include proofs and/or demonstrations, as in the Problem-Solving Competition Contest; prepared oral presentations, as in Chalk-Talk Derbies; and prepared projects, as in mathematical fairs.

TYPES OF COMPETITION

Contest competition includes competition between individuals, competition between school teams, and combinations of these two. Individual competition was a feature of almost all of the contests covered in this report. School-team competition was included in about half of the contests, and about half included both individual and school-team competition. Several people contributing to this report indicated strong feelings on the matter of school-team competition. Both extremes were represented, some feeling that there should be no school-team competition, in order to de-emphasize interschool competition, and others feeling that this should be a major feature of the contest.

PROVISIONS FOR DIFFERENT GRADE LEVELS

Approximately two thirds of these contests make provisions for different grade levels. This is done in either of two ways: In the first, separate sets of
questions are prepared for the different grade levels. In the second, all students work with one set of questions, but the grading is weighted by the individual grade levels. The net result in either situation is that students compete only with others at the same grade level.

In the remaining third, each participating student competes with all the others.

**PREPARATION OF CONTEST QUESTIONS**

A major responsibility associated with any contest is the preparation of questions or problems. The general objective is to keep within the appropriate level of mathematical attainment yet to pose questions that require perception and rational thought by the student to determine a solution.

The methods used in preparing questions generally fall into three patterns. The first and most popular method of the reporting contests is for an individual not affiliated with any of the sponsoring organizations and not otherwise connected with the contest to undertake the responsibility of preparing the questions. Often he is paid for this duty. Second, a team of people from a college prepare the questions. This method is especially prevalent in those contests where the college acts as a sponsor. Third, an individual or a committee, selected from within the contest organizations, is responsible for preparing the questions. Quite often, teachers from the participating schools are invited to submit questions for the contest. From these questions and other sources the individual or committee prepares the questions. (See pages 34–37 for a list of sources for contest questions.)

**PREPARATION OF CONTESTANTS**

About one third of the reports stated that the contest or some phase of it requires preliminary preparation by, or assistance to, the students. Assistance is normally provided by a teacher. It was mentioned most frequently in connection with mathematics leagues, mathematics fairs, and other contests that include an oral or written presentation by the students. Mathematics league teams, in particular, meet periodically to “practice.” Practice may consist of working problems of a nature similar to those included in the contest, receiving instruction in mathematics and problem-solving techniques, practicing these techniques, and other appropriate activities. The practice may take place before or after school or through mathematics club activities.

**GRADING AND JUDGING**

The grading or judging of contest results constitutes an important responsibility in any contest. For those contests where it is appropriate, grading is done promptly if not immediately after the completion of the contest.
Contest Procedures and Practices

Most frequently, teachers from the participating schools act as the graders. Also, college mathematics professors and college students who are mathematics majors (often graduate students) may grade results.

For large-scale contests involving short-answer questions only, the contest may be given and graded by the teachers in an individual school. In this case they are usually required to forward the better papers to an independent committee for regrading. Also, large-scale, short-answer contests can be set up for machine grading.

A practice reported by a majority of the contests is the appointment of an individual or committee to judge appeals and disputes over alternate answers or alternate interpretations to given questions.

Contests involving oral or written presentations by students, or involving student projects, usually require a number of judges. Judges are teachers from the participating schools, professors from area colleges, or people with mathematical backgrounds from business and industry. Where teachers from participating schools serve as judges, the contest is arranged so that the teachers will not be involved with students from their own schools.

ADMINISTRATION OF THE CONTESTS

Once the actual examinations have been constructed, the sponsoring organizations plan and carry out the details necessary to administer the contest: raising funds to pay contest expenses; setting a date and naming a location, registering students, providing the required equipment and space, and planning the system of awards.

FINANCING

Underwriting the cost of a contest is handled in many ways. Sources of funds include contributions from the sponsoring organizations; participation fees for each student, which the schools often pay; other contributions from the operating funds of schools with students who participate; and appropriations from boards of education.

Contests may be operated very economically, primarily because so many people are willing to volunteer their services. The major fixed costs are the preparation and distribution of contest materials, and the purchase of awards. Very few people receive any payment for their services.

