DOCUMENT RESUME

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These materials were written with the aim of reflecting the thinking of The Cambridge Conference on School Mathematics (CCSM) regarding the goals and objectives for school mathematics. This document details the planning and response for each of ten lessons involving symmetry motions. The problems focused on (1) combining motions in a given order, and (2) finding the axis of symmetry for the triangle, rectangle, square, and octagon. Comments on the symmetry motion sessions follow at the end of the notes. [Not available in hard copy due to marginal legibility of original document]. (RP)

Symmetry Motion Classes

From the end of January to the end of term Miss Lyn McLane conducted ten sessions on symmetry motions with a delta team class of Estabrook Elementary School. The class teacher, Mr. David Horton, was also involved in preparing material, reviewing, and supervising the class's worksheet activity. The following notes prepared by Miss McLane detail the planning and response of each session.

Symmetry - Plans for 1st Lesson on Jan. 31, 1967

Estabrook - Dave Horton's Class - Lyn McLane

[Check with Dave Horton about triangles. If each child does not have one, make sure compasses, rulers and crayons are available.]

1. Pass out triangles (or have children make them.) 

2. Discuss various positions of a triangle so that it fits in shape like △ . (Pass out dittoed sheet full of △'s to facilitate listing of positions)
3. Develop language for describing how to get from headquarters position (to be defined) to the other positions. [Be sure descriptions do not depend on previous motions - have 120°, 240° rotations, identity motion (-360° rotations) and flips about each of the three altitudes.]
4. Play games with motions - "Close eyes. I have used two different motions. Open eyes. What were two motions? What one motion would do the same job?"
5. Get into commutativity, inverse motions, identity, closure.
6. Relate some of these findings to arithmetic.

Symmetry - Comments on 1st Lesson - Jan. 31, 1967

McLane (Dave Horton's Class - Estabrook)

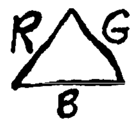
Gave each child a triangle 

and asked children to describe the triangle. Replies were - all sides have same length; 3 vertices and 3 sides; each side has a different color; colors are the same on both sides; smells like magic marker; it has a middle.

Asked each child to lay triangle on desk in position like △ .
Asked different children to draw their △ 's on board - labeling the sides with R, G, B. Some children thought there would be 9 different △'s since the green side could be in 3 positions, the red side in 3 positions and the black side in 3 positions. Others thought there would be only six positions since you had 3 positions on the front and 3 positions on the back. After eliminating the repetitions on the board we found we had six different positions.

[Dave came in with dittoed sheets each with 6 △'s on it.]

Passed out sheets and asked children to copy △ 's from board on the sheets.

Discussed how to get from starting point 

to

(2) 

decided on lower left hand twist or left twist (lt)

(3) 

decided on clockwise rotation (cr^1) [^{1} means once]

(4) 

decided on peak twist (pt)

(5) 

decided on counter clockwise rotation (ccr^1)

(6) 

decided on right twist (rt)

Asked children to close their eyes. I held triangle in starting position and made two motions. I asked children to open eyes and guess which two motions I made. Someone finally came up with two. Found that cr^1, pt gave same thing as pt, ccr^1 (from starting position)

Played coordinate tic-tac-toe in all four quadrants with children acting as plotters after first game.

In talking with Dave later, we developed a new and hopefully easier notation for motions.

Start = I	lt = T_L
Cr! = R_c	pt = T_p
CCr! = $R_{\bar{c}}$	rt = T_R

Dave said he would work next week on practice of the motions, introduce the new notation and start on Session 4 of the notes of Dick Barnes' classes.

Symmetry - Plans for 2nd Lesson Feb. 14, 1967

McLane - Dave Horton's Class, Estabrook School

1. Pass out triangles and sheet of paper with triangles on it.
2. Discuss again the six different positions of the triangle and re-establish notation for symmetry motions. Label positions as H,A,B, C,D,E. Label colors as b,g,r. Label motions as I, T_L, T_R, T_T, R_C, R_{CC}.
   (See Worksheet)
3. Practice making series of symmetry motions.
   What motions can we use twice which will bring $\triangle$ back to H?
   What motions can we use three times to get $\triangle$ back to H?
   If we do a whole string of symmetry motions, do we end up with one of our $\triangle$ positions?

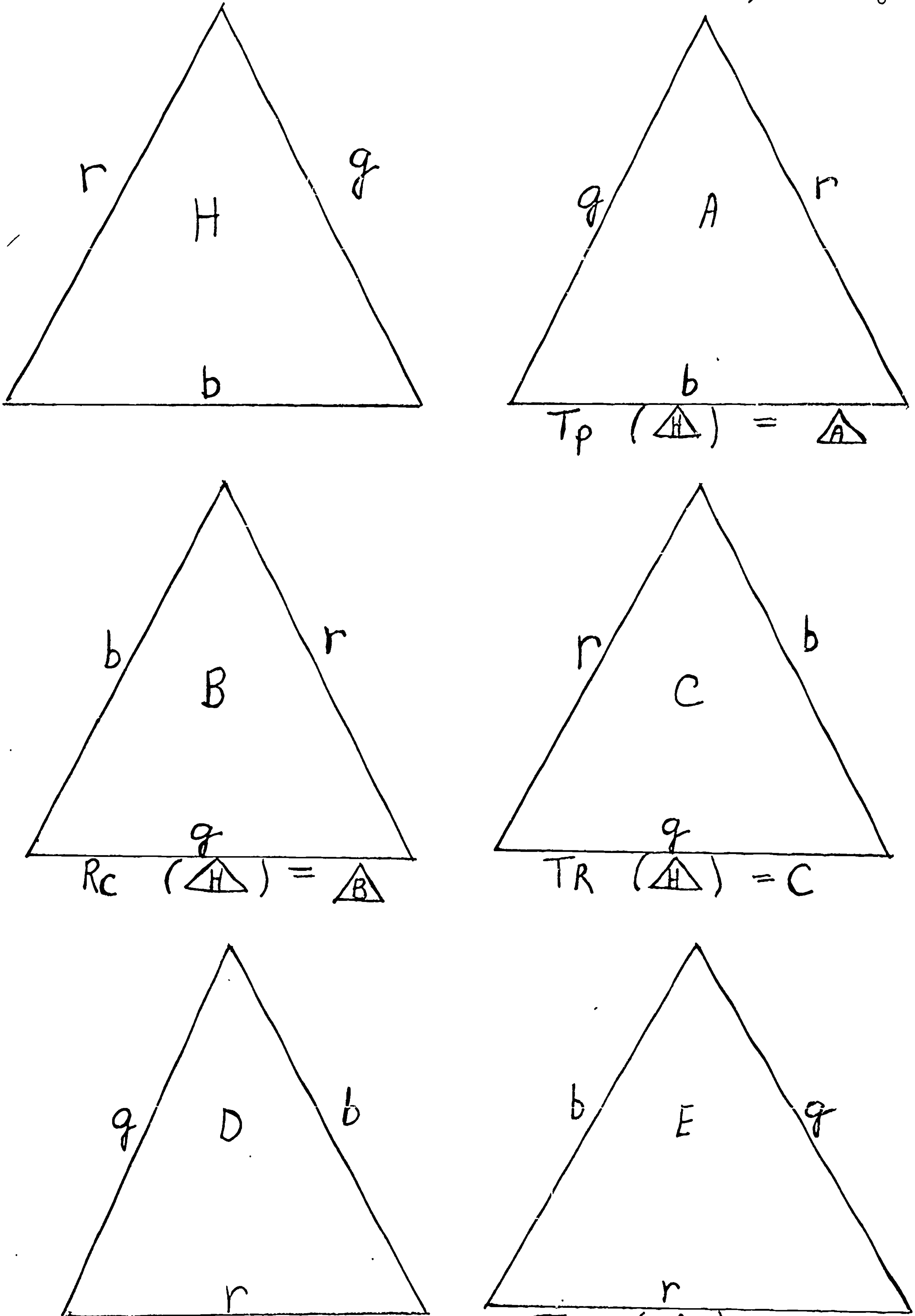
   T_L ($\triangle$ H) =

   T_R (T_L $\triangle$ H) =

   Make chart on board.