In only eighteen of the fifty-nine contests covered in this report were any individuals paid for duties associated with the contest. In these contests, honoraria were given to the person or persons who prepared the contest questions or to the chairman or coordinator of the contest. In a few cases, too, it was necessary to pay for clerical services.
PROCEDURES FOR HANDLING LARGE NUMBERS OF STUDENTS

A necessity stressed in the reports received is extensive planning for a contest and preregistration of contestants so that all necessary scheduling can be prepared prior to the day of the contest.

Some specific procedures reported were the use of automatic data processing (for scheduling and grading), color-coded test forms, and geographically convenient contest sites.

RECOGNITION OF ACHIEVEMENT

Recognition of individual achievement in most contests is quite extensive. Typically, all participants receive a certificate of participation, since in many contests participation alone represents a certain level of achievement. The range of eligibility for individual awards varies from the first three places to as many as the first twelve, in each of the different grade-level categories. If a provision is made for school-team competition, there are also awards to the top school teams.

The most frequently mentioned types of awards are related to mathematics. Included among these are appropriate books on mathematics, mathematical tables, and slide rules. Other awards mentioned are scholarships, medals, ribbons, plaques, trophies, cash, watches, and pen-and-pencil sets.

ORGANIZATION FOR ADMINISTERING CONTESTS

Organization for the administration of contests varies widely, from one extreme of an elaborate committee structure to the other extreme of having a single coordinating committee or sometimes even a single individual to bear the overall responsibility. To some degree, the nature and extent of a contest will affect the organizational structure. Regardless of the makeup and extent of the organizational structure, the following are some of the principal aspects of organizing a contest:

- Question formulation
- Editing
- Scoring and judging
- Administration and monitoring
- Finance
- Registration
- Arrangements
- Awards
- Publicity
- Location
- Rules
- Appeals
- Refreshments

EFFECT OF MATHEMATICS CONTESTS

One of the questions included in the questionnaire to assess the effect of mathematics contests was, "Is there any evidence to indicate that the contest
Contest Procedures and Practices

has caused interest and/or enrollment in mathematics courses to increase?"

No concrete evidence in the form of research studies or similar investigations was reported. There was no reference to tangible evidence such as class enrollments. However, in the opinion of teachers and others involved in contests, judging in part from the feedback received from students, there is an increased interest in mathematics, and students are highly motivated by participation in contests.

Another question on the questionnaire was, "Has there been any follow-up to determine whether the students placing in the contest pursue a career that is mathematically related?" One contest group reported that they are planning to conduct such a study. Other replies to the question indicated that many of these students are following mathematically related careers. For the most part, these replies were based on opinions, feedback from students, and informal observation; however, there is one continuing follow-up study of top-ranked students in the MAA National High School Mathematics Contest. The results so far indicate that a substantial percentage of these winners are following a mathematically related field of study.1

SOME OF the school contests that are national in scope and include competition involving mathematics are listed here.

THE NATIONAL HIGH SCHOOL MATHEMATICS CONTEST

As mentioned earlier, the National High School Mathematics Contest is an annual joint project of the Mathematical Association of America and the National Council of Teachers of Mathematics, and participation in the contest has grown tremendously since it began in 1958.

The contest is a timed test consisting of a number of problems to be answered with numerical or other short-answer forms. Thus the contest can be (and is) set up in multiple-choice or true-false form. This greatly simplifies the problem of grading papers. The contest is given in individual schools, and teachers in the school do the initial grading. The better papers are forwarded to an MAA office for regrading. All information and supplies, including tests, answer keys, and directions for administering, are sent to participating schools well in advance of the date of the contest, which is usually in the spring of the year.

The contest makes the claim that all problems can be solved with the mathematics of intermediate algebra and geometry. It supports this claim by supplying a solution key for all problems. However, while the contest does not require an extensive background in mathematics course achievement, it does place a heavy premium on perception, ingenuity, and reasoning ability on the part of the students. For this reason, the problems have become popular with teachers for use in the classroom. All problems used in past contests have now been published in two volumes. (See Salkind, under “Sources.”)

MAA membership is divided into geographical regions throughout the country, and each of these regions is responsible for conducting the contest for the high schools within its boundaries. Further details on participation in the contest may be obtained from MAA regional representatives.
SCIENCE FAIRS

A science fair is a collection of exhibits, each of which is designed to demonstrate a scientific or technical principle, a laboratory or other procedure, or an industrial development. An exhibit may also be an orderly collection of anything that can be fitted into the broad concept of any branch of any pure or applied science. Each exhibit or project is the result of the students' own work and research.

Science fairs enjoy great popularity and have grown extensively. There are now more than 15,000 science fairs each spring in secondary schools. The best student exhibits from school science fairs are eligible for regional fairs, which now number more than 200. In turn, finalists from regional fairs are eligible for entry into the annual International Science Fair. In 1965, over 400 finalists participated in the International Science Fair in St. Louis.

Information on organizing and conducting a science fair can be obtained from Science Service, 1719 N Street, N.W., Washington, D.C. 20036.

SCIENCE TALENT SEARCH

The Science Talent Search is conducted annually by Science Clubs of America as an activity of Science Service. It is sponsored by the Westinghouse Educational Foundation, an organization endowed by the Westinghouse Electric Corporation for the purpose of promoting education and science. Students compete for scholarships and awards. The competition is based on the judging of science projects developed by the students.

Only students in the last year of an accredited secondary school in the United States are eligible. An entry must consist of (1) a completed science aptitude examination answer sheet, (2) a personal data record and secondary school record, and (3) a report of about 1,000 words on the science project done by the student. Additional information can be obtained from Science Clubs of America, 1719 N Street, N.W., Washington, D.C. 20036.

STATE SCIENCE TALENT SEARCHES

Concurrently with the Westinghouse-sponsored national Science Talent Search and by special arrangement with Science Clubs of America, authorized groups such as State Academies of Science in more than forty states hold state science talent searches. Eligibility for participation in a state-level program is generally the same as for the national program. Details for an individual state can be obtained from the state authorized group or the Science Clubs of America, at the address previously given.

JUNIOR ACADEMY OF SCIENCE PROGRAMS

The Junior Academies of Science in many states sponsor programs providing for the presentation of papers on projects in science and mathematics.
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The papers are usually presented at regional and state meetings. Details for an individual state can be obtained from the state Junior Academy of Science.

FORD–FUTURE SCIENTISTS OF AMERICA AWARDS PROGRAM

This program is sponsored by the Ford Motor Company and is conducted by the National Science Teachers Association and its division, the Future Scientists of America. Students in Grades 7–12 are eligible to enter. An entry in this program consists of a written paper that reports in detail the student project in some branch of science, mathematics, or engineering. The student project must be done individually and must be of an experimental, investigative, research, or field-study nature. Details on this program can be obtained from Ford–Future Scientists of America Awards Program, National Science Teachers Association, 1201 Sixteenth Street, N.W., Washington, D.C. 20036.

ADDITIONAL PROGRAMS

Two additional programs that have the objective of encouraging students to undertake individual work in science or mathematics are the NASA–NSTA Youth Science Congresses and the Science Recognition Program. Details can be obtained from the following addresses: NASA–NSTA Youth Science Congress, 1201 Sixteenth Street, N.W., Washington, D.C. 20036, and Science Recognition Program, Science Materials Center, Inc., Dept. G–1, Box 464, Madison Square Post Office, New York, New York 10010.
VI
Considerations for Beginning and Operating Contests

By all indications, properly conducted mathematics contests provide a medium by which the mathematics curriculum can be supplemented and enriched. Participating students seem to be stimulated by their participation, to enjoy their participation, and to receive encouragement to study mathematics.

However, as with any extracurricular study, care must be exercised to ensure that the extracurricular activity does not begin to outweigh classroom instruction in importance. Therefore, it is important that a contest (like any other extracurricular activity) be examined periodically to see if it is fulfilling its purposes.

The National Association of Secondary School Principals annually publishes an approved list of national contests and activities in the September issue of its official publication, The Bulletin. A set of criteria for judging the worthiness of contests is included. These criteria offer an excellent guide for establishing and conducting contests.