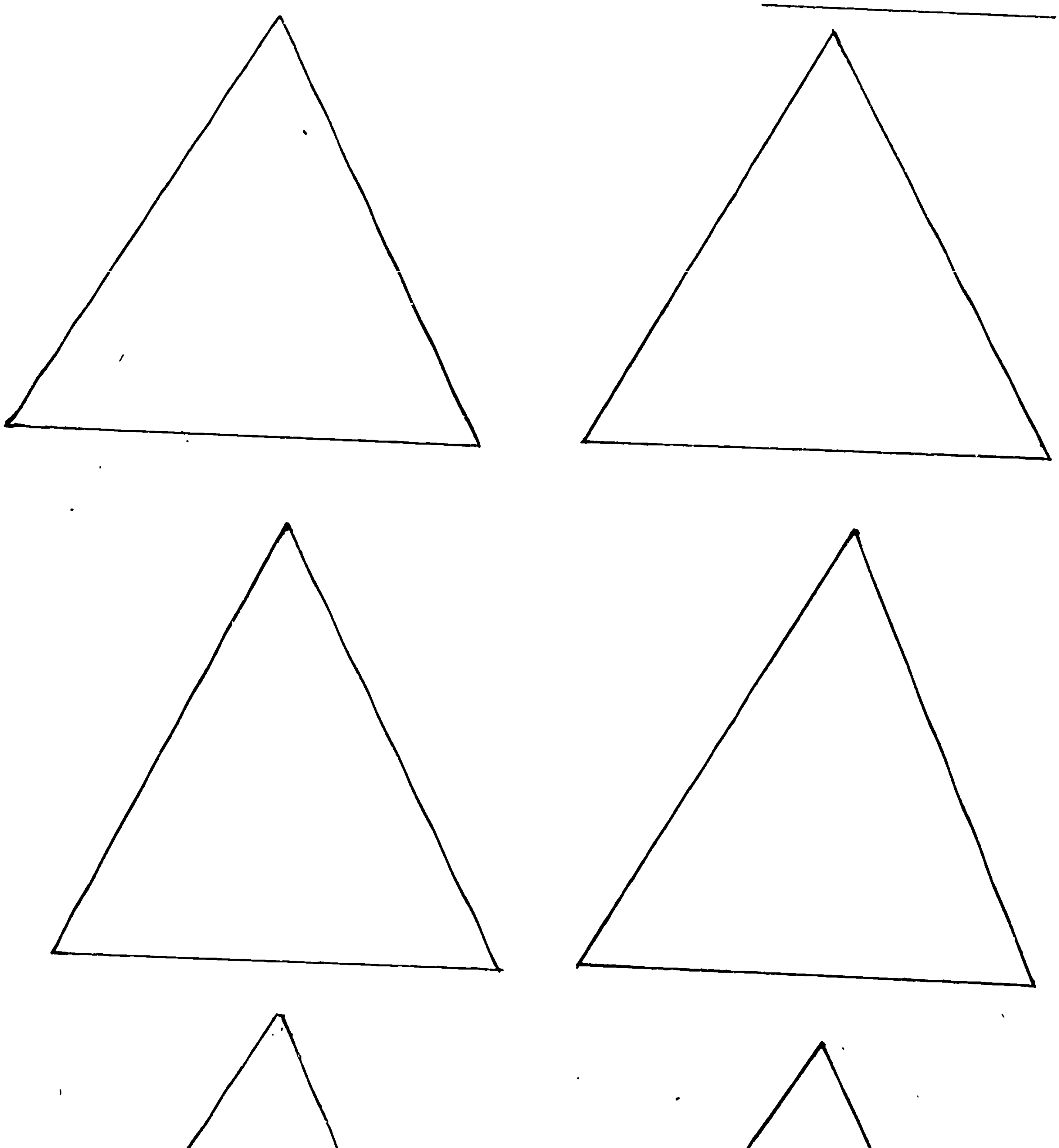
Start	Symmetry Motions	End

Name L. McLane
Date Feb. 14, 1967



Name ____________________

Date ____________________



Plans for 3rd Lesson on Symmetry - Mar. 21, 1967

Estabrook School - Lyn McLane

Practice motions.

Discuss order of colors after a twist.

Identity motion.

Commutativity.

Equivalent motions.

Maybe inverse motions.

Results

Labelled Identity Motion as S : $S(H) = H$

One boy said $T_pT_p\ (H) = S(H)$

$T_pT_p = S$ equivalent motions

[found using word Same for S led into difficulties.]

ST. $R_{cc}R_{cc}R_{cc} = S$

$3R_{cc} = S$ changed to $R_{cc}{}^3 = S$

$R_c{}^3 = S$

$R_cR_{cc} = S$

$R_{cc}R_c = S$

[Should have also mentioned that in order to be sure they were equivalent we will have to try all positions.]

Discussed commutativity - but didn't use word.

$R_cR_{cc} = R_{cc}R_c$

$R_cT_L \neq T_LR_c$

$T_pT_L \neq T_LT_p$

[Should have discussed idea that ↑ were not equivalent motions.]

Dave will do more with Identity, Inverses, Associativity.

Symmetry - Plans for 4th Lesson on April 4, 1967

Estabrook School - Dave Horton's Class - Lyn McLane

Check with Dave on Identity, Inverses, Associativity

Equivalent operations - if starting with the same position they bring us to the same result.

Hit idea of what symmetry transformation is - leaves triangle in same shape.

Have worksheets on making lists of equivalent motions.

What motion is needed to get from A to D?

Connections with real number system - commutativity closure, identity, inverse, associativity.

Results:

$T_p R_c T_R (A) =$ __________ . Start with T_R. Children made up two more - 1st child gave starting position, 2nd child gave operation, 3rd child gave operation, etc.

Let's call "S" (Same) "$\mathcal{I}$" to make the language easier.

Danny's Theory: $T_p 2 = \mathcal{I}$, $T_p 4 = \mathcal{I}$, etc.

Bernie's (?) Theory: $R_c 3 = \mathcal{I}$. $R_c n = \mathcal{I}$ for odd values of n.

T: What about $R_c 6$?

Bernie (?): Well odd, even, odd, even.