Also recommended is an article by Professor R. Creighton Buck in The American Mathematical Monthly.¹ The article comprehensively examines mathematics contests, setting forth possible advantages and disadvantages in contests, and some things to consider in establishing a mathematics contest. As possible advantages, he suggests the following:

1. Awards give official recognition to some of the better students now studying mathematics in high school.
2. Talented students who might otherwise escape attention are discovered and encouraged.

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3. Additional motivation is given some students to take more mathematics in high school.

4. Some students, especially those who are highly gifted and whose talents extend in many directions, are encouraged to consider mathematics as a career.

5. A certain amount of tactful guidance is given to the high school curriculum by indicating the level of competence and maturity of viewpoint that can be expected from the better students.

As possible disadvantages, he suggests the following:

1. If the students are coached by a teacher in preparation for contests, the teacher may neglect the less able students, the ones who need the teacher's help the most.

2. Even for better students, drilling on sample test questions may not be the best form of mathematics education.

3. It is possible that a contest that does not emphasize the individual contestant and that pits school against school and teacher against teacher, will emerge in the end only as a device for rating teacher efficiency.

4. Success, especially where large cash prizes are at stake, may be harmful to both winners and losers.

5. A large-scale contest, such as a state or national contest, may pose a threat to intellectual freedom by the creation of one central committee to administer the contest, especially where this might be tied into a scholarship program.

Other articles listed in the bibliography provide additional considerations for contests and information about experiences of those who are involved with contests.
APPENDIX A: BIBLIOGRAPHY


*The Bulletin of the National Association of Secondary School Principals.* Annually in the September issue, the NASSP publishes its approved list of national contests and activities for the coming school year.


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APPENDIX B: SOURCES FOR MATHEMATICS CONTEST PROBLEMS

The books listed below represent a sampling of materials in addition to textbooks that can be used as sources for contest problems and for ideas that can be adapted to problems.


Appendixes


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Appendixes


In addition to these books, the following periodicals can be used as sources for problems. Some of these periodicals have a section for problems as a regular feature.

*American Mathematical Monthly*. Mathematical Association of America, University of Buffalo, Buffalo, N.Y. 14214.


*Mathematical Log*. Box 504, University of Oklahoma, Norman, Okla. 73069.


*Pi Mu Epsilon Journal*. St. Louis University, St. Louis, Mo. 63103.

*Recreational Mathematics Magazine*. Box 35, Kent, Ohio 44240.

*School Science and Mathematics*. Box 408, Oak Park, Ill. 60303.

*Science and Math Weekly*. Education Center, Columbus, Ohio 43216.

*Scientific American*. 415 Madison Avenue, New York, N. Y. 10017.

*Scripta Mathematica*. 186th Street and Amsterdam Avenue, New York, N.Y. 10033.

APPENDIX C: INFORMATION ABOUT INDIVIDUAL CONTESTS

Following are names and addresses of organizations sponsoring some of the contests reported in this study. Addresses have not been included when the sponsoring group is an affiliated group of the National Council of Teachers of Mathematics. (The names and addresses of the current officers of affiliated groups are available from the NCTM Washington office, 1201 Sixteenth St., N.W., Washington, D.C. 20036.)

Samford University Invitational Mathematics Tournament
Chairman, Department of Mathematics
Samford University
800 Lakeshore Drive
Birmingham, Alabama 35209

Yavapai County Arithmetic Contest
Chairman, Department of Mathematics
Prescott High School
Prescott, Arizona 86301

Annual Redwood Empire Mathematics Tournament
California Mathematics Council—Northern Section
Cal Poly Royal Math Contest
Department of Mathematics
California Polytechnic College
San Luis Obispo, California 93401

Chaffy High School District Junior High School Math Field Day
Math-Science Coordinator
Chaffy High School District
211 West Fifth Street
Ontario, California 91762

Claremont Men's College Mathematics Competition
Chairman, Department of Mathematics
Claremont Men's College
Claremont, California 91711

Desert Interscholastic Mathematics League
Mathematics Field Day
Chairman, Department of Mathematics
Antelope Valley High School
Lancaster, California 93534

Elementary School Mathematics Field Day
Chairman, Department of Mathematics
Los Angeles Pierce College
6201 Winnetka Avenue
Woodland Hills, California 91364