Equivalent operations. $T_p 2 = \mathcal{I}$, $T_p 4 = \mathcal{I}$

Danny said: $T_R 2 = \mathcal{I}$ and $T_L 2 = \mathcal{I}$

T: What are some equivalent operations for T_p?

$T_p(B) = C$

S_1: $T_p^3\ (B) = C$

S_2: How about Rc (B) = C?

S_3: $R_cT_pR_c\ (B) = C$.

S_4: Why couldn't we have triangle like  ?

S_5: That isn't a triangle.

T: What is definition of a triangle?

S_1: Three sides.

S_2: An upside-down triangle if it is like a block won't stand up.

Question of why we couldn't have  was left hanging.

S_1: What about square?

S_2: What do triangles have to do with math?

Didn't get to worksheet.

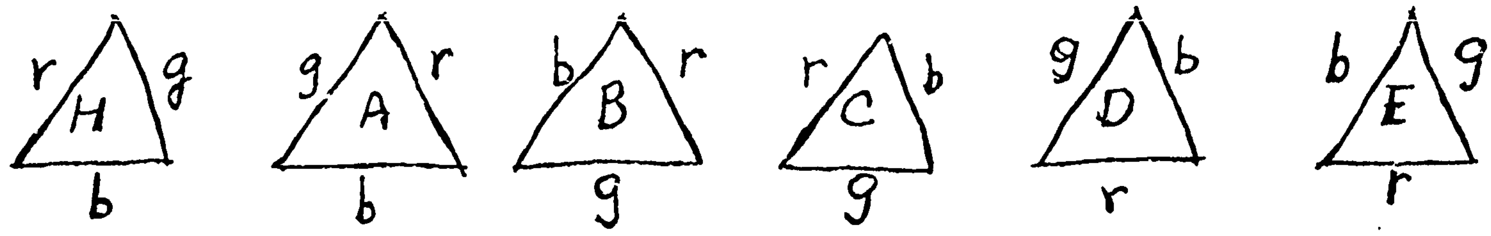
Next time - ask Dave about worksheet - how did children do?

Discuss 

Draw parallels between properties of number system and symmetry motions.

Name ______________________

Date ______________________



1. a. Find the motion needed to get from position A to position D. _____

   b. " " " " " " " " D to position H. _____

   c. " " " " " " " " H to position B. _____

   d. " " " " " " " " C to position B. _____

   e. " " " " " " " " E to position H. _____

2. $T_p \; R_c \; T_{>} \; T_R \; R_{cc} \; I \; T_L$ (B) = ______________ .

3. Make up your own example like the one in problem 2.

____

4. Find some equivalent operations and fill in the table below.

I	T_P	R_c	T_R	R_{cc}	T_L

Worksheet which Dave gave to class in 5th Symmetry Lesson. April 15, 1967.

Name ______________________

(1) $T_L\ R_c\ T_R\ T_p\ R_c\ T_L\ (E) =$

(2) $T_R^2\ I\ T_p^2\ T_L\ R_{cc}^5\ (E) =$

(3) $R_{cc}^3\ T_L\ R_c\ T_p\ (B) =$

(4) $T_L^2\ I\ T_R\ R_{cc}^4\ R_c^2\ T_p\ (E) =$

(5) $R_c\ T_R\ T_L\ T_p^2\ (H) =$

(6) $R_c^2\ T_L^2\ R_{cc}^3\ T_R\ T_p^2\ (C) =$

(7) $T_p^2\ R_c\ T_R\ T_L\ R_{cc}^2\ (D) =$

(8) $T_p^2\ R_{cc}^2\ R_c\ T_R\ T_L\ (A) =$

(9) $I\ R_{cc}^3\ R_c\ T_R\ T_L\ T_p\ (E) =$

(10) $T_R^{10}\ T_L^5\ T_p^5\ R_c\ R_{cc}^2\ (E) =$

(11) $T_L\ R_{cc}\ T_R^2\ R_c\ T_p\ (H) =$

(12) $T_L^2\ T_p^2\ I\ T_R\ R_{cc}\ R_c\ (H) =$

(13) $R_c^3\ T_L\ T_p^2\ R_{cc}\ (A) =$

(14) $R_{cc}^2\ R_c^3\ T_p^2\ T_R^2\ T_p^3\ T_L\ I\ (C) =$

(15) $T_L^2\ T_R\ T_p^2\ R_{cc}^2\ (E) =$

(16) $R_{cc}^6\ R_c^3\ I\ T_L^2\ T_R^3\ R_c^3\ (C) =$

Result of 6th Symmetry Lesson - April 25th 1967

Estabrook School - Dave Horton's Class - Lyn McLane

Drew different position of triangles on board.

Discussed properties of the real number system and how some of them related to the symmetry motions of the triangle. (I had difficulty remembering that the operation was the combining of two symmetry motions- not the symmetry motions.) The children mentioned the distributive property which we could not apply to the symmetry motions because there was only one operation.

Discussed how many examples were needed to disprove a statement.

Mentioned that next time we would try squares or maybe a rectangle. Would we get the same number of positions for each?

Plans for 7th Symmetry Lesson - May 9, 1967

Estabrook - Dave Horton's Class - L. McLane

Look at symmetry motions of square and rectangle.

Results:

See notes which Dave took during class. Dr. Lomon came to visit.

On marking square - children suggested: 1. coloring edges
2. putting letters in corners
(accepted) 3. putting 1,2 on sides

Told Dave I would send him a worksheet for square and rectangle.

Next time:

Find one motion to describe R_c^2 .

Discuss which axis are left fixed.

How many positions for rectangle?

Inverses

Chains

Properties

Dave Horton's notes on 7th Symmetry Lesson - May 9th 1967

Estabrook School - Dave Horton's Class - Lyn McLane

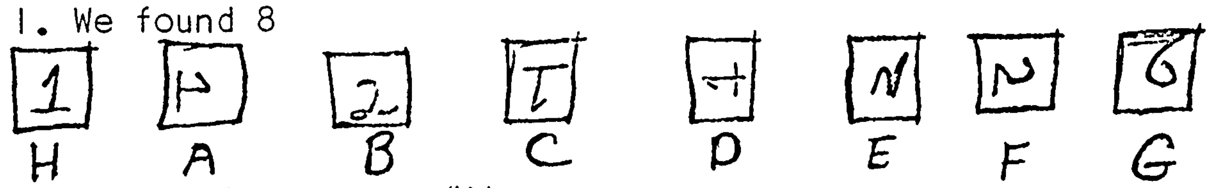
1. Start with square (large square held by teacher)
   A. Close eyes, move square (What did I do to it?)
   B. Ask children to tell how to mark square to determine how it was moved.
      1. No. 1 on one side.
      2. No. 2 on 2nd side.
   C. Describe motions teacher does.

Dave Horton's notes on 7th Symmetry Lesson (continued)

II. Students manipulate small square.

A. How many positions can you find?

1. We found 8



B. Label them (See above #1)

C. Describe motions.

1. B_f (H) = G — (B_f = Back Flip)

2. R_c (H) = A — (R_c = Clockwise rotation)

3. T_T (H) = B — (T_T = Top twist)

4. R_c^2 (H) = C — R_c^2 = 2 clockwise rotations)

5. Rcc (H) = D — (Rcc = counter clockwise rotation)

6. R_T(H) = E — (R_T = Right Twist)

7. $B_f R_c$ (H) = F — (clockwise rotation followed by backflip)

or 8. L_T (H) = F — (L_T = Left Twist)

D. Equivalent Motions

1. $R_{cc}^2 = R_c$

2. $L_T = B_f\ R_c$

3. $L_T^2 = I$

Pulled for "Inverse motions". Inverse of L_T is L_T.

T: What is inverse of R_c?

S: Rcc.

Results of 8th Symmetry Lesson - May 16, 1967

Estabrook School - Dave Horton's Class - Dave Horton

See chart below which was accompanied by a note from Dave saying that the children still do not really "catch on" to the inverse motion:

I. Equivalent Shapes

16 May

H A B C

II. Motions of Symmetry

Vocabulary		Symbols	Examples
Doubleturn clockwise	=	D_C	D_C (H) = A
Doubleturn Counterclockwise	=	D_{CC}	D_{CC}(H) = A
Flip Back	=	F_B	F_B(H) = C
Flip Front	=	F_F	F_F (H) = C
Twist Top	=	T_T	T_T(H) = B
Twist Bottom	=	T_B	T_B(H) = B
I	=	I	I (H) = H