Greater San Diego Mathematics Field Day
Chairman, Department of Mathematics
San Diego State College
San Diego, California 92115

or

Greater San Diego Mathematics Council

Imperial Valley College Mathematics Festival
Chairman, Department of Mathematics
Imperial Valley College
Box 158
Imperial, California 92251

Long Beach Mathematics Field Day
Supervisor of Mathematics
Long Beach Unified School District
Long Beach, California

Occidental College Math Field Day
Chairman, Department of Mathematics
Occidental College
Los Angeles, California 90041

Orange Coast College Invitational Mathematics Meet
Chairman, Department of Mathematics
Orange Coast College
2701 Fairview Avenue
Costa Mesa, California 92626
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Bethany College Mathematics
Scholarship Test
Chairman, Department of Mathematics
Bethany College
Lindsborg, Kansas 67456

Eastern Kentucky State College
Mathematics Achievement Program
Chairman, Department of Mathematics
Eastern Kentucky State College
Richmond, Kentucky 40475

Montgomery County Interscholastic
Mathematics League
Supervisor, Department of Pupil and
Program Appraisal
Montgomery County Public Schools
850 North Washington Street
Rockville, Maryland 20850

The Greater Boston Mathematics
League
Chairman, Department of Mathematics
Brookline High School
Brookline, Massachusetts 02146

Intermediate Mathematics League
Chairman, Department of Mathematics
Emerson Junior High School
Concord, Massachusetts 01742

Massachusetts Mathematics League
Chairman, Department of Mathematics
Hamilton-Wenham Regional High School
Bay Road
Hamilton, Massachusetts 01936

Newton Junior College Annual High
School Mathematics Tournament
Chairman, Department of Mathematics
Newton Junior College
Newtonville, Massachusetts 02160

Bergen County Interscholastic
Mathematics League
Chairman, Department of Mathematics
St. Joseph Regional High School
40 Chestnut Ridge Road
Montvale, New Jersey 07645

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Jersey Hills Conference
Mathematics League
Chairman, Department of Mathematics
Livingston High School
Livingston, New Jersey 07039
Union County Regional High School
District #1 Mathematics Contest
Coordinator of Mathematics
Union County Regional High School
District #1
Berkeley Heights, New Jersey 07922
Long Island (New York) Mathematics Fair
Nassau County Mathematics Teachers' Association
Suffolk County Mathematics Teachers' Association
Nassau County Junior Mathematics League
Chairman, Department of Mathematics
Wantagh High School
Wantagh, New York 11793
New York City Interscholastic Mathematics League
Director of Mathematics
Board of Education
131 Livingston Street
Brooklyn, New York 11201
Suffolk County Interscholastic Mathematics League
Suffolk County Mathematics Teachers' Association
Suffolk County Junior High School Mathematics League
Suffolk County Mathematics Teachers' Association
Suffolk County Mathematics Contest
Suffolk County Mathematics Teachers' Association
The Triple Cities Mathematics League
Chairman, Department of Mathematics
Binghamton Central High School
31 Main Street
Binghamton, New York 13905
Mental Mathematics Contest
Supervisor, Division of Mathematics
Cleveland Public Schools
Cleveland, Ohio
Luzerne County Council of Teachers of Mathematics Contest
Chairman, Department of Mathematics
Nanticoke High School
Nanticoke, Pennsylvania 18634
Western South Dakota Mathematics Contest
Chairman, Department of Mathematics
South Dakota School of Mines and Technology
Rapid City, South Dakota 57701
Tennessee Statewide High School Mathematics Contest
Tennessee Mathematics Teachers' Association
Junior High School Slide Rule Contest
Consultant in Mathematics
Fort Worth Public Schools
3210 W. Lancaster Avenue
Fort Worth, Texas 76107
Annual Mathematics and Science Tournament
Chairman, Department of Mathematics
Wheatley Senior High School
4900 Market Street
Houston, Texas 77020
Director, Number Sense Contest
University Interscholastic League
University of Texas
Austin, Texas 78712
Director, Slide Rule Contest
University Interscholastic League
University of Texas
Austin, Texas 78712
St. Mark's School of Texas Mathematics Tournament
Chairman, Department of Mathematics
St. Mark's School of Texas
10600 Preston Road
Dallas, Texas 75230
### Appendixes

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