III. Equivalent Motions

$F_F = F_B$

$T_T = T_B$

$D_C = D_{CC}$

$D_C 2 = I$

$D_{CC}^2 = I$

$F_B^2 = I$

$F_F^2 = I$

$F_B^2 = F_F^2$

IV. Inverse

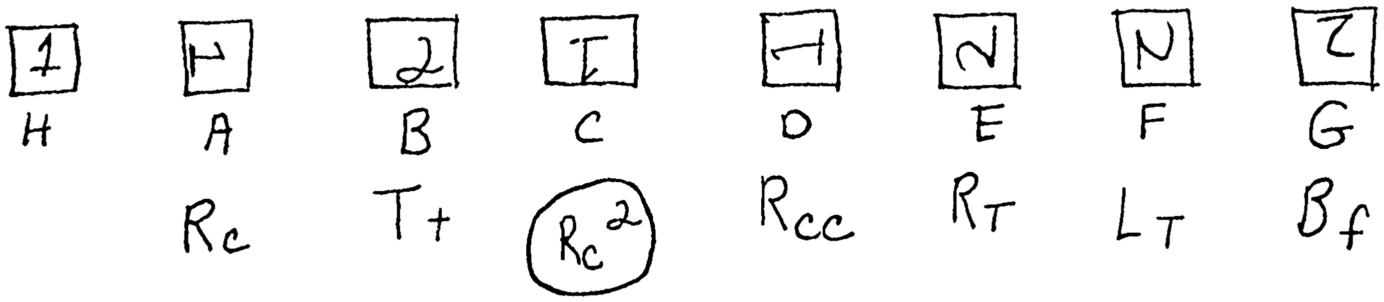
Dcc can be inverse of Dc

e.g. Dcc(H) = A

Dc (A) = H

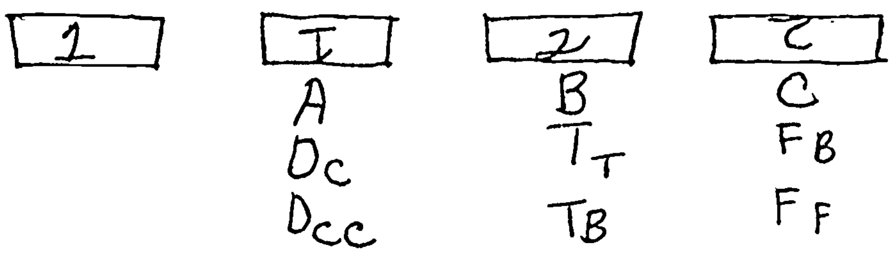
Plans for 9th Symmetry Lesson - May 23, 1967

Estabrook - Dave Horton's Class - L. McLane



Find new name for!

Rectangles



Developed in Dave's lesson

1. Did you find 8 symmetry motions for the rectangle? Why not?

2. Examine axes of symmetry 

   No. of Sym. motions

3. Worksheet for 15 min.

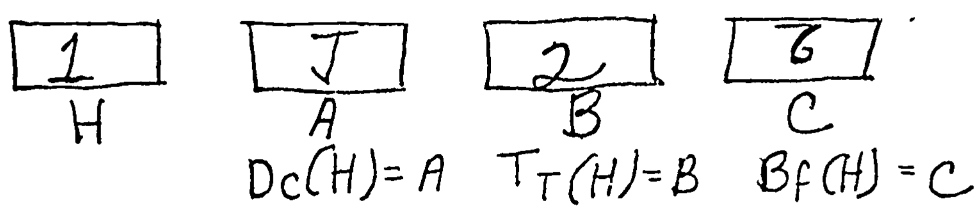
4. Inverse motions

5. Properties - closure
   commutativity

Results:

Listed motions for rectangle to find out how many there were.

Students had suggested 4,6,8,9. Found just the following:



D_c = double clockwise rotation

Asked which line segment remained fixed in top twist.

S: The line in the middle.

T: (Drew line segment in chalk on paper rectangle.) This is called an axis - axis of symmetry. Are there any other axes of symmetry?

Symmetry (cont.) May 23, 1967

S: (Comes up and draws horizontal line)

T: If we twist about this axis which motion are we doing?

S: Backflip.

T: Are there any more axes of symmetry?

S: Yes - (holds corners of diagonal) No, it doesn't work.

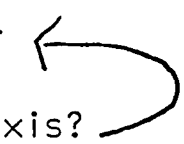
T: How many axes of symmetry for rectangle?

S: Two.

T: How many symmetry motions?

S: Four

T: Let's look at  . Who can come up and draw in an axis of symmetry?

(Students came up and drew  

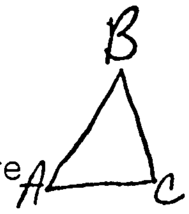
T: What motions do we do when we twist about this axis?

S: R_T.

(Did same for others)

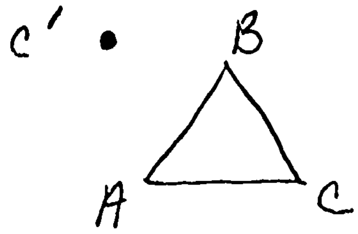
T: Any more axes of symmetry?

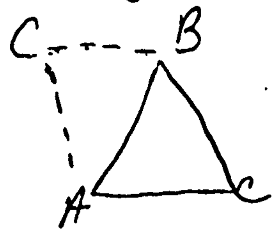
S: (Draws )

T: Who can come up and show me with his finger where 

point C will go when we twist triangle about $\overline{AB}$?

S: (Comes up and shows where C will go.) C will land here.





T: (Sketches in new ) Does this look like what we started with?

S_1: No. S_2: Looks like upside down headquarters.

T: Would $\overleftrightarrow{AB}$ be an axis of symmetry?

S: No.

T: How many axes of symmetry?

S: 3

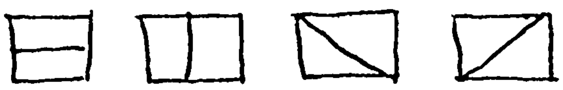
T: How many symmetry motions?

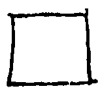
S: 6

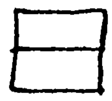
T: How about square - how many axes of symmetry?

S_1: 2

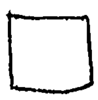
S_2: 4

(Students go to board and draw in axes  and tell which motions (B_f, T_T, R_T, L_T) go with each.)

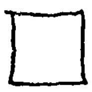
S: (Draws ) This is another axis of symmetry. New square looks just like one you started with)

T: (Referring to earlier diagrams) when we twist about this axis  where do all the points of the square go?

S: Back onto the square.

T: When we twist about this line  where do points of square go?

S: To the left.

T: Then can this  be an axis of symmetry?

(Most of class responded no but the student who brought up the question said "But you didn't say."

Symmetry (cont.) May 23, 1967

T: You are absolutely right. I guess I have to add another limitation and say that when we twist about axes of symmetry we have to end we where we started.

How many axes of symmetry for square?

S: 4

T: How many symmetry motions?

S_1: 8

S_2: 8 or 9

(Some boys were mumbling numbers - 3 + 3 = 6, 2 + 2 = 4, etc. as if they were drawing connections betweeen number of axes and number of motions.)

Listed rotations - used for Double clockwise rotation, S:R_c^4 as I so introduced ∼ for equivalent to.

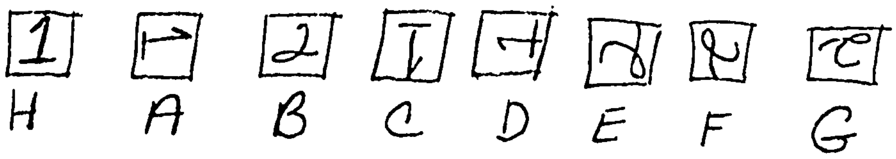
Passed out worksheet. Made correction on 7b to read "B_fR_c (F) = ______".

Some children had trouble with right and left twists but most were able to go ahead. Many were not sure what was expected on 8. They thought they should start at H and do R_cB_f (H) = _____.

Symmetry

Name ____________________

Date ____________________



I = Identity

R_c = Clockwise rotation

R_{cc} = Counter clockwise rotation

- =

T_T = Top twist

R_T = Right twist

L_T = Left twist

B_f = Back flip

Fill in the blanks.

1. $T_T R_{cc} B_f R_c I\ R_T\ T_T$ (D) = __________.

2. ______ $R_T R_c$ (A) = G

3. a. ______T_T (B) = B b. ____R_c (A) = A c. ____R_T (F) = F

4. a. B_f ___ (D) = D b. ___ ___ (H) = H c. ___ ___ (C) = C

5. a. $R_T L_T$ (B) = _____ b. $L_T R_T$ (B) = _____

6. a. $R_{cc}\ R_c$ (A) = _____ b. $R_{cc} R_{cc}$ (A) = _____

7. a. $R_c B_f$ (F) = _____ b. $R_c B_f$ (F) = _____

8. a. $R_c B_f$ ~ _____ b. $R_T T_T$ ~ _____

Plans for 10th Symmetry Lesson - June 6, 1967

Estabrook 4th Dave Horton's Class - Lyn McLane

How many different symmetry motions for triangle?
" " " " " " rectangle?
" " " " " " square?
" " " " " " octagon?

Axis of symmetry for octagon.

Symmetry motions for octagon.

Axis of symmetry and symmetry motions for circle.

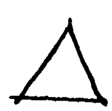
Geo-board - make symmetric figures

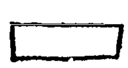
make figure and ask children to add to figure to make it symmetric.

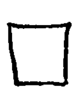
Triangle puzzle.

Results:

Children were quick with responses for number of symmetry motions:

 6

 4

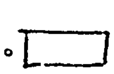
 8

 16

T: How do you know there will be 16 for the octagon?

S_1: 8 notations on one side and 8 on the other side.

S_2: There are 8 sides and they are all equal. 2 X 8 = 16.

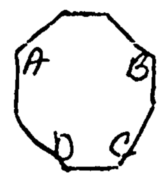
S_3:  has 3 equal sides. 2 X 3 = 6.  has 2 equal sides and 2 X 2 = 4,  has 4 equal sides and 2 X 4 = 8. ∴ Octagon with 8 equal sides has

2 X 8 = 16 symmetry motions. (Others expressed variations on above.)

T: How many rotations?

S_1: 8 (most of class agreed)

S_2: 16, 8 going one way and 8 going the other way. (Labels octagon on both sides)



T: Are those all different? Suppose I do this (turns octagon 1/8 turn clockwise). Will I hit this position when I rotate octagon counterclock-wise?

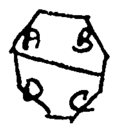
S: Yes. There are only 8 then.

T: How many axis of symmetry are there?

S: 8

T: Who can come and draw one?

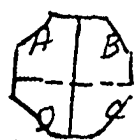
S:



T: Is it an axis of symmetry? (Most agree.)

T: Who can draw another one?

S:



(Uses ruler to measure 6/8" from A and from C)

T: Would you hold the octagon so the top is horizontal. (Student does so) Now would you flip the octagon about your line. (Does so) Did you end the way you started? (Disagreement) [Student rotated octagon so the new top was horizontal.]

(Some students were saying there were lots of them.)

T: Start again this time with your line horizontal. Which way does the top slope?

S: Towards you.

T: Now flip the octagon? Which way does the line slope?

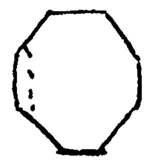
S: The other way?

T: (Draws outline of octagon indicating student's line and asks him to flip octagon.) Do you end up where you started?

S_1: It doesn't fit into pattern.

S_2: If line were half way between them it would work.

(Student who had measured so carefully said that if he cut figure in half then he could show that it fit. Teacher says the rules do not permit us to cut the octagon.)

(Other students draw in remaining axes. One girl tried  which class said wouldn't work.)

T: How many symmetry motions for a circle?

S_1: Lots of them. S_2: Millions. S_3: Trillions. S_4: Infinity.

T: How many rotations?

S: You would have to know where you started otherwise you could keep making more and more.

T: What is smallest rotation you could make?

(Confusion)

T: With the square (holds up square) What is smallest rotation you could make so it will be a symmetry motion.

S: One

T: Well thats true the way we have been labelling our rotations. What part of a full turn would it be? (Rotates  360')

S: 1/4 of a turn.

T: With an octagon, what is the smallest rotation?

Symmetry (cont.)

June 6, 1967

S: 1/8 of a turn.

T: With a circle what is smallest rotation?

S_1: This big (holding fingers close). S_2: $\frac{1}{1,000,000}$, etc.

T: Is there a smallest rotation?

(Most said no.)

T: How many axes of symmetry?

S_1: Lots. S_2: Infinity.

T: Infinitely many. I think I would agree.

T: This figure is symmetric (Shows rectangle (1 X 2) on geo-board).

Who can come up and show me an axis of symmetry?

(One girl comes up and shows both axes)

T: Are there any more?

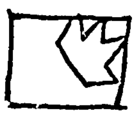
S: No

T: This figure is symmetric. (Has  on geoboard).

(Some students disagree.)

T: Who can find an axis of symmetry?

(One student points out vertical axes.)

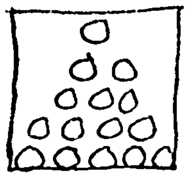
T: (Makes  on geo-board.) Who can find axis of symmetry?

(Student does so.) Who can make another figure with this yellow elastic so that this (the segment on the main diagonal) will be an axis of symmetry for the whole figure?

(Student comes up and eventually gets it with some help from a friend.)

T: I have a game for you to play. It goes like this. (Explains  game -

Symmetry (cont.) June 6, 1967

hands out cardboard  (as form equilateral ) and small

white peas which are to be used as markers.)

(Children ask if they can keep cardboard and peas.)

T: Yes.

(Many of children were successful in doing game)

Last Lesson.

A Few Comments on the Above Symmetry Motion Sessions:

The approach in the first lesson to obtaining all the triangle orientations with "point up" was to have all the children stimultaneously choose a position and then compare. This seems to be faster than asking for one "new one" at a time.

The second worksheet of the fifth session is straight drill, and may have too many examples, each one rather long. It would be better if the practice was obtained in a more game-like way. For example, the class could be divided into two teams - one team could make up a compound motion and challenge the other team to find an equivalent motion compounded from a given number of elementary motions.

In the sixth session it was again apparent that care is required in establishing that the operation is that of combining motions in a given order, and that a motion is an element of the set being operated on. It was interesting that this class began trying each of their arithmetic rules on the operations involved here, and wanted to see if the distributive law was applicable. Most classes never seem to recall the existence of that law. In this session the important question arose "How many examples are needed to disprove a statement?" One notes that with this finite set of orientations one may also discuss simply the question "How many examples are needed to prove a statement?"

In lessons seven and eight it was not clear if the children understood "inverse" motion. Perhaps a more explicit statement of the motion required "the motion which brings the triangle colors back where they were" may have helped. The geometrical visualization of the figure retracing its steps has seemed clear in other classes.

In the last two sessions the class quickly and accurately applied its ideas to all regular polygons, particularly the relation between the number of lines of symmetry and the number of classes of symmetry motions. The discussion of the circle was excellent. The ground covered in ten sessions was most satisfactory.